P H Y S I C A L   R E V I E W   L E T T E R S

Extraction of Singlet States from Noninteracting High-Dimensional Spins

F. Ciccarello,[1,2,3] M. Paternostro,[3] M. S. Kim,[3] and G. M. Palma[2]

[1]*CNISM and Dipartimento di Fisica e Tecnologie Relative, Università degli Studi di Palermo, Viale delle Scienze, Edificio 18,
I-90128 Palermo, Italy*
[2]*NEST-INFM (CNR) and Dipartimento di Scienze Fisiche ed Astronomiche, Università degli Studi di Palermo, Via Archirafi 36,
I-90123 Palermo, Italy*
[3]*School of Mathematics and Physics, Queen's University, Belfast BT7 1NN, United Kingdom*

We present a scheme for the extraction of singlet states of two remote particles of arbitrary quantum spin number. The goal is achieved through post-selection of the state of interaction mediators sent in succession. A small number of iterations is sufficient to make the scheme effective. We propose two suitable experimental setups where the protocol can be implemented.

Achieving control at the quantum level is a pivotal requirement for the grounding of quantum technology and the development of reliable protocols for information processing. Frequently, state manipulation of a quantum device needs the connection of remote nodes of a network and the creation of their entangled state. Such a delocalized architecture has received strong experimental attention, especially at the quantum optics level. The heralded entanglement of remote atomic ensembles or individually-trapped ions has been produced, and atom-photon entanglement has been observed [1]. The transfer of prebuilt entanglement to distant systems has been proposed as a way to distribute quantum channels [2].

A different approach exploits a mediated interaction between two remote nodes, 1 and 2, by means of their sequential coupling to the same ancillary system e: The ancilla can bring to system 2 the information that has been previously impressed on it by its interaction with system 1. Recently, this idea has been used in a solid-state context involving multiple electron scattering between magnetic impurities [3–5]. Interestingly, e can also be used so as to condition the state of 1 and 2. Once a three-body correlated state is established by means of bilocal $1 - e$ and $2 - e$ interactions, by measuring the state of e we could project the remote systems onto entangled states with a nonzero probability [3–6]. In these examples, 1 and 2 are embodied by two-level systems whose finite Hilbert space bounds the entanglement that can be shared [7]. Overcoming such a limitation is an important task deserving attention.

Here we present a scheme that allows the "extraction" of maximally entangled states via an effective nondemolition Bell measurement performed onto the state of two spin-s particles. This occurs through repeated injection and post-selection of simple mediators, each undergoing multiple scattering and spin-flipping between the two spins [8]. Besides achieving the maximum number of ebits allowed to two spin-s systems, the protocol provides a procedure for accumulating entanglement. Remarkably, our protocol does not require interaction-time tuning. In our scheme

maximal entanglement is stable against the parameters of the conditioned dynamics, which is a clear advantage in experimental implementations. In order to fix the ideas, we first describe the protocol in terms of a system composed of a conduction electron and two magnetic impurities. This will allow us to clearly illustrate the relevant features of our scheme. Later, we show how a cavity-quantum electrodynamics (QED) system, consisting of two multilevel atoms interacting with a photon field, can also embody the desired dynamics and allows a prompt experimental implementation.

