

# Radiative recombination in the presence of a few cycle laser pulse

Saverio Bivona, Riccardo Burlon, Gaetano Ferrante and Claudio Leone

*Dipartimento di Fisica e Tecnologie Relative, Università degli Studi di Palermo, Ed.18, Viale delle Scienze 90128 Palermo, Italy*  
[burlon@unipa.it](mailto:burlon@unipa.it)

**Abstract:** We have investigated the laser-assisted radiative recombination in the presence of a few-cycle pulse with the aim of demonstrating means of controlling such process. Within the Coulomb-Volkov approach already employed to describe the radiative recombination assisted by a monochromatic laser field, we have found that the emitted photon spectrum is affected by both the cycle number  $n_c$  and the carrier-envelope relative phase  $\varphi$ . In particular, it has been shown that the minimum and the maximum values of the emitted photon energy may be controlled by varying  $n_c$  and  $\varphi$ . Finally, it has been found that the enhancement of radiative recombination occurring in the presence of a monochromatic field, takes place also by using a few-cycle laser pulse.

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## 1. Introduction

The study of the recombination of a free electron with a positive ion followed by the spontaneous emission of a photon has a long story. The process has been extensively enquired in the past years, as its comprehension is relevant to the analysis of astrophysical and laboratory plasmas [1].

The recent story of radiative recombination (RR) is mainly concerned with the inclusion into the process of the laser radiation to take advantage of the new possibilities offered by the properties of these coherent sources of e.m. radiation. Schematically, advances in RR in the presence of laser radiation have taken place along the following directions: a) RR in the presence of a weak laser field tuned into resonance with an optical transition of the recombining system. Only one photon participates into the elementary process. b) RR in the presence of a moderate intense nonresonant laser field. Several photons participate to the process, but one-photon and few-photon channels are largely dominating. c) RR in the presence of very intense nonresonant laser fields, when a large number of photons may take part into the process, and many different multiphoton channels have comparable probabilities of occurrence. It is intuitive that in this latter case the physics of field-free RR will be strongly modified.

Concerning the direction a) we mention that stimulated radiative recombination experiments have been carried out in ion storage ring by merging electrons and ions with a weak laser field tuned into resonance with an optical transition from the continuum state of the free electron to a bound state of the recombined system. In such experiments several groups have observed significant enhancements of the recombination rate for the case when one photon is emitted at the laser frequency [2-9]. In the stimulated resonant radiative recombination the laser field is weak and only one-photon process occurs. The availability of intense laser fields allows to observe multiphoton processes in radiative recombination and it has prompted investigations i) in the regime of moderate intense fields, when more than one photon is exchanged, but the number of exchanged photons remains still small [10, 11]; and ii) in the strong field regime, when a large number of photons participates to the process [12-19].

In the strong monochromatic laser field context, the main results of the laser assisted radiative recombination (LARR) may be summarized as follows:

- i) The LARR is enhanced when the average translation momentum is of the same order magnitude as or lower than the quiver momentum amplitude [12, 13];
- ii) The spectra exhibit large values at emitted photon energy,  $\omega_x$ , very higher than that possible in the field process [12, 13];

iii) The LARR may be described as a semiclassical model [17] in which the recombination is viewed as a two-step process. In the first step, the free electron propagates towards the ion and its motion is described classically with motion changes ascribed to the action of the laser field; in the second step, the free electron recombines with the ion instantaneously at a given value of the laser field phase. According to this picture, the spectrum of the emitted radiation exhibits large values in the range of the photon energy  $\omega_k$ , where the emission is classically allowed. Moreover, the maxima in the spectra occur when both the instantaneous value of the electric laser field and the instantaneous absolute value of the quiver velocity are, respectively, close to zero and to one of its maximum. Therefore, it turns out that the emission spectra are characterized by a maximum at the furthest edge, as in correspondence of the electron kinetic energy the oscillating electric field is zero.

In the present work the LARR will be reconsidered in the presence of a few-cycle pulse with the aim of demonstrating means for controlling such process. We use the Coulomb-Volkov approach already employed to describe the radiative recombination assisted by a monochromatic laser field. Within this formalism we will investigate how the emitted photon spectrum is affected by the cycle number  $n_c$  and the carrier-envelope relative phase  $\varphi$ . In particular, we will examine how the maximum value of the emitted photon energy may be controlled by varying  $n_c$  and  $\varphi$ . We also will investigate whether the LARR enhancement found in the presence of a monochromatic laser field [12, 13] occurs too by using a few-cycle pulse. Finally, we will show that the LARR in the presence of a few-cycle laser pulse may be described through a semiclassical model.

