

**Talvila Erik, The regulated primitive integral. Illinois J. Math. 53 (2009), no. 4, 11871219, 46Gxx (26A39 46Exx) MR 2 741 185**

A descriptive definition of an integral is a definition which provides a “description” of the space of primitives. The derivatives in some sense of the primitives are the integrands.

In this paper the author introduces a descriptive method of integrating distributions: the regulated primitive integral.

The set  $\mathbf{B}_R = \{F : [-\infty, \infty] \rightarrow \mathbf{R} \mid F \text{ is regulated and left continuous on } \mathbf{R}, F(-\infty) = 0, F(\infty) \in \mathbf{R}\}$  is the family of primitives. The derivative here is in the sense of the distributions (i.e. a distributional or weak derivative). Then the integrable distributions are those distributions (in the Schwartz’s sense) that are the distributional derivative of a function in  $\mathbf{B}_R$ .

The regulated primitive integral is a proper extension of the integral of distribution defined by L. Schwartz [Thorie des distributions. (French) Publications de l’Institut de Mathmatique de l’Universit de Strasbourg, No. IX-X. Nouvelle dition, entirement corrige, refundue et augmente. Hermann, Paris 1966 xiii+420, 46.40 (44.00), MR0209834 (35#730)]. Moreover it is proved that the space of regulated integrable distributions is the completion of the space of signed Radon measures in the Alexiewicz norm, but it is not the completion in this norm of the Henstock-Kurzweil integrable functions. The functions of bounded variation constitute its dual space and also the space of multipliers. In the introduction a wide panorama of descriptive and constructive integration methods is given.

Reviewed by (*L. Di Piazza*)