Università degli Studi di Palermo

Tesi di Dottorato

SUPERCONDUCTIVITY EXPLAINED WITH THE TOOLS OF THE CLASSICAL ELECTROMAGNETISM

Educational path for the secondary school and its experimentation

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Corso di Dottorato di Ricerca

in Storia e Didattica delle Matematiche, della Fisica e della Chimica (XXIV Ciclo) – 2013

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Physics is such stuff as dreams are made on

Contents

Abstract

The work of this thesis describes an educational path for the presentation of superconductivity at the secondary school, together with the experimentation of some particular parts of the path with high school students.

The educational path that we have developed is mainly centred on the phenomenological aspects of superconductivity and has been inspired by the two fluid theory of Gorter and Casimir (1934) where a superconductor is seen as a material in which two fluids are present, the normal fluid described by the Ohm's laws, and the superconductive fluid descibed by the London equation (1935). Both the Ohm's laws and the London equation give respectively the phenomenological descriptions of the conductive and the superconductive fluids. There is no reference to the microscopic structure of the conductor or of the superconductor.

For this reason we have chosen these phenomenological models as the framework in which we contextualized our work and the reconstruction of the contents of superconductivity.

In order to develop these phenomenological models in secondary school, we have reconstructed the contents of the electrical conduction and we have subsequently developed a formal analogy between conduction and superconduction from a mathematical point of view, by means of the introduction of the London equation. Therefore a very conspicuous part of this work pertains the presentation of the London equation, for whose writing it is necessary the introduction of the magnetic vector potential.

Being the magnetic vector potential never treated in secondary schools, its introduction has required a lot of reconstruction work, mainly because the mathematics generally used in the literature for the vector potential is based on differential calculus, with operators as divergence and curl, that are really beyond the knowledge and the possibilities of most of the secondary school students.

We have described the magnetic vector potential in analogy with the electric scalar potential, always present in the secondary school curriculum, we have used a mathematical formalism suitable for the level of the students, and we have realized how the introduction of this new, and somewhat complicated object at first glance, can be not only a suitable way for presenting superconductivity at secondary school level, but a stimulus for a deeper understanding of the electromagnetism as well.

Once that the needed prerequisite are introduced and diffusely discussed, we could afford the actual description of the phenomenological characteristic of superconduc-

tivity. This has been developed in two steps. A first basic step, in which by the London equation a student may describe mathematically both, the resistivity of the superconductor that drops to zero at a certain temperature (called critical temperature) and also the expulsion of the magnetic field from the bulk of a superconductor (called Meissner effect). The second step is addressed to students that have a suitable back-ground in quantum physics, and has the goal of introducing some basic aspects of Ginzburg-Landau theory and move towards the BCS microscopic theory. The lines of this second step are hinted in the conclusive part of this work.

Besides the educational reconstruction of the contents of conduction and superconduction, that have taken the large part of this thesis, we have experimented many parts of the path with high school students. As it has been refined three times in the framework of the design-based research method, in this Phd thesis we report the results of an analysis, both quantitative and qualitative, of the part of experimentations that regards electrical conduction and the considerations that yielded the conclusive proposal we made on electrical conduction. Moreover, it is reported also a first analysis of the experiemtation on the vector potential. This part is primarily focused on investigate how the introduction of the vector potential can be useful to better understanding of electromagnetism in secondary school.

We briefly describe here the structure of the work.

- A preliminary introduction to the Higgs mechanism in order to highlight its tight connections to superconductivity. A chapter that is almost off topic, but that is presented to suggest the immense power of superconductivity in putting together many different physics topics even those that are recently on everyone's lips, as the discovery of the Higgs boson.
- A description of the superconductivity at secondary teachers level (or for everyone with a minimum physics back-ground). Here, the didactical problems are not considered, an the chapter contains a brief summary coming from the standard literature on the subject in order to summarize what a teacher could/should already know when deciding to treat superconductivity with students.
- A schema of the educational path that will be subsequently discussed in the thesis: the essential topics and how they are related to each other.
- The core of the thesis: two educational paths for secondary school, one on the electrical conduction (together with the experimentations and the obtained results that lead us to the actual path) and one on vector potential (together with a first analysis of the obtained results). In fact we retains that these topics are fundamental prerequisites for a meaningful introduction of superconductivity at secondary school .
- Finally our main goal: an educational path for the description of superconductivity by means of the London equation and an its generalization, when the students' back-ground is suited.

1. Introduction

This thesis describes the development of an educational path for the introduction of the superconductivity to secondary school students. Superconductivity is engaging for students and teachers, infact the PLS laboratory of superconductivity of the University of Milan always attracts hundreads of students each year, that voluntarily decide to follow an afternoon entirely focused on superconductivity. The great enthusiasm generated by these afternoons stimulated the research that is reported in these pages.

Indeed, superconductivity in an afternoon lab can be considered just a first step of familiarization with the physics of the low temperatures. During the Ph.D research described in this work, we have tried to go deeper into the physics of superconductivity, and in particular, our effort was oriented to the developing of a mathematical explanation of the phenomenology observed in lab, with tools that were suitable for secondary school students. The reconstruction of the superconductivity for secondary school is, in fact, the main part of this work, we may say, its really new content.

Superconductivity is a very difficult subject, deeply related to almost every physics topic that a secondary school student encounters (or at least should enconuter) in his/her school path. Besides being also an estraordinary fascinanting theme, this is why we believe that the study of superconductivity is surely worth to be done at the end of a physics curriculum. Quite obviously, however, the comprehension of superconductivity is based on many aspects of thermodynamic, of electromagnetism, of modern physics, and also needs some interesting lab work. For these reasons a three year work made by two people cannot face in details all the previous aspects, but, after a hard work to organize a preliminary framework, it has to concentrate only on some aspects. We hope to be able, here, to allow the reader at least a fast look at the physical and educational problems involved, and to convince him/her of the very deep educational reconstructions of some important content that we believe appropriate for a menaningful introduction of superconductivity at secondary school.

The vast literature on superconductivity can be roughly divided into three parts: specialist treatises of various levels [1, 2, 3, 4, 5, 6, 7], many discursive/popular books [8, 9] and websites [10, 11, 12, 13], and few educational approaches[1]. In our opinion, apart from the specialistic literature, most of them give a quite (or even a very) good introduction for what concerns the magnetic proprieties of superconductors, but lack of a suitable explanation of the superconductive mechanism.

[1] The most important european project is well described in www.fisica.uniud.it/mosem/indice.htm

Since superconductivity is a very complex topic, it can be approached meaningfully in two ways: as a funny game, or as a very serious game. Whereas during the afternoon labs the level can be only that of a funny game, in this work we present how we played the very serious game. We then explored the complexity of superconductivity in order to find the conceptual nodes and making possible, from a mathematical point of view, an explanation of the phenomenology observed in lab, even by the students themselves, when they explore the interactions between superconductors and magnetic fields. Infact, in our initial investigation of the educational paths on superconductivity present in the literature, we quite frequently noticed a gap between the profound traitation of the experimental part, with experiments that highlighted in a detailed way the role of the magnetic field and the electromagnetic induction in superconductivity, and the theoretical explanation of the experimental part. Usually the theoretical part consisted in an almost qualitative description of the BCS theory.

It is clear that the BCS theory can not be proposed in the secondary school as it was written by its Nobel laureates (in 1972, just for the BCS theory) Bardeen, Cooper and Schrieffer. And it is clear that a serious and consistent approach to that theory from a mathematical point of view is really beyond the possibilities of the secondary school students. For this reason we decided to change radically our approach. We decided to do not focuse our efforts on the microscopic mechanism of superconductivity, because this would have meant the need of the quantum physics and the familiarization with some mathematical methods for the quantum physics itself. But we focused on the phenomenological aspects of superconductivity and decided to formalize them from a mathematical point of view, thus giving a kind of continuity between the experimental lab and its physical/mathematical interpretation.

We did *a priori* choice to ensure to the addressees of our proposal the possibility of treating coherently all the parts of it. In fact, since the microscopic description of a physical problem is always related to the quantum nature of that phenomenon, it is not possible to investigate the microscopic mechanism of a certain phenomenon without a minimum back-ground in quantum physics. This does not regard only modern physics topics as superconductivity, but it regards many other physics phenomena, as for instance the electrical conduction, mentioned now because it is discussed in this thesis. In this work, we have given much attention expecially to the phenomenological description of the phenomena involved, giving instead only some hints for what concens their microscopic interpretation.

As the reader will see, the entire didactical proposal is divided into two parts, a first part that coherently describes the basic phenomena of superconductivity, from the experimental point of view and from a physical/mathematical point of view, and a second part, that contains the first part as a subset, that describes the essential lines for a deeper understanding of the supeconductivity. While the first part can be addressed to every class of secondary school students, the second part requires a quantum physical back-ground that in the current Italian school could be difficult to find, even if new curricula should improve this situation.

The theory that inspired our work is the two fluid theory of Gorter and Casimir; and the London brothers equation [14, 15], that describes the superconductive fluid of the theory just mentioned, is the key point of our traitation. In order to introduce the London equation it has been necessary to develop two other important educational paths, that can be considered as prerequisites:

- A path on electrical conduction from a phenomenological point of view, in which we introduce the Ohm's laws in their local formulation. We devote much room to the basic phenomena of conduction, both because it remains difficult for students and because the local formulation of the Ohm's laws (that describes the normal fluid) is expressed by a law that is formally equivalent to the London equation (that describes the super fluid). This will allow us to develop a useful analogy between the current flowing in a conductor, and the super-current flowing in the superconductor.
- A path on the magnetic vector potential by which the London equation is expressed. This is the second important part of the work, that is meaningful not only in order to explain the superconductivity, even at its basic level, but it is meaningful for the description of the electromagnetism itself. In a sense, we could say that the vector potential *is* the electromagnetism, even if it is never treated in secondary school. We devoted much room also to this topic, because of its ability in lighting electromagnetism even for secondary school students.

Before entering the core of the superconductivity, describing in detail the topics tightly related with it and needed to introduce it, we dedicate the initial chapter of this work to a subject that, at this time, is of global concern and it is born in the light of the superconductivity: the Higgs boson (Nobel prize to Higgs and Englert in 2013). We hope that this subject may engage teachers for its connection with superconductivity and for its hidden beauty, thus allowing them to deal with superconductivity aware of the role it had in the history of physics even beyond its important practical and theoretical implications.

2. The Higgs boson and the superconductivity

This introductory part is devoted to teachers, with the hope that they could enjoy themselves before starting to work very hard with the superconductivity! In fact, the superconductivity is tightly related to the mechanism of the Higgs boson, that has become so trendy in recent months because of its discovery and the Nobel prize 2013 to Higgs and Englert. The Higgs mechanism was already known to physicists when Higgs developed his model, because it was a model used in superconductivity. Higgs used the superconductivity model in such a way to solve a fundamental problem arisen in particle physics [16, 17].

In the following we will try to describe in the simplest possible way the problem that Higgs had to solve, and its relation with superconductivity. Because of the complexity of this topic we cannot afford directly the description of the problem and a preliminary discussion of the theoretical framework is needed in order to contextualize it. We are going to give a picture of the fundamental interactions (in particular the weak and the strong interactions) and of the standard model.

2.1. The Lagrangian formalism for the description of a system

We briefly recall that when a physical system is described by means of its Lagrangian $\mathcal{L}$ it is possible:

- To get the equations of motion by a simple derivative process:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} = \frac{\partial \mathcal{L}}{\partial q_i}, \tag{2.1}$$

  where q_i are the generalized coordinates and the Lagrangian $\mathcal{L}$ is defined as $\mathcal{L} = T - V$, in which T is the kinetic energy and V is the potential energy of the system.

- To get the constants of motion, or the conserved quantities of the physical system.

2.1.1. An example: the conservation of the linear momentum

We present here a brief example that could help a teacher to recall the meaning of the previous two points, in order to understand the basic mechanisms that will be widely used throughout this chapter.

Let us consider the mono-dimensional problem of a moving particle of mass m in a field of forces represented by the potential V. The Lagrangian for this system is then:

$$\mathcal{L} = \frac{1}{2}m\dot{x}^2 - V. \tag{2.2}$$

Now, we can impose that the system is invariant for a certain group of transformations, and we will see that this assumption will give us the conservation of a particular physical quantity. The simplest group of transformations that we can consider is the group of translations along the x-axis. If we suppose that the system is invariant under the translation along the x-axis, we have that the potential V cannot depend on x. This is quite obvious, if the energy of the system would depend on the position x, the system could not be invariant respect to the x coordinate, as we supposed initially. Hence, since V does not depend on x, the equation of motion along the x-axis is:

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{x}} = 0, \tag{2.3}$$

that is:

$$\frac{\partial\mathcal{L}}{\partial\dot{x}} = constant. \tag{2.4}$$

Thus with eq.(2.4) we have found a conserved quantity. We can get a more convenient expression of the quantity of eq.(2.4) by means of the Lagrangian expression of eq.(2.2), and we can write:

$$\frac{\partial\mathcal{L}}{\partial\dot{x}} = m\dot{x} = constant \tag{2.5}$$

that is, the momentum of the particle is a constant of motion.

What we have discussed is an example of a quite general scheme: once a system is described by means of a Lagrangian, and if the Lagrangian is symmetric with respect to a *continuous* group of transformations, that is, it is invariant with respect to a certain group of transformations (in the present example, the translation along the x-axis), then there is a certain physical quantity that is conserved (in the present example, the momentum of the particle).

This holds in general, because of the Noether theorem (that has been obviously stated in a more precise mathematical form). But we believe that for our aims the previous example can be sufficient to understand the problem in a friendly way. What is clear is that each conserved quantity of a physical system is related to the Lagrangian via a particular symmetry. The strong relation between symmetries and conserved quantities is such that if one observes the conservation of a certain physical quantity, but the Lagrangian supposed to describe the system does not allow the same conservation, one is led to modify the Lagrangian to impose the particular symmetry corresponding the conservation of the physical quantity observed.

2.1.2. The importance of the theoretical framework

A reason that make us to believe that this introductive part is important to be treated with teachers, pertains the fact that it can highlight a physicist typical way of reasoning: from one hand there are experimental evidences that need a theoretical explanation, and hence the need of developing a model in order to describe the experiments; on the other hand there is the theoretical model, developed for reasons that are not necessarily related to experiments, that predicts some experimental facts, that have to be discovered.

We recall here a known and meaningful example from the classical electromagnetism, in order to show the importance of the theoretical approach to physical problems. When in a physical system a continuous additive quantity q is conserved, then it is possible to write a continuity equation for the density of that quantity, that we call ρ. The continuity equation is the well-known eq.(2.6):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0, \tag{2.6}$$

where $\mathbf{J}$ is the current density of the conserved physical quantity. This is an equivalent way to describe the meaning of the Noether's theorem: a symmetry (in a continuous group of transformations of the system) corresponds to a continuity equation for the conserved quantity and this, in turn, means that a variation of the quantity generates a current. In some important cases the current couples with the fields and we have a dynamical symmetry: this is the case in gauge theories like electromagnetism.

In the electromagnetic case, we have the conservation of the electric charge. Thus ρ represents the charge density and $\mathbf{J}$ represents the electrical current density. When an electrical current is generated, then the electric and the magnetic field are excited and their behavior is described by the Maxwell's equations.

A charge density ρ generates an electrical field $\mathbf{E}$, by means of the equation:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}, \tag{2.7}$$

while a current density $\mathbf{J}$ generates a magnetic field $\mathbf{B}$ by means of the equation:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}. \tag{2.8}$$

Maxwell realized that eq.(2.8) couldn't be true, because if it held, the continuity equation, that represents the charge conservation, couldn't have been true. In fact, if we take the divergence of both sides of eq.(2.8) we obtain:

$$0 = \nabla \cdot \mathbf{J} \tag{2.9}$$

that is different from the continuity equation (2.6) that we have supposed initially.

Giving more importance to the conservation principle (or equivalently to a symmetry of the system) than to the fields equations, Maxwell changed one of the fields equations in order to maintain true the continuity equation for the electric charge, and he added the term of the displacement currents:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \varepsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}. \tag{2.10}$$

We see here an example of the fact that, in a theoretical approach, conservation laws could represent the guidelines for the description of the systems in terms of the fields involved. In a similar approach a description of the system in terms of the Lagrangian becomes fundamental, in order to find out the relation between the symmetries of the system and the terms of the Lagrangian that represents the particular interaction.

Moreover, if the aim is a characterization of the fundamental interactions, we can understand how important could be the description of the interacting terms in the Lagrangian in order to delineate, for instance, the strong and the weak interactions.

2.2. The characterization of the unknown interactions

We want now to say something about the description of fundamental interactions. In order to get information about these interactions there are two different approaches, that are briefly discussed in the following: the experimental and the theoretical ones.

2.2.1. Experimental hints for the description of the strong interaction

The Rutherford experiment is the prototype of the experiment that can be realized in order to investigate the structure of matter [18]. The more the energy involved, the smaller the length you can investigate. With energies of the order of some electronvolt we can have information about the external structure of atoms, while with energies of the order of $100 - 1000$ eV we can get information of the inner structure of the atoms. We suppose that the main results of the structure of the matter for what concerns atoms is known, so we can increase more the energy in order to investigate the nucleus.

The experiments carried out in a similar way of the Rutherford experiment (that is by shooting particles such as electrons and so on towards nuclei, nucleons etc), gives results that are very similar to those obtained for the atoms:

- the nucleus has energy levels, of much more high energies than that of the atom (of the order of $10 - 100M$ eV);
- the energy levels of mesons (for example $c\overline{c}$) have the same structure of the equivalent electromagnetic counterpart (for example positronium);
- the proton appears as a complex particle composed by three sub-particles (quarks)
- the collision $p\overline{p}$ gives rise to jet events with almost the same angular dependence as the Rutherford scattering.

These results suggest that the nuclear interaction, or the strong interaction, is very similar to the electromagnetic interaction, that is the interaction that binds the electrons to the nucleus.

But there is a difference: the force binding nucleons together does not distinguish among the nucleons: neutrons and protons are equivalent with respect to this interaction. An evidence of this fact can be obtained from experiments performed with mirror nuclei, that are nuclei with the same number of nucleons but different charges. For mirror nuclei, the energy levels of the spectra are exactly equivalent.

The interaction inside the nucleons has much to do with the electromagnetic interaction, as in the case of the nucleus, but, even in this case there is a difference: a sub-nuclear particle can never be found alone, while an electric charge can stand alone without any problem. This means that the strong interaction is almost equivalent to the electromagnetic interaction for the distances of the order of the nucleus or less, but for distances greater than that of the nucleus its behavior is completely different, thus making impossible to find a quark alone.

By this set of experimental considerations we can describe the potential of the strong interaction with two components:

- an electromagnetic component $V(r) = \alpha/r$, with obvious meaning of the notation;
- a strong component, whose expression is more complex than the previous one, and that, at the moment, we can think to be increasing with the distance r.

2.2.2. Theoretical hints for the description of the strong interaction

One of the most famous principles that describes what happens in the quantum field context is the Gell-Mann's totalitarian principle that states: *Everything not forbidden is compulsory.* The application of this principle defines a way to proceed:

the only prohibitions come from the impositions of determined symmetries, and there is no need to specify what are the forces involved in the interactions, because these are automatically determined by the symmetries and by the vacuum fluctuations.

A possible way to approach the problem from an heuristic point of view, is focused on considerations on the dimensions of the physical quantities. Briefly:

- by relativity *time* and *length* have the same unit of measure, posing the speed of light $c = 1$ and we can then write

$$t \sim L \tag{2.11}$$

by quantum mechanics, *length* and *energy* units of measure are reciprocals of each other, posing the Planck constant $\hbar = 1$ and we can then write

$$L \sim \frac{1}{E}, \tag{2.12}$$

thus time, length and energy can be expressed in terms of a one dimensional parameter that we can choose in cm, m, etc.

Keeping in mind these preliminary considerations we can try to discover the expression of the allowed potentials. Because we already know the Coulomb force, we start from that force, in order to understand the way of proceeding that can be applied subsequently to get information about the unknown forces.

The electromagnetic potential can be written as

$$V = -e^2/r,$$

where r is the distance between the two charges.

In the framework in which these considerations are done, the constant e is dimensionless. In fact, since V must correspond to an energy and an energy E has the dimensions of $1/L$, we must conclude that e, that is called *coupling constant*, must be dimensionless: the electronic charge e is then dimensionless. The fact that the coupling constant is dimensionless is related to the spatial dependence of the potentials by which the electromagnetic force is defined, and it is important to notice that the dimensions of a coupling constant describe the behavior of the force we are dealing with.

In particular, the dimentionlessness of the coupling constant of the electromagnetic fields tells us that the range of the electromagnetic force is infinite and it is always possible to detect an electromagnetic interaction at every distance.

If the coupling constant had the dimensions of L, then one observer at distances greater than L would not see anything. For this reason, in quantum mechanics, forces that have a coupling constant that are not dimensionless are called *irrelevant*: because they cannot be seen, while dimensionless coupling constants can be seen at all the scales.

In other words, this means that the only potentials that can be considered, because they can be meaningful at short distances, must have a dependence on the distance of $1/r$. This conclusion, that comes out from heuristic considerations is the same conclusion obtained from an experimental point of view.

This is the basis on which we have to build our knowledge. Remembering that "Everything not forbidden is compulsory", all that we have are the conservation laws, or better, the symmetries of a system. The way that can be followed consists in the individuation of particular symmetries and from those symmetries one gets the forces. Since we know the electromagnetism, and since the unknown forces must have a behavior quite similar to that of electromagnetism, the physicist have tried to get information about the unknown forces going over the known way again: the way of the electromagnetism. Thus the problem is how to generalize the symmetry (the $U(1)$ symmetry that we will not deal with in these few pages) that gives the electromagnetic force.

So far we have:

- The electromagnetic force can be obtained from the symmetry $U(1)$
- The electromagnetic potential is:

$$V(r) \sim \frac{1}{r} \tag{2.13}$$

- The Maxwell equations can be written in terms of the quadripotential A^μ and the D'Alambertian operator $\Box$ as:

$$\Box A^\mu = J^\mu. \tag{2.14}$$

  (see chapter 7 for details on this last point).

The weak and the strong interactions have to come out from an equation with a structure very similar to that of eq.(2.14) because these unknown interactions have a very similar behavior to that of the electromagnetic interactions. For what concerns the strong interaction, as we have already seen, the similarity is related to the response of the system subjected to experiments as the Rutherford scattering. The difference in the case of the strong interaction is that the strong interaction is not known, while the electromagnetic interaction is. The knowledge of the electromagnetic interaction allows to discover, via the Rutherford experiment, the nucleus and the fact that the Coulomb interaction holds also for distances of the order of 10^{-15}m.

If we have the Rutherford experiment as unique tool to find out information, and the strong interaction is not known, the issue is much more complicated.

For what concerns the weak interactions, the similarity with electromagnetism is related to another behaviour of nature. Here we give only a hint with a simple example. A typical system in which it is implied the weak interaction is the neutron decay, where a neutron decays in a proton, with the emission of an electron and an antineutrino. It can be represented, with obvious meaning of the notation, as:

$$n \to p + e + \overline{\nu}_e. \tag{2.15}$$

Why is this decay an interaction? What is interacting? In electromagnetism we have a very similar decay, in the case of an excited atom that decays to a lower energetic level and a photon is emitted:

$$A^* \to A + \gamma. \tag{2.16}$$

In the atomic decay represented by eq.(2.16) an excited atom couples with the electromagnetic field giving the atom with a lower energy and the mediator of the interaction is emitted. The mediator of the interaction was not inside the atom, but is generated during the interaction with the electromagnetic field.

For the neutron, the situation is very similar, even if the mediators of the interaction do not appear, but the neutron couples with the field of the weak interactions, giving the proton, that in this context appears as a neutron in a lower state. The weak interaction has the same characteristic of the electromagnetic interaction: the currents generated in the rearrangement on the initial system to a system of lower energy couple with the field, excited it, the mediators of the interaction are excited and the system reaches a new configuration.

The similarities between the electromagnetism and the strong (and the weak) interactions are, in quantum field theories, reversed on the gauge structure of these interactions: electromagnetism has $U(1)$ as gauge symmetry group, while weak and strong interactions have respectively $SU(2)$ and $SU(3)$ as symmetry groups. In quantum field theory interactions are mediated by gauge bosons and all the previous symmetries imply that these bosons are massless (as the well known photon). But, as we shall see later, an interaction mediated by massless bosons has an infinite range, contrary to the very short experimental range of the weak and strong interactions. This gives rise (at least in part) to the problem of masses in the Standard Model.

2.3. The problem of the particle masses in the Standard Model

In the previous section we have seen that the non dimensionless coupling constant, with positive dimensions of length can be written, but they give always rise to *irrelevant* interactions. And what about the negative dimensions of length? This is possible, and it is equivalent to have positive dimensions of energies that, in turn, is equivalent to have massive terms.

Hence, for what concerns the fundamental interactions, they can generate terms of mass. Going deeper into the theory, one discovers that the masses generated in this way by the theory, corresponds to the greatest allowed energies, and are therefore of the order of the Planck mass. This is not what happens in our world.

So, in the Standard Model of the fundamental interactions, if a certain symmetry is imposed, then the interactions are very well described in all the experimental evidences, except for what concerns the masses of the particles, being forbidden by the symmetries $U(1)$, $SU(2)$ or $SU(3)$. On the other hand, if the symmetry is not imposed, then the masses allowed from the theory are of the order of the Planck mass, in disagreement with the experimental observations.

This is a very simple description of the problem of the masses in the Standard Model, nevertheless it appears the necessity to invent a mechanism that preserves the symmetry imposed, but that prevents the masses to grow indefinitely as well. The solution of the problem has been found by Higgs, and for this reason it is called *the Higgs mechanism.*

The Higgs mechanism was already known in superconductivity in the years in which Higgs was looking for the solution to the problem of the masses in the Standard Model. This fact is a beautiful example of what happens sometimes in physics. Ideas found in a certain context are used to solve problems in a different one, but often from a completely different point of view. The solution to this problem is called *spontaneous symmetry breaking.*

2.3.1. Short range interactions are mediated by massive quanta

Before giving the description of the Higgs mechanism we consider interesting aspects related to the problem of the particle masses. We report here a first intuitive approximate calculation that relates the range of an interaction with the mass of the quantum that mediates that interaction. For example, the electromagnetic interactions are mediated by photons. The photons are massless and this is in a sense equivalent to say that the electromagnetic interaction range is long.

Let us suppose to have an interaction mediated by quanta of mass m. From the Heisenberg relations we have:

$$\Delta E \cdot \Delta t \approx \hbar. \tag{2.17}$$

Considering the Einstein relation $E = mc^2$, from eq.(2.17) we can heuristically get:

$$mc^2 \cdot \Delta t \approx \hbar \tag{2.18}$$

and hence:

$$\Delta t \approx \frac{\hbar}{mc^2}. \tag{2.19}$$

Now, we can imagine that the quantum of interaction in the time interval Δt travels a distance r and we can roughly estimate that distance supposing that its velocity is equal to the speed of light c:

$$r \approx c \cdot \Delta t \tag{2.20}$$

and by a substitution in eq.(2.19) we get:

$$r \approx \frac{\hbar}{mc}, \tag{2.21}$$

by which we have the relation between the mass m of the mediator of the interaction and the range r of the interaction itself. Moreover, we can consider another important characterization of these interactions, that we will see related with the superconductivity: the quantum interactions are local. This means that the conservation laws have not to be satisfied locally, while the Heisenberg relations have to. This has a very strong implication: an energy ΔE can be created from the vacuum if the time interval Δt for that creation is short enough, that is, if the Heisenberg relation $\Delta E \cdot \Delta t \approx \hbar$ is satisfied.

Hence, recalling eq.(2.19) we can say that a quantum of mass m can be created by a vacuum fluctuation and can travel for a time Δt given by the same eq.(2.19) and for a distance given by eq.(2.21).

In chapter 3, we will see that in a superconductor the magnetic field can penetrate the sample for a very short distance λ_L from the surface of the superconductor itself. This can be interpreted by saying that the range r of the interaction of the magnetic field inside a superconductor is just λ_L. This implies that, from eq.(2.21), it is possible to associate a mass $m = \hbar/\lambda_L c$ to the quantum of the magnetic field inside the superconductor. Since the quantum of the magnetic field is the photon, we can conclude that we can see the superconductive phenomenon in two equivalent ways: we can imagine the existence of screening currents that prevents the magnetic field to enter the superconductor, or we can imagine that the interaction of the electromagnetic field with the superconductor is a short-range interaction mediated by massive photons.

2.3.2. The screening currents and the generation of the photon mass

The topic we are dealing with can be treated in a more general way [16], but for simplicity and in order to show the structure of the problem rather than its mathematical details, we will consider the electromagnetic field and the Maxwell's equations that describe it, in presence of the current densities $\mathbf{J}$, in the stationary case ($\partial \mathbf{A}/\partial t = 0$), with the Coulomb gauge condition $\nabla \cdot \mathbf{A} = 0$. We can write:

$$\nabla^2 \mathbf{A} = -\mathbf{J}, \tag{2.22}$$

and we can say that eq.(2.22) must hold because of the gauge condition imposed.

Moreover, besides the propagation of the electromagnetic field, we can consider also the free propagation of a *massive* field. The free propagation of a massive field is described by the Klein-Gordon equation ($(\Box + M^2)\, A^\mu = 0$, that we can not treat explicitly here). We can rewrite the Klein-Gordon equation in the stationary case:

$$\left(-\nabla^2 + M^2\right) \mathbf{A} = 0, \tag{2.23}$$

and we can say that eq.(2.23) must hold because of the experimental evidences of massive mediators of the interactions, due to the short ranges of the weak and strong interactions.

The reason for which we have called always $\mathbf{A}$ the field that satisfies the two eqs.(2.22) and (2.23) is that we are looking just for a field that at the same time satisfies both the two equations. We notice that eq.(2.22) is satisfied by a massless field, while eq.(2.23) is satisfied by a massive field. How can the two equations be satisfied at the same time?

The two equations can be satisfied at the same time if it is satisfied the condition:

$$\mathbf{J} = -M^2 \mathbf{A}. \tag{2.24}$$

The structure of the eq.(2.24) is identical to the so called *London equation* that we will see in the next chapter 3, and we will use throughout all this work, because it is the first equation introduced by the London brothers in order to describe the behavior of the superconductive current.

Eq.(2.24) is also called *condition of the screening current*, for the characteristic behavior of the current in a superconductor that screens the magnetic field and prevents its penetration. We find again the equivalence between the presence of the screening current and the free propagation of massive photons inside a superconductor.

Going through what we have seen so far, we can resume the main points in order to sort out the ideas.

1. We can not investigate directly the strong and the weak interactions from an experimental point of view, but we can find a great similarities with the electromagnetic interaction.
2. The coupling constants have to be dimensionless, in order to give no irrelevant interactions. This in turn implies that the potential that describes the interactions should have an expression like $V(r) \sim 1/r$.
3. In the quantum fields theory the interactions are mediated by bosons. The bosons of the electromagnetic field are the photons, massless particles.
4. From the Heisenberg relations we have that the range of the interactions is related to the mass of the particle that mediates the interaction, that is $r = \hbar/mc$.

5. The theory that has to describe the weak and the strong interactions has to be modeled starting from electromagnetism, being the known theory which we can refer to. Electromagnetism is a gauge theory, and the theory of the strong and weak interactions will be a gauge theory as well. Since in a gauge theory the interactions come from gauge symmetries, the same thing will hold for the unknown interactions, the weak and the strong interaction. This, eventually predicts massless mediators of the interactions, as photons.

6. Experimental evidences show that the strong and the weak interactions have a very short range (10^{-15}-10^{-18}m), therefore the interactions must be mediated by massive bosons.

7. The point 5. and 6. seems to be irreconcilable, unless a mechanism similar to that of the superconductivity is introduced.

Developing the analogy, Higgs sees the penetration range λ_L of the magnetic field in a superconductor as if the photon were massive (with a mass $m = \hbar/\lambda_L c$) or, equivalently, to each boson of the interactions can be associated a current density $\mathbf{J}$, that we can call the Higgs current and that makes finite the range of the interactions, as it happens in a superconductor.

In a sense, with the Higgs mechanism, our universe appears as a superconductor in which the screening Higgs currents make short the ranges of the strong and the weak interactions. But the amazing thing is that a so strange mechanism has been experimentally verified, because the particle associated to the screening Higgs current has been detected. Infact, the Higgs boson is the particle that corresponds to the screening Higgs current as the Cooper pair corresponds to the screening current for the magnetic field in superconductivity.

2.4. The Higgs mechanism

In the general framework that we resumed in the previous section remains to characterize the Higgs boson from the point of view of the quantum fields theory. We are going to describe in a quite intuitive way the mechanism that involves the Higgs boson, trying to avoid as much as possible heavy calculations.

2.4.1. The chiral symmetry and the fermion masses

The Higgs mechanism is related to the particular symmetry that prevents the (fermion) masses to grow indefinitely, that is called *chiral symmetry*. The chiral symmetry is in turn related to fundamental principles related to the allowed space-time degrees of freedom of a particular system. For instance, an elementary particle is specified when are specified the particular symmetry that it obeys, depending on the fact that it can be charged, what is its spin, etc. The spin of a particle, in particular, corresponds to the space-time degrees of freedom of that particle. For example:

- A particle of spin 0: it is an object with 1 space-time degree of freedom.
- A particle of spin 1/2: it is an object with 2 space-time degrees of freedom, corresponding to the two possibilities +1/2 and -1/2 (that differs by one unit to each other)
- A particle of spin 1: it is an object with 3 space-time degrees of freedom, corresponding to the three possibilities -1; 0 and +1 (that again differs by one unit to each other).

For reasons related to special relativity, when a object is massless then the degrees of freedom diminishes of one unit, and for instance, a massless particle of spin 1 has only two degrees of freedom.

Let us consider the case of fermions, that are particles of spin 1/2. If a fermion is massive, then the particle has two degrees of freedom, but if the fermion is massless, the two degrees of freedom are decoupled, or, in other words it is as if there were two distinct particles that do not interact with each other. Massless fermions correspond to the chiral symmetry, that is a kind of extra symmetry that for example preserves the charge of each of the two objects.

That the fermions obey to the chiral symmetry is a fact that is in agreement with the experimental observations. The problem is that there exist massive fermions, as the electrons. How can we reconcile these two facts? The fundamental idea that allows us to find a solution to this problem is called *spontaneous symmetry breaking*, a theory that was developed in early '60 mainly by Nambu and Jona-Lasimio, even if there are previous works by Gell-Mann and Levi.

2.4.2. The spontaneous breaking of the symmetry

Nambu and Jona-Lasimio developed a model, still used, in which the chiral symmetry of the fermions is respected, but the fermions may have a mass by means of the spontaneous breaking of the chiral symmetry. The main idea is that it is possible to develop a completely symmetric theory in which all the allowed states are not symmetric. This may seem a bit too abstract, but there is a very simple example that can well clarify what we are saying.

Let us imagine to have a perfectly symmetric pen, perpendicular to a perfectly smooth and horizontal table, and the pen is held in equilibrium by an hand. The situation is completely symmetric. When the pen is left to go, from its metastable equilibrium state, it will fall in a particular direction, chosen among infinite different equiprobable directions. All the possible states of the system are not symmetric, while the system is symmetric.

In order to describe mathematically situation similar to that of the pen of the previous example, Nambu and Jona-Lasimio introduced in the Lagrangian the well-known potential with the shape of a mexican hat, as shown in Fig. 2.1. The top of the hat is the point of metastable equilibrium, while the circumference at the bottom of the hat, that gives the infinite possible vacuum states, is constituted by the points of stable equilibrium of the system, that, in the previous example corresponded to the infinite possible directions that can be chosen by the pen when it falls. When one of these infinite points is chosen, the symmetry is spontaneously broken.

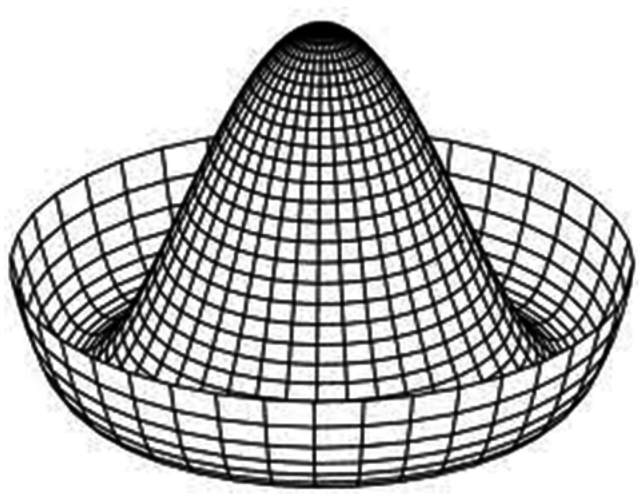


Figure 2.1.: Schematic representation of the potential introduced by Nambu an Jona-Lasimio in order to describe the spontaneous breaking of symmetry.

Nambu and Jona-Lasimio showed that with a chiral symmetry can be realized the same thing, and the fermions can get a mass if they can interact with a potential similar to that represented in Fig. 2.1.

An important consideration on the mexican hat potential is that, once that a particular equilibrium point is chosen on the circumference at the bottom, two degrees of freedom manifest: one of the two degrees of freedom corresponds to the fact that it is possible to move along the circumference, without expending energy, while the second degree of freedom corresponds to the movement in the perpendicular direction, as for an harmonic oscillator, and this is a process that requires energy.

Without going in further details we can read what we have just seen for the degrees of freedom as:

- Moving along the circumference of the minima we get an excitation *without mass*. The excitation without mass is called *Goldstone boson*.
- Moving along the orthogonal direction of the circumference we get an excitation *with mass*. The excitation with mass is related to the existence of the *Higgs boson*.

In other words, the spontaneous breaking of the chiral symmetry, important for giving masses to the fermions, gives also two other particles, two bosons, named the massless Goldstone boson and a second massive particle that is related via the local gauge symmetry breaking mechanism to the Higgs particle.

2.4.3. The Higgs idea

From the previous section we can argue that the spontaneous symmetry breaking of a global symmetry generates one degree of freedom massless, the Goldstone boson. This is stated in the Goldstone theorem, that appeared in a his paper of 1936 for the first time. In that paper the author refers explicitly to the theory of superconductivity.

The Higgs' idea consists in a mechanism that allows systems with spin 1 to get mass, taking the extra degree of freedom from the spontaneous symmetry breaking of the *local* gauge symmetry.

As we have said in the previous sections, systems of spin 1 have three degrees of freedom when massive, while they have two degrees of freedom when they are massless. Therefore, if we want to give mass to a system of spin 1, we can take the extra degree of freedom using the Goldstone boson, that combines with the system with two degrees of freedom to give a system of spin 1 and three degree of freedom, that is a system of spin 1 massive. In order the Goldstone boson to disappear, the local gauge symmetry must be broken. In this way the "the gauge field eats the massless Goldstone boson and acquires mass".

But in this mechanism something is left: the Higgs boson, the excitation with mass that is created whenever a local gauge symmetry is broken in presence of a Goldstone boson. And the Higgs boson was actually found the 4th july 2012 in LHC.

So, Higgs has understood two things: the first is that it is possible to give mass to a particle of spin 1 taking the extra degree of freedom from a system of spin 0, by the spontaneous breaking of symmetry; and second is that the particle left, by the spontaneous breaking of symmetry, must be detected, as it happened.

The educational path that we have developed in the next chapters does not involves the spontaneous symmetry breaking, because it requires such a large amount of mathematical and physical tools that are not suitable for secondary school students. Our journey in the quantum fields theory ends here, but starts the journey in classical electromagnetism.

3. Some basic superconductivity for teachers

Superconductivity is one of the most intriguing phenomena man has ever observed. From its discovery, more than a century ago, the study of superconductivity has given rise to more than ten Nobel laureates, many industrial, medical and research applications and it still continues to astonish us with its thousands effects and subtleties. As we have just seen, the discovery of the Higgs boson is in many and deep ways strictly connected to superconductivity, both because the LHC (Large Hadron Collider) that allowed us to find this particle is a superconductive accelerator and because the same existence of this particle means that our universe behaves like a huge superconductor for what concerns weak interactions!

Moreover, among other fascinating physics topics that can be studied in secondary school, such as general relativity and the problem of the existence of a still unknown dark matter, superconductivity has a great advantage from a didactical point of view, because, contrary to the previous topics, there exist simple, but not trivial and not too expensive, experiments that can be really performed by students in a school lab. Besides, in our experience, the complexity and the witchery of superconductivity can also be a great stimulus for students to take up again (at least some of the) topics of electromagnetism, thermodynamics, waves and quantum physics, in order to deepen and frame them in a new light [19].

For these and other reasons related to quantum physics education research, the Physics Education Research Group of the University of Milan (PERG UNIMI) has been studying effective presentations of superconductivity to secondary school students for nearly ten years. We can safely say that, despite it is a very difficult topic, secondary school students and teachers have always been interested and involved in superconductivity lessons and lab work, both at university and at school.

The following pages are thought for secondary school physics teachers but can be read by everyone interested in superconductivity with a secondary school physics background and with some calculus knowledge.

3.1. Resistivity: few words

Superconductivity deals with the electromagnetic proprieties of matter when it's temperature is cooled down to only few kelvin; in particular it deals with the electric conduction proprieties of matter. This is why we start this notes with considering electric conductivity. In general, when a potential difference ΔV is applied to a material sample or to an electrical device, a current I that depends on ΔV will flow in it: the function $I = I(\Delta V)$ is called the characteristic curve of the apparatus, see for example Fig. 3.1.

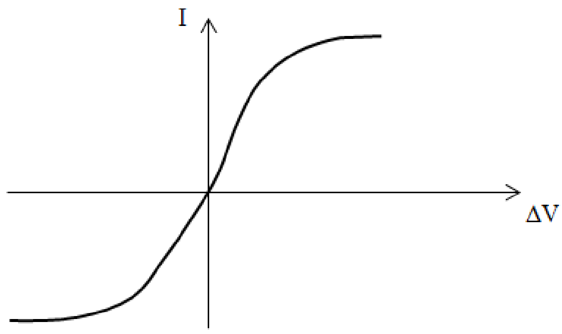


Figure 3.1.: Characteristic curve of an electric motor

The angular coefficient $G(I)$ of the tangent line to this curve is called conductance of the apparatus at the current I; that is:

$$G(I) \equiv \frac{d\Delta V}{dI}, \tag{3.1}$$

while the reciprocal of $R(I) = \frac{1}{G(I)}$ is called resistance. The resistance is a global property of a sample (or of a device) that depends not only on the current, but also on the geometry of the sample and on the points between which the potential difference is applied. If, therefore, we want to study the elecric proprieties of a material, it is more useful to rely on the local physical quantity ρ that is called resisivity. If an electric field is applied at the point $\mathbf{x}$ of a sample of a given material, its resistivity $\rho(\mathbf{x})$ is defined by the relation:

$$\mathbf{E}(\mathbf{x}) = \rho(\mathbf{x})\,\mathbf{J}(\mathbf{x}), \tag{3.2}$$

where $\mathbf{J}(\mathbf{x})$ is the current density at $\mathbf{x}$. It is sometimes useful to reverse equation (3.2) by writing:

$$\mathbf{J}(\mathbf{x}) = \sigma(\mathbf{x})\,\mathbf{E}(\mathbf{x}), \tag{3.3}$$

where $\sigma \equiv \frac{1}{\varrho}$ is called conductivity of the sample.

As well known, the relation between the resistivity of a homogeneous specimen of a material of lenght l and section S and its resistance is given by:

$$R = \rho \frac{l}{S}. \tag{3.4}$$

Resistivity has the SI unit of Ωm; a material has the resistivity of 1Ωm if a cube of 1m size, crossed by a current of 1A, has, at two opposite faces, an applied potential difference of 1V (see equation (3.4)) or, to tell it in another (maybe clearer) manner, if an electric field of $1N/C$ generates in the specimen a density current of $1A/\mathrm{m}^2$ (see equation (3.2)).

In general, the resistivity ρ of a sample of a certain material depends on the temperature T (here T is the absolute temperature). The function $\rho(T)$ that gives such a relation depends, in turn, on the material itself. Quite obviously, as every "good" function, $\rho(T)$ can be approximated by its tangent line in an appropriate neighbourhood of one of its point. For example, in a neighbourhood of 293K (that is the standard room temperature of the so-called "normal conditions") one finds:

$$\rho(T) = \rho_{293}\left[1 + \alpha\left(T - T_{293}\right)\right], \tag{3.5}$$

where ρ_{293} is the resistivity at 293K and α is a constant that depends on the material.

Most of materials can be divided into three categories for what concerns their conductivity. The reason of such a division will become much clearer from the observation that the typical order of magnitude of the room temperature resistivity of, the so-called, *conductors*, *semiconductors* and *insulators* are very different, see Tab. 3.1.

Material	Room temperature resistivity (Ωm)
Conductors	10^{-8}
Semiconductors	1
Insulators	10^{14}

Table 3.1.: Order of magnitude of room temperature resistivity of conductors, semicomductors and insulators.

It is also interesting to have a look at the different values of the αs in Tab. 3.2.

As can be seen, a part from Constantan (and other conductors not mentioned here), whose value is about thousands time smaller, all the other conductors have nearly the same value of α that is, 4.2×10^{-3} K^{-1} while for semiconductors the value is nearly the opposite. This fact is very interesting because it suggests the possibility of an explanation of conductivity that, to the base level, is independent of the specific material. (The fact is even more intriguing if we note that $4.2 \times 10^{-3}\mathrm{K}^{-1} \sim \frac{1}{273}\mathrm{K}^{-1}$, the famous thermal coefficient for perfect gases). But here we will not explore these facts in further details.

MATERIAL	α (K^{-1})
Conductors	
Aluminium	4.7×10^{-3}
Silver	4.1×10^{-3}
Gold	3.8×10^{-3}
Copper	4.3×10^{-3}
Zinc	4.2×10^{-3}
Platinum	3.9×10^{-3}
Constantan	4.0×10^{-6}
Semiconductors	
Germanium	-5.0×10^{-3}
Silicon	-4.5×10^{-3}
Insulators	0

Table 3.2.: Thermal coefficients of some materials. (Data taken from http://www.treccani.it/enciclopedia/resistivita/)

3.2. Superconductivity

Out of the temperature interval into which we can approximate $\rho(T)$ with its tangent line, equation (3.5) is not valid any more. We can, thus, wonder which is the shape of the function $\rho(T)$ when the temperatures are very high or very low. In this way we discover a very interesting phenomenon: many substances (nearly half of the elements of the periodic table, for example) have a resistivity that drops suddenly to zero below a particular temperature T_c, called critical temperature, that is characteristic of the given material. That is: the resistivity does not diminish gradually, but it makes an abrupt jump, going from a finite value, to zero. It does not jump from a finite value to another finite value (that would be already strange enough), but to exactly zero! (See Fig. 3.2).

In general, we are more accustomed with physical quantities that change with continuity. For example if we gradually increase the temperature of a gas in a container, also its pressure will gradually increase. If we slowly decrease the potential difference at the ends of an electrical circuit, also the current in the circuit will slowly decrease. Here, on the contrary the situation is different: as soon as the temperature goes below T_c, the resistivity goes to zero, but as soon as we heat again the sample above T_c, its resistivity jumps immediately to its previous value, without passing for the intermediate values. The process is, therefore, a reversible one. With that, we mean that decreasing the temperature below and then increasing it above the critical

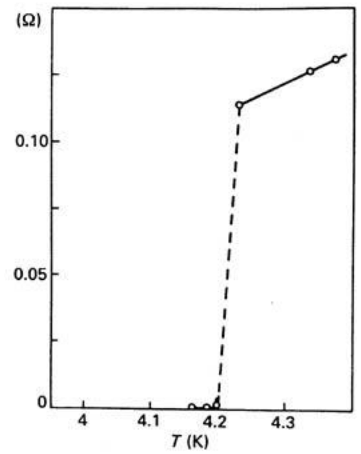


Figure 3.2.: Resistivity vs temperature for a mercury superconducting sample. This curve comes from the first measurements made by Kammerling Onnes.

temperature, we obtain always the same result. To the behaviour we have discussed (resistivity jumping abruptly to zero), we give the name of superconductivity, and we call superconductors the materials behaving precisely in this way (in the next section we will see that the above discontinuity in resistivity, leading to $\rho = 0$, comes always together with another fundamental property of superconductors, that is, more or less roughly, the expulsion of the magnetic field from the sample).

