An Hypervolume based constraint handling technique for multi-objective optimization problems

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Keywords: multi-objective optimization, constraints, hypervolume.

SUMMARY. Formulation of structural optimization problems usually leads to the individuation of one or more objective functions to be minimized under different constraints. Many multi-objective evolutionary algorithms are approached by a Pareto-compliant ranking method, where no a priori information on the problem is needed and the concept of non-dominated solutions is used. In this paper a constraint handling technique based on the concept of hypervolume indicator is presented. Initially proposed to compare different multi-objective algorithms hypervolume indicator is the only single set quality measure to reflects the dominance of solution's sets. The constraint handling technique proposed use an extension of stochastic ranking approach for single-objective optimization problem to multi-objective ones. The extension proposed use the hypervolume indicator to compares different solutions and is tested on a structural constrained multi-objective problems. Results show the suitability of the proposed approach.

1 INTRODUCTION

Formulation of structural optimization problems usually leads to the individuation of one or more objective functions to be minimized under different constraints.

Let us defines a general multi-objective constrained optimization problem as:

$$\begin{aligned} &\min_{x\in\Re^{npar}}\{f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_m(\mathbf{x})\}; \qquad f_j : \Re^{npar} -> \Re \qquad j \in \{1, 2, ..., m\} \\ &subject \quad to : g_i(\mathbf{x}) \le 0 \qquad g_i : \Re^{npar} -> \Re \qquad i \in \{1, 2, ..., p\} \end{aligned} \tag{1}$$

the objective funtion f_j assigns to each variables vector $\mathbf{x}$ a corresponding objective value $f_j(\mathbf{x})$ and, without loss of generality is assumed that $f_j(\mathbf{x}) \in \Re$. The functions g_i are constraints that the variables vector has to satisfy. The feasible region Γ is defined by:

$$\Gamma = \{\mathbf{x} \in \Re^{npar} | g_i(\mathbf{x}) \le 0\} \tag{2}$$

the usual output of this problems is a set of incomparable variables vectors that belong to feasible region Γ. Usually two variables vectors for unconstrained multi-objective are considered incomparable using the notion of Pareto optimality. A solution is said to be Pareto optimal for a multi objective problem if all other solutions have a higher value for at least one of the objective functions, or else have the same value for all objectives. If we consider two solutions $\mathbf{x_1}$ and $\mathbf{x_2}$ the solution $\mathbf{x_1}$ is said to dominate the other solution $\mathbf{x_2}$, if both the following conditions are true:

a) the solution $\mathbf{x_1}$ is no worse than $\mathbf{x_2}$ in all objectives

$$f_j(\mathbf{x_1}) \le f_j(\mathbf{x_2}) \quad for \quad all \quad j = 1, m \tag{3}$$

b) the solution $\mathbf{x_1}$ is strictly better than $\mathbf{x_2}$ in at least one objective

$$f_j(\mathbf{x_1}) < f_j(\mathbf{x_2}) \quad for \quad at \quad least \quad one \quad j \in 1, m \tag{4}$$

If any of the above conditions is violated, the solution $\mathbf{x_1}$ does not dominate solution $\mathbf{x_2}$.

Typically, there is an entire curve or surface of Pareto points or non-dominated points and the shape of this curve indicates the nature of the tradeoff between different objectives.

For single objective problems different approaches was used. Penalty methods used to transform the constrained problem into an uncostrained one [1], constrains are considered as additional objectives [2], constraints violations are used to introduce a rank in the solutions [3]. The classical extension to constrained multi-objective problems, [4], [5], assumes that feasible solutions always dominate unfeasible solutions. For unfeasible solutions is then introduced a nondominated ranking using the overall constraint violation [4] or a more complex technique where a nondomination check of constraints violation is performed, [5].

Choosing always feasible solutions could be defined as $overpenalization$, [3] and, especially when the feasible region is disjointed, could drive the search to local optima.

To overcome these difficulties other approaches try to use a combination of objective function values and constraint violation values, [6], using an adaptive balancing between them in different stages of search.

In the present work the stochastic ranking proposed by [3] for single objective constrained optimization problems is extended to multiobjective ones using the concept of hypervolume indicator [7], [8].

The paper is organized as follows: after presenting the basic of stochastic ranking and of hypervolume indicator, the proposed multi-objective extension is illustrated. Then, the results for a typical structural constrained multi-objective problem are discussed.

2 HYPERVOLUME INDICATOR

The hypervolume indicator H (or S-metric), first introduced by Zitzler et al. [7], is the only known unary quality measure that is compliant with the concept of Pareto-dominance, i.e, whenever a set of solutions dominates another set, its hypervolume indicator value is higher. Thanks to this characteristic was used to to compare the performance of different multiobjective algorithms and more recently also as criterion guidance for multiobjective optimization algorithms itself.

