RC beams retrofitted by FRP oriented in any direction: influence of the effectiveness factors

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Abstract

Shear strength of FRP-retrofitted RC beams is usually influenced by different brittle failure modes characterizing the collapse of the FRP reinforcement. The most significant analytical models for assessing the shear strength of FRP-retrofitted RC beams reflect the effect of brittle failure through an effectiveness factor "*R*", which reduces the ultimate tensile strength of FRP. The brittle collapse of FRP reinforcement often leads to a lower shear contribution by steel stirrups due to hindering of the yielding of all stirrups involved by critical cracking. Some analytical models consider this phenomenon introducing a further effectiveness factor "*r*", which reduces the yielding strength of stirrups. The key differences characterizing many of the shear models are represented by the expressions used for assessment of the effectiveness factors. This paper focuses on the influence of effectiveness factor models in analytical predictions provided by different shear models. The reduction of the shear contribution of steel stirrups due to FRP brittle failure is modeled as dependent on the FRP wrapping scheme, through the ratio between FRP effective strain and steel yielding strain. To highlight the influence of the effectiveness factor for steel stirrups, two analyses are performed through different shear models, considering the effectiveness factor for FRP only, and considering the effectiveness factor for both FRP reinforcement and steel stirrups. The results are discussed considering two databases, one constituted by beams whose FRP reinforcement and steel stirrups are arranged at right angles to the beam axis, and one in which the FRP is arranged at angles different from 90°.

Keywords: Effectiveness factor; FRP strengthening; Stress field; Variable inclination; Inclined FRP.

1 INTRODUCTION

Assessment of the shear capacity of RC beams strengthened with externally bonded Fiber-Reinforced Polymer (EB-FRP) is still a topic widely debated in the literature [1-4]. As a matter of fact, it is challenging to properly interpret and take into account the several resisting mechanisms involved in the shear strength of retrofitted RC beams (e.g., influence of cross-section shape on the compressed concrete strength, aggregate interlock, size effect, dowel effect), as well as the effect of the strengthening system and its real contribution to the overall shear strength. Experimental tests show that most retrofitted RC beams fail in shear when FRP failure occurs, which is mainly due to debonding, or tensile failure [5, 6]. Due to early debonding, or stress concentration around the corners of RC members, the tensile failure of FRP strips/sheets usually occurs at strain values smaller than those attained when testing FRP specimens with direct tensile tests. The tensile strains experienced by those FRP strips/sheets and steel stirrups involved within the shear-critical crack can vary significantly during the loading process [7]. However, most existing models only focus on providing equations to predict strain values at failure, using an effectiveness factor R that aims to limit the contribution provided by FRP reinforcement to shear capacity. Some effectiveness coefficients consider the interaction with the internal transverse reinforcement, which, limiting the widening of the crack, delays debonding of the FRP reinforcement. Moreover, experimental results show that brittle failure of FRP often occurs before all stirrups intersected by shear-critical crack yield, potentially leading to a reduced shear strength contribution provided by the steel stirrups [8-10].

Starting in 2002, Pellegrino and Modena stressed the interaction between internal steel transverse reinforcement and external FRP both for side-bonded and U-wrap configurations [11]. Monti and Liotta [8] noted that “it is not guaranteed that both concrete and steel stirrup can exploit their maximum strength when in the presence of FRP strengthening.” Grande et al. [10] explored the interaction between FRP and steel transverse reinforcement resisting action, analyzing the deformation behavior of shear resisting systems. They found that steel transverse reinforcement at failure yielded only when a reduced amount of FRP was used. Boussalem and Chaallal [12] noted that transverse steel reinforcement contributes to the load-carrying capacity only after diagonal cracking occurs, and in slender specimens they yield well before failure. The addition of FRP reduces the contribution of the transverse steel reinforcement depending on the FRP axial stiffness, although at failure yielding of the transverse steel occurred in most cases [13].

Chen at al. [14] modeled the interaction between externally bonded FRP and internal steel reinforcement by means of a parabolic crack shape function to represent the widening process of an RC beam single shear crack, reproducing the adverse interaction effect in side-bonded FRP reinforcement, and limiting the adverse effect in U-strips as a function of FRP axial stiffness.

Despite the experimental evidence, most shear capacity models still do not take this phenomenon into account, and thus they overestimate the effective contribution to the shear capacity provided by the steel stirrups. It is noteworthy that the reduced efficiency of stirrups worsens when the FRP reinforcement is arranged with a smaller inclination, with respect to the RC member axis, than that of the steel stirrups. In this case, the effective stirrup strain is lower than the effective FRP strain and thus the steel stirrup shear strength contribution is further reduced.

Recently, Colajanni, Guarino, and Pagnotta. [15] formulated a design-oriented analytical model (CGP model) able to calculate the shear strength of RC beams retrofitted with FRP reinforcement arranged in any direction in which both the two aforementioned adverse interactions are considered. The model is formulated to represent an extension of the EN1992 shear model to beams retrofitted with FRP, widening the field of application to FRP reinforcement arranged in different directions than that of the internal pre-existing stirrups, a configuration which when code models are applied to it requires arbitrary approximation.

Within this framework, this paper focuses on the influence of effectiveness factor models in the analytical shear capacity predictions provided by ACI 440.2R-17 [16], CNR DT 200 R1/2013 [17] and CGP [15], which are the best performing models among those compared in [15]. With reference to the effectiveness factor R for FRP reinforcement, the models proposed by ACI [16], CNR [17], fib [18], Chen and Teng (C&T) [6, 7], and Mofidi and Challal (M&C) [19] are used, as well as the model used in Colajanni et al. [15]. The latter was derived by including different formulations proposed by Khalifa and Nanni [20, 21], and Pellegrino and Modena (K&N+P&M) [22]. As regards the effectiveness factor r for steel stirrups, the model described in Colajanni et al. [15] is used, which was developed by modification of the formulation proposed in [9]. The advantage of the r model in [15] is that is able to take into account the fact that, at beam failure, the strain of the most elongated steel stirrup is limited to that of the FRP reinforcement, and that not all the stirrups intercepted by shear critical crack have the same strain state. This effectiveness factor considers the influence of FRP reinforcement on the efficacy of steel stirrups by means of the ratio between the effective strain of the FRP reinforcement at failure and the yielding strain of the steel. The model was extended to include the case in which FRP reinforcement and steel stirrups are arranged with different inclinations.

To perform the comparison between the shear capacity models investigated, a database of 158 specimens was collected, containing RC beams having both rectangular and T-shaped cross-sections, reinforced with both steel stirrups and FRP strips or sheets, the latter oriented in any direction.

To highlight the influence of the effectiveness factor for steel stirrups, two different analytical predictions were carried out by means of the shear capacity models investigated, i.e. considering the effectiveness factor for

FRP reinforcement only, and considering the effectiveness factor for both FRP reinforcement and steel stirrups. The results are discussed considering two partial databases, one constituted by results of specimens with FRP reinforcement and steel stirrups arranged at right angles with respect to the beam axis, and one in which the FRP reinforcement is arranged at an angle β different from 90°.