The values of the critical temperatures depend strongly on the material (even if, in general, they are very low temperature). For many substances such as mercury, lead and niobium (niobium, whose chemical symbol is Nb, is a metal that is very often used in applications of superconductivity, as we shall see later) the critical temperature is of only few kelvin, while it reaches about 100 kelvin for many ceramics such as YBCO, very often used in many experiments on superconductivity an that is a compound of yttrium, barium, copper and oxygen. Some values of the critical temperatures are shown in Fig. 3.2.

We have said that this strange phenomenon of superconductivity is an almost general one: in fact many materials are superconductors. Let's have a look at Fig. 3.3, that shows the elements that at sufficiently low temperatures become superconductors.

We immediately observe that they are really a lot. For example aluminium, mercury and lead are superconductors at atmospheric pressure; and many other materials become superconductors if, besides lowering their temperature, we also greatly increase the pressure which they are subjected to. Even many alloys and, as already said, some ceramic structures become superconductive! We can also observe that the diamond and graphite allotropes of carbon are not superconductive, while the fullerene allotrope does indeed super-conduce (fullerene (C_{60}) is a great molecule made of 60 carbon atoms arranged in a structure very similar to that of a soccer ball). Not every substance, however, becomes superconductive. For example, it

Material	T_c (K)
Gallium	1.1
Aluminium	1.2
Indium	3.4
Tin	3.7
Mercury	4.2
Lead	7.2
Niobium	9.3
Nb_3Sn	17.9
YBCO	92

Table 3.3.: Critical temperature of some common superconductors

is very interesting to observe that very good electrical conductors, such as copper, silver and gold do not exhibit superconductivity, while, as already said, some ceramic insulators have zero resistance below a certain temperature. In Fig. 3.4 we can see the resistivity versus temperature of a pure metal (i.e. copper) and that of the same, but this time "dirty", metal (because of some impurities inside). As one can immediately see, their trend at low temperatures is very different from that of a superconductor. In fact, even if the pure metal has a resistivity that goes to zero with absolute temperature, there is not a jump in its resistivity, nor we can find a finite temperature below which its resistivity is zero.

It is interesting also to observe that resistivity of copper (we often consider copper as an example because it is one of the most common conductors) decreases a lot while going down to very low temperatures: at 4K it is nearly $10^{-11}\Omega$m, that is only a thousand time of that at room temperature $10^{-8}\Omega$m (Fig. 3.4). But having a very low resistance is a completely different thing than having a null resistance. But then, what does it mean to have zero resistance? To start with, let us see two consequences of this fact.

1. In an electrical circuit the current can flow forever without damping, even without a battery that generates an electromotive force in the circuit! In fact, let us consider the electric circuit of Fig. 3.5: the part with the power supply (a battery in our figure) is kept at room temperature while the other part of the circuit, with a coil and a switch, is inside a vessel filled with liquid helium (liquid helium, at atmospheric pressure, is a very cold liquid, with a boiling temperature of only 4.2K) so that it is superconductive, because it is kept at a temperature below T_c (Fig. 3.5)

2. When the switch is open, a current will flow both: in the coil with zero resistance, and in the part with the power supply and the wires that are kept at room temperature. When the switch is closed, the current will be able to flow even in the part of the circuit closed by the switch and immersed in liq-

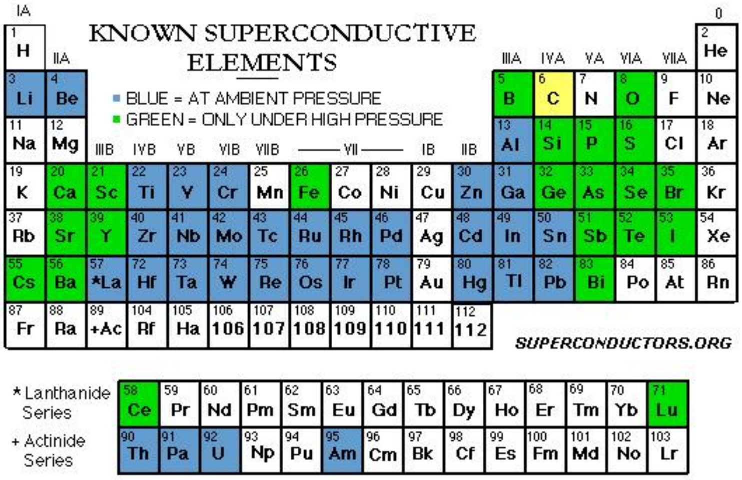


Figure 3.3.: Periodic table of the elements: in blue the elements that become superconductors at sufficiently low temperature. In green those that become superconductors when at low temperatures and high pressures. Carbon is evidenced in yellow because its forms diamond and graphite are not, while fullerene (C_{60}) is, superconductive. (superconductors.org).

uid helium, in this way short-circuiting the battery and the outer part of the circuit that, therefore, will be no more crossed by the current. The current flowing in the coil can be detected measuring the magnetic field induced by the current. Experiments of the kind here discussed have been really performed. The magnetic field has been measured continuously for more than one year: no detectable decrease of the current has been found. In fact the decrease in the current intensity (if ever) is so small that the current would flow for millions of years without a noticeable reduction.

3. We know that in a conductor the relation between the electric field and the current density is given by equation (3.2). Therefore, as in equilibrium conditions, the current density is finite (how could it be different?) and as $\rho = 0$, then also **E** must be zero. Be careful not to be confused. In fact even in electrostatics the electric field inside a (normal) conductor is zero, but this is just a static phenomenon when no current is flowing and nothing is happening. When, instead, some current is flowing inside a conductor, its internal electric field is surely not null, but is given by the same equation (3.2) with $\rho \neq 0$. On the contrary, for a superconductor the internal electric field is always zero (a part from the case of high frequency time-dependent fields, as we will see later), even when a current is flowing.

It is also interesting to observe that there are also some materials that become superconductive at temperature that are well above LHe boiling temperature, typically at LN boilging temperature and also much more above. These materials, that are called high temperature superconductors to distinguish them from the low tempera-

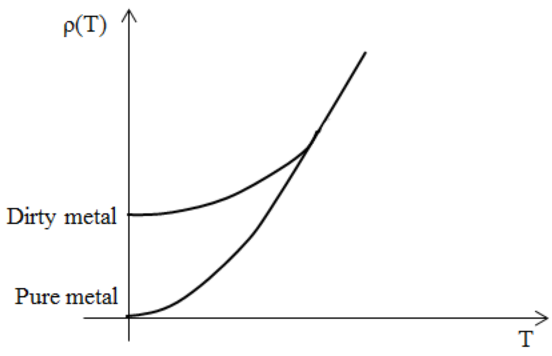


Figure 3.4.: Resistivity vs temperature of a pure and of a dirty metal.

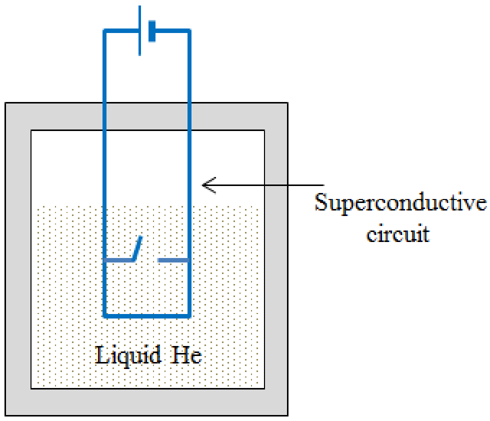


Figure 3.5.: an electrical circuit made of two parts: the part with the power supply (a battery in our figure) is kept at room temperature while the other part of the circuit, with a coil and a switch, is inside a vessel filled with liquid helium so that it is superconductive.

ture ones, are, in general, type II superconductors made of ceramic materials mostly insulating at room temperature.

The practical importance of low type II superconductors such as NbTi or Nb_3Sn lies in their very high B_{C2} (of even more than 10 tesla) that makes them very suitable for the construction of high field magnets.

3.3. Magnetic field expulsion (Meisser effect)

The most evident property of superconductivity (it is the one that has been discovered earlier) has been just discussed in the previous section: the resistivity of a superconducting sample drops to zero as soon as the temperature goes under the critical temperature. But, besides the vanishing of resistivity, a superconducting sample exhibits also another fundamental phenomenon: the expulsion of the magnetic field from its bulk. To understand the meaning of this fact let us consider a long and thin superconducting cylinder put inside a solenoid and at a temperature well below its critical temperature, let's say near the absolute zero.

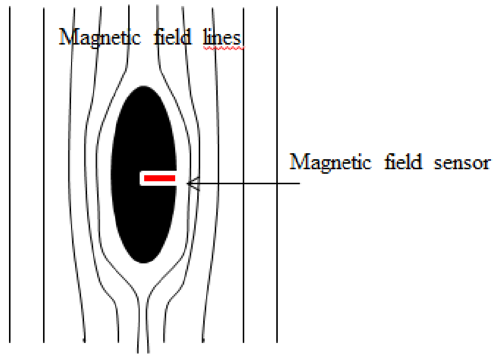


Figure 3.6.: a superconducting sample, in an applied magnetic field, with a magnetic sensor inside.

Let us now begin to circulate some current in the coils of the solenoid, so that to create a weak magnetic field B_{app} called applied magnetic field. If we now measure the magnetic field inside the superconducting sample, we see that it is zero. With the increasing of the applied field B_{app}, the magnetic field inside the sample remains zero. This happens until the field B_{app} reaches a certain value, that depends on the material and that is called critical magnetic field and that we indicate with B_C (we do not put the vector symbol here, because with B_C we indicate only the strength of the field). From the moment in which $B_{app} > B_C$ the magnetic field penetrates the sample, as our sensor indicates. This behaviour is common to every superconductive material. But experimental data show that the behaviour above B_C depends on the particular material. In fact superconducting materials can be divided into two categories: Type I and Type II superconductors.

Type I

In type I superconductors, as soon as the applied magnetic field overcomes the critical value, the magnetic field inside our cylindrical sample goes abruptly from zero to a value very close to B_C. So that we find again a strange behaviour: with the discontinuity of a physical quantity, that in the previous case was the resistivity and that now is the magnetic field. It is fundamental to observe that, for $B_{app} > B_C$, not only the magnetic field penetrates the sample, but also that, when this happens, the sample is no more superconducting (that is, its resistance suddenly jumps to a value different from zero), even if the temperature is kept well below T_C! Anyway this process is reversible: coming back to applied fields such that $B_{app} < B_C$, the magnetic field inside the bulk is again expelled and the material becomes again superconductive.

Type II

As the type I, also the type II superconductors expel the magnetic field from their bulk until the applied field reaches a certain critical value that we can call B_{C1}. But, as B_{app} overcomes B_{C1}, the interior field, that now penetrates the sample grows with continuity from zero up to a certain value B_{C2}. Moreover, for applied fields between B_{C1} and B_{C2}, superconductivity is not destroyed (differently from the case of type I superconductors). For applied fields greater than B_{C2}, the expulsion of the field happens no more, however superconductivity is not completely destroyed, in fact it remains in a thin superficial layer of only about 10^{-7}m, until a value B_{C3} of the applied field is reached. As for the type I case, even the processes here described are anyway reversible processes.

3.4. Few elements of thermodynamic of superconductors

What we have just described is true for temperatures that are well below the critical temperature. With increasing temperature, what already said is still true, but the values of the critical magnetic field decreases with temperature. This is completely reasonable because, as soon the temperature reaches the critical value, the material becomes no more superconductive and therefore the critical magnetic field can nothing but be zero. Experimentally one observes that the critical magnetic field versus the temperature follows the trend given by Fig. 3.7. The curve $B_C(T)$ is a parabola and a good experimental fit is given by:

$$B_C\left(T\right) = B_C\left(0\right)\left[1 + \left(\frac{T}{T_c}\right)^2\right]. \tag{3.6}$$

Up till now we have seen the following facts for a superconductive material:

- The existence of a critical temperature, below which the resistivity goes abruptly to zero in a reversible process.
- The existence of a critical value Bc below which the interior magnetic field is expulsed from the superconductive sample.
- The fact that above B_C (whatever the temperature) the material is no more superconductive.
- The fact that B_C is linked to the temperature T by a specific relation, see eq.(3.6).

All these facts indicate that the transition from the normal to the superconductive behaviour is a phase change. It is a proper thermodynamic phase change as the one we see when, lowering the temperature below 0°C, water transforms from liquid to solid. In this transition we can see that many physical properties change with a jump; for instance the specific heat jumps from 1 for liquid water to about 0.5 for solid water, the density jumps from 1, again for the liquid case, to 0.92 for the ice, and so on... In general we are used to describe thermodynamical phases in terms of P,V and T (pressure, volume and temperature). In our case, instead, V does not play almost any role, while the important variables are P, B and T. In gases, a variation of pressure is very important; it is also important for superconductors, as can be seen from Fig. 3.3. However, we are not going to study the behaviour of superconductors at varying the pressure, but we will always imagine to keep it fixed at the atmospheric value. Eq.(3.6) is, in a sense, the analogue of the state equation of perfect gases at constant pressure:

$$V\left(T\right)=V_0\alpha T. \tag{3.7}$$

In Fig. 3.7, if the point (T,B) lies below the curve $B_C\left(T\right)$ the material is in its superconducting state irrespective of how the point has been reached. If, instead, (T,B) is above the curve $B_C\left(T\right)$ the material is in its normal state.

The notion of superconducting state and the existence of a critical magnetic field allows us to calculate the energy difference between the normal and the superconducting state at the same temperature. In fact, in order to transform from the normal to the superconducting state, the sample has to expel the magnetic field and therefore it has to push out its magnetic energy (we are here referring to type I superconductors, but very similar argumentations can be done also for type II superconductors, considering B_{C1} or B_{C2} when needed). This condition can be achieved only if the energy in the superconducting state is lower than that of the the normal one of an amount equal to the magnetic energy stored in the sample when the field is the critical one. That is, if the sample has a volume V, and reminding that the energy density stored in a magnetic field is $B^2/2\mu_0$ we can write, with obvious symbology, that:

$$E_n-E_s=\frac{B_c^2}{2\mu_0}V. \tag{3.8}$$

Besides temperature, there is clearly another parameter that influences superconductivity, and it is the value of the superconducting current. In fact from Maxwell equations we know that a current density generates a magnetic field, and therefore the existence of a critical magnetic field must also be linked to the value of current density in the superconductor; and even in the absence of an applied magnetic field we can find, for each superconducting material, a critical current density $\mathbf{J}_c$ above which superconductivity is destroyed.

Other important thermodynamical features of superconductors (that we just mention here without further details) are the following.

1. Specific heat: for zero applied magnetic field the normal-superconducting transition is of second order, fact that implies a discontinous specific heat at the transition temperature, but no latent heat. In presence of an applied magnetic field, on the contrary, the transition is of the first order, with a latent heat and a jump in the specific heat that, below T_c vanishes exponentially at $T \to 0$.
2. Isotope effect: even if the crystallographic properties in the superconducting and in the normal state are the same, the ionic lattice play a very important role in superconductivity, in fact the critical temperature depends on the ionic mass M and the dependence is approximately given by $T_c \sim \mathrm{M}^{-0.5}$.

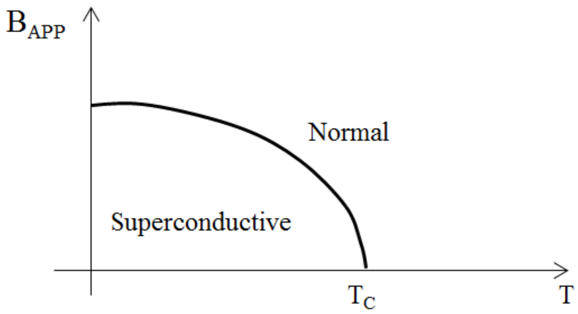


Figure 3.7.: Critical magnetic field versus temperature for a superconductive material.

3.5. A first phenomenological description: the London equation

The first explanation of superconductivity was given in 1935 by the London brothers (Heinz and Fritz London). Let us suppose that in the superconducting state there are two charged fluids that can give rise to two current densities: one is a normal (ohmic) fluid and the other is a superconducting fluid. The normal fluid obeys Ohm's law $\mathbf{J_n}= \sigma\mathbf{E}$ while the superconducting one is composed of superconducting charges q of density n_s and mass m. We want to write down a phenomenological equation that explains both the $\varrho = 0$ condition and the Meissner effect. The supercurrent density is obviously given by:

$$\mathbf{J}_s = n_s q \mathbf{v_s} \tag{3.9}$$

where $\mathbf{v_s}$ is the supercharge velocity. This immediately yields

$$\frac{\partial \mathbf{J_s}}{\partial t} = n_s q = n_s q \frac{q\mathbf{E}}{m} = \frac{n_s q^2}{m}\mathbf{E}. \tag{3.10}$$

Recalling the Maxwell's equation:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{3.11}$$

taking the rotation of eq.(3.10) one gets:

$$\nabla \times \frac{\partial \mathbf{J_s}}{\partial t} = -\frac{n_s q^2}{m} \frac{\partial \mathbf{B}}{\partial t}, \tag{3.12}$$

and therefore:

$$\nabla \times \mathbf{J_s} = -\frac{n_s q^2}{m} \mathbf{B} + \mathbf{B_0}, \tag{3.13}$$

where $\mathbf{B}_0$ is an integration constant vector. The essential *ad hoc* assumption of the London brothers, allowed to take into account that the magnetic field inside the superconducting sample is zero, that is $\mathbf{B}_0$ vanishes. So we are let with:

$$\nabla \times \mathbf{J_s} = -\frac{n_s q^2}{m} \mathbf{B}. \tag{3.14}$$

Eq.(3.14) is a phenomenological equation at the basis of the superconducting behaviour. In fact, going back from equation (3.14) to equation (3.10), it is clear that a supercurrent obeying to equation (3.14) feels no friction. Let's now see how equation (3.14) explains also the Meissner effect. For static fields we can write that:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J_s}, \tag{3.15}$$

and therefore eq.(3.14):

$$\nabla \times \nabla \times \mathbf{B} = -\mu_0 \frac{n_s q^2}{m} \mathbf{B}. \tag{3.16}$$

But, as well known, since $\nabla \cdot \mathbf{B} = 0$:

$$\nabla \times \nabla \times \mathbf{B} = \nabla \nabla \cdot \mathbf{B} - \boldsymbol{\nabla^2} \mathbf{B} = -\nabla^2 \mathbf{B}. \tag{3.17}$$

Putting together eq.(3.16) and eq.(3.17) we arrive at:

$$\nabla^2 \mathbf{B} = \mu_0 \frac{n_s q^2}{m} \mathbf{B}. \tag{3.18}$$

The Meissner effect follows from equation (3.18). Let's us consider a very simple geometry: the superconductors fills the semi-space $x > 0$, therefore its surface is in the yz plane. Form eq.(3.18) for a magnetic field parallel to the surface (and directed along the z-axis) we have:

$$\frac{\partial^2}{\partial x^2} B_z = \mu_0 \frac{n_s q^2}{m} B_z, \tag{3.19}$$

whose solution is:

$$B_z(x) = B_0 e^{-\frac{x}{\lambda_L}}, \tag{3.20}$$

where we have introduced the parameter

$$\lambda_L(x) \equiv \sqrt{\frac{m}{\mu_0 n_s q^2}}, \tag{3.21}$$

that is called London penetration depth and by eq.(3.20) one sees that it represents the thickness of the superconducting surface penetrated by the magnetic field (Fig. 3.8).

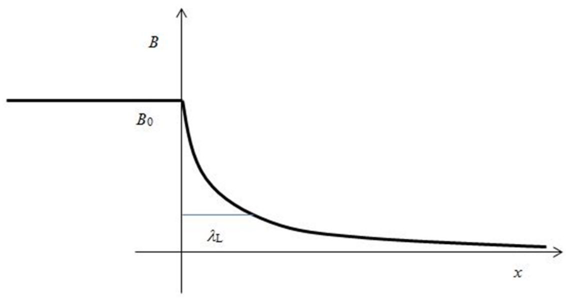


Figure 3.8.: Intensity of the magnetic field inside an infinite superconducting sample.

For most materials the penetration depth is some tens nanometers that is a very small value from a macroscopic point of view; we can therefore say that in the bulk the magnetic field is zero, which is just the Meissner effect we wanted to explain. A consequence is that superconducting currents can flow only on the same thin layer penetrated by the magnetic field.

As can be clearly undesrtood from an heuristic point of view, the temperature dependence of the London penetration depth must be such that, for $T \to T_c$ $\lambda_L \to \infty$ and the sample becomes normal. An explicit expression of such dependence is:

$$\lambda_L(x) = \frac{\lambda_{L0}}{\sqrt{1 - \left(\frac{T}{T_c}\right)^4}}, \tag{3.22}$$

and can visualized in Fig. 3.9.

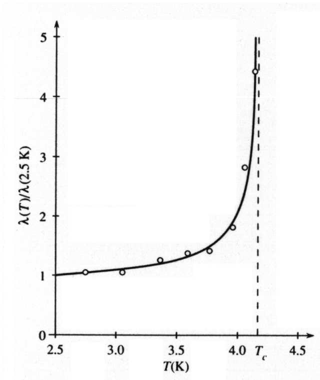


Figure 3.9.: Temperature dependence of the London penetration depth.

Recalling the definition of the magnetic vector potential as the vector **A** such that **B** $= \nabla \times$ A, another way of setting eq.(3.14) is to say that:

$$\mathrm{J} = -\frac{n_s q^2}{m}\mathbf{A}, \tag{3.23}$$

we won't go in more details here, but to say that this formulation of the London equation is much more useful when dealing with high school students, as we shall explain in chapter 11.

3.6. Something more on magnetic fields inside type I and type II superconductors

The situation that we have here described is very ideal. In fact the expulsion of the magnetic field is as simple as we have described only in the case of a thin long sample of type I superconductor parallel to a the applied magnetic field. Actually the situation is strongly geometry dependent. Let us suppose that the sample is a sphere placed in a uniform magnetic field less than B_C. As the filed lines should be expulsed, they tend to thicken towards the "equator" of the sphere. This thickening can be easily so great that the magnetic field can be larger than B_C in that zones and therefore the sample should be no longer superconductive and the field lines should penetrate the sample, thus varying the geometry of the lines and making the field intensity decrease below B_C as we imagined before. The real situation is that the sample splits into normal (and therefore penetraded by the field) and superconduting zones that are roughly regularly distributed in the sample. This is a new state called *intermediate state*.

In a sense similar to this, but thermodynamically completely different, is the *vortex state* of a type II superconductor. In fact, if a magnetic field, in between $B_{C1}$and B_{C2}, ia applied to a type II superconductor, the field lines penetrate the sample in vortex that create a very regular exagonal pattern, seeFig. 3.10.

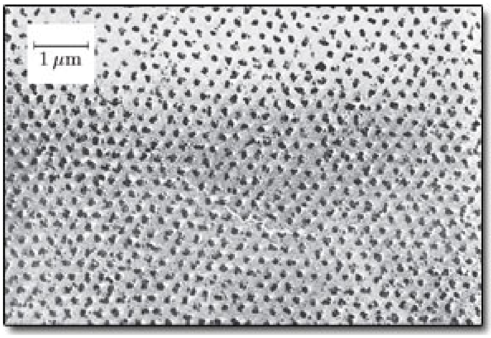


Figure 3.10.: First image of Vortex lattice in a supercondictor. U. Essmann and H. Trauble Max-Planck Institute, Stuttgart Physics Letters 24A, 526 (1967) Vortex state in a superconductor.

Similarity and differences between the intermediate and the vortex states deserve some attention. Let us first compare the magnetic properties of a very thin superconductive sample to that of a very thick one. In the thin sample, provided its dimension are comparable to the London penetration depth λ_L, it is easy to qualitatively understand from eq.(3.19) that the applied external field does not go to zero in the centre, while the bulk of the thick superconductor the field is vanishing. Therefore less energy per unit volume is needed to expel the field in the thin sample case, or to tell it in another way, that the critical field in the thin sample is greater than the critical field of the thick one, even if they are of the same material. From this point of view one can understand why for a thick sample it could be energetically favorable to subdivide itself into thin normal and superconducting slices. For this observation to be correct we have also to take into account the energy given of the normal - superconducting interfaces. In fact the previous subdivison will occur only if the interface energy is less than the gain in the magnetic energy. And here a new fundamental length comes into play. In fact the superconductor charge density does not reach abroupltly the value it has in the bulk (remember that at the surface of the superconductor the mgnetic field is just the one outside), but it rises smoothly from zero over a finite typical length, that we call coherence lenght and indicate with ξ. The ratio between the London penetration depth and the coherence length determines whether a superconductor is type I or type II. In fact the magnetic energy gained per unit area is given by:

$$\Delta E_{mag} = \frac{B_c^2}{2\mu_0}\lambda_L, \tag{3.24}$$

while the loss of condensation energy per unit area is:

$$\Delta E_{con} = \frac{B_c^2}{2\mu_0}\xi. \tag{3.25}$$

Therefore it wolud be energetically favorable for the sample to split into normal and superconducting slices (that is to be a type II superconductors) if $\Delta E_{mag} > \Delta E_{cond}$, that is if $\lambda_L > \xi$. Introducing what is called the Ginzburg-Landau parameter κ, we can say from our heuristic considerations that we have:

$$\kappa < 1 \quad \text{Type I superconductors,} \quad \kappa > 1 \quad \text{Type II superconductors.} \tag{3.26}$$

3.7. Flux quantization

If we keep in mind the idea that superconducing currents can be well explained in terms of an underlying wave description, we can easily understand another peculiar feature of superconductivity. Let us suppose that our superconducting sample is not simply connected as it is the case if it is a ring or, also, if it is a sample of a type II superconductors in the vortex state. Let us also suppose that the supercurrents are described by the wave ψ (with obvious symbology):

$$\psi = Ae^{i\varphi} = Ae^{i2\pi\left(\frac{x}{\lambda} - vt\right)}. \tag{3.27}$$

Therefore the phase difference between the two points X and Y will be given by:

$$\Delta\varphi_{XY} = 2\pi \int_X^Y \frac{\hat{\mathbf{x}}}{\lambda} \cdot d\mathbf{l}, \tag{3.28}$$

where $\hat{\mathbf{x}}$ is the unit vector in the propagation direction. Now, in presence of an electromagnetic field, the relation between the wavelength and the momentum of one quantum of the field is:

$$\frac{1}{\lambda} = \frac{p}{h}; \quad \mathbf{p} = 2m\mathbf{v} + q\mathbf{A} = 2m\frac{\mathbf{J}_\mathrm{s}}{n_s q} + \mathrm{q}\mathbf{A} \tag{3.29}$$

If we have a superconducting ring, and if we choose a closed path inside the ring, as ψ must be single valued, there must result:

$$\oint \frac{d\varphi}{ds} ds = 2n\pi. \tag{3.30}$$

and therefore:

$$2n\pi = \Delta\varphi_{XX} = 2\pi \oint \frac{\hat{\mathbf{x}}}{\lambda} \cdot d\mathbf{l} = \frac{4\pi m}{h n_s q} \oint \mathrm{J_s} \cdot \mathrm{dl} + \frac{4\pi q}{h} \oint \mathbf{A} \cdot d\mathbf{l} = \tag{3.31}$$

$$= \frac{4\pi m}{h n_s q} \oint J_s \cdot dl + \frac{4\pi q}{h} \oint \mathbf{B} \cdot d\mathbf{s}$$

In the bulk there must be no current, and so the first integral at the right hand side of equation (3.31) is identically zero. We thus are left with:

$$2n\pi = \frac{4\pi q}{h} \oint \mathbf{B} \cdot \mathrm{d}\mathbf{s}, \tag{3.32}$$

from which we obtain that the magnetic flux enclosed by the circular path must be quantized and the flux quanta are given by:

$$\phi_0 = \frac{h}{q}. \tag{3.33}$$

Experiments confirm this result with the value of $q = 2e$ where e is the electron charge. See for some other details, chapter 11.

4. Superconductivity at the secondary school, a proposal

4.1. Overview

After we have been recalled the main features of superconductivity in the previous chapter, features that are reported in an almost infinite number of textbooks of different levels (see for example [2] or [9], it is the moment to describe the way in which superconductivity could be proposed to students. In this chapter, after a brief description of the method of the Educational Reconstruction of the Contents that has constituted the general framework that inspired all our work, we will give only the synthetic scheme for an educational path on superconductivity for secondary school students. Instead, in the next chapters we will describe in detail the main parts of the educational path proposed.

Indeed, our proposal can be seen as a first basic proposal with an integrating part that can be added to the basic one, depending on the students' level to whom the proposal is addressed. For this reason we will describe the schema of the basic proposal, and subsequently the schema of the complete proposal that includes the basic one as a subset.

4.2. The Model of the Educational Reconstruction of the Contents

The Educational Reconstruction Model was developed from the mid-90s, with the aim of improving the practice of the teaching together with the aim of the reorganization of the contents of the curriculum. In the framework of the Educational Reconstruction the achievement of the improving of the teaching process is related to the reconsideration of the disciplinary contents in such a way to reconstruct them in a more effective didactical perspective.

In this context, the Model wants to combine the educational aim of the teaching process, that is centred to the personal growth of the student, with the most effective transposition of the scientific concepts. The main steps of the Educational Reconstruction Model are:

1. The analysis of the structure of the contents: from the analysis of the pubblications, to the analysis of the key points, and the conceptual nodes.
2. The developing of the educational path and the realization of the path with groups of students.
3. The empirical investigation of the learning process.

The three steps of the Model are tightly connected to each other. The second step of the design of the educational path have to be related with the previous knowledge of the students that have to be considered and it is thought as the starting point for the learning process of the students themselves: the previous knowledge is not seen as an obstacle to the learning process, but rather an occasion of growth.

For this reason this second step must be related with the first step, in which the contents are studied in detail, both from a didactical point of view and from a disciplinary point of view. Moreover, the third step is related to the second in order to understand if the learning process really occurs and the teaching is, thus, effective. A problem in the third step should elicit a revision of the second step, as a problem in the second step should elicit a revision of the first step.

From the previous considerations it is clear that the structure of the scientific content has to be thought on the basis of the ideas of the teacher for what concerns his/her goals in teaching that particular content, in the particular context in which the content must be taught.

Many teachers think that the structure of the content for students should be simpler than the structure of the scientific content itself. In the Educational Reconstruction framework it is the opposite, in fact, in order to help students in overcoming their difficulties with the introduction of a new abstract content, it is necessary to consider all the facets of the content and describe many of them, in such a way as to have the chance to find a match with the different ways of thinking of the students.

We have referred to the Educational Reconstruction of the Contents throughout the entire work. But we could not develop the three key point of the Method for each topic involved in this research, because of the enormous amount of work that would be needed, and that would take a lot more than three years.

For this reason we completely developed the part on the electrical conduction according with the Educational Reconstruction, following the three key point of the Method. Moreover, we experiemented the Design Based Research Method for that topic, in order to refine in subsequent steps the initial proposal. But, as we shall see in chapter 5, the first proposal has been modified two times: the first time by making small changes, in the framework of the Design Based Research, while the second time, the educational path has been drastically changed, revolutionizing the proposal in the Educational Reconstuction framework.

We applied the first two steps of the Educational Reconstruction Method even for the topics of the magnetic vector potential and the explanation of the basic superconductivity by means of the vector potential. These two topics needed a lot of work, due mainly to the fact that very few is present in the literature for what concerns the vector potential for the secondary school, both for the explanation of superconductivity and for the vector potential itself. This lack in the literature forced us to work very long on structuring the contents, but it has also satisfied us for the originality of the work that has came out.

4.3. Schema of the basic educational path on superconductivity

In this section we will describe the minimal path on superconductivity that we have developed in the light of the Educational Reconstruction of the Contents. As it has been well established in the previous chapter, the basic level to which superconductivity can be treated is the phenomenological level, as the two fluids theory shows and how the London equation simply resume.

The London equation becomes then the key point of our educational path. Whereas the London equation is quite simple for teachers and for who can use differential operators such as curl and divergence, as the London brothers did in their paper, it would need months of work, for secondary school students. Why so much work? Why a student should be motivated in doing all this work?

In order to answer to these questions we would clarify that the months of work needed do not start when the educational path on superconductivity starts. The months needed start mainly in the teacher's approach to the last year curriculum: infact superconductivity may be faced consistently only if the students have already built their back-ground approaching many topics of electromagnetism in a precise way with a great reconstruction of some generally taught content. In the following we will describe all the topics we need and how they are related to each other. In the next chapters we shall describe in detail the approach that we developed in our reconstruction of the contents and that we experimented with students.

In the following Fig. 4.1, we describe the structure of the basic proposal.

The aim of the basic path is the phenomenological explanation of the two main features of the superconductivity: the resistivity that drops to zero at a certain temperature, that is called critical temperature of the particular superconductor, and the expulsion of the magnetic field from the bulk of the superconductor. These two main features can be clearly explained by the London equation. The box 4. in the scheme of Fig. 4.1 contains just the explanation of the basic superconductivity. But boxes 1., 2., and 3. are prerequisite boxes, needed to face box 4..

In other words, in order to introduce the London equation, two pre-requisites are needed:

- Box 1. Phenomenological aspects of the electrical conduction: this topic pertain the description of a normal conducting fluid using the Ohm relation writ-

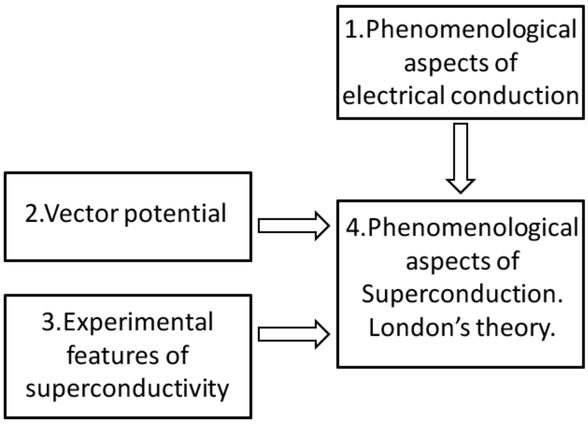


Figure 4.1.: Schema of the basic proposal for superconductivity with high school students, it is represented the way in which the topics are related to each other.

ten in its local formulation $\mathbf{J} = \sigma\mathbf{E}$, with obvious meaning of the notation. This is the first step that will permit students to develop the analogy between normal conduction and super conduction that we will see described by the London relation $\mathbf{J}_S = -k\mathbf{A}$, where $\mathbf{J}_S$ represents the super current density and $\mathbf{A}$ the magnetic vector potential.

- Box 2. Besides the prerequisite about the electrical conduction, from the previous point it appears clearly another, fundamental, prerequisite: the introduction of the magnetic vector potential $\mathbf{A}$. The vector potential is introduced to students developing an analogy with the usual electrical scalar potential. This part is the core of our proposal, infact, despite it is a challenging topic, it is also very meaningful because it allows to read in a new light the contents of electromagnetism, thus deeping electromagnetism, not only from a mathematical but mostly from a physical point of view.

And, in order to give a physical motivation for the London equation, that is for developing the basic phenomenological traitation of the superconductivity, it appears the last prerequisite: it consists in the description of the experimental evidences of superconductivity, and the realization, when possible, of some typical experiments on superconductivity: mostly, the measure of the critical temperature of an YBCO sample and the observation of the interactions between a superconductor and a magnetic field, as we shall better discuss in chapter 9.

4.4. Schema of the complete educational path on superconductivity

As the students working in lab with YBCO samples learn quickly, the magnetic field is not always expelled from the bulk of a superconductor. For this reason the too basic description of superconductivity, centred on the London equation, can no longer hold. The theory of the London brothers have to be extended to a more general case, but for doing this, it is necessary a back-ground both in mathematics and in physics that needs much more time than a few hours to be built.

We resume the complete proposal in the following Fig. 4.2. The complete proposal contains the basic proposal as a subset (into the dashed line).

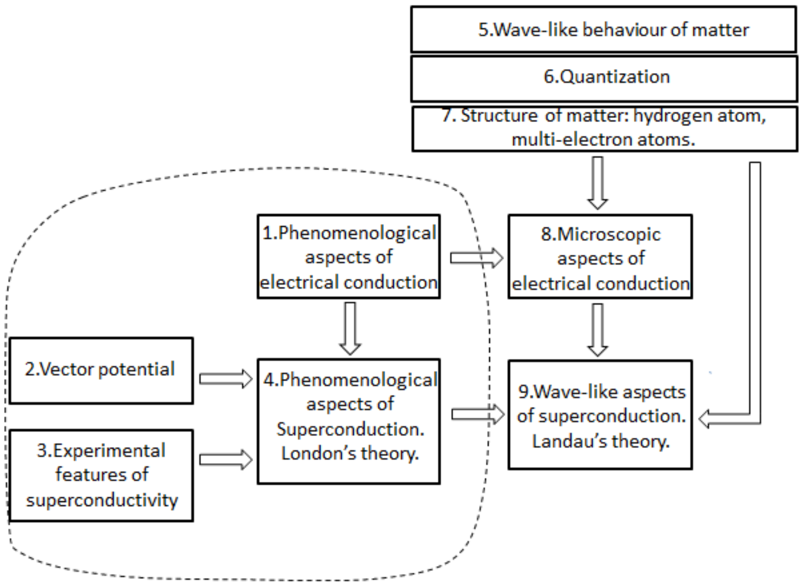


Figure 4.2.: Schema of the complete educational path for the presentation of superconductivity: they are shown the connections between the basic proposal, in the dashed line, and the integrating proposal outside the dashed line.

The integrative part represented in Fig. 4.2 out of the dashed line has the aim of describing the superconductive phenomena at a deeper level than that is mathematically resumed by the London equation. But a generalization of the London equation needs a certain effort. This generalization is only hinted in this thesis work: while a whole and detailed description is beyond the aim of this work, the outline of the possible educational paths that could be treated with students can be useful for teachers. Infact we think that this could provide a more extended vision of the problem both regarding the normal electrical conduction and the super conduction.

Infact, if teachers want to go deeper in their presentation, they will realize that not only superconduction needs the the wave-like interpretation of the superfluid, but even the normal fluid should be described in terms of matter waves. In the proposal schematically reported in Fig. 4.2 we can see that there is a first set of topics pertaining the wave-like behaviour of matter and the structure of matter. These boxes 5., 6., and 7. are not treated in this work because, although they are fundamental in approching wave-properties of matter and quantum physics, they have already been presented by the Physics Research Group of the University of Milan in many other studies and pubblications [20, 21, 22, 23, 24, 25, 26, 27, 28, 29].

Instead, we have dealt with boxes 8. and 9. although in a very schematic way, because the aim of the work has been the development of the basic educational path on superconductivity and the experimentation of some of its parts.

- Box 8. Microscopic mechanism of the electrical conduction: the mechanism of conduction is explained in the frame of the wave-like behaviour of matter. The normal fluid flowing in the wire is represented by an electronic wave bounded in the wire itself. In order to get a simple mathematical traitation of the problem, the wire is considered linear and the allowed energetic levels for the electronic wave, the normal modes of the wave, constitute the Fermi sea. Students may have a picture of the Fermi sea, and of the diffraction of an electronic wave in order to describe the dissipative processes inside the wire, that are the basis of the Ohm's law.
- Box 9. Wave-like aspects of the superconduction: the mechanism of superconduction can be faced mainly with the BCS theory or by the Landau theory. While the BCS theory needs mathematical tools really beyond the possibilities of the secondary school students, the Landau theory is based on the wave-like behaviour of matter, that students could have already treated as a pre-requisite of this path. In this part of the proposal it is possible to generalize the London equation with the new equation $\mathbf{J}_{\mathrm{S}} + k\mathbf{A} = constant$ and, by means of quite simple considerations, it is possible to get the quantization of the magnetic flux inside the superconductor.

The following chapters contain the developing of the main parts of the educational path proposed here: the electrical conduction, the magnetic vector potential, and the explanation of the superconductivity using the magnetic vector potential, together with considerations about the experimentation of some of these parts.

5. Electrical conduction in secondary school

5.1. Overview

In this chapter we start the educational part of the research. In fact, while so far we have taken into account the physics of superconductivity, now we have to develop an educational path suited for secondary school students. As it is clear from the contents of the previous chapter, the first step we have to discuss pertains the electrical conduction. We decided to dedicate so much time to the electrical conduction and circuits only later, after meeting the classes of the first experimentation. As it is widely discussed in the literature [30, 31, 32, 33, 34, 35, 36, 37, 38], the problem of the electrical conduction is still open.

We have been dealing with electrical conduction for three years with secondary school students and we changed our approach a lot during the course of these three years. The first time we encountered the students, we thought that it was possible to take almost for granted that they knew the meaning of the Ohm's laws. Therefore our initial path did not contain explicitly the phenomenology of the electrical conduction, except for a part in which it was described how to pass from the integral approach based on the first two Ohm's laws, $\Delta V = RI$ and $R = \varrho \frac{L}{S}$, to the approach with the local formulation $\mathbf{J} = \sigma \mathbf{E}$ of the Ohm's laws. Our aim was a description in terms of the local quantities $\mathbf{J}$ and $\mathbf{E}$ in order to develop an analogy between the *normal* electrical current and the *super*-current, typical of the superconductive phenomena.

Moreover our initial approach was oriented mainly on the microscopic mechanism of conduction, rather than on a phenomenological description, that we supposed well-known to students. Infact, in the first design of our complete didactical path, the microscopic descriptions of electrical current and super-current were an important part of the path. But when the first step of our *design-based research* was proposed to students the results made us to believe that a great change was needed. In this chapter we report the evolution of our path and the most interesting results that we have obtained from our research with students. We discuss written tests and oral interviews in the light of the *conceptual change* in the Hewson formulation [39, 40, 41, 42], with the aim of clarify common students' beliefs, the critical points of our proposal and some good result in students' understanding.

5.2. A Design-Based Research for the development of the educational path

The theoretical framework we adopted for our developing of the path on electrical conduction has been the Design-Based Research Method [43]. Since we started the path during the first Ph.D year, we have available three years for the experimentation and the revision of the path in reference of the Design-Based Method. As we have already mentioned, the initial path has been profoundly revised. Infact, the main characteristics of the Design-Based Method can be resumed in the following points:

- The Method has two goals: the design of contexts suited for the learning activity and the development of learning theories.
- The Method develops and implements the project in continuous cycles of design, implementation, analysis and redesign.
- The Method is based on a research that takes into account how the project actually works in real contexts: it has to document successes and failures in order to refine the understanding of issues related to learning.

As the previous points show, this Method recognizes the profound complexity of the teaching/learning process, because of the many variables involved. The variables pertain the social context (for instance the group of students and the enviroment in which they learn); the project itself that has to be realized; and the teachers that have to realize the project, in practice.

The irreducibility of these variables makes very interesting the analysis of the relations between the context and the results and becomes very useful an accurate analysis of the results. This could open a discussion about the choice of the assessment tools. For example, in our experimentations we used written tests and we gave the same tests always to control groups. For this reason the question proposed were very general and sometimes expressed by problems or exercises given in a no traditional way. Analyzing the results of the tests, we have found that the students of the control group were disoriented as the others when they had to solve no-traditional problems.

In the following sections it is described the evolution of the initial path on the electrical conduction and the related results of the tests proposed, at least for what concerns the most significative outcomes. The path has been experimented and revised three times, but unfortunately, the main difficulty consisted in the lack of cooperation with teachers, that accepted to perform the experimentation with their students, but they never get involved in the project expecially during our lectures and laboratory, and this have prevented a significant relapse on students' learning.

5.3. The initial path proposed to high school students

Since the last version of the path is presented in detail in the next chapter, here we briefly outline the essential points of the first step of our proposal. It was constituted by two main parts:

- A phenomenological description of the electrical current in terms of local quantities, using the relationship $\mathbf{J} = \sigma\mathbf{E}$
- A microscopic description of the electrical current in terms of propagation of wave of matter in a crystal

Let us briefly deal with these two parts, in order to understand the experimentation that we report in the next sections.

5.3.1. Overview of the phenomenological description in terms of local quantities $\mathbf{E}$ and $\mathbf{J}$

(A) From the integral quantities ΔV and I to the local quantities $\mathbf{E}$ and $\mathbf{J}$

The path starts with a recall of the Ohm's laws, written in traditional terms for the secondary school, that is, with obvious meaning of the notation:

$$\Delta V = RI, \tag{5.1}$$

$$R = \rho\frac{L}{S} \tag{5.2}$$

and

$$\rho = \rho_0\left(1 + \alpha T\right). \tag{5.3}$$

These laws and their meanings are considered as known topics to students. At this point, with the aim of a subsequent development of analogies and differences with the super-current, we rewrite the first two Ohm's laws in terms of the electric field $\mathbf{E}$ and the current density $\mathbf{J}$ in the particular simple case in which the wire is considered of uniform section and homogeneous. We postpone this calculation to the next chapter, and report the results:

$$\mathbf{J} = \sigma\mathbf{E}. \tag{5.4}$$

Eq.(5.4) is important not only for superconductivity, but even to understand the concepts of resistors in series and parallel. We have therefore developed an approach based on a sort of game with *cubes* of conductive material to describe the series and the parallel of resistors using the relation of eq.(5.4).

(B) The cubes representation to determine resistances

The representation that we propose is useful to better understand what the second Ohm's law (eq.(5.2)) states. Even if this representation is explained in detail in the next chapter, here we give the essential lines. We imagine to have a material that in a first approach to the problem can be considered the material of which the wire is made. We than suppose to have a small cube of this material and we can work with it. In a schematic representation as that of Fig. 5.1, we have applied a potential difference ΔV between two opposite faces of the cube. The line that joints each face of the cube to the generator of the potential difference has to be considered as an abstract way of performing the connection, or a connection with a infinite conducibility.

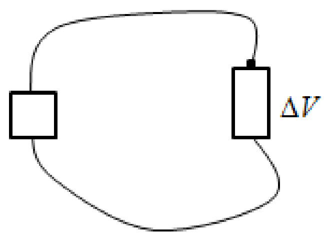


Figure 5.1.: Representation of resistors by cubes. The potential difference ΔV between the two faces of the cube is represented graphically by a battery.

Through this representation it is possible to treat the resistors in series and in parallel. We can have a picture of two resistors in series by placing two resistor side by side as in Fig. 5.2(a), or a representation of two resistors in parallel as in Fig. 5.2(b).

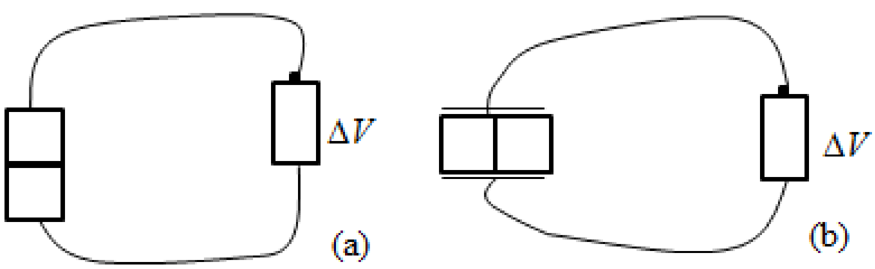


Figure 5.2.: Representation of two resistors in series (a) and two resistors in parallel (b).

Using the relation $\mathbf{J} = \sigma\mathbf{E}$ is then possible to determine quite intuitively the variation of the current flowing in the series and in the parallel with respect the starting situation described in Fig. 5.1. Only later it becomes appropriate the interpretation in terms of the Ohm's laws and hence in terms of resistances. In this way it seems to be more simple to transfer the idea of the conductor as *something that permits the current flow* rather than *something that uses up current* as students often think. Instead, whether the importance is primarily given to the resistance, as the Ohm's laws in their integral form suggest, it becomes increasingly difficult for students to construct a coherent picture of the conduction, even from the phenomenological point of view.

In our experimentation we have investigated students' ideas about electrical conduction before and after our instruction and we will discuss these results in a following section, in the light of the conceptual change method.

5.3.2. Microscopic picture of the electrical conduction

The microscopic picture of the electrical conduction can take place in the conceptual framework of the quantum physics. We have chosen for our path the framework that has been developed for at least 15 years at the University of Milan. The experimentation that we briefly present here is only the third part of a much wider proposal that is structured in three main sections:

- Waves. From the wavelike behaviour of water and light, to the wavelike behaviour of matter: interference and diffraction of electrons and neutrons; the Kapitza-Dirac effect.
- Quantum physics. From the wavelike behaviour of matter and electromagnetic waves to matter quanta and photons. The discrete energy levels of the hydrogen atom. Energy quantization and energy levels of microscopic bound systems such as electrons in atoms, electrons in a metal, nucleons in a nucleus.
- Microscopic mechanism of electrical conduction. The Fermi sea and the diffraction of electrons by the cristal lattice of the metal that carries the current.

