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Damage identification by a modified Ant Colony Optimization for not well spaced frequency systems

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ABSTRACT: Recently, it has been shown , that a damage detection strategy based on a proper functional calculated on the analytical signal of the structural dynamical response, consents to identify very low damage level. In this regard, they stressed the efficiency of Hilbert Transform to obtain the analytical response representation that shows more sensitivity for predicting damage with respect to the simple signal response. Then, a damage identification procedure based on the minimization of the difference between theoretical and measured data was proposed with satisfactory results. Unfortunately, this procedure, since the need of use of band pass filter around the natural frequency of the system, fails for structures having closed natural frequencies. By the way, performing procedures for sharply detecting damage in not well spaced frequency structure is a hoary problem. Aim of this paper is to extend the aforementioned procedure to these systems. To aim at this, it is desirable to go further insight into optimization algorithms, suitable for this kind of systems. For instance, it will be interesting considering, the ant colony optimization algorithm (ACO) that is a probabilistic technique for solving computational problems which can be reduced to finding good paths through graphs. Recently ACO has been extended to continuous domain and labeled as $ACO_{\Re}$. A novel aspect of the proposed paper is introducing $ACO_{\Re}$ into the previous procedure avoiding the use of filters, such that may be available for not well spaced frequency system. Moreover, it will be desirable avoiding the use of Hilbert transform, that means apply the identification procedure directly on the acceleration responses and not on the analytical signal response. Therefore, in this paper it will be introduced a procedure for detecting damage in structures having close frequencies, without using analytical signal response.

KEY WORDS: Damage Identification; Hilbert Transform; Ant colony optimization.

1 INTRODUCTION

Detecting damage for predicting its evolution in time is fundamental in engineering field. This in order to evaluate the remaining life of the structure and the effectiveness of reinforcements and control procedures and to provide a monitoring system using very updated tools [1], [2], [3], [4], [5], [6], [7], [8].

However, when dealing with a very low level of damage, for instance a little variation in structural stiffness, this phenomenon cannot be detected either in time domain or in frequency domain. In fact in [9], [10], [11], it was stressed that using a procedure of damage identification based on the minimization of an objective functional, this functional has to be composed of characteristics of the analytical signal; since the healthy impulse response function (IRF) and the healthy frequency response function (FRF) totally overlaps the IRF or the FRF of the damaged one. This means that both IRF and FRF are not good features in such a damage identification procedure. Then it was developed an identification procedure very useful to detect low variation of structural stiffness.

In this regard, this procedure is based on applying Hilbert Transform, to obtain the analytical representation of the system response to a given input (pulse force load and wide band noise). It is worth reminding that an analytical signal is a complex signal composed of the signal itself as real part and the Hilbert transform of the signal as imaginary part. Further, the characteristics of the response analytical signal (frequency, phase, amplitude) have been proved to be very sensitive even to very small variation strictly connected to the structural stiffness. According to the above considerations, this means that the characteristics of analytical signal are good features in such a damage identification procedure [9], [10], [11].

This damage identification procedure works very well in the case of single degree of freedom (SDOF), and the objective function has a sharp minimum in correspondence of the actual damage parameters, however the identification procedure to signals, which are responses of multi degree of freedom (MDOF) systems, was not satisfactory, because the signals have not a well-behaved Hilbert transform. In fact a mono-component signal, like the SDOF response has a well-behaved Hilbert transform and not the total response of the MDOF system, that is a multi-component signal. In a previous work [10], to overcome the latter problem, the empirical mode decomposition (EMD) was necessary to restore the sharpness of the procedure adopted in SDOF.

The empirical mode decomposition (EMD) method, proposed by N. Huang is widely used for signal detection, damage identification, sensitivity analysis and so on and it was demonstrated to be capable of identifying modal parameters as well as mode shapes and mass, stiffness and damping matrices of linear structures more accurately than the method based on the wavelet transforms [12], [13].

Indeed the EMD is a black-box software, based on empirical observations, hence in recent papers [9], [10], [11], a procedure has been developed for getting rid from the use of the EMD, validity of results being equal.

