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Quantifying, characterizing, and controlling information flow in ultracold atomic gases

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We study quantum information flow in a model comprised of a trapped impurity qubit immersed in a Bose-Einstein-condensed reservoir. We demonstrate how information flux between the qubit and the condensate can be manipulated by engineering the ultracold reservoir within experimentally realistic limits. We show that this system undergoes a transition from Markovian to non-Markovian dynamics, which can be controlled by changing key parameters such as the condensate scattering length. In this way, one can realize a quantum simulator of both Markovian and non-Markovian open quantum systems, the latter ones being characterized by a reverse flow of information from the background gas (reservoir) to the impurity (system).

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Introduction. In past decades, high precision control of ultracold atomic gases has allowed the realization of experiments unveiling fundamental phenomena in the physics of many-body quantum systems at low temperatures. Key examples are the observation of Anderson localization [1], the superfluid-Mott insulator transition [2], the creation of Tonks-Girardeau gases [3], and the atom laser [4], just to mention a few.

More recently, hybrid systems composed of quantum dots, single trapped ions, and optical lattices coupled to Bose-Einstein condensates (BECs) have been studied both theoretically and experimentally [5]. These systems are studied in the framework of open quantum systems [6], effectively described as one or more two-level systems (qubits) interacting with a reservoir consisting of the ultracold gas. The possibility of manipulating crucial parameters of the reservoir, such as the scattering length [7], combined with the continuous improvements in quantum control of qubits, highlights the enormous potential of hybrid systems as quantum simulators of both condensed-matter models and open quantum systems.

In this Rapid Communication, we study a qubit system composed of an impurity atom trapped in a double-well potential, interacting with a BEC environment. This model has been shown to describe an effective pure-dephasing model [8]. Our focus is on the dynamics of quantum information between the qubit system and the ultracold reservoir. We show how information flux can be manipulated by experimentally achievable means, such as changing the scattering length, the effective dimension of the background gas, or the trapping geometry of the qubit.

Recently, dynamics of information flow has been an active area of research in the open quantum systems community due to several proposals to link it to the division of quantum processes into Markovian and non-Markovian ones [9–12]. The latter ones have been defined as processes where an open system recovers some previously lost information and therefore temporarily combats the destructive effect of the environment as a sink for quantum properties. Such effects, limiting the performance of all quantum devices, can indeed be seen as loss of information on the system. Therefore, looking at the dynamics of information flow gives us an indication of the time of usability of a given quantum system for quantum information processing and, more generally, for quantum technologies.

Bose-Einstein condensates are often referred to as typical examples of non-Markovian reservoirs; however, a quantitative and qualitative analysis of such a claim does not exist in previous literature. This is partly due to the fact that non-Markovianity measures have been introduced only recently. Here we present the first characterization of non-Markovian effects in the context of ultracold gases. We derive an analytic expression for the non-Markovianity measure of Ref. [9] valid for general pure-dephasing qubit models. We find a crossover between Markovian and non-Markovian dynamics in an experimentally accessible parameter space of the model, and we uncover the physical mechanisms at the root of non-Markovian phenomena induced by the ultracold background gas.

Our findings pave the way to the realization of quantum simulators for non-Markovian open quantum system models with ultracold atomic gases. It is worth mentioning that the first quantum simulator for Markovian open quantum systems has been experimentally realized very recently in the trapped ion context [13,14].

Experiments on quantum simulators of non-Markovian open quantum systems, on the other hand, have not yet been performed and are, in general, more demanding than their Markovian counterpart. Non-Markovian quantum simulators would allow one to tackle crucial fundamental open questions in the theory of non-Markovian open quantum systems, such as the generalization of the Lindblad theorem.