## 2. Quantum theory of the laser assisted radiative recombination

We outline below the main steps leading to the expression of the transition probability of the recombination of a free electron with a hydrogenic ion with charge  $Z^+$  in the presence of a finite laser pulse, linearly polarized with a sin-square envelope, having the following electric field:

$$E(t) = \begin{cases} \hat{\boldsymbol{\epsilon}}_L E_0 \sin^2\left(\frac{\pi t}{\tau}\right) \cos(\omega t + \varphi) & t \in [0, \tau] \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Here  $\tau$  is the total duration of the pulse,  $E_0$  the electric field laser amplitude,  $\hat{\boldsymbol{\epsilon}}_L$  the polarization vector and  $\varphi$  the carrier-envelope relative phase. In order to have an integer number of cycles we assume  $\tau = n_c T$  with  $T = 2\pi/\omega$  the period of the carrier. We use atomic units throughout the paper. The electric field given by the Eq. (1) is zero before and after the pulse, and  $n_c$  will be taken equal or greater than 2. With this last choice the impulse imparted to the electron by the electric field of the laser pulse during its duration will be zero. Of course, the vector potential, taken as  $\mathbf{A}(t) = -c \int_0^t dt' \mathbf{E}(t')$ , turns to be zero for  $t \leq 0$  and  $t \geq \tau$ .

By assuming an electron, initially, at  $t = 0$ , in a continuum state described by the wavefunction  $\Psi_q^+(\mathbf{r}, t)$ , the transition amplitude of finding it in a bound state  $\Psi_0(\mathbf{r}, t)$ , at the time  $t$ , is given by

$$T_{j_i}(t) = -i \int_0^t dt' \langle \Psi_0(\mathbf{r}, t') | \mathbf{E}_x(t) \cdot \mathbf{r} | \Psi_q^+(\mathbf{r}, t') \rangle \quad (2)$$

with  $\mathbf{q}$  the asymptotic electron momentum and

$$E_x(t) = -\sqrt{\frac{2\pi\omega_x}{V}} \hat{\boldsymbol{\epsilon}}_x \exp[-i(\mathbf{k}_x \cdot \mathbf{r} - \omega_x t)] \quad (3)$$

In Eq. (3)  $\omega_x$  is the energy of the emitted x-ray photon during the recombination process,  $\hat{\boldsymbol{\epsilon}}_x$  the polarization vector,  $\mathbf{k}_x = (\hbar/c) \hat{\mathbf{n}}_x$ , the momentum,  $\hat{\mathbf{n}}_x$  the propagation direction, and  $V$  the quantization volume of the high-frequency radiation. In our calculations  $\Psi_0(\mathbf{r}, t)$  is approximated by the unperturbed ground state of the hydrogenic ion with  $(Z-1)$  charge and energy  $I_0 = -0.5 Z^2$ :

$$\Psi_0(\mathbf{r}, t) = u_0(\mathbf{r}) \exp(-iI_0 t) \quad (4)$$

The continuum state  $\Psi_q^+(\mathbf{r}, t)$ , describes the incoming electron whose momentum, when the laser is off, is  $\mathbf{q}$ . It will be assumed to be approximated by the Coulomb-Volkov ansatz

$$\Psi_q^+(\mathbf{r}, t) = \chi_q^+(\mathbf{r}, t) u_q^+(\mathbf{r}) \quad (5)$$

where  $\chi_q^+(\mathbf{r}, t)$  is the Volkov wavefunction

$$\chi_q^+(\mathbf{r}, t) = \exp\left\{ i[\mathbf{q} + \mathbf{k}_L(t)] \cdot \mathbf{r} - \frac{i}{2} \int_0^t dt' [\mathbf{q} + \mathbf{k}_L(t')]^2 \right\} \quad (5a)$$

with  $\mathbf{k}_L(t) = \mathbf{A}(t)/c$  the momentum imparted to the electron by the radiation field and  $u_q^+(\mathbf{r})$

$$u_q^+(\mathbf{r}) = \frac{\exp(1/2\pi\nu)}{(2\pi)^{3/2}} \Gamma(1-\nu) {}_1F_1[-i\nu, 1, i(\mathbf{q} \cdot \mathbf{r} - qr)] \quad (5b)$$