To do that the response signal is separated in several components, labeled filtered modal responses (FMR), by means of a proper band-pass filter around each modal frequency. In such a way we are in the same condition either as the SDOF case or using EMD for MDOF systems: a signal that has a well-behaved Hilbert Transform.

Unfortunately this procedure together with the Empirical Mode Decomposition is not available for structures having closed natural frequency. By the way, performing procedures for sharply detecting damage in not well spaced frequency structure is a hoary problem. In this regard, aim of this paper is not only to provide a procedure valid in this case, but also getting rid of Hilbert Transform.

It is worth mentioning that when the natural frequencies are not well separated, it is impossible the use of the filter around the own frequencies, that leads to analyze the multicomponent signal response. This feature compromises the sharpness of the identification procedure for the functional minimization, being present several local minima.

To overcome this problem, in this paper the ant colony optimization algorithm (ACO), has been used, this is a probabilistic technique for solving computational problems which can be reduced to finding good paths through graphs.

This algorithm is a member of ant colony algorithms family, in swarm intelligence methods, and it constitutes some metaheuristic optimizations. Initially proposed by Marco Dorigo in 1992 the first algorithm was aiming to search for an optimal path in a graph; based on the behavior of ants seeking a path between their colony and a source of food. The original idea has since diversified to solve a wider class of numerical problems, and as a result, several problems have emerged, drawing on various aspects of the behavior of ants [14], [15].

In the real world, ants (initially) wander randomly, and upon finding food return to their colony while laying down pheromone trails. If other ants find such a path, they are likely not to keep travelling at random, but to instead follow the trail, returning and reinforcing it if they eventually find food.

Over time, however, the pheromone trail starts to evaporate, thus reducing its attractive strength. The more time it takes for an ant to travel down the path and back again, the more time the pheromones have to evaporate. A short path, by comparison, gets marched over faster, and thus the pheromone density remains high as it is laid on the path as fast as it can evaporate. Pheromone evaporation has also the advantage of avoiding the convergence to a locally optimal solution. If there were no evaporation at all, the paths chosen by the first ants would tend to be excessively attractive to the following ones. In that case, the exploration of the solution space would be constrained.

Thus, when one ant finds a good (i.e., short) path from the colony to a food source, other ants are more likely to follow that path, and positive feedback eventually leads all the ants following a single path. The idea of the ant colony algorithm is to mimic this behavior with ”simulated ants” walking around the graph representing the problem to solve.

In this paper a modified version of ACO, Ant Colony Optimization for Continous domains ($ACO_{\Re}$) proposed in [16], well suited for continous optimization problem is applied to check the possibility to perform the minimization of the functional without extrapolating the modal component by filtering the structural response, in order to verify if damage may be identified on structures having closed frequencies. Moreover as features of the response it will be considered structural acceleration responses, in order to test the possibility of getting rid of Hilbert Transform.

2 DAMAGE IDENTIFICATION PROCEDURE

In this paper we restrict our attention to identify damage considering acceleration response of structures having not well separated frequencies. To this aim, let us indicate with $\gamma^{eff} = (\gamma_1^{eff},...,\gamma_n^{eff})$ the unknown damage indexes. They are a set of n real coefficients defined in the interval $[0,1[$ characterizing the damage magnitude on n different structural elements, where the zero bound corresponds to structural integrity.

For example, one can consider $(1-\gamma_j^{eff})$ being the j-th coefficient such that, multiplied by the j-th structural elements flexural stiffness, it characterize, the decrement of stiffness. The apex eff means that the γ^{eff} indexes are effective and our aim is to develop a damage identification procedure capable to identify the value of each coefficient associated to the respective structural element. It has to be noted that the present formulation is not suited to establish location inside the element where damage occurred.

Further, indicate by $\eta^{ex}(\gamma,t)$ a general feature extracted from the real structure, where the apex ex denotes that it is a recorded data, i.e. coming from an experimental test, while $\eta^{th}(\gamma,t)$ is the same feature calculated from a structural model, depending on the γ variables.

