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PME40, Szeged, Hungary, 3-7 August, 2016

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### Mathematics Education: How to solve it?



# Proceedings of the 40<sup>th</sup> Conference of the International Group for the Psychology of Mathematics Education

**Editors** 

Csaba Csíkos Attila Rausch Judit Szitányi



PME40, Szeged, Hungary, 3-7 August, 2016









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#### **WELCOME TO PME 40**

We are delighted to welcome you to the 40<sup>th</sup> Annual Conference of the International Group for the Psychology of Mathematics Education, being held in Szeged, Hungary. PME40 is being hosted by the University of Szeged, and the theme of the conference is *Mathematics Education: How to solve it.* This title reminds all participants that 70 years ago the Hungarian Pólya György (George Pólya) published his seminal book entitled "How to solve it?". This book was used by generations of mathematics teachers as their inspiring source of teaching ideas. Besides commemorating Pólya's oeuvre, the title evokes the everlasting debate on the role of mathematical problem solving in fostering children's thinking. We invite all participants to contribute actively to the discourse and analysis of ideas. We also encourage all of you to foster a welcoming and stimulating atmosphere at the conference, that all participants may feel included as members of the PME community. We extend a special welcome to those attending their first PME conference. Our hope is that the conference will provide a chance to attain some pinnacles and to establish some fruitful connections.

It is the second time Hungary is hosting a PME conference. PME 12 was held in Veszprém, and several presenters of that conference are still active members of the PME community. Another special welcome is due to them!

Do extra-terrestrial beings exist? – the Nobel Prize winning Italian physicist, Enrico Fermi, was once asked by his disciples in California. Of course, Fermi answered – they are already here among us, they are called Hungarians... A headline article published in *Nature* in 2000 ("Genius Loci") claimed that the 20th century was made in Budapest. The article goes on to enumerate all the amazing contributions to progress by Hungarian scientists early in the century. Recognizing and developing mathematical talent has long been and is still a central issue in the Hungarian educational system. However, international assessment projects (TIMSS, PISA) in the last decades warned us that our mathematics education should be reformed according to the principle of evidence-based educational policy. We hope that our educational system will benefit from hosting such a highly prestigious scientific conference.

The Local Organizing Committee has 13 members from different universities, thus making the occasion a national endeavor. The University of Szeged is proud of hosting such a highly prestigious event. Szeged is most famous for its culture, including the University which is among the 500 best universities of the world. The name of the town is also closely intertwined with sport events: the Canoe Sprint World Championships were hosted in Szeged three times. Moreover, the town is a gastronomical and spa event itself worth being discovered.

The Program Committee and the Local Organizing Committee want to express our thanks for the support we have received from members of the PME community, including previous conference organizers and Bettina Rösken-Winter, PME's administrative manager.

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The Proceedings book has four volumes in accordance with PME tradition. The selection of the contributions involved in these volumes reflects the hard work of our reviewers and the Program Committee, listed in full later.

I am personally grateful to my colleagues whose work went beyond what could reasonably be expected. Attila Rausch was my right-hand secretary available all time during the last ten months; Dóra Prievara managed to answer several hundreds of emails; Katalin Molnár took care of all little details the importance of which cannot be overestimated, and Judit Kléner gave help in handling the financial issues of the conference.

We hope you enjoy your stay in Hungary and find your participation at the conference fruitful and memorable.

Csaba Csíkos

Chair of PME 40

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## AN EDUCATIONAL EXPERIENCE OF CULTURAL TRANSPOSITION IN PRIMARY SCHOOL: PROBLEMS WITH VARIATION

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Since several years, politicians, researchers and educators from different countries are debating on possible reasons for the success of Confucian area students in interactional assessment projects (such as PISA). Starting from the study of teaching practices developed in this East area, in Italy we developed an educational research based on the process of cultural transposition (Mellone & Ramploud, 2015). With this term we mean the process of change that develops considering two or more cultural-educational backgrounds with the aim to maintain their differences without "translating" them from one culture to another, but rather highlighting these in order to review their meaning processes and daily use in classrooms. In this article we present two educational experiences developed in a second and fifth grades Italian classroom on the cultural transposition of the *pictorial equations approach* of the Chinese curriculum inserted in the typical structure of Chinese word problems, called problems with variation (Bartolini Bussi et al., 2013). According to Cai and Knuth (2011) in China this symbolic representation is used since the early school years as a possible support for "constructing" a bridge to the "informal algebra". In the Russian curriculum it is possible to find an analogous approach called intermediate strategies of graphic representation (Davydov, 1982) referred to similar pictorial equations. Our experience of cultural transposition in the Italian context have shown the potentialities of the use of problems with variation as well as the crucial role of the figural equation.

#### References

- Cai, J., & Knuth, D. (Eds.) (2011). Early algebraization: A global dialogue from multiple perspectives. Springer.
- Bartolini Bussi, M.G., Sun, X., Ramploud, A. (2013). A dialogue between cultures about task design for primary school. In C. Margolinas (a cura di), *Proc. of ICMI Study 22*.
- Mellone, M. & Ramploud, A. (2015). Additive structure: an educational experience of cultural transposition. In Sun X., Kaur B., Novotná N. (eds.), *Proc. of ICMI Study 23*.
- Davydov, V.V. (1982). The psychological characteristics of the formation of elementary mathematical operations in children. In T.P. Carpenter, I.M. Moser & T.A. Romberg (Eds.), *Addition and Subtraction: A cognitive perspective*. Hillsdale, NJ: Lawrence Erlbaum.