2. Theoretical background

2.1 – Parameters affecting scour phenomenon

Local scour has received great attention from researchers. Not only the longitudinal and transversal extensions of scour hole, but also the rapidity at which the scour develops, depend on a great number of parameters, such as: the typology of the bed material (cohesive or non cohesive) and geometrical characteristics of bed material (size, density, shape, packing and orientation); condition of sediment transport (clear-water and live-bed conditions); the flow characteristics (flow intensity and flow depth); shape, position and typology of structural element inserted along the river reach (bridge piers, abutments, bed sills, spillways, sluice gate, etc…).

Each of the aforementioned factors is characterized by a significant degree of uncertainty (Breusers and Raudkivi, 1991; May, 2002). On the other hand, it has to be considered that the responses of the natural channel to erosion, both in short-term (floods) and in long periods of time, are hard to predict accurately. This is both because of an incomplete understanding of the involved physical processes which, in turn, depend on a huge number of parameters (drainage density, hill-slope angle, soil type, flow discharge, sediment yield, channel pattern and channel depth) and because such processes interact each other in a complex way (Charlton, 2007).

Information on the bed material characteristics is necessary in order to assess scour potential and design protection works. In nature two kinds of bed material can be found: non-cohesive (gravels, sands and silts) and cohesive (silts and clays). Cohesive bed materials require relatively large forces to initiate movement, but relatively small forces to transport the particles away (May et al, 2002). Furthermore, as it is known the threshold condition of movement depends also on the particle size, density, shape, packing and orientation in the bed (Chatterjee et al., 1994; Karim & Ali, 2000; Dey & Westrich, 2003; Davis et al., 2006; Adduce & Sciortino, 2007).

Scouring development can be also different in: clear-water and/or live-bed scour conditions. In clear-water condition the scour evolves with no sediment suspended in water flow (Richardson and Davis, 1995). As the flow discharge increases so that a general bed material movement occurs, a supply of sediment from upstream equalizes the local removal of material. Under these conditions scouring is called as *live-bed scour* (May et al., 2002.). When the flow discharge reduces, clear-water scour may occur again.

The conditions of transport are strictly connected to the shear velocity ratio, u^* / u_C^*, called flow intensity ($u*$ = shear velocity of the incoming flow, u_C^* = critical shear velocity). The influence of flow intensity on equilibrium scour depths, d_s, can be observed (for uniform sediments) in Figure 2.1. In particular, when flow intensity is greater than 1 the scour develops in clear-water conditions; the transition from clear-water to live-bed scour occurs when the flow intensity is equal to 1 (Melville and Coleman, 2000; Chang et al., 2004).

In Figure 2.2 the comparison between clear-water and live-bed scour time evolutions is reported. Clear-water scour reaches the maximum scour depth asymptotically, generally over a longer period than live-bed scour (Richardson and Davis, 1995; May, 2002). But under live-bed conditions (sediments are transported into the scour area), scour reaches equilibrium depth much more rapidly than under clear-water conditions (no sediment transported into the scour area). Richardson and Davis (2001) observed that in the live-bed condition the scour depth tends to fluctuate around a mean (equilibrium) value. The fluctuations are of about 10% of the equilibrium scour depth so that the maximum value of scour depth under clear-water and live-bed condition is almost equal.

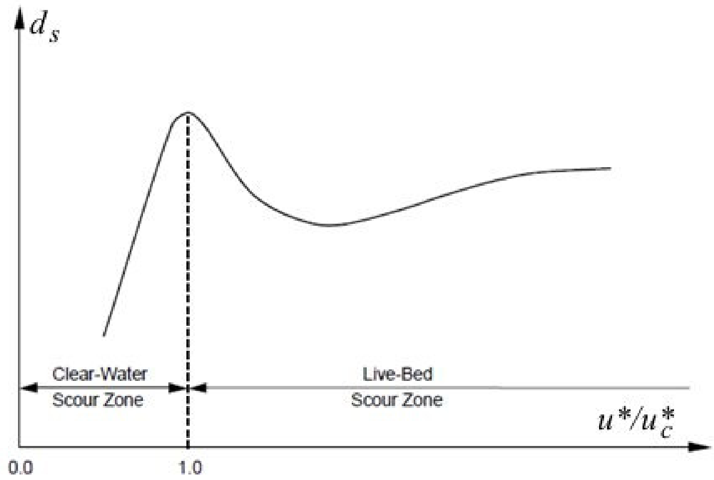

Figure 2.1 - Influence of flow intensity on equilibrium scour depth (from Benedict and Caldwell, 2005).