The objective functional

$$J_\eta = \frac{1}{T_f - T_i}\int_{T_i}^{T_f} (\eta^{th}(\gamma,t) - \eta^{\mathbf{ex}}(\gamma^{\mathbf{eff}},t))^{\mathbf{2}} dt \qquad (1)$$

being the observation temporal window. It has been shown in [9], [17], [11], [18] that the most sensitive feature that consents to identify small damage also in presence of measuring noise affecting the experimental data, is the phase of the analytical signal. However, this needs the use of Hilbert Transform, being the analytical signal a complex signal whose real part is just the signal itself and the imaginary part is its Hilbert Transform.

In this paper, since the optimization algorithm is ant colony optimization algorithm (ACO) it will be verified if damage identification is possible considering as feature the acceleration response so no need of filters and of Hilbert Transform as well.

Two examples will clarify the approach. In the first example this procedure is applied on a structure with very closed natural frequencies and in the second example, the procedure is applied on the experimental system studied in [11] using the recorded experimental responses. The latter, is proposed, in order to asses the robustness of the method, since the real experimental response record is considered to identify a very low level of damage.

2.1 Ant Colony Optimization

Ant Colony Optimization (ACO), originally proposed for combinatorial optimization problems ([14], [15]) has been extended to continuous domains in [16], main reference of this section. It is use to denote the continuous domain formulation as $ACO_{\Re}$ to make a distinction from ACO. The foraging behavior of real ants has motivated the ACO:

i) ants firstly leave the nest for exploring the ambient in random way;

ii) in case one ant finds the food, returns to the nest depositing a pheromone trail proportional to the food quality and quantity;

iii) this trail is the only indirect possible communication with other ants;

iv) pheromone does evaporate, i.e. the shorter the path the higher the appeal of the pheromone trail or, similarly, less attractive paths will disappear.

Let S be the search space and $X_1, X_2, ..., X_n$ the so-called decision variables, where n characterizes the dimension of the functional $f : S \to \Re$ to be minimized. Construction of a solution $\mathbf{s} \in \mathbf{S}$ means to assign values x^i to each X_i, for $i = 1...n$, such that $\mathbf{s} = (\mathbf{x^1}, \mathbf{x^2}, ..., \mathbf{x^n})$. A solution $\mathbf{s}^*$ is a global minimum if and only if $f(\mathbf{s}^*) \le \mathbf{f}(\mathbf{s})$ for every $\mathbf{s} \in \mathbf{S}$. The idea that is central to the way $ACO_{\Re}$ works is the incremental construction of solutions based on the biased probabilistic choice of solution components. The bias is constituted by the pheromone trail guiding the searching process and is taken into account by means of n Gaussian kernel probability densities, $G^i(x)$ defined below.

Practically we construct an archive T of k solutions $T = [\mathbf{s_1}, \mathbf{s_2}, ..., \mathbf{s_k}]$ where the generic solution $\mathbf{s_l}$ has n components, i.e. $\mathbf{s_l} = (\mathbf{x_l^1}, \mathbf{x_l^2}, ..., \mathbf{x_l^n})$. Then, the solutions of the archive T are ordered according to their objective function values $f(\mathbf{s_l})$. A base solution l is selected from the archive T, to be modified, according to the following probability:

$$p_l = \frac{\omega_l}{\sum_{j=1}^{k} \omega_j} \tag{2}$$

where

$$\omega_l = \frac{1}{qk\sqrt{2\pi}} e^{-\frac{(l-1)^2}{2q^2k^2}} \tag{3}$$

which essentially defines the weight ω_l to be a value of the Gaussian function with argument l, mean 1 and standard deviation qk, where q is a parameter of the algorithm. When q is small, the best-ranked solutions are strongly preferred, and when it is large, the probability becomes less dependent on the rank of the solution.

