Learning for allocations in the long-run average core of dynamical cooperative TU Games

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***Abstract*—We consider repeated coalitional TU games characterized by unknown but bounded and time-varying coalitions' values. We build upon the assumption that the Game Designer uses a vague measure of the extra reward that each coalition has received up to the current time to learn on how to re-adjust the allocations among the players. As main result, we present an allocation rule based on the extra reward variable that converges with probability one to the core of the long-run average game. Analogies with stochastic stability theory are put in evidence.**

I. Introduction

This paper is in spirit with a few other recent attempts by the same authors to bring robustness and dynamics within the picture of coalitional TU games [3], [4], [5]. While in [4], [5] we dealt with the robust stabilizability of the excesses, here we are more concerned with their convergence in probability which then translates into the long-run convergence in probability of the average allocation to the core of the average game. Conversely, the main difference with respect to [3] is that we now require convergence to a specific point in the core whereas there we analized convergence to the core, and whichever was the converging point was not really a point of interest.

In particular, we deal with learning in *robust* and *dynamical* coalitional TU games. Learning together with the above two elements naturally arise in all the situations where the coalitions values are uncertain and time-varying.

The issue of "robustness" is also addressed in some literature on stochastic coalitional games [15], [16]. However, we deviate from this stochastic framework since we model coalitions values as Unknown But Bounded variables within a priori known polytopic sets [6]. A similar approach can be found in the recent literature on interval valued games [1], where the authors use intervals to describe coalitions values similarly to what is done in this paper. A new element with respect to [1] and the references therein is the time-varying nature of the coalitions values. We have found inspiration for our approach in Set invariance Theory [7] which provides us some "nice" tools for stability analysis (see, e.g., the resort to a Lyapunov function in the proof of Theorem IV.1).

Issues related to the *dynamical* nature of the coalitions' values enter into the picture in the form of a differential equation involving the system state. Under the assumption that the game is played repeatedly and continuously over time, the state accounts for the accumulated discrepancy between coalitions' values and allocations up to the current time. At each time, different coalitions' values realize which are undisclosed to the Game Designer (GD) who then adjusts the allocations based on information on the system state. Bringing dynamical aspects into the framework of coalitional TU games is an element in common with other papers [9], [11]. The main difference with those works is that the values of coalitions are set exogenously and no relation exists between consecutive samples.

The main contribution of this paper is a constructive way to design an allocation rule converging to a specific point in the average. Convergence conditions together with the idea that allocation rules use a measure of the extra reward that a coalition has received up to the current time by re-distributing the budget among the players are a main issue in a number of other papers [2], [8], [10], [13], [14] as well. However, this paper departs from the aforementioned contributions mainly in that dynamics is there captured by a bargaining mechanism with fixed coalitions' values while we let the values be time-varying and uncertain. This last element adds some robustness in our allocation rule which have not been dealt with before.

This paper is organized as follows. In Section II, we formulate the problem. In Section III, we present the basic idea of our solution approach. In Section IV, we state the main result of this work. In Section V, we provide some numerical illustrations. Finally, in Section VI, we draw some concluding remarks.

II. Problem formulation

Consider a set of players $N=\{1,\dots,n\}$ and all possible *coalitions* $S\subseteq N$ arising among these players. Introduce a *time-varying characteristic function* $\psi(S,t)$ which assigns a real value to each coalition S at time $t\geq 0$:

$$\psi : 2^N\setminus\{\emptyset\}\times\mathbb{R}_+\to\mathbb{R}.$$

If we denote by $m=2^n-1$ the number of possible coalitions, we can view the characteristic function $\psi(.,t)$ as returning a continuous-time signal in the m-dimensional space:

$$v(t)\in\mathbb{R}^m,\quad \forall t\geq 0.$$

Turning from a function to a signal is useful to define the following dynamical coalitional games.

Definition II.1 ***(dynamical TU game)*** *For each time $t\geq 0$, the* instantaneous, integral, *and* average dynamical game *is defined by the pairwise*

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- ***(instantaneous game)*** $< N, v(t) >$, *with* $v(t) \in \mathbb{R}^m$;
- ***(integral game)*** $< N, \tilde{v}(t) >$, *with* $\tilde{v}(t) = \int_0^t v(\tau)d\tau$;
- ***(average game)*** $< N, \overline{v}(t) >$, *with* $\overline{v}(t) = \frac{\tilde{v}(t)}{t}$.