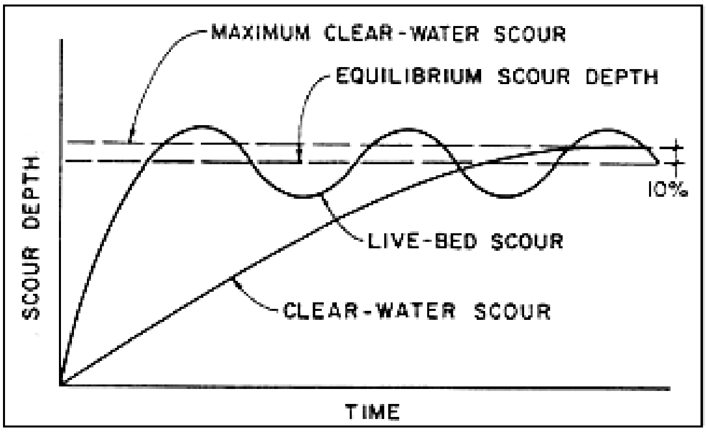

Figure 2.2 - Clear-water and live-bed scour conditions (from Richardson and Davis, 1995).

2.2 – Influence of typology of hydraulic structure on scouring

The evolution of scour phenomenon strongly depends on the typology of the hydraulic structure along the river reach considered. Although several laboratory researches have been carried out on scour evolution around different types of structure, there are still significant gaps in knowledge and general understanding on phenomenon evolution. In fact, the difficulty of making field measurements nearby structures and the consequent lack of reliable field data makes difficult to verify peculiar elements of scouring phenomenon obtained from small-scale laboratory tests.

Literature findings on local scour are available especially around bridge piers, abutments and downstream of apron.

Predictive formulae are available for "equilibrium" scour depth both in clear-water and live-bed conditions. But, the quantitative prediction of localized erosion obtained by using these formulae is still affected by significant uncertainties, because of the large number of control parameters included. Thus, significant differences can be found between predictions and experimental results, as well as among forecast models of different authors. In this part of the work some predictive formulae found in literature by considering different structural element are shortly discussed.

2.2.1 – Equilibrium scour depth prediction

- *Scour at bridge piers*

The complex flow pattern around bridge piers has been described in detail by several researchers (Melville, 1975; Chiew, 1984; Breusers and Raudkivi, 1991). It has been observed that flow around a pier is complex due to its three dimensional nature and the existence of multiple vortices (see Figure 2.3). The flow near the pier generally includes: the surface roller (bow wave) on the water surface around the upstream surface of the pier; the horseshoe vortex located at the upper stream surface of the pier in the vertical plane, that is the result of the interaction of the down flow and the bed; the wake vortex behind the downstream side of the pier. The horseshoe vortex rolling near the bed gives the primary contribution to the scour at the upper stream of the bridge pier. The wake vortex occurs behind the bridge pier and it is due to flow separation. The axes of the wake vortices are nearly vertical and they act like a vacuum to suck the sediments into the flow.