Then every variable s_l^i of the choosen solution $\mathbf{s_l}$ is perturbed by a Gaussian function

$$g_i = \omega_l \frac{1}{\sigma_i\sqrt{2\pi}} e^{-\frac{(x-\mu_i)^2}{2\sigma_i^2}} \tag{4}$$

The standard deviation σ_i is defined as

$$\sigma_i = \xi \sum_{e=1}^{k} \frac{|x_e^i - x_l^i|}{k-1} \tag{5}$$

where ξ is a parameter user-defined in the algorithm ranging from 0 and 1. The higher the value of this parameter the slower the convergence speed.

The parameter, μ_i, is the value of the i-th parameter of the base solution itself ($\mu_i = s_l^i$).

Indeed at each construction step an ant select a value x^i of X^i making a probabilistic choice $G^i(x)$, i.e. producing a sample whose density is determined by the sole knowledge of components of archive T, where $G^i(x)$:

$$G_i(x) = \sum_{l=1}^{k} \omega_l \frac{1}{\sigma_i\sqrt{2\pi}} e^{-\frac{(x-\mu_i)^2}{2\sigma_i^2}} \tag{6}$$

In box 1 the $ACO_{\Re}$ pseudocode is reported.

Algorithm 1 $ACO_{\Re}$ Pseudocode

Random creation of the solutions archive of size k
Choice of ξ, q, k
while not(termination) **do**
for z=1 to m **do**
Choice of one solution from the archive using (2)
for all parameter (Ant construction) **do**
Calculate standard deviation σ_l^i using (5)
Modify the i-th parameter in the following way:
$x^i = x^i + g_i(x)$
end for
Evaluation of the new solution
end for
Archive update
end while

3 NUMERICAL ANALYSIS

3.1 3-DOF structure with not well spaced natural frequencies

Let us consider a three-storey shear-beam type building. The mass, stiffness and viscous damping of each storey are reported in Table **??**.

Damage is simulated by decreasing the stiffness at first and second floors by a damage indexes $\gamma_1^{eff} = 0.010$, $\gamma_2^{eff} = 0.015$ and $\gamma_3^{eff} = 0.000$. The acceleration of the third floor is recorded.

For each combination of $\gamma = (\gamma_1, \gamma_2, \gamma_3)$ it is possible to calculate the model theoretical response, i.e. the acceleration of the third storey $\ddot{x}^{th}(\gamma_1, \gamma_2, \gamma_3; t)$ and the functional in terms of acceleration is computed.

At this point, in previous works the signal response was filtered in order to restore a well-behaved functional as like as in the single DOF previously outlined. But the considered structure presents not well spaced natural frequencies, in such a way that isolating the single modal component from the recorded signal by filtering might request very high order filters with consequent difficulties.

In this paper, without using the pre-filtering of the dynamical response, the objective functional is calculated using the acceleration response at third floor to an impulse excitation.

Then, $ACO_{\Re}$ is used to minimize $J_\eta(\gamma)$ and we show that the algorithm is suited to estimate the assigned level of damage $\gamma = (0.010, 0.015, 0.000)$.

$ACO_{\Re}$ has been implemented with $q = 0.1, \xi = 0.85, k = 50$, the single run of optimization is stopped when the functional is evaluated 1000 times; 50 independent runs have been used to assess the performance of the algorithm. The search space is the domain $[0,0.2]x[0,0.2]x[0,0.2]$. Table 1 resumes the mean and the standard deviation of the damage indexes obtained and of the absolute percentage error:

$$\varepsilon_j = \frac{100(\gamma_j^{eff} - \gamma_j)}{\gamma_j^{eff}} \tag{7}$$

Table 1. Mean and standard deviation of damage indexes.

	γ_1	γ_2	γ_3
mean	$9.84 * 10^{-3}$	$1.51 * 10^{-2}$	$8.50 * 10^{-9}$
st.dev	$1.91 * 10^{-5}$	$8.80 * 10^{-6}$	$9.00 * 10^{-9}$

	$\varepsilon_1\%$	$\varepsilon_2\%$
mean	1.64	−0.64
st.dev	0.19	0.06

Figure 1 shows the variation of the best, the worst, the mean and the median of the functional values for the 50 independent runs vs. the number of functional evaluation (FES). Figure 2 shows the variation of coefficients $\gamma = (\gamma_1, \gamma_2, \gamma_3)$ during a single run. As can be seen $ACO_{\Re}$ succeeds to identify the incipient damage and gives a very good quantitative estimate of the extent of the damage in very few FES.