Concerning shear capacity models based on the truss mechanism with variable inclination of the concrete strut (e.g. CNR), some uncertainties arise when evaluating shear strength provided by the concrete strut, due to the lack of a single method to select the angle of shear reinforcement to be used when FRP reinforcement and steel stirrups are arranged with different angles. To point out the effect of the concrete strut inclination on shear capacity prediction, a comparison is carried out between the CNR model, the one developed by Colajanni et al. [15], and the modified version of the CNR model proposed in [15]. The modified version of the CNR model, named CNRm, calculates an equivalent angle of shear reinforcement involved in the shear capacity assessment of the concrete strut, given by a linear combination of the angles of steel stirrups and FRP reinforcement weighted by the shear capacity provided by each of them.

2 EFFECTIVENESS FACTOR FOR FRP

The shear failure of retrofitted RC beams observed during experimental tests is usually due to failure of the FRP reinforcement. The commonest modes of failure of FRP are the following [6, 7]: FRP tensile failure; debonding of FRP from concrete; excessive shear-critical crack width and subsequent significant reduction of the contribution provided by aggregate interlock; and peeling off of the concrete cover from the concrete core of the beam (e.g. [22]). Shear models usually consider these phenomena by assuming a reduced effective tensile strength of FRP $f_{fe} = E_f \times \varepsilon_{fe}$, smaller than its ultimate tensile strength $f_{fu} = E_f \times \varepsilon_{fu}$, which is obtained by introducing an effectiveness factor R ($\varepsilon_{fe} = R\ \varepsilon_{fu}$). During the last twenty years, several proposals have been formulated to evaluate the R factor. In this paper, six different formulations are compared to evaluate their effectiveness when used in the CGP model and in the ACI and CNR models. Two of these six formulations are those used in the CGP model [15]: the first approach (CGP1) uses the equations provided by Khalifa & Nanni and Pellegrino & Modena [20-22], while the second approach (CGP2) uses the equations provided by Chen & Teng [6, 7]. The other four formulations are those of ACI [16], CNR [17], fib [18], and Mofidi and Chaallal [19].

First approach (CGP1 model [15]): Khalifa and Nanni [20, 21] and Pellegrino and Modena [22]

According to the first approach used in [15], the value of R is the lowest among the three effectiveness factors R_i (i = 1, 2, 3) given in [20, 21] and the fourth R_4 provided in [22], i.e. $R = min\{R_1, R_2, R_3, R_4\}$.

The equations used to describe the four modes of failure are given below. The efficiency factor R_1, which considers FRP tensile failure, is calculated via the following equation:

$$R_1 = 0.56\left(\rho_{fw}E_f\right)^2 - 1.22\left(\rho_{fw}E_f\right) + 0.78 \tag{1}$$

in which E_f is the elastic modulus of the fibers. The coefficient R_2, representing the debonding phenomenon, is calculated using the following equation:

$$R_2 = \frac{\left(f_{ck}\right)^{\frac{2}{3}}\left(d_{ft} - \eta L_e\right)\left[738.93 - 4.06\left(E_f t_f\right)\right]}{\varepsilon_{fu} d_{ft} 10^6} \tag{2}$$

in which L_e is the effective length, which is evaluated using the expression given by ACI and CSA:

$$L_e = 23300 / \left(E_f t_f\right)^{0.58} \tag{3}$$

η is a parameter that considers the anchorage conditions, equal to 1 or 2 if the shear strengthening is U-shaped or side-only, respectively. The range of validity of Eq. (3) is $20 \leq E_f t_f \leq 90$. In the case of complete wrapping or U-shaped strengthening with anchorages able to prevent a debonding effect, the coefficient R_2 is not considered. The coefficient R_3, which takes into account the shear crack width, is:

$$R_3 = 6 \cdot 10^{-3} / \varepsilon_{fu} \tag{4}$$

In the case of side-bonding and U-jacketing reinforcement, FRP failure often involves the separation of the concrete cover along a vertical plane (peeling off). Hence, the factor suggested in [22] is assumed equal to:

$$R_4 = \frac{2 f_{ct} A_c \cos^2 \beta b_{c,v}}{n_f t_f L_f E_f \left[\left(h_f - L_e\right) / \left(h_f\right)\right] b_f \varepsilon_{fu}} \tag{5}$$

Further details about the parameters involved in Eq. (5) can be found in [22].

Second approach (CGP2 model [15]): Chen and Teng [6, 7]

As suggested in [6, 7], the effectiveness factor is the lowest of two effectiveness coefficients, i.e. $R = \min\{R_5, R_6\}$. The effectiveness factor R_5 considers FRP tensile failure, which usually occurs across the critical crack, correlating it with non-uniform strain distribution in the FRP along a shear crack. The authors, assuming proportionality of the fiber strain to the width of the shear crack, assumed an approximate strain linear distribution, where the FRP strain increases linearly from a minimum at the crack tip to a maximum at the lower end. Thus, R_5 can be expressed as:

$$R_5 = \frac{1 + (h_w - d_f)/z}{2} \tag{6}$$

where d_f = height of the FRP. Since experimental observations show that this kind of failure mode usually occurs in fully wrapped or U-wrapped RC beams, this coefficient must only be considered for these arrangements. By contrast, the coefficient R_6 considers failure through debonding of FRP, which may occur when the bond length is not sufficient. According to [6] the stress in the FRP is variable along the bond length, and its maximum stress ($\sigma_{f,max}$) can be given as:

$$\sigma_{f,\max} = 0.427 \beta_w \beta_L \sqrt{\frac{E_f \sqrt{f_c^{'}}}{t_f}} \leq E_f \varepsilon_{fu} \tag{7}$$

where β_w = coefficient of the FRP-to-concrete-width ratio and β_L = bond length coefficient. These values can be calculated by using the following equations:

$$\beta_w = \sqrt{\frac{2 - w_f / s_f \sin \beta}{1 + w_f / s_f \sin \beta}} \tag{8}$$

$$\beta_L = \begin{cases} 1 & \text{if } \lambda \geq 1 \\ \sin \dfrac{\pi \lambda}{2} & \text{if } \lambda < 1 \end{cases} \tag{9}$$

where $\lambda = L_{max}/L_e$ = normalized maximum bond length, in which the maximum (L_{max}) and the effective bond length (L_e) are respectively:

$$L_{\max} = \begin{cases} \dfrac{h_{frp,e}}{\sin \beta} & \text{for U - wrap} \\ \dfrac{h_{frp,e}}{2 \sin \beta} & \text{for side bonded} \end{cases} \tag{10}$$

$$L_e = \sqrt{E_f t_f / \sqrt{f_c^{'}}} \tag{11}$$

where $h_{frp,e}$ is the effective height of the FRP (further details can be found in [6]). Thus, the effectiveness reduction factor R_6 can be expressed as:

$$R_6 = \frac{\sigma_{f,\max}}{E_f \varepsilon_{fu}} \begin{cases} \dfrac{2}{\pi\lambda} \dfrac{1-\cos\dfrac{\pi\lambda}{2}}{\sin\dfrac{\pi\lambda}{2}} & \text{for } \lambda < 1 \\ 1-\dfrac{\pi-2}{\pi\lambda} & \text{for } \lambda \geq 1 \end{cases} \tag{12}$$

3 EFFECTIVENESS FACTOR FOR STEEL STIRRUPS

In the case of shear reinforcement constituted by both FRP reinforcement and steel stirrups, the shear strength contribution provided by them is reduced [8-10], due to their mutual interaction. In this connection, the shear contribution provided by the FRP reinforcement decreases when the axial stiffness ratio between steel stirrups and FRP reinforcement increases [22], while the shear contribution provided by the steel stirrups is limited by the fact that few or no stirrups passing through the shear-critical crack yield due to brittle failure of FRP [8-10]. To take these phenomena into account, several models have been proposed, for which the interaction between the two shear reinforcements is related to their stiffness [13], or to the shear-critical crack width [25], or considering a fixed coefficient $\alpha = 0.75$ [9].