In this thesis work, we propose the third point, on the microscopic mechanism of electrical conduction, as it was prepared for the first step of our design-based research. The first two points are instead out of the aim of this thesis and the reader can find in bibliography some references on these topics, to deepen the work of the physics educational research group of the University of Milan.

Before starting our work with classes, we investigated students' beliefs about the microscopic mechanism of conduction with a written test and with oral interviews in order to better calibrate our educational path.

Unfortunately, students' responses made us to think it would have been better not to extend our proposal also to the description of a microscopic model, because it would have taken too long: an amount of time that we did not have to experiment in real classrooms. Nevertheless, we report here this part on the microscopic mechanism of conduction, because we believe from previous experimentations that it could give good results in students' learning, if presented with a sufficient time available (more or less 30 hours in total) in paths concerning quantum physics, addressed also to the electrical conduction.

(A) Electron gas in a box

In this part we refer to what students have already seen in the section of the path of quantum theory, that is not reported in this thesis.

We start supposing that a student is able to associate a plane wave propagating along the x-axis to the free motion of an electron along the x-axis. This is a result that a student can achieve when he/she has become familiar with the wave-like behaviour of matter, that is the core of the quantum physics path of the University of Milan. We also suppose that a student already knows that a confined wave exchanges quanta of precise discrete energies. Exemples contained in the path are: a guitar string that oscillates only with its normal modes, the stationary waves of light, the energetic discrete levels of the hydrogen atom, etc.

Students can found the order of magnitude for the energy levels of a bound mono-dimensional system. In their previous quantum physics path they have got the relation:

$$E_n = \frac{h^2}{8mL^2}n^2. \tag{5.5}$$

It is possible to obtain the order of magnitude of the energies involved by replacing m and L, repectively with the mass of the quantum and with the length of the bound system. And now we can describe the model: in a first approximation, conduction electrons are supposed to be described by a free electron gas in a box at small (that is in our simplified model zero) temperature. In this case, m is the electron mass and L is the typical length of a conductor, because we suppose that the electron wave is free to move in all the conductor. The energies of the levels are of the order of $10^{-27}eV$, so small as to be considered quite a continuum. What is the energy taken by such electron gas?

From the quantum physics path a students should recall that nature has only two possibilities for attributing energy to the indistinguishable quanta of a wave field, depending on the fact that we can have bosons or fermions. Since the electronic wave field is constituted by fermions, they must have different energies to each other. Each of the electronic waves contained in the system of the conductor will have one of the available energies given by eq.(5.5), all stacked till the maximum energy, called Fermi energy, because the set of all these electron energies is referred to as the Fermi sea.

(B) The Fermi energy

As we said, a conductor can be seen as a portion of the space in which it is present a confined electronic field, that is a fermionic field, whose quanta have the energies defined by an analogue of eq.(5.5) for the three dimensional case. In the mono-dimentional case the Fermi energy is given by:

$$E_F = E_1 \left(\frac{N}{2}\right)^2 , \tag{5.6}$$

supposing that in the conductor are present N quanta, and considering that in each level it is possible to place two quanta. In terms of the physical quantities involved it is possible to write eq.(5.6) for the mono-dimensional case:

$$E_F = \frac{\hbar^2}{2m}\pi^2 \left(\frac{N}{2L}\right)^2 . \tag{5.7}$$

A generalization of eq.(5.7) for the three-dimensional case can be easily deduced even for students, and it is possible to obtain:

$$E_F = \frac{\hbar^2}{2m} \left(3\pi^2\right)^{\frac{2}{3}} \left(\frac{N}{V}\right)^{\frac{2}{3}} , \tag{5.8}$$

where N/V is the number of quanta per unit of volume of the conductor.

By simple calculations (surprisingly not so simple for secondary school students) it is possible to evaluate the Fermi energy, for instance, of a copper conductor, so to get: $E_F(Cu) \sim 7eV$. We report this consideration because in our experimentation we encountered classes of students that were not able to substitute the physical quantity in eq.(5.8) by their numerical values, without being helped by the teacher. It is interesting, at this point, let students to note that the value they have found for the Fermi energy is equivalent to the thermal energy associated to a temperature of about $20000K$. Moreover, from this observation some considerations could arise in order to evaluate the energies involved in the conduction process with respect to the vibrational energies of the lattice. This could be helpful for students above all to dissipate their ideas (that we found in our experimentation) about the fact that the electrical current increases with the temperature, through a mechanism in which the free electrons in the metal can gain more kinetic energy.

(C) The free electron gas in a conductor

If a potential difference ΔV between the endpoints of a circuit is applied, it generates an electric field of intensity E in each point of the circuit and thus a force acting on the charges that are free to move in the wire. A free charge in an electric field accelerates. So, why the current I , that is established by the first Ohm's law in the circuit, does not increase with time? We posed this question to students during our lessons, in the first experimentation, and in the next section some answers are reported.

This is certainly one of the key point of the electrical conduction: how a free electron moving in the electric field inside a wire does not accelerate? It is an experimental fact, and the model must describe it. In this section we briefly outline a possible way to answer to the question posed above. We will not enter into details because it would take too long and it is beyond the puropses of this thesis. Infact, the microscopic aspects of conduction and super-conduction are not the core of our presentation, but a teacher should however be conscious of the possibilities that could exist with an adequate background that can be build with students, working with them for about three years, reconstructing many contents of their curriculum.

We have just said that electrons are fermions and only two fermions can have the same energy. For this reason the energetic levels of the conductor are all occupied till the Fermi energy level. The energy of each electron depends on its momentum p and, being the electron completely free, its energy is a pure kinetic energy $E = p^2/2m$. It is possible to represent this condition of the conductor in its equilibrium state as in Fig. 5.3.

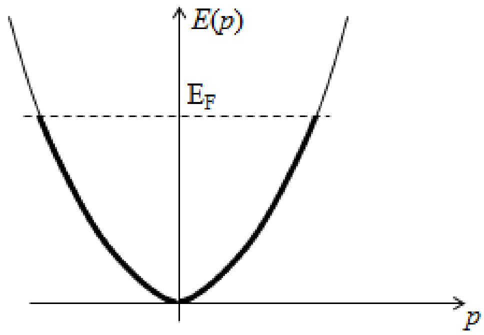


Figure 5.3.: Energetic levels of the conductor in its equilibrium state. The levels are distribuited along the vertical axis quite continuously because the gap between two consecutive levels is very short. The bold line indicates that the linear momentums of the electrons are symmetrically distribuited along the two directions of the conductor, thought mono-dimensional.

In this approximation, the conductor is thought as mono-dimensional and the electrons move with the same probability in the two possible directions of the wire, as the intensities of the velocity in both the directions are the same, from the minimum to the Fermi velocity in both directions. The situation remains in this equilibrium state until a potential difference is supplied to the endpoints of the wire.

When the potential difference is applied, there appears a new dynamic equilibrium state. Infact, as Fig. 5.4 shows, the bold line is no more symmetrically placed with respect to the momentum axis. There is a net contribution to the momentum in a particular direction that is determined by the electric field correspondent to the potential difference applied.

The upward "movement" of the bold line is generated by the fact that electrons with energy near to the Fermi energy can take enough energy to jump from the Fermi

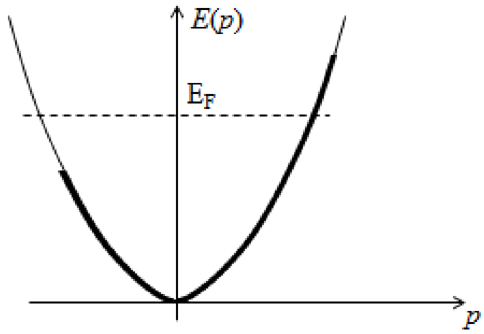


Figure 5.4.: Energies of the electrons in a carrying current conductor: the maximum energy is higher than the Fermi energy and the momentums are shifted on the right (or in the left) and represent the net current that is flowing.

level to a higher level. The hole left in the Fermi level, or near it, will be occupied by an electron of lower energy. This process goes on and on until a maximum level is reached. What determines the maximum energy level? It happens that when the energies become too high a diffraction process occurs. Here, we do not deal with the mechanism of diffraction, but we describe the process that prevents an indefinite increase of the current in its essential lines.

As we have already said, an electron moving in a conductor can be seen as a propagating wave of matter in the crystal lattice. The basic mechanism of the model is represented in Fig. 5.5. Now the lattice must be taken into account. The electron wave propagates until it reaches an inhomogeneity of the positive charge in the crystal lattice. At this point the wave can interact with it (it is a sort of diffraction process) and looses energy by diminishing energetic levels of some of its quanta, (see the diagram of Fig. 5.5): in this way the bold line "moves" downward and the process becomes stationary, infact the energy gained by the waves from the electric field is subsequently lost through interaction with the crystal lattice.

(D) From the model to the phenomenology

Once the qualitative model has been described, it has to be used in a more quantitavie way to obtain the phenomenological Ohm's laws: the first two laws that we have written as $\mathbf{J} = \sigma\mathbf{E}$, but in particular the third law or the right dependence of the resistivity ϱ from the temperature:

$$\varrho \propto T. \tag{5.9}$$

This step is important because in the literature there are many alternative models to describe the electrical conduction from a microscopic point of view. It is quite

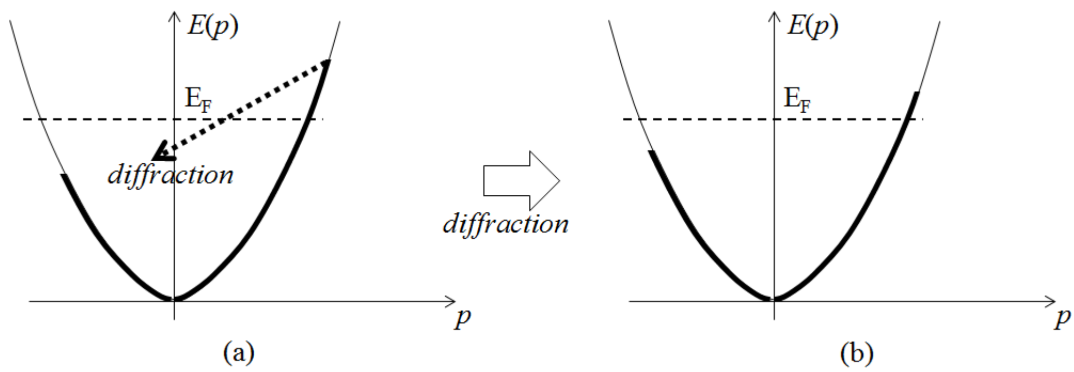


Figure 5.5.: Mechanism by which the current does not increase indefinitely through diffraction processes.

common for instance to find versions of the Drude model, but these models (besides being quantistically wrong) provide a dependance of the resistivity ϱ from the temperature that is not correct. By the Drude model infact we obtain that:

$$\varrho \propto \sqrt{T}, \tag{5.10}$$

where T is the temperature of the conductor.

In our presentation of the microscopic mechanism of the electrical conduction we obtain the correct dependence from the temperature. Here below we outline the sequence that we have proposed to students in order to deduce the third Ohm's law.

- It is needed a pre-requisite on electronic waves: the wave-like behaviour of matter cannot be directly experienced because the wave function that describes the propagation is a complex wave and therefore not perceivable as such. Instead, a complex wave function can be indirectly detected by experiences of interference and diffraction. If $\psi(\mathbf{x}, t)$ represents the complex wave of matter, than it is possible to detect only $|\psi(\mathbf{x}, t)|^2$ that equals the square of the wave amplitude, A^2 that is a constant. Hence, through this wave-like modelization of the electrical current, the complex wave field flowing in a conductor is, in a sense, analogue to water flowing in a duct. We can thus imagine that an electronic wave is associated to a fluid of charge density ρ flowing in a conductor.
- It is necessary to recall the definition of the current density $\mathbf{J} = \rho\mathbf{v}$, where ρ is the charge density and $\mathbf{v}$ is the velocity of the element of charge in the conductor. In our model, that is based on the scattering of electronic waves, we interprete the motion of the charge as the motion of a portion of a fluid, rather than the motion of an electron. And it is for this reason that in our sequence we will never refer to electrons, but we will refer to small volumes of charge with density ρ moving in the conductor.

- A charge placed in an electric field accelerates. From the kinematics of a uniformly accelerated motion we can thus write:

$$v = at \tag{5.11}$$

and

$$v \propto \rho E \tau, \tag{5.12}$$

where τ is the time between two diffractions of the electronic wave with the lattice.

- The time τ between two collisions is related with the mean free path λ. We can evaluate it considering the mean velocity v_F of the waves, or the electronic fluid that is flowing in the conductor. We called v_F this velocity because the only charges that can move must have the velocity very near to that corresponding to the Fermi energy, for what we have said previously. Hence we have:

$$v_F = \frac{\lambda}{\tau}. \tag{5.13}$$

Now, taking together eq.(5.12) and eq.(5.13) we have the relationship:

$$\tau \propto \lambda. \tag{5.14}$$

- In order to get a relation between the conducibility σ and the mean free path λ we have to express the conducibility in terms of the quantity used so far. We have the definition of current density $\mathbf{J} = \rho \mathbf{v}$ and the first two Ohm's laws in their local form $\mathbf{J} = \sigma \mathbf{E}$. We have then:

$$\sigma E = \rho v \tag{5.15}$$

and equivalently:

$$\sigma \propto v \propto \tau, \tag{5.16}$$

for eq.(5.12).

- We can lastly write this chain of proportionalities for the resistance R of the conductor:

$$R \propto \varrho \propto \frac{1}{\sigma} \propto \frac{1}{\tau} \propto \frac{1}{\lambda}, \tag{5.17}$$

and taking the first and the last term in the previous chain of proportionalities, we get:

$$R \propto \frac{1}{\lambda}. \tag{5.18}$$

- The last part of this sequence must involve the temperature T of the conductor in such a way to relate it with the quantity λ given by eq.(5.18). Infact, the temperature is related to the thermal oscillations of the crystal lattice and it is possible to think that each positive center of the lattice is an harmonic oscillator of amplitude r. In this way, the propagating wave enecounters obstacles that can be seen as circles of surface πr^2. Each of those circles represents a cross section for the scattering of the elctronic wave: if the temperature increases, then the cross section increases and the mean free path λ decreases, so we can write:

$$\lambda \propto \frac{1}{r^2}. \tag{5.19}$$

- We can now relate the mean free path λ with the temperature T. Recalling the expression of the elastic energy of an oscillator $E_{EL} = \frac{1}{2}kr^2$, if the oscillation has amplitude r, and the expression of the thermal energy $E_T = k_B T$, where k_B is the Boltzmann constant, we obtain by considering that the eleastic energy equals the thermal energy:

$$r^2 \propto T. \tag{5.20}$$

- Taking together eqs.(5.18), (5.19) and (5.20) we get:

$$R \propto T. \tag{5.21}$$

  We obtained the proportionality between the resistance R and the temperature T, that is what is stated by the third Ohm's law.

We have reported the essential line of the path on the microscopic mechanism of the electrical conduction. As we have already noticed, the path needs a consistent back-ground on the wave-like behaviour of matter. Considering that the current ministerial programs contain modern physics as a important part of them, we think that the presented path could be done. Nevertheless, it cannot be simply added to the program of the last year, and instead it is necessary to restructure the program of the last three years in such a way to treat adequately waves and complex numbers and the necessary basis to deal with quantum physics.

In our research on superconductivity we have however developed an educational path even for students that encounter quantum physics and superconductivity only in their last year. In this case the entire microscopic part should be cut, thus avoiding a too discoursive way of presenting it. We stress this concept because in secondary school happens that after the first two years in which physics is treated in detail and quite rigorously, with exercises and problems, during the last year, in dealing with modern physics, all the rigours, the exercises, disappear, leaving room only to discoursive presentations of something that has lost the typical characteristics of physics.

5.4. The Conceptual Change Method as a framework in which we analyze results

Over the past three decades, research has shown that students come to science classes with pre-instructional conceptions and ideas about the phenomena and concepts to be learned that are not in harmony with science views. Furthermore, these conceptions and ideas are firmly held and are often resistant to change [39, 40].

What is conceptual change? In the context in which we are, it can be seen as an idea that has the possibility of transforming science education. When thinking of conceptual change, it is helpful to recognize that the word "change" is used in different ways.

1. Change can means *extinction* of the former state, when a concept and the context in which the former concept in structured, are completely substituted with another one.
2. Change can means *exchange* of one entity with another one, this happens when a second concept becomes more fruitful than the former.
3. Change can means *extension*, and this happens when the context is simply extended without create competition with the pre-existing knowledge, that is when students learn things they didn't know by making connections to what they already know. It is not a problem when their present views can be reconciled with what they learn.

The three different ways in which the word change can be interpreted in the light of the conceptual change, can highlight three corresponding key points on the process of teaching science. We think that it would be interensting to mention them and to relate them with our experimentation on electrical current.

- "*Conceptual change is inclusive or exclusive?*"

Depending on the ideas and conceptions of the student, the conceptual change can be both inclusive or exclusive, infact when one thinks of a student learning ideas that are the goals of a given curriculum, one needs to consider whether in order to achieve a goal the student may have to give up, reject, or demote an idea particularly if it contradicts the goal idea. Such a case would entail conceptual change and there is common agreement in the literature that the process of a student exchanging one idea for another is conceptual change.

But there is another way to think about the conceptual change: to regard the existing knowledge as "capturing" new knowledge. From this perspective, the conceptual change is an inclusive term, because it includes the extension of the pre-exisring knowledge.

As we will report in the next sections about our experimentation on electrical current, we observed quite frequently during the oral interviews, that students experimented directly the dissatisfaction of their previous knowledge in order to solve the problem proposed. In general, there happened frequently two situations:

1. The question proposed was so general that the student started speaking describing in a so approximate way, and using notations and words so improperly, that a more precise second question of the teacher was needed in a very short time. At the point of the precise question, usually posed by the teacher in order to clarify the meaning of a concept that was introduced in the speech by the student, the silence or the immediate admission of complete ignorance, enter in the interview to indicate the student's dissatissfaction.
2. The question corresponds to an exercise proposed in very general terms, without numbers. The exercise is always very simple, but given in no traditional terms to avoid that the students could apply usual strategies that mask their conceptual difficulties or their ideas. In this way we notice that the dialogue opens wide possibilities for teachers and for students to take a few steps in the process of understanding. In front of these exercise, students can say a thing and its opposite in a few minutes and this can be very useful for them, in order to experiment the dissatisfaction.

In our experience it is very unfrequently that during an oral interview a student does not experiment dissatisfaction, and this happens particularly when the sudent is really refractory, for personal reasons. Instead, even when we interviewed very skilled and motivated students, we have used the dissatisfaction as a way to improve awareness, attention and knowledge.

- "*Conceptual change: teaching and/or learning?*"

It is quite common in the school practice the following way of thinking: "If I taught well, my students will have learned what I wanted them to". But often, this way of thinking leads a teacher to another common feeling of dissatisfaction, because students sometimes do not achieve the goal.

There is, on the other hand, an alternative perspective, in which the relationship between teaching and learning is seen in a different light. While teachers may require their students to carry out learning activities and intend that these activities will lead to particular learning outcomes, it is also necessary that learners share these intentions. In this view, then, teaching is not a cause of learning outcomes: it facilitates them. Thus teaching can take place without learning occurring an vice versa. Possible consequences of this are a sharpening of the distinction between teaching and learning and a focusing of attention on learners and what is involved when they achieve intended learning outcomes.

In performing of our experimentation with a high school class we achieved only a part of the intended outcomes. As we have previously mentioned the outcomes where reached mainly during the oral interviews, because in that context it was possible to establish a closer relationship with the student. Infact, the class to which we addressed our proposal was unknown to us, and we had only a few hours to develop our relationship. We finally can conclude about the teaching/learning that:

1. When the intervention involves students for a limited number of hours, it is necessary that the host teacher participates directly to the activities, being active also out of the context of the hours devoted to the experimentation, reviewing the concepts treated during the experimentation and making them alive, integrating them in the curriculum. In our experience, unfortunately, this did not happened, because the host teacher considered his curriculum and our intervention as two different branches, even if in some cases the topics treated were just the same.
2. Generally, students have difficulties in following lectures. We realized it, in particular, during oral interviews. Only in particular cases in which students are strongly motivated the lectures seem a good way to engage them. It is most common instead, that students need to be directly involved in their learning process. We think that more time is needed to devote to the personal relationship with each student or group of students, in order to clarify their way of reasoning, their problems, their ideas and make more personal their understanding. The opportunity for discussion should become more frequent, and even clarifications and the carrying out of particular goals, even in little groups of students.

- *The status of a person's conception*

The interpretation of student responses as driven by alternative conceptions suggests that learning may involve changing a person's conceptions to adding new knowledge to what is already there.

From this point of view, learning involves an interaction between new and existing conceptions, with the outcomes being dependent on the nature of the interaction. To perform the analysis of our outcomes both in oral interviews and in the written tests, we used the three indicators of the conceptual change introduced by Hewson and Hennessey in 1992 [40]:

1. **Intelligibility**. This level is reached if the student knows what the concept means, or, for instance, knows what the first or the second Ohm's law means. The student is then able to describe the concept using own words, or to decide whether a particular law can be appropiate for the description of a certain physical context.
2. **Plausibility**. This level is reached if the student believes that the concept, or the law in our examples, is true. The concept is plausible if it fits well with the pre-existing knowledge.

3. **Fruibility**. This level is reached when the conceptual change is occurred. In this case a student finds the new concept useful in order to solve problems or to better understand things.

When a student reaches the fruibility of a conception, he/she has reached also the plausibility and the intelligibility of the same conception. In the same way, if a conception is plausible, it is also intelligibile. When the new conception fits all the previous three indicators, learning proceeds without difficulty.

In the following we analyze the outcomes of oral interviews and written tests in the light of the conceptual change model resumed by the three indicators of intelligibility, plausibility and fruibility in order to describe students' understanding of electrical conduction, in particular of the phenomenological aspects of conduction described by the Ohm's laws.

5.5. The first step of the experimentation

The first experimentation involved around 73 high school students that participated to the PLS (Scientific degree Plan). PLS is an italian project founded by the ministry of education and it is created to promote collaboration between high school and university in order to stimulate the interest in science. The main work was done in university lab-room, even though students had some optional homework to help them in re-thinking about the laboratory work.

The students involved in the first experimentation attended the 24 hours PLS laboratory on superconductivity of the Physics Department of the University of Milano. The lessons about electrical conduction took about 4 hours. Three months after the lessons we proposed to students a written test, discussed in the following, and subsequently we proposed the same test to a control group of 79 high school students that did not participate to the PLS activities, to compare the results. From the written test we deduced some quantitative results, but we proposed also a certain number of open question in order to get some qualitative results.

Starting from the literature we focused our attention in understanding students' ideas about some of the most common difficulties, before and after our laboratory. Here below we list the principal students' difficulties that we have investigated:

1. Although many students are able to state definitions, often they do not relate the concepts to each other, or do not apply the concepts to a real circuits when the situation is proposed in no traditional terms.
2. Students are not able to state definitions and hence they are not able to distingush of concepts such as: current, voltage, resistance and energy.
3. Most students are convinced that bulbs, wires or resistors use up current, or make it difficult the passage of current in the conductor.

4. Students do not distinguish an ideal generator from a real generator, and very often they see a generator as a source of constant current.

5.5.1. Quantitative results of the first step

1. Related to the first point of the previous section we have analyzed students' results in order to understand their ability in application of basic definitions, or basic laws, in real contexts. In particular we proposed two experimental situations and asked them a description of the results they expected.

5.5.1.1. Plausibility and Fruibility of the first Ohm's law

We propose here below the questions given to students and the analysis we realized grouping their answers in categories, as it is shown in Tab. 5.1.

(A) Which of the following bulbs will light up? In Fig. 5.6 it is reported the explanatory figure of the corresponding question posed in the test.

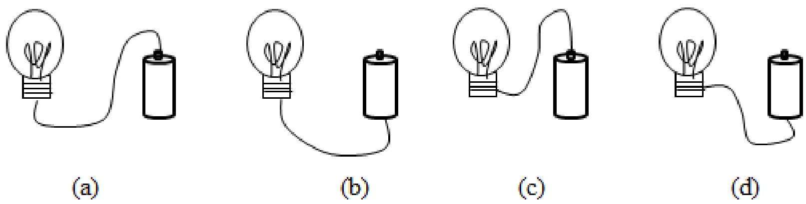


Figure 5.6.: Which of the bulbs will light up?

(B) Which of the following bulbs will light up? In Fig. 5.7 it is reported the explanatory figure of the corresponding question posed in the test.

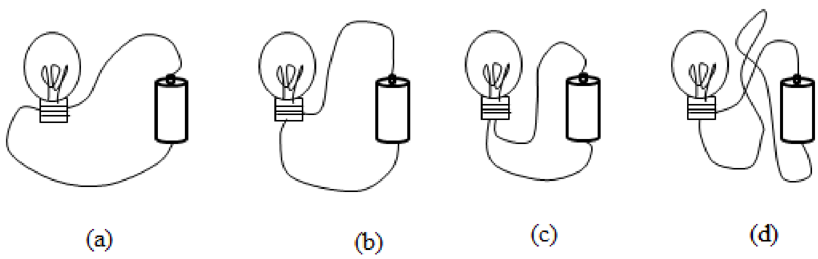


Figure 5.7.: Which of the bulbs will light up?

In appendix A it is reported the full text of the questions, where it was described in detail the functioning of a incandescent bulb. Here below we report the results obtained for students that followed our sequence on electrical conduction (named PLS in Tab. 5.1) and the results obtained from the group of control students (named CONTROL in Tab. 5.1).

Categories	**PLS**	**CONTROL**
Current flows only in a complete circuit	96%	75%
Good comprehension of the first Ohm's law	74%	38%
Contraddictory ideas /Poorly explained ideas	16%	30%
No explanation, hence inconclusive results	8%	33%

Table 5.1.: Comparison of the results obtained by PLS students than CONTROL students. Are students able to apply the first Ohm's law to a real circuit?

We report here, as an example, one of the students' answers that we chategorized as "current can flow in a incomplete circuit".

- "The battery pole that should be connected to the wire, in such a way that the current could flow, is the negative pole, because the charge carriers come out from that pole (negative)"

Data of Tab. 5.1 can be regarded in the light of the ***normalized learning gain*** g that has been defined in order to give a quantitative evaluation of the students' understanding [44].

$$g = \frac{T - C}{100 - C} \cdot 100\%, \tag{5.22}$$

where T is the fraction of students that gave positive results post instruction and C is the fraction of students that gave positive results pre instruction. In our experimentation we did not have the same group of students for the pre instruction and the post instruction, but we gave the same test to two different groups of students and named students pre instruction (C) the CONTROL students, and students post instruction (T) the PLS students. Despite our groups were different, we decided to still use the same Thornton normalized learning gain because our groups were rather numerous and we are sufficiently confident that their starting levels were very similar (they came from similar school etc.).

In Tab. 5.2 we relate the normalized learning gain g with the indicators that are used in the the conceptual change model, that is: intelligibility, plausibility, fruibility for what concerns the students' understanding of the first Ohm's law. The intelligibility of the first Ohm's law was not directly investigated in our written test because in oral interviews during the lessons we already had occasion to observe that students were able to state the first Ohm's law without difficulty.

Categories	Indicators	g
Current flows only in complete circuits	plausibility	84%
Good comprehension of the first Ohm's law	fruibility	58%

Table 5.2.: Students' understanding of the first Ohm's law in terms of the normalized learning gain g and the conceptual change method, that is represented by the column of the indicators.

Tab. 5.2 shows an important gain factor in plausibility and fruibility of the first Ohm's law. This is a good result that appears quite clearly in our analysis and it appears that around the 60% of students become able to use the first Ohm's law to solve the problems posed or to better understand things. But we can analyze a second problem that we gave to students in the written test, a little bit less simple represented in Fig. 5.8.

(C) When you close the switch, what can you say about the current in the copper bar, both in case (a) and (b)?

In Fig. 5.8 it is reported the explanatory figure of the corresponding question posed to students in the test.

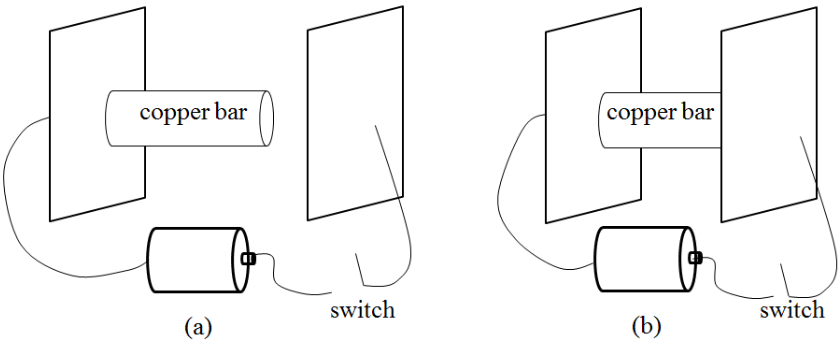


Figure 5.8.: When you close the switch, what can you say about the current in the copper bar, both in case (a) and case (b)?

Students answered this question with a lot of difficulties. The maximum score for the question was 8 (as for all the other questions of the proposed test, half for the content and half for the comments) and we report in Tab. 5.3 the mean score for the PLS students and for the control students. More interesting than the score can be some student comment, reported here below.

Some students' sentences:

1. “Charges enter from one plate and leave, crossing the other, but they don't cross the condenser: the bar is not crossed by the current.”

	PLS	CONTROL
Scores of students that answered	3.6 / 8	2.8 / 8
Percentages of students that answered	89%	48%

Table 5.3.: Application of the first Ohm's law to a real circuit.

2. "The current does not reach the bar."
3. "Charges flow to fill the potential difference, so they move towards the plate at highest potential."
4. "There is the equivalent capacity, and the current flows in the circuit overcoming the plates."

As Tab. 5.3 shows, only a minority of the control students answered to this question. Moreover the explanations of the answers, that were always requested, are almost never given by the two groups of students. The contents of the responses PLS students are only slightly better than those of the control group students, but a very marked difference can be seen in Tab. 5.3 for the percentage of students that tried to answer to this question. We consider this as a real positive consequence of the experimentation, that could have helped students in getting into the game more easily, making them more self-confident.

Results found so far make us to think that while the intelligibility and the plausibility of the first Ohm's law has been sensibly increased through our intervention in PLS laboratory, the fruibility has not been substantially modified, as Tab. 5.3 shows. Instead, it has been modified a psicological condition for the subsequent conceptual change of the first Ohm's law: the students' self-confidence.

5.5.1.2. Intelligibility, plausibility and fruibility of the second Ohm's law

We discuss first an exercise of the questionnaire, that was given in order to test student's plausibility of the second Ohm's law. Though very simple, it consists in a direct application of the second Ohm's law in a concrete case:

(A) The same bar is connected to the same type of battery, but in two different ways, as figures (a) and (b) show. Which is the relation between the two resistances?

In Fig. 5.9 it is reported the explanatory figure of the corresponding question of the test given to students.

We report in the following Tab. 5.4 the results obtained by the two groups of the experimentation, together with the learning gain factor g as in the previous case.

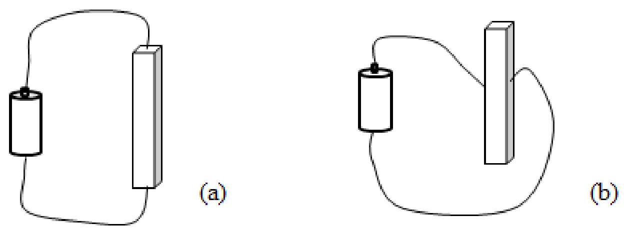


Figure 5.9.: The same bar is connected to the same type of battery, but in two different ways, as figures (a) and (b) show. Which is the relation between the two resistances?

We will consider that the intelligibility of the second law has been reached when students are able to correctly state the law, and we will consider the plausibility of the second law when students are able to correctly apply the law to solve the exercise proposed. Moreover, we have considered, in our analysis of the conceptual change regarding the second Ohm's law, also the comments that students have written in their questionnaires. As Tab. 5.4 shows, the explanations, or the comments to the given responses, are almost absent for the control students and almost present in PLS students. It appears that the students' consciousness of PLS students is three times that of the control students.

Categories	**Indicators**	**PLS**	**CNTL**	**g**
Knowledge of the second law	Intelligibility	86%	65%	60%
Correct application of the second law	Plausibility	60%	10%	55%
Good explanation of the solution		32%	10%	25%

Table 5.4.: Intelligibility and plausibility of the second Ohm's law.

The following question proposed to students was designed to investigate the fruibility of the second Ohm's law. We remember that the fruibility of the second Ohm's law is tightly related with the concept of resistors in series and in parallel, and for this reason, in our analysis, we considered this question twice, the first time in order to investigate the fruibility of the law at a basic level, and the second time in order to investigate students' understanding of the concepts of series and parallel, taking together this question with another, more complex, question.

(B) In the case (a) an ammeter and a hundread meter long insulated copper wire are in series. In the case (b) the situation is similar, but the copper wires are four, each of them identical to the first one in (a). If the ammeter in (a) reads a current intensity i_A, what is the current intensity i_B that the ammeter in (b) will read?

We report in Fig. 5.10 the explanatory figure of the corresponding question posed to the students in the test.

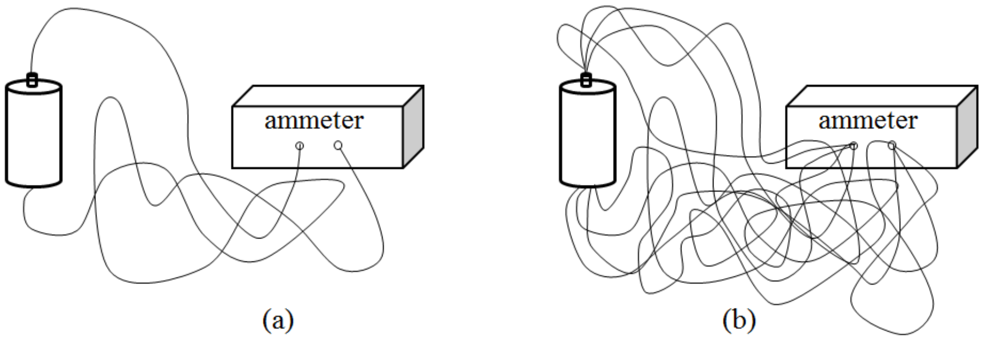


Figure 5.10.: In the case (a) an ammeter and a hundread meter long insulated copper wire are in series. In the case (b) the situation is similar, but the copper wires are four, each of them identical to the first one in (a). If the ammeter in (a) reads a current intensity i_A, what is the current intensity i_B that the ammeter in (b) will read?

Responses to this question are resumed in Tab. 5.5. There are two main considerations that arises from our analysis. First of all, few students have dealt with this question, probably because it was not friendly and it was proposed in terms that are very far away from the traditional ones. Although this does not appear directly in Tab. 5.5, all those students that decided to deal with this question, got the maximum score in the test, or almost the maximum. This circumstance makes us to believe that one big problem for students could have been the lack of time we could devote to this topics. Hence, it seems probable that only the most skilled and motivate students have reached this result.

Categories	Indicator	PLS	CNTL
Good application of the law	Fruibility	18%	27%
Good explanations	Fruibility	18%	27%
Students that answered		18%	27%

Table 5.5.: Fruibility of the second Ohm's law.

As we have just mentioned, this question has been analyzed, also together with another one, in order to better investigate students' understanding of the second

Ohm's law, in particular in relation with the concept of series and parallel circuits. Here below we report the second question and the results of the analysis.

(C) The circuit is supplied by a potential difference of 3V. The three parts of the circuit are indicated by the numbers 1, 2 and 3. Each part has a resistance of 1ohm. In case (b) the wire 2 is splitted in two identical parts, named wire 4 and wire 5.

What is the current flowing in the wire 4 of the circuit (b)?

In (c) the wire 5 is removed. What is now the current flowing in the wire 4 of the circuit?

In Fig. 5.11 is reported the explanatory figure corresponding to the question proposed to the students in the test.

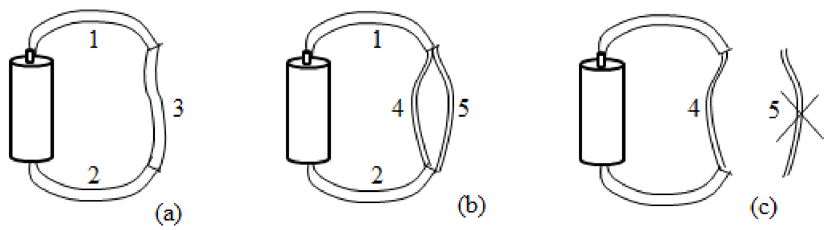


Figure 5.11.: What is the current flowing in the wire 4 of the circuit (b)? In (c) the wire 5 is removed. What is now the current flowing in the wire 4 of the circuit?

The two questions answered by students highlight some difficulties. This can be explicitly seen in Tab. 5.6, through the reported percentage of students who answered to this question, that was so confused that it wasn't possible to deduce anything. Answers of this kind concerns more markedly the control group of students. Nevertheless, the fruibility of the second Ohm's law does not differ substantially for the two groups of students.

Categories	Indicators	PLS	CNTL
Students are able to answer first question	Fruibility	22%	22%
Students are able to answer second question	Fruibility	9%	5%
Series and parallel are recognized	Plausibility	78%	78%
Too confused data to get information		12%	38%

Table 5.6.: Fruibility of the second Ohm's law and its application in solving circuits proposed in no traditional terms.

We notice that in Tab. 5.6 the learning gain factor g does not appear, because it would not be significant. Infact, for what concerns the frubility of the second Ohm's law the scores obtained from the two groups of students are very similar, and quite low too. The only difference that can be appreciated regards the presence of the too confused responses in order to get information. For this category we see again that the PLS students are significantly more able to face the questions and to explain their thoughts.

In addition to these data, we think that it could be interesting to make a comparison between the two groups in other categories related to the second Ohm's law; categories that we have not yet analyzed. In the following Tab. 5.7 we report some of them. Infact, while it is quite common that students are able to state the second Ohm's law, when this law is directly asked to them, it is not just so common that students think to the second Ohm's law when appropriate in order to solve problems. Instead of the second law, they usually use other intuitive considerations, mainly based on common sense; considerations that prevent them to get the right solution of the problem. These students' considerations are just those that a teacher should retain useful in order to improve his/her way of teaching.

Categories	**PLS**	**CNTL**
Battery is a source of constant current	6%	14%
The wire uses up current, or prevent its flowing	44%	32%
Lack of a model to which students may refer	13%	25%

Table 5.7.: Analysis of some categories related with the comprehension of the second Ohm's law.

These categories have been investigated throughout the students' responses of the questions proposed, questions that we considered related with the second Ohm's law. We have got the information reported in Tab. 5.7 expecially in students' comments. As it is reasonable, the considerations reported in Tab. 5.7 have been found even as comments of quite correct solutions: this is not so strange, because it is very frequently that different ideas coexists in students' mind if the conceptual change is not completely or significantly occurred. For example, in Tab. 5.7 we can see that the idea of the wire, or of the electrical conductor, as of an object that uses up the current, is very ingrained, and persists even after the experimentation [45]. Many of the students that have these ideas, on the other hand, are in some case able to solve the exercises proposed in the test, but in oral interviews this contraddiction frequently emerges and could be an occasion for students to improve his/her understanding. By a written test we collect a lot of information but we do not have the possibility of following the way of reasoning of each student in order to understand in a precise way his/her ideas. Even if the lessons were completely open to discussions and many interactions took place among us and the students, with many questions answered and posed, the possibility to interact with our students one by one for sufficiently long time to support each of them in his/her learning process was missing.

We tried to overcome this lack of deep interaction with students during more structured, out of classroom, oral interviews, that took place in the last experimentation, described in the next section. In the first step of the experimentation instead, we focused more on the quantitative analysis of the results, leaving less time for interviews, that are however reported in the next subsection.

From Tab. 5.7 we can focused on two main difficulties, that have to be remembered in the design of the second step of our research:

1. A conductor permits the current flow and does not use up current.
2. The battery in a circuit element that has to be treated in a much more detailed way.

Before the most meaningful results of our oral interviews, we conclude our quantitative analysis on electrical conduction with an overview on students' ideas about the microscopic mechanism of conduction. Since we did not dealt with this topic in our experimentation, the students' responses are not grouped into two, but they are taken all together. Responses were not significantly different between the two groups, and this convinced us that in this analysis a single group containing all the students of the two initial ones could provide an interesting statistics of students' pre-conceptions on the microscopic aspects of the electrical conduction.

We proposed multiple choice questions to have a pictures of students' ideas, we decided to investigate these students' ideas with multiple choice questions rather than with open questions because we had a quite large number of students (152) to analyze, and because we did not treat these topics in our lessons, as we have already said. The complete texts of the questions are reported in appendix A. We report here below the four main kind of answeres obtained.

- Electrical conduction is a threshold effect: it is needed a minimum voltage (68%)
- Electric charges that are responsible for the current are on the surface of the wire, because the electric field inside the conductor must be zero. (60%)
- The current increases when the temperature of the wire rises up, because the velocity of the charges that carries current increases for thermal agitation. (18%)
- The conduction electrons in a potential difference accelerates.

5.5.2. Qualitative results of the first step

As we previously mentioned, we have briefly discussed with students (two or three at a time) asking them to describe their ideas expecially regarding the microscopic mechanism of the electrical conduction. The question we posed to students to start the dialogue has been always the same : "*How do you imagine that the electrical conduction could take place in a metal?*"

In the following we report some exerpts taken from our dialogues. Students are indicated by the letter "S", while the teacher (one of us from PERC Unimi) is indicated by the letter "T".

Exerpt of the first interview

S1: Electrons collide with the walls of the wire.

S2: Electrons do not have a regular motion... yes, because it wouldn't be possible as they are always colliding with the walls...

S3: I imagine that there are many dots that flow under the influence of a force...

T: *If dots are under the influence of a force, they speed up, and if the dots speed up, then the current should rise!*

S3: Yes, but they collide with the end of the conductor!

T: *What do you mean by the expression "end of the conductor"?*

S3: ... [He does not answer...]

Exerpt of the second interview

S4: I think instead, that the current leaves the battery and with the speed of light it is distributed almost instantly around the wire.

T: *The speed of light?*

S4: There is nothing that goes faster than light, isn't it?

S5: No! Current doesn't go with the speed of light!

Exerpt of the third interview

S6: Current is a line made up by something, that I don't know what it is...

T: *Bananas?*

S6: I don't know...

Exerpt of the fourth interview

S7: I think that current is a wave of energy, as the Dragonball wave... do you know Dragonball? When that wave passes, it activates the parts of the wire in succession... but the circuit must be complete, otherwise the Dragonball wave does not start!

Exerpt of the fifth interview

S8: I think that electrons go from "-" to "+" and in order to do this they jump from one atom to the other...

T: *But if the electrons belong to an atom, they need a minimum energy to leave the atom... so why, experimentally, we don't need this threshold voltage?*

S8: I don't know, perhaps in a metal the conduction electrons don't belong to a single atom!

5.5.3. Conclusions about the first step

At the end of the first experimentation we had quite clear that students needed more time to understand the electrical conduction even only in its phenomenological aspects. The low scores reached by students for the questions related to the fruibility of the second Ohm's law did not leave many doubts. Moreover, since our aim was to provide students the minimum background in order to deal with superconductivity, we decided to cut out the microscopic aspects of electrical conduction and to potentiate the phenomenological description of the conduction. It took place an intermediate experimentation during which we gave more room to lab experience with electrical circuits, but we want to describe now the last experimentation, because it gave us many information both quantitative and qualitative and made us to retain that an important change was needed in our path, to significantly improve our results. The final part of this chapter describes the last experimentation on the electrical current, while the next chapter describes the lines of the new path on electrical current that we developed changing fairly radically the approach. Here we describe the intermediate experimentation with the aim of showing the motivations that made us to believe that better results could be achieved by means of important and stimulating changes.

5.6. The second step of the experimentation

The second experimentation took place with an high school class of 21 students, attending their last years. The experimentation presented here on electrical conduction, is a part of a more extended experimentation on superconductivity. The total number of hours devoted to the electrical conduction was about 5 hours, including 1 hour in lab. The hours to collect quantitative data or to perform oral interviews are obviously in addition to those indicated before.

The schema of the five hours we did with students is described below:

- Integral formulation of the Ohm's law: we describe to students the Ohm's laws in order to explain their physical meaning.

- Lab experience during which students measure the internal resistance of a battery.
- Ideal and real batteries, what is the difference? An attempt of dispel students' difficulties in their vision of the battery as a source of constant current.
- Local formulation of the first and the second Ohm's law.
- The concepts of series and parallel with the local formulation of the Ohm's laws.

The way in which the topics have been treated is reported in the previous sections. The only difference pertains the lab experience that has been introduced in this last step of the experimentation. Although it was a very simple experiment, it has been appreciated a lot by students, who worked to their circuit with interest and remembered this experience during their oral interview, as a very useful moment of the path. Below, we report the scheme of the experiment proposed to students, and it will be surprisingly for the reader knowing that a similar experience has been so important for students.

There is a last consideration that we find interesting. During oral interviews we listen and dialogued with many students, many of whom spoke very freely, and baldly expressed their alienation to physics. Nevertheless in almost all these cases, students remembered the lab experience as a happy moment. It is not possible to say that through lab experiences students can understand physics better, but it seems really possible that lab experiences bring students close to the physics, as it obviously should be.

The lab experience proposed: measure of the internal resistance of a battery.

The time available was 1 hour, so we give to students some indications in order to allow them to conclude their measures. The students were divided into groups of four students each, and each group had access to the same material: a battery, wires, a bulb and a tester. In Fig. 5.12 we show the material used by the students.

5.6.1. Quantitative results of the second step

Data analysis of this step of the experimentation is performed in a slightly different way with respect the previous one. We compared the results of the pre-experimentation with respect of the post-experimentation. In this case we proposed two different tests reported in appendix B and C to the same group of students.

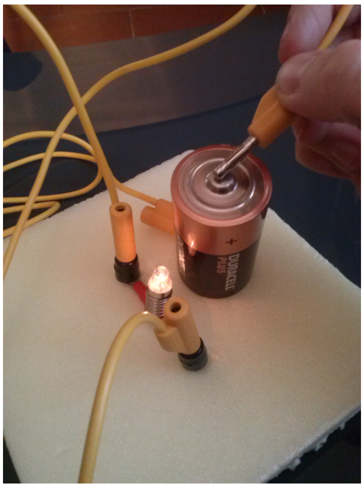


Figure 5.12.: Experimental setup used by the students.

(A) Plausibility and intelligibility of the first Ohm's law: only in a complete circuit the current can flow.

The proposed question is the same given in the test of the first experimentation, see Fig. 5.7. The results are given in Tab. 5.8.

Categorization	PRE	POST	g
Only in a complete circuit the current can flow	70%	91%	70%

Table 5.8.: Comparison between the results pre and post experimentation, about the comprehension of the first Ohm's law.

(B) Intelligibility and plausibility of the second Ohm's law: evaluation of the resistance of a conductor depending on its shape and on the way it is connected to the battery.

As in the first experimentation, we have evaluated the intelligibility and the plausibility of the second Ohm's law with the students' responses of the same question proposed in the first test: "The same bar is connected to the same type of battery, but in two different ways, as figures (a) and (b) show, see Fig. 5.9. Which is the relation between the two resistances?"

We describe our results in the following Tab. 5.9 for what concerns the students' ability in application of the second Ohm's law and in the following Tab. 5.10 for what concerns other categories related with the second Ohm's law.

Categorization	PRE	POST	g
Good application of the second Ohm's law	10%	50%	44%

Table 5.9.: Comparison between the results pre and post experimentation, about the comprehension of the second Ohm's law.

Categorization	PRE	POST	g
The longer the wire, the slower the current	33%	17%	48%
The resistance depends on the particular material	10%	5%	50%
Confused/inconsistent answers	39%	14%	64%

Table 5.10.: Results about students' ideas related to the second Ohm's law.

(C) Fruibility of the second Ohm's law

We posed another question to evaluate the fruibility of the second Ohm's law. The question is identical to that proposed in the first test: "In the case (a) an ammeter and a hundread meter long insulated copper wire are in series. In the case (b) the situation is similar, but the copper wires are four and identical to the first one in (a). If the ammeter in (a) reads a current intensity i_A, what is the current intensity i_B that the ammeter in (b) will read?" (for the drawings the reader can see Fig. 5.10). Results to this question are discussed in Tab. 5.11.