This result leads to consider this procedure valid for damage identification in structures with not well spaced frequencies without the use of Hillbert transform.

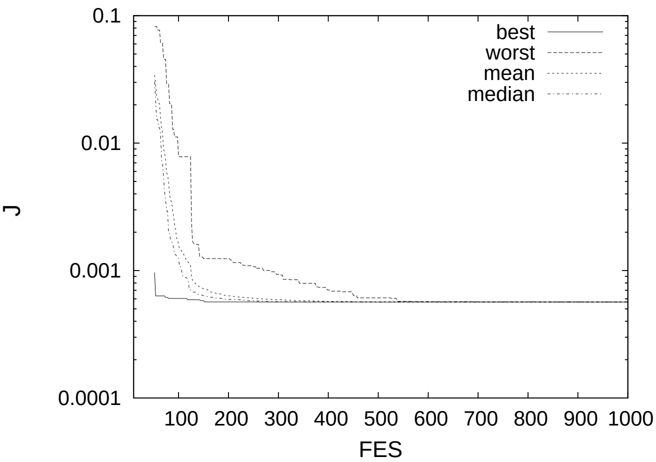

Figure 1. Evolution of the functional along ants generation steps.

3.2 3-DOF structure with well spaced natural frequencies

To assess the robustness of this identification procedure performed together with ACO, the same experimental system developed in [11] has been considered. This system has been built up in the structural Dynamic Lab of university of Palermo,

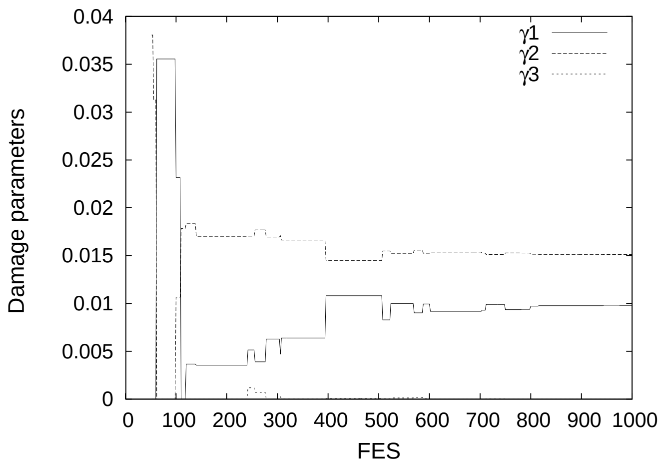

Figure 2. Typical optimization run, evolution of damage parameters.

and of course all recorded signals are restored. The experimental model, consists of a small-scaled plane three-storey shear-type building, whose dynamical characteristics are reported in Table 2. Each storey is composed of a rigid girder made of Anticorodal (an aluminium alloy), with a span of L=350 mm. For the healthy system, the column of all stories, represented by leaf springs with rectangular cross-section (3 x 15 mm), are made of hard steel. The height of each storey is $h_1 = 200$ mm and $h_2 = h_3 = 180$ mm, respectively. The masses, lumped at the floors, are $m_1 = 0.747$ kg, $m_2 = 0.755$ kg and $m_1 = 0.689$ kg, respectively. The excitation was supplied using an Impact Hammer and the signal acquired from the force transducer was used as input signal

Table 2. Dynamical properties of the experimental system.

floor	Mass [kg]	Stiffness [kN/m]
1	0.747	19249.1
2	0.755	25967.1
3	0.689	26768.0

Mode	Frequency [Hz]	Damping ratio [%]
1	12.32	0.87
2	36.04	0.19
3	53.25	0.21

In [11] several damage scenarios have been considered as detailed in the following.

In the set of damage scenarios, named D-type, the reduction in the stiffness of the structure is imposed by substituting the columns at some floor with others having a smaller cross section. In particular, the damage scenario D02 is obtained by replacing the original columns of the first floor (3x15 mm), with slender members, in which a reduction of area was introduced at mid-height (3x7 mm; dog-bone shape). In such a way a very low damage level is achieved only at first floor.