Starting from the r parameter provided by [9], Colajanni et al. [15] derived an r reduction parameter which is expressed as a bi-linear equation. It relates the decrease in shear contribution provided by the steel stirrups to the ratio between the component of the effective strain acting on the FRP reinforcement in the direction of the steel stirrups $\varepsilon_{fe,sd}$ and the steel stirrups' yielding strain ε_{syw}:

$$r = \begin{cases} 0.75\dfrac{\varepsilon_{fe,sd}}{\varepsilon_{syw}} & \text{if } \varepsilon_{fe,sd}/\varepsilon_{syw} \leq 1.33 \\ 1 & \text{if } \varepsilon_{fe,sd}/\varepsilon_{syw} > 1.33 \end{cases} \tag{13}$$

in which $\varepsilon_{fe,sd} = \varepsilon_{fe} \cos(\beta-\alpha)$, where α and β are the angles between steel stirrups or FRP reinforcement and the longitudinal axis of the beam, respectively.

The preceding effectiveness factor takes into account the following phenomena due to the effective strain of

FRP reinforcement at beam failure: the strain limitation of the most elongated transverse steel reinforcement; the different strains attained by the steel stirrups intersected by the shear-critical crack. More precisely, in the case of $\varepsilon_{fe,sd}/\varepsilon_{sy} = 1$, only the most elongated stirrup yields. Even if the ratio $\varepsilon_{fe,sd}/\varepsilon_{sy}$ is greater than 1, the factor *r* can assume values less than 1, so that the different strains acting on the stirrups passing through the shear-critical crack can be considered.

As shown in the following sections, the *r* factor proposed by [9] and modified in [15] can reduce the inaccuracy of the shear models investigated caused by overestimation of the shear strength provided by steel stirrups.

4 COLAJANNI ET AL. SHEAR MODEL [15]

The shear model proposed by Colajanni et al. [15] (CGP model) is based on the stress field theory with variable inclination of the concrete field, and it was developed to evaluate the shear strength of RC beams retrofitted with FRP reinforcement arranged in any direction. The shear strength is calculated by using three different equations, obtained by evaluating the vertical equilibrium of a segment of beam, which is identified by means of three different sections.

Based on the same hypotheses as the shear models reported in [23, 24], the shear strength of a retrofitted RC beam can be evaluated using three different equations, given by the vertical equilibrium of beam segments, which are identified through three different sections parallel to the directions of stress fields of FRP reinforcement, steel stirrups, and concrete stress field (Figure 1).

Two stress fields only are involved in each of the three equilibrium equations, making it easier to solve them:

$$v = r\,\tilde{\sigma}_{sw}\,\omega_{sw}\left(\cot\theta+\cot\alpha\right)\sin^2\alpha + R\,\tilde{\sigma}_{fw}\,\omega_{fw}\left(\cot\theta+\cot\beta\right)\sin^2\beta \tag{14}$$

$$v = \tilde{\sigma}_{cw}\left(\cot\theta+\cot\alpha\right)\sin^2\theta + R\,\tilde{\sigma}_{fw}\,\omega_{fw}\left(\cot\beta-\cot\alpha\right)\sin^2\beta \tag{15}$$

$$v = \tilde{\sigma}_{cw}\left(\cot\theta+\cot\beta\right)\sin^2\theta + r\,\tilde{\sigma}_{sw}\,\omega_{sw}\left(\cot\alpha-\cot\beta\right)\sin^2\alpha \tag{16}$$

where *v* is the non-dimensional shear strength, while $\omega_{sw} = \left(A_{sw}f_{syw}\right)/\left(b_w s_w \sin\alpha f'_{cm}\right) = \rho_{sw}f_{syw}/f'_{cm}$, $\omega_{fw} = \left(2b_f t_f f_{fu}\right)/\left(b_w s_f \sin\beta f'_{cm}\right) = \rho_{fw}f_{fu}/f'_{cm}$, $\tilde{\sigma}_{sw} = \sigma_{sw}/f_{syw}$, and $\tilde{\sigma}_{fw} = \sigma_{fw}/f_{fu}$ are the mechanical ratios and the non-dimensional stresses of steel stirrups and FRP reinforcement, respectively, f_{fu} is the ultimate stress of the fiber, *R* the coefficient which considers the "effective" strain and stress acting on the

FRP at failure (effective strain $\varepsilon_{fe}=\varepsilon_{fu}\,R$, effective stress $f_{fe}=f_{fu}\,R=E_f\,\varepsilon_{fe}$), and r is the coefficient which considers the efficiency of the steel stirrups involved by shear critical crack. The mechanical ratios of shear reinforcements ω_{sw} and ω_{fw} are equal to the product between the geometrical ratios of the shear reinforcements and the ratio between their ultimate/yielding stress and the reduced compressive strength of the concrete.

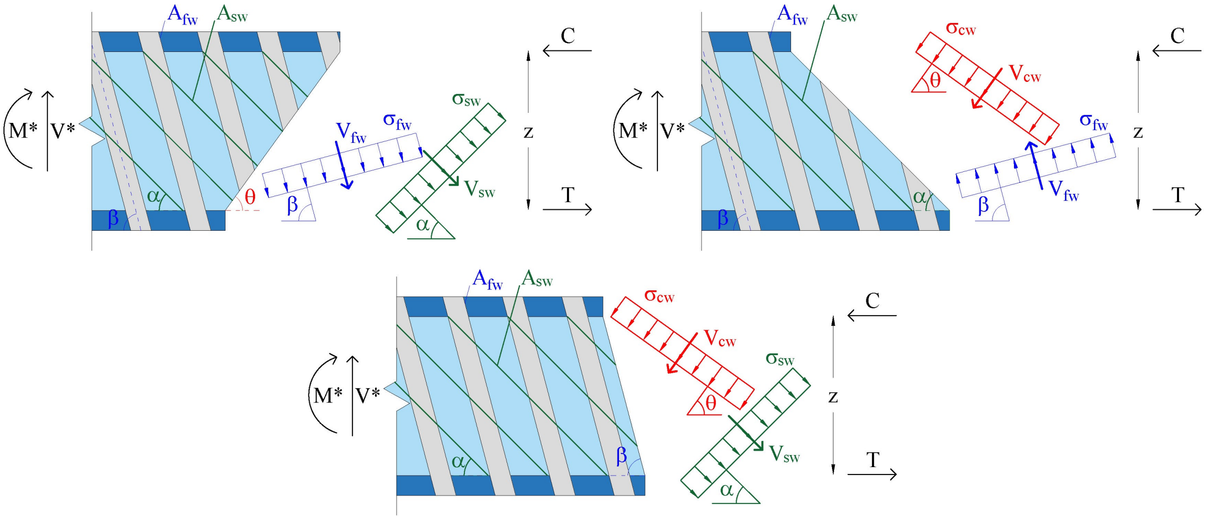

Figure 1. Beam segments identified via three sections parallel to the stress field directions of (a) concrete strut; (b) steel stirrups; (c) FRP reinforcement