In the present work a binary quality indicator I_{HD} based on hypervolume indicator is used, [9]. A binary quality indicator could be defined as real-valued function that compares two set of solutions of a multi-objective problem, could then be seen as a continous extension of Pareto dominance concept. To preserve characteristics of Pareto dominance the indicator has to be dominance preserving:

$$\begin{aligned} &assigned \quad \mathbf{x_1}, \mathbf{x_2} \\ &\mathbf{x_1} \quad dominate \quad \mathbf{x_2} \quad \Rightarrow I_{HD}(\mathbf{x_1}, \mathbf{x_2}) < I_{HD}(\mathbf{x_2}, \mathbf{x_1}) \\ &assigned \quad \mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3} \\ &\mathbf{x_1} \quad dominate \quad \mathbf{x_2} \quad \Rightarrow I_{HD}(\mathbf{x_3}, \mathbf{x_2}) \leq I_{HD}(\mathbf{x_3}, \mathbf{x_1}) \end{aligned} \tag{5}$$

When only two solutions are compared the indicator I_{HD} is defined as:

$$\begin{aligned} &I_{HD}(\mathbf{x_1}, \mathbf{x_2}) = \\ &\mathbf{x_2} \quad dominate \quad \mathbf{x_1} \Rightarrow H(\mathbf{x_2}) - H(\mathbf{x_1}) \\ &\mathbf{x_2} \quad notdominate \quad \mathbf{x_1} \Rightarrow H(\mathbf{x_1} + \mathbf{x_2}) - H(\mathbf{x_1}) \end{aligned} \tag{6}$$

I_{HD} represents the volume of space dominated by $\mathbf{x_2}$ but not by $\mathbf{x_1}$. In figure 1 is represented the I_{HD} indicator for two objectives. In figure 1a $\mathbf{x_1}$ and $\mathbf{x_2}$ are not comparable, in figure 1b $\mathbf{x_2}$

dominates $\mathbf{x_1}$.

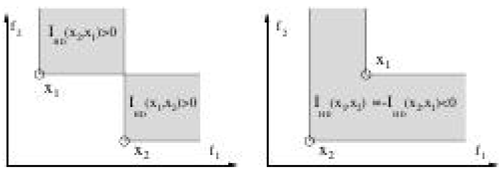

Figure 1: I_{HD} indicator for two objective, left no dominance between $\mathbf{x_1}$ and $\mathbf{x_2}$, right $\mathbf{x_2}$ dominates $\mathbf{x_1}$

Using I_{HD} is possible to assign a fitness F to each solution according to the contribution to the optimization objectives:

$$F(\mathbf{x_j}) = \sum_{i \neq j} I_{HD}(\mathbf{x_i}, \mathbf{x_j}) \tag{7}$$

The algorithm ACOR$_L$ with Lévy [10] perturbation was applied as underlying optimization algorithm.

3 MULTIOBJECTIVE STOCHASTIC RANKING APPROACH

The central idea of stochastic ranking [3] is to obtain a right balance between considering constraint violation and considering fitness value in the search process. If the correct balance is achieved the search is driven toward the optimum value in the feasible region using not only informations made available by feasible solutions but also by unfeasible ones.

To this aim in comparing two solutions is used an assigned probability P_f to not consider constraint violations but only fitness values. The ranking of different solutions is obtained using a bubble-sort-like procedure where two solutions are compared using a probability P_f to not consider constraint violations in solution's ranking.

When $P_f = 0$, constraint violation is always considered, $overpenalization$ approach is used, and for $P_f = 1$ no constraint violation is considered, $underpenalization$ approach is used. More details are available in [3].

Using I_{HD} is possible to introduce a stochastic ranking also for multiobjective probles as reported in Box 1, obviously if the number of objectives $m = 1$ the original stochastic ranking procedure is recovered.

4 STRUCTURAL CASE STUDY

In order to test the proposed approach the optimization of a composite laminate for maximum buckling loads and minimum weight was analyzed. Considering a rectangular composite plate simply supported and subjected only to normal compressive loads the plate buckles into m and n half waves in the x and y direction, respectively, when the loads reach the values $\lambda_b N_x$ and $\lambda_b N_y$.

