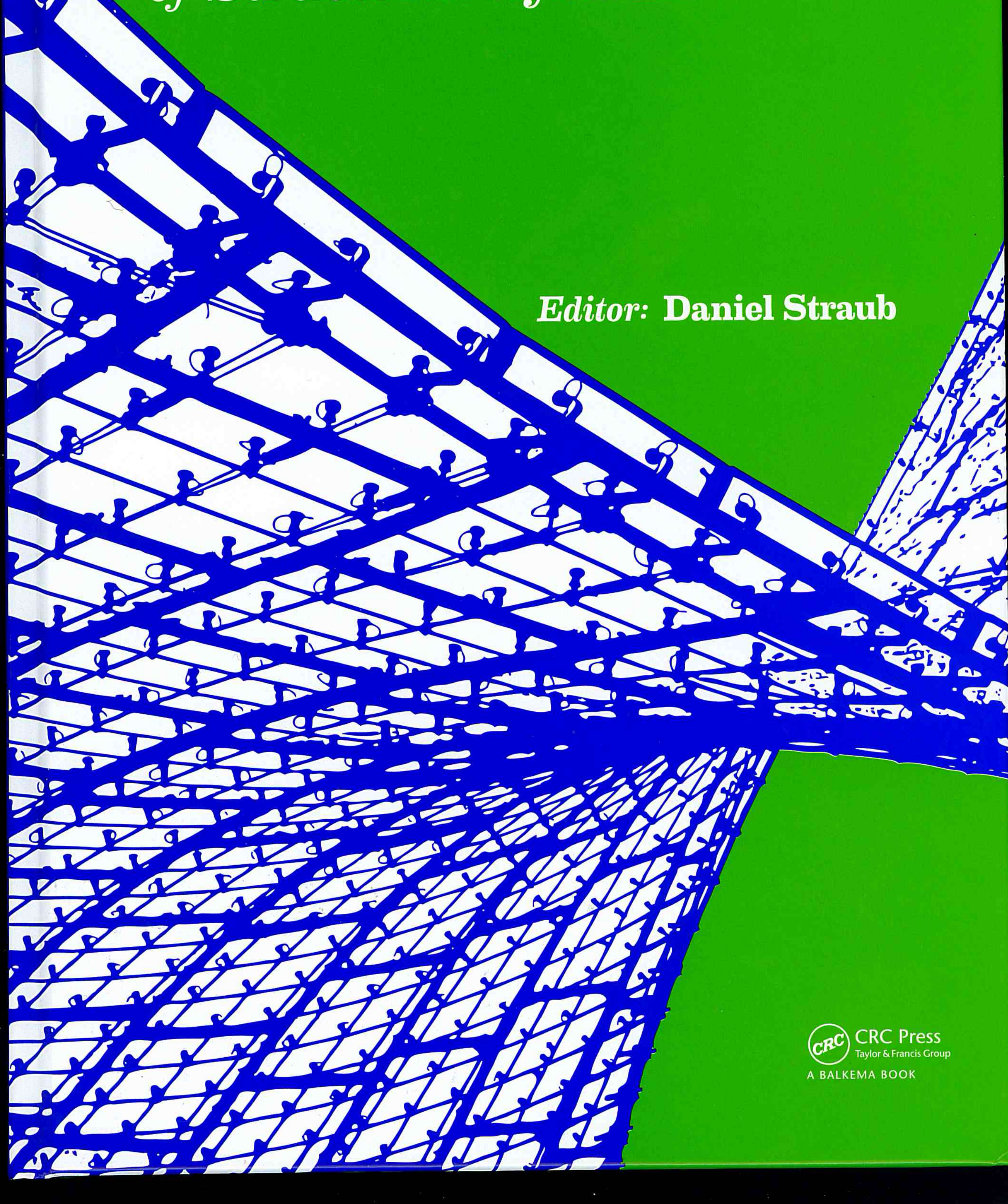
Reliability *and* Optimization *of* Structural Systems



Editor: **Daniel Straub**

CRC Press
Taylor & Francis Group
A BALKEMA BOOK

CRC Press/Balkema is an imprint of the Taylor & Francis Group, an informa business

Typeset by MPS Ltd. (A Macmillan Company), Chennai, India

Printed and bound in Great Britain by
Antony Rowe (A CPI Group Company), Chippenham, Wiltshire

Published by: CRC Press/Balkema
P.O. Box 447, 2300 AK Leiden, The Netherlands
e-mail: Pub.NL@taylorandfrancis.com
www.crcpress.com – www.taylorandfrancis.co.uk – www.balkema.nl

ISBN: 978-0-415-88179-1 (Hbk)
ISBN: 978-0-203-84141-9 (eBook)