The shear strength of an RC beam is evaluated by means of the static theorem of plasticity, through which the evaluation of the shear strength is given as the maximum value among the solutions satisfying not only the equilibrium equations (14)-(16), but also the plastic admissibility equations given below:

$$0 \le \tilde{\sigma}_{cw},\ \tilde{\sigma}_{fw} \le 1 \qquad -1 \le \tilde{\sigma}_{sw} \le 1 \tag{17}$$

By combining (14), (15) and (17), the following inequality is obtained, which clarifies the relation between the stress fields of the FRP reinforcement, steel stirrups and concrete strut:

$$0 \le \tilde{\sigma}_{cw} = \left(r\,\tilde{\sigma}_{sw}\omega_{sw}\sin^2\alpha + R\,\tilde{\sigma}_{fw}\,\omega_{fw}\sin^2\beta\right)\left(1+\cot^2\theta\right) \le 1 \tag{18}$$

To assess the shear capacity, the model initially assumes that, at failure, the three stress fields of FRP reinforcement, steel stirrups and concrete strut reach their stress limits at the same time (i.e. $\tilde{\sigma}_{cw} = \tilde{\sigma}_{fw} = \tilde{\sigma}_{sw} = 1$). Hence, by using the upper limit given by Eq. (18), the value of the inclination of concrete strut is calculated as follows:

$$\cot\theta = \sqrt{\left(r\,\omega_{sw}\sin^2\alpha + R\,\omega_{fw}\sin^2\beta\right)^{-1} - 1} \qquad (19)$$

Considering $\cot\theta_{lim} = 2.5$, three cases are identified:

- $1 \leq cot\theta \leq 2.5$: the three stress fields reach their stress limits at the same time, and the shear strength is calculated via Eq. (14) considering $\tilde{\sigma}_{cw} = \tilde{\sigma}_{fw} = \tilde{\sigma}_{sw} = 1$;

$cot\theta > 2.5$: the amount of shear reinforcement does not lead the compressed concrete stress field to fail, and hence $cot\theta = 2.5$ is assumed, and the shear strength is calculated via Eq. (14), considering $\tilde{\sigma}_{fw} = \tilde{\sigma}_{sw} = 1$.

The stress value acting on the concrete stress field can be obtained via Eq. (19), assuming $cot\theta = 2.5$;

- $cot\theta < 1$: failure is attributable to the stress limit being attained in the concrete stress field and in only one shear reinforcement. $\alpha > \beta$ is considered. The shear strength of the beam is given by the maximum value obtained via Eq. (15) assuming the FRP reinforcement attains the maximum effective strain in tension ($\tilde{\sigma}_{fw} = 1$) or (an improbable case) via Eq. (16) assuming the steel stirrups yield in compression ($\tilde{\sigma}_{sw} = -1$).

It is noteworthy that, when $\beta = 90°$, the CGP model coincides with the CNR model if $r = 1$ and the same formulations of R are retained.

5 MODIFIED VERSION OF CNR-DT 200 R1/2013 SHEAR MODEL [17]

The shear capacity model for RC beams strengthened by FRP suggested by CNR [17] represents a direct extension of the model suggested by EN1992-1-1 [26] to assess the shear strength of common RC members. The shear strength of each element constituting the truss mechanism, as well as the shear strength of retrofitted members, can be written in non-dimensional form, as follows:

$$v_s = \omega_{sw}\left(\cot\theta + \cot\alpha\right)\sin^2\alpha \qquad v_f = \omega_{fw}\left(\cot\theta + \cot\beta\right)\sin^2\beta$$
$$v_c = \left(\cot\theta + \cot\psi\right)/\left(1 + \cot^2\theta\right) \qquad v = \min\left(v_s + v_f, v_c\right) \qquad (20\ a,b,c,d)$$

To improve the model, in Eq. (20c), the angle α given by the CNR model is replaced with the to-be-determined angle ψ. In the original CNR model the angle ψ is set equal to the angle β, assuming that in a beam to be strengthened in shear the pre-existing transverse steel reinforcement provides a modest contribution compared to the contribution provided by the FRP reinforcement to be added. The inclination of the concrete stress field is evaluated in order to maximize the shear strength of the strengthened RC member. However, considering $\psi = \beta$ (or $\psi = \alpha$) would result in neglecting the presence, and the effect, of the other shear reinforcement when evaluating the shear capacity provided by the concrete stress field. In the modified version of the CNR model, the angle ψ is defined as the angle of inclination of the equivalent shear reinforcement, obtained as that of the direction of the vector summing shear contributions made by the FRP reinforcement V_{fw} and the steel stirrups V_{sw} (Figure 2). It can be evaluated using Eqs. (20a), (20b) and (20c), once the $cot\theta$ value is known. To this aim, a simple iterative procedure was suggested in [15] to compute the angle ψ, based on the static theorem of plasticity. The $cot\theta$ value that maximizes the shear strength is evaluated by equating the shear strength that determines the failure of the concrete stress field in equation (20c) to the shear strength that determines the failure of the FRP and steel reinforcement (the sum of Eqs. (20a) and (20b)) according to the following equation:

$$\begin{aligned}&\cot^3\theta\left(\omega_{sw}\sin^2\alpha+\omega_{fw}\sin^2\beta\right)+\cot^2\theta\left(\omega_{sw}\sin^2\alpha\cot\alpha+\omega_{fw}\sin^2\beta\cot\beta\right)+\\&+\cot\theta\left(\omega_{sw}\sin^2\alpha+\omega_{fw}\sin^2\beta-1\right)=\cot\psi-\omega_{sw}\sin^2\alpha\cot\alpha-\omega_{fw}\sin^2\beta\cot\beta\end{aligned} \tag{21}$$

A tentative $cot\theta$ value is considered that can be evaluated by means of the ω_{fw} and ω_{sw} values using Eq. (19) in which r equal to 1 has to be retained (otherwise a value of 1.75 can be adopted, which is the average value of the range of variation of $cot\theta$ given by CNR); 2) Through Eqs. (20a) and (20b), the shear capacities v'_f and v'_s provided by the FRP reinforcement and steel stirrups are computed, respectively. These values are used to approximate ψ' as the weighted value of α and β as $\psi' = (\alpha v'_s + \beta v'_f)/\left(v'_s + v'_f\right)$; 3) The angle ψ' is used in Eq. (21), and a new $cot\theta$ value is calculated.

This procedure can be applied iteratively until a negligible difference between two consecutive values of $cot\theta$ is obtained. The analyses carried out with the above procedure demonstrated that one iteration is enough to yield reliable values of $cot\theta$.