Henceforth, we use the symbol $\tilde{\psi}(t)$ and $\overline{\psi}(t)$ to indicate the integral and average up to time t respectively of any given function $\psi(t)$. Also, the underlying assumption throughout this paper is that $v(t)$ is unknown to the GD but confined within a convex set at any time. We also assume that $v(t)$ is a mean ergodic stochastic process.

Assumption 1 ***(UBB and mean ergodic)*** *Signal $v(t)$ is UBB within a given convex set $\mathscr{V}$: $v(t) \in \mathscr{V} \in \mathbb{R}^m$. Furthermore, the expected value of $v(t)$ coincides with the long term average, i.e., $E[v(t)] = lim_{t\to\infty} \overline{v}(t)$.*

Under the above assumption, the core of the instantaneous game can be empty at some time t. Even if the above is true, we can still suppose that the core of the average game is non empty on the long run.

Assumption 2 ***(balancedness)*** *The core of the average game is non empty in the limit:* $\lim_{t\to\infty} C(\tilde{v}(t)) \neq \emptyset$.

We can view the above assumption as introducing some steady-state (average) conditions on a game scenario subject to instantaneous fluctuations.

Now, assume that the GD can take actions in terms of instantaneous allocations denoted by $a(t) \in \mathbb{R}^n$ and suppose the following budget constraints.

Assumption 3 ***(bounded allocation)*** *The instantaneous allocation is bounded within a hyperbox in $\mathbb{R}^n$*

$$a(t) \in \mathscr{A} := \{a \in \mathbb{R}^n : a_{min} \leq a \leq a_{max}\},$$

with apriori given lower and upper bounds a_{min}, $a_{max} \in \mathbb{R}^n$.

Let us turn to comment on the information structure of the problem. To do this, we need to introduce some new terminology which is useful to clarify the information available to the GD.

For any coalition $S \subseteq N$, we define *excess (extra reward) at time* $t \geq 0$ as the difference between the total integral reward, given to it, and the integral value of the coalition itself, i.e.,

$$\varepsilon_S(t) = \sum_{i\in S} \tilde{a}_i(t) - \tilde{v}_S(t).$$

Furthermore, we say that S *is in excess at time* $t \geq 0$ if the excess is non negative, i.e., $\sum_{i\in S} \tilde{a}_i(t) \geq \tilde{v}_S(t)$. In one word, coalitions in excess are those with respect to which the grand coalition of the integral game is stable. With the above clarification in mind, we henceforth assume that the GD has access to the limit of the average coalitions' values, call them *nominal coalitions' values*, stored in the vector $v_{nom} := \lim_{t\to\infty} \overline{v}(t)$. For future purposes, let us introduce an opportune deviation $\Delta v(t)$ so that we can express $v(t) = v_{nom} + \Delta v(t)$. Furthermore, the GD also observes the *vector of coalitions' excess* $\varepsilon(t) := [\varepsilon_S(t)]_{S\subseteq N} \in \mathbb{R}^m$.

	$\mathbf{v}_1$	$\mathbf{v}_2$	$\mathbf{v}_3$	$\mathbf{v}_4$	$\mathbf{v}_5$	$\mathbf{v}_6$	$\mathbf{v}_7$
S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$

TABLE I
CORRESPONDENCES VERTICES-COALITIONS

Assumption 4 ***(partial information)*** *The GD knows v_{nom} and $\varepsilon(t)$ at each time $t \geq 0$. Furthermore, signal $v(t)$ and excess $\varepsilon(t)$ are non correlated.*

In [3] we have solved the problem of finding an allocation rule based on the available partial information so that if the instantaneous allocation is selected from this rule then the average allocation converges to the core of the average game on the long run. The problem in the present paper is slightly different. Indeed, we first select apriori a specific point in the core of the average game, say it *nominal allocation* and denote it

$$a_{nom} \in C(v_{nom}).$$

Hence we look for an allocation strategy that converges exactly to the nominal allocation in the average.