Categories	POST
$i_B = i_A$, the current i_A is splitted in 4 parts, one part in each wire	44%
$i_B = 1/4\, i_A$, the total length of the wire is quadrupled	28%
$i_B = 4\, i_A$, the correct answer	22%

Table 5.11.: Fruibility of the second Ohm's law.

Tab. 5.11 shows quite clearly that the fruibility of the second Ohm's law has not yet been achieved by the majority of the students. And it is not so surprisingly, since we found a similar result in the first experimentation. A curious fact is the equal percentage of students that seem to have reached the results of fruibility of the second Ohm's law, that is again the 22%.

Moreover, from Tab. 5.11 we can see that the 28% of students is again not able in distinguishing between a circuit in series and in parallel, if it is proposed in no traditional terms. This is an outcome that interrogates us and stimulates us to understand how it happens. The students of the experimentation have already dealt with the Ohm's laws and the solution of the electrical circuits with their curricolar teacher even before our intervention. Nevertheless, when we dialogued with students during our lessons we often had the feeling that they were very far from the comprehension of the electrical conduction, and they answered as they saw those topics for the first time. But it was clearly not true, because they had already actually done at school a written test about it!

It is mostly for this reason that at the end of this last experimentation we decided to change drastically our approach. This new, last step of our design based research is developed in the next chapter, where the reasons of our change of course are diffusely explained.

For completeness, we report below also the results of the last part of the proposed questionnaire, in order to have a picture of the effects of the experimentation on the student's way of thinking. As we have already said, we gave to students two similar questionnaires before and after the experimentation. The question that we report here below has been proposed only in different terms in the two cases: before the experimentation we proposed the question in traditional terms as it is shown in Fig. 5.13, while in the post-test we proposed the same question in a no traditional way, as we did in the first experimentation, and this has already been reported in Fig. 5.11.

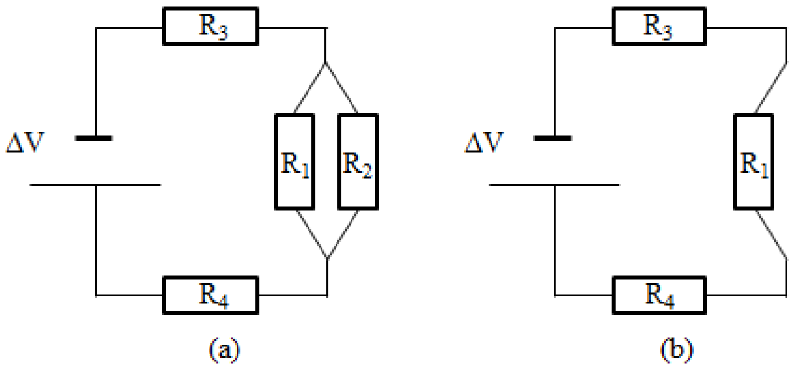


Figure 5.13.: Question on electrical circuits and second Ohm's law in the pre-test

We report the analysis of this question in Tab. 5.12, where we describe students' ideas in the case (a), when the resistor is not yet removed from the parallel, and in Tab. 5.13, when we describe students' responses in the last part of the problem, when the resistor is removed, this corresponds to the case (b) in Fig. 5.13 and to the case (c) in Fig. 5.11. Each of the categories reported in the tables is described for what concerns both the pre-test and the post-test and the learning gain g is indicated in the table only if the corresponding category represent a positive goal to be achieved by the students.

In Tab. 5.12 are compared the most diffuse students' responses of the pre-test and the post-test for what concerns the first part of the question, that is an analysis

Categories	**PRE**	**POST**	g
The current is halfed when it reaches the parallel	24%	43%	25%
The resistance of the parallel is twice than the single resistor	38%	23%	
Confused or absent responses	33%	38%	

Table 5.12.: Frequent categories of the part (a) of the problem proposed in Fig. 5.13.

of the circuit with resistors in series and parallel. The category of the "correct answer" does not appear in the table, because no student completed this question in such a way to achieve this result. In the first category, infact, are contained those students who have intuitively divided into two parts the current flowing in the parallel, without specifying nothing about the resistors: none of them for example has written something like "$i_1 = \Delta V/R_1$ and $i_2 = \Delta V/R_2$", that is nothing that could make us thinking that the students were referring to a precise theory.

Categories	**PRE**	**POST**	g
The current flowing through R_1 remains unchanged	38%	28%	
The current flowing through $R_1$is double	38%	28%	
Confused or absent responses	38%	33%	
Correct answer	0%	14%	

Table 5.13.: Frequent categories of the part (b) or (c) of the problem proposed respecively in Fig. 5.13 and Fig. 5.11

The categories reported in Tab. 5.13 are instead referred to the students' responses of the last part of the question, both in the pre-test and the post-test. In this table we can see that a minority of the students has correctly answered to the post-test. As for the first part of the question the percentages of the confused responses is quite high, just for what concerns the first two categories in Tab. 5.13. From this table we can see that the main ideas of students about this electrical circuit are:

1. If the resistor R_2 is removed from the parallel, nothing changes: the current that flowed before, through the resistor R_1, will flow after, when the resistor R_2 is cut out. An interpretation that we give in relation to this category is that students uses integral quantity to describe the circuit, such as the potential difference ΔV and the current I, but they do not think to the circuit as a system that changes in each of its parts, if something is changed in a certain point. Students' thinking is *local*, whereas students' physical tools are *integral*.
2. If the resistor R_2 is removed from the parallel, the current flowing in the resistor R_1 left, doubles. The interpretation we give in relation to this category is that students' thinking is far from the guide of the reference theory: their

reasoning is intuitive and related to a sort of conservation principle for the electrical current flowing to a constant rate from the batttery, indipendently from the resistance of the circuit.

While in the point 1. there appears a certain students' attempt in following the theory, in the point 2. the theory as a guide to solve the problem is completely neglected. From Tab. 5.13 we can see that a slight improvement in students' understanding seems to be present, but in our opinion it is yet too small.

5.6.2. Qualitative results of the first step

Here we report exerps of the oral interviews that we realized with students.

5.6.2.1. Pictures of electrical conduction.

These dialogues occurred in different moments of the path, before, during and after the test. They do not differ substantially from each other depending on the moments. The sequence on electrical conduction was quite short, and it could be for this reason that students may have had some difficulties in elaborate the new concept explained, and in their differentiation from the old concepts explained by their curricular teacher. We proposed to all the students the same question in order to start the dialogue:

T: What is your picture of the electrical conduction?

First interview

S1: The first thing I think of is a copper wire...

T: *Neutral or charged?*

S1: I think it is neutral because inside it nothing is happening.

T: *What happens if the wire is connected to a battery? Does it remain neutral?*

S1: I don't know... the electrons... as the battery keeps a constant potential difference, the electrons moves in the direction indicated by the current.

T: *Do the electrons know the direction of the current?*

S1: The electrons move in the opposite direction indicated by the current.

T: *How can electrons know the direction of the current?*

Second interview

S2: Conduction suggests me the idea of transport and movement, so I relate it to the transport of electrons, and hence to the concept of energy transmitted from an electron to another.

T: *Do the electrons move?*

S2: Yes! But the very high velocity of the conduction is not the velocity of the electrons.

T: *What gives the conduction velocity?*

S2: The interaction.

T: *The interaction between what?*

S2: Between the conductor and... please give me a hint...

S3: Well, the wire contains an energy that is pushed to a certain direction by a force which I don't know what it is.

Third interview

S4: I imagine electrons as particles flowing inside the wire. There are forces acting on the particles, but I can't understand what is the potential difference even if I know that's a force pushing the electrons.

5.6.2.2. The concept of potential difference and the fruibility of the first Ohm's law

In order to investigate the first Ohm's law we proposed to students always the same exercise represented in Fig. 5.14:

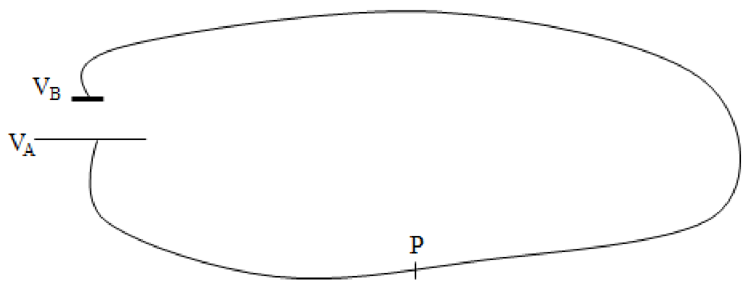


Figure 5.14.: Scheme of the proposed circuit in order to discuss the fruibility of the first Ohm's law.

"Given the draw of Fig. 5.14, what is the relation between the potential difference ΔV_{AB} and the potential difference ΔV_{AP}, where P is a generic point along the circuit proposed? We are interested only to a qualitative reasoning, hence have not given the intensities of the drawn quantities". (As before, S indicates a student while T one of us that was experimenting the path).

First interview

S1: We can use the Ohm's laws!

T: *Yes! Let's try!*

She is a very perky student at the beginning of the conversation, but after a few seconds...

S1: I don't want to hazard...

[...]

T: *You have just said that if there is a potential difference in the circuit, then there is an electrical field througout the wire. Now, tell me: is this electric field uniform in all the wire?*

S1: Oh... yes! I only feel it istinctively!

S2: I'm not so sure that the field is uniform along the wire...

S1: ... and which parameter would change the electric field?

S2: Well, if we have to evaluate the difference between ΔV_{AB} and ΔV_{AP} this will mean something... As the electric charge changes from pole A to pole B, I believe that the electric field cannot be uniform.

T: *Is the wire charged?*

S2: Yes!

T: *Is the sign positive or negative?*

She laughs, but she is not able to provide any answer to the teacher's question. At this point we need a moment to explain why the electric wire is neutral and why the electric field does not vary along the wire. We are not reporting these considerations here.

When the teacher repeats the question proposed at the beginning of the interview, the discussion starts again:

S1: There are no resistances, that is we have not placed a resistor in P and hence the potential difference is the same in both cases.

She reflects silently, then she says:

S1: We can also think that the resistance is generated point by point, that is, the farther one moves away from the electrode A, the more one feels the opposition of the wire.

This is the first step that helps the student S1 to solve the problem, but only after the teacher has recalled her the first Ohm's law.

Second interview

S1: there are no resistances, even if we have a current.

S2: It is subjected to a force.

T: *What?*

S2: At the electrode A a current is present because that point is subjected to a force.

T: *I don't understand... what is subjected to the force?*

S2: The electrons in the electric field.

We add a new point Q to the drawing of and ask:

T: *Tell me, is the force on the electrons in P the same or different from the one in Q?*

S2: I have no tool to know it...

T: *And you Matteo?*

S1: In my opinion it's the same.

S2: umh...

These interviews that we have done to investigate the fruibility of the first Ohm's law make us to believe that the students' reasoning is hindered when the exercise is proposed in no traditional terms. In all the cases listened during the oral interviews, the students have never recognized the resistance in the proposed circuit, despite they have already faced the Ohm's laws and the electrical circuits in their curricolar hours, with their curricular teacher. Nevetheless, once the possibility of using the Ohm's law is suggested by the teacher, then they are in a sense forced in recognizing the resistance in the proposed circuit and become able to solve the problem. We can not describe this students' understanding as belonging to the level of fruibility of the first Ohm's law, but only to a the level of plausibility.

5.6.2.3. Intelligibility, plausibility and fruibility of the second Ohm's law

An exercise is proposed to students always in the same terms, represented in Fig. 5.15:

"Imagine a wire with a certain section S that is carrying a current I, as indicated in Fig. 5.15. At a certain moment the section of the wire instantly decreases, while all the other experimental parameters remain unchanged. What happens to the current?"

First interview

S1: The current in the second configuration decreases with respect to the first one.

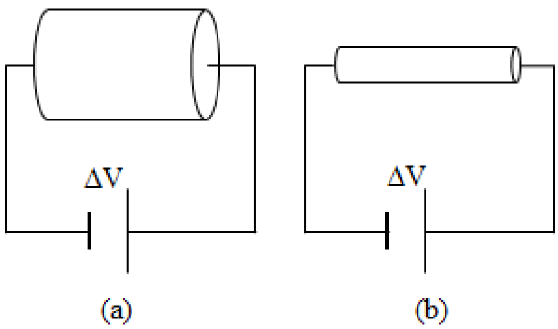


Figure 5.15.: Exercise proposed to students during oral interviews: If the section of the conductor decreases, what happens to the electrical current flowing if all the other experimental parameters remain unchanged?

T: *Can you explain me your thought?*

S1: ... [She keeps silent]

Few seconds later:

S1: If there were a resistance, but... there isn't any... isn't there?

T: *There is no resistance, is that what you say?*

S1: The resistance is an obstacle in the circuit, it is already present in a precise point of the circuit, it is not generated gradually while moving along the circuit...

The teacher, addressing the other student:

T: *And you? What do you think?*

At that point, Dalila, the first student, cuts in to precise her thought:

S1: In order to change the resistance you have to change the resistor placed in the circuit with a different one... (she draws the zig-zag by which a resistor is conventionally indicated in electrical circuits).

T: *Is there a resistor in this circuit?*

S1: no... no, I don't see any ... (and indicates the zig-zag)...

She returns silent.

S2: In my opinion a resistance is a "utilizer" of the circuit... hence, since there is no "utilizer" here, then... it's useless... there is nothing to do with this circuit!

S2: Anyway, in my opinion the velocity of the conduction electrons increases similarly to the water velocity inside a reed: if the section decreases, then the velocity increases.

T: *You guys are saying two opposite things: Dalila (S1) says that the current decreases and Nicolò (S2) says that the current increases. We have to sort this problem out!*

S1: In the circuit the current is unchanged, because the battery is unchanged, but only the wire connected is changed.

S2: I agree.

S1: Well, it must be so... othewise, where does the energy go? That is, the battery provides a certain quantity of energy and if the section decreases...

At this point, they are rather confused. They keep saying a thing and immediately after its opposite, without realising it. The fruibility of the second Ohm's law is absent. Let's understand if they have got the plausibility of the law.

T: *Ok, let us recall the second Ohm's law.*

S1: I've never understood physics... so... the resistivity is related to the particular material...ok...

Dalila returns silent, although she loves speaking, and... very freely... They look at the drawings, for a certain time. Dalila notices that in the second Ohm's law it is contained the letter S. She immediately relates to the section of the wire, therefore she argues that:

S1: ...a resistance is present in the circuit!

T: *Where?*

S1: Somewhere, in the circuit...

T: *Do you think that it could be possible that the resistance is represented by the cylinder itself?*

She is very surprised... happy I dare say...

S1: Do you mean that the conductor itself is a resistor? That the cylinder is a resistor?

After these considerations Dalila uses the second Ohm's law to solve the exercise.

Second interview

S1: In the second cylinder flows the current $I/2$ if in the first one flows the current I.

T: *What is the reason of the factor* 2*?*

She keeps silent.

T: *Do you think that the section of the second cylinder is half the first one?*

S1: No...

T: *If I reduce the section more, will there the current* $I/2$ *flow again?*

S1: Oh... I don't know... I think... no?

The second student intervenes:

S2: I think that the current decreases, but I don't know how much, even if the length does not vary.

T: *What happens to the current if the length increases?*

S2: Most probably, following the same reasoning, the current increases...

T: *Why? What is your reasoning? It's not so clear to me...*

S2: Oh, I don't know... I don't understand physics! ... Only... it seems to me that an increase of any parameter of the wire gives a decrease in the current...

[...]

T: *Remember the Ohm's laws.*

S2: $\Delta V = RI$.

T: *Is this law useful in this case?*

S2: No.

T: *Why not?*

S2: Because there is no resistance in the circuit.

S1: I agree.

T: *So, and the second Ohm's law?*

S1: It is usless, because there is no resistance.

S2: But in the formula there are L, S... yes, I think this law may be used.

The two students analyse the experimental situation in the light of the first two Ohm's laws, then, they get the correct solution of the problem and without further interventions of the teacher they realise their own mistakes.

In this second dialogue it has been briefly discussed the relation between the resistance and the length of the conductor, represented in Fig. 5.16 and not only the relation between the resistance and the section of the wire.

We have simply followed the flow of the discussion, and we noticed that the intuitive way of reasoning by which the only physical law is that of the proportionality between two physical quantities is somewhat diffuse: "*the more the length, the more the resistance*" (and this is true!) but we found equivalently "*the more the section, the more the resistance*" (and this is not true!). In the written tests that we proposed, infact, we found almost frequently that students intuitively relate the resistance with:

- The length of the material;
- The quantity (in a not well defined way) of the matter through which the current flows.

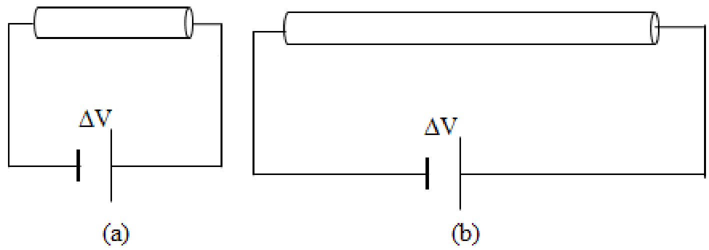


Figure 5.16.: Exercise proposed to students during oral interviews: If the length of the conductor increases, what happens to the electrical current flowing if all the other experimental parameters remain unchanged?

In this work we reported only some exerpts of our oral interviews. Besides the very useful considerations on students' ideas that arise clearly in a dialogue rather than reading a written test, we would like to emphasize an important outcome of the oral interviews. When students came to address their oral interview, in many cases they appeared almost disoriented and insecure. They kept silent for long time and they needed to be helped very frequently to build even simple sentences, or concepts.

During the time of the oral interview, it always happened that sudents at a certain point realized for the first time what they heard in classroom some days ago, or simply they understood the meaning of sentences or formula that they have already seen previously many times.

Another common situation that we encountered in our oral interviews was the student dissatisfaction when he/she was not able to solve an exercise apparently harmless.

These two circumstances are very important in order to improve the understanding of a physical concept. Infact, whereas at the beginning of the interview, students typically say things as "I've never known anything about physics", during the interview usually say things as "Oh, now I begin to understand what we were saying in classroom!". In our opinion this is a first, needed step in the process of knowledge, but this step needs a time to be accomplished, time that the teacher should devote to students, one by one, or for groups of students that work together in solving problems.

5.6.3. Conclusions about the second step

In this second step we have deepen our qualitative analysis, and we have obtained more information about students' understanding. In fact, whereas the written tests can provide information about a high number of students, the oral interviews can provide a very accurate understanding of the students' ideas, even if the number of interviews is not very high, 14 in our second step of the experimentation.

As we have reported in these previous sections of this chapter, students' understanding is not significantly improved from the first step to the second for what concerns the written tests. From the first step to the second, infact, we have only made some changes to the first step, leaving almost completely its structure.

In conclusion, we felt the need to change radically our educational path on electrical conduction, in order to try to radically change the student's understanding. Our last attempt in the development of the new proposal is the content of the next chapter.

6. A path on electrical conduction

6.1. Overview

In the previous chapter some common students' difficulties on electrical conduction have been discussed, as a result of an experimentation developed in three years work. The aim of this chapter is therefore the proposal of an educational path on electrical current, that takes into account the shown students' difficulties and that can also provide a basis for the subsequent developing of the super-conduction. So far, we have only outlined the essential contents of superconductivity, to present the problem of how to develop a path on this complex topic. Referring to what we have said in chapter 3 we argue that the magnetic vector potential is the mathematical tool and the physical quantity that is suited for treating the problem of superconductivity at high school level from a phenomenological point of view.

But, the vector potential can be introduced to students if the entire path on electromagnetism is revised so to permit that the vector potential can be done and can be helpful for understanding electromagnetism, instead of making its treatment even more heavy. In our attempt to describe some important topics needed to have a coherent picture of superconductivity for high school students, the electrical current plays an important role. In fact, on the one hand the relation between the current and the electric field is analogue to that between the super-current and the magnetic vector potential, and on the other hand a description of the electrical current in terms of the electric field allows you treat the electrical current as closely linked to electrostatics, rather than an argument poorly connected with other parts of electromagnetism and mostly addressed to the electrical circuits. As we shall see, in the path that we present in this chapter, the electrical circuits are thought mainly as a way to become familiar with the electric field, to become familiar with mathematical tools as the circulation of a vector field and hence a way to become familiar with electromagnetism in order to prepare students to face superconductivity in a similar way.

Although we have investigated students' ideas and difficulties about the microscopic mechanism of electrical conduction, and we have also developed a path on the microscopic mechanism of conduction, we prefer to do not deal in detail with this part in this thesis. Indeed, we have chosen to avoid the microscopic approach to conduction and superconduction in order to develop at a unique phenomenological level both conduction and superconduction, so to give to the reader a minimum level to which it is possible to provide a consistent description of superconductivity. The experience we had with high school students has led us to believe that, given the time available, it is more productive to have the goal of a greater understanding of electromagnetism, rather than the goal of the discussion at the microscopic level.

6.2. Description of the electrical current by the electric field

As we have just said, our aim is to give to students a descriprion of the phenomenological aspects of the electrical conduction in the frame of electromagnetism. Generally, the electrical conduction takes place in an electrical circuit, and it is for this reason that we begin our path on conduction with electrical circuits: to physically contextualize our problem.

Before starting with the description of the path, we think it is important to discuss some preliminary notions.

- Since we are interested in the phenomenological description of currents, but we are not interested in giving a description of its microscopic nature, we do not speak in terms of electrons, or of elementary charges. Instead, in our path we hope to familiarize students with the fact that the current can be seen as a flow of charge. Thus, without deepen in this path the experimental reasons to get a similar conclusion, we simply use this idea and we will illustrate how, through this idea, it is possible to build a picture of the electrical conduction. Arons and many other works in the literature [46] treat diffusely the problem of the nature of the current and it is for this reason that we do not linger further.

- Students are supposed to be acquainted of the fact that forces on electric charges are, in general, due to the presence of an electric field, and know some basic electrostatics.

- Another important preliminary notion is that the current intensity in a stationary resistive loop (fixed by the experimental conditions) does not depend on the particular point of the loop (it is the same at every point). Suppose for absurd the current I_A at the point A of the circuit is greater than the current I_B flowing at the point B (for instance students often think that the current flowing before a bulb is greater than after the bulb). Due to the fundamental law of the conservation of the electric charge, this would mean that the charge is accumulated between A and B. This progressive piling up of charges would make the trait AB more and more charged, and this would eventually prevent the same flow of charge, in contraddiction with the hypotesis of stationary current.

6.2.1. The experimental evidence of dissipation in electrical circuits

To obtain information about how the current flows in a circuit we can start with an experimental result: a simple resistive circuit, in which current flows, heats up. It is possible to relate the amount of the heat released by a piece of wire of the circuit with the potential difference ΔV present at its ends. If P is the power dissipated in the piece of wire, that is the thermal energy per unit of time, we get experimentally:

$$P = k\Delta V^2, \tag{6.1}$$

where k is a constant that depends on the piece of wire, and is obtained from the experimental setup.

Named W the work done by the electric field along the wire during the motion of the charge q contained in a small flowing volume, we can thus write:

$$P = \frac{dW}{dt} = \frac{d(q\Delta V)}{dt} = I\Delta V. \tag{6.2}$$

By eq.(6.1) and eq.(6.2) we obtain:

$$\Delta V = \frac{1}{c} I, \tag{6.3}$$

that is just the first Ohm's law, after the substitution $R = 1/c$. From eq.(6.3) we have a confirmation of the fact that the current I does not vary in a loop if ΔV is kept fixed. Keeping in mind that the piece of wire can be chosen as needed, we can conclude that it is equivalent to say that (1) the current does not vary in a loop, (2) the energy of the electric field is entirely dissipated in a loop and (3) the first Ohm's law holds.

As written before, in this path we suppose that the electric field has already been introduced to students, as it is usual in the classroom practice. The main goal (and difference) in our path is that we retain that it is very important to use the electric field as the main protagonist to understand electrical conduction. Thus, not only we apply the electric field to describe the energy dissipation of the charge in the electrical circuit, but we shall see that the electric field can be the guide for the interpretation of circuits and also to become familiar with the Maxwell's equation in the stationary case.

6.2.2. A relation between the electric field and the current density

The first Ohm's law that we have just presented, is an integral law, that is, a law whose physical quantities involved are not defined point by point. What we are going to introduce now is a local version of eq.(6.3). The distinction between local and integral laws must be carefully taken into account in dealing with circuits, infact students often mix local descriptions of and integral descriptions, without being aware of this. For instance, as we discussed before, it is very common that students do not see the difference between the currrent flowing in a resistor, which in parallel with another one, in the case the second resistor is removed from the circuit. Students apply the integral Ohm's law as if it could be valid locally.

When we decide to apply local equations we have to use local quantities, as the density current $\mathbf{J}$ and the electric field $\mathbf{E}$, and we can use integral quantities, as the current I and the potential difference ΔV, when we decide to apply integral equations. The first Ohm's law in terms of integral quantities can be written in a very usual way as:

$$\Delta V = RI. \tag{6.4}$$

Eq.(6.4) has an analogue in terms of local physical quantities. To find it, we can start from the integral first Ohm's law and replace the integral quantities with the corresponding local ones, supposing to have a homogeneus wire, of constant section S for all its length L. We re-write eq.(6.4): since the current must be the same at every section of the conductor, the electric field must be uniform. We thus have:

$$EL = RJS, \tag{6.5}$$

from which, taking also into account the direction of the electric field and of the current density, we obtain definitively:

$$\mathbf{J} = \sigma \mathbf{E}, \tag{6.6}$$

if we pose $\sigma = L/RS$ (that is the second Ohm's law, where σ is the usual conductivity).

The relationship of eq.(6.6) may be applied to describe the electric field inside a wire and can be used in many different situation, helping students' understanding, as we shall see throughout this chapter.

6.2.3. The electric field in electrical circuits

The electric field does not vary with time in a stationary circuit, therefore we are in presence of an electrostatic field, therefore a conservative field. This is a condition that should be clarified as soon as possible when students begin their sequence on circuits, because it can be very useful in the description of the problems. There is much confusion on this point. Infact, although teachers have no doubt about the conservativity of the electrostatic field, they often say that "the circulation of the electric field in a circuit is equal the electromotive force" or, sometimes, they are confused because they don't know if it is possible to apply the Maxwell's equations to a circuit or not. A typical source of difficulty is given by the presence of the battery in the circuit, that can be the element because of which it can be no longer possible to think to a conservative electric field.

Discussing with teachers we found two main positions. The first is an attempt of adjusting their ideas through the introduction of the schema of the "electromotive field" acting throughout the circuit, while the second is the fact that they imagine that there are two different electric fields acting in different parts of the circuit. In any case, we found that the electric field of an electrical circuit is a physical quantity somewhat vague in their way of reasoning.

We will return on the concept of electromotive force later, while now we prefer to provide a description of the electric field generated by a circuit, using the main property of the electric field involved:

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0. \tag{6.7}$$

The property defined by eq.(6.7) can dispel many students' and teachers' misconceptions. First of all it is clear that the electric field must be a physical quantity well defined in each point and in each position of the circuit. By the stationary conditions that we suppose and from the Maxwell's equations we can be sure that eq.(6.7) describes the electric field of an electrical circuit. Secondarily, we can use the same property of the electric field to get informations about the electric field outside a wire carrying steady current. The majority of students and teachers is convinced that there is no electric field outside a wire. In fact, while the magnetic field is immediately associated to the space outside a wire carrying current, the electric field is associated to the inside of the same wire. Most of students and teachers we interviewed never think to a magnetic field inside the wire or to an electric field outside the wire. But, if we imagine a portion of a wire carrying current, as in Fig. 6.1, we know that there is an electric field $\mathbf{E}$ inside the wire and, to determine the electric field outside, we can evaluate the circulation of the field along a the rectangle γ, of sides a and b.

Supposing the field $\mathbf{E}$ uniform inside and null outside the wire, we find:

$$\oint \mathbf{E} \cdot d\mathbf{l} = \int_A^B \mathbf{E} \cdot d\mathbf{l} = Ea, \tag{6.8}$$

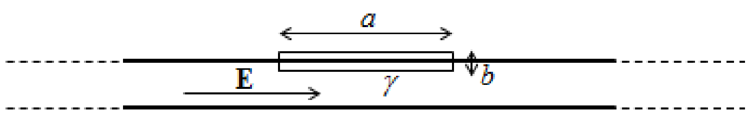


Figure 6.1.: Scheme of a section of a wire carrying current. It is represented a path γ along which it is possible to calculate the circulation of the electric field.

that is in contrast with eq.(6.7).

If the rectangle has the sides b short enough to do not consider the contributes of the electric field along them, we get that the electric field outside the wire must be equal to the field inside the wire itself. By similar arguments, we can also deduce that in the stationary case we can assume that the electric field is uniform even on each section of the wire. The analysis of this problem could be refined, because if the circuit bends in some points, the electric field outside the wire becomes more complex, because we should take into account the contributions to the electrostatic field given by the charges placed on the surface of the wire. But, in a first approximation in which the wire is very long and thus the electrodes are far from the point where is placed γ, the electric field is parallel to the surface of the wire. The qualitative determination of the electric field outside circuits is certainly very instructive and can be carried out with students as a game with simple rules, but few is present in literature [47]. We simply mention that the electric field inside the wire is, in general, parallel to the wire itself, while outside the wire, the field is the sum of the field parallel to the wire and the field generated by the charges on the surface of the wire, placed in such a way to maintain the electric field uniform inside. We do not linger more on this point because a very interesting work has already be done by Chabay and Sherwood [45, 48] and we simply use the fact that the electric field in stationary conditions is established by opportune charged distributions on the surface of the wire.

For the stationarity of the current, we have thus to conclude that in each point of the circuit, the charge must exchange energy in order to keep its flowing with uniform velocity, but it has to give away the surplus of energy that would cause an acceleration, so that the circuit must heat up. By considerations about the distribution of the charges on the surface of the wire it is possible to clarify that the electric field is uniform and parallel to the direction of the wire, but it can be interesting to interprete in terms of electric and magnetic fields the energy dissipation. This part involves the concept of electric and magnetic fields and the Poynting vector. Usually the Poynting vector is not part of the electromagnetism path in high school, if not in a marginally way. If students have the concepts of electric and magnetic fields it is possible to introduce the Poynting vector in this context, being the vector that describe the propagation of energy. Although we are dealing with stationary circuits, and there is no propagation of energy, it can be meaningful that a student can imagine where does the energy that moves the charge in a circuit comes from.

This is not a deepening on the Poynting vector, but an appropriate time to apply the concept, for a better understanding of circuits.

As it is well-known, the Poynting vector gives the direction and the intensity of the electromagnetic energy per unit of time and surface area, and can be written as:

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}. \tag{6.9}$$

By eq.(6.9) we see that it can be simple for students to determine the direction of the Poynting vector in each point of the circuit considered, and it can be a useful exercise. If we refer to Fig. 6.2 we can imagine that each point of a circuit can be seen as a straight trait of wire in which the electric field is directed parallel to the wire and the magnetic field has the well-known circular shape around the wire.

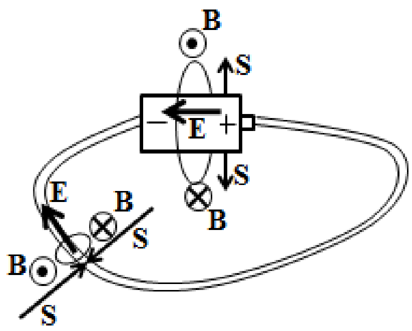


Figure 6.2.: Scheme of a simple resistive circuit. In two traits of the circuit, the battery and a trait of the wire, are represented the electric field, the magnetic field and the Poynting vectors.

By the rule through which we determine the direction and the verse of the vector product we can argue that the electromagnetic energy is always orthogonally directed towards the inside of the wire.

Always referring to Fig. 6.2 we can note that while the current maintains the same verse inside and outside the battery, the electric field is opposite, so that it is immediate to deduce that the Poynting vector is directed towards the oustside of the battery. By this simple calculation a students can have a picture of the exchanges of energy: it is as if the energy came out of the battery and reach each point of the circuit. It is also possible to calculate the energy absorbed by a trait of the circuit per unit of time, that is the power P dissipated by a trait of the circuit. It is interesting that we will find exactly what we obtained in our previous experiment.

Let us consider a trait of wire of length L and section of radius r, as it is represented in Fig. 6.3.

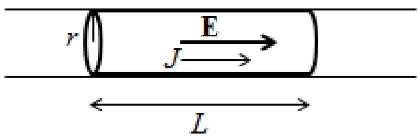


Figure 6.3.: Representation of a trait of a wire carrying current: its length is L and its radius is r. The Poynting vector is orthogonal to each point of the lateral surface of the cylinder in bold.

We want to determine the energy per unit of time that enters the circuit from the lateral surface $A = 2\pi rL$. This is nothing else than the Poynting vector multiplied by the lateral surface A. That is:

$$P = |\mathbf{S}|\, A = \frac{1}{\mu_0} E \cdot B \cdot A, \tag{6.10}$$

being the electric and the magnetic field orthogonal to each other in each point of the circuit. Hence, taking into account the expression of the intensities of the electric field and the magnetic field on the surface of the wire:

$$E = \varrho J, \tag{6.11}$$

and

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}, \tag{6.12}$$

we obtain:

$$P = \frac{1}{\mu_0} \cdot \varrho J \cdot \frac{\mu_0}{2\pi}\frac{I}{r} \cdot 2\pi rL = \frac{\rho L}{\pi r^2} I^2, \tag{6.13}$$

where we have expressed the current density J in terms of current I, by the relationship $I = J \cdot \pi r^2$.

And hence:

$$P = RI^2, \tag{6.14}$$

as we have altready found experimentally, if we suppose that L is the length of the entire loop that we are considering in our example. In the same way we can proceed with a similar argument for the part of the circuit between the ends of the wire, that is for the battery. In that trait of the circuit, we know that there is a potential difference ΔV and a current I flows through it, so that we can be sure that the power supplied by the battery is $P = \Delta V \cdot I$, that is equal to eq.(6.14) after the substitution $\Delta V = RI$, the first Ohm's law that we have found previously. We can thus conclude that the power absorbed by the wire, that allows the charge to move along the wire, is the power that comes out from the battery in the form of electric field established in each point of the circuit, as the Poynting vector indicates.

6.2.4. The e.m.f. of a circuit

A very important source of misunderstanding for students and teachers concerns the electromotive force (*e.m.f.*) of a circuit. Most of the teachers we have interviewed said, as we have pointed out previously, that "the circulation of **E** equals the electromotive force, because **E**, in a circuit, is not conservative". Also in many textbooks for high school and for university we can find:

$$e.m.f. = \oint \mathbf{E} \cdot d\mathbf{l}, \tag{6.15}$$

where **E** sometimes is defined as the "electromotive field". But it is hard to define the electromotive field: it depends on the particular device used to maintain the potential difference in the circuit. By eq.(6.15) this field is clearly not conservative. This, in turn, implies that for the electromotive field, it is not possible to define a potential, and then a potential difference between, for instance, the poles of a battery, as it happens in the practice.

Therefore, we think it should be convenient to remove the idea of the electromotive field, and use only the electrostatic field that is well defined inside and outside the circuit. We propose a way to define the electromotive force *e.m.f.* that bypasses the "electromotive field" and that uses the mathematical tool of line integral (as well known, necessary in electromagnetism and for the work definition).

To introduce the concept of *e.m.f.* we consider a stationary resistive circuit, in which we do not specify the device used to maintain the potential difference, so we can refer to this generic object as the "device". It can be a mechanical device that transports charges by a conveyor belt, or it can be a chemical device as a battery, or other. We will simply apply the work-energy theorem, that can be written as $W = \Delta K$, where W is the work done by the force acting on a charge q between two points A and B of the circuit, and ΔK is the variation of kinetic energy of the same charge between the two points A and B. We can immediately observe, for the stationarity of the circuit, that $\Delta K = 0$, so we can write:

$$\oint \mathbf{F} \cdot d\mathbf{l} = 0, \tag{6.16}$$

where $\mathbf{F}$ is the resultant force acting on the charge.

We can describe in detail the line integral of eq.(6.16) along the circuit. If A and B are the endpoints of the citcuit connected to the device as shown in Fig. 6.4, we have:

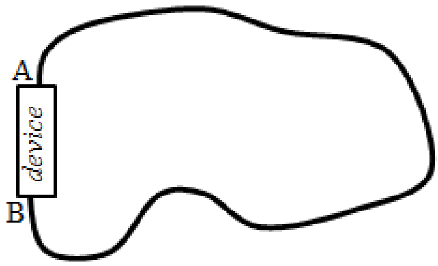


Figure 6.4.: Scheme of a generic resistive circuit

$$\oint (\mathbf{F}_{el} + \mathbf{F}_{diss} + \mathbf{F}_{device}) \cdot d\mathbf{l} = 0, \tag{6.17}$$

where $\mathbf{F}_{el}$,$\mathbf{F}_{diss}$ and $\mathbf{F}_{device}$ are respectively the electrostatic force, the dissipative force and the force that the device exerts on the charge q. Eq.(6.17) can be also written as:

$$\int_A^B q\mathbf{E} \cdot d\mathbf{l} + \int_A^B \mathbf{F}_{diss} \cdot d\mathbf{l} + \int_B^A q\mathbf{E} \cdot d\mathbf{l} + \int_B^A \mathbf{F}_{device} \cdot d\mathbf{l} + \int_B^A \mathbf{F}_{diss} \cdot d\mathbf{l} = 0, \tag{6.18}$$

with obvious meaning of the notation used.

In terms of the potential difference, dividing by the charge q, we obtain:

$$\Delta V_{AB} + \frac{W_{AB}^{diss}}{q} - \Delta V_{AB} + \frac{W_{BA}^{diss}}{q} + \frac{W_{BA}^{device}}{q} = 0. \tag{6.19}$$

By the work-energy theorem, we can get:

$$\Delta V_{AB} + \frac{W_{AB}^{diss}}{q} = 0 \tag{6.20}$$

and hence:

$$-\Delta V_{AB} + \frac{W_{BA}^{diss}}{q} + \frac{W_{BA}^{device}}{q} = 0. \tag{6.21}$$

Both eqs.(6.20) and (6.21) allow us to describe a resistive circuit. In particularly, eq.(6.20) says that the energy dissipated by the circuit from A to B (that is in the wire) equals $q\Delta V_{AB}$, while eq.(6.21) describes the two unknown forces acting in the circuit through the well-known quantity ΔV_{AB}, that is:

$$\Delta V_{AB} = \frac{W_{BA}^{diss}}{q} + \frac{W_{BA}^{device}}{q}. \tag{6.22}$$

Now, we can pose:

$$e.m.f. = \frac{W_{BA}^{device}}{q}, \tag{6.23}$$

that is the *e.m.f.* of a circuit is the work done by the device for unity charge. We can also express this quantity by:

$$e.m.f. = \Delta V_{AB} - \frac{W_{BA}^{diss}}{q}. \tag{6.24}$$

From eq.(6.24) we observe that, in the case in which there is no dissipation inside the device (the work in eq.(6.24) is done from B to A), the *e.m.f.* coincides with the potential difference ΔV_{AB} at the endpoints of the circuit. If, on the contrary, we consider a dissipation inside the device, then we have $-W_{BA}^{diss}/q > 0$, being the work done by the dissipative forces inside the device surely negative, and the *e.m.f.* is not just equal to ΔV_{AB}.

In many cases, with the aim of simplifying the vision of the problem, the dissipation inside the device $-W_{BA}^{diss}/q$ can be thought as if there were an internal resistance r in the same device, then eq.(6.24) can be written, as usual:

$$e.m.f. = \Delta V_{AB} + rI. \tag{6.25}$$

Another interesting consideration may be obtained from eq.(6.24) when it is written as:

$$\Delta V_{AB} = e.m.f. + \frac{W_{BA}^{diss}}{q}. \tag{6.26}$$

In fact, using eq.(6.26) we observe that the potential difference ΔV_{AB} is the sum of two quantities, *e.m.f.* and W_{BA}^{diss}/q, that are not potential differences taken singularly. Eq.(6.26) can be obtained from thermodynamical considerations. In fact a circuit is a thermodynamical system in which a certain heat Q is dissipated and a certian amount of work W is done and for which we can write (with obvious notation):

$$\Delta U = W + Q. \tag{6.27}$$

In this framework the *e.m.f.* represents the work W done on the system per unit charge, while W_{BA}^{diss}/q represents the heat Q dissipated by the Joule effect, per unit charge. With students it could be very interesting to point out that an electrical circuit is a dissipative system in which it is possible to define a sort of internal energy, that is a state function (the electric potential) which has a precise value in each point of the circuit.

By eq.(6.20) we can express the energy dissipated per unit charge along the circuit as a potential difference ΔV_{AB}. This, in turn, makes it possible to write for each part of the circuit that causes dissipation a relation like:

$$\Delta V = -\frac{W^{diss}}{q}, \tag{6.28}$$

and supposing that there are N different elements of the circuit that dissipate the energies W_1^{diss}, W_2^{diss},...W_N^{diss}, then the sum of the losses of energy divided by q, must be equal to the potential difference given by eq.(6.20) and combining with eq.(6.28) we can write:

$$\Delta V_i = -\frac{W_i^{diss}}{q} \tag{6.29}$$

and therefore:

$$\Delta V_{AB} = \Delta V_1 + \Delta V_2 + ... + \Delta V_N, \tag{6.30}$$

that is the well-known Kirchhoff's second law.

6.3. Resistors in series and in parallel

As we have seen in the previous chapter students have great difficulties with the concepts of resistors in series and parallel. Moreover, at a deeper analysis, it appears that the difficulty is in the understanding of the meaning of resistance at all. In this regard, we would remember the oral interviews in which we asked the students what would be the current flowing in a wire if we considered a wire with a smaller section. In our path we believe that it is necessary to propose a sequence on the concept of resistance, in such a way to overcome the difficulties observed in our experimentation. In the first part of this sequence we will introduce a way to represent graphycally a resistor, while in the second part we will see a way to treat resistors in series and in parallel. The second part of the sequence, after a description of the concepts of series and parallel, uses a new approach to better understand these concepts through the use of cubes of the material of the wire.

6.3.1. A useful representation for resistors

In discussing electrical circuits it seems very important to give students a concrete representation of some of the quantities involved in the treatment of this topic, even if it is developed in a traditional way. In particular it can be very useful to give a representation for:

- resistors
- the dependence of the electric field inside the wire from the section of the wire itself

In the following we will see how it can be described to students, using the basic concepts they already know. It is only a question of different perspective through which it is possible to observe all the known concepts.

Let us consider a stationary electrical circuit with a cylindrical wire of section S_1 that in a certain point decreases its section to the value S_2. The current I must be the same in each point of the loop, for the charge conservation in a stationary circuit. We know that I is an integral quantity, whereas J is a local quantity and, since I is fixed, J depends on the section of the wire. Therefore, being I the same in all the points of the wire, we can write:

$$J_1 S_1 = J_2 S_2 \tag{6.31}$$

and, from the proportionality between E and J we obtain:

$$E_2 = \frac{S_1}{S_2} E_1. \tag{6.32}$$

So, for what concerns the electric field, from eq.(6.32) we observe that the electric field is not uniform inside a wire with a variable section and, in particularly, that it increases where the section decreases, just as it happens for the current density J, see Fig. 6.5.

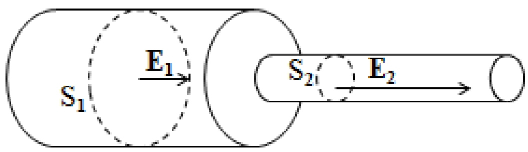


Figure 6.5.: Electric field (proportional to the current density) in a wire with a decreasing section.

On the other hand, it is quite common to see the electrical current as a flow of a fluid-like entity, and from the Bernoulli principle we know that the fluid flows more quickly through the narrow sections.

Recalling the second Ohm's law $R = \rho L/S$ we can think to a resistor as a piece of wire with a section much thinner then the rest of the wire [45]. With a similar graphical representation, for students it will be much more difficult to think that “the resistance of a wire increases with the quantity of material” or that “a wire has no resistance”, when they do not see the conventional notation for resistors and the sketch shows a circuit constituted by a wire and a battery. Moreover, it becomes simpler for students to be aware of the fact that a conductor permits the flowing of the current and does not “use-up the current”, that is a very common students’ misconception, as it is described in the previous chapter. With a mental representation of resistors like that of Fig. 6.5 it appears very clearly that the resistance of a conductor can increase when the section decreases and, in the limit in which the section becomes zero the conduction is, in practice, prevented or equivalently, the resistance is almost infinite.

After we have developed such a representation for resistors, we found that also Chabay and Sherwood use a very similar approach. This concurrence confirmed the effectivness of the idea we had, for this reason we have reported it in this work as a important part of our path.

6.3.1.1. The approach by cubes

We consider cubes constituted by the same material of the wire, or by a material that can be thought as inserted in the wire in a certain point of the loop. The dimensions of the cubes are not microscopic, but, depending on the situation we have to model, the cubes can have macroscopic dimensions or mesoscopic dimensions. For mesoscopic dimensions we mean dimensions large enough to avoid the consideration of the microscopic structure of the material of the cube, but short enough with respect to the wire, so to consider a cube as a point in the material in which the electric field is approximately uniform. We propose in Fig. 6.6 two similar ways of representing cubes of material in order to visualize the problem and treating resistors in series and in parallel.

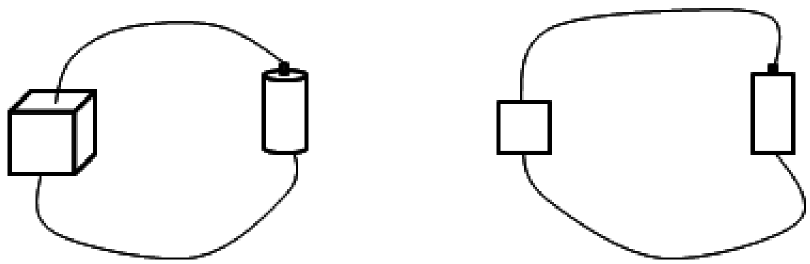


Figure 6.6.: Two ways of representing cubes in order to visualize and carry out calculations with resistors in series and parallel.

In each of the two sketches the resisteance is thought as to be placed only in the cube, while the two wires that connect the cube with the battery are thought as abstract tools for the application of the potential difference to the cube, and hence they have no resistance. We have to take care that students understand this point and the abstraction of this representation in which the cube may represent from time to time experimental situations very different: for instance a cube may represent all the loop, a trait of loop or a very small piece of a trait of a loop.

6.3.2. Resistors in series

Once the idea of resistor is put in relation with a constriction of the wire, rather than with a big piece of material placed in the wire that causes a slow down in the current flow, it becomes possible to utilize the cubes to treat resistors in series and in parallel. The simpler case we face is the case of resistors in series. Resistors are in series when the current that crosses them is the same. We suppose that the definition of series, as well as the definition of parallel, is already been given to students.

Our problem is how to give a representation of the series of two resistors and how to treat them throgh the relationship already introduced $\mathbf{J} = \sigma\mathbf{E}$, that in most cases we will use in its non-vector formulation $J = \sigma E$.

For this purpose, let us consider the simpler case of two identical resistors in series. In a graphical representation similar to the left part of Fig. 6.6 in which we have drawn a sort of section, a square, we get for the series the situation represented in Fig. 6.7.

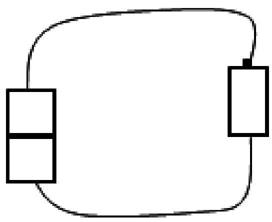


Figure 6.7.: Representation of two identical resistors in series by two sections of cubes placed side by side in the same direction of the current.

We can imagine that each square has a side of length L. Using Fig. 6.7 as a reference, we can apply the relation $J = \sigma E$ with the aim of understanding the meaning of resistors in series.

If we suppose that the potential difference does not vary in the series, then it is possible to evaluate instead what varies in the new circuit with the resistors in series. The total length of the cubes is changed, infact in the series we have $L_S = 2L$. So we can write:

$$E_S = \frac{\Delta V_S}{L_S} = \frac{\Delta V}{2L}, \tag{6.33}$$

and this clarifies a relation for the electric field:

$$E_S = \frac{1}{2}E. \tag{6.34}$$

By eq.(6.34) we get immediately that the electric field E decreases in a series and in the same way decreases the current density J, being E and J proportional. This first step allows a characterization of the electric field in a circuit when the length of the circuit increases. It happens when two resistors are placed side by side, that is when two resistors are in series.