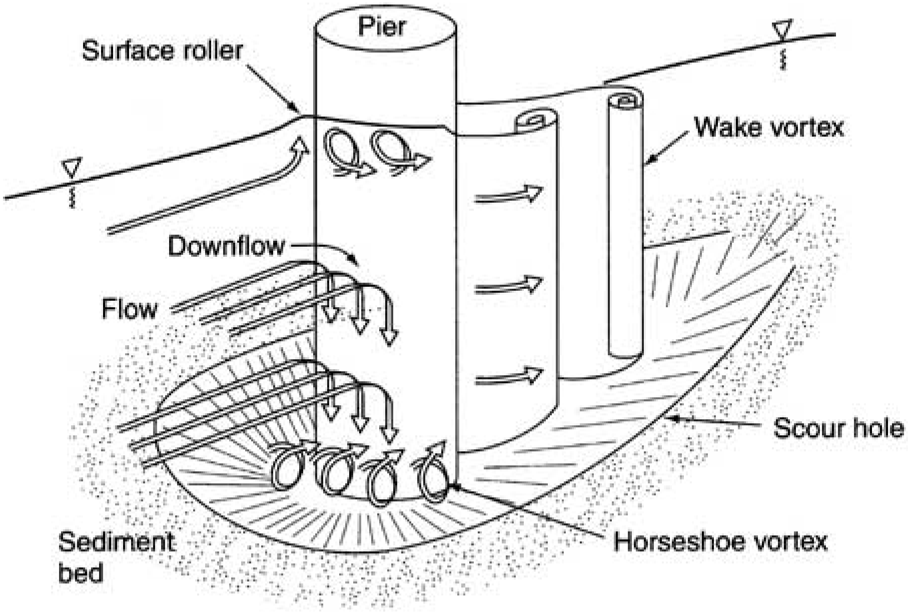

Figure 2.3 - Flow around a bridge pier.

The estimation of the scour depth around bridge piers has attracted considerable research interest and a number of prediction equations exist. Various scour equations are based on laboratory observations (Breusers et al., 1977; Melville, 1992; Melville and Chiew, 1999; Richardson and Davis, 2001). Mainly, the scouring process around the bridge piers is affected by parameters related to the approaching flow (flow depth and mean flow velocity) and parameters related to the geometry of the bridge piers. The bridge piers geometry is defined by the size, shape in plan, roughness of the pier surface, number and spacing of piers, orientation to the approaching flow direction. Considering constant relative density of the bed material, neglecting the viscous, the contraction effects, most of the literature researches (Breusers, 1977; Melville, 1992; Hoffmans and Verheij, 1997; Cardoso and Bettess, 1999; May et al., 2002) leads to the conclusion that the equilibrium scour depth, d_{se}, around a pier can be written as:

$$\frac{d_{se}}{h_0} = f\left(K_y, K_f, K_D, K_\sigma, K_s, K_\theta, K_g\right) \tag{2.1}$$

where h_0 is the water depth upstream the structure, K_y is a parameter related to the relative water depth b/h_0 (being b the horizontal width of the bridge pier), K_f is a function of the flow intensity, K_D is a function of the sediment diameter D_{50} (diameter of the bed material corresponding to 50% passing by weight), K_σ is a function of the geometric standard deviation of the sand σ_g; K_s is a function of the pier shape and K_θ depends on the bridge piers alignment. Different expressions of the parameters have been proposed in literature. As an example, K_y can be estimated by the following formula due to May et al. (2002):

$$\begin{cases} K_y = 0.55 \cdot \left(\dfrac{h_0}{b}\right)^{0.6}, & \text{for } \dfrac{h_0}{b} \le 2.7 \\ K_y = 1, & \text{for } \dfrac{h_0}{b} > 2.7 \end{cases}; \tag{2.2}$$

K_f may be calculated (May et al., 2002) as:

$$\begin{cases} K_f = 0 & \text{for } \dfrac{u^*}{u_C^*} \le 0.375 \\ K_f = 1.6 \cdot \left(\dfrac{u^*}{u_C^*}\right) - 0.6 & \text{for } 0.375 \le \dfrac{u^*}{u_C^*} \le 1.0 \\ K_f = 1.0 & \text{for } \dfrac{h_0}{b} > 1.0 \end{cases}; \tag{2.3}$$

K_D can be estimated (Melville, 1997; Coleman et al., 2003) as:

$$\begin{cases} K_D = 0.57 \cdot log\left(2.24 \dfrac{b}{D_{50}}\right) & \text{for coarse sediment of } \dfrac{b}{D_{50}} \le 25 \\ K_D = 1 & \text{for fine sediement of } \dfrac{b}{D_{50}} > 25 \end{cases}; \tag{2.4}$$

The parameter K_σ represents the influence of the sediment gradation on phenomenon evolution. Raudkivi and Ettema (1977) found that ratios b/D_{50} above 0.3 scour depth decreases dramatically with σ_g. The reduction in scour depth is due to the armoring effects in the scouring hole. Figure 2.4 summarizes the results of the equilibrium scour depth in term of σ_g (Raudkivi and Ettema, 1977).