The model. The setup we consider (see Fig. 1) consists of an impurity atom trapped in a deep double-well potential

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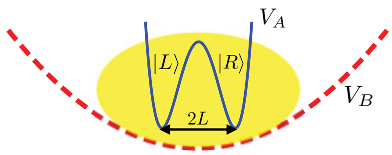

FIG. 1. (Color online) Setup of the impurity-BEC system. The impurity is trapped in a double-well potential V_A (solid) and immersed in a BEC confined by a shallow harmonic potential V_B (dashed). The impurity occupies mostly the two states $|L\rangle$ and $|R\rangle$, in which the atom is localized in the left and right well, respectively. Finally, the distance between the two wells is $2L$.

$V_A(\mathbf{r})$. The impurity atom forms a qubit system with the two qubit states represented by the occupation of the impurity atom in the left or the right well: $|L\rangle$ and $|R\rangle$, respectively. The impurity atom couples to a bosonic background gas B trapped in a shallow potential $V_B(\mathbf{r})$, which forms a Bose-Einstein-condensed environment for the qubit system. The Hamiltonian for this system, derived in Ref. [8], is

$$H=\sum_{\mathbf{k}} E_{\mathbf{k}} c_{\mathbf{k}}^{\dagger} c_{\mathbf{k}}+\sigma_z \sum_{\mathbf{k}}(g_{\mathbf{k}} c_{\mathbf{k}}^{\dagger}+g_{\mathbf{k}}^{*} c_{\mathbf{k}})+\sum_{\mathbf{k}}(\xi_{\mathbf{k}} c_{\mathbf{k}}^{\dagger}+\xi_{\mathbf{k}}^{*} c_{\mathbf{k}}), \tag{1}$$

where $\sigma_z = |R\rangle\langle R| - |L\rangle\langle L|$ and $E_{\mathbf{k}} = \sqrt{\epsilon_{\mathbf{k}}[\epsilon_{\mathbf{k}} + 2g_B^{(D)} n_D]}$ is the energy of kth Bogoliubov mode $c_{\mathbf{k}}$ of the condensate with boson-boson coupling frequency $g_B^{(D)}$ and condensate density n_D. D denotes the effective dimension of the environment. The energy of a free mode is $\epsilon_{\mathbf{k}} = \hbar^2 k^2/(2m_B)$, where $k = |\mathbf{k}|$ and m_B is the mass of a background gas particle. Furthermore, $g_{\mathbf{k}}$ and $\xi_{\mathbf{k}}$ are coupling constants that depend on the spatial form of the states $|L\rangle$ and $|R\rangle$ and on the shape of the Bogoliubov modes. Their specific form is elaborated in Ref. [8]. When the background gas is at zero temperature, the reduced dynamics of the impurity atom is captured by the following time-local master equation (ME):

$$\frac{d\rho(t)}{dt} = \Lambda(t)[\sigma_z,\rho] + \gamma(t)\left[\sigma_z\rho(t)\sigma_z - \frac{1}{2}\{\sigma_z\sigma_z,\rho(t)\}\right]. \tag{2}$$

Quantity $\Lambda(t)$ renormalizes the energy of the qubit but has no qualitative effect on the dissipative dynamics. Instead, in this work we are interested in the decay rate,

$$\gamma(t) = \frac{4g_{AB}^2 n_0}{\hbar}\int \frac{d\mathbf{k}\,\sin^2(\mathbf{k}\cdot\mathbf{L})}{(2\pi)^D}\frac{\sin(E_{\mathbf{k}}t/\hbar)}{\epsilon_{\mathbf{k}} + 2g_B^{(D)} n_D} e^{-k^2\tau^2/2}, \tag{3}$$

where g_{AB} is the impurity-boson coupling frequency, τ is a trap parameter, and $\mathbf{L}$ is half the distance between the two wells of the double-well potential.