Problem 1 *Find an allocation rule $f : \mathbb{R}^m \to \mathscr{A} \in \mathbb{R}^n$, such that if $a(t) = f(\varepsilon(t))$ then $\lim_{t\to\infty} \overline{a}(t) = a_{nom}$.*

III. Flow transformation

The basic idea of our solution approach is to turn the problem into a flow control one. To do this, consider the hypergraph $\mathscr{H}$ with vertex set V and edgeset E as:

$$\mathscr{H} := \{V, E\}, \quad V = \{\mathbf{v}_1, \ldots, \mathbf{v}_m\}, \quad E := \{\mathbf{e}_1, \ldots, \mathbf{e}_n\}.$$

The vertex set V has one vertex per each coalition whereas the edge set E has one edge per each player.

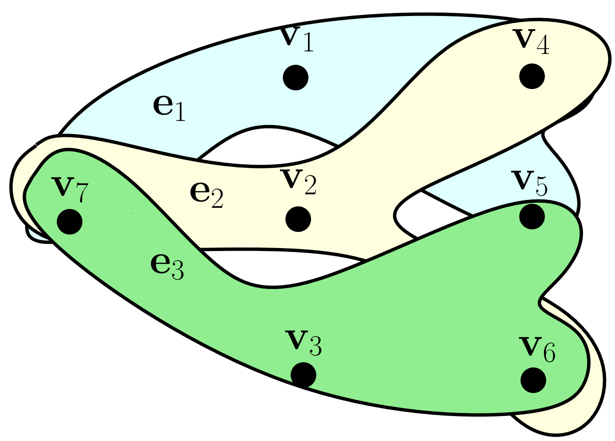

Fig. 1. Hypergraph $\mathscr{H} := \{V, E\}$ for a 3-player coalitional game.

A generic edge i is incident to a vertex $\mathbf{v}_j$ if the player i is in the coalition associated to $\mathbf{v}_j$. So, incidence relations are described by matrix $B_{\mathscr{H}} = [c_S^T]_{S\subseteq N} \in \mathbb{R}^{m\times n}$ whose rows are the characteristic vectors of all coalitions $S \subseteq N$.

The flow control reformulation arises naturally if we view allocation $a_i(t)$ as the flow on edge $\mathbf{e}_i$ and the coalition value

$v_S(t)$ of a generic coalition S as the demand $d_j(t)$ in the corresponding vertex $\mathbf{v}_j$, namely $v_S(t) = d_j(t)$. At this point we can introduce a d_{nom} and $\Delta d(t)$ so that, in the future, we can again use the expression $d(t) = d_{nom} + \Delta d(t)$ in analogy with $v(t) = v_{nom} + \Delta v(t)$.

In view of this, allocation in the core translates into satisfying in excess the demand at the vertices. Specifically,

$$\tilde{a}(t) \in C(\tilde{v}(t)) \quad \Leftrightarrow \quad B_{\mathscr{H}} \tilde{a}(t) \geq \tilde{d}(t) \tag{1}$$

Now, since $\tilde{d}(t)$ is unknown at time t, we need to introduce some error dynamics which accounts for the derivatives of excesses:

$$\dot{\varepsilon}(t) = B_{\mathscr{H}} a(t) - d(t), \quad d(t) \in \mathscr{V}.$$

With the above in mind, the problem can be turned into a flow control problem where a controller wishes to drive the error $\varepsilon(t)$ (the excesses) to a target set

$$\mathscr{T} := \{\varepsilon \in \mathbb{R}^m : \varepsilon_m = 0, \varepsilon_j \geq 0, \forall j = 1, \ldots, m-1\}.$$

But we can do more than this to simplify the tractability of

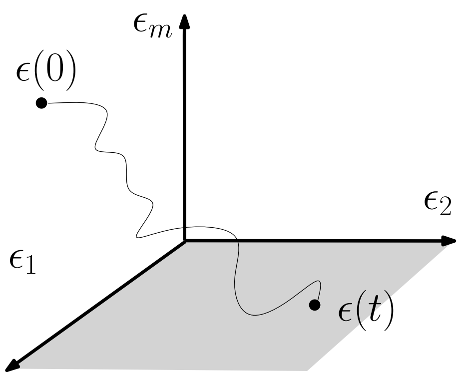

Fig. 2. Trajectory for $\varepsilon(t)$.