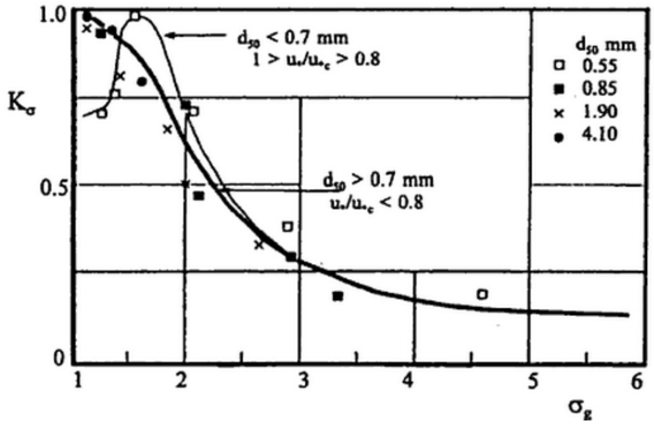

Figure 2.4 - Relationship between K_σ and σ_g (Raudkivi and Ettema, 1977).

The parameter K_s, which takes account of the piers shape effect on the depth scour of local scour, can be determined as reported in table 2.1 (May et al., 2002). In the case of complex piled structures K_s has to be determined from testes with physical model.

TYPE OF BRIDGE PIER SHAPE

Pier shape in plan	K_s
Circular	1.5
Lenticular	1.0 -
Elliptic	0.9 -
Square	2.0
Rectangular	1.5 -
Rectangular with semi-circular nose	1.35
Rectangular with chamfered corners	1.5
Pier shape in evevation	K_s
Pyramid (widest at base)	1.15
Inverted Pyramid (narrowest at base)	1.8

Table 2.1 - Different values of K_s given by May et al.(2002).

K_θ takes in account the effect of the angle between the approaching flow and the longitudinal axis of the bridge pier, θ. It can be estimated (Richardson and Davis, 2001) as:

$$K_\theta = cos\,\theta + \frac{L}{b} sin\,\theta \qquad (2.5)$$

where L is the plan length of the pier and b is the width of bridge pier.

K_g defines the channel geometry effect (Melville and Coleman, 2000). This factor is much more important the case of abutment structures.

- *Scour at bridge abutments*

The obstruction due to the bridge abutment to the flow determines the formation of a horizontal vortex which starts at the upstream end of the abutment and runs along the toe of the abutment. A vertical wake vortex also forms at the downstream end of the abutment. The vortex at the toe of the abutment is very similar to the horseshoe vortex forming near the piers, and the vortex that forms at the downstream end is similar to the wake vortex forming downstream of the pier (Richardson et al., 1993). A scheme of the complex turbulent flow is reported in figure 2.5 (by Hamil, 1999). Numerous abutment scour equations have been developed to predict the scour depth around abutment due to horseshoe vortex. Few research to determine the erosion from the wake vortex has not been conducted.

In literature, several equations for predicting abutment scour depths can be found (Liu et al., 1961; Laursen, 1963; Froehlich, 1989; Melville, 1992) essentially based on laboratory data (Richardson and Davis, 1995). Liu et al. (1961) developed equations by applying the dimensional analysis to laboratory data. Laursen(1963) proposed equations on the basis of inductive reasoning related to flow acceleration near the abutment.

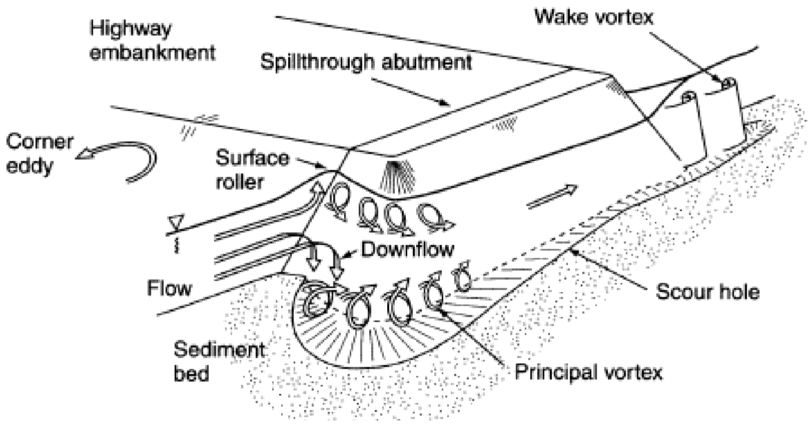

Figure 2.5 - Flow at the spill-through abutment (by Hamill, 1999).

Froehlich (1989) and Melville (1992) derived other relationships through the dimensional analysis and by using laboratory data.