Reliability and Optimization of Structural Systems – Straub (ed)
© 2010 Taylor & Francis Group, London, ISBN 978-0-415-88179-1

Table of Contents

Reliability and Optimization of Structural Systems – Straub (ed)
© 2010 Taylor & Francis Group, London, ISBN 978-0-415-88179-1

Damage identification by Lévy ant colony optimization

G. Cottone
Engineering Risk Analysis Group, TU München, Germany
DISAG, Università di Palermo, Italy

A. Pirrotta & G. Fileccia Scimemi
Dipartimento di Ingegneria Strutturale, Aerospaziale e Geotecnica, Università di Palermo, Italy

E. Riva Sanseverino
Dipartimento di Ingegneria Elettrica, Elettronica e delle Telecomunicazioni, Università di Palermo, Italy

ABSTRACT: This paper deals with the identification of incipient damage in structural elements by non-destructive test based on experimentally measured structural dynamical response. By application of the Hilbert transform to the recorded signal the so-called phase of the analytical signal is recovered and a proper functional is constructed in such a way that its global minimum gives a measure of the damage level, meant as stiffness reduction. Minimization is achieved by applying a modified Ant Colony Optimization (ACO) for continuous variables, inspired by the ants' foraging behavior. The modification consists in the application of a new perturbation operator, based on alpha stable Lévy distribution. Numerical application to three degrees of freedom system is proposed to show the method and test the performance.

1 INTRODUCTION

Detecting and quantifying structural damage at early stage by means of non-destructive tests is of primary concern in many engineering applications. Most of non-destructive techniques are based on the correlation between measured data (in the majority of cases from dynamic tests) and the response of an analytical model of the damaged structure (model-based procedures). Different structural response characteristics can be used for this purpose including modal data (Adams et al. (1991), Davini et al. (1992) Gounaris et al. (1996), Cerri & Vestroni (2000)), curvature measures (Pandey et al. (1991), Luo & Hanagud (1997), Ratcliffe (1999)), frequency response functions (Wang et al. (1997), Thyagarajan et al. (1998), Sampaio et al. (1999), Lee & Shin (2002)) and strain energy (Hearn & Testa (1991), Shi et al. (1998), Cornwell et al. (1999)). However, frequency-based identification procedures often fail to give a satisfactory answer for very low damage levels. Indeed, especially for small damage extent, the response in terms of impulse response function and amplitude is almost insensible to the system perturbation. Conversely, it has been recently shown, in Cottone et al. (2008) that a damage detection strategy based on a proper functional calculated on the analytical signal of the structural dynamical response consents to identify very low damage levels. Moreover, it has been shown that if one applies a system of filters to the experimental signal in order to isolate the single modal component of the dynamical response, the method can be effective also in presence of instrumental noise and for damages occurring in more points of the structure. The necessity of the filtering operation relies on the fact that, in the above mentioned case, the functional to be minimized may have many local minima and most of the optimization procedures do not succeed to find a correct value of the damage level. It has been shown that a way to circumvent the excess of local minima problem is by isolating the structural modal components, through filtering operations on the signals (refer to Cottone et al. (2008) and Barone et al. (2008)). On the other hand, in some practical case, for example dealing with structures whose principal frequencies are too narrow, the procedure based on the filtering is not suitable.

In this paper, we apply the Ant Colony Optimization for continuous domains ($ACO_{\mathbb{R}}$) already proposed in Socha & Dorigo (2008), introducing a perturbation operator with α-stable Lévy distribution. The choice of the α-stable Lévy distribution, with its particular inverse power-law tails, is justified because it has been observed that the wider exploration potential of such a distribution, makes the algorithm to attain better performance than $ACO_{\mathbb{R}}$ based on Gaussian distribution for function showing many local minima. In this way, we show that it is possible to perform the minimization of the functional without extrapolating the modal component by filtering the structural response. Applications to structures with narrow principal frequencies show the good performance in detecting an incipient damage.

The paper is organized as follow. Firstly, the Hilbert transform of the structural response is introduced in order to represent the analytical signal, and then a proper functional in terms of the phase of the analytical signal whose minimum corresponds to the damage level, is constructed. Then, a modified $ACO_{\mathbb{R}}$ with Lévy perturbation will be presented following the papers Candela et al. (2009) and Cottone et al. (2010). Lastly, numerical examples are given, considering a 3 degrees of freedom shear type structure.

