Measurement of the Convective Heat-Transfer Coefficient

Rosaria Conti, Aurelio Agliolo Gallitto, and Emilio Fiordilino, University of Palermo, Italy

We propose an experiment for investigating how objects cool down toward the thermal equilibrium with their surroundings. We describe the time dependence of the temperature difference of the cooling objects and the environment with an exponential decay function. By measuring the thermal constant τ, we determine the convective heat-transfer coefficient, which is a characteristic constant of the convection system.

In the last years, an increasing number of researchers are involved in programs aimed at saving energy and reducing emission of carbon dioxide in the atmosphere. In this context, the challenge to sustainability was issued by the U4energy Project,[1] an initiative funded through the Intelligent Energy Europe Programme of the European Union.[2]

Recently, scientific interest in the study of energy efficiency of houses has increased substantially since, in many countries, a large percentage of energy consumption is due to heating and cooling of buildings.[3] For this reason, research to improve the energy efficiency of buildings is a challenge for the future. The main process of energy exchange involved in home conditioning is convection. Both natural and forced convection systems are employed in a common home heating system, e.g., hot water is circulated through radiators by forced convection, while warmed air rises by natural convection.

In this article we propose an experiment to measure the convective heat-transfer coefficient, which is a characteristic constant of convection systems. This experiment can also be used as a laboratory introduction to exponential decay functions.[4] The activity has been carried out together with students of secondary school, in the framework of the Italian National Plan for Scientific Degrees.[5]

Simplified model for cooling of objects

Thermal energy is exchanged between two systems via conduction, convection, and radiation.[6] In the case of cooling objects, from calorimetric relations one has that the heat released by the object is proportional to the temperature variation of the body, assuming that the heat transfer is slow enough to maintain the temperature uniform within the object

$$Q = -mc\Delta T, \tag{1}$$

where m is the mass and c the specific heat of the object.

The energy flow H can be written as

$$H = \frac{dQ}{dt} = -mc\frac{d}{dt}T(t), \tag{2}$$

obtained by Eq. (1) for $\Delta T \to 0$. The negative sign indicates that the temperature decreases when the energy flows from the body toward outside (that is, when $H > 0$).

A comprehensive study of the heat transfer problem has been discussed by Vollmer.[7] The author has shown that for convective heat-transfer coefficient $h_{conv} \approx 30\ \mathrm{W{\cdot}m^{-2}{\cdot}K^{-1}}$, a simple exponential cooling seems to work quite well for $\Delta T < 100$ K. In this case, the contribution to the energy exchange through radiation is about 10% of convection and it becomes larger in the case of lower heat-transfer coefficients.[8] Therefore, for $h_{conv} \gtrsim 30\ \mathrm{W{\cdot}m^{-2}{\cdot}K^{-1}}$, small ΔT, and for systems in which the emissivity coefficient ε is smaller than the ideal blackbody coefficient ($\varepsilon = 1$), the energy exchange by radiation can be neglected.

Following the simplified model for cooling of objects proposed by Vollmer, neglecting the contribution of radiation, one has

$$mc\frac{d}{dt}T(t) = -h_{conv}A[T(t) - T_a], \tag{3}$$

where A is the surface through which the heat flows, $T(t)$ the temperature of the external surface of the body at the time t, and T_a the ambient temperature.

Equation (3) can be rewritten as follows

$$\frac{d}{dt}T(t) + \frac{T(t)}{\tau} = \frac{T_a}{\tau}, \tag{4}$$

where we have introduced the thermal time constant τ as

$$\tau = \frac{mc}{h_{conv}A}; \tag{5}$$

$1/(h_{conv}A)$ is the "thermal resistance" and mc the "thermal capacitance."[9] The time constant τ indicates that large masses m and large specific heats c (large thermal capacitances) give rise to slow temperature changes, whereas large surfaces A and large thermal transfer coefficients h_{conv} (small thermal resistances) give rise to fast temperature changes. By supposing that T_a is constant and replacing $\Delta T \equiv T(t) - T_a$, the solution of Eq. (4) is

$$\Delta T(t) = \Delta T_0\, e^{-t/\tau}, \tag{6}$$

where ΔT_0 is the initial temperature difference at $t = 0$.

In other words, the object reaches the same temperature of the ambient following an exponential law determined by the characteristic thermal time constant τ. By measuring τ, from Eq. (5) we can determine the convective heat-transfer coefficient.

