# A shoreline model for breaking waves

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## 1 Introduction

In order to simulate the wave motion and, in turn, the flow, within the nearshore region, in the last decades the derivation and the application of *depth-integrated* type of models have been widely investigated and developed. However, in such models, the problems of facing wave breaking and the moving shoreline are not trivial and therefore several approaches have been proposed. About wave breaking, approaches both based on the adoption of an artificial eddy viscosity [1] and on the concept of roller ( [2], [3], [4]) have been implemented. As regards the shoreline boundary condition, a couple of numerical techniques have been mainly adopted, namely the porous beach method, also known as slot method ([5]), and the extrapolating method proposed by [6].Such methods seems to be not very fisically based.

In the present work an effort toward a more physically based model of the surf and the swash zone has been accomplished. In particularly, a new version of the fixed grid shoreline model introduced by [7] is proposed here and implemented in a Boussinesq type model for breaking waves [4]. Moreover, in order to get over the numerical instabilities generated at the time of rapid variation of the flow, the aforementioned shoreline model has been coupled with the extrapolation method presented by [6] and a bottom friction term has been also included. To validate the model a classical test which adopts monochromatic waves along with other application with non breaking and breaking solitary waves have been performed.

## 2 Derivation and numerical solution

The dynamics of the wave propagation within the surf zone is here represented through the weakly dispersive fully nonlinear *Boussinesq*-type of model developed by [4]. The flow is assumed rotational after breaking and the governing equations are derived with no assumptions on the order of magnitude of the non-linear effects. In the present work the bottom friction effects have been considered by adopting the following quadratic model:

$$F^{att} = \frac{f}{(h+\zeta)} \cdot \overline{u} \cdot |\overline{u}| \tag{2.1}$$

where $f$ is the friction factor, $u$ is the mean horizontal velocity, $\zeta$ is the wave surface elevation and $h$ is the local depth.

The proposed shoreline boundary condition is developed with a fixed grid method with a wet-dry interface. In order to solve the problems due to the numerical scheme during the onshore movement of the shoreline, a linear extrapolation [6] near the wet-dry boundary has been used and coupled with the shoreline equations.( In Figure 1 the logical algorithm of the proposed strategy is showed and compared with the previous approach).

It is known that to develop a moving boundary algorithm the velocity and the position of the shoreline at each time step must be known; however, at the shoreline, where the water depth goes to zero, the volume fluxes also become zero, but the velocity of the fluid particles may not be null. Therefore following the approach introduced by [7] for the non-linear shallow water inviscid case, the equations for the shoreline motion for the 1DH problem have been here adopted. Moreover the effects of bottom friction has been introduced here in the shoreline momentum equation as well.

The shoreline equations state for the kinematic condition at the shoreline that the fluid particles at the shoreline remains at the shoreline; thus named $\xi(t)$ the $x$ coordinate of the shoreline it follows that

$$\frac{\partial \xi}{\partial t} = u^s \tag{2.2}$$

where the velocity of the shoreline $u^s$ is obtained from the momentum equation written as follows:

$$\frac{du^s}{dt} = -g\frac{\partial \zeta}{\partial x}^s + F^{att} \tag{2.3}$$

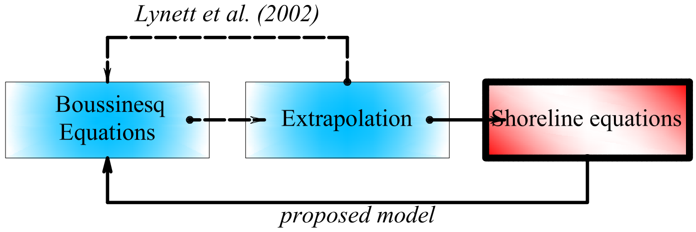


Figure 1: Sketch of the adopted strategy for the shoreline model. [6].

with $\zeta^s$ being the surface elevation of the shoreline. It should be noticed that the shoreline position is spatially continuously resolved (i.e. the shoreline may not stay on the numerical grid).