We consider a quasi one-dimensional (1D) wire, such as a semiconductor quantum wire [9] or a single-wall carbon nanotube [10], where two identical spin-s magnetic impurities 1 and 2 are embedded at positions $x_1 = 0$ and $x_2 = x_0$ [see Fig. 1(a)]. Left-incident single electrons undergo multiple scattering between the two impurities and simultaneous spin-flipping. Assuming that the electron's coherence length exceeds x_0 and that each electron occupies only the lowest sub-band, the Hamiltonian reads (we set $\hbar = 1$) $\hat{H} = \hat{p}^2/(2m^*) + J\hat{\boldsymbol{\sigma}} \cdot [\hat{\mathbf{S}}_1 \delta(x) + \hat{\mathbf{S}}_2 \delta(x - x_0)]$. Here, $\hat{p} = -i\nabla$, m^*, and $\hat{\boldsymbol{\sigma}}$ are the electron momentum, effective mass, and Pauli spin operator, respectively. $\hat{\mathbf{S}}_i$ is the spin-s operator of the impurity $i = 1, 2$, and J is the Heisenberg exchange coupling constant whose dimensions are frequency times length. Because of the elastic nature of the interactions, the energy spectrum reads $E = k^2/2m^*$
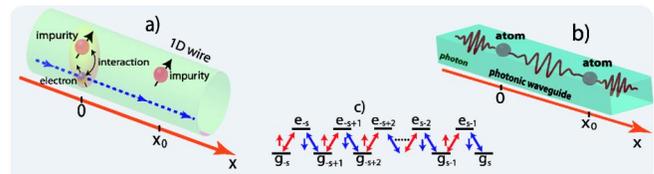

FIG. 1 (color online). Setups for the implementation of our scheme in nanowire (a) and cavity QED (b). (c) Multilevel atom embodying a spin-s particle for the setup in panel (b), where symbols ↑, ↓ indicate, abstractly, proper polarization of a photon.

(k is the electron wave vector). We label with $\hat{\mathbf{S}} = \hat{\boldsymbol{\sigma}} + \hat{\mathbf{S}}_1 + \hat{\mathbf{S}}_2$ the total spin of the system, while m_i and $m_e = \pm 1/2$ are the quantum numbers associated with $\hat{S}_{iz}$ and $\hat{\sigma}_z$, respectively. From now on, we denote $\{1/2, -1/2\}$ by $\{\Uparrow, \Downarrow\}$ and, for convenience, we use the basis of product states $|m_e, \{m_i\}\rangle = |m_e\rangle_e |m_1, m_2\rangle_{12}$. We prepare the impurities in $|\{m_i'\}\rangle_{12}$. An incoming electron of wave vector k and spin state $|m_e'\rangle_e$ is reflected (transmitted) in the state $|m_e\rangle_e$, while the impurities' spin state changes into $|\{m_i\}\rangle_{12}$ with probability amplitude r (t) (we omit the dependence of r and t on $m_{e(i)}$ and $m_{e(i)}'$). As $\hat{S}_z$ is a constant of motion, the only nonzero amplitudes are those obeying the selection rule $m_{12}' + m_e' = m_{12} + m_e$ with $m_{12} = m_1 + m_2$. We solve this scattering problem by finding the steady states $|k, m_e', \{m_i'\}\rangle$ with input part $\langle x|k, m_e', \{m_i'\}\rangle_{\text{in}} = e^{ikx}\theta(-x)|m_e', \{m_i'\}\rangle$, where $\theta(x)$ is the Heaviside step function. Their output part reads $\langle x|k, m_e', \{m_i'\}\rangle_{\text{out}} = \sum_{\alpha=r,t}\langle x|k, m_e', \{m_i'\}\rangle_\alpha$ with $\langle x|k, m_e', \{m_i'\}\rangle_\alpha = \sum_{m_e,\{m_i\}} \alpha f_\alpha(x)|m_e, \{m_i\}\rangle$ and $f_\alpha(x) = e^{i\eta_\alpha kx}\theta(\eta_\alpha x - \frac{1+\eta_\alpha}{2}x_0)$ ($\eta_r = -\eta_t = -1$). The steady states are computed at all orders in J solving the time-independent Schrödinger equation and imposing the matching of the wave function at x_i's [4]. We now derive how an (in general mixed) initial state of the impurities ρ_{12} is transformed after scattering of an electron incoming in an arbitrary statistical mixture ρ_e of the spin states $|\Uparrow\rangle_e$ and $|\Downarrow\rangle_e$. To this aim, we consider the state having $|k\rangle\langle k|\rho_e\rho_{12}$ as the input part, where $\langle x|k\rangle = e^{ikx}\theta(-x)$. The output part of such state is found by expanding it in the basis $\{|k, m_e', \{m_i'\}\rangle\}$ and replacing each component of this expansion with the corresponding output part. A further projection onto the electron's position eigenstates far from the impurities $|x_r\rangle$ and $|x_t\rangle$ ($x_r \ll 0$, $x_t \gg x_0$) yields $\sum_{\alpha=r,t}\langle x_\alpha|k, m_e', \{m_i'\}\rangle_\alpha \langle k, m_e', \{m_i'\}|x_\alpha\rangle |x_\alpha\rangle\langle x_\alpha|$. After tracing over the electron's degrees of freedom, the impurities' state becomes