Richardson and Davis (1995) recommended to use the Froehlich (1989) equation, determined in live-bed conditions for the calculation of both clear-water and live-bed abutment scour depths. This equation was obtained by data acquired during a great number of laboratory experiments. By using the regression analysis Froehlich (1989) estimated the equilibrium scour depth d_s as:

$$\frac{d_{se}}{h_0} = 2.27 \cdot K_s \cdot K_\theta \cdot \left(\frac{L'}{h_0}\right)^{0.43} Fr^{0.61} + 1 \qquad (2.6)$$

where h_0 is the mean water depth in the floodplain, Fr is the Froude number, K_s is the abutment shape factor, K_θ takes into account the angle θ of the approaching flow (Figure 2.6) and L' is the length of embankment blocking "live" flow. The abutment geometries include: spill-through abutments, vertical walls without wing walls and vertical-wall abutments with wing walls. The values of K_s for the considered shapes are reported in table 2.2 (Richardson and Davis, 1995).

The coefficient K_θ may be estimated (Richardson and Davis, 1995) as:

$$K_\theta = \left(\frac{\theta}{90}\right)^{0.13} \qquad (2.7)$$

The angle θ of the abutment is reported in Figure 2.6. The abutment pointing downstream, the scour depth is less than the scour depth of an abutment angled upstream.

Type of bridge abutments	K_s
spill-through abutments	1.0
vertical walls without wing walls	0.82
vertical-wall abutments with wing walls	0.55

Table 2.2 - Different values of K_s-factor given by Richardson and Davis (1995).

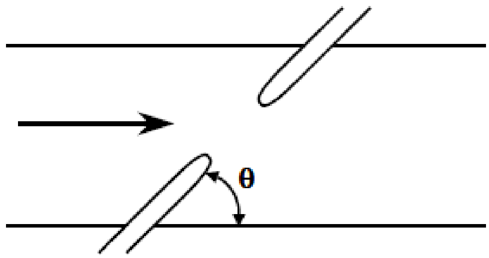

Figure 2.6 - Angle of the approaching flow.

- *Scour downstream a control structure*

The scouring process close to control structures (such us spillways, low head, high head structures, drops, culverts and pipe outlets) is typically caused by turbulent water jets. In order to protect the river bed by the action of the jet stilling basin or rigid apron are often realized downstream of the structure itself. The velocity of the water jet and the turbulence of flow over these protections assume intensities so high to determine local scour downstream of them.

Because the complexity of the physical processes involving, most of scour depth equations have been developed by using data collected during laboratory experiments under various flow conditions (two and three dimensional turbulent jets) and structure configurations.

Schoklitsch (1932) based on flume experiments established the follow empirical equation for overflow gates with horizontal sills:

$$d_s + h_d = c_s \cdot H^{0.2} \cdot q^{0.57} \cdot D_{90}^{-0.32} \tag{2.8}$$

where h_d is the downstream flow depth, c_s is a dimensional coefficient (c_s = 4.75 $s^{0.57}/m^{0.02}$), H is the difference between upstream and downstream water surfaces, q is the water discharge per unit width and D_{90} is the sediment size (90% passing by weight). The equation 2.8 is only for coarse sediment (D_{90} = 9÷36 mm).

Shalash (1959) proposed a scour equation similar to equation 2.8, in which the effect of the apron length L was introduced:

$$d_s + h_d = 9.65 \cdot H^{0.5} \cdot q^{0.6} \cdot D_{90}^{-0.4} \left(\frac{1.5H}{L} \right)^{0.6} \tag{2.9}$$

Uyumaz (1988) conducted a model study in order to investigate when water flows simultaneously under and over a movable vertical gate. During the experiments two different homogeneous non cohesive soils were used (D_{90} = 8.4 mm and 13 mm). The results of the tests highlighted that the length of scour and the maximum scour depth were smaller for simultaneous flow under and over the gate case than for underflow case. Uyumaz (1988) proposed the following equation:

$$\begin{cases} d_s + h_d = w \cdot H^{0.5} \cdot q^{0.6} \cdot D_{90}^{-0.4} \\ w = A \cdot \left(\frac{q_o}{q_u} \right)^2 + B \cdot \left(\frac{q_o}{q_u} \right) + C, \text{ with } \frac{q_o}{q_u} < 2 \\ w = E \cdot \left(\frac{q_o}{q_u} \right) + F, \text{ with } \frac{q_o}{q_u} \geq 2 \end{cases} ; \tag{2.10}$$

where q_o is the overflow discharge, q_u is the underflow discharge and the coefficients *A*, *B*, *C*, *E* and *F* were determined for each material used in the experiments.

