INTERFACE MODEL FOR THE NONLINEAR ANALYSIS OF BLOCKY STRUCTURES OF ANCIENT GREEK TEMPLES

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ABSTRACT: The presence of singularity surfaces with reference to the displacement field is a characteristic of a number of structural systems. Strong discontinuities are present in old masonry structures where dry joints connect the blocks or the mortar ageing suggests to neglect the adhesion properties.
These structures cannot be considered a continuum but rather an assembly of blocks. These discontinuous structures could be modelled as an assembly of blocks interacting trough frictional joints whose mechanical behaviour is described by appropriate interface laws.
In the present work an interface model present in literature is adopted, the double asperity model, which has been implemented in a standard finite element code with the principal aim to develop structural analysis of old monumental masonry structures.
The interface model is briefly illustrated and the numerical implementation of the interface laws is described in detail.
Numerical examples are presented to simulate the behaviour of a couple of greek temples of Agrigento Italy. These old monumental structures, IV-VI sec. BC, are inserted in the world heritage list by Unesco.

1 INTRODUCTION

There is a wide interest in the research community to develop mechanical models and numerical tools capable to reproduce the structural behaviour of historical monumental buildings. The presence of strong discontinuities, typical of blocky structures where dry joints connect the blocks, and the way in which the deformation modes of the joints are modelled is the crucial point in the choice of the analysis tool.

Different approaches could be found in literature. In the *Continuum* approach the structure is modelled as an anisotropic but homogenous continuum with smeared characteristics, typical examples are *notension* material (Fuschi et al. 1995), Cosserat continuum theory (Cerrolaza et al. 1995), nonsmooth multisurface plasticity (Mistler et al. 2006). All these approaches tries to incorporate the deformation modes of the joints in the constitutive laws even though considering an homogenous system.

In the *Discrete* approach the structure is considered as an assembly of blocks connected by contact joints. In this way the deformation modes of the joints could be modelled by apposite interface laws (Chuhan et al. 1997). In the present work the second approach is followed using interface laws derived from the *double asperity* interface model (Mróz and Giambanco 1996).

The remainder of the paper is organized as follows. In Section 2, the simplified version of the interface model is briefly illustrated. Section 3 describes the local integration of the constitutive laws. In Section 4 numerical examples regarding a couple of famous historical greek temples are presented.

2 INTERFACE MODEL

The model adopted to simulate the contact between the masonry blocks is the double asperity model (Mróz and Giambanco 1996).

Referring to figure 1 the classical interface model assumes that the contact surface between two bodies Ω^1 and Ω^2 could be assumed as a as a contact layer of thickness *h*. In the double asperity model the planar joint is characterized by spherical asperities of different radius.

Considering a local reference system (O, $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$) with $\mathbf{e}_1$ and $\mathbf{e}_2$ on the middle plane π of the contact layer and $\mathbf{e}_3$ oriented along the normal to π and directed in to the body Ω^1, the discontinuity displacement vector at the contact layer is expressed by the *sliding* displacement discontinuities $[u_1]$ and $[u_2]$, along $\mathbf{e}_1$ and $\mathbf{e}_2$, respectively, and by the *separation* displacement discontinuity $[u_N]$, along $\mathbf{e}_3$:

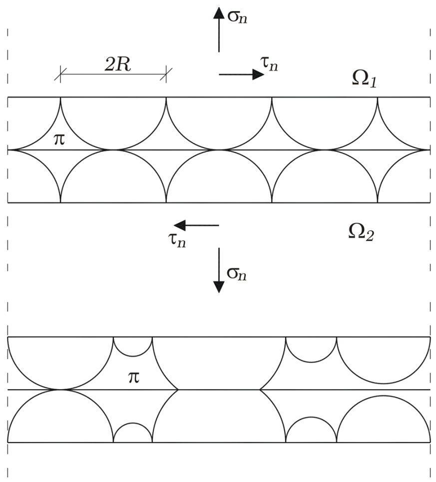

Figure 1: Spherical asperity model.

$$[\mathbf{u}] = [u_1]\mathbf{e_1} + [u_2]\mathbf{e_2} + [u_N]\mathbf{e_3}. \tag{1}$$

Similarly the traction vector $\boldsymbol{\sigma}$ can be expressed as:

$$\boldsymbol{\sigma} = \tau_1\mathbf{e_1} + \tau_2\mathbf{e_2} + \sigma_N\mathbf{e_3}. \tag{2}$$

Following the formulation of the spherical asperity model, (Mróz and Giambanco 1996), and simplifying the problem by neglecting micro-slip effects, the elastic displacement discontinuities are related to the contact stresses by the constitutive law which takes the form:

$$[\boldsymbol{\sigma}] = \beta\mathbf{K}[\mathbf{u}]^e; \tag{3}$$

where β is the contact factor which provides a measure of the actual contact area with respect to the nominal area within the plane π, defined as:

$$\beta = \left(\frac{-3 < [u_N] >}{h}\right)^{\frac{p}{3-p}}, \tag{4}$$

where $< x > := \dfrac{(x + |x|)}{2}$ denotes the Mac Auly brackets,

for the parameter p, introduced to account for complex asperity interaction with initial gap distribution, assumes the unit value only in the case of uniform spherical asperities, like in the present work, figure 1.