$$\mathcal{E}_{\rho_e}(\rho_{12}) = \sum_{\mu,\nu=\Uparrow,\Downarrow} \rho_{e\nu\nu}(\hat{R}_\nu^\mu \rho_{12}\hat{R}_\nu^{\mu\dagger} + \hat{T}_\nu^\mu \rho_{12}\hat{T}_\nu^{\mu\dagger}), \quad (1)$$

where $\sum_\mu (\hat{R}_\nu^{\mu\dagger}\hat{R}_\nu^\mu + \hat{T}_\nu^{\mu\dagger}\hat{T}_\nu^\mu) = \mathbb{1}_{12}$. Each Kraus operator R_ν^μ (T_ν^μ) depends only on r's (t's) and is physically interpreted as the effect on ρ_{12} due to the detection in spin-state $|\mu\rangle_e$ of a reflected (transmitted) electron incoming in state $|\nu\rangle_e$. We want to show that, conditioning the map in Eq. (1)

and iterating it for n electrons (injected in succession in the same spin state), singlet-state extraction is efficiently performed. To achieve this, we first describe what is induced by post-selecting the state of $n = 1$ scattered electrons. Preparation and post-selection of a given electron-spin state, say $|\Uparrow\rangle_e$, can be accomplished using spin-filtering contacts at the input or output ports of the wire [11], each selecting the same spin state. We obtain the final impurities' state $\varrho_{12}^{(1)} = \mathcal{E}_{\Uparrow\Uparrow}(\rho_{12}) = (\hat{R}_\Uparrow^\Uparrow \rho_{12}\hat{R}_\Uparrow^{\Uparrow\dagger} + \hat{T}_\Uparrow^\Uparrow \rho_{12}\hat{T}_\Uparrow^{\Uparrow\dagger})/P_{\Uparrow\Uparrow}^{(1)}(\rho_{12})$ with success probability $P_{\Uparrow\Uparrow}^{(1)}(\rho_{12}) = \text{Tr}_{12}(\hat{R}_\Uparrow^\Uparrow \rho_{12}\hat{R}_\Uparrow^{\Uparrow\dagger} + \hat{T}_\Uparrow^\Uparrow \rho_{12}\hat{T}_\Uparrow^{\Uparrow\dagger})$. The state $\varrho_{12}^{(n)}$ corresponding to n electrons being prepared and post-selected in $|\Uparrow\rangle_e$ is obtained as $\varrho_{12}^{(n)} = \mathcal{E}_{\Uparrow\Uparrow}^n(\rho_{12})$ with conditional probability $P_{\Uparrow\Uparrow}^{(n\geq 1)}(\rho_{12}) = \prod_{j=1,n} P_{\Uparrow\Uparrow}(\varrho_{12}^{(j-1)})$ and $\varrho_{12}^{(0)} = \rho_{12}$ [12]. Here, the rate of electron-injection is chosen so that, as an electron reaches the impurities, the previous one has been already scattered off.