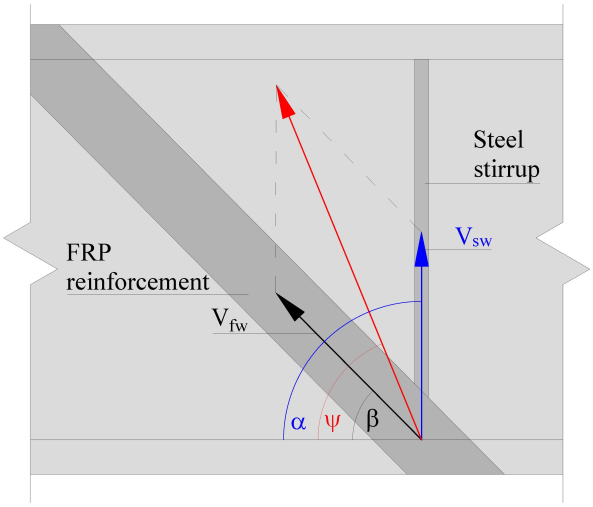

Figure 2: Physical meaning of the angle ψ

6 DESCRIPTION OF THE DATABASE

In order to evaluate the reliability of the model described above, a database with 158 specimens of RC beams characterized by T-shaped or rectangular cross-sections, retrofitted in shear with FRP sheets or strips, was collected [8-10, 22, 27-44]. The main specimen characteristics and the experimental shear strengths are reported in Table 1 in the Additional Data. The database comprises beams retrofitted with U-jacketing or complete wrapping schemes made of glass or carbon fibers. The shear span ranges between 2.3 and 3.8, while the effective beam depth is between 155 and 831 mm. The database only contains beams having both FRP reinforcement and steel stirrups, the latter constituted by vertical stirrups with a maximum geometrical ratio of 0.48%. Regarding the FRP reinforcement, the elastic modulus ranges between 8 and 640 GPa, the ultimate tensile strength is between 106 and 4361 MPa, and the geometrical ratio varies between 0.04% and 3.00%.

To highlight the influence of different angles of arrangement of FRP reinforcement and steel stirrups, two partial databases were identified from the above-described database: DTB1, containing 138 specimens, in which the FRP reinforcement and steel stirrups are arranged with the same angle with respect to the beam longitudinal axis ($\beta = \alpha = 90°$); DTB2, containing 20 specimens, where the FRP reinforcement is characterized by an angle β smaller than $\alpha = 90°$. In each database six different subgroups were detected based on the cross-section shape (rectangular "R" or T-shaped "T") and the retrofitting scheme (U-jacketing "U", U-jacketing with partially effective anchorages "U*", U-jacketing with fully effective anchorages "U/C", complete wrapping "C"). In the case of U*, the R parameter was calculated using formulations proposed for U-jacketing schemes, while for U/C, the formulations for complete wrapping were used.

7 INFLUENCE OF EFFECTIVENESS FACTORS

Firstly, the influence of the effectiveness factors in shear capacity prediction of retrofitted RC beams is investigated, by comparison of the efficiency of the design-oriented model proposed by Colajanni et al. [15] when different formulations of effectiveness factors are adopted. The efficiency and reliability are assessed through the average ratio between experimental shear strength v_{exp} and analytical prediction v_{the} of test outcome (Avg) and the Coefficient of Variation (CoV). Initially, the models are used in their original form, i.e. considering the effectiveness factors R proposed by ACI [16], CNR [17], fib [18] and Mofidi and Challaal [19] (M&C) in which $r = 1$, as well as the two models of effectiveness factors proposed in Colajanni et al. [15] where both effectiveness factors for FRP reinforcement R and steel stirrups r are retained.

Generally speaking, most of the approaches employed to compute the effectiveness factor give acceptable results, as illustrated in Figure 3, where both the databases were considered, and the square symbol refers to specimens with $\alpha = \beta = 90°$, and the triangular one to specimens with $\alpha = 90°$, $\beta \neq 90°$. Reduced effectiveness is found only for the fib and M&C models.

However, the average and CoV values provided by the compared models are significantly different, pointing out the remarkable effect of the R factor in evaluating the shear strength of retrofitted beams. Indeed, the effectiveness factors proposed by ACI or CNR provide the best average value of the efficiency ratio (0.97), whereas that of fib provides the worst one (0.83), producing a significant overestimation, on average, of the beam shear strength. Conversely, the M&C effectiveness factor gives an overall underestimation of the shear strength, characterized by an average value of 1.12. Regarding the effectiveness factors used in the CGP model and described in the Additional Data (K&N + P&M model, C&T model), they give analogous average values, with general slight overestimation of the shear strength. With reference to the CoV values, it can be seen that the C&T model combined with the effectiveness factor for steel stirrups described in Section 3 gives the lowest scatter (0.20). Though the effectiveness factor suggested by ACI provides the closest average value to 1, it leads to the highest CoV (0.32), like that obtained with the M&C model (0.29). Concerning the effectiveness factor used in the CGP1 model, along with those suggested by fib and CNR, the CoV values vary between 0.25 and 0.27. In view of the foregoing, it can be affirmed that the formulation to compute the effectiveness factors which combines the highest average accuracy and the best reliability is the C&T + r one, which adopts the equations suggested by Chen & Teng [5, 6] to assess the FRP effectiveness factor, and the equations reported in Section 3 to evaluate the steel stirrup effectiveness factor. By focusing on the subgroups, the analytical predictions of the shear strength provided in the case of a rectangular shape of the cross-sections (subgroups “R U”, “R U*”, “R C”) are in general more markedly overestimated than those obtained for T-shaped specimens (subgroups “T U, “T U*”, “T U/C”). This is because the model does not explicitly consider the shear contribution provided by the flange of the T-shaped section, which was found to be relevant in recent studies (e.g., [40]), and thus in the cases of T sections overestimation of the model is balanced by an increment of the shear resistance due to the flange. Moreover, the results obtained in the case of FRP reinforcement oriented differently with respect to the steel stirrups (triangular symbols in Figure 3) are generally more markedly overestimated than those in which FRP reinforcement and steel stirrups are arranged parallel. This negative effect is reduced for the CGP1 and CGP2 models, thanks to the introduction of the effectiveness factor for steel stirrups, which reduces their contribution to the overall shear strength.