We have derived ME (2) using the time-convolutionless projection operator technique to second order in the coupling constant g_{AB} [6]. Remarkably, in this case the second-order ME describes the reduced dynamics exactly [15]. Solving the ME reveals that the impurity atom dephases without exchanging energy with the background gas. More precisely, $\rho_{ii}(t) = \rho_{ii}(0)$ and $\rho_{ij}(t) = e^{-\Gamma(t)}\rho_{ij}(0)$ when $i \neq j$, where $\rho_{ij} = \langle i|\rho|j\rangle$ and $i,j = R,L$. The decoherence function $\Gamma(t) = \int_0^t ds\,\gamma(s)$ coincides with that derived in Ref. [8]; however, here we wish to stress the connection between the decay rate and the non-Markovian features. The authors of Ref. [8] discovered situations when the decoherence function $\Gamma(t)$ is nonmonotonic and conjectured that this is due to non-Markovian effects in the reduced dynamics. Already the form of the ME (2) supports this intuition; the theory of non-Markovian quantum jumps has shown that there is a profound connection between non-Markovian effects and negative regions of the decay rates of Lindblad-structured MEs as the one of Eq. (2) [16]. The full characterization of non-Markovian systems, however, is usually not an easy task. In the following, we derive an analytic expression for the non-Markovianity measure, discover the existence of a Markovian–non-Markovian crossover, and expose the physical mechanisms at the root of this transition in the system dynamics.

Non-Markovianity measure. Breuer, Laine, and Piilo (BLP) have proposed a rigorous definition for non-Markovianity of a quantum channel Φ based on the dynamics of the so-called information flux $\sigma(t) = dD[\rho_1(t),\rho_2(t)]/dt$ [9]. This is the temporal change in the distinguishability $D[\rho_1(t),\rho_2(t)] = \frac{1}{2}||\rho_1(t) - \rho_2(t)||_1$ of two evolving quantum states $\rho_{1,2}(t) = \Phi(t)\rho_{1,2}(0)$ as measured by the trace distance. Negative information flux describes information leaking from the system to its environment and it is associated to Markovian dynamics. Instead, if it is possible to find a pair of states $\rho_{1,2}(0)$ for which the information flux is positive for some interval of time, that is, the system regains some of the previously lost information, then process Φ is considered non-Markovian. The amount of non-Markovianity is defined to be the maximal amount of information that the system may recover from its environment, formally $\mathcal{N}_{\rm BLP} = \max_{\rho_{1,2}}\int_{\sigma>0} ds\,\sigma(s)$.

Generally, calculating $\mathcal{N}_{\rm BLP}$ is difficult because of the optimization over all pairs of initial states. Indeed, an analytic expression for the non-Markovianity measure has been calculated, until now, only for the Jaynes-Cummings model, the driven qubit model, and the depolarizing channel [9,17,18]. For the model studied in this Rapid Communication, we find that $\sigma(t) > 0$ if and only if $\gamma(t) < 0$; that is, the process is non-Markovian precisely when the decay rate can take temporarily negative values. Within experimentally relevant values of the physical parameters, we have discovered at most a single time interval $t \in [a,b]$, when the decay rate is negative and information flows back to the system after an initial period of information loss. Therefore, instead of using the original measure faithfully and quantifying non-Markovianity as the maximal amount of information that the system may recover, we introduce a normalized quantity that reveals the maximal *fraction* of the previously lost information that the system can recover:

$$\mathcal{N} = \max_{\rho_{1,2}}\frac{D[\rho_1(b),\rho_2(b)] - D[\rho_1(a),\rho_2(a)]}{D[\rho_1(0),\rho_2(0)] - D[\rho_1(a),\rho_2(a)]}. \tag{4}$$

Unlike $\mathcal{N}_{\rm BLP}$, the modified quantifier $\mathcal{N}$ is bounded between zero (system only leaks information) and one (system regains all previously lost information) and is therefore more meaningful as a number. We have confirmed numerically that in the relevant case of dephasing noise the above quantity is maximized for the same pair of initial states that maximize $\mathcal{N}_{\rm BLP}$. These are the states whose Bloch vectors lie on the opposite sides of the equator of the Bloch sphere [17]. Using these states in the general expression of Eq. (4), we find the analytic expression of the non-Markovianity measure for a dephasing qubit to be

$$\mathcal{N}_{\rm deph} = \frac{e^{-\Gamma(b)} - e^{-\Gamma(a)}}{e^{-\Gamma(0)} - e^{-\Gamma(a)}}, \quad \Gamma(t) = \int_0^t ds\, \gamma(s). \tag{5}$$

We are now ready to study how changes in the background scattering length and in the dimensionality of the BEC affect the dynamics of information flow.