Let $|\Psi_s^-\rangle$ be the singlet state of two spin-s impurities. Using resonance conditions (i.e., $kx_0/\pi \in \mathbb{Z}$), in Figs. 2(a) and 2(b) we consider the case $s = 1/2$ and plot the fidelity $F^{(n)}$ of $\varrho_{12}^{(n)}$ with respect to the singlet $|\Psi_{1/2}^-\rangle$ together with $P_{\Uparrow\Uparrow}^{(n)}$ as functions of n and J/v for the initial product state $|1/2, -1/2\rangle_{12}$ ($v = k/m^*$ is the electronic group velocity). Clearly, $F^{(n)} \to 1$ for a range of values around $J/v \simeq 1.5$ that becomes a plateau when n increases ($n < 7$ iterations are enough to get fidelity higher than 0.95). For a fixed value of J/v, such convergence is exponential in n. Remarkably, although our protocol is conditioned on the outcomes of n projective measurements all with the same outcome, the probability of success converges exponentially to 0.5. Differently from [3–5], the scheme is still efficient for a nonoptimal J/v. Only a larger n is required, for a fixed s. Moreover, the process is robust against discrepancies of k with respect to resonance conditions and the use of a stream of mediators with mutually different wave vectors. In fact, by considering a Gaussian distribution of wave vectors centered at k with variance σ, we have found that the fidelity (probability) is larger than 0.9 (0.35) for $kx_0 \in [0.9, 1.03]\pi$ and σ/k up to $\simeq 5\%$.

We now address the dependence of our figures of merit on the dimensionality of the impurities' spin. While the optimum ratio J/v depends slightly on s, the efficiency of singlet extraction persists, as shown in Fig. 2(c) for
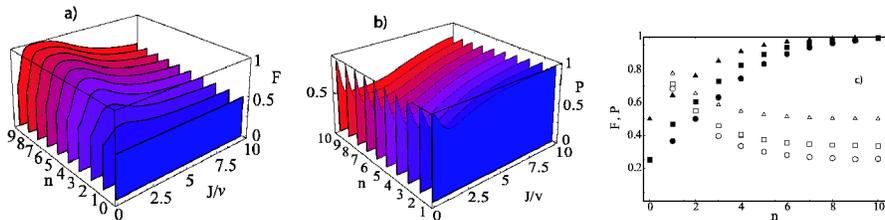

FIG. 2 (color online).   (a) and (b) Fidelity and success probability vs. J/v and n for $s = 1/2$. (c) $F^{(n)}$ (filled symbols) and $P_{\Uparrow\Uparrow}^{(n)}$ (empty symbols) vs n for $s = 1/2$ and $J/v = 1.5$ ($\blacktriangle$, $\triangle$), $s = 1$ and $J/v = 1.2$ ($\blacksquare$, $\square$), and $s = 3/2$ and $J/v = 1.1$ ($\bigcirc$, $\bullet$) at $kx_0/\pi \in \mathbb{Z}$ (J/v is optimized for each s).

$\rho_{12} = |s, -s\rangle\langle s, -s|$ with $s = 1/2, 1, 3/2$. Evidently, $\varrho_{12}^{(n)}$ rapidly converges to the singlet state regardless of s (for instance, $F^{(n>5)} > 0.95$ for $s = 1$) while $P_{\uparrow\uparrow}^{(n)}$ approaches a finite value according to $P_{\uparrow\uparrow}^{(n\gg1)}(\rho_{12}) \rightarrow |\langle\Psi_s^-|s, -s\rangle|^2 = (2s + 1)^{-1}$, exponentially in n. Our scheme thus asymptotically performs an effective projective measurement onto the spin-s singlet state. As the singlet state has the maximum number of ebits allowed by the dimension of the Hilbert space of each impurity, the scheme provides a way to extract more than one ebit by considering sufficiently high-dimensional impurities' spins. Moreover, an entanglement accumulation mechanism is achieved [7]. For instance, for $s = 2$ and $J/v = 1$ the impurities' entanglement [measured by the logarithmic negativity, which is upper-bounded by $\log_2(d)$ for a d^2-dimensional Hilbert space] after $n = 2, 4$, and 5 is, respectively, 1.2, 1.8, and 2. These are larger than the bound given by $\log_2(2s + 1)$ for $s = 1/2, 1$, and $3/2$, making our system an iteratively exploitable quantum channel: The impurities' entanglement can be extracted to many pairs of qubits [7]. Similar results hold for any initial eigenstate of $\hat{S}_{12z} = \hat{S}_{1z} + \hat{S}_{2z}$ with null eigenvalue.