Bormann and Julien (1991) analyzed theoretically local scour downstream of grade-control structures by considering the analogy between the local scour process and jet diffusion in a plunge pool. The authors proposed the following equation of the equilibrium scour depth d_s:

$$\begin{cases} d_s + H_D = K \cdot q^{0.6} \cdot \left(\dfrac{U_e}{g^{0.8} D_{90}^{0.4}} \right) \cdot sin\alpha \\ K = C_d^2 \left[2 \dfrac{sin\phi}{sin(\phi+\alpha)} \cdot \dfrac{\gamma}{\gamma_s - \gamma} \right]^{0.8} \end{cases} ; \qquad (2.11)$$

where ϕ is the submerged angle of repose of the granular material (ϕ = 25°), α is water jet angle near the free surface, C_d is the jet diffusion coefficient, H_D is the drop height of the control structure, U_e is the mean velocity jet entering in tailwater. The jet diffusion coefficient C_d depends on inlet conditions and it remains nearly independent of the jet orientation (range from 2.0 to 2.4).

Chatterjee et al. (1994) developed an empirical relationship for the maximum scour depth due to a two-dimensional horizontal water jet issuing from a sluice gate and flowing over a rigid apron to an erodible bed (gravel bed – D_{50} = 4.3 mm; sand bed – D_{50} = 0.76 mm). The authors obtained a scour equation in which the maximum scour depth d_s is a function of the Froude number F_r:

$$\frac{d_s}{B_0} = 0.775 \cdot F_r ; \qquad (2.12)$$

where B_0 is the water jet width. The equation 2.12 highlights that the maximum scour depth is independent of grain size effect.

Hoffmans (1997) proposed an equation to estimate the scour depth produced by a plunging jet from a weir or drop structure discharging on to an unprotected erodible bed:

$$d_s + h_d = \left(\frac{20}{\kappa}\right) \cdot \sqrt{\frac{q \cdot U_e \cdot sin\vartheta}{g}} \quad ; \tag{2.13}$$

where κ is the scour factor. The non dimensional scour factor κ depends on the D_{90} size of the sediment particles and it could be estimated as:

$$\begin{cases} \kappa = 2.95 \cdot D_{90}^{1/3}, \text{for } 0.1\,\text{mm} < D_{90} < 12.5mm \\ \kappa = 6.85 \qquad , \text{for } D_{90} > 12.5mm \end{cases} \tag{2.14}$$

Dargahi (2003) performed a series of experiments about scouring downstream a spillway. The variables of the runs were the sediment size (sand bed - D_{50} = 0.36 mm; gravel bed D_{50} = 4.9 mm), the plate roughness (smooth apron and roughness apron) and the water discharge (20÷100 l/s). Dargahi (2003), by using the non-linear multi-regression analysis to fit equation to experimental data, obtained the follow expression for the maximum scour depth d_s and for its location, X_{max}:

$$\begin{cases} \dfrac{d_s}{h_s} = 1.7 \cdot \left(\dfrac{h_s}{D_{50}}\right)^{1/4.5} \\ \dfrac{X_{max}}{h_s} = 5 \cdot \left(\dfrac{h_s}{D_{50}}\right)^{1/3} \end{cases} ; \tag{2.15}$$

where h_s is the operating water head.

Oliveto and Comuniello (2009), based on experimental data, proposed the following empirical relationship:

$$\begin{cases} \dfrac{d_s}{S} = 46.5 \cdot \left(\dfrac{h_t}{S}\right)^{3/5} \cdot \left(\dfrac{D_{50}}{S}\right) \cdot (F_d - 1) \\ \dfrac{X_{max}}{S} = 133.3 \cdot \left(\dfrac{h_t}{S}\right)^{3/5} \cdot \left(\dfrac{D_{50}}{S}\right) \cdot (F_d - 1)^{4/3} \end{cases} ; \tag{2.16}$$

where F_d is the densimetric particle Froude number, S is the end-sill height (S = 0.092 m), h_t is the tail water depth and X_{max} is the streamwise horizontal distance downstream the edge end of a positive stilling basin.