Box 1 - MULTIOBJECTIVE STOCHASTIC RANKING

```
For each solution i in archive
   For each solution j in archive
   sample u in [0 1]
      If x_i and x_j are feasible
      or u ≤ P_f
         compares F(x_i)andF(x_j)
      Else
         compares violation(x_i)andviolation(x_j)
   end for
   end for
   Take the first k solutions
   Increase number of iterations
while termination not met
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In the general case of laminate with multiple anisotropic layers and without any stacking sequence symmetry the problem doesn't admit a simple solution. If we assume particular constraints on the stacking sequences, i.e. plates for which the bending twisting coefficients are zero are so small in respect to the other coefficients to be assumed zero, using the classical laminate theories [12] the buckling load factor λ_b could be found as:

$$\lambda_b(m,n) = \frac{\pi^2}{a^2} \frac{m^4 D_{11} + 2(D_{12} + 2D_{66}) r^2 m^2 n^2 + r^4 n^4 D_{22}}{m^2 N_x + r^2 n^2 N_y} \tag{8}$$

where a and b are the lamina dimensions; $r = \frac{a}{b}$ the aspect ratio; N_x and N_y the applied loads; D_{ij} the bending stiffness of the composite plate depending from the assumed stacking sequence of the laminate.

The smallest value of λ_b over all possibles values of m and n represents the lowest value of loads for which the buckling conditions are reached and hence the critical buckling load factor λ_{cb}. According to [11] limiting the values of m and n to 1,2 gives a good estimation of critical buckling load, so for an assigned plate geometry the first objective could be stated as:

$$\max(f_1) = \max_{D_{ij}} \left(\min_{m,n} \lambda_b(m,n); \quad m,n \in 1,2 \right) \tag{9}$$

According to the classical laminate theories [12] before the buckling condition is reached the plane stress condition is assumed valid for each ply of the laminate. In the generic lamina k the constituive equations could be expressed as:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{13} \\ \overline{Q}_{21} & \overline{Q}_{22} & \overline{Q}_{23} \\ \overline{Q}_{31} & \overline{Q}_{32} & \overline{Q}_{33} \end{bmatrix}_k \cdot \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{bmatrix} \tag{10}$$

where $\overline{Q}_{ij}$ are the lamina stiffness components expressed in the plate reference axis. The bending stiffness D_{ij} of a plate made by n lamina could be now expressed as

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \overline{Q}_{ij} \left(z_k^3 - z_{k-1}^3 \right) \tag{11}$$

Fiber directions	Constraints	No. design variables
$P_1 = [0, 45, 90]$	symmetric, balanced	16
$P_2 = [0, 30, 60, 90]$	symmetric	32
$P_3 = [0, 15, 30, 45, 60, 75, 90]$	symmetric	32

Table 1: Design problems analyzed.

where z_k and z_{k-1} are the coordinate of the k lamina through the laminate thickness.

The terms $\overline{Q}_{ij}$ could be expressed knowing the fiber orientations θ_k and the elastic properties of the material along the principal directions E_{11}^k, E_{22}^k, G_{12}^k, ν_{12}^k of each lamina, [12].

The weight W is assumed proportional to the laminate thickness, hence the second objective could be expressed as:

$$\min(f_2) = \min_{z_k} \sum z_k \tag{12}$$

The laminate strains are imposed to be less than allowable values for each lamina k, the constraints could be stated as:

$$(\epsilon_i)_k \leq \bar{\epsilon}_i \tag{13}$$

Eq. (9) is used to evaluate laminate strains for the composite plate.

For an assumed plate geometry the design variables are hence the elastic properties, the fiber orientations and thickness of each lamina.

In this paper a laminate made by graphite epoxy lamina was considered, the elastic properties of the material are: E_{11} = 127.6 GPa; E_{22} = 13.0 GPa; G_{12} = 6.4 GPa; ν_{12} = 0.3.

The maximun ply thickness is $\bar{t}$ = 0.254 mm.

The laminate has length $a = 0.508$ m, width $b = 0.254$ m, and is made by 64 plies. Table 1 shows the different set of possible fiber orientations, the constraint adopted on the stacking sequence and the number of independent variables for each case analyzed in the present paper. The *continuous relaxation approach* is adopted in the optimization algorithm, i.e. the discrete variables are replaced by continuous ones and in the evaluation of the objective function are transformed in the allowed discrete values. This choice is suitable due to the natural order in the design variables space.

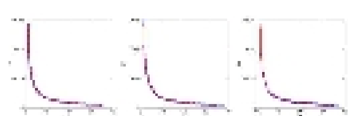

Figure 2: Nondominated fronts obtained for a typical run. Design problems P_1, P_2, P_3.

In Figure 2 are reported non dominated front, obtained for a typical run, for different design problems analyzed. The objective functions are normalized. The proposed algorithm was able to find feasible individuals and solutions are well spread on the sub-optimal pareto front. Use of informations from unfeasible individuals helps algorithm to explore search space, avoiding the confination effect due to $overpenalization$ approach.

5 CONCLUSIONS

In the present work an extension to classical constraint handling technique used for single objective problems was used. The peculiar characteristic of fitness definition, using binary indicator, and of constraint handling technique frees from the introduction of specialized operators to take into account problem's characteristics. The technique was applied to a structural optimization problems and results are promising. Comparisons with other constrained multiobjective algorithms are needed to better understand capabilities and limitations of the proposed approach.

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