2 DAMAGE IDENTIFICATION BY ANALYTICAL SIGNAL

2.1 *Hilbert transform and analytical signals*

In this section we briefly recall some features of the analytical signal in order to set the appropriate symbols. Let $x(t)$ denote the response of a linear system, where t indicates the time. Its Hilbert transform is defined as

$$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(t)}{t-\tau}\,d\tau \tag{1}$$

where the integral is meant in the Cauchy principal value sense. The so-called *analytical signal* (refer to Oppenheim & Schafer, (1989)) is the complex signal defined as $z(t)=x(t)+\mathrm{i}\,\hat{x}(t)$, with $\mathrm{i}=\sqrt{-1}$. The function $z(t)$ can also be expressed in terms of the amplitude function $A(t)$ and the phase function $\theta(t)$ as

$$z(t) = A(t)\exp(\mathrm{i}\theta(t)) \tag{2}$$

with

$$A(t) = \sqrt{x(t)^2 + \hat{x}(t)^2} \qquad \theta(t) = tan^{-1}\left(\frac{\hat{x}(t)}{x(t)}\right) \tag{3a,b}$$

It is useful to recall the very remarkable property of the Hilbert transform, useful in applications. Let us denote with $X(\omega)=\mathcal{F}\{x(t);\omega\}$ the Fourier transform of the function $x(t)$, then it is possible to show that the property

$$\mathcal{F}\{\hat{x}(t);\omega\} = -\mathrm{i}\,\mathrm{sgn}(\omega)\,X(\omega) \tag{4}$$

where $\mathrm{sgn}(\omega)$ is the signum function, holds true. Practically, one can compute $X(\omega)$, apply the latter and finally perform an inverse Fourier transform to obtain $\hat{x}(t)$. Moreover, it is easy to prove from the definition and the latter equation that the analytical signal can be computed making the inverse Fourier transform of the function $2X(\omega)U(\omega)$, being $U(\omega)$ the unit step function.

2.2 *Incipient damage identification procedure*

In this paper we restrict our attention to identify incipient damage in structural elements by analyzing a recorded dynamical response. To this aim, let us indicate with $\gamma^{\mathrm{eff}}=(\gamma_1^{\mathrm{eff}},\ldots,\gamma_n^{\mathrm{eff}})$ the unknown damage indexes. They are a set of n real coefficients defined in the interval [0,1] characterizing the damage magnitude on n different structural elements, where the zero bound corresponds to structural integrity. For example, one can consider $(1-\gamma_j^{\mathrm{eff}})$ being the j-th coefficient such that, multiplied by the j-th structural element's flexural stiffness, it characterize the decrement of stiffness. The apex eff means that the γ^{eff} indexes are effective and our aim is to develop a damage identification procedure capable to identify the value of each coefficient associated to the respective structural element. It has to be noted that the present formulation it is not suited to establish location inside the element where damage occurred.

Let us indicate by $\eta^{ex}(\gamma^{eff}; t)$ a general feature extracted from the real structure, where the apex ex denotes that it is a recorded data, i.e. coming from an experimental test. Further, indicate by η^{th} (γ; t) the same feature calculated from a structural model, depending on the γ variables.

The objective functional

$$J_\eta(\boldsymbol{\gamma}) = \frac{1}{T_f - T_i}\int_{T_i}^{T_f} \left(\eta^{th}(\boldsymbol{\gamma}; t) - \eta^{ex}(\boldsymbol{\gamma}^{eff}; t)\right)^2 dt \qquad (5)$$

where $[T_i, T_f]$ is the observation temporal window, consents to detect the set of damage indexes γ^{eff}. It has been shown in Cottone et al. (2008), Barone et al. (2008) that the most sensitive feature that consents to identify small damage also in presence of measuring noise affecting the experimental data is the phase of the analytical signal. Two examples will clarify the approach.

2.2.1 *Example: Incipient damage identification in a cantilever bar*

Consider a mass-less cantilever bar with a lumped mass at the free end subjected to axial displacement. The bar stiffness is EA/l, being E the Young's modulus, A the section area and l the bar length. An occurring damage will be modeled by a reduction of the axial stiffness $EA(1-\gamma)/l$ where $\gamma \in [0, 1]$ is the previously defined damage index. The equation governing the vibration of the given system under an impulsive force in horizontal direction modeled as a Dirac's delta function $\delta(t)$ is given in the form

$$\ddot{x}(t) + 2\Lambda\dot{x}(t) + \omega_0^2\,(1-\gamma)x(t) = \delta(t)/m \qquad (6)$$