If we want to discover what happens to the resistance of the series it is necessary to introduce the resistance by the first Ohm's law. We have:

$$\Delta V = RI = RJS \tag{6.35}$$

and, for the circuit with the series of resistors, the relation similar to that of eq.(6.35) becomes:

$$\Delta V = R_S \frac{1}{2} JS, \tag{6.36}$$

da cui:

$$R_S = 2R. \tag{6.37}$$

We have to highlight that the result obtained from eq.(6.37) depends on the fact that we have chosen two identical resistors. In any other case it is clear that is possible to get the value of the series of resistors simply by adding the resistances to each other.

Using many cubes it is possible to describe every situation in which resistors are put in series, even if they are not equal to each other. Moreover, in this way it is possible to describe the dipendance of the resistance on the length of the material, as the second Ohm's law contains. In fact, by the equation:

$$R = \varrho \frac{L}{S}, \tag{6.38}$$

we immediately see that the resistance R is proportional to the length L of the material considered. Anyway, this relation between length and resistance seems to be quite clear to students when are proposed written tests. Instead, it seems less clear during oral interviews, when it is possible to deepen the discussion. In this case, what seem to emerge with a certein frequence is the fact that there is not a significant difference if it is increased the length or the thickness of the material, because what is determinant is often the quantity of material that the current "sees" during its flow. With the next discussion about the resistors in parallel we can continue the understanding of the second Ohm's law, and in particular we can examine with attention the dependance of the resistance on the section od the wire in which the current is flowing.

6.3.3. Resistors in parallel

As we did for the resistors in series, we want to do for resistors in parallel. Using a definition that we imagine that students already know, that is "two resistors are in parallel if they are placed between the same potential difference, we can represent two resistors in parallel as in Fig. 6.8.

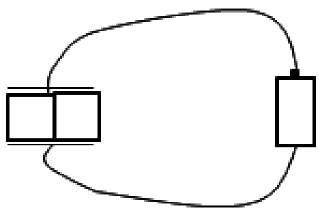


Figure 6.8.: Representation of two identical resistors in parallel by two sections of cubes placed side by side in a transverse direction respect the direction of the current.

Now, we want to deduce the properties of two resistors in parallel using together the representation of Fig. 6.8 and the usual relation between the electric field and the current density $J = \sigma E$.

As in the previous case we suppose that the potential difference does not vary for the parallel, with respect to the simple circuit with a cube only. It is possible to evaluate what are the physical quantities that vary in this case, with the resistors in parallel. Differently from the case of the series, in the case of the parallel the total length of the cubes does not change, and is therefore equals to L and thus we have $L_P = L$.

In this case we have a change in the current that flows in the circuit, in fact, despite the current density is the same, being J related with the electric field that is not changed, the integral quantity $I = JS$ is changed, because the surface S_P crossed by the current is increased, and exactly we have:

$$S_P = 2S, \tag{6.39}$$

and the current becomes:

$$I_P = JS_P = 2JS = 2I. \tag{6.40}$$

Eq.(6.40) states that placing in a circuit two identical resistors in parallel, the current flowing in the circuit doubles, with respect the case in which there is only one resistor. Such a discussion is very clear and by the relation $J = \sigma E$ a first comprehension of the concept of resistors in parallel is got immediately. In this way students can see without difficulties that resistors do not use up current, and that the current in a circuit may be affected by the resistors depending on their disposition in the circuit itself. If we want to bring us back to face the problem from the point of view of the resistance, as in the previous case, we have to invoke the first Ohm's law.

From the first Ohm's law we have that:

$$R_P = \frac{\Delta V}{I_P}. \tag{6.41}$$

Using eq.(6.40) we can express the resistance of the parallel in terms of the current I that flows in the circuit with one resistor only:

$$R_P = \frac{\Delta V}{2I} = \frac{1}{2}R. \tag{6.42}$$

This equation states that the resistance of the parallel is halved, and this because we have considered two identical resistors. It is possible to deduce a general relationship for the equivalent resistance for resistors in parallel, even in this case that is less imediate than the previous. We show that it is possible to follow a similar way of reasoning, based on the relation $J = \sigma E$, to get the equivalent resistance. In order to consider the general case of two resistors in parallel we refer to Fig. 6.9.

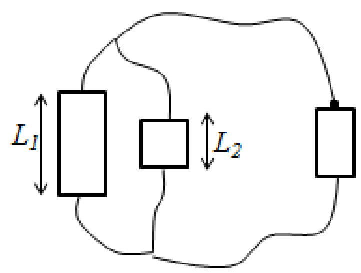


Figure 6.9.: Two resistors in parallel. The length of the resistors are different, so to get two different values for the resistances.

Being the potential difference the same accross the two resistors, and being the lengths of the resistors different, as Fig. 6.9 shows, we have two different electric fields in the two traits of the circuit. Named E_1 and E_2 the intensities of the two fields, with obvious meaning of the subscripts used, we have:

$$J_1 = \sigma_1 E_1 = \frac{\Delta V}{\varrho_1 L_1} \tag{6.43}$$

and

$$J_2 = \sigma_2 E_2 = \frac{\Delta V}{\varrho_2 L_2}. \tag{6.44}$$

Now, we can write the current densities J_i in terms of currents I_i, and write the total current I that is fowing in the parallel circuit:

$$I = I_1 + I_2 = (J_1 + J_2)\, S, \tag{6.45}$$

because we suppose that each element in the parallel has the same section S, for simplicity. Eq.(6.45) becomes:

$$I = \left(\frac{\Delta V}{\varrho_1 L_1} + \frac{\Delta V}{\varrho_2 L_2}\right) S = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) \Delta V, \tag{6.46}$$

from which we obtain the equivalent resistance R that is defined as

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}. \tag{6.47}$$

What is so easily visible in this way is also visible by the second Ohm's law. In fact the presence of the factor S at the denominator, tells us that when the section of the material crossed by the current increases, then the resistance decreases, in a way that is conterintuitive for students, in general. On the contrary, the current must increase, infact, remaining the electric field unchanged as the current density, the current increases because the surface S increases. Moreover, it is useful that students notice that the present decrease in the resistance is yet another way of saying that the greater the thickness S of the material, the lower its resistance R.

As we have seen in the case of resistors in parallel, we can use the circulation of the electric field to get another description of the circuit. This is important for students, which can become familiar with a mathematical tool as circulation, fundamental in electromagnetism and often source of problems. Secondarily, this approach allows students to become familiar also with the electric field.

We can imagine that the wire is divided into two parts, so that, from the total section S we obtain two sections S_1 and S_2, with $S_1 \cup S_2 = S$. This is not the general case, but it is enough to describe the concept of resistors in parallel. The resistance of the parallel is $R = \rho L/S$, because it corresponds to the trait of the wire, of length L before we have divided it, while each of the two traits will have the resistance $R_1 = \rho L/S_1$ or $R_2 = \rho L/S_2$. For the conservation of the charge we can say that the current I must be divided into two parts when the wire is divided. How the current will be shared between the two resistors?

If we represent the conductor as in Fig. 6.10, we can calculate the circulation of the vector $\mathbf{E}$ along the line γ.

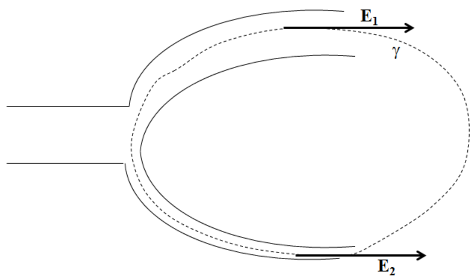


Figure 6.10.: Scheme of two generic resistors in parallel. The circulation of the electic field E is done along the dashed path γ.

Its value must be zero, so that we can say with good approximation that the intensity of the electric field E_1 in the branch of section S_1 of the sketch, must be equal to the intensity E_2 in the branch of section S_2, being equal the lenghts of the two branches. Since even the material of the two branches is the same we can conclude that J_1=J_2 and therefore:

$$\frac{I_1}{I_2} = \frac{S_1}{S_2}. \tag{6.48}$$

Eq.(6.48) says that if you have two resistors in parallel, the higher intensity of current will flow in the resistor with the smaller resistance, that is in the thicker piece of wire. And this, on the other hand, is intuitive because the larger is the section of the wire, the larger is the current that can pass through it. Despite of this, students often think that a thin wire "opposes less resistance" than a thick one, because it contains less opportunities of consuming current. The inconsistency of this way of thinking can be easily appreciated imagining a wire whose cross-section goes gradually to zero. In the limit in which the section of a branch is zero, the current will also become zero, and will not become infinite, as it would be if the "conductor" was a "material that makes hard the current to flow".

6.3.4. An example of circuit with resistors in series and parallel

We feel the need of dealing with this example because, in our investigation, students encountered great difficulties. They had a circuit with two identical resistors in parallel, called R_1 and R_2 and two resistors in series called R_3 and R_4, as it is shown in Fig. 6.11. For simplicity it is possible to think that the values of the four resistances are equivalent to each other.

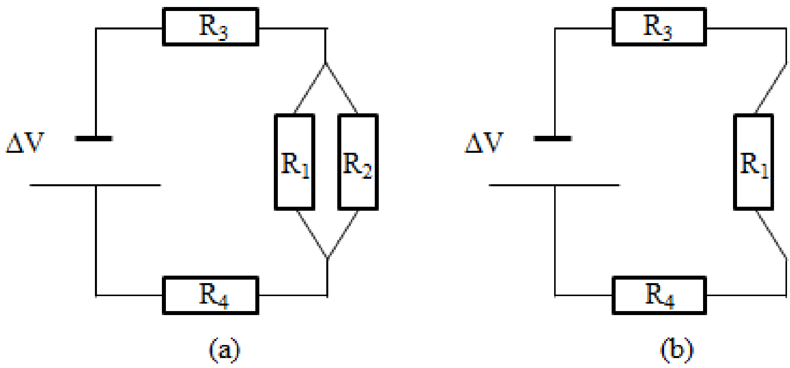


Figure 6.11.: An example of a circuit with four resistors a). In the case b) a resistor is removed.

As we have already seen previously; in the first part of the question they were asked to determine the current flowing in one of the two resistors, R_1 or R_2, while in the second part, one of the two resistors was removed from the parallel and the students were asked to determine the new current flowing in the resitor remained. As it is reported in the previous chapter, most students stated that "after the resistor is removed, through the other resistor the current does not vary".

An interpretation of this common mistake is that students used the integral first Ohm's law locally, losing sight of the fact that the circuit is changed in another point, different from the point in which they are applying the integral law as if it was a local law. Following a coherent *integral* approach, a student may say that when the resistor is cut out, the total resistance is changed, as well as the current flowing, being the differential potential unchanged. It is instead very difficult for students to mix the two approaches. To illustrate this difficulty, we used a simple example with students. In Fig. 6.12 it is shown a wire of section S, that, for a certain trait is divided into two parts of sections S_1 and S_2 such that $S_1 + S_2 = S$. In this example we have reproposed a similar experimental situation to that we proposed in various formulations in the written tests we proposed to the students of our experimentation, see appendix A, B and C.

In the test we asked to students what happened to the current flowing when the branch (3) is cut out. Before answering this question, we want to examine in detail this problem.

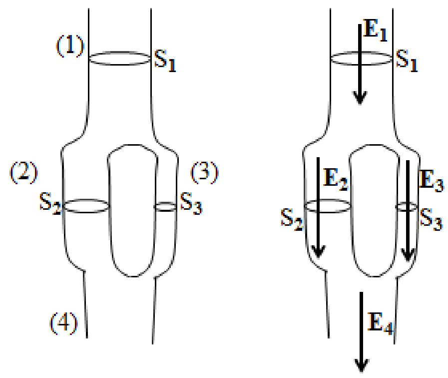


Figure 6.12.: The wire is divided into two parts for a trait, as if there were two resistors in parallel in (2) and (3), and two resistors in series in (1) and (4).

The first preliminary consideration pertains the fact that the current I that flows in the branches (1) and (4) is the same, for the conservation of the electric charge, and for what we already dealth with about the fact that the circuit does not use up current. It turns out that:

$$I = I_2 + I_3, \tag{6.49}$$

but this is quite obvious and quite known to students. It could be less obvious that students deduce the values of the currents in relation to the experimental situation, as it is reported in the previous chapter. For a complete description we can say, named R the resistance of the whole wire through which is applied a potential difference ΔV, that:

$$I_2 = \frac{\Delta V}{R_2} = I\left(\frac{R}{R_2}\right) \tag{6.50}$$

and similarly

$$I_3 = I\left(\frac{R}{R_3}\right). \tag{6.51}$$

For the second Ohm's law we can rewrite eq.(6.50) and eq.(6.51) in terms of the sections of the two branches, that is:

$$I_2 = I\left(\frac{S_2}{S}\right) \tag{6.52}$$

and

$$I_2 = I\left(\frac{S_3}{S}\right). \tag{6.53}$$

What happens to the electric field, or equivalently to the current density, in the branches of the circuit? With very simple calculations we can find the values of the current densities. Let us consider for instance the branch (2). We have:

$$J_2 = \frac{I_2}{S_2} = \frac{{}^{IS_2}/_{\mathrm{S}}}{S_2} = J$$

and an identical result holds for the other branch.

We can conclude that both the current densities and the electric fields do not vary in any point of the circuit. In other words, when a wire is divided into two parts for a trait, the local quantities do not vary. In Fig. 6.13 we have thus represented the electric field **E** equals in each point of the circuit. On the other hand, the application of the circulation of the electrostatic field as in Fig. 6.11 gave us the same result.

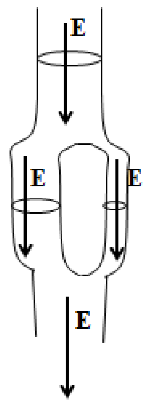


Figure 6.13.: Diagram of the electric field in each of the four branches of the circuit.

Let us imagine now that the branch (3) is cut out, as in Fig. 6.14. What happens to the circuit?

In general, it is possible to think of two main ways in which to remove parts of a circuit. It is possible to remove longitudinally, as in Fig. 6.14, or transversely. In the first case it equals to remove a resistor from the parallel, while in the second case it equals to remove a resistor from a series. It is clear that in the first case the resistance of the circuit will increase, while in the second case the resistance will decrease. Therefore, what one expects is that the current flowing in the branch (1) or (4) of the circuit in Fig. 6.14 is decreased than that flowing in the same branches of Fig. 6.12.

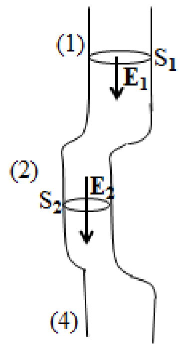


Figure 6.14.: The circuit of the example of Fig. 6.13 in which the branch (3) is cut out.

It can be instructive for students to deduce information about the electric field present in the branches of Fig. 6.14. In fact, with very simple arguments, we can get a relation between the electric fields $\mathbf{E}_1$ and $\mathbf{E}_2$ of the modified circuit and the relation between the fields of the modified circuit with respect to the field $\mathbf{E}$ of the initial circuit in Fig. 6.12.

Relation between E_1 and E_2

We have already treated this problem in this chapter. Recalling eq.(6.32) we immediately have the result needed. Since $S_1/S_2 > 1$ we get:

$$E_2 > E_1. \tag{6.54}$$

Relation between E_1 and E

The relation between E_1 and E is less obvious. Is the electric field acting in the same trait of the circuit (1) changed after the branch (3) has been removed? Being the total resistance increased, we expect a decrease in the current and hence a decrease in the electric field, since the section of the wire is not changed. This can be confirmed by simple calculations.

In the circuit of Fig. 6.12 we have:

$$E = \frac{I}{\sigma S}, \tag{6.55}$$

while in the circuit of Fig. 6.14 we have:

$$E_1 = \frac{I_1}{\sigma S}. \tag{6.56}$$

Named R the total resistance of the circuit in Fig. 6.12 and R_1 the total resistance of the circuit in Fig. 6.14 we can write:

$$\frac{E_1}{E} = \frac{I_1}{I} = \frac{R}{R_1}. \tag{6.57}$$

From the relation $R < R_1$ we get:

$$E_1 < E. \tag{6.58}$$

Relation between E_2 and E

Starting from eq.(6.32) and eq.(6.57) we can write:

$$\frac{E_2}{E} = \frac{1}{E} \cdot E_1 \frac{S_2}{S_1} = \frac{R}{R_1} \cdot \frac{S_2}{S_1}. \tag{6.59}$$

Both the factors R/R_1 and S_2/S_1 are smaller than 1, thus we can conclude:

$$E_2 < E. \tag{6.60}$$

And, in the end we can write:

$$E_2 < E_1 < E. \tag{6.61}$$

6.4. Phenomenology of the electrical current: syntesis of the path

(A) Clarification of some preliminary concepts

- In a phenomenological description the current is seen as a flow of charge and it is not seen as electrons in motion: the concept of electron is not needed and for this reason never mentioned.
- The circuits considered in this approach are only stationary resistive circuits.
- The current in a loop does not depend on the particular point of the loop.

(B) The first Ohm's law

- The first Ohm's law is introduced taking together experimental considerations about the power dissipated in a circuit and the assumption that the dissipation pertains the energy of the electric field that moves the charge in the circuit.
- The first Ohm's law introduced in terms of integral quantities such as the potential difference and the electrical current, $\Delta V = RI$, can be proposed also in terms of local quantities, such as the electric field and the current desitiy, $\mathbf{J} = \sigma \mathbf{E}$.

(C) The electrostatic field in electrical circuits

- The electric field that causes the electrical current in a stationary circuit is a conservative field and for this field holds the relation $\oint \mathbf{E} \cdot d\mathbf{l} = 0$.
- The conservativity of the electric field in a circuit allows a description of the electric field inside and outside the circuit.
- The use of the electric field and the circulation of the electric field allows to improve both the familiarity with the electric field and with the mathematical tool of circulation, fundamental in the continuation of the path and, in general, for the comprehension of the electromagnetism.

(D) Deepening of the circuits by the use of the Poynting vector

- During the treatment of circuits, it is possible to introduce the Poynting vector, or later, as it is usual, with the electromagnetic waves. By means of the Poynting vector it is possible to develop this part in which it is considered the problem of how the energy is exchanged and dissipated in a circuit.
- Through the Poynting vector it is also possible to get on theoretical basis a relation for the power dissipated in a circuit equal to that deduced previously in the path, experimentally. In this way it is possible to return to the initial point (B) so to reinforce it.

(E) The e.m.f. of a circuit

- The definition of the *e.m.f.* of a circuit is a crucial point of the path and it is given from two basic relations. The first is our usual $\oint \mathbf{E} \cdot d\mathbf{l} = 0$, while the second is the general work-energy theorem: $W = \Delta K$, that in the stationary circuits assumes the simpler expression: $W = 0$.
- By the definition of the *e.m.f.* it is possible to write the potential difference of a circuit ΔV in terms of two quantities that are not two potential differences:

just the *e.m.f.* and the energy dissipated inside the device that maintains the current flowing in the circuit W^{diss}/q.

- It is possible with very simple calculations to get the Kirchhoff second law: $\Delta V_{AB} = \Delta V_1 + \Delta V_2 + ... + \Delta V_N$.

(F) Resistors in series and parallel, the second Ohm's law

- By the charge conservation and for the stationarity of the circuit it is possible to represent graphically a resistor with a constriction of the wire.
- By an approach, here defined "by cubes", together with the relation $\mathbf{J} = \sigma\mathbf{E}$ it is possible to investigate the second Ohm's law.
- Using the circulation of the electrostatic field it is possible to describe locally circuits in series and in parallel, and to get information about the electric field and the current density in each point of a circuit.
- As in other sequences of the path, the use of the circulation of the electric field allows a better familiarization with the electric field itself and the mathematical tool of circulation.
- By the use of the first Ohm's law in its integral formulation $\Delta V = RI$, it is also possible to get the usual relations of the equivalent resistance for the series and the parallel.

Although a path on electrical conduction, from the phenomenological description to the microscopic description was ready for students of different levels, from high school to university, we encountered a lot of difficulties in its experimentation. Infact, most of our students were really unprepared on the basic concepts of electrostatics needed for the path. Thus we started a deeper analysis of the students' and teachers' underlying problems.

The main problem we have tried to overcome with the subsequent path pertains the clear separation of electrostatics from the electrical circuits, or from the electrical conduction. For instance, the circulation of the electric field, that is one of the main topics of electromagnetism, is never mentioned when students deal with circuits. This, on the one hand, prevents students to become aware of the concept of electric field and its circulation, and hence it hinder students to build gradually the Maxwell's equations. On the other hand, students lose motivation in physics because they spend much time in solving circuits as a game stand-alone, without understanding the physics involved, that should be the main reason to study circuits, at least for those students that will not be electricians.

As it is resumed in the previous chapter, through several oral interviews we have perceived that also many teachers have sometimes doubts about the basic concepts of electromagnetism, and this has been the reason that made us believe that this part of the path could be helpful.

7. Essential lines of the magnetic vector potential

7.1. Overview

With the aim of treating superconductivity, besides the electrical conduction it is necessary to introduce the megnetic vector potential. In fact, through this physical quantity it becomes possible a phenomenological description of the superconductivity in a way that has been inspired to the works of the London brothers or to the two fluids theory. Besides the traitations of the vector potential in the university textbooks of electromagnetism, something is present in the literature on this topic [49, 50, 51, 52, 53, 54] with the purpose of describing the characteristics of the vector potential, so that it is not thought only as a mathematical tool. Many other pubblications pertains instead the ability of the vector potential in describing particular physical situations [55, 56, 57].

In this chapter, we introduce the vector potential at a level suitable for teachers, while we will present a path on the vector potential for secondary school students in the next chapter. We believe that this first part for teachers can be of great help to let them an understanding of the potentiality of this tool in physics, and, in turn it can be an incentive for teachers to motivate them and bring them closer to the charme of the physics: the first step to elicit the same reaction in their students.

7.2. The concepts of field and potential

7.2.1. The concept of field

Why is the concept of field so important in physics? Let us indicate a field by $\psi(\mathbf{x}, t)$, an expression that can represent a scalar, a vector, a tensor, a spinor, etc. That expression means that in each point of the space and at each time a physical quantity, or a property of the space, ψ, is defined. The concept of field is introduced to eliminate the concept of action at a distance. We assume that this is well-known to teachers. But, when the concept of field is meaningful? In other words, when the concept of field is more convenient in the description of the experimental evidences, rather than the concept of force acting between two points?

An example could be useful to give the idea of what we are saying.

1. Let us suppose to have two charges Q and q at a certain distance, and suppose that we know that there is a force F between the two charges, given by the Coulomb's law. In this case we have to accept the concept of action at a distance and if we introduce the concept of electrostatic field E generated by the charge Q, defined by $E = F/q$ our vision is almost unchanged. That is, the field just introduced does not affect the vision of the world: the action at a distance is avoided, but the central point is again the force acting between the two charges and the field is only another way to describe the world through the forces.

2. Let us now suppose that the field just introduced is a physical quantity that changes with precise laws, for which it is possible to write equations that describe the variations on space and on time of the field itself. In this way, the field intoduced changes the vision of the world. That is, if we can write equations that describe how the field behaves, the field we have introduced is meaningful, otherwise it is not. In this second case, the field is a physical system per se, with its equations that describe it.

Therefore we can say that a field becomes meaningful when its behaviour is decribed by equations and when the field propagates by waves. The idea is that when the field $\psi(\mathbf{x}, t)$ is changed in a certain point and at a certain time, then its variation propagates in all the space in which the field is defined. The law that describes the propagation of the field in space and time is a wave equation.

It is possible to mention many examples of fields that behave as reported in point 2.; for instance, we can consider a scalar field, as the temperature field. If the temperature is changed in a point, then every point of the space will change its temperature following the heat equation. Moreover the change of the temperatures in all the points of the space will occur in a delayed time with respect the initial change, depending on the distance of each point.

An other common example is the field of the positions of a vibrating string. If the string is plucked in a certain point, the deformation of the string will propagate along the string following the D'Alembert wave equation. Even in this case the deformation will travel along the string with a finite velocity, depending on the tension and the linear density of the string.

But there are also examples of fields as described in the point 1.; let us consider for instance the gravitational field. If we suppose to change the mass in a certain point of the space-time, then instantly, in each point of the space the gravitational field $\psi(\mathbf{x}, t)$, would be changed. If there exist a propagation of the gravitational field, its velocity is infinite. In other words, the gravitational field appears as an unnecessary artifice, being the concept of wave propagation difficult to use.

Instead, the electric and the magnetic fields are two fields that can be described by a set of equations, the Maxwell's equations, and a perturbation of that fields propagates in all the space with the velocity of light (in vacuum) by a wave equation. We can see quite easily that the Maxwell's equations can be written in terms of potentials, rather than in terms of fields, and this permits to emphasize many aspects of the electromagnetism. In this framework the vector potential, besides the scalar potential, becomes very helpful.

7.2.2. The Maxwell's equations in terms of potentials

In the present introduction for teachers we resume the standard introduction of the Maxwell's equations in terms of potentials, as can be found in most textbooks. With usual notation, we can write the Maxwell's equations in their differential form:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \tag{7.1}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{7.2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{7.3}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} - \varepsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}. \tag{7.4}$$

We have to introduce now the concept of vector potential, while we suppose that the scalar potential is already known.

Let **F** be a vector field. If there exist a vector field **P** such that:

$$\mathbf{F} = \nabla \times \mathbf{P}, \tag{7.5}$$

then **P** is called vector potential of the field **F**.

It is straightforward to derive that a solenoidal field admits a vector potential and more generally:

$$\nabla \cdot \mathbf{F} = 0 \Leftrightarrow \mathbf{F} = \nabla \times \mathbf{P}. \tag{7.6}$$

Therefore, from eq.(7.2), there exist a vector **A** called vector potential such that:

$$\mathbf{B} = \nabla \times \mathbf{A}. \tag{7.7}$$

By eq.(7.7) we can rewrite eq.(7.3) for the electric field $\mathbf{E}$ in terms of the vector potential $\mathbf{A}$ and we obtain:

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}(\nabla \times \mathbf{A}) \tag{7.8}$$

and rearranging the terms of eq.(7.8):

$$\nabla \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t}\right) = 0. \tag{7.9}$$

As for every scalar field V we have $\nabla \times (\nabla V) = 0$, from eq.(7.9) we have also that there exists a scalar field V such that:

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla V, \tag{7.10}$$

so that we can write also the electric field in terms of the potentials:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla V. \tag{7.11}$$

Hence, from the Maxwell's equations, we have concluded that both the electric and the magnetic fields can be expressed through their potentials. We have to specify that in the expressions given by eq.(7.7) and eq.(7.11), the potentials are not uniquely determined in terms of the fields, this because they are determined through a process of integration. More precisely, we can add to the magnetic vector potential the gradient of an harmonic function and get the same magnetic field and, similarly, we can add to the scalar potential a constant function to get the same electric field; in formula:

$$V' = V - \nabla \chi \tag{7.12}$$

$$\mathbf{A}' = \mathbf{A} + \nabla \chi \tag{7.13}$$

where χ is a generic scalar function that admits the second derivative.

If V and $\mathbf{A}$ are the potentials respectively for the fields $\mathbf{E}$ and $\mathbf{B}$, then also V' and $\mathbf{A}'$ given by eq.(7.12) and eq.(7.13) are potentials of the same fields. The transformations given by eq.(7.12) and eq.(7.13) are called gauge transformations.

In this way, while the eqs.(7.1) and (7.4) are the Maxwell's equations that give the potentials, the eqs.(7.2) and (7.3) give the relation between the potentials and the field expressed by eqs.(7.7) and (7.11).

It is very interesting now, to rewrite the Maxwell's equations in terms of the potentials. From eq.(7.7) we can write eq.(7.4) as:

$$\nabla \times (\nabla \times \mathbf{A}) = \mu_0 \mathbf{J} - \varepsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}. \tag{7.14}$$

Now, using the vector identity:

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} \tag{7.15}$$

and eq.(7.11), we obtain:

$$\nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} = \mu_0 \mathbf{J} - \varepsilon_0 \mu_0 \frac{\partial}{\partial t} \left(-\frac{\partial \mathbf{A}}{\partial t} - \nabla V \right). \tag{7.16}$$

Rearranging the terms of the previous equation we get:

$$\nabla \left(\nabla \cdot \mathbf{A} + \varepsilon_0 \mu_0 \frac{\partial V}{\partial t} \right) = \nabla^2 \mathbf{A} - \varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} \mathbf{A} + \mu_0 \mathbf{J}. \tag{7.17}$$

We have just noted that the potentials are not fixed uniquely by the field, and for this reason we can choose them with a certain freedom. A convenient possible choice can be done by the assumption:

$$\nabla \cdot \mathbf{A} + \varepsilon_0 \mu_0 \frac{\partial V}{\partial t} = 0, \tag{7.18}$$

that is called Lorenz gauge.

In the Lorenz gauge we rewrite the Maxwell eq.(7.17) as:

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{A} - \nabla^2 \mathbf{A} = \mu_0 \mathbf{J}. \tag{7.19}$$

that is eq.(7.4) written in terms of the vector potential.

What happens to the other Maxwell's eq.(7.1)? We can rewrite eq.(7.1) replacing the electric field by its expression in terms of the magnetic vector potential:

$$\nabla \cdot \left(-\frac{\partial \mathbf{A}}{\partial t} - \nabla V \right) = \frac{\rho}{\varepsilon_0}. \tag{7.20}$$

Taking the divergence of both members, we obtain:

$$\nabla \cdot \left(-\frac{\partial \mathbf{A}}{\partial t} \right) - \nabla^2 V = \frac{\rho}{\varepsilon_0}. \tag{7.21}$$

Now, it is convenient to add and to subtract the quantity $\varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} V$ to eq.(7.21) to get:

$$\left(\varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} V - \nabla^2 V \right) - \frac{\partial}{\partial t} \left(\nabla \cdot \mathbf{A} + \varepsilon_0 \mu_0 \frac{\partial V}{\partial t} \right) = \frac{\rho}{\varepsilon_0}. \tag{7.22}$$

In the Lorenz gauge (eq.(7.18)) the quantity in the second brackets in eq.(7.22) is zero, therefore we obtain the differenetial equation for V:

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}V - \nabla^2 V = \frac{\rho}{\varepsilon_0}, \tag{7.23}$$

that is eq.(7.1) in terms of the potentials.

We observe that the Lorenz gauge besides simplifying the Maxwell's eqs.(7.17) and (7.22) decouples them, giving:

$$\varepsilon_0\mu_0\frac{\partial^2}{\partial t^2}\mathbf{A} - \nabla^2\mathbf{A} = \mu_0\mathbf{J} \tag{7.24}$$

for the vector potential, and:

$$\varepsilon_0\mu_0\frac{\partial^2}{\partial t^2}V - \nabla^2 V = \frac{\rho}{\varepsilon_0} \tag{7.25}$$

for the scalar potential.

We already know ρ and $\mathbf{J}$ as the sources of electric and magnetic field, and now they can be considered as the sources of the scalar and the vector potentials.

The solutions of eq.(7.19) and eq.(7.23) can be written in terms of the sources, and we get:

$$\mathbf{A}\left(\mathbf{r},t\right) = \frac{\mu_0}{4\pi}\int_{\mathrm{V}}\frac{\mathbf{J}(\mathbf{r}',t)}{|\mathbf{r}-\mathbf{r}'|}d\mathrm{V}' \tag{7.26}$$

as a solution of the differential equation in $\mathbf{A}$, and

$$V\left(\mathbf{r},t\right) = \frac{1}{4\pi\varepsilon_0}\int_{\mathrm{V}}\frac{\rho(\mathbf{r}',t)}{|\mathbf{r}-\mathbf{r}'|}d\mathrm{V}' \tag{7.27}$$

as a solution of the differential equation in V, with obvious meaning of the notation.

Moreover, we can write eqs.(7.24) and (7.25) in a more compact form, that is:

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}A_\mu - \nabla^2 A_\mu = J_\mu, \tag{7.28}$$

where the subscript μ runs from 0 to 3, $A_0 = V$, $\mathbf{A} = (A_1, A_2, A_3)$, $J_0 = \frac{\rho}{\varepsilon_0}$ and $\mathbf{J} = (J_1, J_2, J_3)$.

When the Maxwell's equations are expressed in the Lorenz gauge and with the notation of eq.(7.28) they appear in all their elegance and show the importance of the potentials.

To get an even more sophisticated shape of eq.(7.28) it is possible to introduce the D'Alembertian operator $\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$ and write that equation as:

$$\Box A_\mu = J_\mu. \tag{7.29}$$

This equation in the framework of relativity can be very meaningful in order to describe the properties of the electromagnetic field, since eq.(7.29) is clearly covariant.

7.2.3. A new approach for the introduction of the vector potential

We want now to reconsider the problem of the introduction of the vector potential, because we think that it could be more interesting from an educational point of view. So far we have treated the vector potential as it is usual in university physics courses. In particular the gauge fixing that bring us to the definition of the vector potential as defined by eq.(7.26) seems to be a pure mathematical attempt of simplification. Instead, we will see that the next approach allows us to define an identical vector potential starting from physical considerations, thus better understanding the meaning of a certain gauge fixing.

For a general distribution of conduction current density $\mathbf{J}$ and taking also into account the displacement current density, the magnetic field $\mathbf{B}$ at position $\mathbf{r}$ and time t, in vacuum, is given by:

$$\mathbf{B}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\int_{V'} \frac{\left[\mathbf{J}(\mathbf{r}',t) + \varepsilon_0 \frac{\partial \mathbf{E}(\mathbf{r}',t)}{\partial t}\right] \times \Delta\mathbf{r}}{(\Delta r)^3} dV', \tag{7.30}$$

where we posed $\Delta r = |\mathbf{r} - \mathbf{r}'|$.

If we adopt the quasi-static approximation we can neglect the time derivatives multiplied by $\varepsilon_0\mu_0 = 1/c^2$, then the expression for the magnetic field becomes:

$$\mathbf{B}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\int_{V'} \frac{\mathbf{J}(\mathbf{r}',t) \times \Delta\mathbf{r}}{(\Delta r)^3} dV', \tag{7.31}$$

and this is the starting point of our approach.

Observing that:

$$\nabla\left(\frac{1}{\Delta r}\right) = -\frac{\Delta\mathbf{r}}{(\Delta r)^3}, \tag{7.32}$$

and commutating the factors of the vector product in the integrand of equation (7.31), we can write:

$$\mathbf{B}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\int_{V'} \nabla\left(\frac{1}{\Delta r}\right) \times \mathbf{J}(\mathbf{r}',t)\, dV'. \tag{7.33}$$

Keeping in mind that if f is a scalar field while $\mathbf{v}$ is a vector field one has the identity:

$$\nabla \times (f\mathbf{v}) = \nabla f \times \mathbf{v} + f\nabla \times \mathbf{v} \tag{7.34}$$

and using the fact that $\nabla \times \mathbf{J}(\mathbf{r}',t) = 0$ because $\mathbf{J}$ depends on primed variables while the curl is done with respect to unprimed ones, we obtain:

$$\mathbf{B}(\mathbf{r},t) = \nabla \times \left(\frac{\mu_0}{4\pi}\int_{V'} \frac{\mathbf{J}(\mathbf{r}',t)}{\Delta r} dV'\right). \tag{7.35}$$

Eq.(7.35) clearly shows that we can introduce a vector:

$$\mathbf{A}\left(\mathbf{r},t\right) \equiv \frac{\mu_0}{4\pi}\int_{V'} \frac{\mathbf{J}\left(\mathbf{r}',t\right)}{\Delta r} dV' \tag{7.36}$$

such that

$$\nabla \times \mathbf{A}\left(\mathbf{r},\mathbf{t}\right) = \mathbf{B}\left(\mathbf{r},\mathbf{t}\right). \tag{7.37}$$

Eq.(7.37) shows that the vector $\mathbf{A}$ of eq.(7.36) is a magnetic vector potential and that, moreover, in the framework of our slowly varying time-dependent approximation, it is the magnetic vector potential to which one is naturally led by physical considerations.

In addition, eq.(7.36) proves a clear analogy between magnetic vector potential and electric scalar potential

$$V\left(\mathbf{r},t\right) = \frac{1}{4\pi\varepsilon_0}\int_{\mathrm{V}} \frac{\rho(\mathbf{r}',t)}{|\mathbf{r}-\mathbf{r}'|} d\mathrm{V}' \tag{7.38}$$

where $\rho\left(\mathbf{r}',t\right)$ is the charge density at point $\mathbf{r}'$ and time t. The vector potential given by eq.(7.36) is a precise function of the current density. Therefore (in our slowly varying field approximation) once the currents are known, $\mathbf{A}$ is univocally determined. Instead, if we start from eq.(7.37), $\mathbf{A}$ is not unique. In fact, the Helmholtz theorem [53] states that a quasi-static vector field vanishing at infinity more quickly than $1/r$, is completely determined once both its curl and its divergence are known. Therefore, if we want to fix $\mathbf{A}$ an additional condition (the so-called gauge condition) is clearly needed. This is generally done by arbitrary fixing the divergence of $\mathbf{A}$. On the contrary, in our approach, we have no need to fix a gauge. Nevertheless it is interesting to directly calculate $\nabla \cdot \mathbf{A}$.

With the same symbology of eq.(7.34), we have the following vector identity:

$$\nabla \cdot \left(f\mathbf{v}\right) = \nabla f \cdot \mathbf{v} + f\nabla \cdot \mathbf{v}, \tag{7.39}$$

therefore, from eq.(7.36) we obtain:

$$\begin{aligned}\nabla \cdot \mathbf{A}\left(\mathbf{r},t\right) &= \frac{\mu_0}{4\pi}\int_{V'} \nabla\left(\frac{1}{\Delta r}\right) \cdot \mathbf{J}\left(\mathbf{r}',t\right) dV' + \frac{\mu_0}{4\pi}\int_{V'} \frac{1}{\Delta r}\nabla \cdot \mathbf{J}\left(\mathbf{r}',t\right) dV' \qquad (7.40)\\ &= -\frac{\mu_0}{4\pi}\int_{V'} \nabla'\left(\frac{1}{\Delta r}\right) \cdot \mathbf{J}\left(\mathbf{r}',t\right) dV',\end{aligned}$$

where the operator ∇' acts on the primed variables. We note that $\nabla \cdot \mathbf{J}\left(\mathbf{r}',t\right) = 0$, because $\mathbf{J}$ depends only on the primed variables while the divergence is done with respect to the unprimed ones and $\nabla\left(\frac{1}{\Delta r}\right) = -\nabla'\left(\frac{1}{\Delta r}\right)$. Moreover (again keeping in mind eq.(7.39)) the integrand of the last term in eq.(7.40) can be written as follows:

$$\nabla'\left(\frac{1}{\Delta r}\right)\cdot\mathbf{J}\left(\mathbf{r}',t\right)=\nabla'\cdot\left[\frac{\mathbf{J}\left(\mathbf{r}',t\right)}{\Delta r}\right]-\frac{1}{\Delta r}\nabla'\cdot\mathbf{J}\left(\mathbf{r}',t\right) \tag{7.41}$$
$$=\nabla'\cdot\left[\frac{\mathbf{J}\left(\mathbf{r}',t\right)}{\Delta r}\right]+\frac{1}{\Delta r}\frac{\partial\rho\left(\mathbf{r}',t\right)}{\partial t},$$

where, in the last equality, we have used the continuity equation:

$$\nabla\cdot\mathbf{J}\left(\mathbf{r},t\right)+\frac{\partial\rho\left(\mathbf{r},t\right)}{\partial t}=0. \tag{7.42}$$

Therefore we have:

$$\nabla\cdot\mathbf{A}\left(\mathbf{r},t\right)=-\frac{\mu_0}{4\pi}\int_{V'}\nabla'\cdot\left[\frac{\mathbf{J}\left(\mathbf{r}',t\right)}{\Delta r}\right]dV'-\frac{\mu_0}{4\pi}\int_{V'}\frac{1}{\Delta r}\frac{\partial\rho\left(\mathbf{r}',t\right)}{\partial t}dV'. \tag{7.43}$$

The first integral in eq.(7.43) is zero thanks to the divergence theorem. In fact

$$-\frac{\mu_0}{4\pi}\int_{V'}\nabla'\cdot\left[\frac{\mathbf{J}\left(\mathbf{r}',t\right)}{\Delta r}\right]dV'=-\frac{\mu_0}{4\pi}\oint_{\Sigma'}\frac{\mathbf{J}\left(\mathbf{r}',t\right)}{\Delta r}\cdot\mathbf{n}d\Sigma', \tag{7.44}$$

where $\Sigma'\equiv\partial V'$ is the boundary of the region V' and $\mathbf{n}$ is the outer normal to Σ'. Since V' must contain at each time all the currents that generate $\mathbf{A}$, it can be taken so large that $\mathbf{J}$ can be considered zero upon Σ' and therefore the right hand side integral in eq.(7.44) vanishes. For the second integral in the right hand side of eq.(7.43), since V' is time-independent, we get:

$$\frac{\mu_0}{4\pi}\int_{V'}\frac{1}{\Delta r}\frac{\partial\rho\left(\mathbf{r}',t\right)}{\partial t}dV'=\frac{\mu_0}{4\pi}\frac{\partial}{\partial t}\int_{V'}\frac{\rho\left(\mathbf{r}',t\right)}{\Delta r}dV'=\frac{\mu_0}{4\pi}\frac{\partial}{\partial t}\left[4\pi\varepsilon_0 V\left(\mathbf{r},t\right)\right], \tag{7.45}$$

where $V\left(\mathbf{r},t\right)$ is the electric scalar potential given by eq.(7.38). From equations (7.43) - (7.45) we finally obtain:

$$\nabla\cdot\mathbf{A}\left(\mathbf{r},t\right)=-\varepsilon_0\mu_0\frac{\partial}{\partial t}V\left(\mathbf{r},t\right)=-\frac{1}{c^2}\frac{\partial}{\partial t}V\left(\mathbf{r},t\right). \tag{7.46}$$

Eq.(7.46) is the well-known Lorenz gauge that we have already treated in the previous section. In the quasi-static approximation that we are adopting here, the right hand term of eq.(7.46) can be considered zero, and therefore we are left in the so-called Coulomb gauge:

$$\nabla\cdot\mathbf{A}\left(\mathbf{r},t\right)=0. \tag{7.47}$$

The most common attitude is to arbitrary fix $\nabla\cdot\mathbf{A}\left(\mathbf{r},t\right)=0$ from the beginning, so that eq.(7.36) is obtained as a final result. On the contrary, we have followed an inverse path in which we have been naturally led to select the magnetic vector potential expressed in terms of the current density (that can therefore be seen as the source of the potential), as we have done in eq.(7.36). As a consequence of this choice we found that the magnetic vector potential of eq.(7.36) is given in the Coulomb gauge, that therefore can be seen as the "natural" gauge for slowly varying fields.

As it is well known the relations which give a link among the fields $\mathbf{E}(\mathbf{r},t)$, $\mathbf{B}(\mathbf{r},t)$ and the potentials are:

$$\mathbf{E}(\mathbf{r},t) = -\frac{\partial}{\partial t}\mathbf{A}(\mathbf{r},t) - \nabla V(\mathbf{r},t) \tag{7.48}$$

and eq.(7.37). In the general case, when the following trasformations, that we have called gauge transformations, are performed:

$$V \to V' = V - \frac{\partial \chi}{\partial t}, \tag{7.49}$$

$$\mathbf{A} \to \mathbf{A}' = \mathbf{A} + \nabla\chi, \tag{7.50}$$

the electric and the magnetic fields remain unchanged. Here we want to stress that our choice of a vector potential given in terms of the conduction current (eq. (7.36)) is not completely equivalent to the choice of the Coulomb gauge. Infact, while $\mathbf{A}$ expressed by eq.(7.36) is completely determined once the currents are known, a vector potential in the Coulomb gauge is determined only up to the gradient of an harmonic function, as can be seen from eq.(7.50).

In this approach the problem relative to the gauge fixing is considered only after treating the introduction of a vector potential for a quasi-static approximation, and therefore the Coulomb gauge comes out as the natural gauge for a quasi-static magnetic field.

7.2.4. The physical meaning of the magnetic vector potential

We cannot directly assign a physical meaning to the electric scalar potential itself. The physical meaning can only be assigned to its spatial derivative or to the difference between the potentials at two separate points. However, in many physical situations, we can choose one of the two different points at the spatial infinity, where we can fix the value of the scalar potential equal to zero. A particularly interesting case is when the magnetic vector potential $\mathbf{A}$ is time independent. In fact, in this condition, we can define the potential energy $U(\mathbf{r},t)$ of a point-like charge q set at position $\mathbf{r}$ at time t, as the work (independent of the chosen path) necessary to move the charge q from infinity, where the electric field is zero, to the point $\mathbf{r}$, against the forces of the electric field; that is:

$$U(\mathbf{r},t) = -\int_{\infty}^{\mathbf{r}} q\mathbf{E}(\mathbf{r}',t)\cdot d\mathbf{r}'. \tag{7.51}$$

The electric scalar potential can therefore be written as:

$$V(\mathbf{r},t) = -\int_{\infty}^{\mathbf{r}} \mathbf{E}(\mathbf{r}',t)\cdot d\mathbf{r}'. \tag{7.52}$$

Therefore, the scalar potential has the clear physical meaning of potential energy per unit charge and can be identified with the function V of eq.(7.48). We note that the integrals of equations (7.51) and (7.52) are performed only over the spatial coordinates while the time coordinate is a fixed parameter.

Likewise, we cannot assign a physical meaning even to the magnetic vector potential itself, while it can be assigned to its time derivative or to the difference between the vector potentials at two different times. Similarly to what is done for the scalar potential, in many physical situations we can choose one of the two different times at $t = -\infty$ and fix $\mathbf{A}(\mathrm{t} = -\infty) = 0$. An interesting situation arises when $\nabla V(\mathbf{r},t) = 0$ and the magnetic field is slowly varying in time.

In this case we can give a physical meaning to the vector potential $\mathbf{A}$ of eq.(7.48), which is univocally determined and naturally given by eq.(7.36). To do this, we have to exchange the roles of the variables $\mathbf{r}$ and t; that is, we have to perform an integral over the time coordinate while the point $\mathbf{r}$ remains fixed. Let us consider a point-like charge q in the position $\mathbf{r}$ at a time, which we will indicate as $-\infty$, when the currents and consequently the magnetic field are zero. Let us now slowly switch the currents on. They will generate a magnetic field $\mathbf{B}$, a vector potential $\mathbf{A}$ and therefore an electric field $\mathbf{E}(\mathbf{r},t) = -\frac{\partial}{\partial t}\mathbf{A}(\mathbf{r},t)$ that will act on q. In order to keep q fixed in $\mathbf{r}$, an impulse must be applied against the field forces, and this is given by:

$$\boldsymbol{\Upsilon}(\mathbf{r},t) = -\int_{-\infty}^{t} q\mathbf{E}(\mathbf{r},t')\,dt' = \int_{-\infty}^{t} q\frac{\partial}{\partial t'}\mathbf{A}(\mathbf{r},t')\,dt' = q\mathbf{A}(\mathbf{r},t), \tag{7.53}$$

where we have put $\mathbf{A}(\mathrm{t} = -\infty) = 0$, since the currents are zero at that time.

The impulse against the magnetic force does not depend on the time dependence of $\mathbf{A}$, provided the field is slowly varying. The magnetic vector potential can thus be interpreted as the total momentum per unit charge that must be transferred to a charge, during the time interval $(-\infty, t)$, in order to keep this charge at rest at the point $\mathbf{r}$ while the field varies slowly from zero to the value $\mathbf{B}$ in that time interval. Magnetic vector potential can be therefore considered as a “momentum vector” per unit charge, while the electric scalar potential can be seen as an energy component. As qV is called potential energy, $q\mathbf{A}$ can be called potential momentum (of the charge q, at point $\mathbf{r}$ and time t).

7.2.5. A stimulating hint

Besides its physical meaning, the magnetic vector potential gives us also the possibility to write in a clearer and more understandable way some physical relations. For instance, a mechanical harmonic plane wave of amplitude S_0 and angular frequency ω, propagating in a medium of density ϱ with velocity v, carries an intensity given by:

$$I = \frac{1}{2}\rho\omega^2 S_0^2 v. \tag{7.54}$$

If we consider an electromagnetic linearly polarized, harmonic, plane wave of amplitude E_0, angular frequancy ω, propagating in a medium of absolute dielectric permittivity ε with velocity v, its intensity is usually written without explicitly showing the angular frequency, that is as:

$$I = \frac{1}{2}\varepsilon E_0^2 v. \tag{7.55}$$

The magnetic vector potential gives the possibility to write eq.(7.55) in a form completely similar to eq.(7.54). In fact, from the first term of eq.(7.48) and denoting the vector potential amplitude with A_0, we immediately have:

$$I = \frac{1}{2}\varepsilon\omega^2 A_0^2 v. \tag{7.56}$$

Eq.(7.56) shows that the vector potential plays for the electromagnetic field the same role played by the displacement from the equilibrium position for a mechanical wave propagating in a medium (see equation (7.54)). Moreover, since the intensity, the frequency and the velocity of propagation can be all measured, eq.(7.56) immediately yields A_0.