The elastic stiffness matrix reported in equation (3), is easily calculated from the elastic properties of the material constituting the asperities:

$$\mathbf{K} = \frac{1}{h}\mathrm{diag}\left[\begin{array}{ccc} E_{t1} & E_{t2} & E_N \end{array}\right]; \tag{5}$$

with:

$$E_{t1} = E_{t2} = \frac{2G}{2-\nu}; \qquad E_N = \frac{2G}{1-\nu}; \tag{6}$$

where G is the tangential elastic modulus and ν is the Poisson's coefficient of the material.

The sliding limit condition used is the classical Mohr-Coulomb one, thus :

$$\Phi(\boldsymbol{\sigma}) = |\boldsymbol{\tau}| + \sigma_N \tan\phi = 0, \tag{7}$$

where ϕ is is the friction angle between the asperities surfaces. The anelastic part of the discontinuity displacement can be obtained from the following non associative flow rule:

$$[\dot{\mathbf{u}}]^s = \dot{\lambda}\frac{\partial\Gamma(\boldsymbol{\sigma})}{\partial\boldsymbol{\sigma}}, \tag{8}$$

where $\Gamma(\boldsymbol{\sigma})$ is defined as:

$$\Gamma(\boldsymbol{\sigma}) = |\boldsymbol{\tau}|; \tag{9}$$

and $\dot{\lambda}$ is the sliding multiplier which satisfies the following complementarity conditions:

$$F(\boldsymbol{\sigma}) \leq 0; \qquad \dot{\lambda} \geq 0; \qquad \dot{\lambda}F(\boldsymbol{\sigma}) = 0. \tag{10}$$

3 DISCRETE INTERFACE LAWS

The present section concerns with the evaluation of the state variables obtained by integration of the interface laws for a given sequence of total strain increment history and with assigned initial conditions.

Let $[0, T] \subset \mathbb{R}$ be the time interval of interest and assume that the state variables at time $t \in [0, T]$ or:

$$\{[\mathbf{u}]_k, [\mathbf{u}]_k^s\} \tag{11}$$

are known given data at time t_k.

The elastic displacement discontinuities $[\mathbf{u}]_k^e$, the contact factor β_{k+1}, and the contact tractions $\boldsymbol{\sigma}_k$ can be regarded as dependent variables that can be evaluated from equation (3)-(6) and (7)-(10).

Let us consider a subsequent time $t_{k+1} \in [0, T]$ and let denote with $\Delta[\mathbf{u}]_k$ the respective incremental displacement discontinuity field which is assumed to be given. The basic problem is to update the field variables at t_{k+1} in a manner consistent with the interface constitutive equations presented in the previous sections. Equation 8 define a rate non linear evolution problem with initial conditions (11). This rate problem can be transformed into a discrete one by applying an implicit backward-Euler difference integration scheme:

$$[\mathbf{u}]_{k+1} = [\mathbf{u}]_k + \Delta[\mathbf{u}]_k; \tag{12}$$

$$[\mathbf{u}]_{k+1}^s = [\mathbf{u}]_k^s + \mathbf{R}^t\frac{\partial G}{\boldsymbol{\sigma}}\Delta\lambda_k; \tag{13}$$

$$\boldsymbol{\sigma}_{k+1} = \frac{1}{h}\beta_{k+1}\mathbf{R}^T\mathbf{E}\mathbf{R}([\mathbf{u}]_{k+1} - [\mathbf{u}]_{k+1}^s); \tag{14}$$

$$\beta_{k+1} = \sqrt{3\frac{< -[u_N]^{*,e}_{k+1} >}{h}}. \tag{15}$$

The above reported discrete equations may be regarded in the context of a two-step-algorithm, splitting the previous problem, in additive manner, in an *trial elastic predictor* stage and in a *plastic corrector* stage. In the time step $[t_k, t_{k+1}]$ the elastic predictor stage leads to the following:

$$predictor \qquad \| \; [\mathbf{u}]^{s,trial}_{k+1} = [\mathbf{u}]^s_k. \tag{16}$$