the problem. Using standard LP techniques we can introduce $m-1$ surplus variables (one per each coalition other than the grand coalition) so to project the allocation space into a one of higher dimension. In particular, let us expand control $u(t) = [a(t)^T | s(t)^T]^T \in \mathbb{R}^{(n+m-1)}$. This technique has the advantage of turning the inequalities (1) into equalities of type (see, e.g., [4], [5]):

$$\dot{x}(t) = Bu(t) - d(t), \quad d(t) \in \mathscr{V},$$

where matrix B is defined as

$$B = \left[\begin{array}{c|c} B_{\mathscr{H}} & \begin{array}{c} -I \\ 0 \end{array} \end{array} \right] \in \mathbb{R}^{m \times (n+m-1)}$$

and I is an identity matrix of compatible dimensions. Variable $x(t) \in \mathbb{R}^m$ is now the *state* of the system. In order to rephrase the original problem in terms of a flow control problem, we still have to introduce the feasible controls set

$$U := \{u \in \mathbb{R}^{n+m-1} : a \in \mathscr{A}, s \geq 0\} \in \mathbb{R}^{n+m-1} \tag{2}$$

and define a new vector $u_{nom} = [a_{nom}^T | s_{nom}^T]^T \in U$ satisfying

$$Bu_{nom} = v_{nom}.$$

We hereafter abbreviate "with probability one" and write "w.p.1".

We are now ready to restate the original problem as follows. Find a control strategy $\phi : \mathbb{R}^m \to U$ which drives the state $x(t)$ to zero in probability and returns the desired average control u_{nom}:

$$u(t) := \phi(x(t)) \in U \Leftrightarrow \lim_{t \to \infty} x(t) = 0 \text{ w.p.1, and } \lim_{t \to \infty} u(t) = u_{nom}.$$

Stating differently, we require the dynamics $\dot{x}(t) = B\phi(x(t)) - d(t)$ to be stochastically stable with $\phi(x(t))$ satisfying certain average constraints.

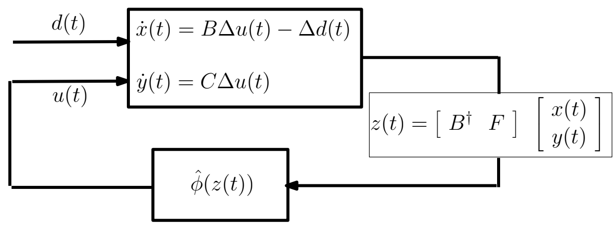

Fig. 3. Dynamical system.

IV. Stochastic stability and average constraints

In this section, we state the main result of this work which proposes a solution to Problem 1. Before stating the result we need to modify the dynamics (8) in the way explained next. First denote by $B^\dagger$ a generic pseudo inverse matrix of B and complete matrices B and $B^\dagger$ with matrices C and F such that

$$\begin{bmatrix} B \\ C \end{bmatrix} \begin{bmatrix} B^\dagger & F \end{bmatrix} = I. \tag{3}$$

Then, building upon the new square matrix $\begin{bmatrix} B \\ C \end{bmatrix}$, let us move to consider the augmented system

$$\begin{array}{rcl} \dot{x}(t) & = & B\Delta u(t) - \Delta d(t) \\ \dot{y}(t) & = & C\Delta u(t). \end{array} \tag{4}$$

After integrating the above system (see (5), right) we come up with a new variable $z(t)$ (see (5), left), that plays a central role for the problem at hand:

$$z(t) = \begin{bmatrix} B^\dagger & F \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \quad \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} B \\ C \end{bmatrix} z(t). \tag{5}$$

Indeed, it turns out that to drive $x(t)$ to zero w.p.1, and obtain u_{nom} as average allocation on the long run, we can rely on a simple function $\hat{\phi}(.)$, which depends on $z(t)$. Before introducing this function, for future purposes observe that the dynamics for $z(t)$ satisfies the first-order differential equation:

$$\begin{array}{rcl} \dot{z}(t) & = & \begin{bmatrix} B^\dagger & F \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \end{bmatrix} \\ & = & \begin{bmatrix} B^\dagger & F \end{bmatrix} \begin{bmatrix} B \\ C \end{bmatrix} \Delta u(t) - \begin{bmatrix} B^\dagger & F \end{bmatrix} \begin{bmatrix} \Delta d(t) \\ 0 \end{bmatrix} \\ & = & \Delta u(t) - B^\dagger \Delta d(t). \end{array} \tag{6}$$

