Multicriterion design of frames with constraints on buckling

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SUMMARY. The present paper is devoted to the optimal design of frame structures subjected to static and dynamic loading assuming the material behaviour as elastic perfectly plastic. The relevant optimal design problem is formulated as a minimum volume search problem. The minimum volume structure is determined under suitable constraints on the design variables as well as accounting for different resistance limits: the elastic shakedown limit and the instantaneous collapse limit, considering for each limit condition suitably chosen amplified load combinations. The effects of the dynamic actions are studied on the grounds of the dynamic features of the structure taking into account the structural periods referring to the actual Italian Codes related to the structural analysis and design. The minimum volume design is developed at first as the search for the optimal structure with simultaneous constraints on the elastic shakedown behaviour and on the instantaneous collapse. Moreover, in order to avoid undesired further collapse modes, the structure will be constrained to prevent element buckling. The numerical applications are related to steel frames.

1 INTRODUCTION

On the grounds of an ever increasing knowledge of material and structure behaviour, in addition to the better capability of effecting a reasonable prediction of the actions that a structure must suffer during its lifetime, and on the grounds of the continuous technological development, structural optimization has been object of several studies devoted to the proposing of new search problem formulations as well as appropriate computational methods. Such a technical and scientific effort had several positive effects; in particular, the consciousness of the structural engineers and of the institutions in controlling the related activities has strongly grown. The present paper is devoted to the formulation of a multicriterion optimal design problem of elastic perfectly plastic structures subjected to different combinations of fixed, quasi-statical (wind) and dynamic (seismic) loads. Wind and seismic effects will be both considered as perfectly cyclic. The structure must be designed in such a way to be able to simultaneously elastically shakedown and prevent the instantaneous collapse considering acting for each different limit condition a suitably chosen amplified load combination. Moreover, in order to avoid undesired further collapse modes, the structure will be constrained to prevent element buckling.

The optimal (usually minimum weight) structural design has been pursued by several researchers. The formulation of the problem strongly depends on the particular chosen resistance criterion, namely it depends on the special limit behaviour required for the structure. Several formulations have been proposed for the elastic optimal design (see, e.g. [1]), for the elastic shakedown optimal design (see, e.g., [2]), and for the standard limit design (see, e.g., [3]), always accounting for fixed and/or quasi statical loads. Each one of these formulations takes into account just the corresponding structural limit state, related to a special load condition, disregarding the observance of suitable safety factors for other possible limit states, related to as many dangerous load conditions. As a consequence, further formulations, the so-called multicriterion optimal design formulations, have been proposed (see, e.g., [4]). Furthermore, for load conditions above the elastic shakedown limit, an alternating plasticity behaviour is certainly preferable with respect to a ratchetting one, so several different formulations of the so-called plastic shakedown optimal design have been proposed (see, e.g., [5]). Finally, more recently, some further formulations have been proposed in which the dynamic behaviour of the structure is taken into account and where the results obtained by a rigorous application of the Italian Code [6] are critically examined and several contributions are provided aimed at the improving of the design (see, e.g., [7]). Anyway, whatever the special formulation is utilized, substantially depending on the special limiting criterion imposed on the structure behaviour, it is very useful to know if the optimal structure, at the prescribed limit state, fulfils special limits on its functionality. Among such bounds, and in particular making reference to frame structures, an effective limit is related to the buckling of the elements. Some contributions on this topic have been proposed just for elastic shakedown design and for standard limit design (see, e.g., [8]).

Aim of the present paper is to propose a formulation of a special multicriterion optimal (minimum volume) design problem devoted to elastic perfectly plastic frame structures subjected to a combination of fixed, cyclic and dynamic loads, imposing simultaneously constraints on the elastic shakedown behaviour (related to serviceability conditions), on the instantaneous collapse (under the combination of fixed and cyclic loads due to the wind effect), on the instantaneous collapse (under the combination of fixed and seismic loads) and preventing the undesired phenomenon of buckling. Several applications, performed by utilizing a suitable iterative technique, based on an appropriate linearization of the minimum volume problem formulated on the grounds of the statical approach, conclude the paper.

2 THE MODEL

Let us consider now a shear plane frame just subjected to an horizontal ground acceleration $a_g(t)$ and modeled as a Multi-Degree-Of-Freedom (MDOF) structure starting from zero initial conditions, such that the total number of degrees of freedom is equal to n_f.

The dynamic equilibrium equations can be written in the following form:

$$\boldsymbol{M}\,\ddot{\boldsymbol{u}}(t)+\boldsymbol{A}\,\dot{\boldsymbol{u}}(t)+\boldsymbol{K}\,\boldsymbol{u}(t)=\boldsymbol{f}(t) \tag{1}$$

being $\boldsymbol{f}(t)=-\boldsymbol{m}\,a_g(t)$. In equation (1) $\boldsymbol{M}$, $\boldsymbol{A}$ and $\boldsymbol{K}$ are the mass, damping and stiffness matrices (with dimensions $n_f \times n_f$), respectively, which are assumed to be positive ones; $\boldsymbol{m}=\boldsymbol{M}\,\boldsymbol{\tau}$, being $\boldsymbol{\tau}$ the (n_f ×1) influence vector; $\boldsymbol{f}(t)$ is the (n_f ×1) excitation vector, while $\boldsymbol{u}(t)$, $\dot{\boldsymbol{u}}(t)$ and $\ddot{\boldsymbol{u}}(t)$ are the displacement, the velocity and the acceleration (n_f ×1) vectors of the system, respectively, and the over dot means time derivative of the relevant quantity.