where $\Lambda = \zeta_0\omega_0\sqrt{1-\gamma}$ denotes the damping term, ζ_0 the damping ratio and $\omega_0 = \sqrt{EA/ml}$ is the undamaged bar circular frequency. Assume that the experimental system is characterized by a damaged index $\gamma_{eff} = 0.01$, that is a very low stiffness reduction. Performing an hammer test on the experimental set-up, the acceleration data $\ddot{x}^{ex}(\gamma^{eff}; t)$ is recorded and the phase of the analytical signal is calculated and indicated by $\theta^{ex}(\gamma^{eff}; t)$. Then, the functional in eq.(5) is constructed by selecting a value of the unknown damage index γ and solving the theoretical model ruled by eq.(6) and finding the signal $\ddot{x}^{th}(\gamma, t)$. From this acceleration response the $\theta^{th}(\gamma; t)$ to be introduced in eq.(5) is calculated and one value of the functional $J_\theta(\gamma)$ is obtained. The absolute minimum of the functional is located at $\gamma = \gamma^{eff}$ In this simple case, the functional is very well-behaved, there is only a global minimum and its minimization is very easy. Conversely, in multi degrees of freedom (DOF) structures or in recorded signal affected by measurements and/or ambient noise, the functional $J_\theta(\gamma)$ has many local minima due to the multi-component behavior of the Hilbert transform (see Cottone et al. (2008)). This aspect is stressed in the next example.

2.2.2 *Example: Incipient damage identification in a 3 DOF structure*

Let us consider a three-storey shear-beam type building. The mass, stiffness and viscous damping of each storey are assumed to be the same, with $m_j = 1000$ kg, $k_j = 980$ kN/m, $c_j = 2814$ Ns/m, respectively, for j = 1, 2, 3. Damage is simulated by decreasing the stiffness at first and second floors by a damage indexes $\gamma_1^{eff} = 0.010$ and $\gamma_2^{eff} = 0.015$, while $\gamma_3^{eff} = 0$. The acceleration of the third floor is recorded and the phase component from the analytical signal is extracted and indicated as $\ddot{\theta}_3^{ex}(\gamma_1^{eff}, \gamma_2^{eff}, \gamma_3^{eff}; t)$.

For each combination of $\gamma = (\gamma_1, \gamma_2, \gamma_3)$ it is possible to calculate the model theoretical response, i.e. the acceleration of the third storey $\ddot{x}_3^{th}(\gamma_1, \gamma_2, \gamma_3; t)$ and the functional in terms of phase is computed. Both the noise and the multimodality of the response negatively affect $J_\theta(\gamma)$ and many local minima render the minimization to be a difficult task. At this point, two strategy of attack to the problem are possible. The first is to process the signal in order to restore a well-behaved functional as like as in the single DOF previously outlined. In Cottone et. al (2008), Barone et al. (2008) this has been achieved by filtering the single modal components both from the experimental and the theoretical signals and calculating the objective function. This strategy proved to perform very well also in experimental tests in Navarra & Pirrotta (submitted). Limits of this approach rely on the ability of the filter in isolating the modal responses. In case of structures with narrow principal frequencies such procedure might not be effective.

The second strategy to solve the problem is to use an optimization algorithm able to deal with strong presence of local minima. This is investigated in this paper, exploiting the Ant Colony Optimization with Lévy perturbation already proposed by some of the authors in Candela et al. (2009) and Candela et al. (2010).

3 LÉVY ANT COLONY OPTIMIZATION

3.1 *α-stable Lévy distribution in searching and optimization*

α-stable Lévy distributions are characterized by heavy-tailed probability density function that causes infinite variance and are defined by four coefficients. A random variable X is said to have a α-stable distribution if there are parameters $0 < \alpha \leq 2$, $\sigma > 0$, $1 \leq \beta \leq 1$, $\mu \in \mathbb{R}$ such that its characteristic function $\phi_X(\theta)$ has the form:

$$\phi_X(\theta) = \begin{cases} \exp\left[-\sigma^\alpha|\theta|^\alpha\left(1 - i\beta\text{sgn}(\theta)\tan\left(\frac{\pi\alpha}{2}\right)\right) + i\mu\theta\right], & \alpha \neq 1 \\ \exp\left[-\sigma\,|\theta|\left(1 + \frac{2}{\pi}\text{i}\beta\text{sgn}(\theta)\ln|\theta|\right) + i\mu\theta\right], & \alpha \neq 1 \end{cases} \quad (7)$$

The four parameters affect the shape of the distribution in an essential way and it is common to introduce an appropriate notation to take them into account. We denote with the symbol $X \sim S_\alpha(\sigma, \beta, \mu)$ an α-stable random variable with assigned parameters characterizing eq.(7). The coefficient α is called stability index and characterizes the tails of the stable distribution, decreasing as $x^{-1-\alpha}$. Lower values of α correspond to higher probability of having realization on the tails, while for $\alpha = 2$ the distribution $S_2(\sigma, \beta, \mu)$ coincides with the Gaussian distribution $N(\mu, \sqrt{2}\sigma)$. The scale σ, the skewness β and the location μ are parameters defining the shape of the distribution. Such a great variability makes Lévy distribution to be very suited in searching algorithms (see Gutowski (2001), Lee & Yao (2004), Lomholt et al. (2007)). Moreover, heavy-tailed distributions consent to have less localized space of search. In this paper, these features are introduced in the Ant Colony Optimization in continuous domain to minimize the objective function in eq.(5).