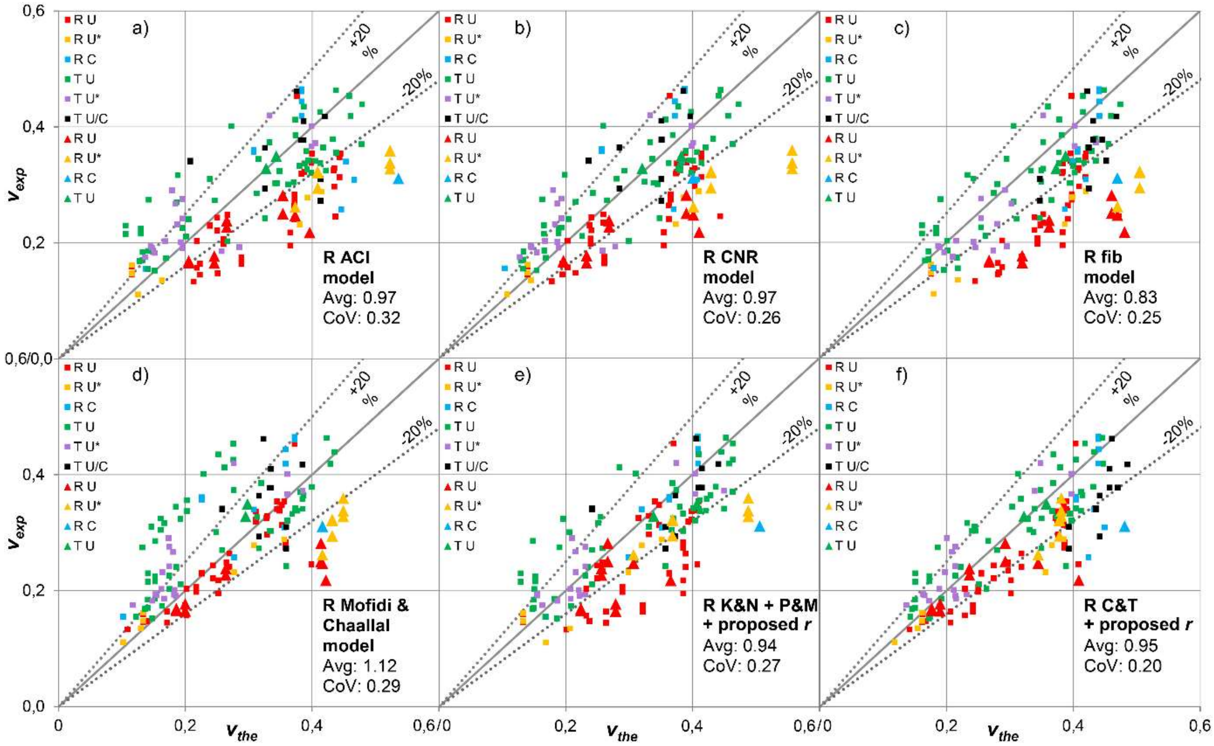

Figure 3: Experimental vs. theoretical shear strength calculated via the CGP model with effectiveness factors proposed by ACI (a); CNR (b); fib (c); Mofidi & Chaallal (d), Khalifa & Nanni + Pellegrino & Modena + *r* (CGP1) (e); Chen & Teng + *r* (CGP2) (f)

To have a broader understanding of the influence of the two effectiveness factors, namely *R* and *r*, in analytical predictions of shear capacity models, a comparison between the models proposed by ACI [16], CNR

[17], and Colajanni et al. [15] is carried out considering the two subsets of the preceding database. The comparison is performed considering the same six different effectiveness factor models for FRP reinforcement used before, including or excluding the effectiveness factor for steel stirrups proposed in [15]. The results are summarized in Table 1 and Table 2. In the case of DTB1 ($\alpha = \beta = 90°$) without effectiveness factor r, for which the CNR and CGP models coincide, the models provide average values close to 1, while in the case of DTB2 ($\alpha = 90°$, $\beta \neq 90°$) the three models significantly overestimate the shear capacity. This is due to overestimation of the contribution provided by the steel stirrups to the shear capacity of beams, and in the first two models due to overestimation of the capacity of the compressed concrete strut, as will be shown in the next sections, where the trend of effectiveness factor R and the effect of approximate evaluation of the strength of the compressed concrete will be investigated. When the effectiveness factor for steel stirrups proposed in [15] is considered in the shear models, the latter provide results which are slightly improved in terms of average values (+1 / +6 %) in the case of DTB1 (excluding the results provided by Mofidi and Chaallal [19], for which the differences are +12 / +24 %). In the case of DTB2, an overall significant improvement is obtained when the r factor is used (average values increase of +5 / +40 %). Except for the results obtained by means of R fib, the average values increase much more in the case of truss models with variable inclination of the concrete strut (CNR and CGP models) than in the case of the additive model (ACI model). This is due to the fact that, when truss models with variable concrete strut inclination are considered, the r effectiveness factor affects both the strength of the steel tensile tie of the truss and inclination of the compressed concrete strut θ, while in the additive model only the former contribution is influenced by the effectiveness factor r. As for the CoV values, except for the values given by the CNR model using the R factor given by CNR or K&N+P&M R, and the CGP model using the R factor given by ACI, CNR and K&N+P&M, the use of the r factor can reduce the scatter of the values, improving the reliability of the shear models.

Table 1: DTB1 - Average and CoV values obtained with the ACI model, the CNR model and the CGP model with different effectiveness factors for FRP including or excluding the effectiveness factor for steel stirrups

DTB 1 ($\beta = 90°$) (without effectiveness factor r)							
		R ACI	R CNR	R fib	R M&C	R K&N+P&M	R C&T
ACI model	Avg	0,97	1,00	0,92	1,07	0,96	0,95
	CoV	0,21	0,20	0,20	0,21	0,19	0,19
CNR model	Avg	1,00	1,00	0,85	1,16	0,93	0,92
	CoV	0,31	0,24	0,23	0,27	0,27	0,21
CGP model	Avg	1,00	1,00	0,85	1,16	0,93	0,93

	CoV	0,31	0,24	0,23	0,27	0,27	0,21
DTB 1 ($\beta = 90°$) (with effectiveness factor r)							
		R ACI	R CNR	R fib	R M&C	R K&N+P&M	R C&T
ACI model	Avg	0,99	1,03	0,93	1,20	0,98	0,97
	CoV	0,21	0,21	0,20	0,27	0,20	0,20
CNR model	Avg	1,03	1,06	0,86	1,45	0,96	0,94
	CoV	0,31	0,26	0,23	0,27	0,27	0,20
CGP model	Avg	1,03	1,06	0,86	1,45	0,96	0,96
	CoV	0,31	0,26	0,23	0,27	0,27	0,20

Table 2: DTB2 - Average and CoV values obtained with the ACI model, the CNR model and the CGP model with different effectiveness factors for FRP including or excluding the effectiveness factor for steel stirrups

DTB 2 ($\beta \neq 90°$) (without effectiveness factor r)							
		R ACI	R CNR	R fib	R M&C	R K&N+P&M	R C&T
ACI model	Avg	0,84	0,83	0,74	0,86	0,80	0,82
	CoV	0,16	0,15	0,17	0,16	0,15	0,17
CNR model	Avg	0,74	0,72	0,59	0,77	0,67	0,70
	CoV	0,19	0,19	0,20	0,23	0,17	0,21
CGP model	Avg	0,75	0,74	0,61	0,79	0,70	0,73
	CoV	0,17	0,17	0,18	0,21	0,15	0,18
DTB 2 (b $\neq$ 90°) (with effectiveness factor r)							
		R ACI	R CNR	R fib	R M&C	R K&N+P&M	R C&T
ACI model	Avg	0,95	0,93	0,78	0,98	0,87	0,86
	CoV	0,14	0,11	0,14	0,14	0,13	0,14
CNR model	Avg	0,95	1,01	0,63	1,05	0,79	0,86
	CoV	0,19	0,27	0,17	0,17	0,18	0,15
CGP model	Avg	1,02	0,95	0,64	1,05	0,79	0,87
	CoV	0,26	0,18	0,16	0,17	0,17	0,14

Moreover, the formulation for the R factor proposed by ACI provides small scattering when applied to the ACI additive model, while it produces higher scattering of the results when it is applied to the models with variable inclination of the concrete strut (CNR and CGP model). It can be concluded that the additive model proposed by ACI, in which the influence of the R and r effectiveness factors is limited to FRP reinforcement and steel stirrups, not involving the contribute of concrete V_c, seems the most effective one. When $\beta \neq 90°$ is considered, all the models tend to overestimate the strength of the retrofitted beams; the apparent effectiveness of the M&C R factor is due to its generalized tendency to underestimate the contribution provided by FRP, which is wiped out with the overall overestimation tendency stressed before.