We would like to remark that the exponential dependence is valid for small temperature differences between the object and its surroundings. In case of large temperature differences, different behaviors are observed.[7,9] An analytical solution

Fig. 1. Containers used in the experiment: plastic (polypropylene), glass, and expanded polystyrene, shown from left to right. The heater used to warm up water is also shown on the right side.

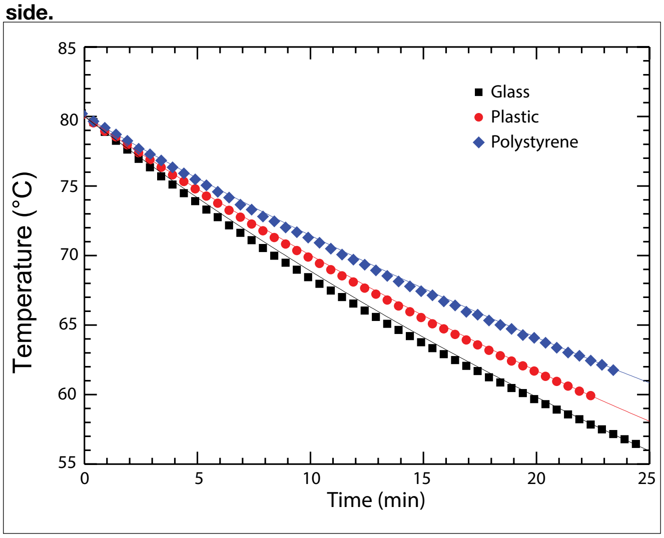

Fig. 2. Temperature of the water as a function of time for the three different containers. Symbols are experimental data. Lines are theoretical curves obtained as explained in the text. The plots have been shifted in time such that they all start at the same temperature of 80 °C.

of cooling and warming laws has recently been proposed by Besson.[10]

Experimental apparatus and results

The experimental apparatus consists of three containers of different material and a computer equipped with an interface and a temperature sensor. The interface used is Vernier LabPro,[11] which works under the Windows operating system and is controlled by the software Vernier Logger Pro; it allows one to automatically acquire experimental data by using sensors like the Vernier temperature sensor. The interface can acquire data with maximum sampling rate of 5×10^4 readings per second. The Vernier temperature sensor is based on an NTC resistor, which has a resolution of about 0.1 °C in the temperature range from 40 °C to 100 °C, accuracy of about ±0.5 °C at $T \sim 100$ °C, and a response time of about 10 s, when immersed in water.[12] Three different containers made of glass, plastic (polypropylene), and expanded polystyrene were used; they are shown in Fig. 1.

In order to measure the time dependence of the temperature during the cooling process, we filled the containers with $m_w = 140$ cm^3 of water at $T \approx 90$ °C. Soon after, we measured the temperature of the water as a function of time for about 30 minutes. The temperature values were automatically acquired using the software Logger Pro. In order to prevent the energy flows from the top and bottom, the containers were thermally insulated from the bottom and from the top by layers of expanded polystyrene with thickness of about 2 cm. The cooling of the system was done in still air under natural convection. The results obtained with the different containers are reported in Fig. 2.

Discussion

The variation with time of the temperature of a liquid that cools in an environment at lower temperature can be described by an exponential law if the temperature difference between the object and the ambient is small. An exponential law is characteristic of many transitory phenomena that concern the evolution of the system toward an equilibrium state or a stationary state. Similar temporal dependencies can be observed in mechanical systems such as a falling body in a viscous medium as well as in electric circuits such as the charging and discharging process of electric capacitors in an RC circuit, etc. Since there is a common mathematical description for all these phenomena, it would be very fruitful for students to note that the same equations can describe experiments lying in different realms of physics, which, therefore, admit the same solution: in this case an exponential decay.[4,7,8]

By fitting the experimental data with Eq. (6), we have obtained the values of τ. As can be seen in Fig. 2, the temperature variation follows an exponential decay quite well. To determine the values of h_{conv}, we have to include also the mass m_c and heat capacity c_c of the containers used in the experiment. From Eq. (5) we have

$$h_{conv} = \frac{m_c c_c + m_w c_w}{\tau A}. \qquad (7)$$

Since we need the lateral surface A of the containers in contact with water through which the heat flows, we have determined the lateral surfaces by measuring the height l of the water in the three containers and multiplying it by the external circumference of each container. The values are shown in Table I.