3.2 *The metaphor of ants foraging in combinatorial and continuous optimization and its Lévy based version*

Ant Colony Optimization (ACO), originally proposed for combinatorial optimization problems (Dorigo (1992); Dorigo & Gambardella (1997)) has been extended to continuous domains in Socha & Dorigo (2008), main reference of this section. It is use to denote the continuous domain formulation as $\text{ACO}_\mathbb{R}$ to make a distinction from ACO. The foraging behavior of real ants has motivated the ACO: i) ants firstly leave the nest for exploring the ambient in random way; ii) in case one ant finds the food, returns to the nest depositing a pheromone trail proportional to the food quality and quantity; iii) this trail is the only indirect possible communication with other ants; iv) pheromone does evaporate, i.e. it the shorter the path the higher the appeal of the pheromone trail. Although the algorithmic translation of this concepts into ACO are out of the scope of this paper, nevertheless it is worth to briefly outline how $\text{ACO}_\mathbb{R}$ works in order to better explain the role of the previously outlined Lévy distribution.

Let S be the search space and $X_1, X_2, \ldots, X_n$ the so-called decision variables, where n characterizes the dimension of the functional $f\colon S \to \mathbb{R}$ to be minimized. Construction of a solution $s_l \in S$ means to assign values $x^i \in \mathbb{R}$ to each X_i, for $i = 1, \ldots, n$, such that $s = (x^1, x^2, \ldots, x^n)$. A solution $s*$ is a global minimum if and only if $f(s*) \leq f(s)$ for every $s \in S$. The idea that is central to the way $\text{ACO}_\mathbb{R}$ works is the incremental construction of solutions based on the biased probabilistic choice of solution components. The bias is constituted by the pheromone trail guiding the searching process and is taken into account by means of n Gaussian kernel probability densities, $G^i(x)$ defined below. Practically we construct an archive T of k solutions $T = \{s_1, \ldots, s_l, \ldots, s_k\}$ where the generic solution s_l has n components, i.e. $s_l = (x_l^1, x_l^2, \ldots, x_l^n)$. Then, the solutions of the archive T are ordered according to their objective function values $f(s_l)$. For each $l = 1, \ldots, k$ we calculate the weights

$$\omega_l = \left(qk\sqrt{2\pi}\right)^{-1}\exp(-(l-1)^2/2(qk)^2) \quad (8)$$

that are dependent on the rank of the solution and are needed to calculate the Gaussian kernel

$$G^i(x) = \frac{1}{\omega_{tot}} \sum_{l=1}^{k} \omega_l g_l^i(x) = \frac{1}{\omega_{tot}} \sum_{l=1}^{k} \omega_l \left(\sigma_l^i \sqrt{2\pi}\right)^{-1} \exp\left(-(x-\mu_l^i)^2 / 2(\sigma_l^i)^2\right) \tag{9}$$

where $\mu_l^i = s_l^i$ and $\omega_{tot} = \sum_{l=1}^{k} \omega_l$ is a normalization factor. It must be noted that each density $g_l^i(x)$ is centered in s_l^i. The standard deviations σ_l^i are defined as average distance between s_l^i and the other solutions in T as

$$\sigma_l^i = \xi \sum_{r=1}^{k} |s_l^i - s_r^i| / (k-1) \tag{10}$$

The size of the archive k, ξ and q are user defined parameters of the algorithm whose correspondence to the ant metaphor is described in Socha & Dorigo (2008). Following this approach, Lévy distributed $g_l^i(x)$ could not be introduced, because Lévy densities are not known in analytical form. Yet, the following consideration is fundamental to introduce Lévy perturbations. Indeed, at each construction step i an ant selects a value x^i of X^i making a probabilistic choice $G^i(x)$, i.e. producing a sample whose density is given in eq.(9) and determined by the sole knowledge of the components $s_1^i, \ldots, s_k^i$. This corresponds to choose one distribution $g_l^i(x)$ with probability $p_l = \omega_l / \omega_{tot}$ and then sampling the chosen density by generating a random sample following the density $g_l^i(x)$. Therefore, the $ACO_{\mathbb{R}}$ with Lévy densities $g_l^i(x)$ can be implemented, simply generating samples of $S_\alpha(\sigma_l^i, 0, \mu_l^i)$ as described in Samorodnitsky & Taqqu (1994).

