

IL MODELLO DI INTERFASE APPLICATO ALLA MESO-MODELLAZIONE DI STRUTTURE ETEROGENEE.

A. Spada¹, G. Fileccia Scimemi¹, and G. Giambanco¹

¹Dipartimento di Ingegneria Strutturale, Aerospaziale e Geotecnica,
Università di Palermo, Viale delle Scienze, edificio 8, 90128, Palermo
antonino.spada@unipa.it, giuseppe.filecciascimemi@unipa.it, ggiamb@unipa.it

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Abstract. *All those structures that are constituted by heterogeneous materials exhibit a complex anisotropic behaviour strictly related to the static and kinematic phenomena occurring in each constituent and at their interfaces. The overall macroscopic approach may be not appropriate to describe the elastic and post-elastic response of the structures. A more rigorous approach is the meso-modelling approach. In literature, usually the thin joints of the structure are simulated by applying the so-called 'zero-thickness interface', whose behaviour is expressed in terms of contact tractions and displacement discontinuities. However, because of the real thickness of the joint, the response depends also on internal stresses and strains within the bulk material. In this direction the enhancement of the zero-thickness interface is represented by the interphase model where strain and stresses are separated into internal and contact components.*

1 INTRODUCTION

Two main approaches have been used in literature to analyze structures made up of heterogeneous materials: the macroscopic approach and the mesoscopic approach.

The overall macroscopic approach ([1]) considers the structure as an homogeneous anisotropic continuum, so it consists in formulating phenomenological constitutive laws expressed in terms of macroscopic stress and average strain.

The meso-modelling approach takes the materials' and interfaces' properties separately. In composite structures it is also possible to distinguish units and joints. The joints in most cases represent the weakness areas of the heterogeneous material where fractures appear and propagate. In literature, a common way to simulate the joint is by applying the so-called 'zero-thickness interface', where the joint collapses to its middle surface that is in consequence considered as a simple contact layer between units ([2]-[4]).

For practical applications the mesoscopic approach may be in some cases not appropriate to describe the elastic and post-elastic response of the structures since it requires the introduction of strong simplifications. In fact, most of the nonlinearities of the overall response depend on local phenomena occurring in weaker joints such as debonding, sliding and dilatancy. On the other side, depending on the real thickness of the joint, the meso-modelling approach, based only on contact static and kinematic quantities, may not catch particular failure mechanisms. A

typical example is the squeezing effect of a mortar joint interposed between two rigid blocks and subjected to a compression load. This phenomenon can be captured only if the response includes internal stresses and strains within the weaker material.

In this work we present the enhancement of the zero-thickness interface, represented by the 'interphase model' ([5]). By employing the term interphase, we shall mean a layer separated by two interfaces from the bulk material or a multilayer structure with varying properties and several interfaces. The constitutive laws of the new mesoscale interphase model are written in terms of internal state of stresses and contact tractions and related kinematic variables. The model is implemented in a research oriented finite element code. Numerical simulations are provided to show the main features of the model and novelties introduced with respect to the common interface model.

2 THE INTERPHASE FINITE ELEMENT

With regards to the general formulation showed by [5], in this work the theory has been recasted to face 2D problems in plane stress conditions. In this particular case, the mean values of the stress and strain components are: $\hat{\sigma} = \begin{bmatrix} \sigma_x & \sigma_z & \tau_{xz} \end{bmatrix}$, $\hat{\varepsilon} = \begin{bmatrix} \varepsilon_x & \varepsilon_y & \varepsilon_z & \gamma_{xz} \end{bmatrix}$. The equilibrium equations are represented by:

$$\mathbf{t}^+ = \hat{\sigma} \cdot \mathbf{I}_3 - \frac{h}{2} \text{div} \hat{\sigma} \quad \text{on } \Sigma; \quad \mathbf{t}^- = -\hat{\sigma} \cdot \mathbf{I}_3 - \frac{h}{2} \text{div} \hat{\sigma} \quad \text{on } \Sigma, \quad (1)$$

$$\mathbf{m} \cdot \hat{\sigma} = 0 \quad \text{on } \delta\Sigma. \quad (2)$$

where \mathbf{t}^+ and \mathbf{t}^- are the traction components generated by the interaction of the interphase element with the upper and lower materials, h is the interphase thickness, $\mathbf{I}_3 = \{\delta_{i3}\}$, and Σ is the interphase middle plane.

The finite element is characterized by 4-node rectangular shaped. The displacement at each point of the finite element is obtained as a function of the nodal displacements. If we define the vector \mathbf{u} collecting all the nodal displacements as:

$$\mathbf{u} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 & \mathbf{u}_4 \end{bmatrix}, \quad (3)$$

the total strain at the Gauss point is given by:

$$\boldsymbol{\varepsilon} = \mathbf{B}\mathbf{U} \quad (4)$$

where

$$\mathbf{B} = \begin{bmatrix} N'_1/L & 0 & N'_2/L & 0 & N'_1/L & 0 & N'_2/L & 0 \\ 0 & -N_1/h & 0 & -N_2/h & 0 & N_1/h & 0 & N_2/h \\ -N_1/h & N'_1/L & -N_2/h & N'_2/L & N_1/h & N'_1/L & N_2/h & N'_2/L \end{bmatrix}. \quad (5)$$

In Eq. 5 L is the interphase's length, while N_1 and N_2 are the two shape functions given by:

$$N_{1,2}(\xi) = \frac{1}{2}(1 \mp \xi); \quad \xi \in \begin{bmatrix} -1, & 1 \end{bmatrix}. \quad (6)$$

By introducing N_1 and N_2 into Eq. 5 and 4 the compatibility matrix can be decomposed in the following form:

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1\xi \quad (7)$$

with

$$\mathbf{B}_0 = \frac{1}{2h} \begin{bmatrix} -\eta & 0 & \eta & 0 & -\eta & 0 & \eta & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 & 0 & 1 \\ -1 & -\eta & -1 & \eta & 1 & -\eta & 1 & \eta \end{bmatrix}, \quad \eta = \frac{h}{L} \quad (8)$$

and

$$\mathbf{B}_1 = \frac{1}{2h} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \end{bmatrix}. \quad (9)$$

Finally, by using two Gauss points, the stiffness matrix can be exactly integrated as:

$$\mathbf{K} = bh \frac{L}{2} \int_{-1}^1 \mathbf{B}^T \mathbf{E} \mathbf{B} d\xi = bhL \left(\mathbf{B}_0^T \mathbf{E} \mathbf{B}_0 + \frac{1}{3} \mathbf{B}_1^T \mathbf{E} \mathbf{B}_1 \right) \quad (10)$$

3 ELEMENT PERFORMANCES

To assess the element goodness a simple patch test has been faced. The model considered is that one showed in Figure 1.

Different behaviours could be found in dependence on the relative elastic moduli between

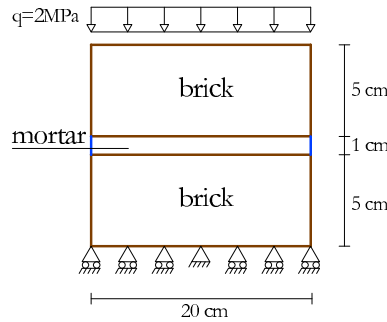


Figure 1: The patch test.

bricks and mortar. In particular, when the mortar is stiffer than bricks a two Gauss point integration leads to oscillations for internal and tangential stresses (Figure 2).

These stress oscillations have been observed, using conventional Gauss-quadrature scheme, in zero-thickness interface elements. In that case the remedy is to locate the sampling points at the nodes (Lobatto quadrature). In the case of the interphase element, instead, because of volumetric and tangential locking, two methods have been used as remedies: 1) the Reduced or Selective Integration method (RSI), which provides the necessary singularity of the constraint part of the stiffness matrix which avoids locking; 2) the Enhanced Assumed Strain method (EAS), where the strain field is enlarged and the element shows extra deformation modes. For example, by using the Selective Reduced Integration method with one and two Gauss points for the integration along the tangential and normal direction respectively, the results showed in Figure 3 are obtained. It can be observed that the previous oscillations disappear. The same results can be obtained by applying the Enhanced Assumed Strain.

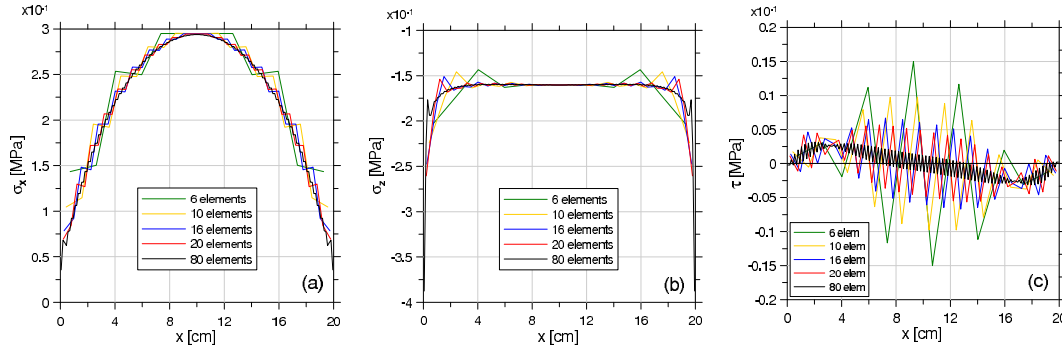


Figure 2: $E_m = 30 E_b$ and two Gauss point integration: (a) internal, (b) normal and (c) tangential stresses.

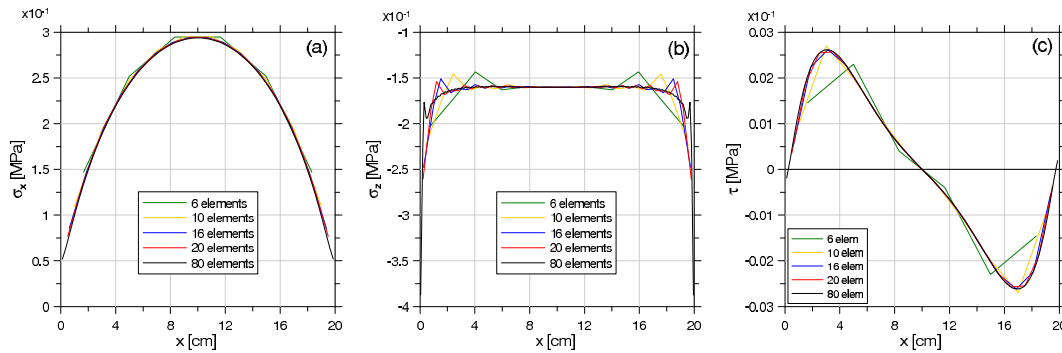


Figure 3: $E_m = 30 E_b$ and Selective Reduced Integration: (a) internal, (b) normal and (c) tangential stresses.

4 CONCLUSIONS

The interphase model represents the enhancement of the zero-thickness interface model. The numerical implementation using standard Gauss quadrature shows stress oscillations that can be solved making use of the RSI or of the EAS. The RSI approach is preferred because no additional variables are required and the stiffness matrix is not modified which is important in non linear applications.

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