According to the actual Italian Standard on Structural Design, for the structure under examination it is possible to define the structural design making use of the so-called response spectrum $S_d(T)$. In order to do this it is necessary to calculate the fundamental periods (or alternatively the frequencies) of the structure which, as it is well known, can be determined once the mass and stiffness matrices of the structure are known. Further it is also assumed that the structure is a classically-damped one, that is

$$\tilde{\boldsymbol{\Phi}}\, \boldsymbol{A}\, \boldsymbol{\Phi} = \boldsymbol{\Xi} \tag{2}$$

In equation (2) $\boldsymbol{\Phi}$ is a diagonal matrix whose j^{th} component is equal to $2\,\zeta_j\,\omega_j$, being ω_j and ζ_j the j^{th} natural frequency and the j^{th} damping coefficient, respectively. In equation (2) is the modal matrix whose columns are the eigenvectors of the stiffness matrix normalized with respect to the mass matrix. According to the Italian Code a study is performed taking into account all structural modes and assuming a constant damping coefficient equal to 5%. The displacement vector due to the j^{th} mode can be determined as follows:

$$\boldsymbol{u}_j = \boldsymbol{\Phi}_j \frac{\boldsymbol{\Phi}_j^{\mathrm{T}} \boldsymbol{M} \boldsymbol{\tau}\, S_d\left(T_j\right)}{\omega_j^2} \tag{3}$$

According to the above referred guidelines the displacements $\boldsymbol{u}$ and the generalized stresses $\boldsymbol{P}$ are combined in a full quadratic way following the equation:

$$E_\ell = \sqrt{\sum_k \sum_j \rho_{jk}\, E_{j\ell}\, E_{k\ell}} \tag{4}$$

being E_ℓ the ℓ^{th} component of the combined effect of the relevant quantity, $E_{j\ell}, E_{k\ell}$ the ℓ^{th} component of the effect due to j^{th} and k^{th} modes, respectively, and ρ_{jk} the correlation coefficients between j^{th} and k^{th} modes expressed by the equation:

$$\rho_{jk} = \frac{8\,\xi^2\,\beta_{jk}^{3/2}}{\left(1+\beta_{jk}\right)\left[\left(1-\beta_{jk}\right)^2 + 4\,\xi^2\,\beta_{jk}\right]} \tag{5}$$

in which $\beta_{jk} = \mathrm{T}_k / \mathrm{T}_j$ and T_j, T_k are the periods of the j^{th} and k^{th} modes.

According with the guidelines of the great part of international codes, in particular with the Italian one, the design of the relevant structure must be performed taking into account a fixed action mainly related with the gravitational loads, a quasi statical load related to the wind effect, and a dynamic perfect cyclic load related to seismic actions, suitably combined. In the present context even the wind load is modelled as a perfect cyclic load; actually, in any case a generic cyclic load can be described through the superposition of a fixed and a perfect cyclic load.

For the aim of the present paper, we now assume that the actions are represented by appropriate combinations of the above referred loads each of which related to different limit conditions; combination C1: fixed load $\boldsymbol{F}_0$ and (reduced) seismic action related to the response spectrum S_d^S, function of the up-crossing probability in the lifetime selected for the structure; combination C2: amplified fixed load $\boldsymbol{F}_{0w}$ and perfect cyclic load related to the wind action $\boldsymbol{F}_{ciw}$; combination C3: fixed load $\boldsymbol{F}_0$ and seismic action related to the response spectrum S_d^I, function of a different (lower) up-crossing probability in the lifetime selected for the structure.

Obviously, the structure must be capable of suffer the above described load combinations according to different limit conditions; in particular, it must respond in an elastic manner (elastic shakedown) when subjected to load combination C1, it must prevent the instantaneous collapse when subjected, alternatively, to combinations C2 or C3.

In the above defined combinations, $\boldsymbol{F}_0$ and $\boldsymbol{F}_{0w}$ are special combinations of gravitational loads as prescribed by the referenced code, S_d^S and S_d^I are the response spectra related to serviceability and instantaneous collapse conditions, respectively, while the reference mechanical cyclic loads related to the wind action are defined as two opposite and independent load conditions $\boldsymbol{F}_{ciw}$, ($i = 1,2$), such that $\boldsymbol{F}_{c1w} = \boldsymbol{F}_w$ and $\boldsymbol{F}_{c2w} = -\boldsymbol{F}_w$; therefore, $\boldsymbol{F}_{ciw}$ is a perfect cyclic load.