In the following the $ACO_{\mathbb{R}}$ with Lévy perturbations will be denoted as $ACO_{\mathbb{R}L}$, whereas it is plain that for $\alpha = 2$, $ACO_{\mathbb{R}L}$ coincides with $ACO_{\mathbb{R}}$. In Candela et al. (2009) some of the authors already attested the efficiency and the performances of the $ACO_{\mathbb{R}L}$ with respect to a set of test function. Briefly the results can be summarized as: The $ACO_{\mathbb{R}L}$ performs better or no worse than $ACO_{\mathbb{R}}$ for functions with many local minima for $\alpha = 1.6$ and $\alpha = 1.8$; lower values of the stability index give rise to higher degree of diversification in the searching process due to the tendency of the Lévy flights to escape from a region. In applying the $ACO_{\mathbb{R}L}$ to the minimization of eq.(5) we will select the value of $\alpha = 1.8$.

4 NUMERICAL ANALYSIS

4.1 *3-DOF structure with well spaced natural frequencies*

Let us consider the 3-DOF structure with parameters already specified in paragraph 2.2.2. The structure principal frequencies are $\omega_1 = 13.6$ rad/s, $\omega_2 = 38.5$ rad/s and $\omega_3 = 56.2$ rad/s, respectively. This example has been already studied in Cottone et al. (2008) by application of Butterworth filter to both the experimental and the theoretical signals, in order to extract the modal component. In that work, this was necessary because a well behaved functional without local minima was searched. We examine the same example with the perspective presented in this paper, without using the pre-filtering of the dynamical response. Instead, $ACO_{\mathbb{R}L}$ is used to minimize $J_\eta(\gamma)$ and we show that the algorithm is suited to estimate the assigned level of damage $\gamma^{\text{eff}} = (0.010, 0.015, 0.00)$. Just to visualize the problem, in Figure 1 the logarithm of the functional calculated on the phase of the analytical signal is depicted: It can be noted that many local minima are present, all along a line dependent on the structure parameters, and that a unique global minimum corresponds to the assigned damage parameters γ^{eff} In this simple example, the value of the functional in γ^{eff} is zero, i.e. in Figure 1 the effective value of the functional at the minimum is negative infinity.

$ACO_{\mathbb{R}L}$ has been implemented with $q = 0.1$, $\xi = 0.85$, $k = 50$ for 5000 ant's generation steps. 30 independent runs have been used to assess the performance of the algorithm. The search space has been confined just to the damage index γ_1 and γ_2 in the domain $[0, 0.2] \times [0, 0.2]$. Table 1 resumes the mean and the standard deviation of the damage indexes obtained and of the absolute percentage error $\epsilon_j = 100(\gamma_j^{\text{eff}} - \gamma_j)/\gamma_j^{\text{eff}}$. Figure 2 shows the variation of the best, the worst, the mean and the median in a single run. In this simple case, $ACO_{\mathbb{R}L}$ succeeds to identify the incipient damage and gives a very good quantitative estimate of the extent of the damage.

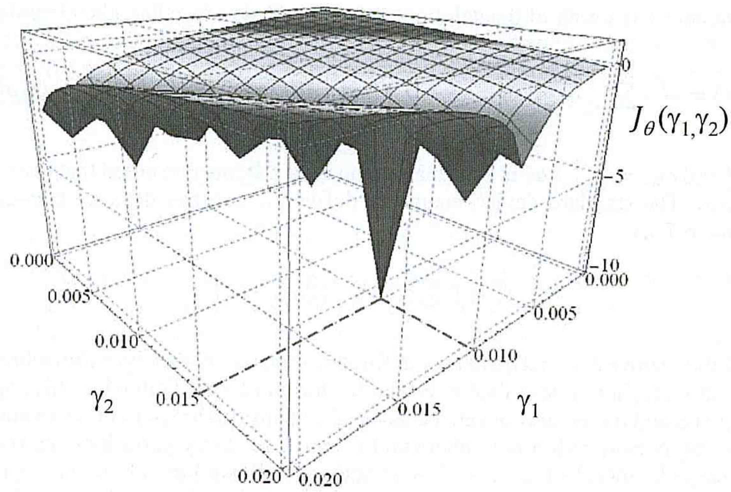


Figure 1. Logarithm of the functional $J_\theta(\gamma)$ calculated on the phase of the analytical signal. Thick dashed lines correspond to the global minimum to be identified. (right panel).

Table 1. Mean and standard deviation of the damage indexes.