The CGP model appears the most effective among the models with variable inclination of the concrete strut when beams strengthened with FRP fiber with different inclination from that of the existing steel stirrup are considered. None of the models explicitly considers the shear contribution provided by the flange of the T-shaped section, which was found to be relevant, as confirmed in recent studies (e.g., [40]).

To provide a further insight into the above-described differences, in Figure 4 there are plotted the FRP effectiveness factor *R* values provided by the above-mentioned procedures for each specimen of the database. Moreover, power regression in the form $R = Ax^{-B}$, as well as coefficient of determination, usually named R^2, are provided for each effectiveness factor with respect to the U-shaped strengthening scheme. The results shown in Figure 4 facilitate understanding of those shown in Figure 3. Indeed, the M&C and fib formulations, which on average underestimate and overestimate the shear strength of RC beams, respectively, are characterized by parameters *A* of power regression equal to 0.08 and 0.19, respectively.

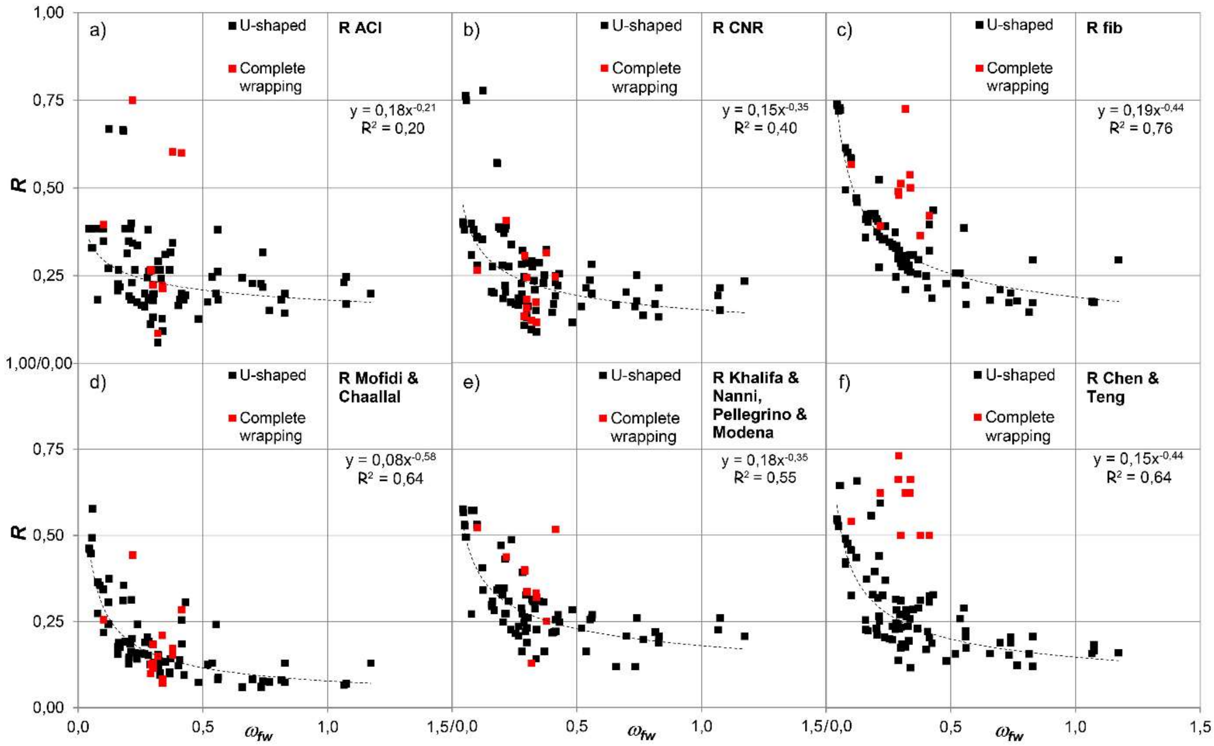

Figure 4: Effectiveness factors for FRP computed as proposed in ACI (a); CNR (b); fib (c); Mofidi & Chaallal (d); Khalifa & Nanni, Pellegrino & Modena (e); Chen & Teng (f)

The three *R* coefficients that provide the best performance, when applied to the three models considered, namely the ACI, CNR, and C&T *R* coefficients, are characterized by different power regression functions. The ACI formulation for *R* is characterized by the lowest coefficient of determination R^2, and the fib formulation by the highest, proving that correlation of *R* with the mechanical ratio of transverse FRP reinforcement is weak.

Concerning the complete wrapping strengthening scheme (red symbols in Figure 4), significant differences between the formulations can be observed. As a matter of fact, the formulations proposed by C&T and fib tend to give significantly greater *R* values than those provided for the U-shaped scheme. In contrast, the formulations proposed by CNR, M&C, and K&N+P&M provide *R* values comparable to those given in the case of the U-shaped configuration. Regarding the ACI formulation, no trend is observed due to the high scatter of the *R* values.

Lastly, to clarify the influence of the ψ angle on evaluation of concrete strut inclination, in Figure 5 there are shown the $cot\theta$ values versus the mechanical ratio of total transverse reinforcement ($\omega_{fw} + \omega_{sw}$) for specimens characterized by steel stirrups and FRP reinforcement arranged with different inclinations, and assessed by means of the CNR, CNRm and CGP2 models, the latter considering C&T and the *r* effectiveness factor. It can be observed that, when the sum of mechanical ratios of shear reinforcements increases, the $cot\theta$ values tend to decrease when ψ angle is calculated using the modified version of the CNR model described in Section 5. This helps to increase the accuracy of the CNR model, decreasing its tendency to overestimate the shear strength of beams. Furthermore, the CGP model gives $cot\theta$ values that are equal to 2.5 in the bulk of the specimens considered. The reason is the different equation adopted to compute the FRP *R* value, which is also characterized by the presence of the *r* factor for steel stirrups, which decreases the value of the denominator of Eq. (19).

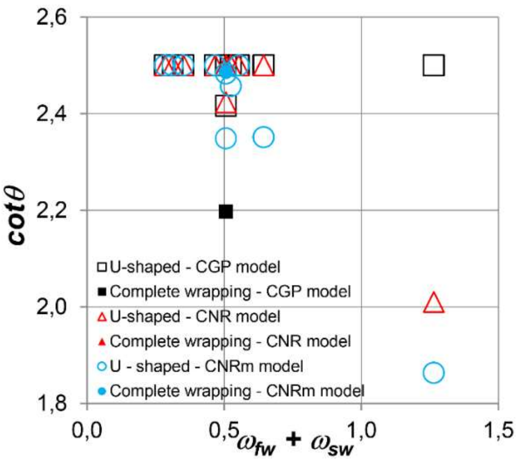

Figure 5: Concrete strut inclination $cot\theta$ vs. mechanical ratios of shear reinforcements $\omega_{fw} + \omega_{sw}$ in the case of DTB2: CGP2 model (a); CNR (b); CNRm (c)

8 PARAMETRIC ANALYSIS

Here a parametric analysis is performed, by changing the angle β of the FRP reinforcement, to demonstrate firstly that a consistent evaluation of the effect of different inclination of internal steel stirrup and external FRP reinforcement (e.g., [15]) affects shear strength assessment, against the approximation of the strength of concrete compression strut according to the CNRm model, or the approximation $\psi = 45°$ suggested in the CNR code [17]; and secondly to show the influence of FRP reinforcement orientation on the shear strength of beams, computed via the preceding models characterized by a concrete strut with variable inclination.