In Table 3. the identified stiffness at each floor and the expected damage values for D2-type damage scenarios are reported [11].

Table 3. Identified stiffness and expected damage parameters for D2-type damage scenarios.

	Healthy	D02	D03	D04
k_1	19249.1	18398.4	19067.6	20484.5
k_2	25967.1	26114.5	25305.9	11362.9
k_3	26768.0	26744.9	12335.7	27197.1

	Healthy	D02	D03	D04
γ_1	-	$4.42 * 10^{-2}$	$0.94 * 10^{-2}$	0.00
γ_2	-	0.00	$2.55 * 10^{-2}$	$56.24 * 10^{-2}$
γ_3	-	0.00	$53.92 * 10^{-2}$	0.00

Also in this case no pre-filtering of the dynamical response was used and the objective functional is calculated using the acceleration response at third floor to the recorded hammer test.

$ACO_{\Re}$ has been implemented with $q = 0.1, \xi = 0.85, k = 50$, the single run of optimization is stopped when the functional is evaluated 5000 times; 50 independent runs have been used to assess the performance of the algorithm. The search space is the domain $[0, 0.6]x[0, 0.6]x[0, 0.6]$. The first case analyzed was the D02 pertaining with a low damage level, Table 4 resumes the mean and the standard deviation of the damage indexes obtained and of the absolute percentage error.

Table 4. D02 - Mean and standard deviation of damage indexes.

	γ_1	γ_2	γ_3
mean	$4.70 * 10^{-2}$	$6.50 * 10^{-8}$	$4.24 * 10^{-8}$
st.dev	$1.55 * 10^{-5}$	$3.56 * 10^{-8}$	$4.06 * 10^{-8}$

	$\varepsilon_1\%$
mean	$-1.31 * 10^{-2}$
st.dev	$3.30 * 10^{-2}$

Figure 3 shows the variation of the best, the worst, the mean and the median of the functional values for the 50 independent runs vs. the number of functional evaluation (FES) for the D02 scenario. Figure 4 shows the variation of coefficients $\gamma = (\gamma_1, \gamma_2, \gamma_3)$ during a single run for the D02 scenario.

As can be seen $ACO_{\Re}$ succeeds to identify the incipient damage and gives a very good quantitative estimate of the extent of the damage. The results found moreover show very little dispersion between best and worst solution with a standard deviation on the parameters found really low.

The second scenario analyzed was the D03, Table 5 resumes the mean and the standard deviation of the damage indexes obtained and of the absolute percentage error.

Figure 5 shows the variation of the best, the worst, the mean and the median of the functional values for the 50 independent runs vs. the number of functional evaluation (FES) for the D03 scenario. Figure 6 shows the variation of coefficients $\gamma = (\gamma_1, \gamma_2, \gamma_3)$ during a single run for the D03 scenario.

Also in this case the results are very satisfactory for γ_2 and γ_3 parameters and same considerations of case D02 apply, instead for γ_1 the identification is less accurate. The presence of γ_3,

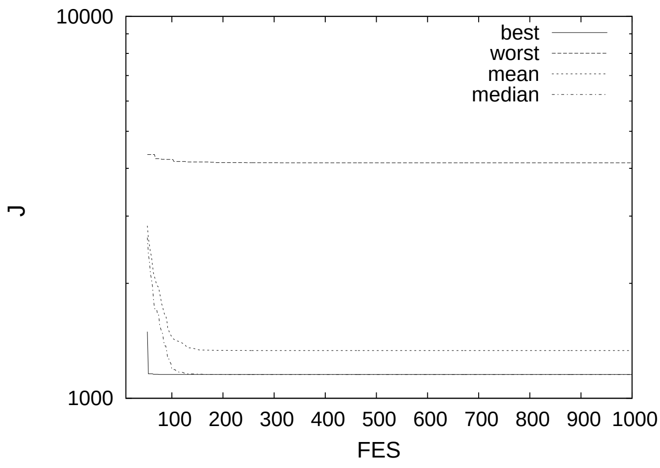

Figure 3. D02 - Evolution of the functional along ants generation steps.