In order to perform the structural design and, to the aim of the present paper, to perform the structural optimization, a FEM-like approach has been adopted discretizing the relevant structure into n finite elements constituted by elastic perfectly plastic material. The typical v^{th} element geometry is fully described by the s components of the vector $\boldsymbol{d}_v\,(v = 1,2,\ldots,n)$ so that $\tilde{d} = \left[\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_v, \ldots, \tilde{d}_n\right]$ represents the $n \times s$ supervector collecting all the design variables.

3 OPTIMAL DESIGN FORMULATION

Let us make reference to the elastic plastic structure described in the previous section. According to the Italian code and to the assumed load model, it is subjected to fixed mechanical loads, quasi statical perfect cyclic loads (wind effect) and perfect cyclic dynamic seismic loads. The minimum volume design problem formulation, where suitable constraints are imposed on the elastic shakedown behaviour, on the instantaneous collapse and on the element buckling, can be written as follows:

$$\min_{\left(\boldsymbol{d},\boldsymbol{u}_0,\boldsymbol{u}_{0w},\boldsymbol{u}_{cw},\boldsymbol{u}_{jce}^S,\boldsymbol{u}_{jce}^I,\boldsymbol{Y}_0^S,\boldsymbol{Y}_{0iw}^I,\boldsymbol{Y}_{0ie}^I\right)} V \tag{6a}$$

$$\boldsymbol{d} - \bar{\boldsymbol{d}} \geq \boldsymbol{0} \tag{6b}$$

$$\boldsymbol{T}\,\boldsymbol{d} - \bar{\boldsymbol{t}} \geq \boldsymbol{0} \tag{6c}$$

$$\boldsymbol{P}_0 = \tilde{\boldsymbol{B}}\,\boldsymbol{u}_0\,, \quad \boldsymbol{K}\boldsymbol{u}_0 - \boldsymbol{F}_0 = \boldsymbol{0} \tag{6d}$$

$$\boldsymbol{P}_{0w} = \tilde{\boldsymbol{B}}\,\boldsymbol{u}_{0w}\,, \quad \boldsymbol{K}\boldsymbol{u}_{0w} - \boldsymbol{F}_{0w} = \boldsymbol{0} \tag{6e}$$

$$\boldsymbol{P}_{cw} = \tilde{\boldsymbol{B}}\,\boldsymbol{u}_{cw}\,, \quad \boldsymbol{K}\,\boldsymbol{u}_{cw} - \boldsymbol{F}_{cw} = \boldsymbol{0} \tag{6f}$$

$$\boldsymbol{P}_{jce}^S = \tilde{\boldsymbol{B}}\,\boldsymbol{u}_{jce}^S\,, \quad \boldsymbol{u}_{jce}^S = \boldsymbol{\Phi}_j \frac{\tilde{\boldsymbol{\Phi}}_j \boldsymbol{M}\boldsymbol{\tau}\, S_d^S\left(T_j\right)}{\omega_j^2}\,, \quad P_{ce\ell}^S = \sqrt{\textstyle\sum_j \sum_k \rho_{kj}\, P_{kce\ell}^S\, P_{jce\ell}^S} \tag{6g}$$

$$\boldsymbol{P}_{jce}^I = \tilde{\boldsymbol{B}}\,\boldsymbol{u}_{jce}^I\,, \quad \boldsymbol{u}_{jce}^I = \boldsymbol{\Phi}_j \frac{\tilde{\boldsymbol{\Phi}}_j \boldsymbol{M}\boldsymbol{\tau}\, S_d^I\left(T_j\right)}{\omega_j^2}\,, \quad P_{ce\ell}^I = \sqrt{\textstyle\sum_j \sum_k \rho_{kj}\, P_{kce\ell}^I\, P_{jce\ell}^I} \tag{6h}$$

$$\boldsymbol{\varphi}_{ie}^S \equiv \tilde{\boldsymbol{N}}\boldsymbol{P}_0 + (-1)^i\, \tilde{\boldsymbol{N}}\boldsymbol{P}_{ce}^S - \boldsymbol{S}\boldsymbol{Y}_0^S - \boldsymbol{R} \leq \boldsymbol{0}\,, \quad \boldsymbol{Y}_0^S \geq \boldsymbol{0} \tag{6i}$$

$$\boldsymbol{\varphi}_{iw}^I \equiv \tilde{\boldsymbol{N}}\boldsymbol{P}_{0w} + (-1)^i\, \tilde{\boldsymbol{N}}\boldsymbol{P}_{cw} - \boldsymbol{S}\boldsymbol{Y}_{0iw}^I - \boldsymbol{R} \leq \boldsymbol{0}\,, \quad \boldsymbol{Y}_{0iw}^I \geq \boldsymbol{0} \tag{6j}$$

$$\boldsymbol{\varphi}_{ie}^I \equiv \tilde{\boldsymbol{N}}\boldsymbol{P}_0 + (-1)^i\, \tilde{\boldsymbol{N}}\boldsymbol{P}_{ce}^I - \boldsymbol{S}\boldsymbol{Y}_{0ie}^I - \boldsymbol{R} \leq \boldsymbol{0}\,, \quad \boldsymbol{Y}_{0ie}^I \geq \boldsymbol{0} \tag{6k}$$

where equations (6a,i,j,k) hold for $i = 1,2$, $j = 1,2,\ldots\ldots.,n_{sm}$, being n_{sm} the number of structural modes and $\ell = 1,2,\ldots\ldots.,n_p$, being n_p the total number of plastic nodes.