Three-dimensional BEC. As a first step, we consider a three-dimensional (3D) background BEC with equal confinement of the background gas in all directions. We consider a ^{87}Rb condensate of density $n_3 = n_0 = 10^{20}$ m^{-3} and ^{23}Na impurity atoms trapped in an optical lattice with lattice wavelength $\lambda = 600$ nm and trap parameter $\tau = 45$ nm. The impurity-boson coupling is $g_{AB} = 2\pi\hbar^2 a_{AB}/m_{AB}$, where $m_{AB} = m_A m_B/(m_A + m_B)$ and m_A and m_B are the masses of the impurity atoms and the bosons, respectively, and $a_{AB} = 55\, a_0$, where a_0 is the Bohr radius. Similarly, the boson-boson coupling frequency is $g_B^{3D} = 4\pi\hbar^2 a_B/m_B$, but now we assume that the s-wave scattering length of the background gas can be tuned from its natural value $a_B = a_{\rm Rb} \approx 5.3$ nm via Feshbach resonances. We explore a range of values of a_B consistent with the assumption of dilute gas and with the regime of weakly interacting gases. The latter is a stronger condition, requiring $\sqrt{a_B^3 n_0} \ll 1$. As a consequence, we can tune the scattering length up to a maximum value given by $a_B \approx 3\, a_{\rm Rb}$.

Figure 2 shows the non-Markovianity measure $\mathcal{N}_{\rm deph}$ as a function of a_B for three different values of the well separation L. Increasing L magnifies the fraction of recovered information flow due to the increased ability of the condensate to resolve the qubit system. Similarly, non-Markovian effects are amplified for stronger interaction of the condensate. However, we find that the scattering length alone plays a crucial role in the emergence of non-Markovian reservoir memory effects. When the background gas is free or very weakly interacting, $0 \leqslant a_B \leqslant a_B^{\rm crit} \approx 0.034\, a_{\rm Rb}$, the qubit only leaks information to the BEC environment. Instead, for a strong enough interaction strength of the background gas, $a_B > a_B^{\rm crit}$, the condensate can take on the role of information storage and feed some information back to the qubit. This

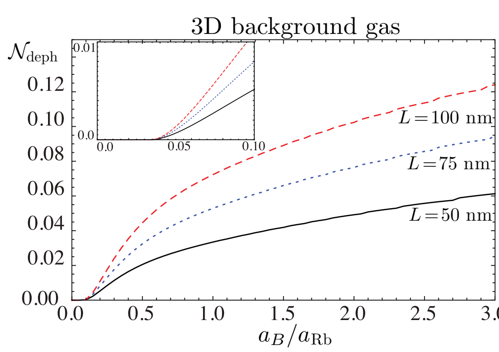

FIG. 2. (Color online) Non-Markovianity measure $\mathcal{N}_{\rm deph}$ as a function of the scattering length of the background gas a_B with values of well separation $L = 50$ nm (red dashed line), $L = 75$ nm (blue dotted line), and $L = 100$ nm (black solid line). The inset shows $\mathcal{N}_{\rm deph}$ for very small values of the scattering length.

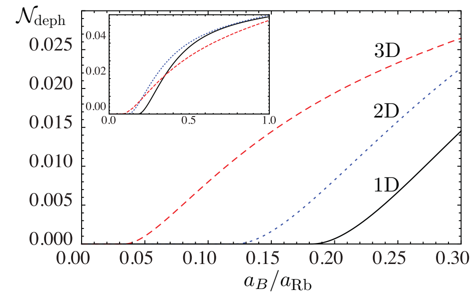

FIG. 3. (Color online) Non-Markovianity measure $\mathcal{N}_{\rm deph}$ as a function of the scattering length of the background gas a_B when the background gas is three-dimensional (red dashed line), quasi-two-dimensional (blue dotted line), and quasi-one-dimensional (black solid line). The inset shows a longer range of the scattering length a_B. In all figures, the well separation is $L = 75$ nm.

result holds for any value of L. This finding challenges the conclusion of Ref. [8], where the scattering length dependent Markovian–non-Markovian crossover was only attributed to the 1D case. We have discovered that the situation is indeed more subtle and we will show next that the crossover point exists in all three dimensions.