We now show how the efficiency of singlet-state extraction relies on resonance-induced selection rules. Let $|s, s, s_{12}, m_{12}\rangle$ be the coupled basis of common eigenstates of $\hat{S}_1^2, \hat{S}_2^2, \hat{S}_{12}^2$, and $\hat{S}_{12z}$ (the singlet state thus reads $|\Psi_s^-\rangle = |s, s, s_{12} = 0, m_{12} = 0\rangle$). Let $\mathcal{E}_\uparrow(\rho_{12})$ be the unconditioned map in Eq. (1) for $\rho_e = |\uparrow\rangle\langle\uparrow|$. Clearly, with the additional output-filtering of $|\uparrow\rangle_e$, $\mathcal{E}_\uparrow(\rho_{12})$ becomes $\mathcal{E}_{\uparrow\uparrow}(\rho_{12})$. Notice that in general the product state $|s, s\rangle_{12}$ is the only fixed point of $\mathcal{E}_\uparrow(\rho_{12})$. However, at resonance ($kx_0 = n\pi$), $\hat{S}_{12}^2$ is conserved due to the equal probabilities of the electron to be found at each of the x_i's [4]. Thus, repeated applications of the unconditioned map cannot drive the system out of the eigenspace associated with a set value of s_{12}. This and the conservation of $\hat{S}_z$ imply that the singlet-state $|\Psi_s^-\rangle$ becomes an additional fixed point of $\mathcal{E}_\uparrow$. Let $p_{s_{12}}$ be the probability for an injected electron prepared in $|\uparrow\rangle_e$ to be flipped down when the impurities are prepared in $|s, s, s_{12}, 0\rangle$. The selection rules at resonance yield the evolved impurities' state $p_{s_{12}}|s, s, s_{12}, 1\rangle\langle s, s, s_{12}, 1| + (1 - p_{s_{12}})|s, s, s_{12}, 0\rangle\langle s, s, s_{12}, 0|$. If we post-select $|\uparrow\rangle_e$ at the output ports, each state $|s, s, s_{12}, 0\rangle$ with $s_{12} \neq 0$ is left unchanged with probability $1 - p_{s_{12}}$. Under application of $\mathcal{E}_{\uparrow\uparrow}^{n\gg1}$, it thus vanishes as $(1 - p_{s_{12}})^{n\gg1} \simeq 0$, which clarifies the exponential convergence exhibited by $F^{(n)}$ and $P_{\uparrow\uparrow}^n$ (cf. Fig. 2). Differently, $|s, s, s_{12} = 0, 0\rangle = |\Psi_s^-\rangle$ survives to the application of $\mathcal{E}_{\uparrow\uparrow}^{n\gg1}$ since the selection rules ensure that $p_{s_{12}=0} = 0$ [4]. If we consider an element of the uncoupled basis $|\xi\rangle$ such that $\hat{S}_{12z}|\xi\rangle_{12} = 0$ and expand it over $|s, s, s_{12}, 0\rangle$'s, we find that, under application of $\mathcal{E}_{\uparrow\uparrow}^{n\gg1}$, $|\xi\rangle\langle\xi| \rightarrow |\Psi_s^-\rangle$ with a probability $P_{\uparrow\uparrow}^{(n\gg1)}$ that asymptotically becomes $|\langle\Psi_s^-|\xi\rangle|^2$. When $|\xi\rangle = |s, -s\rangle_{12}$, as in Fig. 2, the asymptotic probability is $(2s + $

$1)^{-1}$. Our clear interpretation of the physics behind our protocol is an important feature for the development of novel schemes.