Eq.(7.53) and (7.56) and their interpretations clearly show that the magnetic vector potential is not a simple mathematical tool, but it has a deep physical meaning and can greatly help visualization.

8. The magnetic vector potential in secondary school

8.1. Overview

In this chapter we propose an educational path on vector potential. In this path the mathematical tools used are the circulation and the flux of a vector field, the same tools that students already encountered in their traditional courses of electromagnetism. In this part we do not care if students are really familiar with circulation and flux, in fact we will see in the next chapter that they have great difficulties with them, but, so far we suppose that they have already understood their meaning and that thay are able to manage these tools. Besides the fact that the vector potential is fundamental for a treatment of superconductivity in a mathematical consistent way, we are confident that the vector potential can be useful also to better understand the electromagnetism concepts that, in our experience, are frequently sources of difficulties. This last thing will be treated in detail in the next chapter and we hope that it could encourage teachers in facing a topic that is never present in the secondary school curriculum.

8.2. The concept of potential

The concept of scalar potential is challenging for students, and teachers are often tempted to postpone its explanation and its use as much as possible. This happens usually in mechanics, when teachers introduce the concept of kinetic and potential energy, but on the contrary they don't speak about the scalar potential until the electromagnetism course.

The electric scalar potential, in turn, is often treated as a kind of necessary step in order to solve electrical circuit where appears the potential difference ΔV. Instead, in the educational path that we present here, the concept of potential is of fundamental importance and should become a unifying concept rather than a difficulty to be avoided!

For this reason the path contains an introductory part in which the concept of potential is developed starting from mechanics. Here, we describe this part: the introduction of the vector potential for secondary school students is made in strict analogy with the introduction of the electric scalar potential. Therefore, it is very important that students have a clear understanding of the concept of scalar potential and therefore, in the following we will start with a general introduction on the scalar potential in order to recall concepts that students should already have treated and know, for instance from their previous courses of mechanics.

8.2.1. The definition of potential

Among the various forces we encounter in nature, there is a particular class of them that has a very special feature: the work W_{AB} done by these forces from a point A to a point B along the path γ does not depend on the path, but only from the points A and B. To these forces we give the name of *conservative forces*. From this property of the conservative forces, we can observe that if the points A and B coincide, that is γ is a closed path, the work done by a conservative force is zero.

In more formal terms, we say that the circulation of the force is zero and we write:

$$\oint \mathbf{F} \cdot d\mathbf{s} = 0. \tag{8.1}$$

The work-energy theorem states that:

$$W_{AB} = \Delta K_{AB}, \tag{8.2}$$

where ΔK_{AB} is the variation of the kinetic energy from the point A to the point B. Through this theorem, that holds in general, even for not conservative forces, it is possible, when the forces are conservative, to define the potential. We can refer to Fig. 8.1 in order to perform our calculations.

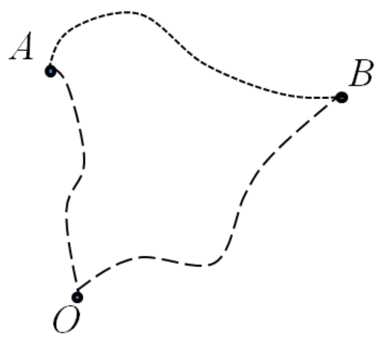


Figure 8.1.: A general path along which it is possible to calculate the work and the kinetic energy.

From the work-energy theorem we have:

$$W_{AB} = K_B - K_A. \tag{8.3}$$

We now suppose of being in the particular case of conservative force, hence the work done depends only from the endpoints of the chosen path, and does not depend on the path itself. Therefore, referring to Fig. 8.1 we have that the work done along the higher path AB is equal to the work done along the lower path AOB, thus we can write:

$$W_{AB} = W_{AO} + W_{OB}. \tag{8.4}$$

Rearranging the terms of eq.(8.4) and using eq.(8.3) we obtain:

$$K_B - W_{OB} = K_A + W_{AO}. \tag{8.5}$$

Now, for the property of the line integral through which the work is defined we have that $W_{OA} = -W_{AO}$ and hence:

$$K_B - W_{OB} = K_A - W_{OA} \tag{8.6}$$

and it clearly appears that the quantity $K_A - W_{OA}$ is a constant of the motion, that is a physical quantity that is conserved during the motion, that we call mechanical energy. Besides the mechanical energy it is possible to define the potential energy of the object that is in the position A respect its potential energy in the position O, that we choose as a reference. If we call U_{OA} this potential energy, we have:

$$U_{OA} = -W_{OA}. \tag{8.7}$$

In this way the sum of the kinetic energy and the potential energy of the object in each point of the path is conserved.

From the calculation performed so far it appears that the mechanical energy of an object can be defined only if:

- it is defined a frame of reference in order to define the kinetic energy, being the kinetic energy expressed by the velocity of the object (and the velocity clearly depends on the reference);
- within the frame of reference, it is chosen a point O of reference in order to define the potential energy of each other point of the space.

Keeping in mind this conceptual framework a students can now follow some applications of the general theory seen so far.

8.2.2. The gravitational potential

We describe initially the gravitational potential, because we guess that it is well-known to students. Treating this case, we would like to show to students a general way of reasoning in physics, and let students sort out the concepts of energy, work, line integral and circulation, and potential. This part needs the mathematical concept of integral and a calculus based notation, therefore it can be considered a stimulus to review the most important ideas covered so far and to practice the mathematical tool just introduced. It is for this reason that this part is thought for students that are dealing with integrals in mathematics, so to apply the abstract notions in a physical context and make more profitable both the topics.

8.2.2.1. Approximation of uniform gravitational field

The gravitational force is conservative. Nevertheless, in the present case in which the field considered is uniform, we have anyhow a conservative field.

For simplicity, we start from the gravitational field, in its approximation of uniform field of acceleration indicated by $\mathbf{g}$. As well-known, in each point of the space (sufficiently close to the ground) the force acting on a mass m is given by $\mathbf{F} = m\mathbf{g}$. If the expression of the force is known, then it is possible to calculate the work done by the gravitational field on the mass m along a path γ whose endpoints are A and B. The work is a line integral and is indicated by:

$$W_{AB} = \int_A^B \mathbf{F} \cdot d\mathbf{x} = \int_{\mathbf{A}}^{\mathbf{B}} m\mathbf{g} \cdot d\mathbf{x}. \tag{8.8}$$

In two-dimensions, we can represent the convenient path that we have chosen for our calculations as in Fig. 8.2.

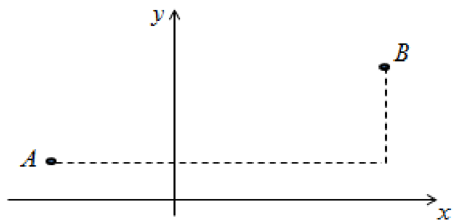


Figure 8.2.: Scheme of the path along which to calculate the work done by the gravitational force.

Solving the integral we obtain:

$$W_{AB} = mg\,(y_A - y_B) = -mgh, \tag{8.9}$$

where $h = y_B - y_A$, that is the variation of the height of the mass m. As we seen in the previous section, it is possible to apply the work-energy theorem, thus obtaining:

$$mgy_A - mgy_B = \frac{1}{2}mv_B - \frac{1}{2}mv_A \tag{8.10}$$

and hence the constant of motion:

$$E = \frac{1}{2}mv^2 + mgx. \tag{8.11}$$

It is now possible to give meaning to the physical quantity $U(y) = mgy$, the gravitational potential energy, from which it is possible to deduce the gravitational force by a derivative process:

$$F = -\frac{dU(y)}{dy}. \tag{8.12}$$

Indeed, this is a mathematical simplification. Infact the force $\mathbf{F}$ is a vector quantity, and cannot be obtained from the derivative of the scalar quantity $U(x)$, as eq.(8.12) shows. Eq.(8.12) shows the mono-dimensional case, but if the mathematics were more precise, eq.(8.12) should be written in terms of the gradient, a differential operator in which are present all the three derivatives, $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial y}$,$\frac{\partial}{\partial z}$. Using the gradient we could obtain the vector $\mathbf{F}$ directly by the scalar potential $U(x)$:

$$\mathbf{F}(x) = -\nabla U(x), \tag{8.13}$$

where $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y},\frac{\partial}{\partial z})$. This can be mentioned to students, even if in their calculations they deal with mono-dimentional cases.

By the potential energy $U(x)$ it is possible to define the potential $V(x)$ that differs from the potential energy by the fact that it does not depend from the particular object considered, but depends only from the particular point of the space. As we have seen in the previous section the potential can be defined with a certain freedom, because it can vary with the particular point of reference O chosen. In this case, if we want to get the field of acceleration $\mathbf{g}$, by a simple derivative process we can define $V(x) = -U(x)/m$.

8.2.3. The general scheme

The example discussed in the previous section allows the definition of a general scheme in which the concept of potential is meaningful.

- Description of the force $\mathbf{F}(x)$, that is related with the field we are dealing with.
- By a line integral it is possible to determine the work W_{AB} done by the force $\mathbf{F}(x)$ along a path γ whose endpoints are A and B.
- By the fundamental work-energy theorem: $W_{AB} = \Delta K_{AB}$, it is possible to define (when the field is conservative) the potential energy $U(x)$.
- If one does not have the force but has the potential $U(x)$, one can obtain the force by: $\mathbf{F}(x) = -\nabla U(x)$, so that from $\mathbf{F}$ one obtains the potential U by integration. The descriptions in terms of force or potentials are equivalent.
- By the potential energy it is possible to define the potential $V(x)$ that differs from the potential energy by a multiplicative factor.

8.2.4. Examples of potential

Following the scheme of the previous section we can get many other potentials. In our educational path at least three hours of lessons are on this topic, and here we briefly report the potentials that we discussed with students.

8.2.4.1. Gravitational potential, in the general case

In this case the gravitational force depends on the distance r of the mass m from the centre of mass of the earth, that we suppose having mass M. Also in this case we have a conservative field and we can suppose to have a mono-dimensional problem thus avoiding the use of the gradient. The force is well-known and its intensity has the expression:

$$F = G\frac{m \cdot M}{r^2}, \tag{8.14}$$

from which we can get the work:

$$W_{AB} = GmM\left(\frac{1}{r_B} - \frac{1}{r_A}\right) \tag{8.15}$$

and applying the theorem of the kinetic energy we can get the potential energy:

$$U(r) = -G\frac{mM}{r}. \tag{8.16}$$

It is very simple for students to obtain the force by a derivative process:

$$F = -\frac{dU(r)}{dr} = G\frac{m \cdot M}{r^2} \tag{8.17}$$

or define the gravitational potential:

$$V(r) = G\frac{M}{r}. \tag{8.18}$$

By eq.(8.18) it is possible to get the gravitational field by a derivative process:

$$g(r) = -\frac{d}{dr}\left(G\frac{M}{r}\right). \tag{8.19}$$

8.2.4.2. Elastic potential

This is another example that comes from mechanics, and it should be known to students.

The elastic force is given by the Hook's law and has the expression:

$$\mathbf{F} = -k\mathbf{x}, \tag{8.20}$$

from which we can get the work:

$$W_{AB} = -\frac{1}{2}k\left(x_B^2 - x_A^2\right) \tag{8.21}$$

and applying the theorem of the kinetic energy we can get the potential energy:

$$U(r) = \frac{1}{2}kx^2. \tag{8.22}$$

It is very simple for students to obtain the force by a derivative process:

$$F = -\frac{dU(r)}{dr} = -kx \tag{8.23}$$

In this particular case the potential is equal to the potential energy, because the potential energy is already given as a property of the space, rather than a property of the particular object.

It is thus possible to obtain the elastic force by a derivative process:

$$F(x) = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx, \tag{8.24}$$

where the minus sign appears even if we are dealing with the intensity F, and this because it should be only the x-component of the vector $\mathbf{F}$.

8.2.4.3. Electric potential

The determination of the electric potential is a very useful and instructive exercise. While the concept of potential is seldom introduced in mechanics, it is different in electromagnetism. Infact, students usually follow their teacher doing this calculation in electrostatics, but in our experience, very little remains to them. It is important instead, that this calculation is performed by student in the two different cases of potential generated by a positive or negative charge. This could appear boring and ripetitive, but it should be considered as an exercise to practice with integral calculations.

Following the scheme of the previous examples, we have:

$$F = k\frac{Q \cdot q}{r^2}, \tag{8.25}$$

with obvious meaning of notation, and where we are considering the case in which the charge Q generates the field and attracts or repels the charge q.

We can get the work:

$$W_{AB} = kQq\left(\frac{1}{r_A} - \frac{1}{r_B}\right), \tag{8.26}$$

where we can notice that the sign is opposite to that of the gravitational case: infact, when the charges are of the same sign the force is repulsive, while the force between two masses is always attractive. Now, applying the theorem of the kinetic energy we can get the potential energy:

$$U(r) = k\frac{Q \cdot q}{r}. \tag{8.27}$$

It is very simple for students to obtain the force by a derivative process:

$$F = -\frac{dU(r)}{dr} = k\frac{Q \cdot q}{r^2} \tag{8.28}$$

or define the gravitational potential:

$$V(r) = \frac{U(r)}{q} = k\frac{Q}{r}. \tag{8.29}$$

By eq.(8.29) it is possible to get the electric field by a derivative process:

$$E(r) = -\frac{d}{dr}V\left(r\right), \tag{8.30}$$

that, for teachers, or for students that can appreciate some hints on vector operators:

$$\mathbf{E}(r) = -\nabla V(r). \tag{8.31}$$

8.2.4.4. Potential inside a capacitor

This is the last example that we present to students in our path. It needs a very simple calculation. Nevertheless, when in our written tests we asked to students the determination of the electric scalar potential inside a capacitor, they never gave an answer. It will be described in detail later the answers that we obtained in our experimentation.

We imagine to have a uniform electric field inside the condenser. Since the field is uniform we have that the field is conservative and hence the work can be evaluated between to points aligned with the field lines, to simplify the calculations. We obtain:

$$W_{AB} = qE\left(x_B - x_A\right), \tag{8.32}$$

where E is the intensity of the electric field, and q is the charge that moves from x_A to x_B.

Applying the work-energy theorem we obtain the potential energy:

$$U(x) = -qEx \tag{8.33}$$

and the potential, simply dividing by the charge q, as it is usual for the electric potentials, so we have:

$$V(x) = -Ex, \tag{8.34}$$

with the graphical representation in (Fig. 8.3), that was asked to students besides the mathematical evaluation.

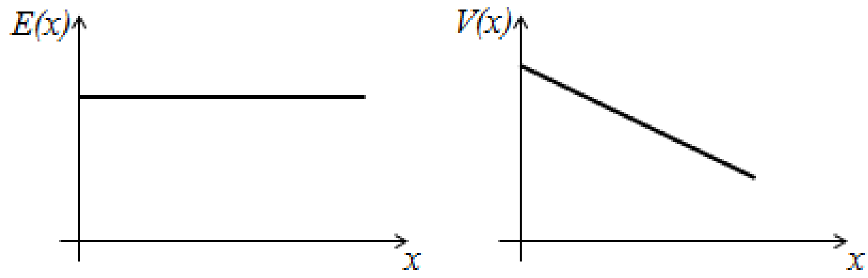


Figure 8.3.: Diagram of the intensity of the electric field $E(x)$ and the electric potential $V(x)$ inside a capacitor.

8.3. The magnetic vector potential for secondary school students

8.3.1. Motivations to face the magnetic vector potential

So far we have given a description of the magnetic vector potential for teachers, using some mathematics and in particular differential operators. In these terms the vector potential seems to be out of the students' possibilities. The aim of this section is to show that the vector potential can be introduced to secondary school students in a way that is symple and effectiveness, as for the scalar potentials seen in the previous section. Infact we will give many examples in such a way to allow students to familiarize with the new physical quantity, even without using too difficult mathematical tools [58, 59].

The previous part that we addressed to teachers, on the contrary, has been more complex and more complete on its theoretical basis, and this with the purpose of motivating teachers in treating the vector potential with their students. In our experience, most of the teachers we met during the PLS activities do not remember to have dealt with the vector potential in their university studies. Only teachers with a degree in physics remember to have treated the vector potential and they associate it with a complicated mathematical tool that have no physical meaning.

It is quite sure that italian physics teachers have never thought to introduce the vector potential to their students and it is never part of students textbooks. So, what are the advantages of introducing the vector potential at secondary school?

From a physical point of view the vector potential is needed for a presentation of modern physics. Infact, in each presentation of the quantum physics, although elementary and discoursive, one of the protagonists is the photon. But the photon is nothing more than the quantum of the electromagnetic field, whose quantization infact needs the vector potential. The new curricula for the secondary school recommend to devote the last year to the modern physics, thus it seems very difficult to deal with the modern physics avoiding the presentation of the photon and, in turn, it seems very difficult to deal with the photon without the vector potential.

But, even remaining in the domain of the quantum physics, there is another meaningful phenomenon that can be presented to students: it is the Aharanov-Bohm effect that can be explained only by the use of the vector potential. There are a lot of important examples in modern physics in which the vector potential plays a crucial role, but this is not the place to discuss it.

Besides the quantum aspects of the vector potential, our interest in it is certainly linked to the fact that the vector potential is fundamental in the description of the superconductive phenomena at a level that is suitable for the secondary school. But this is not sufficient in our opinion, to justify all the work that this would require. Infact, as we shall see, the introduction of the vector potential by means of integral mathematical tools, allows students to familiarize with the classical electromagnetism and better understand many aspects of the classical electromagnetism. Moreover, it allows to improve the understanding of the concepts of flux and circulation, through which the Maxwell's equations are expressed.

In our experience, even if students have already dealt with the Maxwell's equations, they continue to have great difficulties on the concepts of flux and circulation, in their definition and above all in the comprehension of their physical meaning. This, in turn, makes us to believe that the educational path for the electromagnetism has to be revised in order to achieve a better understanding of a fundamental topic as electromagnetism and we are confident that the vector potential could help in this goal.

8.3.2. A definition of the vector potential in analogy with the scalar potential

A very important thing in education is to anchor the new knowlwdge to the old one. In this way the new knowledge has the dual role of setting the old knowledge better and better entrench itself. For this reason we think that one possible way of introducing the vector potential is by an analogy with the electric scalar potential, that students certainly already know.

It is quite surprisingly that whereas the electric scalar potential is always part of the secondary schools physics curriculum, the magnetic vector potential is never mentioned. It is as if in the common mentality of teachers, the two potentials did not have the same utility in understanding electromagnetism and, moreover, it seems that teachers do not think that the vector potential is tightly linked to the magnetic field, just as the scalar potential is linked to the electric field. In this section we want to carry out the analogy between the two potentials, in such a way that is presentable for secondary school.

With secondary school students it is possible to write quite simply the scalar potential, supposing that it is null at infinite and that it is generated by a generic charge distribution that that we have shown in Fig. 8.4. In that image it is highlighted a particular element of the charge distribution ΔQ_i that is indicated by the generic subscript i.

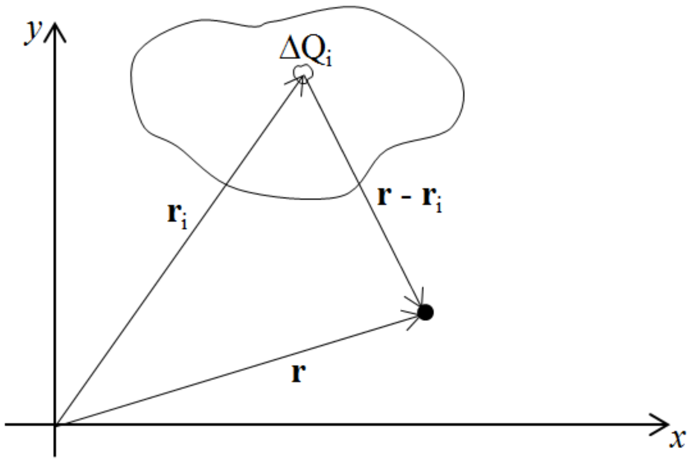


Figure 8.4.: A generic charge distribution for the determination of the scalar potential in the point P.

With obvious meaning of the notation used, we can write the expression for the potential V in the point P:

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{\Delta Q_1}{|\mathbf{r}-\mathbf{r}_1|} + \frac{\Delta Q_2}{|\mathbf{r}-\mathbf{r}_2|} + ... + \frac{\Delta Q_N}{|\mathbf{r}-\mathbf{r}_\mathrm{N}|}\right). \tag{8.35}$$

The analogy with the vector potential can be established when students are able to recognize what are the sources of the fields they have to consider, that is once they realize that the electric charge is the source of the electric field and the currents are the sources of the magnetic field.

Infact, when the students define the magnetic vector potential they have already treated the magnetic field and they should already know that the currents are its sources. One could object that a magnetic field can be generated by the time variation of an electric field. This is true, but in this way of reasoning the magnetic field is a magnetostatic field, just as the electric field above is an electrostatic field (infact its sources are the charges and are not the time variations of a magnetic field).

Referring to Fig. 8.5, we then introduce a new physical quantity in which we substitute the sources of the electric field with the sources of the magnetic field, thinking that the currents we are considering are generated by wires:

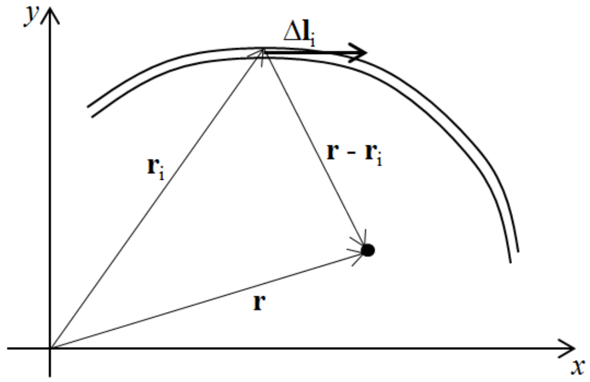


Figure 8.5.: A generic current distribution for the determination of the magnetic vector potential in the point P.

$$\mathbf{A}\left(\mathbf{r}\right) = \frac{\mu_0}{4\pi}\left(\frac{i_1\Delta\mathbf{l}_1}{|\mathbf{r}-\mathbf{r}_1|} + \frac{i_2\Delta\mathbf{l}_2}{|\mathbf{r}-\mathbf{r}_2|} + ... + \frac{i_N\Delta\mathbf{l}_N}{|\mathbf{r}-\mathbf{r}_\mathrm{N}|}\right), \tag{8.36}$$

where the quantities $\Delta\mathbf{l}_i$ are unit vectors in the same direction and in the same verse of the current i in the particular trait of the current distribution placed in the point $\mathbf{r}_\mathrm{i}$.

Depending on the level of the students, the teacher could write eqs.(8.35) and (8.36) using an integral rather than the finite sum, in such a way one obtains:

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{\rho(\mathbf{r}')dV'}{|\mathbf{r}-\mathbf{r}'|} \tag{8.37}$$

and

$$\mathbf{A}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{\mathbf{J}(\mathbf{r}')dV'}{|\mathbf{r}-\mathbf{r}'|}. \tag{8.38}$$

The vector nature of the magnetic vector potential defined by the eq.(8.36) arises as a consequence of the fact that the contributions of the elements of current cannot be considered as scalar quantities as for the charges. This is quite intuitive, but it gives to eq.(8.38) or equivalently to eq.(8.36) a much more complicated structure than that of eq.(8.37) or equivalently of eq.(8.35). It is also quite obvious that eqs.(8.36) or (8.38) cannot be applied to determine the vector potential of an arbitrary current distribution, because it would be too complicated.

We will see later that we can solve this problem using another property of the vector potential that we have not yet mentioned. Instead, it could be interesting for teachers to have some considerations about eq.(8.36) or (8.38).

- Usually, as we have already seen, the vector potential is defined by the relationship $\mathbf{B} = \nabla \times \mathbf{A}$, that is a not direct definition. This implies that it is difficult for a student to get an empircial referent to this physical quantity and secondarily that the definition of the potential is not unique, as we have already seen dealing with the gauge invariance.
- If the definition of the vector potential is given by eq.(8.38) or eq.(8.36) then we have a direct definition of the potential, and hence a relation between the potential and its sources: the electrical currents. In this case we can write "$\mathbf{A} = ...$" and we determine uniquely the vector potential despite of the gauge invariance.
- The relation between the vector potential and the current densities allows to have an empirical referent in such a way to give a picture the vector potential, even for students, because the behaviour of the vector potential follows the behaviour of the electrical currents.

Therefore, following this approach for the presentation of the vector potential, it remains to clarify something about the gauge invariance because it does not appear as a problem to solve in the definition given by eq.(8.36) or (8.38), but this can be done subsequently, specially for students that ignore the problem of the gauge fixing.

8.3.3. A definition of the vector potential from the Maxwell's treatise

In his treatise on electromagnetism, Maxwell introduces the vector potential in a very simple way, that can be suitable for secondary school students [60]. We report here the essential lines of Maxwell's approach.

Let us imagine to have a magnetic field $\mathbf{B}$ in a certain region of space. Let us consider a closed path γ in that region of space and two surfaces S_1 and S_2 that have γ as a boundary. The two surfaces are such that $S = S_1 \cup S_2$ is a closed surface, as Fig. 8.6 shows.

From the solenoidality of the magnetic field $\mathbf{B}$ we have that the flux of the magnetic field through the closed surface S must be zero, that is:

$$\Phi_S(\mathbf{B}) = 0. \tag{8.39}$$

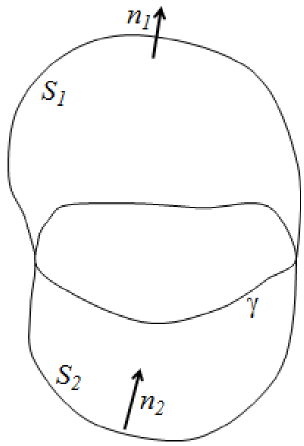


Figure 8.6.: The two open surfaces S_1 and S_2 and their common boundary γ. The two surfaces are such that their union gives the closed surface S.

Now, we choose the normal vectors to the surfaces in such a way to give the same orientation for S_1 and S_2 with respect the orientation of γ as it is shown in Fig. 8.6. Therefore, for the property written in eq.(8.39) together with the orientation of S_1 and S_2 we obtain:

$$\Phi_{S_1}(\mathbf{B}) = \Phi_{S_2}(\mathbf{B}) . \tag{8.40}$$

We recall that S_1 and S_2 are two surfaces arbitrarily chosen, and we can now follow the same way of reasoning of Maxwell. Since the flux of the magnetic field through a surface bounded by a closed line depends only on the closed line and does not depend on the surface bounded by that closed line, it must be possible to determine the flux of the magnetic field through an open surface, by a process that involves only the closed path and that does not involve the surface that has that closed line as a boundary.

The evaluation of the flux of the magnetic field **B** can be done if it is possible to find a vector **A**, related to the magnetic field **B**, whose circulation along the closed line is equal to the flux of the magnetic field through a surface that has the closed line as a boundary. This can be mathematically synthesized as:

$$\int_\gamma \mathbf{A}\cdot d\mathbf{l} = \int_\Sigma \mathbf{B}\cdot d\mathbf{a}, \tag{8.41}$$

in which $d\mathbf{l}$ is the oriented element of the closed line γ and $d\mathbf{a}$ is the oriented element of the surface Σ that has γ as a boundary.

In the continuation of this work it is possible that we will refer to eq.(8.41) by the next more concise formula:

$$C_\gamma(\mathbf{A}) = \Phi_\Sigma(\mathbf{B}) . \tag{8.42}$$

At this point we have two different definitions for the vector potential. They are both possible and effectiveness for students and, moreover, each of them allows to emphasize a particular aspect of the magnetic vector potential. Through the definition given by eq.(8.38) in which it has been carried out a parallelism between the scalar potential and the vector potential, we can focus on the methodological analogies that permit to better understand the properties of the electric and the magnetic field; instead, through the definition given by eq.(8.42) we do not take into account the properties of the electric field and we focus only on the magnetic field, obtaining an integral definition of the vector potential that can be used for the determination of the vector potential in a similar way to the use of the Gauss theorem for the determination of the electric field in some simple situations.

We also notice that eq.(8.42) is the integral version of the relation $\mathbf{B} = \nabla \times \mathbf{A}$, and that it is much more general than the definition given by eq.(8.38). Infact the relation (8.38) can be used only with slowly time-varying fields. But the combination of eq.(8.38), that establishes the symmetry of $\mathbf{A}$ in terms of the currents, and eq.(8.42), allows in some cases a simple determination of vector potentials. The use of eq.(8.42) alone would imply a not uniquely defined vector potential, while eq.(8.38) alone would imply too complicated calculations.

Ultimately, the path we suggest starts from eq.(8.38) to give eq.(8.42) as a property of the vector, but without demonstration. Infact, it is very important to know that the two definitions are related by a theorem that states the equivalence between them, if the fields considered are slowly varying with time and goes to zero at infinity. This theorem is beyond the level of a secondary school, but it can be enunciated to students, thus allowing them to have a framework coherent and complete of this problem.

It could be useful to recall the analogy with the case of the electric field. That is, while from the one hand it is usually presented the definition that gives the coulombian electric field generated by N charges or by a charge distribution, on the other hand, the calculations to determine the electric fields generated by certain charge distributions is carried out through the Gauss theorem. This is a way to overcome the mathematical difficulty to solve the problem starting from the definition. It is exactly the same for the vector potential.

8.3.4. A new formulation for the electromagnetic induction using the vector potential $\mathbf{A}$

The effort that teachers may feel for the introduction of the magnetic vector potential with their students has yet even some advantages. The first we present pertains the possibility of rewrite the complicated Maxwell's equation:

$$C_\gamma (\mathbf{E}) = -\frac{d}{dt}\Phi_\Sigma (\mathbf{B})\,. \tag{8.43}$$

By eq.(8.42) we can simplify eq.(8.43) and we obtain:

$$C_\gamma\left(\mathbf{E}\right) = -\frac{d}{dt}C_\gamma\left(\mathbf{A}\right) = C_\gamma\left(-\frac{\partial}{\partial t}\mathbf{A}\right). \tag{8.44}$$

But for the arbitrariety of the closed path γ we can conclude that:

$$\mathbf{E} = -\frac{\partial}{\partial t}\mathbf{A}. \tag{8.45}$$

This is the new expression for the Maxwell's eq.(8.43) and it is possible to appreciate its simpler expression: the electric field can be found by a simple derivative process, if it is known the vector potential.

In eq.(8.45) appears the partial derivative symbol. Even if it is never used in secondary schools, we think it is possible to specify to students that, whereas the vector potential in general depends on space and on time, that is $\mathbf{A} = \mathbf{A}(\mathbf{r}, t)$, in the Maxwell's equation the derivative is only on the temporal coordinate.

The relation expressed by eq.(8.45) gives only the non-conservative part of the electric field. Obviously, if one is interested to determine *all* the electric field that can be present, one have to complete the expression of eq.(8.45) by a part in which it is taken into account the scalar potential V. This can be deduced with a certain difficulty to sudents that have no experience with the differential operators such as the gradient. For this reason we complete this part for teachers or for those particularly motivated students. If there are only fixed charges, then $C_\gamma\left(\mathbf{E}\right) = 0$ and, as we have already seen, $\mathbf{E} = -\nabla V$. The not conservative part of $\mathbf{E}$ is instead given by the Maxwell's equation (8.43) that is equivalently defined through the vector potential by eq.(8.45). Therefore, in general, besides free charges there are also time varying fields and $\mathbf{E}$ will be given by the sum of these two components:

$$\mathbf{E} = -\nabla V - \frac{\partial}{\partial t}\mathbf{A}. \tag{8.46}$$

8.4. Physical meaning of the vector potential

In the educational path for secondary school students, the physical meaning of the vector potential is very important, just as for the electric scalar potential. The description of the physical meaning of the vector potential may take place at this point of the path and can be carried out following what has been already described in the previous chapter, at the section 7.2.4, that is, through an analogy between the scalar and the vector potential.

8.5. Examples of determination of the magnetic vector potential

In this section we will determine the vector potential in some particularly simple cases. But these are also particularly interesting from a physical point of view.

8.5.1. Vector potential of a solenoid carrying a steady current

Suppose to have an indefinite solenoid of radius a carrying a steady current i, such that inside the solenoid there is a uniform magnetic field $\mathbf{B}$ of intensity $B = \mu_0 ni$, where n is the number of loops per unit of length of the solenoid. We precise for students that in the practice it is possible to be in the approximation of indefinite solenoid if we suppose that the radius a of the solenoid is such that $a << L$, where L is the length of the solenoid.

The aim is the determination of the vector potential inside and outside of the solenoid, generated by this current distribution.

Our guides in the determination of the vector potential are eq.(8.38) and eq.(8.42): the first equation says that the vector potential follows the direction of the electrical current, while the second equation allows us in the mathematical determination of the vector potential.

From considerations on the symmetry of this configuration, we can deduce that the system is invariant for traslation along the solenoid axis and for rotation around the same axis. Moreover, eq.(8.38) assures us that $\mathbf{A}$ is placed along concentric circumferences centred on the solenoid axis and orthogonal to it, moreover the intensity of the vector potential will be the same along each of the circumpherences centred on the axis and will be tangential to each point of each circumpherence, as Fig. 8.7 shows.

The role of eq.(8.36) ends here, with the previous qualitative analysis. We can get quantitative information about the vector potential using eq.(8.42). We have to choose the closed paths along which calculate the circulation of $\mathbf{A}$. The circumpherences of Fig. 8.7 appear convenient for the given problem.

For a generic circumpherence of radius r placed inside the solenoid (therefore with $r < a$) we can apply eq.(8.42) and obtain:

$$2\pi r A(r) = \pi r^2 B, \tag{8.47}$$

and rearranging the terms of the equation:

$$A(r) = \frac{1}{2}Br = \frac{1}{2}\mu_0 nir. \tag{8.48}$$

Eq.(8.48) pertains only the modulus of the vector potential.

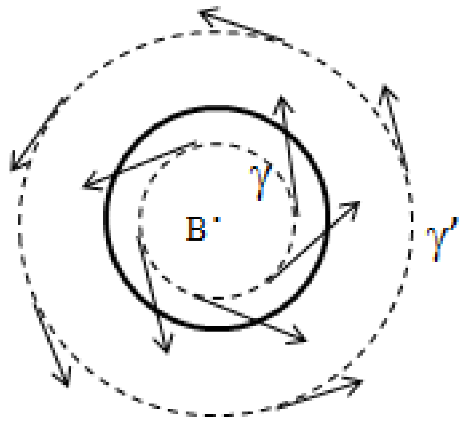


Figure 8.7.: Section of a solenoid carrying current. The circumpherences γ and γ' are the closed path along which perform the calculation of the circulation of the vector potential $\mathbf{A}$.

If we want to describe the vector potential $\mathbf{A}$ completely we have to use the vector product, and it is easy to realize that it can be written as:

$$\mathbf{A}(r) = \frac{1}{2}\mathbf{B} \times \mathbf{r}, \tag{8.49}$$

or, in terms of currents, rather then of magnetic field, we can write:

$$\mathbf{A}(r) = \frac{1}{2}\mu_0 nir\mathbf{u}_\varphi, \tag{8.50}$$

where $\mathbf{u}_\varphi$ is the unit vector oriented tangentially to the closed line γ in the verse of the electrical current flowing in the loop of the solenoid.

For a generic circumpherence of radius r placed outside the solenoid (therefore with $r > a$) we can apply eq.(8.42) and obtain:

$$2\pi r A(r) = \pi a^2 B, \tag{8.51}$$

and rearranging the terms of the equation:

$$A(r) = \frac{1}{2}\frac{a^2}{r}B = \frac{1}{2}\mu_0\frac{a^2}{r}nir. \tag{8.52}$$

Eq.(8.52) pertains only the modulus of the vector potential.

If we want to describe the vector potential $\mathbf{A}$ completely we have to use the vector product, and it is easy to realize that it can be written as:

$$\mathbf{A}(r) = \frac{1}{2}\frac{a^2}{r}\mathbf{B} \times \mathbf{r}, \tag{8.53}$$

or, in terms of currents, rather then of magnetic field, we can write:

$$\mathbf{A}(r) = \frac{1}{2}\mu_0 \frac{a^2}{r} nir\mathbf{u}_\varphi. \tag{8.54}$$

By these calculations we can observe that the vector potential goes always around circumpherences and its intensity increases linearly inside the solenoid getting closer to the currents, while it decreases outside the solenoid as $1/r$ going away from the currents. Recalling that the currents are the sources of the field $\mathbf{A}$, it seems quite reasonable that the intensity of the field decreases going away from its source. In Fig. 8.8 it is shown the trend of the vector potential determined so far, for the solenoid.

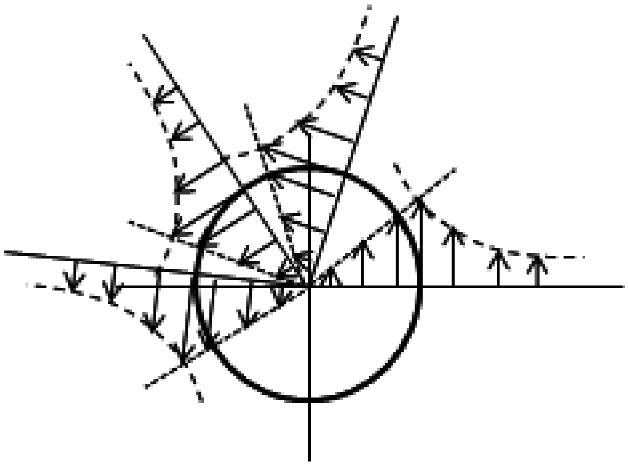


Figure 8.8.: The arrows represent the trend of the magnetic vector potential $\mathbf{A}$ inside and outside the solenoid.

It is interesting to note that while the magnetic field is null outside the solenoid, in our approximation of indefinite solenoid, the magnetic vector potential is defined in all the space, even where the magnetic field is absent.

8.5.2. Vector potential of two parallel planes carrying opposite currents

The second example that can be treated with students is the case of two parallel planes carrying opposite steady currents uniformly distribuited with the same intensity all over the planes, as it is schematically shown in Fig. 8.9.

By a procedure similar to that used in the previous example we need to know the expression of the magnetic field generated by the current distributions. In this way it will be possible to apply eq.(8.42) once that considerations on the symmetry of the problem have been clarified the qualitative trend of the vector potential by eq.(8.36).

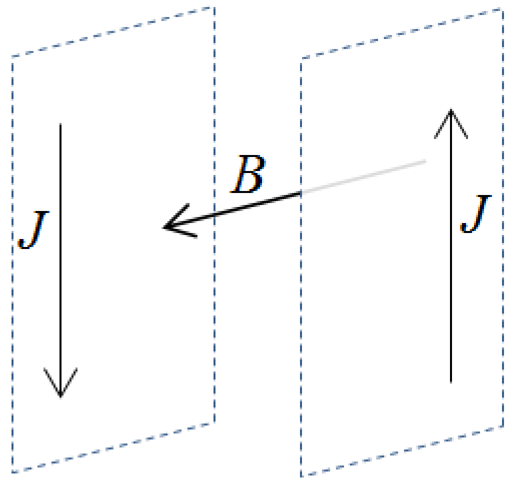


Figure 8.9.: Schematization of two parallel planes carrying currents indicated by the two arrows in the planes. It is also shown the direction of the magnetic field that is generated by this current distribution.

In order to determine the magnetic field of two parallel planes carrying current, we will refer to Fig. 8.10. Suppose that we want to get the magnetic field in a generic point P between the two planes. We can think that the plane on the left is constituted by infinite wires carrying current placed side by side. Therefore, chosen a wire placed in P_1 it will generate a magnetic field B_1 tangential to the circumpherence centred in P_1 of radius P_1P, whose intensity is $B_1 = \frac{\mu_0}{2\pi}\frac{i}{r}$, where r is just the distance P_1P.

Once this contribution to the magnetic field is determined, it is possible to choose another wire, simmetrically placed with respect to P_1 that we call P_2, that generates the contribution B_2 as Fig. 8.10 shows. B_2 has the same intensity of B_1 but its direction is such that the sum of the two contributions is a vector parallel to the plane. The same way of reasoning can be used for each slice of the two planes and for each point between the two planes. The direction of the magnetic field is thus parallel to the planes.

It is not yet determined the intensity of the magnetic field. Let us suppose that the field is not uniform inside the planes, as it is represented in Fig. 8.11.

In order to verify if this hypotesis holds we can apply the Ampère law:

$$C(\mathbf{B}) = \mu_0 i, \tag{8.55}$$

where i is the current passing through the path along which we calculate the circulation of the magnetic field.

Since in the path γ there is no current, we can conclude that for our problem holds:

$$C(\mathbf{B}) = 0. \tag{8.56}$$

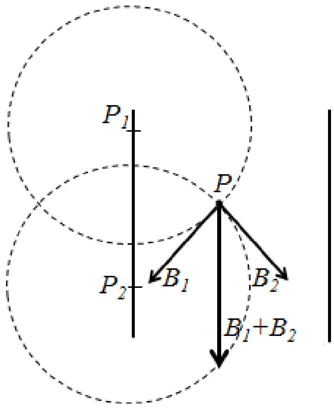


Figure 8.10.: The two carrying current planes are seen from above. Hence, for the plane at the left of the figure there is an inward current, while for the plane on the right of the figure there is an outward current. It is represented a generic point P and the two magnetic fields generated by two slices of the plane, placed in P_1 and P_2, symmetric wuth respect to the point P.

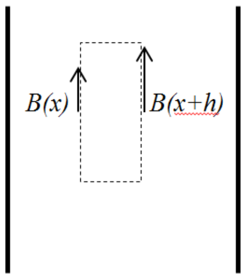


Figure 8.11.: Representation of the path γ along which it is possible to calculate the circulation of the magnetic field in order to determine its intensity.

If we evaluate the circulation, referring to Fig. 8.11 we can write:

$$B(x) - B(x+h) = 0 \tag{8.57}$$

and, therefore:

$$B(x) = B(x+h). \tag{8.58}$$

From eq.(8.58) we can thus conclude that the field inside the planes is uniform and parallel to the planes.

This is the second example of a uniform magnetic field, as the field of the solenoid, that we propose in our path. In this case the uniform field is generated by a different current distribution with respect to the first one. In the educational path that we are developing, this fact is not incidental: here we only point out this, but in section 8.6 we will discuss it in a more detailed way.

We have now all the elements to apply eq.(8.42). We suppose initially to be inside the plane, where we can use the closed line γ of Fig. 8.12 for the determination of the circulation of the vector potential, that is:

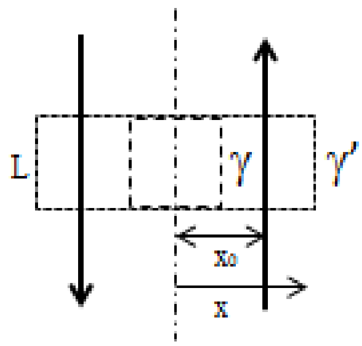


Figure 8.12.: Diagram of the two closed paths γ and γ' chosen for the determination of the vector potential respectively inside and outside the two planes.

$$2A(x)\mathrm{L} = 2x\mathrm{L}B. \tag{8.59}$$

Inside the planes we thus have:

$$A(x) = Bx, \tag{8.60}$$

and we can see that the more the vector potential gets close to the planes, the more its value increases, as Fig. 8.13 shows.

We can carry out a very similar calculation for the closed path γ' and we obtain:

$$2A(x)\mathrm{L} = Bx_0\mathrm{L}, \tag{8.61}$$

and therefore:

$$A(x) = Bx_0. \tag{8.62}$$

It is also possible to write a vector version of eq.(8.62): a useful exercise on vectors for students. Inside the planes we can write:

$$\mathbf{A}(x) = Bx\mathbf{u}_y, \tag{8.63}$$

where $\mathbf{u}_y$ is the unit vector of the y-axis, that in Fig. 8.12 is directed as the arrows that indicate the currents, or the sections of the plane. Outside the planes we have:

$$\mathbf{A}(x) = -Bx_0\mathbf{u}_y, \tag{8.64}$$

for $x < 0$, and:

$$\mathbf{A}(x) = Bx_0\mathbf{u}_y, \tag{8.65}$$

for $x > 0$.

Through eq.(8.62) we see that outside the planes the vector potential is uniform, and its intensity is the maximum value taken by the vector potential of this physical situation. The value taken from the vector potential is therefore maximum on the planes where the currents are (that is, where the sources of the potential are). A graphical representation of the result is given in Fig. 8.13.

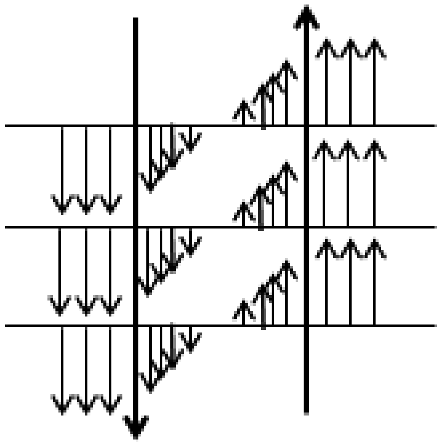


Figure 8.13.: The arrows show the behaviour of the vector potential inside and outside two indefinite planes carrying opposite currents of the same intensity.

8.5.3. Vector potential of a wire carrying steady current

This example involves integral calculations a little bit more complicated than that reported in the previous cases, because of the fact that the magnetic field is not uniform. Moreover, during the execution of the exercise we will see that some approximations are needed in order to write the solution in its meaningful form. These approximations are obviously not spontaneous for the secondary school students, nevertheless we think that this exercise could be all the same interesting and stimulating just for those students.

It is well-known to students that the intensity of the magnetic field generated by an indefinite straight wire carrying a steady current is given by:

$$B(x) = \frac{\mu_0}{2\pi}\frac{i}{x}. \tag{8.66}$$

If we choose the path γ as in Fig. 8.14, we will apply eq.(8.42) remembering that the field is inward in the path and therefore it will give a positive flux. Infact, the unit vector $\mathbf{n}$ of the surface Σ, that is the surface that has γ as a boundary, has the same verse of the magnetic field $\mathbf{B}$. But, it is clear from eq.(8.66) that the field is not uniform on that surface, so care we must taken in writing the integral for the calculation of $\Phi(\mathbf{B})$.

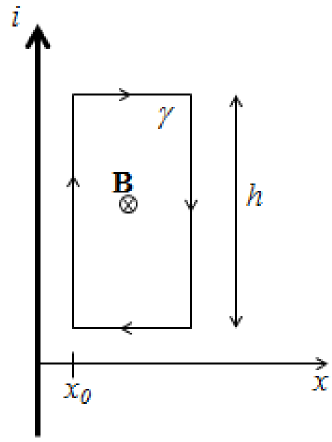


Figure 8.14.: Representation of the wire carrying current (in bold) and the closed path γ along which we calculate the circulation of the vector potential.

We have:

$$C(\mathbf{A}) = h\left[A(x_0) - A(x)\right] \tag{8.67}$$

and:

$$\Phi(\mathbf{B}) = \int_\Sigma \mathbf{B}\cdot\mathbf{n}d\Sigma = \int_{x_0}^{x} B(x)hdx = \frac{\mu_0}{2\pi}ih\left[logx - logx_0\right]. \tag{8.68}$$

Now, using eq.(8.42) we obtain:

$$A(x) = \left[A(x_0) + \frac{\mu_0}{2\pi}ilogx_0\right] - \frac{\mu_0}{2\pi}ilogx. \tag{8.69}$$

Apart from the constant that is contained in the square brackets, this vector potential has a logarithmic trend and decreases going away from the wire. Depending on the experimental conditions in which the wire is placed, it is possible to choose the constant on the square brackets large enough to give a vector potential always positive. From a mathematical point of view, the fact that the vector potential becomes negative has no importance, because the magnetic field that is described remains the same, but this could be bothersome because of the fact that we found that the currents are an empirical referent for the vector potential, and in this case the current flows always in the same verse, while the vector potential changes its verse.

There is a second way that we can follow to treat this problem: we can use directly eq.(8.38), being this case particularly simple, referring to Fig. 8.15.