Back to the function $\hat{\phi}(z(t))$, let Δu^{min} and Δu^{max} be the minimal and maximal values of $\Delta u(t)$ for the following constraints to hold true: $u(t) = u_{nom} + \Delta u(t) \in U$. Then, let us formally define $\hat{\phi}(z(t))$ as:

$$\hat{\phi}(z(t)) := u_{nom} + \Delta u(t) \in U, \quad \Delta u(t) = sat_{[\Delta u^{min}, \Delta u^{max}]}(-z(t)), \tag{7}$$

where with $sat_{[a,b]}(\xi)$ we denote the saturated function that, given a generic vector ξ and lower and upper bounds a and b of same dimensions as ξ, returns

$$sat_{[a,b]}(\xi) \doteq \begin{cases} b_i & \text{for all } i \quad \xi_i > b_i \\ a_i & \text{for all } i \quad \xi_i < a_i \\ \xi_i & \text{for all } i \quad a_i \leq \xi_i \leq b_i \end{cases}.$$

Now, taking the control $u(t) = \hat{\phi}(z(t))$, we obtain the dynamic system $\dot{x}(t) = B\hat{\phi}(z(t)) - d(t)$ as displayed in Fig. 3. With the above preamble in mind, we are ready to state the following convergence property.

Theorem IV.1 *The dynamic system (8) with $\hat{\phi}(z(t))$ as in (7) converges to zero w.p.1 and satisfies $\lim_{t\to\infty} \overline{u}(t) = u_{nom}$:*

$$\dot{x}(t) = B\hat{\phi}(z(t)) - d(t). \tag{8}$$

Proof: Consider a candidate Lyapunov function $V(z(t)) = \frac{1}{2} z^T(t) z(t)$. The idea is to inspect that $E[\dot{V}(z(t))] < 0$ for all $t \geq 0$. To see that this last is true, observe that from (6) we have

$$\begin{aligned} E[\dot{V}(z(t))] &= E[z^T(t)\dot{z}(t)] \\ &= E[z^T(t)\Delta u(t)] - E[z^T(t) B^\dagger \Delta d(t)] \\ &= E[z^T(t) sat(-z(t))] < 0, \end{aligned}$$

where condition $E[z^T(t) B^\dagger \Delta d(t)] = 0$ is a direct consequence of Assumption 4 which translates into $\Delta d(t)$ being uncorrelated with $z^T(t)$. But the above condition implies that $\lim_{t\to\infty} z(t) = 0$ w.p.1, which, from (5)-left, and $[B^\dagger F]$ being non singular and square, leads to $\lim_{t\to\infty} x(t) = 0$ w.p.1 as well.So far we have proved the first part of the statement, i.e., that the dynamic system (8) converges to zero w.p.1. For the second part, after integrating dynamics (6), we have

$$\lim_{t\to\infty} \frac{\int_0^t [\Delta u(\tau) - B^\dagger \Delta d(\tau)] d\tau}{t} = \lim_{t\to\infty} \frac{z(t) - z(0)}{t} = 0.$$

This last condition together with the assumption $v_{nom} := \lim_{t\to\infty} \overline{v}(t)$ yield

$$\lim_{t\to\infty} \frac{\int_0^t B^\dagger \Delta d(\tau) d\tau}{t} = \lim_{t\to\infty} \frac{\int_0^t \Delta u(\tau) d\tau}{t} = 0$$

from which we can conclude $\lim_{t\to\infty} \overline{u}(t) = \lim_{t\to\infty} \frac{\int_0^t u_{nom} + \Delta u(\tau) d\tau}{t} = u_{nom}$ as claimed in the statement. ■

In the next corollary, we use the previous result to provide an answer to Problem 1.