Unlike previous proposals [3–5], a remarkable advantage of our protocol is that it can be applied to magnetic impurities of spin higher than $1/2$. For instance, we could use a 1D semiconducting wire with embedded Mn impurities having $s = 5/2$. Although impressive progress has been made, a major obstacle in spintronics implementations is the current lack of high-efficiency electron-spin filters [11]. As a way to overcome such difficulties, we discuss an alternative system [see Fig. 1(b)] able to act as an accurate simulator of $\hat{H}$ and holding the promises for not far-fetched experimental implementation. The basic idea is to replace the electron with a single photon propagating in a 1D photonic waveguide sustaining two frequency-degenerate orthogonally polarized modes. For consistency of notation, we denote circular polarizations by $\uparrow$ and $\downarrow$. Each impurity is now embodied by a multilevel atom [see Fig. 1(c)] having a $(2s + 1)$-fold degenerate ground level spanned by $\{|g_{-s}\rangle, \ldots, |g_s\rangle\}$ and a $2s$-fold degenerate excited level spanned by $\{|e_{-s}\rangle, \ldots, |e_{s-1}\rangle\}$. The standard three-level Λ and five-level M configurations are recovered, for instance, by taking $s = 1/2$ and $s = 1$, respectively. Such a configuration may be found in the rich hyperfine spectrum of alkali atoms. We assume electric-dipole selection rules such that each $|e_m\rangle$ ($m = -s, \ldots, s - 1$) is connected to the pair of nearest-neighbor ground states $\{|g_m\rangle, |g_{m+1}\rangle\}$ via coherent scattering of a photon between the two orthogonally polarized modes. To fix the ideas, we take the transition $|e_m\rangle \leftrightarrow |g_m\rangle$ ($|e_m\rangle \leftrightarrow |g_{m+1}\rangle$) to be driven by the $\uparrow$-polarized ($\downarrow$-polarized) mode. Each atom can thus undergo a transition between two adjacent ground states $|g_m\rangle \leftrightarrow |g_{m+1}\rangle$ via a two-photon Raman process with associated coherent scattering of a photon between states $|\uparrow\rangle$ and $|\downarrow\rangle$. Assuming a linear dispersion law $E = v_{ph}k$ with v_{ph} the group velocity of the photon and E its energy, the free Hamiltonian of the field in the waveguide is [13] $\hat{H}_{ph} = -i\sum_{\beta=R,L}\sum_{\gamma=\uparrow,\downarrow}\int dx v_\beta \hat{c}_{\beta,\gamma}^\dagger(x)\partial_x\hat{c}_{\beta,\gamma}(x)$ with $v_R = -v_L = v_{ph}$ and $\hat{c}_{R,\gamma}^\dagger(x)$ [$\hat{c}_{L,\gamma}^\dagger(x)$] the bosonic operator creating a right (left) propagating photon of polarization γ at position x. Considering dipole transitions with Rabi frequencies and natural excited-state linewidth smaller than the corresponding detuning from the excited state, each state $|e_m\rangle$ is only virtually populated and the effective atom-photon coupling reads $\hat{V} = \sum_{i=1,2}\int dx[\hat{c}_\uparrow^\dagger(x)\hat{c}_\downarrow(x)\hat{\mathcal{S}}_{i-} + \text{h.c.}]\delta(x - x_i)$ with $c_\gamma^\dagger(x) = \sum_{\beta=R,L}c_{\beta,\gamma}^\dagger(x)$ and $\hat{\mathcal{S}}_{i+} = \hat{\mathcal{S}}_{i-}^\dagger = \sum_{m=-s}^{s-1}J_{s,m}|g_{m+1}\rangle_i\langle g_m|$. Here $J_{s,m}$ is the effective transition rate of the Raman process leading the ith atom from $|g_m\rangle_i$ to $|g_{m+1}\rangle_i$, assuming identical atoms. We map the photonic polarization into an effective pseudospin s as $\hat{\boldsymbol{\sigma}} = \int dx\hat{\boldsymbol{\sigma}}(x)$ with $\hat{\sigma}_+(x) = \hat{\sigma}_-^\dagger(x) = c_\uparrow^\dagger(x)c_\downarrow(x)$ and $\hat{\sigma}_z(x) = [\hat{c}_\uparrow^\dagger(x)\hat{c}_\uparrow(x) - \hat{c}_\downarrow^\dagger(x)\hat{c}_\downarrow(x)]/2$. Provided that $J_{s,m} = J\chi_{s,m}$ with