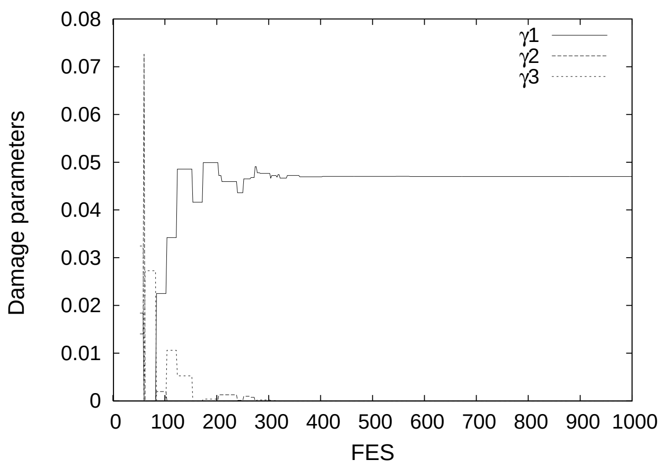

Figure 4. D02 - Typical optimization run, evolution of damage parameters.

Table 5. D03 - Mean and standard deviation of damage indexes.

	γ_1	γ_2	γ_3
mean	$2.26 * 10^{-2}$	$2.82 * 10^{-2}$	$5.38 * 10^{-1}$
st.dev	$2.89 * 10^{-5}$	$2.96 * 10^{-5}$	$6.02 * 10^{-6}$

	$\varepsilon_1\%$	$\varepsilon_2\%$	$\varepsilon_3\%$
mean	$-1.41 * 10^{2}$	$-1.07 * 10^{1}$	$1.59 * 10^{-1}$
st.dev	$3.08 * 10^{-1}$	$1.16 * 10^{-1}$	$1.12 * 10^{-3}$

more than an order of magnitude greater, leads to bigger relative errors on others parameters.

The last scenario analyzed was the D04, in this case to avoid the experimental errors that leads to the apparent increasing of k_1 and k_3 only γ_2 is permitted to vary. Table 6 resumes the mean and the standard deviation of γ_2 and the absolute percentage error.

Figure 7 shows the variation of the best, the worst, the mean and the median of the functional values for the 50 independent runs vs. the number of functional evaluation (FES) for the D04

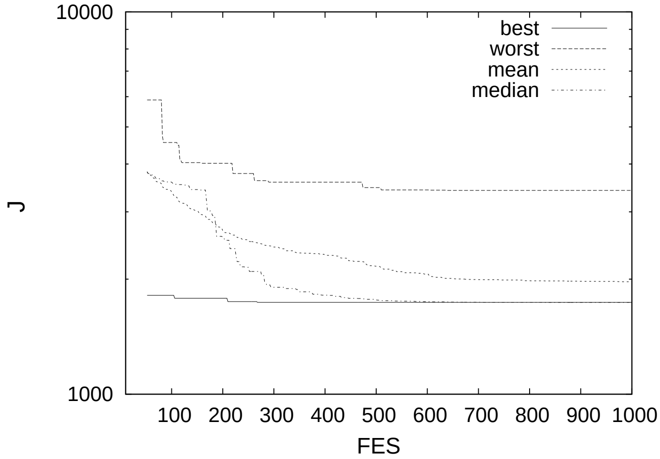

Figure 5. D03 - Evolution of the functional along ants generation steps.