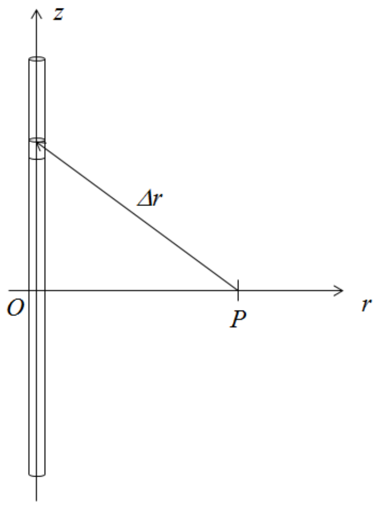


Figure 8.15.: The indefinite wire is represented by the cylinder. It is also represented the point P in which the vector potential is calculated.

We use a slightly different drawing with respect to the previous case, because the mathematical approach is different, although we will see that it will get a very similar result.

From eq.(8.38) we can write:

$$A(r) = 2\frac{\mu_0}{4\pi}i\int_0^L \frac{dz}{\sqrt{r^2+z^2}}, \tag{8.70}$$

and solving the integral we obtain:

$$A(r) = \frac{\mu_0 i}{4\pi}log\frac{L+L\sqrt{1+r^2/L^2}}{r}. \tag{8.71}$$

Now, if we imagine that the wire is very long so that $L \gg r$, we can approximate eq.(8.71) and we get:

$$A(r) \approx \frac{\mu_0 i}{4\pi}log\frac{2L}{r} \tag{8.72}$$

that, for the properties of the logarithm, becomes:

$$A(r) \approx \frac{\mu_0 i}{4\pi}log2L - \frac{\mu_0 i}{4\pi}logr. \tag{8.73}$$

As we have seen in the previous case with eq.(8.69) the dipendence of the vector potential with the distance r of the wire is $-logr$, and even in this second case we have to choose the positive constant large enough to compensate the negative logarithm. Being the wire very long if compared with the distance r at which we want to determine the vector potential, we can therefore suppose that $2L \gg r$ and hence that the vector potential does not change its verse.

Seen in this second way it comes more natural the condition supposed in order to grant the vector potential positive.

8.6. Vector potentials: starting from current distributions or from magnetic fields

In the previous examples we found the expression of the vector potential starting from a known current distribution. It can be interesting now to determine $\mathbf{A}$ starting from a known magnetic field $\mathbf{B}$. In the section for teachers we have already seen that the problem is not univocally determined, infact from the equation $\mathbf{B} = \nabla \times \mathbf{A}$ we have that the same magnetic field $\mathbf{B}$ can be related to an infinite number of vector potentials $\mathbf{A}$.

If we recall the sections in which we described the vector potential generated by the solenoid or by the two parallel planes, we realize that we obtained two different vector potentials for a uniform magnetic field. However, what discussed in the previous section allows us to shed some light on the physical implications of this fact.

Let us imagine to calculate $\mathbf{A}$ for a uniform $\mathbf{B}$. When we choose a particular class of closed paths to calculate the circulation of the vector potential, the symmetry of the problem is broken and a particular $\mathbf{A}$ is found. To recover the lost physical symmetry, one generally considers equivalent all the vector potentials generating the same field $\mathbf{B}$; and in a sense this is one of the physical meaning of the gauge invariance. Back to our example, if $\mathbf{B}$ is really uniform in the whole space we don't know whether we are inside an infinite solenoid of infinite radius or between a couple of current carrying planes, infinite distance apart. Therefore, even if we are in the same Coulomb gauge (in our appoximation the gauge is fixed), $\mathbf{A}$ is not univocally determined by $\mathbf{B}$ because the currents that could generate this field do not vanish at infinity. It is clear that the currents determine both $\mathbf{B}$ and $\mathbf{A}$; the potential $\mathbf{A}$ determines $\mathbf{B}$, while the viceversa is not true.

8.7. The vector potential: synthesis of the path

The sections of this chapter have described many aspects of the vector potential and are developed for students, even if they are however addressed to their teachers, for the shortness and the lack of particulars in the discussion of the topics. Moreover, the order in which the topics are displayed for the students' educational path is instead resumed here below.

(A) The concept of potential

- The concept of potential is fundamental in physics starting from mechanics to electromagnetism.
- A general scheme to get the potential energy starting from the particular conservative force.
- The potential definition from the potential energy.
- Examples of potenials in mechanics: gravitational potential, elastic potential.
- Examples of potentials in electromagnetism: electrostatic potential, potential in a condenser.

(B) Definition of the vector potential

- A description of the electric potential in terms of its sources, the charge densities.
- The definition of a new potential carrying out an analogy with the electric potential: the vector potential has the current densities as its sources.
- It is recalled the physical meaning of the electric scalar potential.
- By the same way of reasoning it is attribuited a physical meaning to the vector potential.
- By the Maxwell's definition of the vector potential, in terms of integral mathematical tools, can be obtained a fundamental property that relates the vector potential to the magnetic field: $C(\mathbf{A}) = \Phi(\mathbf{B})$ and it is possible to show that the property is equivalent to the initial definition.

(C) Examples of determination of vector potentials

- For didactical reasons are described vector potentials generated by some particular current distributions and not by particular magnetic fields.
- The first two examples show the magnetic vector potential of a solenoid carrying steady current and two parallel planes carrying steady currents. Each of the two current distributions generates a uniform magnetic field, but the two vector potentials got are different. This is the first step to build an idea of the gauge invariance.

- The third example of magnetic vector potential is the potential generated by an indefinite wire carrying steady current. This is an example in which a not uniform magnetic field has to be integrated and for this reason there could be some mathematical difficulty.
- Critical discussion of the examples: Is it possible to deduce the vector potential starting from the magnetic field rather than the current distributions? Here students have the possibility of experimenting the concept of gauge invariance without facing the topic from a mathematical, or too abstract, point of view.

9. Experimentation of the vector potential in secondary school

9.1. Overview

In the previous chapters we have widely discussed the vector potential and a way to introduce it in secondary school. During our experimentations with students we collected many data on students' understanding for what concerns their beliefs in electromagnetism. In particular, during the last experimentation we wanted to understand whether the learning of the basic concepts of electromagnetism would improve with the introduction of the vector potential.

We gave students the same written test, before and after the introduction of the vector potential. The complete test is reported in appendix D while in this chapter we will discuss only some questions pertaining basic concepts of electromagnetism and in particular the concepts of electric and magnetic fields and the ability of using the concepts of flux and circulation of a vector field. The analysis could be deepened, but for the moment we write in this work only some preliminary results.

This test was given to the last year high school students. The pre-test has been done by the students of two classes (that in total were 45). As the experimentation was carried out in one of the two classes, the post-test has been performed only by the same 21 students of that class.

9.2. The electric and the magnetic fields for a current-carrying wire

We report the first question of the written test:

You have a very long conducting wire carrying direct current. Describe the fields inside and outside the wire. Provide with words, formulas and drawings as much information as possible.

We consider first the answers about the electric field, and we report the results in Tab. 9.1.

Categories about E	**PRE**	**POST**	g
The electric field is present inside the wire	28%	43%	21%
The electric field is not present outside the wire	0%	6%	6%
Students that answer to the question	64%	91%	42%

Table 9.1.: Some students' ideas on the electric field for an indefinite wire carrying current.

These results were particularly meaningful for us. Infact we have been very surprised in recognizing that students never consider the electric field outside the wire. This particular result made us to believe that the educational path on electrical current had to be changed, and this is the reason of our last proposal in the framework of the design-based research, so different from the first two.

As we can see from the results of the pre-test in Tab. 9.1, the basic students' ideas on the electric field inside and outside a wire, are still very poor and confused at the end of their traditional physics courses. Nevertheless we can notice a certain students' improvement for what concerns their understanding about the electric field inside the wire in the post-test, and this may be due to the fact that in our experimentation on electrical current we insisted on the local formulation of the Ohm's laws $\mathbf{J} = \sigma\mathbf{E}$, with obvious meaning of the notation.

Anyway, to further improve students' understanding, our proposal for the electric current has been considerably changed in the last version of the path just in the direction suggested by this results: we described infact the electrical current in such a way to relate it with electrostatics, the electric field, and the concept of circulation of the electric field. See chapter 6.

In order to give to the reader a the picture of the context of the students' ideas when they began the experimentation, we report here some typical sentences that we recorded during the oral interviews. (The letter T indicates, as usual, the experimenter teacher, while the letter S indicate the student).

T: *What are the fields around a wire carrying direct current?*

S: The electric field?

T: *What is an electric field in your mind?*

S: An electric shock?

T: *When you imagine a charge near a wire carrying current, do you think that something happens to the charge?*

S1: Oh, yes, because the current is made up of charges!

S2: Yes, the charge will tend to be closer to the wire to re-charge itself more and more!

S3: The charge runs around the wire, along the circular lines of the field.

From these oral interviews we get a picture of the students' ideas that seem to be completely separate from a theoretical framework, these ideas look like to the picture of a child, more than to the picture of a high school student.

We consider now something about the magnetic field, and we report the results in Tab. 9.2.

Categories about B	**PRE**	**POST**	g
Circular field lines around the wire	38%	93%	89%
The magnetic field is present inside the wire	0%	0%	0%

Table 9.2.: Some students' ideas on the magnetic field for an indefinite wire carrying current.

Besides this result in which the students seem to have well understood that around a wire a magnetic field is present, it appears quite clear that they have a poor familiarity with the magnetic field. As we have done for the electric field, we present here below also some exerpts of oral interviews on the magnetic field.

T: *What generates a magnetic field?*

S1: A charge?

S2: A magnetic field?

T: *Is there a magnetic field in this room?*

S1: I don't know...

T: *If I consider a simple circuit, with a led connected to a battery, which are the fields involved in this situation?*

S1: If there is a circuit, then there will be a magnetic field, but if the wire is insulated by rubber, the magnetic field does not go outside... otherwise our home would be filled of magnetic fields!

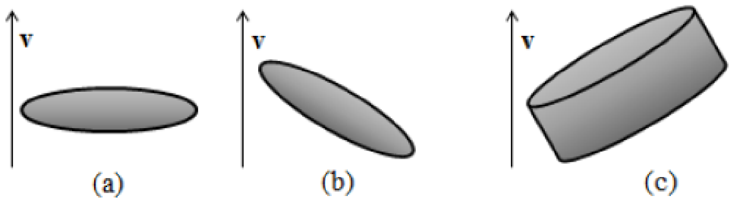
Even for the concept of magnetic field the students' ideas are very confused and they mix the concept of electrical insulation, by means of a rubber sheath, with a kind of magnetic insulation. We found as indistinct also the concepts of static magnetic field and of magnetic field generated by the variation of an electric field.

The experimentation on the vector potential has not been sufficient in order to sort out the students' ideas about the magnetic field. At the end of the path they are able to describe with circular lines the magnetic field outside the wire carrying a direct current, but their way of reasoning still seems naive.

9.3. The flux of a vector field

We report the second question of the written test:

A vector field v is uniform and its intensity is $v = 10^{-2}u$ (where u is a general unit of measure for the vector field v). The field fills all the space where the surfaces are placed, see the following figures (a), (b) and (c), and it is directed as the figures show. Determine the value of the flux of the field through the surfaces in each of the three cases, taking into account that: in (a) the radius of the surface is $R = 3m$, in (b) the surface is inclined at $30°$ to the horizontal, and in (c) The surface is a cylinder of radius $R = 3m$ and heigth $h = 2m$.



We resume the results of the three cases (a), (b) and (c) in three different tables, as follows.

The case (a) is the most simple case, and usually students write quite unconsciously $flux = BS$, with B the intensity of the magnetic field and S the surface, and they do not pay attention to its orientation with respect to the direction of the field. We can notice a certain improvement in their results, see Tab. 9.3.

Categories	PRE	POST	g
$\Phi(\mathbf{v}) = BS$	57%	90%	77%
$\Phi(\mathbf{v}) = BScos(90°)$	36%	5%	

Table 9.3.: Students' results about the concept of flux through a surface orthogonally placed with respect the field direction, case (a) of the exercise.

Tab. 9.3 also shows that after the teaching on the vector potential, students take more care in dealing with the definition of the flux for a uniform vector field, considering more correctly the angle between the field and the unit vector of the surface. The case (b) of the exercise gave a result quite similar to the previous. It is reported in Tab. 9.4.

For the case (c) of the exercise, the instruction have given no results. Infact, no one has answered the question correctly, either before or after. One possible explanation

Categories	PRE	POST	g
$\Phi(\mathbf{v}) = BScos(30°)$	71%	71%	0%
$\Phi(\mathbf{v}) = BScos(60°)$	24%	16%	

Table 9.4.: Students' results about the concept of flux through a surface inclined by 30° with respect the horizonthal, case (b) of the exercise.

of this fact may be that in our path on the vector potential, or the superconductivity, the surfaces that we use are always open. In this way it may even be possible that students do not distinguish between a closed surface by an open surface.

Before the instruction at the beginning of our lessons, we have interviewed some of the students of the class on the concept of flux and we report here below some of the most meaningful exerpts of the conversation.

T: *What surface can I refer to, if I want to determine the flux of a river?*

S1: To the surface of the river, that is... its highest part.

T: *So, is the surface to which I refer for the flux calulation a part of the river?*

S1: Yes, it is!

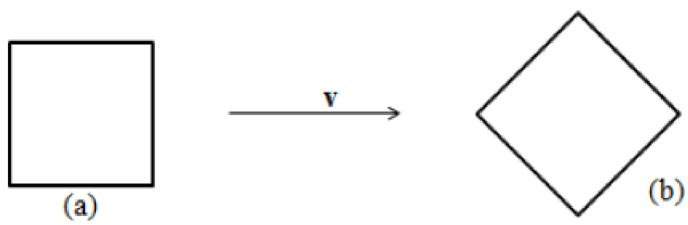
T: *And what do you imagine when I say "the water flows through a surface?*

S2: I imagine some water moving on that surface, in a lot of different directions.

9.4. The circulation of a vector field

We report the third question of the written test:

A vector field v is uniform and its intensity is $v = 30u$ (where u is a general unit of measure for the vector field v). The field fills all the plane where the closed paths are placed, see the following figures (a) and (b), and it is directed as the figures show. Determine the value of the circulation of the field along the paths in each of the two cases, taking into account that: in (a) the closed path is a square of side $L = 2m$ and in (b) the closed path is identical to the case (a) except for the fact that it is rotated of $45°$.



The concept of circulation has been more incisive with respect to the concept of flux, for students. From the one hand it was almost unknown, and this caused a certain dscouragement, but on the other hand, the fact that it was a concept through which one could express the work stimulated their reasoning. They have never defined the work by means of an integral and it was very hard for students to review that known quantity in new mathematical terms. Many of them continued to ask questions, very similar to each other, in order to disentangle their doubts, both from a mathematical point of view and from a physical point of view. And it was stimulating also for the teacher.

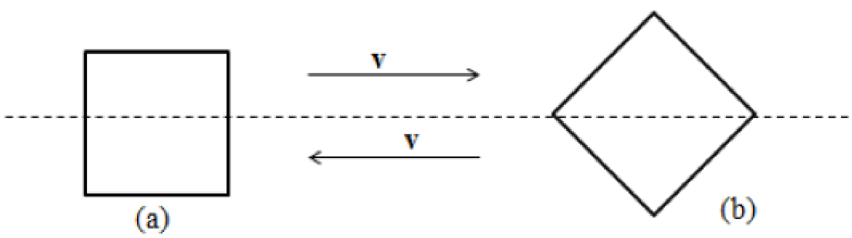
But surprisingly, as it is shown in the following Tab. 9.5, while the teacher was disheartened because of the continuous repetition of the same concepts, and while the students continued in apologizing with the teacher for their poor comprehension of the physics, the post-test gave completely unexpected results!

	Categories	**PRE**	**POST**	g
(a)	$C(\mathbf{v}) = 0$	0%	76%	76%
	answer	19%	81%	77%
(b)	$C(\mathbf{v}) = 0$	0%	76%	76%
	answer	33%	76%	64%

Table 9.5.: Students' results about the concept of circulation of a uniform field along a closed square path, there are shown both case (a) and (b) of the exercise.

We notice that all the students that responded right to the question 3(a), responded right also to the question 3(b), whereas not always concluding the calculations perfectly. We propose here below a second question on the circulation that was part of the same written test.

The situation is identical than in the previous exercise, except for what regards the vector field $\mathbf{v}$. In this case infact, it is not uniform in all the plane: it changes its verse in correspondance of the dashed line, whereas its intensity remains $v = 30u$ (where u is a general unit of measure for the vector field $\mathbf{v}$). Along the dashed line the field is zero. Determine the value of the circulation of the field along the paths in each of the two cases.



Also the results of this exercise show a net improvement after instruction, see Tab. 9.6.

	Categories	**PRE**	**POST**	g
(a)	$C(\mathbf{v}) = 2Lv$	0%	38%	38%
	answer	14%	57%	50%
(b)	$C(\mathbf{v}) = 2\sqrt{2}Lv$	0%	38%	38%
	answer	24%	57%	43%

Table 9.6.: Students' results about the concept of circulation of a non uniform field along a closed square path, there are shown both case (a) and (b) of the exercise.

In Tab. 9.6 we have reported as if they were perfectly right also those answers in which the calculations were not carried out correctly till the end of the exercise. We decided to do not differentiate between the perfect ability in the calculations and the right setting of the calculation, because of the fact that the students were really not used to make calculations and perform exercises or problems in their curricular hours.

Finally, we report the students' ideas about the circulation of a vector field that we collected during some of the lessons we have done on this topic.

T: *Did you look for the definition of circulation?*

S1: The Maxwell's equations?

T: [The teacher tells the difference between a definition and a theorem]

T: *What is the meaning of performing the circulation integral?*

S2: The evaluation of an area?

S3: Please, would you explain again what is the circulation?

This last question was a catchphrase of our lessons on the vector potential, but nevertheless, something was grabbed.

10. An experimental lab on superconductivity for secondary school

10.1. Overview

For completness, we report in this chapter a brief description of the experimental lab that is part of our educational path on superconductivity. In this part we do not consider the preliminary set of experiments pertaining the electrical conduction, but only those minimal experiments that are more tightly related with the superconductivity. Hence, after an introductive set of experiments on the electromagnetic induction, by which the students may acquire familiarity in particular with the magnetic field, the students may perform a measure of the critical temperature of an YBCO sample and the exploration of the interaction between a type II superconductor (YBCO) with magnetic fields. Here below we report a schematic description of the experiments proposed to students in this path.

10.2. Electromagnetic induction lab

In this first part we recall some aspects of the electromagnetic induction, that should have already been treated, during their curricolar lessons with their usual teacher. The central point to be recognized by students in this set of experiments is that all these experiments that seem quite different to each other, has instead a common denominator. We believe that it may be very useful and meaningful for students, to realize what is the common underlying physics in all the presented experiments.

Moreover we have noticed how can be interesting for students to describe the observed phenomenology in different ways:

- By means of the Lorentz force and considerations about eddy currents;
- By means of the magnetic flux variation.

In order to get an idea for the reader, we give a list of experiments useful for the educational path.

1. Using a $2-3m$ long copper tube, whose inner diameter is about $1,5cm$, and magnets. It is possible to observe the way in which a small magnet (for example spherical) inserted at one end of the tube, falls down the tube, or observe, externally, the fall of a magnetic ring with its inner diameter greater than the tube diameter. Moreover, it is possible to compare the falls of magnetic objects with the falls of plastic objects.
2. A very similar experiment to that of point 1. can be carried out with an inclined copper plate above which magnets or plastic objects slide.
3. Observations using the Waltenhofen pendulum.
4. Using a setup as that represented in Fig. 10.1, it is possible to observe the magnet starting its rotation when the disk is put in rotation.
5. Using the same experimental setup, as that represented in Fig. 10.1, it is possible to eliminate the magnets, put in rotation the disk, and stopping it getting close a magnet to it (the electromagnetic brake).

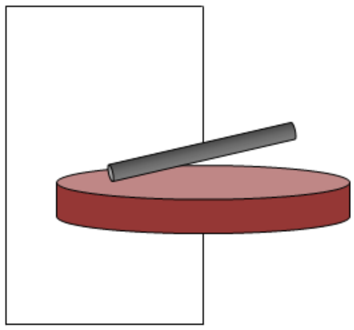


Figure 10.1.: Set up for some experiences on electromagnetic induction. The magnet is suspended above the copper disk by means of a cotton thread. Both the disk and the thread can rotate around the same axis, that is the symmetry axis of the disk.

There is another experiment that can be performed in a school lab. The underlying physics pertains again the electromagnetic induction, but in this case the explanation is slightly different than in the previous five cases. We proposed this experiment for its pleasantness and simplicity.

1. It is needed a circuit supplied by an alternate current, and a LED connected to a second electrical circuit. It must take care in the practical realization of such circuits, so that they can work well, but once things are made correctly, just bring the circuit with the LED to the oscillating circuit and see the LED that lights up without having established any contact between the wires.
2. Since the most difficult part in the experiment of point 1. is related to the realization of the oscillating circuit, it can be skipped using, for example, a power supply of an electric toothbrush.

3. A second (very simplified version) of the same experiment can be carried out with a solenoid, a tester, and a magnet. The endpoints of the solenoid have to be connected with the tester. Approaching and moving away quickly the magnet, a current will be generated in the solenoid, and the tester will measure that current.

4. A third (less simple from a practical point of view) version of the experiment can be carried out with a long tube around which are placed some solenoids, each of whom connected with a led. A strong magnet is inserted at the top of the tube and left to fall. When the magnet from the inside of the tube will pass next to each solenoid, the correspondent LED will light up, with an intensity that increases with the distance from the top of the tube.

10.3. Measure of the critical temperature

After becoming acquainted with the electromagnetic induction, the students may start to use superconductors, YBCO, in particular. The first experiment that students can perform using YBCO samples, is the measure of the critical temperature of the sample. In Fig. 10.2 it is shown in black the YBCO sample and with the letters A, B and C are indicated the point where are established the connections for the measurement of the dependence of the resistivity from the temperature.

A software elaborates the electronic signals from the sample and gives on a screen the graph of the resistivity as a function of the temperature of the sample. The sample is progressively cooled by liquid nitrogen and the temperature decreases until the critical temperature is reached. At this temperature, that is called critical temperature, the graph of the resistivity drops to zero and the thermodynamic phase transition is reached, that is, the sample is now superconductive. In this way, students may perform a very similar experiment to that performed by K. Onnes when discovered the superconductivity.

In order to carry out this measurement, there must be a four-points connection to the YBCO sample:

- The connection indicated by the two letters A is established in such a way to maintain constant the current I that passes through the YBCO sample, during the entire time interval in which the measurement takes place (several minutes). Students should realize that, as the temperature decreases, the resistance of the sample decreases but the electronics of the system maintains the intensity of the electrical current unchanged all the same.
- The two letters B indicate the two points between which we measure the potential difference ΔV. Students should realize that by the first Ohm's law, the measurement of the potential difference allows the instantaneous determination of the resistance of the sample, being constant the current I.

- The connection indicated by the letter C is a thermocouple, for the instantaneous measurement of the temperature of the sample.

The apparatus takes every few seconds a sampling of the system measuring the temperature and the resistance of the material, thus allowing, through a plot of resistance vs temperature, the determination of the critical temperature.

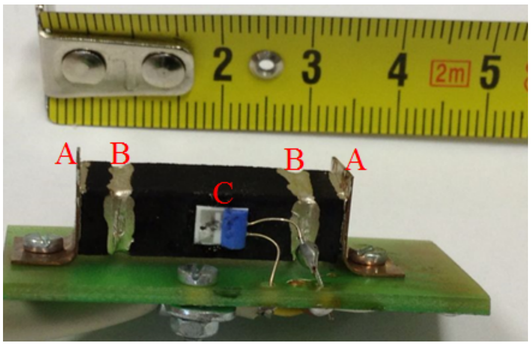


Figure 10.2.: The YBCO sample with all its connections for the measure of the critical temperature.

It is important that students realize that the critical temperature measured through this method is not exactly equivalent to the ctitical temperature that the same superconductor shows in absence of magnetic fields. Infact, since this method for the measurement of the resistance is based on the flow of the current in the superconductive sample, this implies that a magnetic field will be generated in the bulk of the superconductor, thus changing the value of the critical temperature. For this reason the measurement of the critical temperature will be the better, the lower is the intensity of the current used in the apparatus.

10.4. Observations of superconductors in interaction with magnets

Lab experiences with high temperature superconductors are very common in secondary school and they are very important for the students' comprehension of the superconductive phenomenology [61, 62, 63, 64]. For this reason we elaborated a lab sheet, in order to guide students in their activities. The lab sheet is reported in appendix E, therefore we will not describe in detail the experiments and we shall confine ourselves only to clarify the essential points that we would like the students perceived during this lab.

Although the superconductor used in our lab is YBCO, its properties may vary a lot depending on its manufacturing process. Each of the students' kits contains two YBCO superconductors.

One of these has a very poor capacity to trap the magnetic field lines, and for this reason when it interacts with magnetic fields it expells the field as in a almost pure Meissner effect. The other superconductor provided in the kit can trap the field lines, pinning them. Therefore, with the second superconductor the students may experiment the mixed state of a superconductor of type II.

The fundamental steps that a student should realize in this lab are resumed here below:

1) The time at which the phase transition to the superconductive state takes place

At a certain time, a phenomenon in which the liquid nitrogen starts to boil and covers the superconductor (if the superconductor is not completely submerged in the liquid nitrogen) occurs, and it is a peculiar characterization of superconductors such as YBCO which would be nice if students observed.

2) The Meissner effect

In a Meissner effect there is the complete expulsion of the magnetic field. Many descriptions of this phenomenon are present in the literature [65], and the situation, from a practical point of view is not simple, because of its numerous facets. For the superconductors as YBCO, a perfectly pure Meissner effect can not occur, but we can observe an almost perfect elastic levitation of the magnet, due to the fact that the magnetic field is almost completely expelled from the superconductor, as Fig. 10.3 shows.

Figure 10.3.: The elastic levitation of a magnet above a superconductor.

3) Rigid levitation

When a strong magnet interacts with a superconductor that can trap the magnetic flux lines, in certain conditions, that students have to discover, it happens that the magnetic field penetrates the YBCO sample and it behaves as if it was a magnet itself [66, 67, 68]. There is a wide series of experimental situations that students may realize in order to become familiar with the mixed state of a superconductor of type II. A typical situation is represented in Fig. 10.4 where the tweezers hold the magnet that attracts the superconductor as if it were a magnet itself.

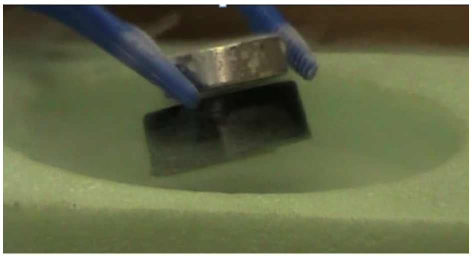


Figure 10.4.: The rigid levitation of a magnet above a superconductor.

4) The visualization of the magnetic field trapped in a superconductor

Using ferrofluid it is possible to visualize the magnetic field trapped in a superconductor in various situations, depending on the students' fantasy, as it is shown in Fig. 10.5.

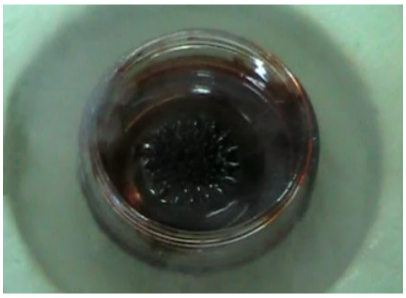


Figure 10.5.: Ferrofluid in a small plastic container is placed on top of the superconductor.

11. An educational path on superconductivity for secondary school

11.1. Overview

In this last chapter we propose an educational path on the basic superconductivity, based on the magnetic vector potential previously introduced. As we did for the treatment of the vector potential, here we will use integral mathematical tools as the circulation and the flux of a vector field, the same tools that students use to write the Maxwell's equations, that we consider a student's back-ground.

In the framework of the method of the educational reconstruction of the contents we have found that the introduction of the vector potential can be of great help in students' understanding of the basic phenomenology of superconductivity. Infact, in this chapter will see how the use of the vector potential allows a phenomenological and consistent explanation of superconductivity at a level suitable for high school students. We will deal with the two main aspects of superconductivity: the resistivity of the superconductor that drops to zero at the critical temperature and the expulsion of the magnetic field from the bulk of a superconductor, that is called Meissner effect. By the use of the vector potential, students can build a phenomenological interpretation of superconductivity, always remaining in the frame of electromagnetism and thus avoiding the use of too complicated mathematical tools that the explanation of the microscopic mechanism would require.

11.2. Main guidelines of our proposal

The didactical proposal on superconductivity that is reported in this chapter is born into our physics education research group, who has been dealing with superconductivity for 8 years with high school students expecially within the PLS activities. During the PLS activities the hours available in the laboratory was about four per class. In a so few time to devote to superconductivity, the activities were focused mainly on the hystorical introduction of the phenomenon, on the physics of the low temperatures and to the carrying out of experiments by the participating students.

But during the last three years, for the occasion of the PhD research, our work has deepened the rigorous and mathematical aspects of superconductivity, in order to present consistently the superconductivity to high school students which do not have the mathematical tools by which superconductivity is usually described. This new approach to the superconductivity has made it necessary to devote much time to lectures and laboratory, and the total amount of time is increased to about 30 hours, where it is included also the time necessary for the vector potential, that is the mathematical tool by which we can describe the main superconductive phenomena.

The most common proposal of superconductivity for the secondary school are usually focused on the phenomenological aspects rather than on their theoretical explanation, and this is due mainly to the difficulty of the mathematics involved. For this reason the explanation of the phenomenology is often treated giving information about the BCS theory, that is clearly beyond the mathematical possibilities of a secondary school student. In this way there is a very marked difference between the level of the experimental/phenomenological part and its mathematical description. This difference in the level of the two approaches captured our attention at the beginning of this thesis work, and our aim was to develop an educational path in which the experimentalphenomenological part could have a consistent mathematical explanation.

Despite superconductivity is a very difficult topic, in our experience students have been always interested and involved and we hope that the complexity and the witchery of superconductivity can be a stimulus for students not only to face superconductivity, but also to take up topics covered till that moment, deepen them and frame them in a new light. Before entering the core of our proposal we resume the main prerequisites needed to follow this path on superconductivity.

1. *Electromagnetism.* In particular, the Maxwell's equations and the vector potential **A**, through which we can introduce the foundamental equation (London equation) that can describe the basic phenomenological evidences in superconductivity.
2. *Thermodynamics.* Although in this work we never focused on these thermodynamic aspects, they are very important, at least for what concerns the definition of a thermodynamic state as a function of some variables: temperature T, pressure P, volume V and also the magnetic field **B**.
3. *Waves.* This last point is a pre-requisite only in the case in which a teacher plans to address the superconductivity at a deeper lever respect the minimum level that will be treted in this chapter. In fact it is possible to describe the superconductive current in terms of material waves (i.e. to generalize the London equation and to get the quantization of the magnetic flux in a superconductor).

The point 3. will be treated only at the final part of the present chapter because before tackling this more complicated part it is convenient to describe with attention the Meissner effect, that is one of the fundamental characteristic fenomena that appears in the superconductive states. In the second part of this chapter we will see that the Meissner effect occurs only in particular conditions, while it is very often to find experimentally the penetration of the magnetic field inside the bulk. The mathematical description of this second part is really much more complicated than the Meissner effect, and for this reason can be addressed only to those students that have an adequate back-ground in quantum physics as indicated in the plan of the educational path of chapter 4.

We have developed the educational path here presented in the frame of electromagnetism, by the use the mathematical tools of flux and circulation that students should have already known from their study when dealing with the Maxwell's equations and we hope that this new application of the mathematical tools of circulation and flux may improve students' understanding of electromagnetism.

11.3. The two fluid model to explain superconductivity

As we have seen in chapter 3, the two main features of superconductivity are:

- the resistivity ϱ of the superconductor that drops to zero below a particular temperature T_C, called critical temperature, that is characteristic of the given superconductive material.
- the magnetic field applied to a superconductive sample remains always zero inside the bulk (Meissner effect), unless the applied field B_{app} overcomes a critical intensity B_C.

In order to construct a meaningful path for secondary school, we have to develop a model able to explain the previous features. The theory that inspired our work is the two-fluid theory, that has been developed in 1934 by Gorter and Casimir. In this theory the superconductor is treated as containing a mixture of two fluids: the normal fluid, with the same properties of the ohmic electrical current in a metal, and the superconducting fluid, that flows without any friction (see Tab. 11.1). The two fluids are always present at the same time, therefore the superconducting sample can be thought as a circuit with two branches in parallel, the superconductive one and the normal one, as can be seen in Fig. 11.1.

Hence, if a constant current I is flowing in the superconductor, than the presence of the superconducting branch shunts the circuit, so that all the current runs in that branch, as the arrows in Fig. 11.1 show. We now suppose that the properties of the normal fluid have already been treated in the lessons on electrical conduction, as its behaviour is explained by the Ohm's laws, that can be summarized in the local formulation

$$\mathbf{J}_N = \sigma \mathbf{E}, \tag{11.1}$$

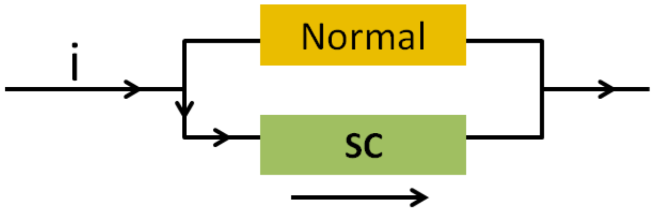


Figure 11.1.: . Schematic representation of a superconductor, where SC stands for superconductor.

where $\mathbf{J}_N$ is the density current and $\mathbf{E}$ is the electric field. The behaviour of the superconducting fluid is, instead, the main fact to be modelled.

11.3.1. A description of the superconductive fluid

As we have already seen in chapter 3, in 1935, the brothers F. and H. London proposed for the first time a phenomenological equation for the frictionless motion of the super-current

$$\mathbf{J}_S + k\mathbf{A} = 0. \tag{11.2}$$

This equation can be obtained from students quite simply, in a similar way to that the London used.

In general the current density $\mathbf{J}$ is proportional to the velocity of the fluid that is flowing, so we can write:

$$\mathbf{J}_S = k' \cdot \mathbf{v}. \tag{11.3}$$

By a time derivative of both members we obtain:

$$\frac{\partial}{\partial t}\mathbf{J}_S = k' \cdot \mathbf{a}, \tag{11.4}$$

where $\mathbf{a}$ is the acceleration of the fluid. But, supposing a frictionless motion, we can associate a force to an acceleration, thus finding:

$$\frac{\partial}{\partial t}\mathbf{J}_S = k''\mathbf{F}. \tag{11.5}$$

Recalling that we have the motion of charges, the force that can accelerate a charge must be associated to an electric field $\mathbf{E}$ that we can express by means of the vector potential $\mathbf{A}$, hence from the relation $\mathbf{E} = -\partial\mathbf{A}/\partial t$ we can write eq.(11.5) as:

$$\frac{\partial}{\partial t}\mathbf{J}_S = -k'''\frac{\partial}{\partial t}\mathbf{A}. \tag{11.6}$$

Rearranging the terms of eq.(11.6) and using for the constant k''' the new symbol k, we have:

$$\frac{\partial}{\partial t}\left(\mathbf{J}_S + k\mathbf{A}\right) = 0. \tag{11.7}$$

The London brothers, at this point, in order to explain the phenomenology, impose an hypothesis *ad hoc*, choosing equals to zero the constant of integration.

Inversely, assuming true the London equation, one can verify that it contains the hypothesis of motion without friction of the super-current, that is the zero resistivity of a superconductor. We will see later that this equation contains much more than the condition of zero resistivity, but, for the moment we can see how to obtain the frictionless motion of the super-current.

We have to operate inversely than at the beginning of this section, thus we have to take the time derivative of eq. (11.2). We get

$$\frac{\partial}{\partial t}\mathbf{J}_S = k\left(-\frac{\partial}{\partial t}\mathbf{A}\right). \tag{11.8}$$

Recalling what we have already seen in chapter 8, we know that $\mathbf{E} = -\partial\mathbf{A}/\partial t$ and hence eq.(11.8) is equivalent to

$$\frac{\partial}{\partial t}\mathbf{J}_S = k\mathbf{E}, \tag{11.9}$$

where $\mathbf{E}$ is the electric field inside the sample.

So, the Ohm's equation (11.1), describing the normal fluid, states that the current density $\mathbf{J}_N$ is proportional to the electric field, so that it is the velocity of the fluid that is proportional to the force acting on it: we thus have motion in presence of friction. On the contrary, from the equations (11.2) and (11.9) we see that it is the time derivative of the current density that is proportional to the electric field, so that it is the acceleration of the superfluid that is proportional to the force acting on it and hence the superfluid is frictionless, and we find the first experimental evidence about the resistivity of the superconductive fluid $\varrho = 0$.

It is interesting to observe that the frictionless condition described by eq.(11.9) is not equivalent to the London condition given by eq.(11.2). In fact, to deduce eq.(11.2) from eq.(11.9) we have to choose the integration constant equal to zero. This is a new phenomenological assumption, made by the London brothers, that is needed to explain the Meissner effect. It is also necessary to stress that eq.(11.2) is valid only if the superconductor is simply connected (it has no holes), and therefore it describes only a narrow, although very important, set of experimental situations: roughly speaking, very weak magnetic fields applied to samples without holes. In the following, we will hint the necessity of a generalization of eq. (11.2) while, in Tab. 11.1, we summarize what we have found so far.

Normal fluid	**Superconducting fluid**
$\mathbf{v}_N \propto \mathbf{E}$	$\mathbf{a}_S \propto \mathbf{E}$
$\varrho \neq 0$	$\varrho = 0$
$\mathbf{J}_N - \sigma\mathbf{E} = 0$	$\mathbf{J}_S + k\mathbf{A} = 0$
friction	*no friction*

Table 11.1.: Characterization of the two fluids

11.3.2. The Meissner effect for secondary school students

Although the problem of describing the Meissner effect is still debated [69, 70, 71, 72, 73] for what concerns the possibility of getting a classical derivation of it, in this thesis we describe the Meissner effect using integral operators, such as circulation and flux, that they should have already used in their previous path of electromagnetism. It should be convenient to divide this explanation into two parts:

1. the first part is preparatory to the second and pertains the fact that an applied magnetic field orthogonal to the superconductor surface cannot penetrate the sample;
2. the second part pertains the expulsion from the bulk of the superconductor of a magnetic field parallel to the surface.

To follow the sequence that we propose, students have to know some properties of the magnetic vector potential and some basic properties of the magnetic field, that we resume below:

- The integral relation between the the magnetic vector potential and the magnetic field

$$C_\gamma\left(\mathbf{A}\right) = \Phi_{S_{OPEN}}\left(\mathbf{B}\right), \tag{11.10}$$

  where $C_\gamma\left(\mathbf{A}\right)$ indicates the circulation of the vector potential along a closed line γ, while $\Phi_S\left(\mathbf{B}\right)$ indicates the flux of the magnetic field through the surface S that has γ as a boundary. This property has already been explained within the path on the vector potential in chapter 8.

- The Ampère-Maxwell law

$$C_\gamma\left(\mathbf{B}\right) = \mu_0 I, \tag{11.11}$$

  where I is the current passing through γ.

- The Maxwell's equation

$$\Phi_{S_{CLOSED}}\left(\mathbf{B}\right) = 0, \tag{11.12}$$

  that states the solenoidality of the magnetic field.

1. An orthogonal magnetic field cannot penetrate the superconductor

For our purpose we choose a superconductor extended in a semi space, as the London did in their derivation of the Meissner effect that we have recalled in chapter 3, where we described the main phenomena related to the superconductivity. Our aim is a description analogue to that of the London, but using integral mathematical tools, suitable for secondary school students.

We suppose that the magnetic field applied, B_{app}, is always uniform and orthogonal to the superconductor surface, as shown in Fig. 11.2(a).

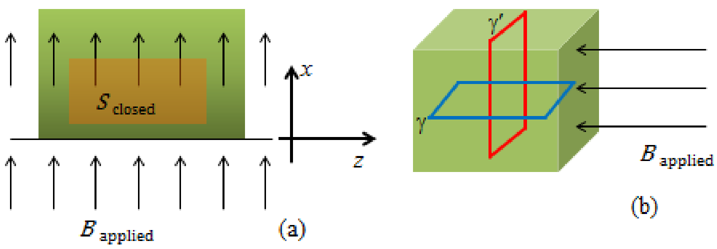


Figure 11.2.: (a) The superconductor, in green, is infinitely extended along the x-axis. It is also shown a section of the closed surface S and the arrows indicate the direction of the magnetic field. (b) A portion of the infinite superconductor and the two chosen pathsγ and γ'.

We also suppose initially, that the magnetic field can penetrate the superconductor. If the superconductor is homogeneous, isotropic and infinitely extended along the yz plane, then we can conclude that the problem has a symmetry for translation along the y and z-axes. For this reason the magnetic field will be uniform over planes parallel to the yz plane, also inside the superconductor.

If we consider a closed surface S, for example a parallelepiped, whose section is represented in Fig. 11.2(a), and we use eq.(11.12), we can immediately deduce that the intensity of the magnetic field $\mathbf{B}$ is also invariant along the x-axis, and the magnetic field must therefore be uniform inside the entire superconductor.

Now, if the magnetic field is uniform, its circulation along any closed path γ is zero, so we can write, referring to Fig. 11.2(b), that

$$C_\gamma(\mathbf{B}) = 0, \tag{11.13}$$

and, therefore, if we take a little loop γ orthogonal to J, from eq.(11.11) we get

$$0 = \mu_0 I = \mu_0 J S, \tag{11.14}$$

where I is the current passing through the line γ along which we calculate the circulation, that is sketched in Fig. 11.2(b) and S is the surface having γ as boundary.

Thus, from eq.(11.14) we can conclude

$$J = 0. \tag{11.15}$$

If we recall the London equation (11.2), we immediately get

$$\mathbf{A} = 0. \tag{11.16}$$

In order to obtain another relation that could relate the vector potential $\mathbf{A}$ with the magnetic field that is the field we are interested to, we can choose a second closed line γ' parallel to the surface of the superconductor, that is parallel to the yz plane and apply eq.(11.10), see Fig. 11.2(b).

Using eq.(11.16) we have:

$$C_{\gamma'}(\mathbf{A}) = 0 \tag{11.17}$$

and hence we obtain

$$\Phi_{S'}(\mathbf{B}) = 0, \tag{11.18}$$

where the surface S' has γ' as a boundary.

Therefore $B = 0$, and since the surface S' can be taken next to the superconductor surface as you want, hence by eq.(11.18) we have that no orthogonal magnetic field can penetrate the sample.

So far we can conclude that in order to have a picture of the behaviour of a superconductor it is necessary to consider together the London equation and the equations of the electromagnetism. This is what we did in the present part 1. and in the next part 2. we will do the same: we will use together electromagnetism and the London equation that evidently states something that is a kind of condition more tighten than the electromagnetism. We stress this concept here because it is important that students understand that the London equation is the key point in the development of their path on superconductivity and it is important that they are able to recognize where this equation is implied.

2. The expulsion of a parallel magnetic field from the bulk of a superconductor

Having just showed that the orthogonal component of a magnetic field cannot penetrate the surface of a superconductor, we now suppose that the magnetic field is applied parallel to the surface of the superconductor, as it is represented in Fig. 11.3. We than suppose that the parallel field enters the superconductor.

Figure 11.3.: Section of the superconductor, in green. The red arrow represents the magnetic field applied parallel to the surface.

First of all, we have to make a preliminary consideration. If the superconductor is isotropic, as we shall assume for simplicity, the field that penetrates inside the superconductor must have the same direction as the field applied, for symmetry reasons. We want to demonstrate that the inside field cannot be uniform. Let us suppose for absurd that the field inside of the superconductor is uniform, as represented in Fig. 11.4(a). We precise that what we are going to say holds in general for uniform fields, although in Fig. 11.4(a) we have drawn the particular case in which the field is uniform and has the same intensity of the applied field.

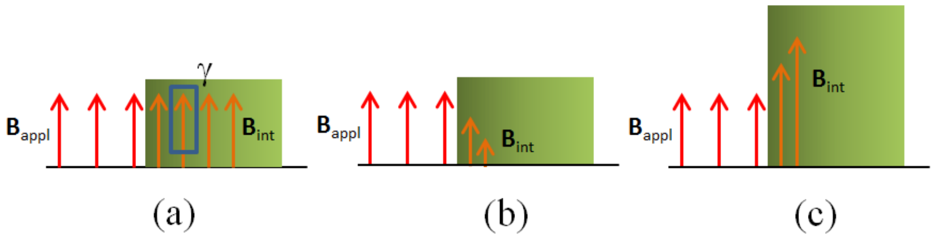


Figure 11.4.: The magnetic field applied is represented in red, while the magnetic field that enters the superconductor is represented in orange. The three figures show three hypothetical possibilities for the intensity of the magnetic field inside the superconductor: in (a) the field remains of the same intensity of the applied field, in (b) the field decreases to zero as it enters the superconductor and in (c) the field increases indefinitely.

If we choose a closed line γ, as the line γ represented in Fig. 11.4(a), we have, for the uniformity of the magnetic field, that

$$C_\gamma(\mathbf{B}) = 0, \tag{11.19}$$

and hence, if we use eq.(11.11) for the line γ just chosen and the surface S that has

γ as a boundary, we obtain

$$\mu_0 I = 0 \tag{11.20}$$

Now, as we did in the previously, we can rewrite the current I in terms of the current density J and apply the London equation (11.2), so to get

$$-\mu_0 kSA = 0, \tag{11.21}$$

that is equivalent to say

$$A = 0. \tag{11.22}$$

But the result $A = 0$ implies the other condition $B = 0$, because we can apply eq.(11.10). In fact, from $A = 0$ we have also that $C_\gamma(\mathbf{A}) = 0$, and thus:

$$\Phi_S(\mathbf{B}) = 0. \tag{11.23}$$

Since S is a open surface that can be chosen as you want, eq.(11.23) implies that the magnetic field must be zero. In other words, there can be no region inside the superconductor where the magnetic field is uniform, or tends asymptotically to a finite value different from zero.

Then, it is reasonable to suppose that the field increases or decreases with the distance from the surface. For obvious reasons related to the conservation of the energy density, we have to reject the case of a indefinitely increasing field, as represented in Fig. 11.4(c). Therefore the only possibility is represented in Fig. 11.4(b), in which the magnetic field decreases to zero. Referring to what we have just said, we can notice that the slower the spatial variation of the magnetic field, the more the intensity of the field approaches zero. Taking into account that the intensity of the magnetic field is different from zero at the surface, it turns out that $\mathbf{B}$ must vanish quickly inside the superconductor. Therefore, the magnetic field $\mathbf{B}$ will be present only very close to the surface where there will be also the current density $\mathbf{J}$, the source of the magnetic field that the superconductor produces in opposition to the applied field.

In fact, we note that the London relationship $\mathbf{J}_S = -k\mathbf{A}$ contains the minus sign just because the current density $\mathbf{J}$, that is established on the superconductor surface, flows in such a way to generate just the magnetic field opposite to the applied field in order to cancel the field inside the superconductor. Although we have seen that the magnetic field must vanish in a small distance very close to the surface, we have not been able to demonstrate that the magnetic field decreases exponentially, differently from what we have shown for teachers. Nonetheless the concept of the penetration depth λ_L introduced in chapter 3 can be easily introduced to students all the same. Moreover, we note that in the practice the penetration depth is evaluated by experiments because its mathematical expression is often unusable.

11.4. The validity of the London equation

At the beginning of this chapter, we have listed among the prerequisites for superconductivity the knowledge of the wave behaviour of matter, for which it is necessary to develope a separate educational path when the teacher wanted to introduce the modern physics in the secondary school. In this case, the wave behaviour of matter becomes the basis for a deeper understanding of superconductivity in which the superconducting fluid is described by a complex wave field, as in the Ginzburg-Landau theory. Even without taking into account any other equation, the description of the superconducting fluid as a wave field allows a generalization of the London equation in a quite clear and easily understandable way. Moreover, from the same generalization, it becomes also possible the discussion of the quantization of the magnetic field in a superconductor, and the introduction of the concept of fluxoid.

If the students have had the possibility of treating the wave behaviour of matter, it becomes possible that they can have an explanation of the most complex phenomenology that they observe in lab, when they perform experiments using YBCO samples in which the magnetic field is not always expelled as in the Meissner effect, but can penetrate in fluxoids throughout the sample in the mixed state.