$\chi_{s,m} = [s(s + 1) - m(m + 1)]^{1/2}$, each $\hat{\mathcal{S}}_{i\pm}$ becomes the effective pseudospin s operator $\hat{\mathcal{S}}_{i\pm} = J\hat{S}_{i\pm}$, where $\hat{S}_{i\pm}$ obeys the standard algebra of angular momentum. Under these conditions, this model can be regarded as the second quantization version of $\hat{H}$ with the exchange electron-impurity coupling replaced by an isotropic XY interaction. It is easily checked that $[\hat{H}_{\mathrm{ph}} + \hat{V}, \hat{S}_z] = 0$ and, provided $kx_0/\pi \in \mathbb{Z}$, $[\hat{H}_{\mathrm{ph}} + \hat{V}, \hat{\mathbf{S}}_{12}^2] = 0$. Through standard procedures [13], we have derived the stationary states $|k, m'_{\mathrm{ph}}, \{m'_i\}\rangle$ for a single photon with wave vector k (m'_{ph} is the quantum number of $\hat{\sigma}_z$). The input (output) part of $|k, m'_{\mathrm{ph}}, \{m'_i\}\rangle$ is formally analogous to $|k, m'_e, \{m'_i\}\rangle_{\mathrm{in}}$ ($|k, m'_e, \{m'_i\}\rangle_{\mathrm{out}}$). Here, $\mathcal{E}_{\uparrow\uparrow}(\rho_{12})$ is obtained analogously to what is done for the previous model with photonic polarization detection used for the post-selection. Plots analogous to those in Figs. 2 are reproduced with only negligible quantitative differences. Practically, $\mathcal{E}_{\uparrow\uparrow}(\rho_{12})$ is obtained using Geiger-like photodetectors at the input or output ports of the waveguide combined with polarizing beam splitters to realize $\mathcal{E}_{\uparrow\uparrow}(\rho_{12})$. Each $J_{s,m}$ depends on the product of the Clebsch-Gordan coefficients associated with the far-detuned (one-photon) transitions involved in the process $|g_m\rangle \leftrightarrow |g_{m+1}\rangle$. The condition $J_{s,m} = J\chi_{s,m}$ is clearly fulfilled for $s = 1/2$, involving only $\chi_{1/2,-1/2} = 1$. For $s \geq 1$ the pattern of $J_{s,m}$'s might in general deviate from the ideal one dictated by the $\chi_{s,m}$'s. However, we have assessed $F^{(n)}$ and $P^n_{\uparrow\uparrow}$ finding that our scheme is strikingly robust against such deviations [14]. For instance, for $s = 3/2$, the ideal pattern yields $J_{3/2,1/2}/J_{3/2,-3/2} = 1$ and $J_{3/2,-1/2}/J_{3/2,-3/2} = 2/\sqrt{3}$. By taking $J_{3/2,-3/2}/v_{\mathrm{ph}} = J_{3/2,1/2}/v_{\mathrm{ph}} = \sqrt{3}$ and $J_{3/2,-1/2}/v_{\mathrm{ph}} = 4\sqrt{3}$, which are far from ideal, we obtain $F^{(n>6)} = 0.97$, and $P^{(n>6)} = 0.26$. These values are basically identical to the values obtained with the ideal ratios. This alternative model turns out to be also robust against deviations of k from the ideal resonance conditions [14]. Our protocol is thus resilient and flexible to the actual working conditions.