2.2.2 – Prediction of time evolution of scouring

Breusers (1966) showed that the scour hole generally develops following four phases: initial phase, development phase, stabilization phase and equilibrium phase. During the initial phase the erosion is severe and during this a certain amount of bed material near the upstream scour hole goes into suspension; in the development phase, the scour hole increases considerably. The scour hole results in a progressive reduction in near-bed velocities and rate of erosion. During the stabilization phase, the rate of erosion is very small, but erosion continues resulting in the scour hole lengthening until equilibrium is nearly achieved. In the equilibrium phase, the dimensions of the scour hole are virtually fixed. The importance of time on scouring evolution has been analyzed especially in recent years (Termini, 2011).

Many equations describing temporal variation of scour around different hydraulic structures have been proposed. Generally, it was found that scour depth is a logarithmic (or an exponential) law of time (Anderson,1963; Franzetti et al., 1982; Lim, 1997; Melville and Chiew, 1999; Coleman et al., 2003).

Anderson (1963) stated "*By virtue of the logarithmic character of the development of the scour region with time, a practical equilibrium is reached after a relatively short time, after which the increase in the depth and extent of scour becomes virtually imperceptible*". Franzetti et al. (1982) considered the equilibrium as the state of scour where no further change occurs with time. In the attempt to standardise the criteria for reaching the equilibrium state, Chiew and Melville (1999) collected data from about 35 experiments covering a wide range of pier diameter, flow depths, and approaching flow velocities. The tests were allowed to run until equilibrium was reached. The authors defined the time to equilibrium as the time when the rate of scour was reduced to 5 % of the pier diameter in a 24 hour period. This criterion yielded values of time to equilibrium as high as three days for some cases. But the experimental data indicated that, for a given approach flow depth and velocity ratio, the time to equilibrium increases with increasing pier diameter. Some researchers suppose that in the scour phenomena the equilibrium scour depth does not exist (Rouse, 1965; Kohli and Hager, 2001; Oliveto and Hager, 2002). Rouse (1965) claimed that scour is an ever-increasing phenomenon and there is no real equilibrium scour depth.

- *Bridge piers and abutments*

Franzetti et al. (1982) studied the influence of test duration on the scour depth around a circular pier. He suggested the following exponential equation:

$$\begin{cases} d_s = d_{se} \cdot \left[1 - exp\left(-0.02 T_R^{\;1/3}\right)\right] \\ T_R = {u \cdot t}/{D} \end{cases} \tag{2.17}$$

where d_s is the scour depth at time t, d_{se} is the equilibrium scour depth, T_R is the dimensionless time, u is the mean velocity of the approaching flow, t is time and D is the pier diameter.

Equation 2.17 has been obtained by using a great number of experimental data, which D_{50} = 2.13 mm (synthetic cohesionless soil), the mean velocity of the approach flow u = 0.13 ÷ 0.19 m/s, the pier diameter D = 26.7÷48.0 mm and the critical flow velocity u_c = 0.19 m/s. The authors also used data from Chabert and Engeldinger (1956) characterized by D_{50} = 0.26 ÷ 0.52 mm, the mean velocity of the approach flow u = 0.18 ÷ 0.37 m/s, the pier diameter D = 50 ÷ 150 mm and the critical flow velocity u_c = 0.21 ÷ 0.38 m/s.

Melville and Chiew (1999), based on series of experiments conducted under clear-water conditions, presented:

$$\frac{d_s}{d_{se}} = exp\left(-0.03 \cdot \left|\frac{u_c}{u} ln\left(\frac{t}{t_e}\right)\right|^{1.6}\right) \tag{2.18}$$

where t_e is the time to develop equilibrium scour depth.

Kohli and Hager (2001) conducted laboratory experiments to study the influence of the test duration on the scour depth near vertical–wall abutments placed in floodplain. They found that the densimetric particle Froude number F_d has a significant effect on the scour depth, suggested the following equation of time-variation of scour depth d_s:

$$d_s = \frac{F_d^2}{10} \cdot \left[\frac{h_0 \cdot L}{cos(\vartheta)} \right]^{0.5} \cdot log\left[\frac{t(\Delta g D_{50})^{0.5}}{10 h_0} \right] \tag{2.19}$$

in which h_0 is the approaching flow depth, L is the transverse length or protrusion length of abutment, D_{50} is the median diameter of sediment particles and Δ is the relative submerged density of sediments ($\Delta = [\rho_s - \rho] / \rho$, being ρ the density of water and ρ_s the density of sand).