Lower dimensions. By a suitable modification of the potential of the condensate $V_B(\mathbf{r})$, we can create a quasi-2D background gas where the gas is trapped in a slightly anisotropic, pancake-shaped harmonic trap. Assuming that the scattering length is still much smaller than the axial length of the condensate, $a_B \ll a_z$, the coupling term is modified to $g_B^{2D} = \sqrt{8\pi}\hbar^2 a_B/(m_B a_z)$ and the 2D condensate density is $n_2 = \sqrt{\pi} n_0 a_z$ [19]. Within the limits of a dilute gas, we can increase the scattering length up to $a_B \approx 2\, a_{\rm Rb}$. The potential $V_B(\mathbf{r})$ can be also modified to create a cigar-shaped quasi-1D background gas with transversal width $a_\perp$. The consequent coupling is $g_B^{1D} = 2\hbar^2 a_B/(m_B a_\perp^2)$ and the 1D density is $n_1 = n_0 \pi a_\perp^2$, again provided that gas is weakly enough confined, $a_B \ll a_\perp$ [20]. In the quasi-1D regime, diluteness of the gas allows at most $a_B \lesssim a_{\rm Rb}$.

In Fig. 3, we plot the non-Markovianity measure $\mathcal{N}_{\rm deph}$ in the quasi-1D, quasi-2D, and 3D cases. In lower dimensions, we find the critical values $a_B^{\rm crit,2D} \approx 0.122 a_{\rm Rb}$ and $a_B^{\rm crit,1D} \approx 0.183 a_{\rm Rb}$. Clearly, when the dimensionality of the background gas is lowered, the crossover value of the scattering length $a_B^{\rm crit}$ increases. Crucially, as we remarked before, $a_B^{\rm crit,3D} > 0$ and therefore it is possible to create both Markovian and non-Markovian dynamical processes in all three dimensions.

We note here that the quantities we have chosen to vary, namely, the scattering length a_B and the well separation L, are indeed the most relevant quantities for manipulating the information flowback. The trap parameter τ determines the trapping frequency of the double-well trap $V_A(\mathbf{r})$ and acts as a natural cutoff parameter in the decay rate of Eq. (3). As long as the double-well trap is deep enough to prevent hopping between the two sites, the particular value of τ has only a minor effect on information flow. It is also clear from the form of the decay rate that the boson-impurity coupling g_{AB} cannot affect the Markovian–non-Markovian crossover. Moreover, we have found that its value has negligible effect on the non-Markovianity quantifier

$\mathcal{N}_{\rm deph}$. Intuitively, it affects the amount of outgoing and incoming information almost equally but leaves their ratio unchanged. In order to explain the key non-Markovian features in the dynamics of the qubit system when the dimensionality and the scattering length of the background gas vary, we need to take a closer look at the spectrum of the BEC reservoir.