In equations (6) $\boldsymbol{d}$ is the design variable vector while $\bar{\boldsymbol{d}}$ represents the vector collecting the imposed limit values for $\boldsymbol{d}$, $\boldsymbol{T}$ is the technological constraint matrix with $\bar{\boldsymbol{t}}$ a suitably chosen technological vector, $\boldsymbol{u}_0$ and $\boldsymbol{P}_0$, $\boldsymbol{u}_{0w}$ and $\boldsymbol{P}_{0w}$, $\boldsymbol{u}_{cw}$ and $\boldsymbol{P}_{cw}$, $\boldsymbol{u}_{jce}^S$ and $\boldsymbol{P}_{jce}^S$, $\boldsymbol{u}_{jce}^I$ and $\boldsymbol{P}_{jce}^I$ are the purely elastic response to the assigned fixed loads, the mechanical cyclic load, the reduced dynamic load related to the j^{th} structural mode, the full dynamic load related to the j^{th} structural mode, respectively, in terms of displacements and generalized stresses, $\boldsymbol{P}_{ce}^S$ and $\boldsymbol{P}_{ce}^I$ the combined generalized stress vectors related to reduced and full seismic actions, $\boldsymbol{\varphi}_{ie}^S$, $\boldsymbol{\varphi}_{iw}^I$ and $\boldsymbol{\varphi}_{ie}^I$ are the plastic potential vectors related to the elastic shakedown limit (apex *S*) and to the instantaneous collapse limit (apex *I*), respectively, $\boldsymbol{Y}_0^S$, $\boldsymbol{Y}_{0iw}^I$ and $\boldsymbol{Y}_{0ie}^I$ are fictitious plastic activation intensity vectors related to the elastic shakedown limit and to the impending instantaneous collapse, respectively. Finally, $-\boldsymbol{S}$ is a time independent symmetric structural matrix which transforms the plastic activation intensities into the plastic potentials.

Problem (6) can be improved in order to take into account the buckling effect on the pillars and on the cross bracing elements, if present, by writing equations (6i-k) as follows:

$$\boldsymbol{\varphi}_{ie}^S \equiv \tilde{\boldsymbol{N}}\boldsymbol{P}_0^{(b)} + (-1)^i\,\tilde{\boldsymbol{N}}\boldsymbol{P}_{ce}^{(b)S} - \boldsymbol{S}\boldsymbol{Y}_0^S - \boldsymbol{R} \le \boldsymbol{0}\,,\ \boldsymbol{Y}_0^S \ge \boldsymbol{0} \tag{7i}$$

$$\boldsymbol{\varphi}_{iw}^I \equiv \tilde{\boldsymbol{N}}\boldsymbol{P}_{0w}^{(b)} + (-1)^i\,\tilde{\boldsymbol{N}}\boldsymbol{P}_{cw}^{(b)} - \boldsymbol{S}\boldsymbol{Y}_{0iw}^I - \boldsymbol{R} \le \boldsymbol{0}\,,\quad \boldsymbol{Y}_{0iw}^I \ge \boldsymbol{0} \tag{7j}$$

$$\boldsymbol{\varphi}_{ie}^I \equiv \tilde{\boldsymbol{N}}\boldsymbol{P}_0^{(b)} + (-1)^i\,\tilde{\boldsymbol{N}}\boldsymbol{P}_{ce}^{(b)I} - \boldsymbol{S}\boldsymbol{Y}_{0ie}^I - \boldsymbol{R} \le \boldsymbol{0}\,,\quad \boldsymbol{Y}_{0ie}^I \ge \boldsymbol{0} \tag{7k}$$

where the apex (*b*) indicates that the generalized stress vectors, and in particular the bending moment values, are amplified. The amplifies bending moment acting on the typical pillar will be evaluated as follows:

$$M^{(b)} = \frac{M}{1-\beta\dfrac{N}{N_c}}\,,\qquad \left(\text{with}\quad N_c = \frac{\pi^2\,EI_{min}}{\ell_0^2}\right) \tag{8}$$

being M and N the standard values deduced from (6d-h), β a suitable safety factor and N_c the relevant critical load (Euler's formula). Furthermore, in order to take into account the buckling effect on the cross bracing elements the following constraints are introduced:

$$\alpha\,\hat{\boldsymbol{L}}\hat{\boldsymbol{I}}_m - \hat{\boldsymbol{A}}\sigma_y \ge 0 \tag{9}$$

where, besides the already defined symbols, $\alpha = \pi^2 E$, being E the material Young's modulus, $\hat{\boldsymbol{L}}$ is a diagonal square matrix collecting terms as $1/\ell_m^2$, $m \in I(n_{cp})$,being ℓ_m the length of the m^{th} compressed element and n_{cp} the total number of compressed bars, $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{I}}_{min}$ are the cross-section area and the related minimum moments of inertia vectors of potentially compressed bars.