Test	γ_1	γ_2	$\epsilon_1\%$	$\epsilon_2\%$
mean	0.010	0.015	0.37	0.34
st.dev	3.8E−5	6.0E−5	–	–

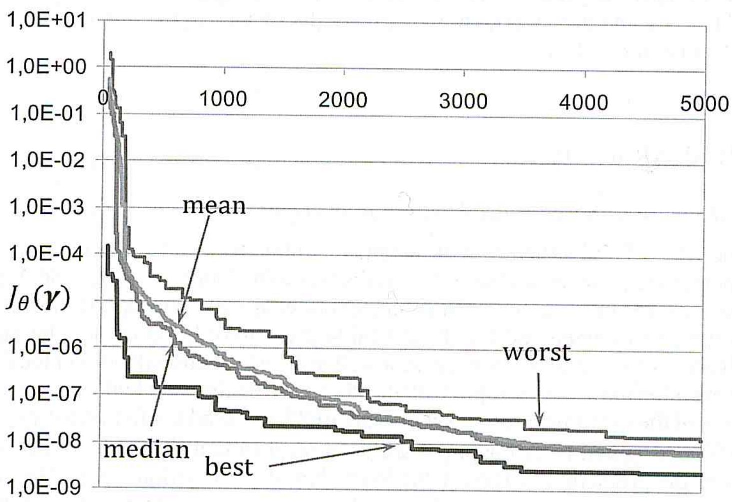


Figure 2. Evolution of the solution along 5000 ants generation steps.

4.2 *3-DOF structure with not well spaced natural frequencies*

Consider a 3-DOF with the parameters of the sound structure that are: $m_1 = 1000\,\text{kg}$, $m_2 = 1300\,\text{kg}$, $m_4 = 500\,\text{kg}$, $k_1 = 980\,\text{kN/m}$, $k_2 = 130\,\text{kN/m}$, $k_3 = 140\,\text{kN/m}$ $c_j = 2814\,\text{Ns/m}$, respectively, for $j = 1, 2, 3$. Such a structure presents not well spaced natural frequencies, in such a way that isolating the single modal component from the recorded signal by filtering might request very high order filters with consequent difficulties. It is assumed that the structure is characterized by the damage indexes $\gamma^{\text{eff}} = (0.010, 0.015, 0.00)$ as in the previous example. As outlined in section 2, the

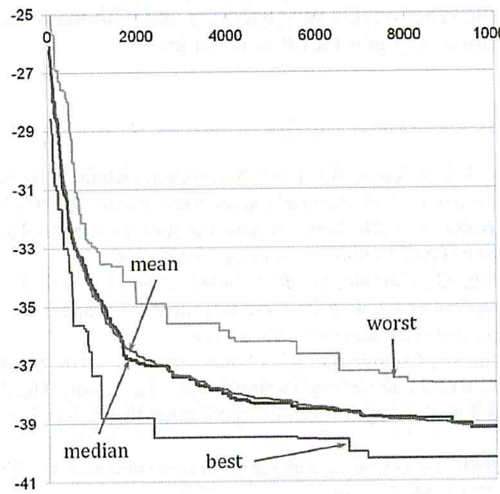


Figure 3. Evolution of the solution along 10000 ants generation steps.

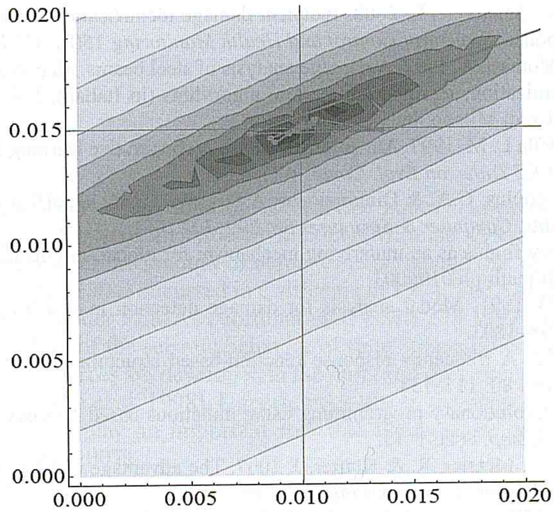


Figure 4. Trajectory (in red, thick line) of the best ant solution for one run, contrasted with the contour plot of the functional in the space of parameters γ_1, γ_2.

experimental analytical signal is calculated and the functional in eq.(5) is constructed. The search space in $ACO_{\mathbb{R}L}$ is the domain $[0, 0.2] \times [0, 0.2] \times [0, 0.2]$ in the variable parameters $\gamma_1, \gamma_2, \gamma_3$.

Also in this case, $ACO_{\mathbb{R}L}$ succeeds in locate the global minimum. Figure 3 shows the space of the objective function revealing that, in this case, the span between the best and the worst ant is greater than before. More interesting is Figure 4, where the trajectory of the best ant, that is the best ranked at each construction step, highlights the process of search captured into the groove where global and local minima are present.

5 CONCLUSIONS

In this paper we presented a method for the incipient damage identification based on the minimization of a functional calculated by some features of the analytical signal. Once the functional has been introduced, the minimization is performed by the meta-heuristic method of the Ant Colony Optimization with Lévy perturbation. Numerical results on 3 degree of freedom system confirm that, even in cases where the functional is very irregular and with many local minima, the damage index can be quantified with a very good level of accuracy.