Oliveto and Hager (2002) presented a large data set collected at ETH Zurich (Switzerland) on scour around a bridge pier and abutment. By using six different bed materials both with uniform and not uniform sediments (characteristic range), the following equation for the temporal scour evolution, in clear water condition, they proposed:

$$\frac{d_s}{L_R} = 0.068 \cdot N_S \cdot \sigma_g^{-0.5} \cdot F_d^{1.5} \cdot log(T_R) \tag{2.20}$$

which $L_R = L^{2/3} h_0^{1/3}$ is the reference length (where L is the transverse length or protrusion length of abutment), N_S is the shape number (N_S = 1 for circular pier and N_S = 1.25 for rectangular abutment or pier) and T_R is the dimensionless time (T_R = $[(g \cdot D_{50})^{0.5}/(h_0^{1/3} \cdot b^{2/3})] \cdot t$).

Coleman et al. (2003) analyzed the time-variation of scour depth under clear-water condition. They proposed the following equation:

$$\begin{cases} d_s = d_{se} \cdot K_t \\ K_t = exp\left(-0.07 \cdot \dfrac{u_c}{u}\left| ln\left(\dfrac{t}{t_e}\right)\right|^{1.5}\right) \end{cases} ; \qquad (2.21)$$

where d_s is the scour depth, d_{se} is the equilibrium scour depth, and K_t is called time parameter. Equation 2.21 is similar to equation 2.18 for local scour at bridge piers.

– *Equations for bed control structures*

Breuser (1966,1967) performed a great number of experiments in order to study scouring phenomena downstream of a fixed bed. Breusers (1966) proposed a formula for the maximum scour depth d_s downstream a bed protection, in which d_s is related to time t as an exponential law as:

$$\begin{cases} \dfrac{d_s}{h_0} = \left(\dfrac{t}{t_1}\right)^a \\ t_1 = \dfrac{K\left(h_0^2 \cdot \Delta^{1.7}\right)}{\left(\alpha \cdot u - u_c\right)^{-4.3}} \end{cases} ; \qquad (2.22)$$

where h_0 is the water depth at the end of the bottom protection, a is a dimensionless parameter (a = 0.2 ÷ 0.4 for 2D flows) and t_1 is the characteristic time of the scouring process (time at which $d_s = h_0$), α is a dimensionless coefficient depending on the bed roughness and the local turbulence intensity (Hoffmans and Booij, 1993), u is the mean velocity of the approaching flow, u_c is the critical mean velocity, K is a calibration coefficient (K = 330 hr $m^{2.3}s^{4.3}$) and Δ is the relative submerged density of sediments.

Hoffmans and Pilarczyk (1995) demonstrated that the value of a in equation 2.7 is not constant, but it depends on the intensity of turbulence into the scour hole. In particular, the authors highlighted that a increases with turbulent intensity of the flow into the scour hole.

Gaudio and Marion (2003) conducted laboratory experiments on time evolution of the scouring phenomenon at bed sills, with steady-flow and clear-water conditions on uniform sediment beds. The experiments were performed in the *Sloping Sediment Duct* at HR Wallingford Ltd. The authors proposed the following experimental relationship:

$$\frac{d_s}{\overline{d}_{se}} = 1 - exp\left(-0.418 \cdot \frac{t}{t_S}\right) \tag{2.23}$$

where d_s is the maximum scour depth corresponding to time t and $\overline{d}_{se}$ is the equilibrium scour depth assumed as the specific average obtained after 20 hours. The authors defined a time scale for the scouring phenomena, named morphological time, t_S (given by the ratio between an eroded volume and a sediment discharge).

Recently, Termini and Misuraca (2006) on the basis of data collected during experimental runs on scouring processes downstream a bed sill, estimated the value for the parameter γ at different values of the water discharge q, and obtained the following equation for the maximum depth d_s of the scour hole:

$$\begin{cases} \dfrac{d_s}{h_0} = m(q) \cdot \left(\dfrac{t}{t_1}\right)^{\gamma(q)} \\ \gamma(q) = 16.85q + 0.335 \quad ; \\ m(q) = 17.38q + 0.452 \end{cases} \tag{2.24}$$