In Eq. (5b)  $\nu = Z/q$ ,  $\Gamma(x)$  and  ${}_1F_1$  are, respectively, the gamma and the confluent hypergeometric functions. The ansatz Eq. (5) and slightly modified version of it [20-23] are in use since many years in dealing with elementary atomic and electrodynamics processes in the presence of strong laser fields. The reverse process of the one considered in the present paper, i.e. the laser assisted single photoionization process, has also been discussed in Refs. [21] and [24]. In the former reference the ejected electrons have been described by the ansatz Eq. (5), while in Ref. [24] the electron energy spectra have been obtained by an accurate numerical integration of the time-dependent Schroedinger equation. A comparison between the results found through Eq. (5) and the one found in Ref. [24] shows that the ansatz Eq. (5) provides a fair representation of the dressed continuum electron states. By using the above approximate wavefunction for  $\Psi_q^+(\mathbf{r}, t)$  and  $\Psi_0(\mathbf{r}, t)$  in Eq. (2) and proceeding in the usual way, the double differential probability of electron-ion recombination accompanied by the x-ray photon emission in the solid angle  $d\Omega_x$  and having energy within the interval  $(\omega_x, \omega_x + d\omega_x)$ , for  $t \in [0, \tau]$ , is equal to

$$DP(t) = \frac{d^2 P(t)}{d\Omega_x d\omega_x} = \frac{\omega_x^3}{(2\pi)^2 c^3} \times \left| \int_0^t dt' \exp\left\{ i[(\omega_x + I_0)t'] - \frac{i}{2} \int_0^{t'} dt'' [\mathbf{q} + \mathbf{k}_L(t'')]^2 \right\} M_q(t') \right|^2 \quad (6)$$

with

$$M_q(t) = \langle u_0(\mathbf{r}) | \hat{\boldsymbol{\epsilon}}_x \cdot \mathbf{r} \exp(i\mathbf{k}_x \cdot \mathbf{r}) | \exp\{i[\mathbf{q} + \mathbf{k}_L(t)] \cdot \mathbf{r}\} u_q^+(\mathbf{r}) \rangle \quad (7)$$

By integrating Eq. (6) over  $\omega_x$  the single differential probability is obtained as

$$SP(t) = \frac{dP(t)}{d\Omega_x} = \int_0^\infty d\omega_x DP(t) \quad (8)$$

By concluding this section we remark that the double and single differential probability is not invariant under the transformation  $\mathbf{q} \rightarrow \mathbf{q}$ , while it is invariant when the contemporaneous transformations  $\mathbf{q} \rightarrow -\mathbf{q}$  and  $\varphi \rightarrow -\varphi$ .

### 3. A semiclassical model

Below we follow the method used by Kroll and Watson for treating the charged-particle scattering in the presence of a strong electromagnetic field within a semiclassical approach [25]. Accordingly, the incoming electron propagates towards the ion under the action of the laser pulse and its motion is described classically. By assuming an electron with kinetic momentum  $\mathbf{q}$ , when the laser pulse off, by solving the Newton's equation its kinetic momentum at some time  $t \in [0, \tau]$  is given by

$$\mathbf{q}(t) = \mathbf{q} + \mathbf{k}_L(t) \quad (9)$$

The electron recombines instantaneously with emission of a photon whose energy is given by

$$\omega_x(t) = \frac{1}{2} q^2(t) + |I_0| \quad (10)$$

Denoting  $P$  the double differential probability that the recombination takes places in the whole pulse duration  $\tau$ , the double differential recombination probability in the time interval  $dt$  reads

$$dP = P \frac{dt}{\tau} \quad (11)$$

By setting the probability per unit time,  $P/\tau$ , equal to the LARR rate in the low-frequency approximation by using a quantum mechanical treatment (see Eq. (12) of Ref. [17])

$$\frac{P}{\tau} \approx \frac{\omega_x^3 |M_q(t)|^2}{2\pi c^3} \delta[\omega_x - \omega_x(t)], \quad (12)$$

upon integration over the time interval  $t$ , we find the counterpart of Eq. (6) obtained by full quantum mechanical treatment

$$CDP(t) = \int_0^t dt' \frac{P}{\tau} = \sum_l \frac{\omega_x^3}{2\pi c^3} \frac{|M(t_l)|^2}{|\mathbf{E}(t_l) \cdot [\mathbf{q} + \mathbf{k}_L(t)]|} \quad (13)$$