REFERENCES

Adams, R.D., Cawley, P., Pye, C.J. & Stone, B.J. 1991. A vibration technique for non-destructively assessing the integrity of structures. *Journal of Mechanical Engineering Science* 20: 93–100.

Barone, G., Marino, F. & Pirrotta, A. 2008. Low stiffness variation in structural systems: Identification and localization. *Structural Control and Health Monitoring* 15:450–470.

Candela, R., Fileccia Scimemi, G., Cottone, G., Riva Sanseverino, E. 2009. Lévy flights for Ant Colony Optimization in continuous domains. CiE 2009. Computability in Europe 2009, Mathematical Theory and Computational Practice. Heidelberg (Germany). 19–24 July.

Candela, R., Fileccia Scimemi, G., Cottone, G., Riva Sanseverino, E. 2010. Composite laminates buckling optimization through Levy based Ant Colony Optimization. The Twenty Third International Conference on Industrial, Engineering & Other Applications of Applied Intelligent Systems. IEA-AIE 2010, Cordoba (Spain). 1–4 June.

Cerri, M.N. & Vestroni, F. 2000. Detection of damage in beams subjected to diffused cracking. *Journal of Sound and Vibration* 234: 259–276.

Cornwell, P., Doebling, S.W. & Farrar, C.R. 1999. Application of the strain energy damage detection method to plate-like structures. *Journal of Sound and Vibration* 224: 359–374.

Cottone, G., Pirrotta, A., Salamone, S. 2008. Incipient damage identification through characteristics of the analytical signal response. *Structural Control and Health Monitoring* 15(8): 1122–1142.

Davini, C., Gatti, F. & Morassi, A. 1992. A damage analysis of steel beams. *Meccanica* 22: 321–332.

Dorigo, M. 1992. Optimization, learning and natural algorithms (in Italian). PhD thesis, Dipartimento di Elettronica, Politecnico di Milano, Italy.

Dorigo, M. & Gambardella L. M. 1997. Ant colony System: A cooperative learning approach to the traveling salesman problem. *IEEE Trans. on Evol. Comp.* 1(1): 53–66.

Gounaris, G.D., Papadopoulus, C.A. & Dimarogonas, A.D. 1996. Crack identification in beams by coupled response measurements. *Computer & Structures* 58: 299–302.

Gutowski, M. 2001. Lévy flights as an underlying mechanism for global optimization algorithms. Available at: http://arxiv.org/pdf/math-ph/0106003.

Hearn, G. & Testa, R.B. 1991. Modal analysis for damage detection in structures. *Journal of Structural Engineering* 117: 1798–1803.

Lee, U. & Shin. J. 2002. A frequency response function-based structural damage identification method. *Computers & Structures* 80: 117–132.

Lee, C., Yao, X. 2004. Evolutionary programming using mutations based on Lévy probability distribution. *IEEE Trans. on Evol. Comp.* 8(1): 1–13.

Lomholt, M.A., Koren, T., Metzler, R. & Klafter, J. 2007. The advantage of Lévy strategies in intermittent search processes. Available at: http://arxiv.org/abs/0709.2352v1.

Luo, H. & Hanagud, S. 1997. An integral equation for changes in the structural dynamics characteristics of damaged structures. *International Journal of Solids and Structures* 34: 4557–4579.

Oppenheim, A.V. & Schafer, R.W. 1989. Discrete-Time Signal Processing, Prentice Hall, Englewood Cliffs, NJ.

Pandey, A.K., Biswas, M. & Samman, M.M. 1991. Damage detection from changes in curvature mode shapes. *Journal of Sound and Vibration* 145: 321–332.

Ratcliffe, C.P. 1999. A frequency and curvature based experimental method for locating damage in structure. *Journal of Vibration Acoustics* 226: 1029–1042.

Samorodnitsky, G. & Taqqu, M.S. 1994. Stable non-Gaussian random processes: Stochastic models with infinite variance. Chapman and Hall, New York.

Sampaio, R.P.C., Maia N.M.M. & Silva, J.M.M. 1999. Damage detection using frequency-response-function curvature method. *Journal of Sound and Vibration* 226: 1029–1042.

Shi, Z.Y., Law, S.S. & Zhang, L.M. 1998. Structural damage localization from modal strain energy changes. *Journal of Sound and Vibration* 218: 825–844.

Socha, K. & Dorigo, M. 2008. Ant colony optimization for continuous domains. *European Journal of Operational Research* 185: 1155–1173.

Thyagarajan, S.K., Schulz, M.J. & Pai, P.F. 1998. Detecting structural damage using frequency response functions. *Journal of Sound and Vibration* 210: 162–170.

Wang, Z., Lin, R.M. & Lim, M.K. 1997. Structural damage detection using measured FRF data. *Computer Methods in Applied Mechanics* 147: 187–197.