4 NUMERICAL RESULTS

The optimal designs of steel frames have been obtained referring to the formulations previously proposed. At first, the multicriterion design problem (6) has been solved for the two six floor frames plotted in Fig. 1a,b constituted by square box section elements ($\ell = 300$ mm for flexural frame, $\ell = 250$ mm for cross bracing one and $\ell = 100$ mm for cross bracing elements).

The constant thickness s is assumed as design variable and cross bracing elements are weakened by holes. Furthermore, $L_1 = 700\ \text{cm}$, $L_2 = 400\ \text{cm}$ and $H = 400\ \text{cm}$, Young modulus $E = 21\ \text{MN}/\text{cm}^2$, yield stress $\sigma_y = 23.5\ \text{kN}/\text{cm}^2$.

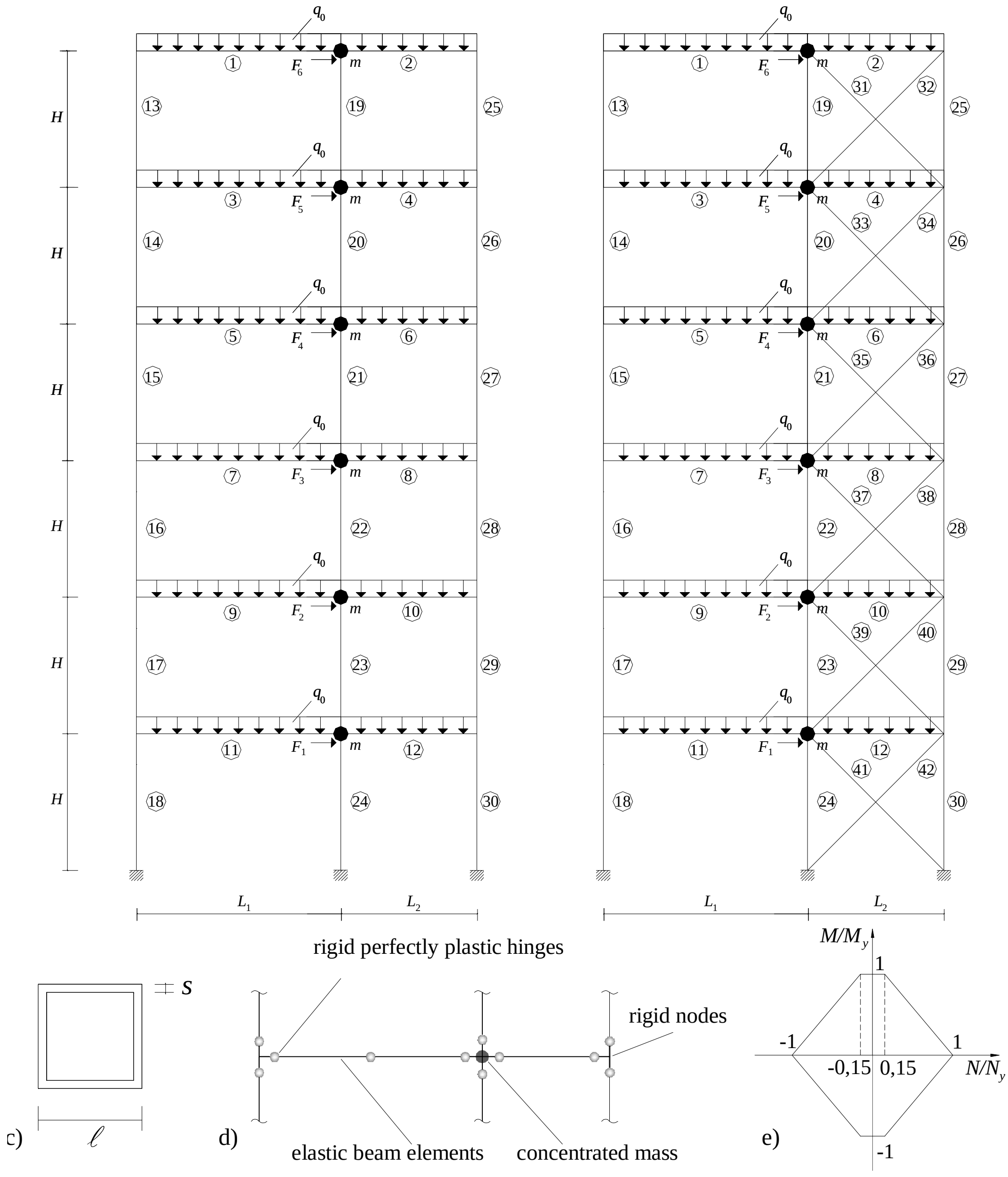

Fig. 1 Steel frames: a) flexural frame geometry and load condition; b) cross-braced frame geometry and load condition; c) typical box cross section; d) structural scheme; e) rigid plastic domain of the typical plastic hinge.