The index  $l$  numbers the real solution of equation  $\omega_x(t) = 0$  in the interval  $[0, \tau]$ . We remark that, as discussed extensively in Ref. [17] and outlined in the Introduction, the low frequency treatment amounts to describe the recombination process as an event occurring at a given value of the assisting monochromatic laser phase. The  $CDP(t)$ , Eq. (13), exhibits a divergence when  $\mathbf{E}(t_l) = 0$ , so that the semiclassical approaches cannot be used when the photon energy  $\omega_x \approx \omega_x(t_0)$ , Eq. (10), where  $t_0$  is a time instant in which the electric laser field instantaneous is close to zero. However, the above discussion allows predicting the position of the maxima in the double differential probability.

The single differential probability, counterpart of the Eq. (8), is equal to

$$CSP(t) = \int_0^t dt' \frac{\omega_x^3(t')}{2\pi c^3} |M_q(t')| \quad (14)$$

#### 4. Calculations

In all the calculations reported below the following geometry will be considered: the incoming electron kinetic momentum with the laser pulse off is oriented along the z-direction ( $\mathbf{q}=q \hat{z}$ ;  $q>0$ ), the propagation direction of the emitted radiation will be chosen in the x-direction, the X-ray and the laser polarization vectors will be taken along the z-axis ( $\hat{\mathbf{e}}_x = \hat{\mathbf{e}}_L = \hat{z}$ ). Further we will choose a Ti: sapphire laser pulse of intensity  $I_L=6 \times 10^{14}$  W cm<sup>-2</sup> ( $\omega=1.5498$  eV) and the hydrogen as model atom ( $Z=1$ ).

