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The fractal model of non-local elasticity with long-range interactions

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The mechanically-based model of non-local elasticity with long-range interactions is framed, in this study, in a fractal mechanics context. Non-local interactions are modelled introducing long-range central body forces between non-adjacent volume elements of the elastic continuum. Such long-range interactions are modelled as proportional to the product of interacting volumes, to the relative displacements of the centroids and to a distance-decaying function that is monotonically-decreasing with the distance.

The choice of the decaying function is a key aspect of the model and it has been proved that any continuous function, strictly positive, is thermodynamically consistent and it leads to a material that satisfy the Drucker stability criterion [2]. Such a mathematical model of non-local elasticity has an interesting mechanical counterpart that is described by a point-spring network with multiple springs with distance-decaying stiffness.

As the functional class of the distance-decaying function is modelled as a power-law function of the distance of interacting particles, then, in the 1D case, the governing operators are Marchaud-type fractional derivatives as proved by the authors in previous studies [1].

In this study we aim to show that, as we assume that the stiffness associated to long-range interactions is modelled as a self-similar transformation of the Euclidean distance with anomalous and real scaling exponent, the mechanical model of the non-local elasticity is a self-similar fractal object.

In more detail, assuming a non-integer power-law decay of the long-range forces between adjacent volumes of an ideal next nearest (NN) model, the scaling law of the stiffness of the long-range bonds is readily obtained. The Hausdorff-Besitckovich (HB) fractal dimension provides the appropriate bounds of the decay coefficient necessary to maintain the self-similarity of the obtained fractal set. The NN model, however leads to mathematically inconsistent governing operator for general class of continuous displacement function and it is proved that in this case only one integer value of the long-range force decay is admissible leading to classical second-order differential operators.

A different scenario is involved as we introduce, on mechanical grounds, the long-range interaction concept so that as we refine observation scale, the interactions between particles is still involving the presence of all the new, non-adjacent particles so that the original NN lattice is turned into a more refined and realistic next to the nearest next (NNN) lattice model. Such a model is equivalent to the mechanical model of the long-range interactions introduced by the authors to describe non-local elasticity. The model is constituted of self-similar copies of elastic chains and henceforth it may be considered as a mechanical fractal as we assume an unbounded domain. This fractal set is not coalescing with usual fractals since it retains all the informations of previous observation scales and henceforth it has been dubbed as *multiscale fractal*. In this context the HB dimension of the mechanical fractal may be obtained as a function of the decaying exponent of the long-range interactions and it may be proved that the governing equation of the problem are Marchaud fractional-type differential operator as already postulated by the authors in a previous study [1]. Some conclusions about the use of fractional operators in the context of multiscale approach to non-local mechanics may be also withdrawn from previous considerations.

References

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