The dependent variables assume the values:

$$[\mathbf{u}]^{e,trial}_{k+1} = [\mathbf{u}]_{k+1} - [\mathbf{u}]^{s,trial}_{k+1}, \tag{17}$$

$$\boldsymbol{\sigma}_{k+1} = \frac{1}{h}\beta_{k+1}\mathbf{R}^T_k\mathbf{K}\mathbf{R}_k([\mathbf{u}]_{k+1} - [\mathbf{u}]^s_{k+1}). \tag{18}$$

If $f(\boldsymbol{\sigma}^{*,trial}_{k+1}) \leq 0$ the process is elastic in the step and no *corrector* stage is further required. Otherwise the *corrector* stage is performed:

$$\left\| \begin{array}{c} \Delta\boldsymbol{\gamma}^s = \mathbf{R}_\gamma \left(\boldsymbol{\gamma}^{*e,trial} + [u_N]^* \frac{E_n}{E_t} \tan\phi \hat{\boldsymbol{\gamma}}^{*e,trial}\right) \\ [u_N]^* = \cos(\alpha)[u_N]^{e,trial} - \sin(\alpha)\hat{\boldsymbol{\gamma}^{sT}}\boldsymbol{\gamma}^{e,trial} \\ \boldsymbol{\gamma}^{*e,trial} = [u_N]^{e,trial}\hat{\boldsymbol{\gamma}}^s \sin(\alpha) + \mathbf{R}_\gamma\gamma_{e,trial}. \end{array} \right. \tag{19}$$

Equations (19) represent a non-linear equation system in the unknowns $\Delta\boldsymbol{\gamma}^s$, that can be solved with a local Newton Rhapson procedure.

The interface laws presented in this section was implemented in an open source finite element code OOFEM (Patzák and Bittnar 2001).

4 NUMERICAL EXAMPLES

The numerical applications regard the analysis of a couple of greek temples of Agrigento in Italy. The temples of *Giunone Lacinia* and *Concordia* are old monumental structures that belong to the ancient greek city of *Akragas*, examples of extraordinary monumental complex of *Valle dei Templi di Agrigento* inserted in the world heritage list by Unesco from 1997.

The material used for the temples is a sandstone rock typical of the zone called *calcarenite*. Mainly formed by erosion and re-deposition of other rocks contains elevate amount of shell fragments, near 50%, cemented by calcium carbonate.

The blocks of *calcarenite* are modelled by brick elements and are considered elastic. The planes of contact between block are modelled by interface elements and are considered the source of all nonlinearities of the behaviour.

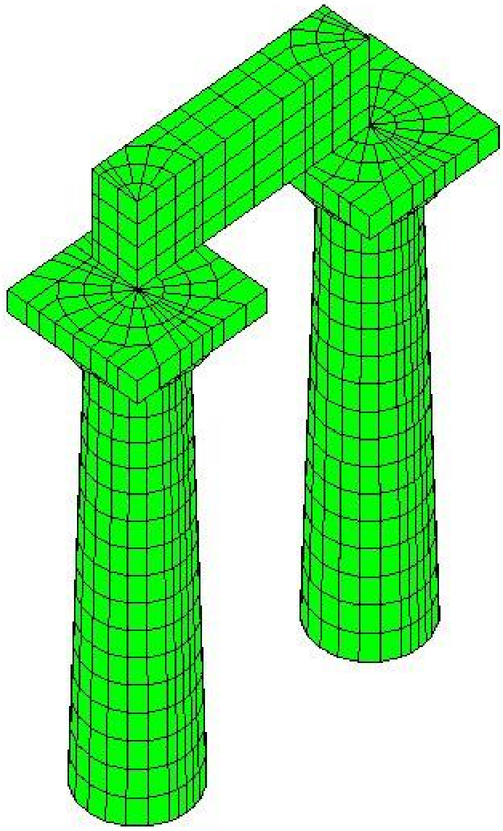

Figure 2: Temple of Giunone Lacinia. Trilite finite element model mesh.

For the temple of *Giunone Lacinia* it was modelled a structural element composed by two columns and the architrave, *trilite*. In Figure 2 the finite element mesh for the *trilite* is reported, with 2648 bricks, 632 interface elements at the interface plans and 4244 nodes in total.

Six different planes of contact have been discretized: one plane at the base, three planes between the blocks of the columns, one plane between the columns and the capital, and one between the capital and the architrave, figure 3.

The trilite was subjected to dynamic analysis and a time-history of acceleration, obtained by a response spectrum defined according to European Standard EC 8, applied to the interface plan at the base where the nodes are fixed.