Two rigid perfectly plastic hinges are located at the extremes of all elements, considered to be purely elastic (Fig. 1d), and an additional hinge is located in the middle point of the beams. The interaction between bending moment *M* and axial force *N* has been taken into account. In Fig. 1e the dimensionless rigid plastic domain of the typical plastic hinge is plotted in the plane (N/N_y , M/M_y), being N_y and M_y the yield generalized stress corresponding to N and M , respectively. The structure is subjected to a fixed uniformly distributed vertical load on the beams, $q_0 = 30\ \mathrm{kN/m}$, to perfect cyclic concentrated horizontal loads (kN) applied on the nodes (wind effect) $\tilde{\boldsymbol{F}}_w = |24 \quad 26.2 \quad 28.4 \quad 30.5 \quad 32.7 \quad 34.9|$, and to seismic actions. We assume that the seismic masses are equal for each floor, $m = 33.64\ \mathrm{kN \cdot sec^2/m}$, and located in the intermediate node at each floor, (Fig. 1a,b). The selected response spectra for serviceability conditions (up-crossing probability in the lifetime 81%) and instantaneous collapse (up-crossing probability in the lifetime 5%) are those corresponding to Palermo, with a soil type B, life time 100 years and class IV. The optimal multicriterion design has been computed solving problem (6), assuming $F_{0wj}/F_{0j} = 1.25$, with F_{0wj} and F_{0j} the j^{th} components of the relevant vectors. The obtained designs have been investigated. The relevant Bree diagrams have been determined for seismic and wind load conditions and plotted in Fig. 2a,e and 2c,g, respectively. As it is easy to observe, as known, a dangerous condition of ratchetting is reached even for load multipliers lower than the prescribed ones. Furthermore, in order to analyze the structural response with regard to instability, the same structure has been studied taking into account the P-Delta effects and the buckling effects performing an elasto-plastic analysis with a selected load history. It has been verified that the structure shows incremental collapse for load multipliers lower and lower. In Table 1 such results are synthesized for suitably chosen couples of load multipliers where F and C indicate Flexural and Cross braced frame. It is worth noticing that the effect of buckling in the case of the cross braced frame is so influent that for load amplifier even very low (and, as a consequence, meaningless for technical aims) the structure shows a very fast collapse (these last values are not reported in the cited Table).

Frame		Analysis	Vol.	$\xi_0;\xi_c$	u^r	w_1^r	w_2^r	w_3^r	w_4^r	w_5^r
F	earth.	standard	1.007	1;0.95	11.2	28.1	32.9	34.7	34.2	34.4
	wind			1.25;1	9.88	11.1	13.7	12.3	8.36	3.1
F	earth.	P-Delta	1.007	1;0.85	26.0	15,8	23,4	24,9	23.0	18.6
	wind			1.25;1	9.62	12.5	15.8	14.3	9.95	4.12
F	earth.	buckling	1.007	1;0.70	35.5	27.7	36.3	37.9	40.9	35.2
	wind			1.25;1	37.6	31.0	34.6	32.2	26.6	17.3
C	earth.	standard	0.907	1;0.85	7.73	8.58	16.1	20.0	173	327
	wind			1.25;1	0.004	166	246	313	360	387
C	earth.	P-Delta	0.907	1;0.60	6.57	8.39	15.8	20.0	8.17	216
	wind			1.25;0.5	0.129	655	528	202	255	267

Table 1. Results of the analyses performed for optimal flexural and cross braced frames deduced by solving problem (6). (u^r = horizontal residual displacement of the upper floor, w_i^r , $i = 1,2,..,5$, are the vertical residual displacements of the middle point of the longer beams at floor 1 to 5).

In order to improve the obtained design, problem (6) has been modified substituting constraints (6i-k) with equations (7i-k) and introducing constraint (9).