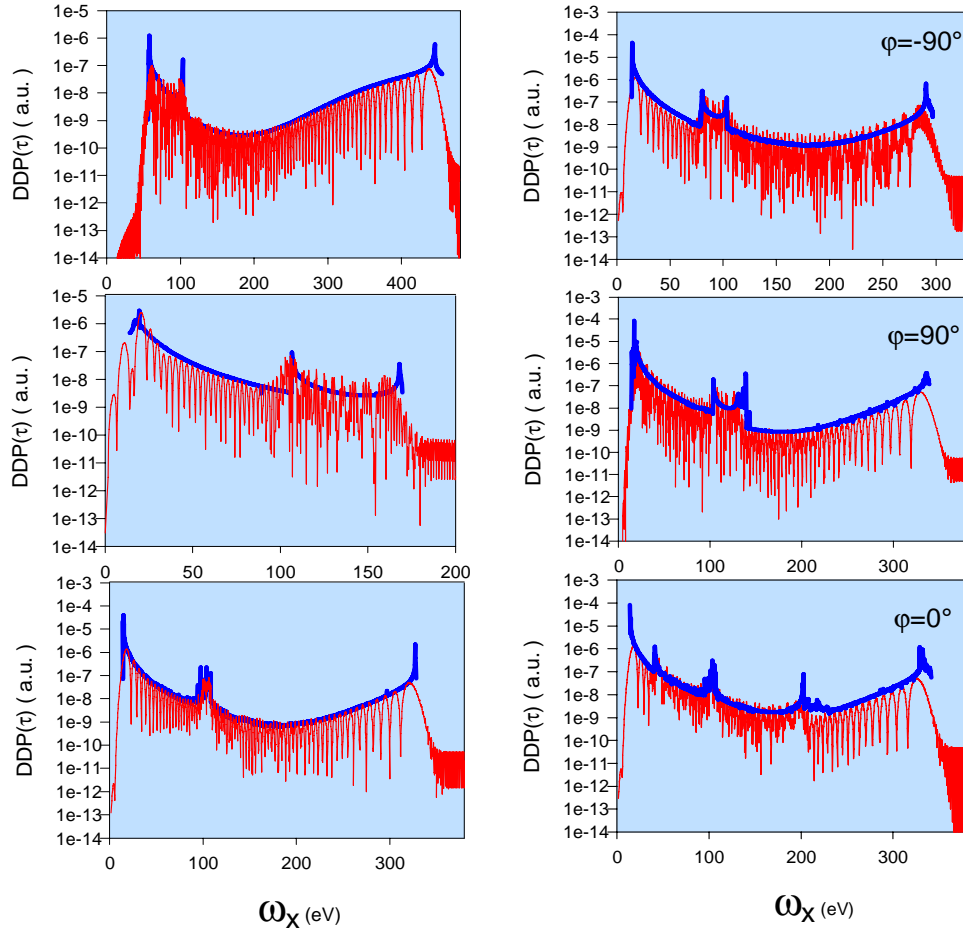


Fig. 1. Double differential probability (DDP) at the end of the laser pulse as a function of the X-ray energy  $\omega_x$  for three different values of the carrier envelope relative phase ( $\varphi = -90^\circ, 0^\circ, 90^\circ$ ), at the energy of the incoming electron  $\varepsilon_q=90$  eV. Red line: DDP calculated in the framework of the quantum theory, Eq. (6). Blue line: the corresponding quantity obtained by the semiclassical model, Eq. (13). Left panel:  $n_c=2$ . Right panel:  $n_c=3$

In Fig. 1 we show, in the left panel, the double differential probability (DDP) at the end of the laser pulse as a function of the emitted photon energy,  $\omega_x$ , for three different values of the carrier envelope relative phase ( $\varphi = -90^\circ, 0^\circ, 90^\circ$ ), and the energy of the incoming electron  $\varepsilon_q=90$  eV. The red line corresponds to DP ( $\tau$ ), Eq. (6), calculated by the quantum theory. The blue line corresponds to CDP ( $\tau$ ), Eq. (13). The left and the right panels show, respectively, the probabilities for  $n_c=2$  and  $n_c=3$ .

We note that, as for the case of a monochromatic wave [17], for a few cycle laser pulse the semiclassical model reproduces the main features of the quantum mechanical results. In particular, the semiclassical result predicts correctly the number and the position of the maxima of the double differential spectra, though it can not reproduce the rapid oscillations found by the quantum calculations. This circumstance follows from the fact that the transition amplitudes at different times, pertaining to emitted photons with the same energy, are summed incoherently, as it can be seen by inspection Eq. (13). The oscillations are, in fact, due to quantum mechanical interferences, originated by the fact that in Eq. (6) the transition amplitudes at different times are summed coherently through the time integration. According to Eq. (13), all the maxima of the DDP occur at the energy  $\omega_x(t)$ , Eq. (10), corresponding to the time instant where the electric field is zero, that is when the instantaneous kinetic momentum  $q(t)$ , Eq. (9), exhibit a minimum or a maximum value. For instance the two maxima at the edges of the DDP correspond, respectively, to the absolute maximum and minimum values of  $q(t)$ . We note that the DDP spectra are practically confined in the range of  $\omega_x$  where the emission is classically allowed, while the DDP is strongly reduced elsewhere. The shape of the reported spectra is in agreement with one reported in Ref. [18] obtained by solving numerically the time dependent Schroedinger equation. As the absolute maximum and minimum values of  $q(t)$  depend on  $\phi$ , we may conclude that the DDP spectra may be controlled by varying  $\phi$ .

The curves reported in Fig. 1 show that for a fixed value of  $n_c$  their shape depends on the value of  $\phi$ . However this dependence is reduced when  $n_c$  increases, as confirmed also by calculations here not reported.

In order to confirm that the LARR occurs in very short time intervals, we display in Fig. 2 the evolution of the double differential probability when the laser pulse is on for the photon energies corresponding to the maxima of the curve of Fig. 1 for  $n_c=2$  and  $\phi = -90^\circ$ .

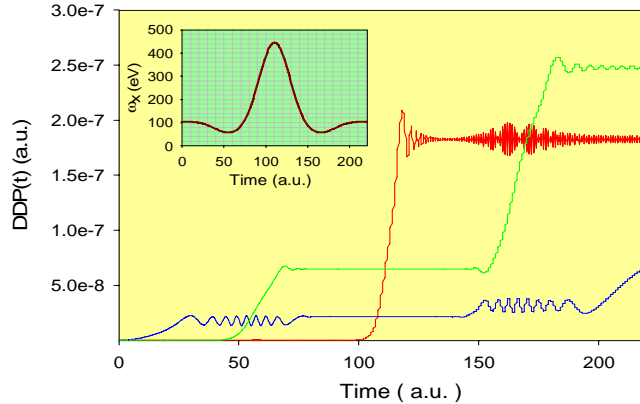


Fig. 2. The evolution of the double differential probability when the laser pulse is on for the photon energies corresponding to the maxima of the curve of Fig. 1 corresponding to  $n_c=2$  and  $\phi=-90^\circ$ . The red curve corresponds to  $\omega_x=437.5$  eV, the blue curve to  $\omega_x=101.4$  eV and the green curve to  $\omega_x=62.4$  eV. In the insert we report the time dependence of the X-ray emitted photon  $\omega_x(t)$ , Eq. (10), evaluated classically.