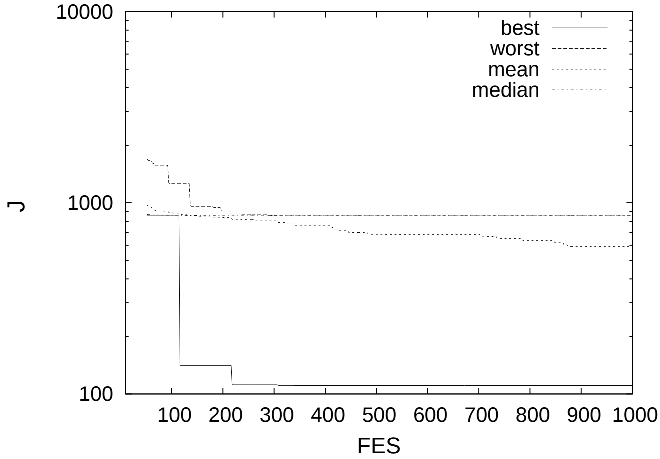

Figure 7. D04 - Evolution of the functional along ants generation steps.

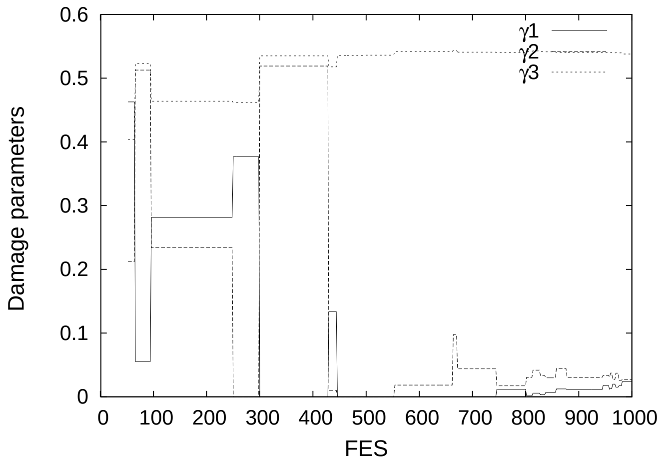

Figure 6. D03 - Typical optimization run, evolution of damage parameters.

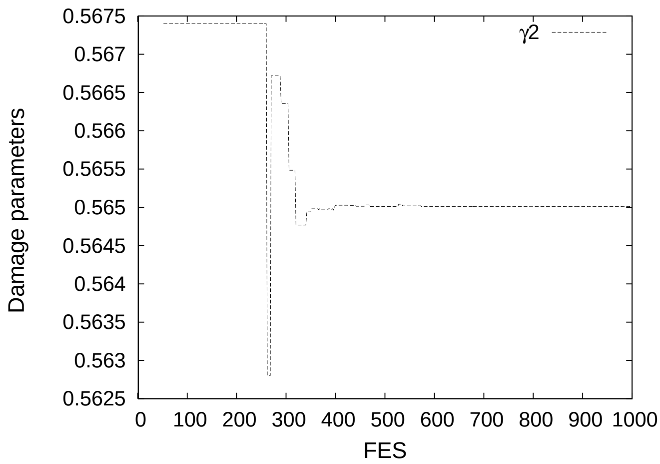

Figure 8. D04 - Typical optimization run, evolution of damage parameter.

Table 6. D04 - γ_2 - Mean and standard deviation.

	γ_2	ε_2
mean	$56.40*10^{-2}$	$4.32*10^{-1}$
st.dev	$1.60*10^{-3}$	$2.85*10^{-1}$

scenario. Figure 8 shows the variation of coefficient γ_2 during a single run for the D04 scenario.

Also in this case the identification results gives a very good quantitative estimate of the extent of the damage.

4 CONCLUSIONS

In this paper we presented a method for detecting damage in structures having close frequencies, without using analytical signal response. Then the damage identification is based on the minimization of a functional calculated on acceleration response, without using filter or Hilbert transform. Once the functional has been introduced, the minimization is performed by the meta-heuristic method of the Ant Colony Optimization (ACO) extended to continuous domain and labeled as $ACO_{\Re}$, having considered inside a Gaussian distribution for function showing many local minima.

Numerical results on 3 degree of freedom system confirm that, even in cases where the functional is very irregular and with many local minima, the damage index can be quantified with a very good level of accuracy. It is woth stressing that this procedure works very well even if real experimental signal responses have been considered as shown in the numerical applications.

Further studies are necessary to better understand the capacity of the algorithm to deal with very different level of damages and the influence of different parameters on the algorithms performances.

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