Table I. Physical parameters of the materials used in the experiment; D_e is the external diameter of the containers.

Material	m_c (g)	c_c (J·g^{-1}·K^{-1})	D_e (mm)	l (mm)	A (cm^2)
Glass	152	0.50–0.84	69 ± 1	39 ± 2	84 ± 4
Plastic	23.5	0.46	65 ± 1	44 ± 2	90 ± 3
Polystyrene	8.2	1.3	80 ± 2	31 ± 2	78 ± 5
Water	140	4.18			

Table II. Values of the thermal parameters obtained from the fitting of the experimental data.

Material	τ (min)	h_{conv} ($W \cdot m^{-2} \cdot K^{-1}$)
Glass	49 ± 2	28 ± 2
Plastic	55 ± 2	21 ± 2
Polystyrene	65 ± 3	19 ± 2

In Table II, we report the values of τ determined by best fitting the experimental data and h_{conv} obtained by Eq. (7), for each container. In Fig. 2, we report the curves (lines) calculated by Eq. (6) using the estimated characteristic time τ for each data set.

We would remark that we have considered infinite the thermal conductivity of the containers, that is the temperature of the outer surface of the container is equal to the temperature of the inner surface, and that it uniformly changes with time. Furthermore, the proposed model does not take into account energy exchange through radiation; therefore, the values of the heat-transfer coefficients we have determined could be affected by an uncertainty larger than that indicated in Table II, especially for plastic and polystyrene containers since for them h_{conv} is considerably less than 30 $W \cdot m^2 \cdot K$.

Conclusion

We have performed an experiment for investigating how objects cool down toward the thermal equilibrium with their surroundings. We have described the time dependence of the temperature difference of the cooling objects and the environment with an exponential decay function. By measuring the thermal constant τ, we have determined the convective heat-transfer coefficient characteristic of the system. As expected, water in the container made of expanded polystyrene cools down at the slowest rate, whereas in the container made of glass it cools down at the fastest rate. The proposed model can be applied for the experimental determinations of such coefficients using different containers under various experimental conditions. This activity can be also used as a laboratory introduction to exponential decay functions for high school students. Although the simplified model is valid for convective heat-transfer coefficient of about 30 $W \cdot m^{-2} \cdot K^{-1}$ and for $\Delta T < 100$ K, that is when the energy exchange by radiation is negligible, it could easily be performed at schools allowing teachers to discuss energy exchange processes, and it might contribute to increase the awareness of students toward energy saving and sustainability issues.

Acknowledgment

This work was carried out under the financial support of the Italian Ministry of Education, University and Research.

References

1. U4energy Project, http://www.u4energy.eu.
2. Intelligent Energy – Europe, http://ec.europa.eu/energy/intelligent/.
3. Paul Hewitt, "Figuring Physics: Newton's law of cooling," *Phys. Teach.* **40**, 92 (2002).
4. John W. Dewdney, "Newton's law of cooling as a laboratory introduction to exponential decay functions," *Am. J. Phys.* **27**, 668–669 (Dec. 1959).
5. E. Fiordilino and A. Agliolo Gallitto, *Il laboratorio di fisica nel Progetto Lauree Scientifiche* (Aracne, Rome, 2010).
6. S. Blundell and K. M. Blundell, *Concepts in Thermal Physics* (Oxford University Press, New York, 2006).
7. M. Vollmer, "Newton's law of cooling revisited," *Eur. J. Phys.* **30**, 1063 (2009) and references therein.
8. V. Thomsen, "Response time of a thermometer," *Phys. Teach.* **36**, 540-541 (Dec. 1998).
9. A. P. French, "Newton's thermometry: The role of radiation," *Phys. Teach.* **31**, 310–312 (May 1993).
10. U. Besson, "Cooling and warming laws: An exact analytical solution," *Eur. J. Phys.* **31**, 1107 (2010) and references therein.
11. Vernier Software & Technology, http://www.vernier.com.
12. See the instrument manual on the website http://www.vernier.com.
13. R. W. Lewis, P. Nithiarasu, and K. N. Seetharamu, *Fundamentals of the Finite Element Method for Heat and Fluid Flow* (Wiley, New York, 2004).

Aurelio Agliolo Gallitto, Dipartimento di Fisica e Chimica, Università di Palermo, via Archirafi 36, I-90123 Palermo, Italy; aurelio.agliologallitto@unipa.it