Corollary IV.1 *The average allocation converges to the nominal allocation:*

$$\lim_{t\to\infty} \overline{a}(t) = a_{nom}.$$

Proof: This is a direct consequence of the result proved in the previous theorem: $\lim_{t\to\infty} \overline{u}(t) = u_{nom}$. ■

V. Numerical illustrations

Consider a 3 player coalitional TU game, so $m = 7$, with the following intervals for values of coalitions:

$$\begin{aligned} &v(\{1\}) \in [0,4],\ v(\{2\}) \in [0,4],\ v(\{3\}) \in [0,4], \\ &v(\{1,2\}) \in [0,4],\ v(\{1,3\}) \in [0,6], \\ &v(\{2,3\}) \in [0,7],\ v(\{1,2,3\}) \in [0,12]. \end{aligned}$$

The convex set $\mathscr{V}$ is then a hyperbox characterized by the above intervals. >From Assumption 4, the GD knows the long run average game, i.e., $\lim_{t\to\infty} \overline{v}(t) = v_{nom}$. Without loss of generality we take the balanced nominal game be as $v_{nom} = [1\ 2\ 3\ 4\ 5\ 6\ 10]^T$. In other words, during the simulations we randomize the instantaneous games $v(t) \in \mathscr{V}$ so that it satisfies the average behavior given by:

$$\lim_{t\to\infty} \frac{1}{t} \int_0^t v(\tau) d\tau = v_{nom}. \tag{9}$$

Next, we describe an algorithm to generate $v(t) \in \mathscr{V}$ such that the above condition holds true.

Algorithm:

1) Generate m random points, $r_i \in \mathscr{V} \subset \mathbb{R}^m,\ i = 1,2,\cdots,m$.
2) Solve $R.p = v_{nom}$, with $R = [r_1,\ r_2,\cdots\ r_m]$.
3) If $p \geq 0$ and $\mathbf{1}^T p > 0$, then go to **(4)** else go to **(1)**.
4) Rescale R as $R = (\mathbf{1}^T p) R$ and p as $p = \frac{p}{(\mathbf{1}^T p)}$
5) If $r_i \in \mathscr{V},\ i = 1,2,\cdots,m$, then go to **(6)** else go to **(1)**.
6) STOP

By construction of the algorithm, v_{nom} is in the relative interior of the convex hull generated by the columns of the matrix R. If an instance of the game $v(t)$ is chosen as r_i with probability p_i from the pair (R,p), Assumption 4 is satisfied. For simulations we ran the algorithm 20 times to generate a total of 140 points (or 20 (R,p) pairs) in $\mathscr{V}$. Further, from each of the 20 pairs we take 2000 random selections (using Matlab `randsrc` function), which amounts to $40,000$ instantaneous games $v(t)$. The nominal choice of allocations and surplus is taken as $u_{\text{nom}} = [2.5\ 3\ 4.5\ 1.5\ 1\ 1.5\ 1.5\ 2\ 1.5]^T$. It can be verified that $Bu_{\text{nom}} = v_{\text{nom}}$.

For simulations, the saturation thresholds Δu^{min} and Δu^{max} are chosen so as to ensure $u(t) \in U$. This condition translates into $U_{\min} \leq u_{\text{nom}} + sat_{[\Delta u^{min},\ \Delta u^{max}]} \leq U_{\max}$. Denote $\mathbf{1}$ as a vector with all entries equal to 1. For the instantaneous game a negative allocation/surplus is not allowed, so $U_{min} \geq 0 \cdot \mathbf{1}$. Further, an allocation/surplus greater than the value of grand coalition is not allowed, so $U_{max} \leq v_{nom}(N) \cdot \mathbf{1}$. For the given game parameters, we see that the lower and upper thresholds for the saturation function are -1 and 5.5 respectively. The robust allocation rule is implemented numerically with a step size of $\Delta = 0.01$. Next, we present some significant performance results of the robust control law given by equation (7). >From Theorem IV.1, $\lim_{t\to\infty} \overline{z}(t)$ converges to zero with the aforementioned specific choice of the control law. Fig.

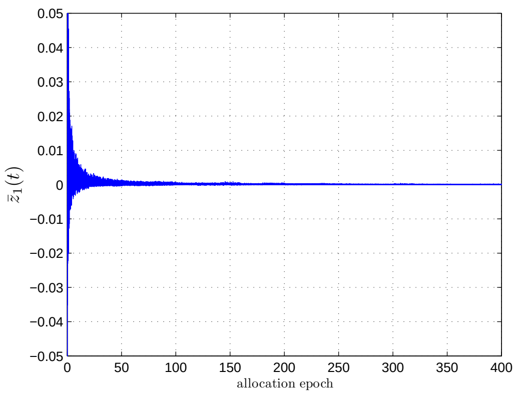

Fig. 4. Performance of the robust control law $\hat{\phi}(z(t))$: time plot of first component of $\bar{z}(t)$.