Environmental spectrum. The crossover between Markovian and non-Markovian processes is best understood in terms of the environmental spectrum $J(\omega)$. Consider the general dephasing model introduced by Palma, Suominen, and Ekert describing qubit dynamics: $\rho_{ii}(t) = \rho_{ii}(0)$ and $\rho_{ij}(t) = e^{-\tilde{\Gamma}(t)}\rho_{ij}(0)$, where $\tilde{\Gamma}(t) \sim \int d\omega\, J(\omega)(1-\cos\omega t)/\omega$ [21]. The dynamical process is non-Markovian if and only if $\tilde{\gamma}(t) = d\tilde{\Gamma}(t)/dt < 0$ for some interval of time. We assume an Ohmic-type spectrum $J(\omega) \sim \omega^s$ and recall the convention that the spectrum is sub-Ohmic when $s < 1$, Ohmic when $s = 1$ or super-Ohmic when $s > 1$. Introducing an *ad hoc* exponential cutoff so that $J(\omega) = \omega^s \exp\{-\omega^2/\omega_C^2\}$, where ω_C is the cutoff frequency, it is straightforward to show that the dynamics is non-Markovian when $s > s^{\rm crit} = 2$. Therefore, in a general setting, a qubit dephasing under the effect of either a sub-Ohmic or an Ohmic environment can only leak information to its environment. If the environment has a super-Ohmic spectrum, the issue is less straightforward: only if the spectrum is sufficiently super-Ohmic with $s > s^{\rm crit}$, information can flow back to the system from the environment.

The qubit in an ultracold bosonic environment considered in this work is a special case of the model above with $J(\omega) = \sum_{\mathbf{k}} |g_{\mathbf{k}}|^2 \delta(\hbar\omega - E_{\mathbf{k}})$. The reservoir spectrum $J(\omega)$ does not have a simple analytical expression due to the complicated form of the coupling constant $g_{\mathbf{k}}$. However, it can be shown that, in the case of a free background gas in one, two, or three dimensions, the spectrum is sub-Ohmic, Ohmic, or super-Ohmic, respectively [8]. The spectrum changes critically when one considers the boson-boson coupling quantified by the scattering length a_B. In this case, increasing the scattering length effectively increases the value of s. Hence when we increase a_B in the 1D case, the spectrum changes from sub-Ohmic to Ohmic to super-Ohmic and once a critical threshold of super-Ohmicity is reached the environment can feed information back to the system. In the 2D non-interacting case, the spectrum is Ohmic and a weaker interaction is required to reach the crossover point $s^{\rm crit}$, leading to $a_B^{\rm crit,2D} < a_B^{\rm crit,1D}$. Finally, in the 3D case, the spectrum is already super-Ohmic in the noninteracting case, although not super-Ohmic enough to give rise to non-Markovian dynamics. Already a small increase in the scattering length modifies the spectrum so that the direction of information flow can be temporarily reversed.

Conclusion. We have studied quantum information flux in an ultracold hybrid system of an impurity atom immersed in a BEC environment. We have shown explicitly how precise control of the ultracold background gas affects the spectrum felt by the qubit and therefore enables the manipulation of the qubit dynamics and the information flux. An important discovery is the existence of a controllable crossover between Markovian and non-Markovian dynamics. In particular, we have discovered experimentally accessible means of reaching non-Markovian dynamical regimes, where the background gas may feed information back to the qubit instead of acting only as a sink for information. Such quantum reservoir engineering is fundamental for understanding decoherence processes in quantum information processing and, more specifically, for the realization of quantum simulators.

The loss of any quantum property—be it quantum superposition or entanglement or quantum discord—can be seen as due to information lost by the system because of its interaction with the environment. In this sense, studying information flux is a convenient way to quantify the tendency of the quantum system to retain those quantum properties necessary for quantum technologies. Non-Markovian systems are able to regain previously lost information and, therefore, compared to Markovian systems, they guarantee a longer operational time for quantum devices. A natural future direction, which will constitute the subject of a followup paper, is the study of the optimal conditions for entanglement-keeping in quantum registers of impurities, as indicated by information flow.

Finally, we would like to stress that the ideas presented in this work can be realized in present-day experiments. The measurement of the degree of non-Markovianity can be achieved by measuring the impurity coherence, since $\mathcal{N}_{\rm deph} = [\rho_{LR}(b) - \rho_{LR}(a)]/[\rho_{LR}(0) - \rho_{LR}(a)]$. This can be done by mapping the states $|L\rangle$ and $|R\rangle$ to the superpositions $|L\rangle \pm |R\rangle$. This atomic "beam splitter" would allow one to infer, from the measurement of the population imbalance of the two wells, the coherence ρ_{LR} [22].

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