For a realization of the scheme in the case $s = 1/2$, the impurities can be embodied by Λ configurations encompassed in the (single-electron charged) trionic picture of semiconducting quantum dots (QDs), which have been the center of extensive studies [15]. Positioning QDs within a waveguide or a cavity is now achievable with high accuracy (~ 30 nm). A back-of-the-envelope calculation shows that for a photonic wavelength of 780 nm in a GaAs structure (400 nm in a GaN nanowire), $x_0 \sim 0.1$ μm (1 μm) is required for the resonance condition, which is achievable. Strong coupling between a single QD and a cavity field has been demonstrated [15] and current experimental efforts make the achievement of $J/v \sim 1$ realistic in large refractive-index structures, without the need of a waveguide's band gap. We consider GaInN (InAs) QDs in GaN (GaAs) nanowires as potential candidates for our scheme. Their typical quality factor is $\simeq 10^3$, implying a single-photon lifetime $\tau_p \sim 1$ ps at 400 nm

wavelength. The refractive index of GaN is ~ 2, so that a photon travels $x_0 = 1$ μm in $\tau_p/100$. Ongoing experimental progress makes the controlled growth and positioning of two QDs in μm-long waveguides quite realistic.

We have proposed a scheme for the conditional extraction of singlet states of two remote spin s's based on projective measurements over interaction mediators. The protocol does not require the demanding recycling of the same mediator. It achieves $s + 1/2$ ebits with finite probability, a small number of steps, weak requirements on the parameters entering the dynamics, and no interaction-time tuning. We have proposed a realistic setup where the mediators are embodied by photons and the spins to be entangled by artificial atoms.

[1] D. N. Matsukevich, *et al.*, Phys. Rev. Lett. **96**, 030405 (2006); B. Julsgaard *et al.*, Nature (London) **432**, 482 (2004); J. Volz *et al.*, Phys. Rev. Lett. **96**, 030404 (2006).

[2] M. Paternostro, W. Son, and M. S. Kim, Phys. Rev. Lett. **92**, 197901 (2004).

[3] D. Yang, S.-J. Gu, and H. Li, arXiv:quant-ph/0503131; A. T. Costa, Jr., S. Bose, and Y. Omar, Phys. Rev. Lett. **96**, 230501 (2006); G. L. Giorgi and F. De Pasquale, Phys. Rev. B **74**, 153308 (2006).

[4] F. Ciccarello *et al.*, New J. Phys. **8**, 214 (2006); J. Phys. A **40**, 7993 (2007); Laser Phys. **17**, 889 (2007); F. Ciccarello, G. M. Palma, and M. Zarcone, Phys. Rev. B **75**, 205415 (2007)

[5] K. Yuasa and H. Nakazato, J. Phys. A **40**, 297 (2007).

[6] H. Nakazato, M. Unoki, and K. Yuasa, Phys. Rev. A **70**, 012303 (2004); L.-A. Wu, D. A. Lidar, and S. Schneider, *ibid.* **70**, 032322 (2004); G. Compagno *et al.*, *ibid.* **70**, 052316 (2004).

[7] M. Paternostro, M. S. Kim, and G. M. Palma, Phys. Rev. Lett. **98**, 140504 (2007).

[8] For quantum state engineering via iterated quantum operations, see D. Burgarth and V. Giovannetti, New J. Phys. **9**, 150 (2007).

[9] S. Datta, *Electron Transport in Mesoscopic Systems* (Cambridge University, Cambridge, England, 1997).

[10] S. J. Tans *et al.*, Nature (London) **386**, 474 (1997).

[11] D. D. Awschalom, D. Loss, and N. Samarth, *Semiconductor Spintronics and Quantum Computation* (Springer, Berlin, 2002).

[12] For the single-impurity case without spin-flip see W. Kim, R. K. Teshima, and F. Marsiglio, Europhys. Lett. **69**, 595 (2005).

[13] J.-T. Shen and S. Fan, Phys. Rev. Lett. **95**, 213001 (2005); **98**, 153003 (2007).

[14] F. Ciccarello *et al.* (to be published).

[15] M. Atatüre *et al.*, Science **312**, 551 (2006); K. Hennessy, *et al.*, Nature (London) **445**, 896 (2007).