Numerical insights of an improved SPH method

Elisa Francomano[1], Marta Paliaga[2], Guido Ala[3]

[1] *Università degli Studi di Palermo, Scuola Politecnica, DIID*
[2] *Università degli Studi di Palermo, Scuola Politecnica, DIID*
[3] *Università degli Studi di Palermo, Scuola Politecnica, DEIM*

Abstract

In this paper we discuss on the enhancements in accuracy and computational demanding in approximating a function and its derivatives via Smoothed Particle Hydrodynamics. The standard method is widely used nowadays in various physics and engineering applications [1],[2],[3]. However it suffers of low approximation accuracy at boundaries or when scattered data distributions is considered. Here we reformulate the original method by means of the Taylor series expansion and by employing the kernel function and its derivatives as projection functions and integrating over the problem domain [3]. In this way, accurate estimates of the function and its derivatives are simultaneously provided and no lower order derivatives are inherent in approximating the higher order derivatives. Moreover, high order of accuracy can be obtained without changes on the kernel function avoiding to lead unphysical results such as negative density or negative energy that can lead to breakdown of the entire computation in simulating some problems [1]. The modified scheme obtains the required accuracy, but the high computational effort makes the procedure rather expensive and not easily approachable in the applications. To this aim we make use of fast summations to generate a more efficient procedure, allowing to tune the desired accuracy. Working with the Gaussian function we proceed by applying the improved fast Gaussian transform as valid alternative to efficiently compute the summations of the kernel and its derivatives [4]. We discuss about the accuracy and the computational demanding of the improved method dealing with different sets of data and bivariate functions.

References

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