In figure 4 the finite element model response is showed in terms of normal discontinuities (opening-closing of the joint) at various interface plans. The maximum values are obtained for interface plan 2 between the first and the second block of the column south and not for the interface plan 1 as for monolitical structure could be expected. Due to the lack of space the results for column north are not reported but the behaviour is similar to south column.

For the temple of *Concordia* the complete west front, composed by six columns, the architrave and the *timpano* was modelled. In this case the structure was subjected to a pseudo-static analysis with increas-

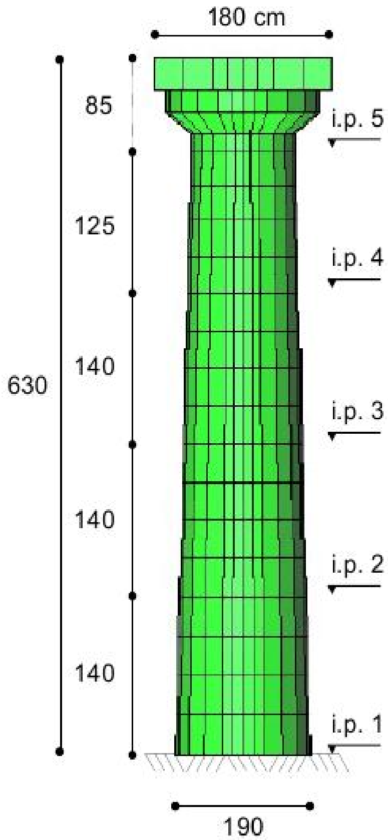

Figure 3: Doric column dimensions and interface plans.

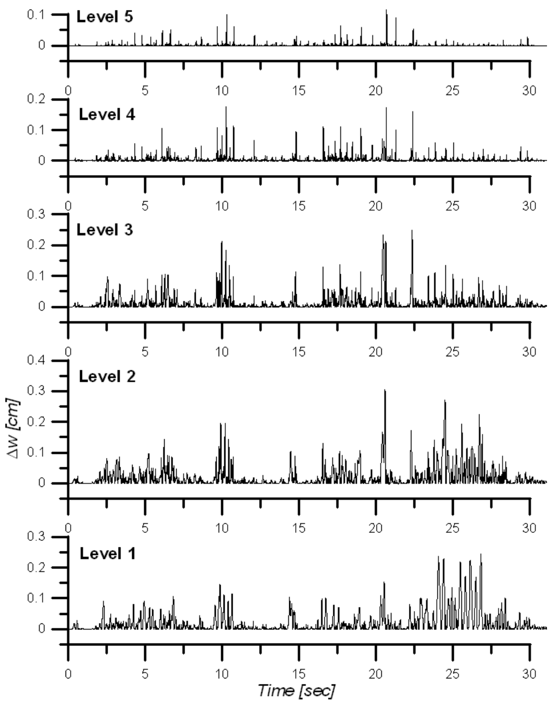

Figure 4: Map of normal discontinuities, column south, at various interface plans.

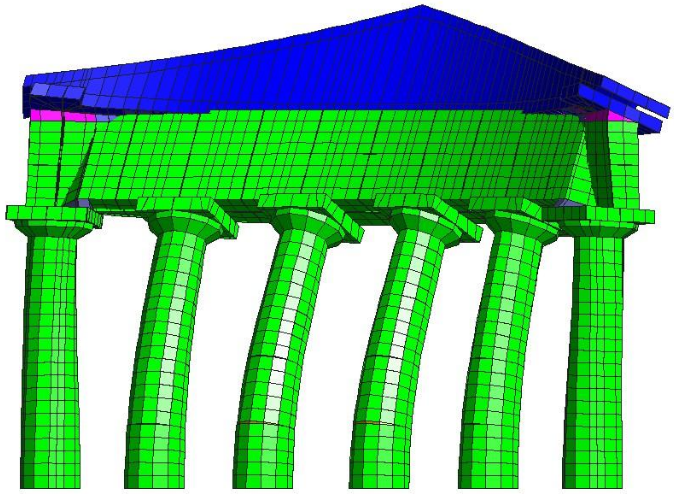

Figure 5: Temple of Concordia, West front. Displacements at incoming collapse.

ing horizontal forces till the collapse. In figure 5 the deformed finite element model at the collapse, obtained from this analysis, is showed.

5 CONCLUSIONS

The paper presents a procedure for analyze the seismic behaviour of historical blocky structures.

The structure is suitably modelled as an assembly of blocks interacting through frictional joints whose nonlinear mechanical behaviour is described by appropriate interface laws.

The pratical and efficient application of the procedure is showed by the seismic analysis of a couple of old monumental masonry structures.

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