The red curve corresponds to  $\omega_x=437.5$  eV, the blue curve to  $\omega_x=10.4$  eV and the green curve to  $\omega_x=62.4$  eV. In the insert we report the time dependence of the emitted photon energy  $\omega_x(t)$ , Eq. (10), evaluated classically. By inspection of the curves shown in Fig. 2, we may conclude that a given X-ray frequency is emitted within particular short time intervals that are a small fraction of the laser period. These short intervals are located around the instant in which the electric field is zero, as predicted by the semiclassical model.

In the upper panel of Fig. 3 we show the single differential probability at the end of the pulse  $SDP(\tau)$  as a function of the incoming electron energy when the laser pulse is off for  $n_c=2$  and three different values of  $\varphi$  (blue curve  $\varphi=-90^\circ$ , red curve  $\varphi=0^\circ$ , green curve  $\varphi=90^\circ$ ). The lower panel show the corresponding data for  $n_c=6$

We remark that for  $n_c=2$  considerable modification occurs in the single differential probability by varying  $\varphi$ , while the dependence on  $\varphi$  is strongly reduced for  $n_c=6$ . Calculations here not reported show that the dependence on  $\varphi$  may be neglected for very large values of  $n_c$ , as expected on the basis of the results in the presence of a monochromatic laser field.

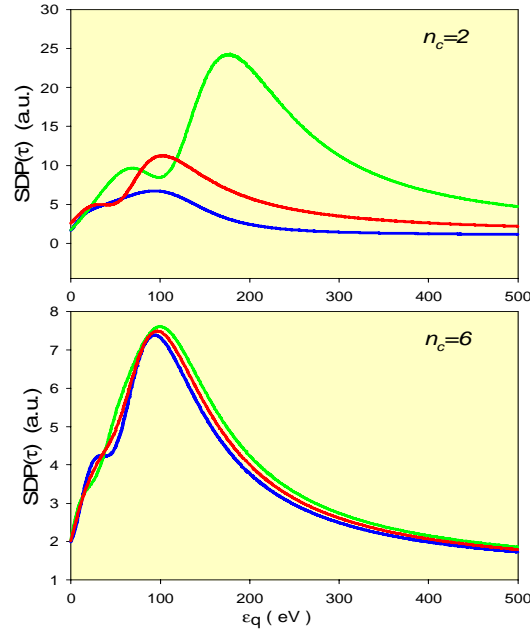


Fig. 3 The single differential probability at the end of the pulse  $SDP(\tau)$  as a function of the incoming electron energy when the laser pulse is off for  $n_c=2$  and 6 at three different values of  $\varphi$  (blue curve  $\varphi=-90^\circ$ , red curve  $\varphi=0^\circ$ , green curve  $\varphi=90^\circ$ )

## 5. Concluding remarks

In this work we have carried out a study of the radiative recombination of a free electron with a hydrogenic ion using a few-cycle pulse.

Our analysis has been restricted to the geometry in which the incoming electron kinetic momentum is along the positive z-axis, the propagation direction of emitted photon along the x-axis and the x-ray and the laser polarization vectors along the z-axis.

The reported results in Fig. 1 show that the double differential probability is strongly affected by the cycle number  $n_c$  and the carrier envelope relative phase  $\varphi$  only for very small values of  $n_c$ . The mean features of the reported curves may be reproduced by the semiclassical model, in which the recombination occurs instantaneously at a given value of the kinetic momentum, obtained by solving the Newton's equation. According to this picture the double differential probability exhibits two maxima at the emission spectrum edges, corresponding to the absolute minimum and maximum values of the kinetic momentum  $q(t)$ . Therefore, as the time evolution of  $q(t)$  depends on  $\varphi$ , the emitted photon energy spectrum may be controlled by varying the carrier-envelope relative phase.



The validity of the semiclassical picture is also confirmed by the quantum calculations of the time evolution of the probability to emit a photon at a given energy (see Fig. 2). Finally, we have shown that the LARR enhancement, found in the presence of a monochromatic laser field, occurs also for a few-cycle laser pulse and it is controlled by the carrier-envelope phase when the cycle number is very small .

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