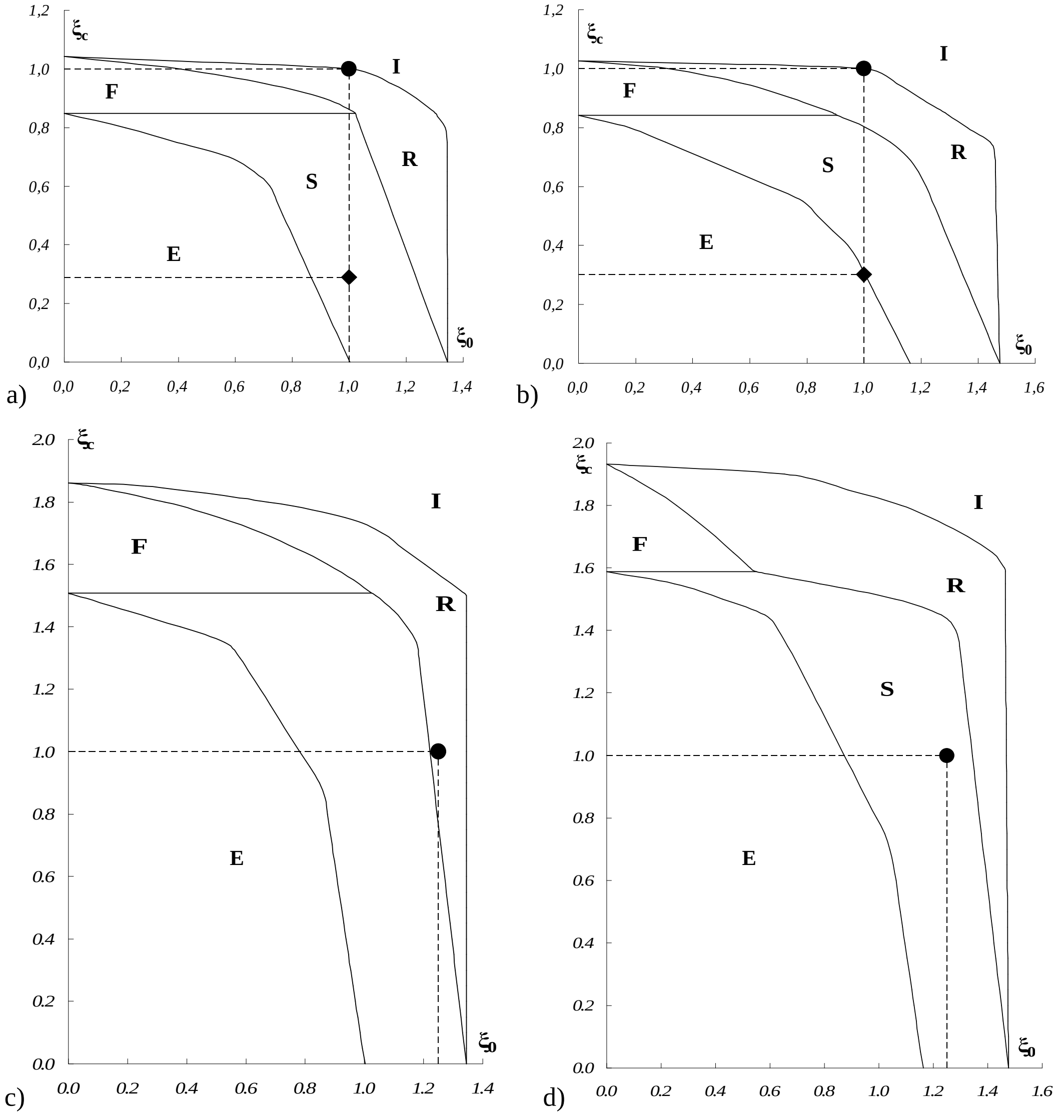

Fig. 2 Flexural frame Bree diagrams: a) optimal structure problem (6), seismic loads; b) optimal structure improved problem, seismic loads; c) optimal structure problem (6), wind actions; d) optimal structure improved problem, wind actions.

The Bree diagrams of the improved design have been determined and plotted in Fig. 2b,f and Fig. 2d,h for seismic and wind load conditions, respectively. In order to verify the goodness of the obtained design, the analysis of the relevant structure has been performed taking into account the P-Delta effect. The results are encouraging and show that the structure prevents the collapse for couples of multipliers even very close to the prescribed ones. In Table 2 same results are summarized in terms of residual displacements.