Because of the fact that many students may have not dealt with the wave behaviour of matter and with the quantum physics, we will discuss the validity of the London equation in two different ways:

1. In the first way we will guide the students to some experimental observations, in order to understand that the London equation no longer holds and we will describe in detail when this happens;
2. In the second way, after the observations of the previous point, we will delineate a path through which it is possible to explain mathematically the penetration of the magnetic field in the bulk of the superconductor.

11.4.1. Experimental evidences of the penetration of the magnetic field in the bulk of the superconductor

From an experimental point of view, there are many situation in which the magnetic field can penetrate a superconductor. Here below we summarize the most common situation in which this happens.

11.4.1.1. The superconductor has holes in its shape

The first example of a superconductor with holes is a *ring* superconductor [74]. It can trap the magnetic field applied in its hole, as indicated in Fig. 11.5. In this example the magnetic field is applied to the ring when the superconductor is at a temperature $T > T_C$; in this case the magnetic field can penetrate the sample. But if the temperature goes under the critical temperature, then the super-currents are activated to preserve the field variations, following the Maxwell's equations of the induction. In this way the magnetic field remains inside the hole of the ring because the super-currents flow along the ring, see Fig. 11.5(b).

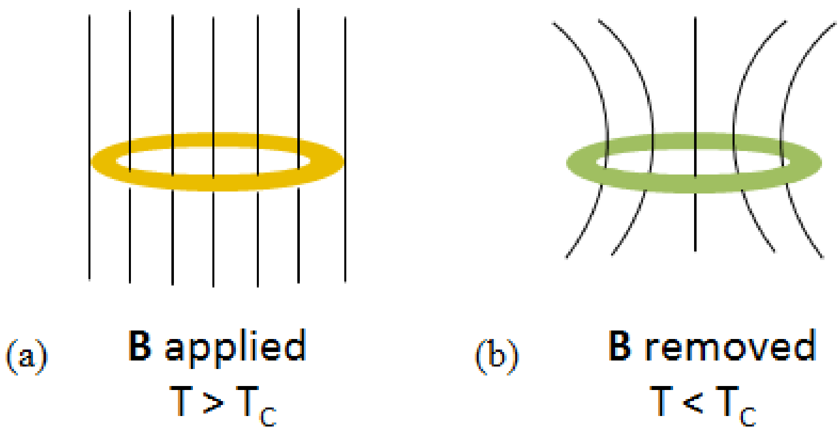


Figure 11.5.: The magnetic field can be trapped in a ring superconductor.

11.4.1.2. Holes in superconductors of type I due to the interaction with a magnetic field

In this section we will see a pair of situations in which a superconductor of type I can get its intermediate state, that is, it can be penetrated by the magnetic field in some regions, although remaining superconductive in the complementar regions. Unfortunately, since superconductors of type I are low-T_C superconductors, they cannot be used in lab with secondary school students, hence the following experiments cannot be performed directly by the students, but the teacher can anyway describe them with drawings and other multimedia material, in order to allow that students can familiarize with these phenomena.

We then present here a second set of examples in which a superconductor of type I can reach its *intermediate state*. In the first example, the intermediate state can occur when the deformation of the magnetic field flux lines determines a penetration of the magnetic field into the superconductor, as it is shown in Fig. 11.6 for a spherical superconductor in a uniform magnetic field. It is due to the fact that there are zones in which the density of the magnetic field lines increases because the superconductor tends to expell the magnetic field. In this way, if the intensity of the magnetic field applied is never beyond the critical value B_C, the superconductor remains completely superconductive, as in Fig. 11.6(a). But the intensity of the magnetic field can increase in certain zones, if the field applied increases, as Fig. 11.6(b) shows inside the yellow ellipses. If, within the zones schematically bounded in yellow the intensity of the magnetic field overcomes the critical value B_C, then the superconductor in the proximity of those zones tends to become normal, while in other zones it tends to remain superconductive, and a kind of thermodynamic equilibrium state is reached, in which normal zones (in yellow in Fig. 11.6(c)) are regularly interspersed by superconductive zones (in green in Fig. 11.6(c)).

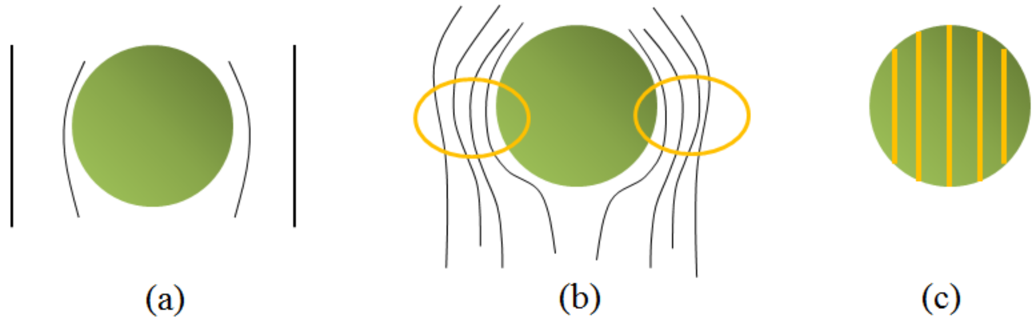


Figure 11.6.: A spherical superconductor (a) reaches its intermediate state (c) if the magnetic field increases enough (b) so that in some points of the surface its value is grater than the critical value B_C.

A second example in which the *intermediate state* of a superconductor of type I can be reached, is when although the magnetic field applied remains unchanged and lower than B_C, the orientation of the superconductor changes respect the direction of the magnetic field lines. It is clear that the orientation of the superconductor can lead to the change in its thermodynamic equilibrium state if the superconductor has at least a dimension much greater than the others, as it is schematically represented in Fig. 11.7.

Another typical way in which a superconductor of type I reaches its *intermediate state* is when an electrical current passes through it, as it is shown in Fig. 11.8. In this case the magnetic field is generated by the current inside the superconducting wire, and for the Maxwell's law the intensity of the magnetic field increases getting closer to the surface. For this reason one expect that the magnetic critical value is reached first in the proximity of the surface of the wire. Infact, when this critical value a new thermodynamic equilibrium configuration is reached in which the superconductive region is more internal than the normal one, as it is sketched in Fig. 11.8(b).

11.4.1.3. Holes in superconductors of type II due to the interaction with a magnetic field

This section recalls the experimental evidences pertaining superconductors of type II, that are those superconductors used by students in lab, of which YBCO is the most widespread one. Also in this case, the behaviour of the superconductor is complex and students have to be guided in order to catch the experimental details, that we resume here briefly giving their basic characterization.

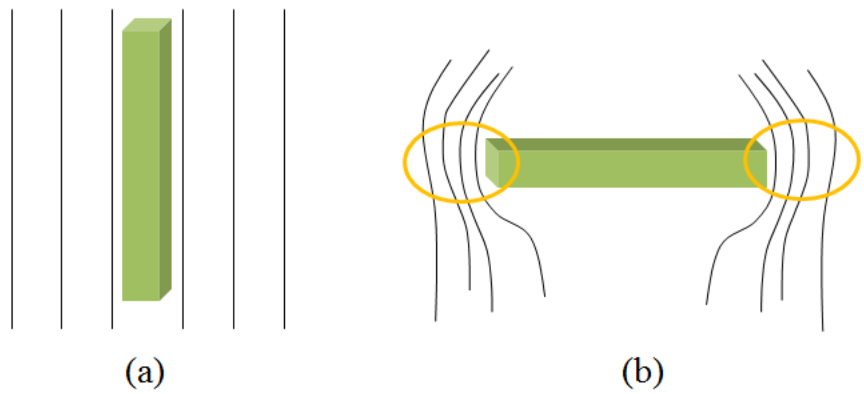


Figure 11.7.: (a) A magnetic field is applied and the deformation of the field lines is small enough to leave the superconductor entirely in its superconductive state; (b) The rotation of the superconductor respect the field lines produces a very high density of magnetic field to the endpoints of the superconductive bar and if in those points the intensity reaches the critical value B_C, then the transition to the intermediate state may occur.

As we have just seen in chapter 3, superconductors of type II are in general ceramic materials, that are insulating at room temperature, but in correspondance of their critical temperature they undergo to a the thermodynamic transition and become superconductive. Moreover, depending on their chemical composition of the alloy that constitutes the superconductor, there exists a characteristic critical field that can penetrate the sample in discrete microscopic regions uniformly distributed as it can be seen in Fig. 11.9. The new state of the superconductor, that is a state where both the superconductive and the normal state cohexist. This state is called *mixed state* or *vortex state* and it is a typical state of the superconductors of type II.

An increase in the strength of the external field affects neither the dimension of each vortex nor the magnitude of the magnetic field flux transmitted by it: simply the number of vortices increases. The magnetic field compresses the vortex lattice until it is destroyed: the vortices flow together and transition into normal state occurs [9].

In the next section we will describe mathematically the generation of the vortices in a superconductor by mean of the interpretation of the super-currents as a matter wave. But as a conclusion of this section we summarize the two most common situations that students see in their lab practice: the elastic levitation and the rigid levitation of a magnet.

11.4.1.4. Elastic levitation

Usually students can work with YBCO samples, different in their chemical structure due to their processing, and each of them exhibits a particular behaviour in

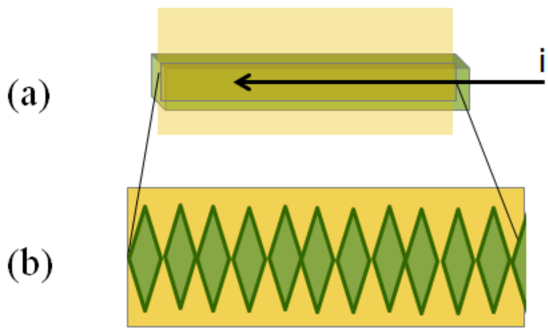


Figure 11.8.: A piece of superconductive wire carrying current. In (a) it is shown the current direction and in yellow the section. In (b) it is shown the section of the wire, with the distribution of the superconductive region, in green, and the normal region, in yellow.

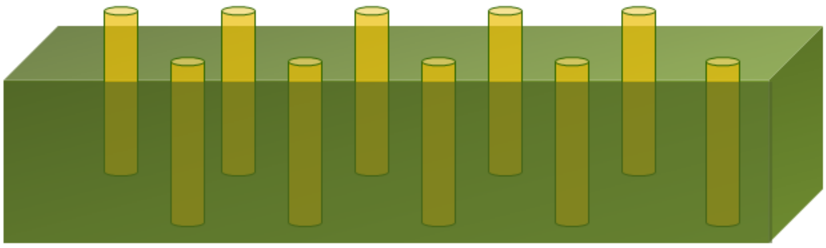


Figure 11.9.: The yellow cylinders represent schematically the vortices present in a superconductor of type II, drawn in green.

interacting with magnetic fields. One of the simpler behaviour observed in lab is the *elastic levitation*, that is the most similar phenomenon to the Meissner effect. It is never a pure Meissner effect, but if the sample is adequately processed one can obtain a very good Meissner effect, that is an almost complete expulsion of the magnetic field from the bulk of the superconductor [66, 67].

A diagram of the forces acting on the magnet suspended above the superconductor is shown in Fig. 11.10. The elastic levitation is characterized by:

- The levitated magnet has a quite well defined equilibrium position about which it can oscillate and rotate;
- There is no energy dissipation.

A student may appreciate these features of the elastic levitation quite simply, but the teacher should put the student in the condition to understand the causes of what he/she observes, and make opportune counterexamples in order to clarify the observations. Never, infact, the superconductivity experiments are trivial. Here below we resume the main causes of the elastic levitation obtained with an experimental set-up as that represented in Fig. 11.10.

- The magnet and the superconductor are cylindrical, thus there is a symmetry along the cylinder axis: this allows the rotation of the suspended magnet. In a different symmetry the rotation could be more difficult.
- The magnetic field does not penetrate the sample: this allows the elasticity of the movement of the magnet respect the superconductor. If the magnetic field penetrated, there would appear friction effects, and the motion of the magnet would be gradually reduced until stop.

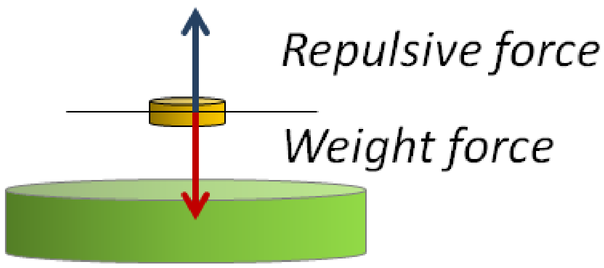


Figure 11.10.: Elastic levitation. The superconductor is represented in green, while the magnet is in yellow. The forces acting of the magnet are represented by the two arrows: the blue arrow represents the repulsive force of the superconductor on the magnet, due to the Meissner effect, while the red arrow represents the weight force of the magnet itself.

11.4.1.5. Rigid levitation

A very common observation for student in lab, when they use YBCO samples, is the *rigid levitation* of a magnet [67, 68, 65]. A student immediately may understand that somethin is changed respect the elastic levitation because he/she can turn upside down his/her sample and the magnet remains close to the superconductor, as if there was an attraction between two magnets, although they remain at a certain distance from each other.

A diagram of the forces acting on the magnet suspended above the superconductor is shown in Fig. 11.11. The rigid levitation is characterized by:

- The levitated magnet stays rigidly suspended in the air in a wide range of positions. When pushed, it remains motionless and does not swing;
- There is energy dissipation.

Here below we resume the main causes of the rigid levitation obtained with an experimental set-up as that represented in Fig. 11.11.

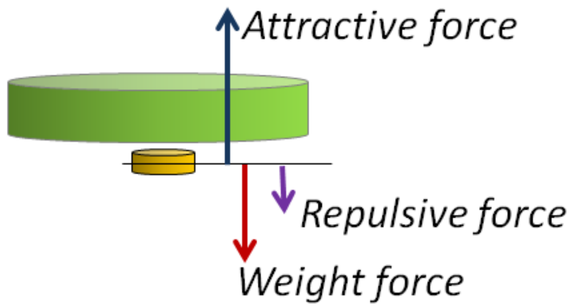


Figure 11.11.: Rigid levitation. The superconductor is represented in green, while the magnet is in yellow. The forces acting on the magnet are represented by the three arrows: the lilac arrow represents the repulsive force of the superconductor on the magnet, due to the Meissner effect, the blue arrow represents the attractive force of the superconductor to the magnet, due to the penetration of the magnetic field, while the red arrow represents the weight force of the magnet itself

- The magnet is strong and its magnetic field penetrates the sample, because its values is beyond the first critical value (the value that characterizes the mixed state of a superconductor of type II) of the particular YBCO used.
- The number of magnetic field lines in the sample does not vary, hence the damping is very weak.
- The particular alloy by which the superconductor is constituted, let the magnetic field inside the bulk to remain fixed even when the magnet is removed: this is called *pinning of the flux lines*.

The pinning is always present when students use YBCO samples and this is a further complication of their experimental work. However, if it becomes clear to students the scheme of the forces involved, see Fig. 11.11, they can be sufficiently aware of what they are doing.

In the lab practice it is possible to experience with these two types of levitation, that can be thought as the two extremes: the extreme in which the magnetic field does not penetrate at all, and the extreme in which the magnetic field penetrates markedly. Sometimes it is possible to experience a levitation that can be classified in the middle of these two extremes, thus making more difficult for a student to understand what is really happening inside the superconductor.

Anyway, the observations in lab may take even 4 hours, or more, in our experience, because if the students are motivated and guided they can discover a very rich phenomenology and have their first picture of superconductivity. We think that in most cases this appoach in lab, besides the description of the phenomenology that cannot be directly experienced by students, can be sufficient in order to get a first approach to superconductivity.

Moreover, through these examples, we have highlighted the fact that the London equation cannot be enough to describe what students observe in lab. For this reason in the next section we propose a possible way to deal with the generalization of the London equation and its critical points. And, if the students have the necessary prerequisites already discussed, they could afford also the part described here below.

11.4.2. Hints for the theoretical explanation of the fluxoids in a superconductor

This last part of the thesis is an outline of the sequence on the generalization of the London equation. It has been experimented one year only, with skilled and motivated high school students, who were familiar also with differential operators as divergence and curl. We think that it should be revised in certain critical points, for the "everyday students" and, in the following, these critical points will be indicated.

When we introduced the London equation:

$$\mathbf{J} + k\mathbf{A} = 0, \qquad (11.24)$$

we posed an *ad hoc* hypothesis on the integration constant, that is we posed that constant equals to zero, see eq.(11.7).

We can now generalize eq.(11.24) avoiding to choose that integration constant equals to zero, that is:

$$\mathbf{J} + k\mathbf{A} = constant. \qquad (11.25)$$

With simple calculations it is possible to obtain a new expression of eq.(11.25) that is very meaningful.

Recalling the definition of the current density, we can write:

$$\mathbf{J} = nq\mathbf{v}, \qquad (11.26)$$

that can be substituted in eq.(11.25) thus obtaining:

$$nq\mathbf{v} + k\mathbf{A} = constant. \qquad (11.27)$$

With simple operations of multiplication and division, we can write:

$$m\mathbf{v} + q\mathbf{A} = new\ constant. \qquad (11.28)$$

The *new constant* that appears in eq.(11.28) can be easily replaced by the generalized momentum $\mathbf{p}$, that corresponds exactly to the expression in the left member of the equation. In this way we obtain that the generalized London equation (11.25) is the definition of the generalized momentum:

$$\mathbf{p} = m\mathbf{v} + q\mathbf{A}. \tag{11.29}$$

At this step, at least two things have to be discussed, both with teachers and students, at different levels.

- The introduction of the generalized momentum, that we did in eq.(11.29) should be part of an educational path on quantum physics, that we have mentioned among the prerequisites of this section. High school students can approach this concept intuitively if they see the generalized momentum as the momentum that a charge must have in an electromagnetic field. In this case it is not enough to consider the quantity $m\mathbf{v}$ that comes from mechanics, but becomes essential the addition of the term $q\mathbf{A}$ that expresses the interaction with the magnetic field.
- With eq.(11.26), without the appropriate knowledge of the wave-like behaviour of matter, we might be led to imagine that the super-current is a stream of particles of charge q, and velocity $\mathbf{v}$, as it is usual to imagine even for the electrical current. Although the microscopic mechanism of the normal current or the super-current is only briefly outlined in this thesis work, being beyond the purposes of this thesis, we would like the reader to perceive the importance of the wave-like model of the super-current, as we already discussed for the normal current, and the necessity of treating current as a matter wave of velocity $\mathbf{v}$ and charge q, rather than a stream of charged particles with velocity $\mathbf{v}$.

Hence, supposing that the super-current is a material plane wave, propagating in the superconductor, and supposing to have the needed back-ground already discussed, we can express the super-current by the following plane wave:

$$\psi(\mathbf{x}, t) = a e^{\frac{i}{\hbar}\mathbf{p}\cdot\mathbf{x}}, \tag{11.30}$$

where $\mathbf{p}$ is the generalized momentum defined by eq.(11.29).

If we indicate the phase of the wave with ϕ, we can write:

$$\phi = \frac{1}{\hbar}\mathbf{p}\cdot\mathbf{x} \tag{11.31}$$

and rewrite the momentum $\mathbf{p}$ in a very useful way, in terms of the phase ϕ by means of a derivative process:

$$\mathbf{p} = \hbar\nabla\phi. \tag{11.32}$$

Eq.(11.32) can be obtained by students in two subsequent steps. In fact, before they write the vector expression, they can try to get the expression in the mono-dimensional case, that is very simple.

If

$$\phi = \frac{1}{\hbar}px, \tag{11.33}$$

it is possible to rearrange the terms of the equation and obtain:

$$px = \hbar\phi \tag{11.34}$$

and

$$\frac{d}{dx}(px) = \frac{d}{dx}(\hbar\phi), \tag{11.35}$$

from which a student may obtain very easily:

$$p = \frac{d}{dx}(\hbar\phi). \tag{11.36}$$

At this point, the generalization to the three-dimensional case, is immediate, and even a secondary school student may obtain eq.(11.32). So, we have to put our new expression of the momentum in eq.(11.29) and to use the equation obtained (11.37):

$$m\mathbf{v} + q\mathbf{A} = \hbar\nabla\phi. \tag{11.37}$$

Both members of eq.(11.37) can be integrated along a closed line: we are describing a relation in which it is implicated the phase of the wave that represents the super-current. For this reason, since the material wave has to turn around the surface of the superconductor, we can calculate the circulation of both members and we can then write:

$$\oint(m\mathbf{v} + q\mathbf{A}) \cdot d\mathbf{l} = \oint(\hbar\nabla\phi) \cdot d\mathbf{l}. \tag{11.38}$$

But, despite the mathematical complexity of eq.(11.38) we can simplify it by mean of two physical considerations, as we already outlined in chapter 3.

- For the good definition of the phase of the wave function ϕ, the circulation in the right member of eq.(11.38) can be equal only to integer multiples of 2π. So we can write:

$$\oint(\hbar\nabla\phi) \cdot d\mathbf{l} = 2n\pi\hbar; \tag{11.39}$$

- For the fact that we are considering the circulation along a closed path γ in the bulk of the superconductor, we can imagine that the current is null, that is $\mathbf{v} = 0$, being different from zero only on the surface of the superconductor. So we can write:

$$\oint (m\mathbf{v} + q\mathbf{A}) \cdot d\mathbf{l} = \oint (q\mathbf{A}) \cdot d\mathbf{l}. \tag{11.40}$$

Now, taking together eq.(11.39) and (11.40), we have:

$$\oint \mathbf{A} \cdot d\mathbf{l} = \frac{2n\pi\hbar}{q}. \tag{11.41}$$

But eq.(11.41) can be easily expressed in terms of the magnetic field, by virtue of the equation $\oint_\gamma \mathbf{A} \cdot d\mathbf{l} = \int_S \mathbf{B} \cdot \mathbf{n}\, d\sigma$, where S is the surface that has γ as a boundary. For shortness in notation, we replace $\int_S \mathbf{B} \cdot \mathbf{n}\, d\sigma$ with $\Phi(\mathbf{B})$ and we finally write:

$$\Phi(\mathbf{B}) = \frac{nh}{q}, \tag{11.42}$$

that represents the quantization of the magnetic flux inside the superconductor, and also appears the quantum of the magnetic flux, the *fluxon* Φ_0, so to have $\Phi_0 = \frac{h}{q}$.

Experimental measurements of the fluxon, give:

$$\Phi_0 = 2 \cdot 10^{-15} weber. \tag{11.43}$$

From the numerical value of eq.(11.43) it is possible to obtain the value of the charge q of the carrier of the supercurrent. One discovers that $q = 2e$ (as the microscopic BCS theory for superconductors of type I predicts) suggesting a kind of coupling between two electronic waves in the process of superconduction.

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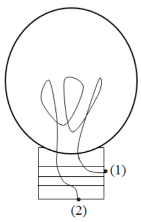
Acknowledgments

When I started this research I couldn't imagine how hard it would be nor how engaging and full of surprises. My thanks go to physics for its beauty, a beauty that never ceases to amaze me. And to its patience in waiting for someone, that sometimes, can find out something about its nature. Pure joy. My thanks also go to all the people who supported me and helped me to get close to physics, without considering my endless limits. I thank Marco Giliberti for his beautiful insights and for heated discussions which were always sincere and full of affection. I thank my sister Marta, for looking after my daughter Ludovica while I was at work or at conferences. I thank my mother for teaching me to never give up no matter what difficulties I faced, and my father for teaching me about beauty and the simple joy of living. I thank 18 11, two numbers that make my life just that I love. Without them, this work would have never been possible and would never have been so profound for me.

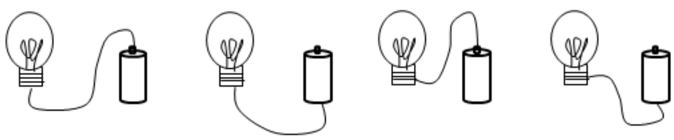
A. Test on Electrical Conduction

We report here the translation in english of the written test given to the students for the first two years of the experimentation. The same test has been given at the end of the experimentation of the group that followed the instruction and in the same period to a control group. The control group was similar to the first group for age and kind of secondary school followed by the students, but did not follow our lessons. For shortness, we omitted the space that was given at the bottom of each question for the students' answers, as we have omitted the heading of the sheet, for the students' data.

Preliminary notions. The image shows the structure of an incandescent bulb: one of the ends of the filament is connected to the lateral metal thread (1), while the other is connected at the bottom of the bulb to the metal protruding (2), that is electrically insulated from the thread. In all the following questions we suppose that the bulb used in the particular experimental situation described is always suited for the battery chosen. Moreover, we suppose that all the wire used in the experimental situation described are always insulated.

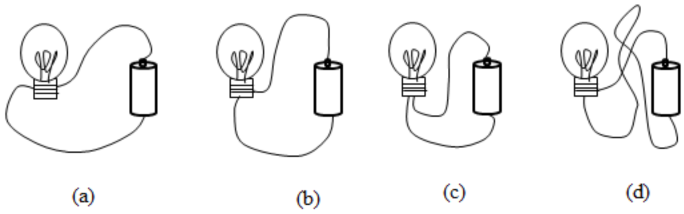


Question 1. Which of the following bulbs will light up? Explain your answer.

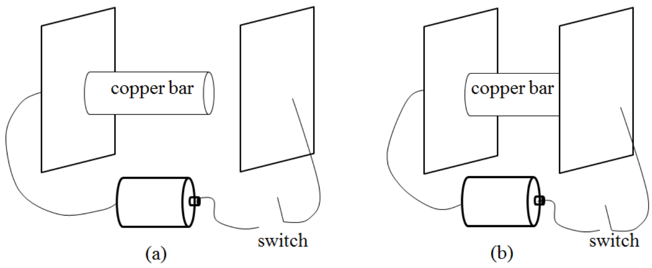


(a) (b) (c) (d)

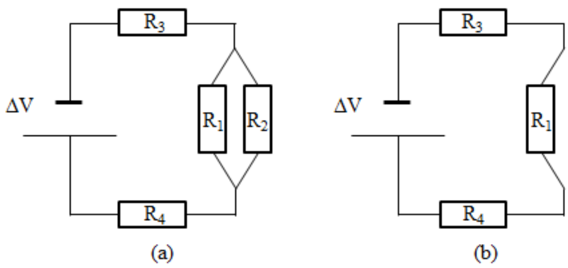
Question 2. Which of the following bulbs will light up? Explain your answer.



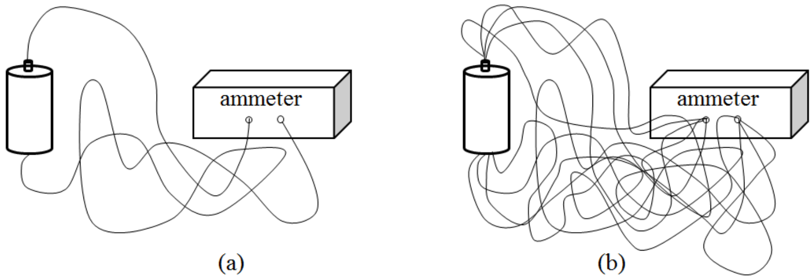
Question 3. The following pictures show two similar experiments. In the case (a) a bar of copper is placed in between the two conductive plates of a capacitor, but not in contact with them. On the contrary, in (b) the bar touches the plates. When you close the switch, what can you say about the current in the copper bar? Explain both case (a) and case (b).



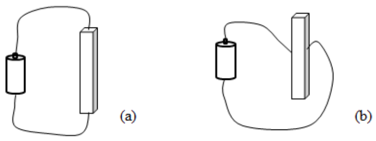
Question 4. In the circuit represented in the following figure, the potential difference is $\Delta V = 6$ V, the resistors $R_3 = R_4 = 2$ ohm, and the parallel of the resistors R_1 and R_2 is again 2 ohm. What is the current $i_1$that flows in the resistor R_1 in the sketch (b) of the figure?



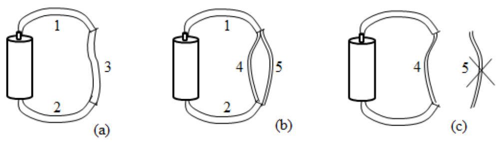
Question 5. In the case (a) a battery, an ammeter and a hundread meter long insulated copper wire are in series. In the case (b) the situation is similar, but the copper wires are four and identical to the first in (a). If the ammeter in (a) reads a current intensity i_A, what is the current intensity i_B that the ammeter in (b) will read?



Question 6. The same bar is connected to the same type of battery, but in two different ways, as figures (a) and (b) show. Which is the relation between the two resistances?



Question 7. The circuit is supplied by a potential difference of 3V. The three parts of the circuit are indicated by the numbers 1, 2 and 3. Each part has a resistance of 1ohm. In case (b) the wire 2 is splitted in two identical parts, named wire 4 and wire 5.



What is the current flowing in the wire 4 of the circuit (b)? In (c) the wire 5 is removed. What is now the current flowing in the wire 4 of the circuit?

Question 8. An electrical current is flowing in a copper wire with the ends connected to a battery. All the parameters of the experiment remains unchanged, except for the temperature that increases. What happens?

1. The current in the wire increases, because the electrons of conduction have more energy than in the initial situation and therefore theor speed may increase.
2. The current in the wire decreases, because the ions in the crystal lattice oscillate more quickly, therefore the electrons pass with difficulty.
3. The current remains unchanged, because the battery is unchanged, therefore it provides always the same current.
4. The right answer is not present, hence write down here below what is the right answer in your opinion.

Question 9. Where are placed the charge carriers in a wire?

1. On the surface of the wire, infact the relation that expresses the second Ohm's law contains an "S" of surface in the denominator.
2. On the surface of the wire, infact in a conductor the charged are always placed on its surface, in such a way that the electric field inside the conductor is zero.
3. The right answer is not present, hence write down here below what is the right answer in your opinion.

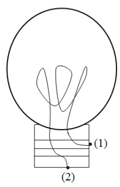
Question 10. An electrical current flows in a wire if:

1. The potential difference does not exceed a maximum threshold, on the contrary, if it happens, the wire overheats so much to prevent the current flow.
2. The potential difference exceeds a minimum threshold; an example of this is represented by a bulb that lights up only if it is connected to minimal potential difference.
3. For every potential difference the current may flow.

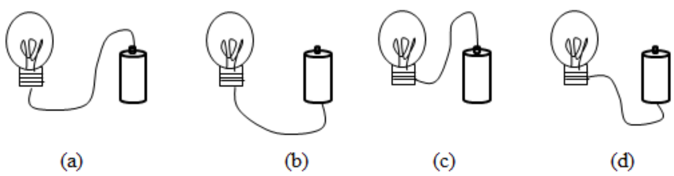
B. Pre-test on Electrical Conduction

We report here the translation in english of the written pre-test given to the students participant to the last experimentation. The same test has been given at the end of the experimentation of the group that followed the instruction and in the same period to a control group. The control group was similar to the first group for age and kind of secondary school followed by the students, but did not follow our lessons. For shortness, we omitted the space that was given at the bottom of each question for the students' answers, as we have omitted the heading of the sheet, for the students' data.

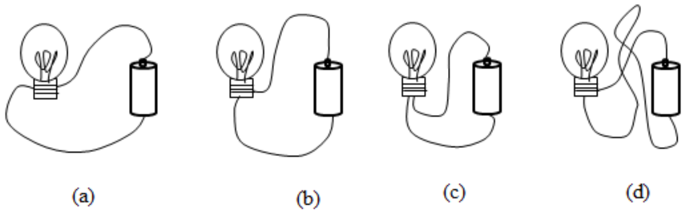
Preliminary notions. The image shows the structure of an incandescent bulb: one of the ends of the filament is connected to the lateral metal thread (1), while the other is connected at the bottom of the bulb to the metal protruding (2), that is electrically insulated from the thread. In all the following questions we suppose that the bulb used in the particular experimental situation described is always suited for the battery chosen. Moreover, we suppose that all the wire used in the experimental situation described are always insulated.



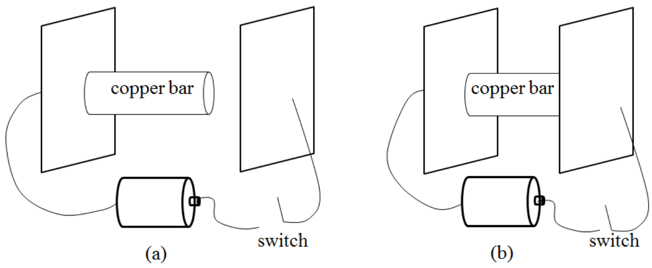
Question 1. Which of the following bulbs will light up? Explain your answer.



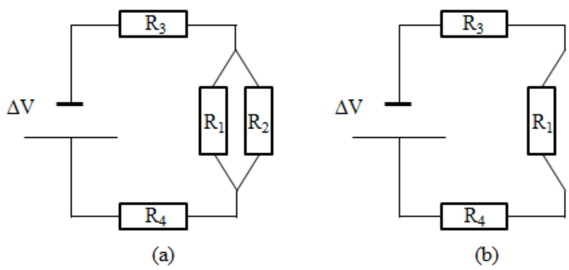
Question 2. Which of the following bulbs will light up? Explain your answer.



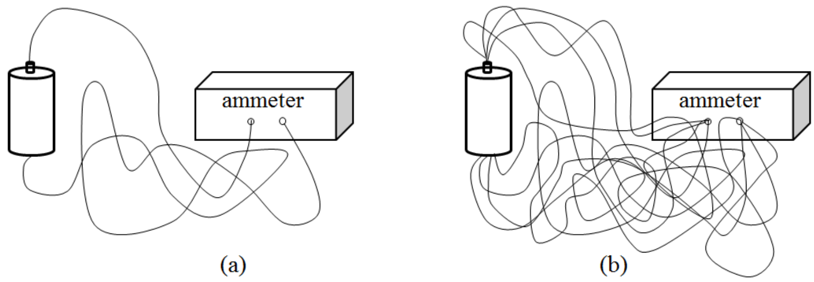
Question 3. The following pictures show two similar experiments. In the case (a) a bar of copper is placed in between the two conductive plates of a capacitor, but not in contact with them. On the contrary, in (b) the bar touches the plates. When you close the switch, what can you say about the current in the copper bar? Explain both case (a) and case (b).



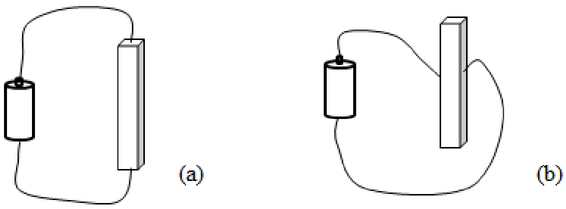
Question 4. In the circuit represented in the following figure, the potential difference is $\Delta V = 6$ V, the resistors $R_3 = R_4 = 2$ ohm, and the parallel of the resistors R_1 and R_2 is again 2 ohm. What is the current $i_1$that flows in the resistor R_1 in the sketch (b) of the figure?



Question 5. In the case (a) a battery, an ammeter and a hundread meter long insulated copper wire are in series. In the case (b) the situation is similar, but the copper wires are four and identical to the first in (a). If the ammeter in (a) reads a current intensity i_A, what is the current intensity i_B that the ammeter in (b) will read?



Question 6. The same bar is connected to the same type of battery, but in two different ways, as figures (a) and (b) show. Which is the relation between the two resistances?



Question 7. An electrical current is flowing in a copper wire with the ends connected to a battery. All the parameters of the experiment remains unchanged, except for the temperature that increases. What happens?

1. The current in the wire increases, because the electrons of conduction have more energy than in the initial situation and therefore theor speed may increase.
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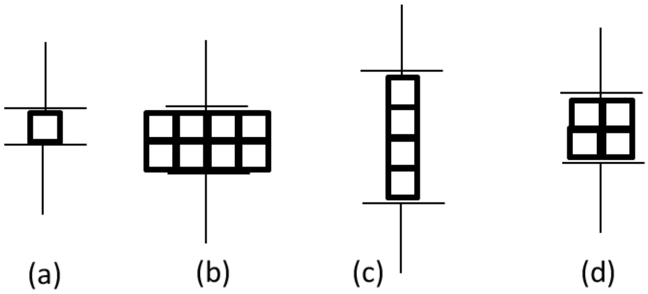
2. The potential difference exceeds a minimum threshold; an example of this is represented by a bulb that lights up only if it is connected to minimal potential difference.
3. For every potential difference the current may flow.

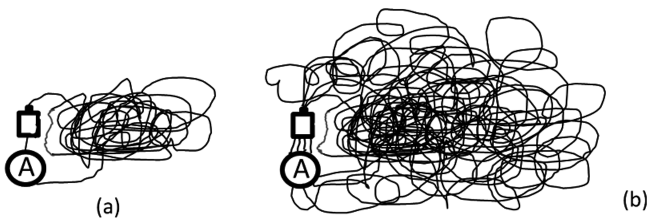
C. Post-test on Electrical Conduction

We report here the translation in english of the written post-test given at the end of the last year experimentation. The first two years the post-test was identical to the first step, but the group of students changed, and it was thus considered as the control group. For shortness we omitted the space that was given at the bottom of each question for the students' answers, as we have omitted the heading of the sheet, for the students' data.

Question 1. A certain material may have four different shapes, represented the following figure (a), (b), (c) and (d). In the four cases the same potential difference ΔV allows a current to flow through the material. We suppose that the thickness of the material does not vary in the four cases considered. There are differences among the currents flowing in the four cases? Describe in detail your answer.

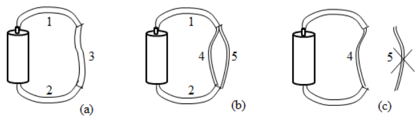


Question 2. In the case (a) a battery, an ammeter A and a hundread meter long insulated copper wire are in series. In the case (b) the situation is similar, but the copper wires are four and identical to the first in (a). If the ammeter in (a) reads a current intensity i_A, what is the current intensity i_B that the ammeter in (b) will read?



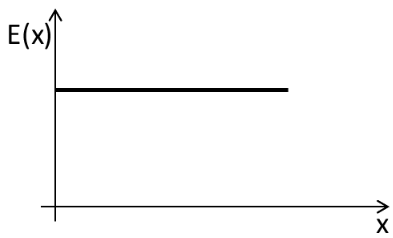
Question 3. Recall the experiment you have performed in lab. Now, imagine having available 9 ohm bulbs (when heated) and a 1.5 volt battery. How many bulbs can you turn on, by connecting them in parallel?

Question 4. The circuit is supplied by a potential difference of 3V. The three parts of the circuit are indicated by the numbers 1, 2 and 3. Each part has a resistance of 1ohm. In case (b) the wire 2 is splitted in two identical parts, named wire 4 and wire 5.



What is the current flowing in the wire 4 of the circuit (b)? In (c) the wire 5 is removed. What is now the current flowing in the wire 4 of the circuit?

Question 5. You have a uniform electric field $E(x)$ represented in the following figure, as the electric field inside a plane condenser. Draw on the same graph of the given figure, the graph of the elecric potential. Explain your drawing.



Question 6. A current is flowing in a wire. While the other experimental parameters remain unchanged, the temperature increases gradually in a certain time interval, and it is possible to measure the current in that time interval. Draw a current-temperature graph, and explain your drawing.

Question 7. Where are placed the charge carriers in a wire carrying currents?

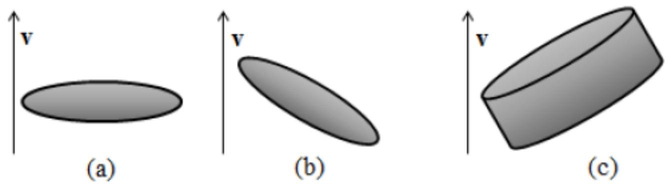
Question 8. There must be a potential difference threshold so that current flows in a conductor?

D. Test on the basic electromagnetism

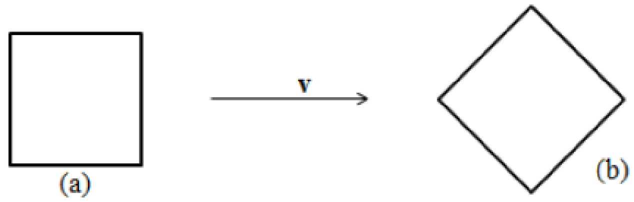
We report here the translation in english of the written test on electromagnetism. The same test has been given as pre-test and post-test, even to the same group of students. For shortness we omitted the space that was given at the bottom of each question for the students' answers, as we have omitted the heading of the sheet, for the students' data.

Question 1. You have a very long conducting wire carrying direct current. Describe the fields inside and outside the wire. Provide with words, formulas and drawings as much information as possible.

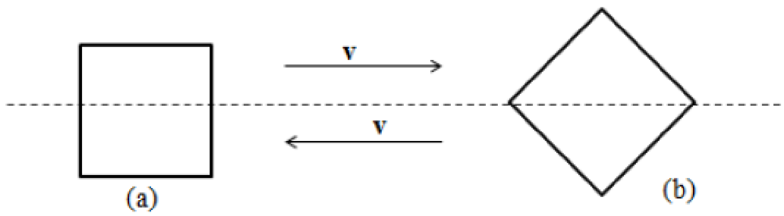
Question 2. A vector field **v** is uniform and its intensity is $v = 10^{-2}u$ (where u is a general unit of measure for the vector field **v**). The field fills all the space where the surfaces are placed, see the following figures (a), (b) and (c), and it is directed as the figures show. Determine the value of the flux of the field through the surfaces in each of the three cases, taking into account that: in (a) the radius of the surface is $R = 3m$, in (b) the surface is inclined at 30° to the horizontal, and in (c) The surface is a cylinder of radius $R = 3m$ and heigth $h = 2m$.



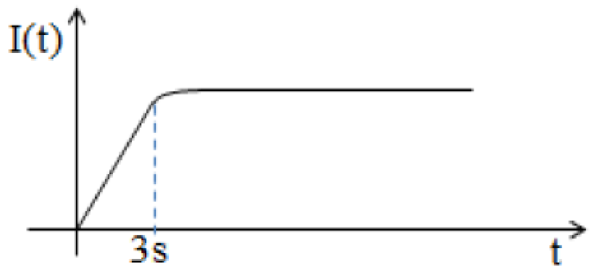
Question 3a. A vector field **v** is uniform and its intensity is $v = 30u$ (where u is a general unit of measure for the vector field **v**). The field fills all the plane where the closed paths are placed, see the following figures (a) and (b), and it is directed as the figures show. Determine the value of the circulation of the field along the paths in each of the two cases, taking into account that: in (a) the closed path is a square of side $L = 2m$ and in (b) the closed path is identical to the case (a) except for the fact that it is rotated of 45°.



Question 3b. The situation is identical than in the exercise 3a, except for what regards the vector field **v** . In this case infact, it is not uniform in all the plave: it changes its verse in correspondance of the dashed line, whereas its intensity remains $v = 30u$ (where u is a general unit of measure for the vector field **v**). Along the dashed line the field is zero. Determine the value of the circulation of the field along the paths in each of the two cases.



Question 4a. You have a very long solenoid (that for simplicity you can consider infinite). The solenoid is carrying a current $I = I(t)$ represented in the following figure. You have also a circular copper loop, coaxial with the solenoid, that is placed outside the solenoid. Determine what happens to the loop, during a certain time interval in which the current in the solenoid varies with time as represented in the figure.



Question 4b. Referring to exercise 4a, determine quantitatively what happens to the loop in the time interval between the time $t_1 = 0s$ and the time $t_2 = 2s$, taking into account that the current increases linearly with time: $I(t) = at$, with $a = 5A/s$. Consider that the radius of the loop is $R = 2cm$, the radius of the solenoid is $r = 1cm$, the solenoid has 300 loops per cm and the loop outside has a resistance of $3m\Omega$.

Question 5. It is given an infinite plane carrying a uniform direct current, whose density for unit of length is J. Describe the magnetic field generated by this current distribution, using words, formulas and drawings.

Question 6. An infinite solenoid is carrying current. The magnetic field outside the solenoid is null, even if it is present inside the solenoid. For a certain time interval, the magnetic field inside changes, because of the changing of the current in the solenoid. Suppose now that a charge is placed outside the solenoid, next to it. What happens to the charge during the time interval in which the field inside changes? Explain also, what the previous considerations have to do with the concept of action at a distance.

E. Lab sheet: the Meissner effect

We report here the translation in english of the main parts of the lab sheet that we gave to students in order to help them become familiar with the complex and wide set of phenomena that are involved in superconductivity.

Preliminary notions.

The kit contains two different types of YBCO superconductors:

- A thick superconductor, that we name SC_{TK}
- A thin superconductor, that we name SC_{TN}

and contains two different magnets:

- A big cylindrical magnet, that we name M
- A very small cylindrical magnet, that we name m

Observations of the interactions between superconductors and magnets

1) In these 4 cases you have to put the magnet on the superconductor and ***only after*** you pour the liquid nitrogen.

Write down all your observation in each of the following cases.

1. $SC_{TN} + m$
2. $SC_{TN} + M$
3. $SC_{TK} + m$
4. $SC_{TK} + M$

2) In these 4 cases you have to pour the liquid nitrogen on the superconductor and ***only after*** you approach the magnet.

Write down all your observation in each of the following cases.

1. $SC_{TN} + m$
2. $SC_{TN} + M$
3. $SC_{TK} + m$
4. $SC_{TK} + M$

3) Interaction between the thick superconductor and the magnet M at a fixed distance

The kit contains a thin plastic plate that can be inserted between the superconductor SC_{TK} or SC_{TN} and the magnet M before pouring the liquid nitrogen, in order to fix the distance between the superconductor and the magnet. At a later time, the plate can be removed with the appropriate plastic tweezers.

Write down all your observations in each of the following cases.

1. SC_{TN} + plate + M $\rightarrow$Liquid nitrogen $\rightarrow$removal of the plate
2. SC_{TK} + plate + M $\rightarrow$Liquid nitrogen $\rightarrow$removal of the plate

Observations of the interactions between two superconductors

In this second part, you can observe two superconductors in interaction between them. In order to do this it is necessary that at least one of the two superconductors has been previously penetrated by the magnetic field and has not lost its magnetization, being remained into the liquid nitrogen. Use the thick superconductor for its property of *pinning* of the magnetic flux lines. To perform some steps of this part you can need two thick superconductors: you can borrow it to some other group.

1) Two superconductors at the liquid nitrogen temperature: one of them previously magnetized

Write down all your observations in each of the following cases.

1. SC_{TK} + plate + M $\rightarrow$ Liquid nitrogen $\rightarrow$ removal of the plate $\rightarrow$removal of the SC_{TK} $\rightarrow$ approach the cold SC_{TN}
2. SC_{TK} + plate + M $\rightarrow$ Liquid nitrogen $\rightarrow$ removal of the plate $\rightarrow$removal of the SC_{TK} $\rightarrow$ approach the cold SC_{TK}

2) A superconductor magnetized interacts with an YBCO sample at room temperature

The superconductor at room temperature has to be approached to the magnetized thick superconductor, putting it directly in the liquid nitrogen bath that contains the magnetized superconductor.

Write down all your observations in each of the following cases.

1. SC_{TK} + plate + M → Liquid nitrogen → removal of the plate →removal of the SC_{TK} → approach the SC_{TN} at room temperature
2. SC_{TK} + plate + M → Liquid nitrogen → removal of the plate →removal of the SC_{TK} → approach the cold SC_{TK} at room temperature

Try to make an estimate

During all the previous experimental part you have seen that when you appoach the magnet to a superconductor previously cooled in liquid nitrogen, if the intensity of your force is enough, you can observe a drastic change in the behaviour of the two objects: from the initial repulsion, they attract each other as two magnets.

Now, suppose that between the two objecs there is a magnetic interaction, and call B the intensity of this interaction.

1. Write down the expression of the force between the two object, taking into account only the physical quantity involved in your opinion, and considering the possible multiplicative constants only from a dimensional point of view.
2. From the formula hypothesized in the previous step, estimate the intensity of the magnetic field B present in the superconductor, at the time in which the to object attract each other as if they were two magnets.

In the case in which the group of students have difficulty in finding an expression of the force between the two objects, can ask to the teacher.

F. Homework on superconductivity

Question 1.

Remembering the experiment on the Meissner effect that you have performed in lab, in your opinion what have been the most meaningful steps that you observed?

Question 2.

Is it always possible to observe the mixed state in the YBCO samples that you have used in lab? What are the steps you have to do if you want to observe the mixed state in a YBCO superconductor sample? How can you recognize the mixed state in a YBCO superconductor?

Question 3.

Was sufficient the time you've got available to carry out the experiment? Did you want to do something, but you did not had time?

Question 4.

Describe with your own word what the Meissner effect is. Try to hang up the experiment in lab.

Question 5.

Did you performed something in your experiment, that you may call properly "Meissner effect?"