The inclination of the transverse steel reinforcement ($\alpha = 90°$) and mechanical ratios of both FRP reinforcement and steel stirrups (ω_{fw} and ω_{sw}) are kept fixed in the analyses, whereas the angle β of the FRP reinforcement changes between $45° \leq \beta \leq 90°$. To point out some particular features of the CGP2, CNR and CNRm models, four different layouts of shear reinforcement are analyzed, assumed to be typical of practical cases: the first two, the former having $\omega_{fw} = 0.15$; $\omega_{sw} = 0.05$ and the latter having $\omega_{fw} = 0.15$; $\omega_{sw} = 0.15$, are chosen to point out how the variation of the amount of steel stirrups affects the shear strength in the case of a small amount of FRP reinforcement; the other two, the former having $\omega_{fw} = 0.20$; $\omega_{sw} = 0.40$ and the latter having $\omega_{fw} = 0.40$; $\omega_{sw} = 0.20$, are chosen to point out how the shear strength varies when high amounts of shear reinforcements are considered. The curves of the theoretical prediction of the shear strength of beams by varying the angle β given by the preceding models are shown in Figure 6.

In general, consistent evaluation of the effect of FRP reinforcement inclination, i.e. in the Colajanni et al. model [15], gives shear strength values that are, in the majority of cases, lower than those provided by the CNR and CNRm models, corroborating the results reported in Table 2. In addition, the shear strength values provided by the CNRm model are near those obtained with the CNR model for a small amount of transverse steel reinforcement or a large amount of FRP reinforcement, whereas they are close to those of the CGP model in the case of a large amount of transverse steel reinforcement or reduced FRP reinforcement. For values of the angle β approaching 45°, the difference between the shear strength values provided by the CNR and CNRm models increases. This phenomenon is attributable to the incapacity of the CNR model to simultaneously consider the inclination of FRP and transverse steel reinforcements when evaluating the shear capacity of the concrete strut.

In the case of a high amount of steel stirrups, this difference is more evident. In detail, the analytical predictions given by the CNR model are close to those given by the CNRm and CGP2 models in the case of a higher amount of FRP reinforcement than steel stirrups ($\omega_{fw} = 0.40$; $\omega_{sw} = 0.20$), whereas significant differences are observed in the opposite case, namely when the amount of steel stirrups is considerably higher than that of FRP ($\omega_{fw} = 0.20$; $\omega_{sw} = 0.40$). This is because the CNR model does not simultaneously consider the presence of steel stirrups and FRP reinforcement in the assessment of the shear strength of the concrete strut.

With respect to shear strength variation, the angle for which the shear capacity is maximized changes based on the mechanical ratios of FRP reinforcement and steel stirrups. In the case of a small amount of shear reinforcement, the maximum shear capacity is given when the angle β ranges between 65° and 90°, whereas for a high amount of shear reinforcement, the maximum shear capacity is obtained when the angle β is lower than 55°.

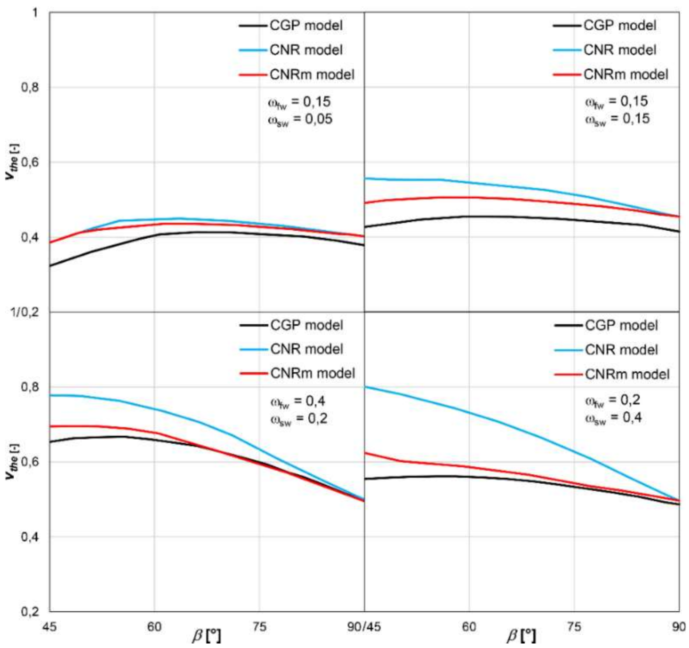

Figure 6: Shear strength curves given by the CGP model (black line), the CNR model (cyan line), and the CNRm model (red line), changing the FRP reinforcement inclination

9 CONCLUSIONS

This paper compares different formulations for evaluation of the effectiveness factors used to decrease the contribution made by FRP reinforcement to the shear capacity of strengthened RC beams. Recently proposed formulations, obtained by modifying already existing procedures, are compared. Regarding the effectiveness factor that aims at reducing the of steel stirrup strain based on the strain acting on the FRP reinforcement, the efficiency of a formulation that takes into account the possible different inclination of FRP and steel shear reinforcement is discussed.

After comparison of the shear strength values obtained with the model proposed by Colajanni et al. [15] using the preceding formulations of the R factor together with those proposed by ACI [16], CNR [17], fib [18], and Mofidi and Chaallal [19], the efficiency of this shear model is compared with those suggested by ACI and CNR. The main results are the following:

- the effectiveness factor formulation which provides the highest average accuracy (Avg near 1) and the best reliability (low scatter) is the one with the equations suggested by Chen & Teng to assess the FRP effectiveness factor, and the equations reported in Section 3 to evaluate the steel stirrup effectiveness factor;
- when the effectiveness factor for steel stirrups proposed by Colajanni et al. [15] is considered in the shear models, the latter provide results which are slightly improved in terms of average values (+1 / +6 %) in the case of beams having FRP reinforcement and steel stirrups arranged at right angles with reference to the beam axis, while, in the case of FRP reinforcement and steel stirrups arranged at different angles, an overall significant improvement is obtained when the preceding factor is used (average values increase of +5 / +40 %);
- the analytical predictions given by the CNR model, and its modified version proposed in [15], CNRm, showed that calibration of the ψ angle, namely the angle involved in the assessment of the shear capacity of the concrete strut, helps to increase the reliability of the CNR model. Yet CNRm model continues to greatly overestimate the shear strength of FRP-retrofitted RC beams.
- the parametric analysis shows that the CNRm model gives shear strength values which are similar to those provided by the CNR model for a small amount of transverse steel reinforcement, while it approaches that of the CGP model when the amount of steel stirrups increases.
- regarding the trend of shear strength by varying the angle of inclination of FRP, it can be seen that the angle for which the shear capacity is maximized differs according to the amount of FRP reinforcement and steel stirrups used. In the case of a small mechanical ratio of shear reinforcement, the shear capacity is maximized for an angle between 65° and 90°, whereas for a high mechanical ratio, the maximum shear capacity is obtained in the case of FRP reinforcement arranged at angles smaller than 55°.

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