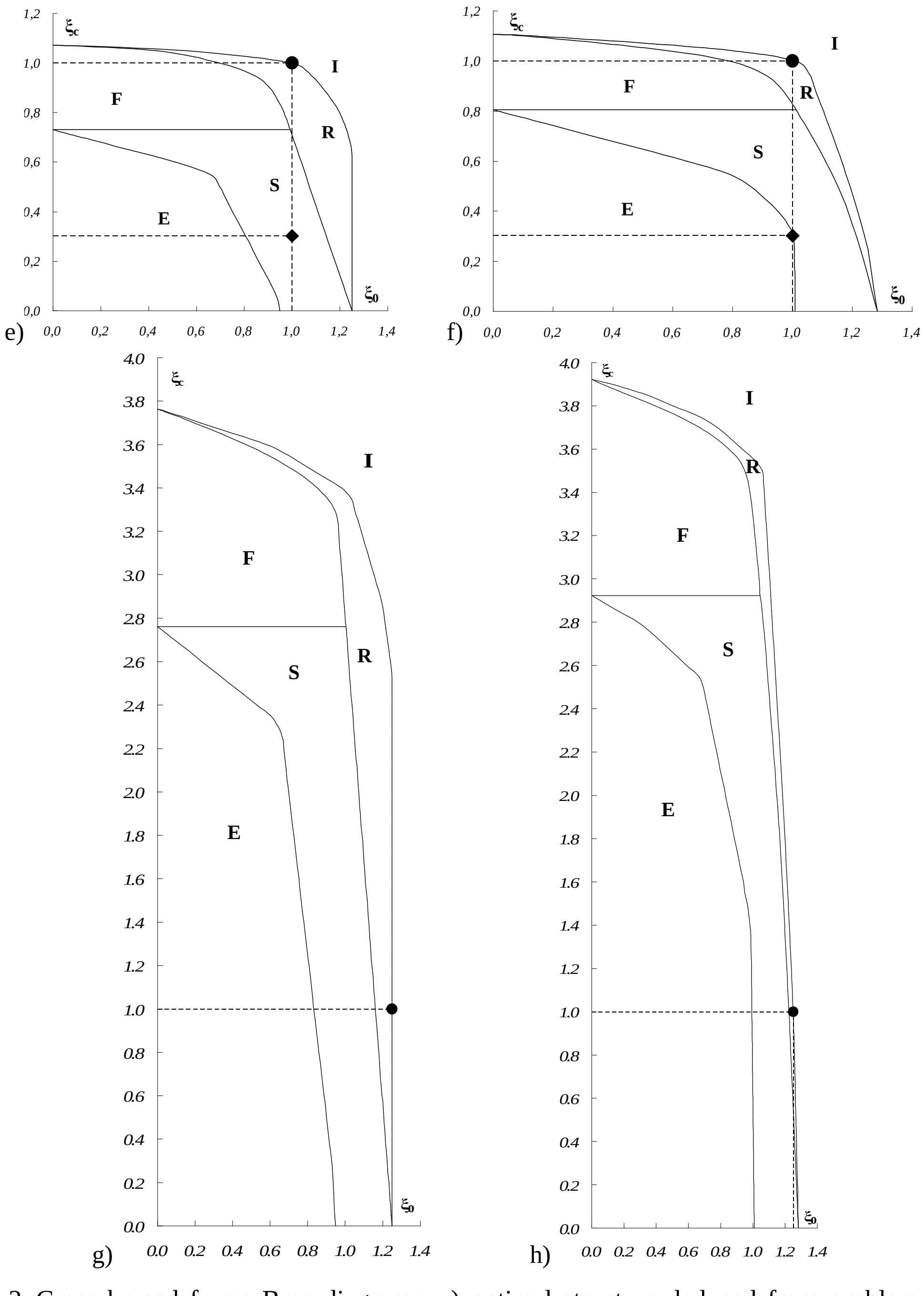

Fig. 2 Cross braced frame Bree diagrams: a) optimal structure deduced from problem (6), fixed and seismic loads; b) optimal structure deduced from improved problem, fixed and seismic loads; c) optimal structure deduced from problem (6), fixed and wind actions; d) optimal structure deduced from improved problem, fixed and wind actions.

Frame		Analysis	Vol.	$\xi_0;\xi_c$	u^r	w_1^r	w_2^r	w_3^r	w_4^r	w_5^r
F	earth.	P-Delta	1.187	1;0.96	51.6	12.3	29.2	32.5	31.8	140
	wind			1.25;1	6.79	0.12	10.5	12.1	9.29	8.05
C	earth.	P-Delta	1.131	1;0.95	34.2	2.81	13.3	17.9	20.8	19.7
	wind			1.25;1	0.124	6.37	13.2	12.7	11.6	7.71

Table 2. Results of the analyses performed for optimal flexural and cross braced frames deduced by solving the improved problem.

5 CONCLUSIONS

The present paper has been devoted to the optimal design of elastic perfectly plastic frames subjected to fixed, perfectly cyclic and dynamic actions. The optimal design problem has been formulated as the search for the minimum volume structure and two different resistance limits have been simultaneously considered: the elastic shakedown limit and the instantaneous collapse limit. In the proposed formulation reference has been made to the Italian codes related to the structural analysis and design; actually, the serviceability conditions have been defined as the combination of fixed and reduced seismic loads, the ultimate limit loads have been defined alternatively as the combination of fixed and perfect cyclic loads, or as the combination of fixed and dynamic loads. Two different formulations of the minimum volume design have been proposed: the first one is devoted to the optimal design of the structure with constraints on the elastic shakedown behaviour related to serviceability condition loads and on the instantaneous collapse related to suitably alternative combinations of fixed and perfectly cyclic or dynamic actions, the second one is devoted to the optimal design with the same conditions as before but introducing new constraints related to buckling. With the introduced further constraints it has been verified that the relevant structure exhibits a behaviour preventing the collapse even when the loads reach values very close to the prescribed ones. Two six plane frames have been investigated. The obtained results are encouraging and furthermore they show that the new designs are characterized by just a very modest cost increment with respect to the safety improvement.

References

[1] Cinquini C.; Sacchi G. A note on the optimal elastic design for given deflection. In Optimization of Distributed Parameter Structures, Hang, E.J. and Ca,J. Ed. 1, 383-398, 1981.

[2] König, J.A., "On optimum shakedown design", in *Optimization in structure design*, Edit by A.Sawczuk and Z. Mroz, Springer-Verlag, Berlin, Germany, 405-414, (1975).

[3] Save, M and Prager, W., *Structural Optimization*, Plenum Press, New York, U.S. (1985).

[4] Palizzolo L. Optimization of continuous elastic perfectly plastic beams. Computers & Structures, 82, 4-5, 397-411, 2004a.

[5] Palizzolo L. Optimal design of trusses according to a plastic shakedown criterion. Journal of Applied Mechanics, ASME, 71, 2, 240-246, 2004b.

[6] Repubblica Italiana, Consiglio Superiore dei Lavori Pubblici, Norme Tecniche per le Costruzioni, D.M. 14 gennaio 2008.

[7] S. Benfratello, F. Giambanco, L. Palizzolo: P-Delta effect in structural optimization, 2nd Int. Conf. on Eng. Opt., Lisbona - Portogallo, 6-9 settembre 2010.

[8] Gajewski A.; Zyezkowski, M. Optimal structural design under stability constraints. Kluwer, Dordrecht, 1988.