The numerical scheme adopted for the shoreline equations solution is the same adopted for the solution of the governing equations. Indeed, an Adam-Bashfort-Moulton scheme of $3^{th}$ order in time for the predictor step and of $4^{th}$ order in time for the corrector step. In such a scheme problems arise in the numerical solution during the run-up stage. Indeed, when a new dry point is included in the computational domain, at that point information on velocity at the time steps are required by the ABM scheme, but they are actually undefined. To overcome such a problem a linear extrapolation of the last two wet point as proposed by [6] was implemented as well. It is worth pointing out that the linear extrapolation method allows for the same finite difference scheme to be used also at the last wet point.

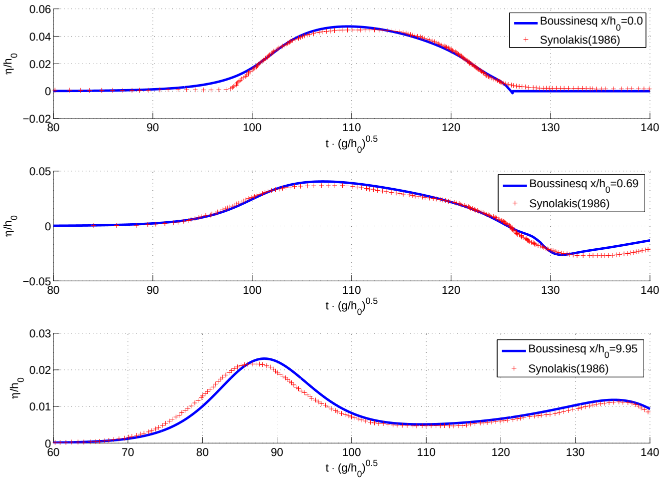


Figure 2: *Run up* of non-breaking solitary wave, on a 1:19.85 beach, with $H/h_0 = 0.0185$ at (a) $x/h0 = 0.0$; (b) $x/h0 = 0.69$; (c) $x/h0 = 9.96$ for different $t = t * \sqrt{gd}$. The red crosses are the experimental results, the blue continue line is the numerical results of the proposed model.

## 3 Model Results

To validate the model a classical test which adopts a monochromatic wave train over a plane beach has been performed. In particular, the analytical solution derived by [8] which makes use of the Airy's approximation of NLSW equations has been used for comparison. As a test, a wave train with an height of 0.006 m and a period of 10 s which travels in a one dimensional channel with a depth of 0.5 m and a slope of 1:25 has been considered. The comparison between the analytical and numerical horizontal shoreline movements provides a very good agreement.

Solitary breaking and non-breaking wave run-up and run-down was also investigated and the numerical results have been compared with the experimental data by [9] using also the bottom friction. In the Figure 2 the numerical results of the run-up of a non-breaking solitary wave with $H/h_0 = 0.0185$ as shown. The mentioned Figure shows the surface elevation versus time at different position in the flume. These results are in good agreement with the experimental data. A further comparison was performed in the case of a solitary breaking waves. In particular it was studied the runup and rundown of a solitary wave characterized by a ratio H/h0 = 0.30. As reported in Figure 3 the model reproduces quite well, at different time steps, the free surface elevations, the dissipation produced by breaking along with the process of runup and rundown at the beach. These results confirm that model is very suitable for studying the risk areas for flooding by Tsunami.

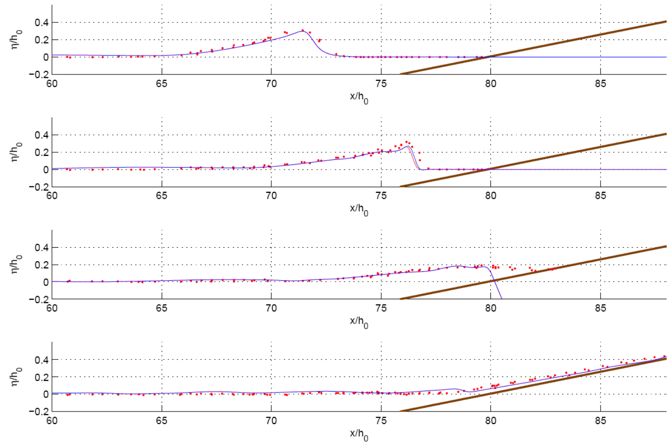


Figure 3: Breaking solitary wave run-up and run-down on a 1:19.85 beach at different time steps (a) $t^* = t(g/h)1/2 = 15$, (b) $t^* = 20$, (c) $t^* = 25$, (d) $t^* = 45$. The solid line represents the numerical results and the points indicate the experimental data from [9].

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