4 illustrates this behavior for the first component of $z(t)$. Further, by Corollary IV.1, the same control law ensures that the average game is balanced in the long run, in other words $\lim_{t\to\infty} \bar{x}(t) = 0$. The control law ensures that $E[\dot{V}(z(t))]$ is

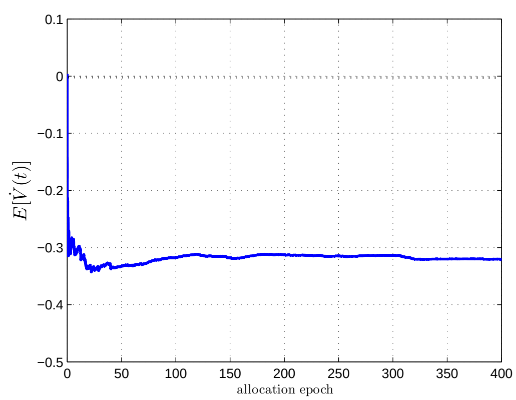

Fig. 5. Time plot of $E[\dot{V}(x(t))]$

negative for all $t > 0$; we illustrate this behavior in Fig. 5. From Corollary IV.1, we also know that the average allocation vector converges to the nominal allocation vector. We illustrate this fact in Fig. 6. Here, we notice that convergence occurs with an approximation error of about 10^{-2}. This reflects the fact that, in generating the instantaneous games $v(t)$ through the above algorithm, the average value of $B^{\dagger}\Delta d(t)$ converges to zero with, more or less, the same error as evident from looking at Fig. 7. We can interpret the average value of $\Delta d(t)$ as a measure of the uncertainty in learning the nominal game v_{nom}, as given by (9). So, we believe an improvement in the numerical precision while generating the instantaneous games $v(t)$ will result in the exact convergence of the allocations.

VI. Conclusions and Future Work

In this paper we derived a robust control law that ensures that the average allocation vector converges to the nominal

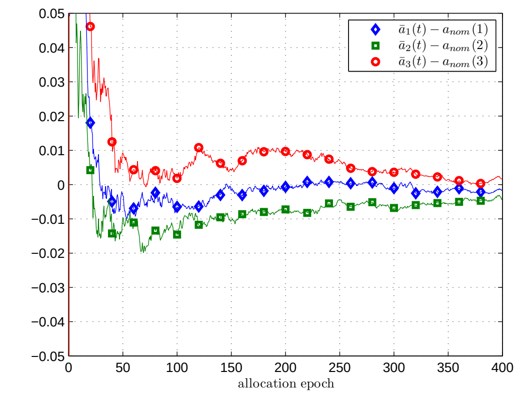

Fig. 6. Time plot of $\bar{a}(t) - a_{nom}$.

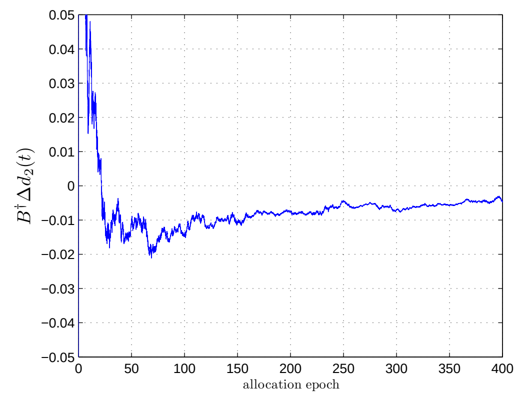

Fig. 7. Time plot of second component of $B^{\dagger}\Delta d(t)$.

allocation vector on the long run. However, the control law is derived on the premise that the GD knows apriori, the nominal allocation vector. If this information is not available the derivation of the control law implicitly involves solving an LP problem. By further relaxing the information requirement, the problem can be treated as a learning process where the GD is trying to learn the nominal game from the instantaneous games. We postpone working in this direction for the future.

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