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On modelling damage in composite laminates using the Ritz method and continuum damage mechanics

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Abstract. In this work, a Ritz formulation for the analysis of damage initiation and evolution in composite plates under progressive loading is presented. The proposed model assumes a first order shear deformation theory and considers geometric non-linearities through the von Karman assumptions. The damage is modelled through Continuum Damage Mechanics. A set of results is presented to show the potential of the method and highlight some issues to be addressed by suitable developments of the method.

Introduction

Composite materials are widely used in several engineering fields such as automotive, naval, and aerospace, due to their superior strength-to-weight ratio and other desirable properties. To further enhance their performance, new manufacturing techniques have been developed, including automated fiber placement and additive manufacturing, which have enabled the design of composite structures with variable fiber angle placement, known as variable angle tow (VAT) laminates. These composites offer numerous advantages and have been extensively studied [1,2].

Due to the wider design space enabled by such innovative manufacturing methods, there is the need, by designers and engineers, of modeling and developing computational tools capable of predicting the structural behavior of components with a satisfactory level of accuracy, including the prediction of their damaging and failure modes. Modeling progressive damage in composite materials is challenging due to the different damage mechanisms that may arise. Damage can be modeled at different scales, from the micro- to macro-scale, depending on the idealization's scale. For example, at the micromechanical level, representative volume elements (RVEs) are used to represent damage in the form of matrix softening or fiber breaking [3]. Instead, at the macro-scale level, damage is often modeled as a hard discontinuity.

Continuum Damage Mechanics (CDM) is one of the most popular frameworks used to handle the damage process at the meso-scale level, where individual plies are represented as homogenous. CDM models are based on the works of Matzenmiller and Ladeveze [4,5] among others, which describe degradation as a progressive loss of material stiffness. Several models have been developed and used in finite element (FE) approaches within the CDM framework [6,7]. In addition to FE-based analysis methods, single domain meshless approaches, such as the Ritz method, have been shown to be competitive, especially when dealing with smeared damaged zones [8].

However, in some cases, damage tends to localize in a narrow band due to loading conditions or initial imperfections [9,10]. This phenomenon can cause the numerical model to suffer from spurious effects when reconstructing the damage state, known as the Gibbs effect, which leads towards a non-physical response. Thus, designers and engineers must carefully consider the limitations of different modeling and computational tools when predicting the structural behavior of composite materials and components.

This work aims at developing an adaptive multi-domain Ritz method to avoid spurious numerical effects and maintain the advantages of the Ritz method in terms of reducing the degree of freedom compared to FE models, while still achieving objective physical response.

Despite the benefits granted by the Ritz method, there are only few works in the literature addressing damage initiation and evolution using the Ritz approach. Few works address the topic [11,12], where the damage models are often overly simplified and offer an on/off representation of damage, more suitable for identifying damage initiation than damage evolution, which tend to be excessively conservative. These considerations highlight the novelty of the proposed method.

Methods

In the present formulation, the kinematic is based on the First order Shear Deformation Theory with von Kármán assumptions accounting for geometric non-linearities. Following a CDM approach, the onset of the damage is predicted using the Hashin's criteria [13] and four damage indices are computed and used for the degradation of the constitutive relations at the ply level, in fiber and matrix directions, both for tension and compression. The evolution of damage is based on a linear softening law, as shown in Fig. 1, and it is assessed by means of equivalent strains.

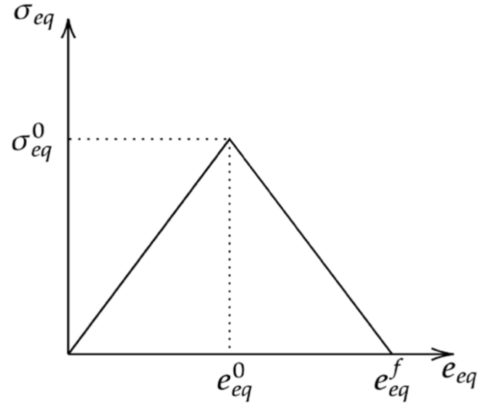

Figure 1. Linear strain softening constitutive law.

To ensure consistent energy dissipation with respect to the Ritz kinematic approximation during the failure process, a smeared approach is adopted by distributing the fracture energy over an area associated with the gauss points used in the numerical integration [14]. Referring to Fig. 1, the fracture energy dissipated per unit area can be computed as

$$G_c = \int_0^w \int_0^{e_{eq}^f} \sigma_{eq}\, de_{eq}\; dx = w\frac{1}{2}\sigma_{eq}^0 e_{eq}^f, \tag{1}$$

where *w* is the square root of the area associated with a generic gauss point. Using orthogonal polynomials for the approximation of the primary variables [15] and applying the principle of the minimum potential energy, the governing equation can be written as,

$$(\boldsymbol{K}_0 + \overline{\boldsymbol{K}}_0 + \boldsymbol{K}_1 + \boldsymbol{K}_2 + \overline{\boldsymbol{K}}_1 + \boldsymbol{R})\boldsymbol{X} = \boldsymbol{F}_D + \boldsymbol{F}_L \tag{2}$$

where, $\boldsymbol{X}$ is the vector collecting the unknown Ritz coefficients, $\boldsymbol{K}_0, \boldsymbol{K}_1, \boldsymbol{K}_2, \overline{\boldsymbol{K}}_0, \overline{\boldsymbol{K}}_1$ are the stiffness matrices of the problem, where the subscripts 1 and 2 refer to the geometric non-linear terms, and the over-bar refers to prescribed initial imperfections. Furthermore, in Eq. (2) $\boldsymbol{R}$ is the penalty matrix related to the enforcement of the BCs thorough a penalty approach and the vectors $\boldsymbol{F}_D$ and $\boldsymbol{F}_L$ collect the discrete terms associated with the external loads. To solve the non-linear problem given in Eq.(2) the incremental form of the governing equation may be expressed as

$$\boldsymbol{R}\Delta\boldsymbol{X} + \Delta\left[\left(\boldsymbol{K}_t^{\boldsymbol{g}} + \boldsymbol{K}_t^{\boldsymbol{d}}\right)\boldsymbol{X}\right] = \Delta\boldsymbol{F}_D + \Delta\boldsymbol{F}_L. \tag{3}$$

where, $\boldsymbol{K}_t^{\boldsymbol{g}}$ and $\boldsymbol{K}_t^{\boldsymbol{d}}$ are the tangent stiffness matrix contributions related to geometric non-linearities and initial imperfections as well as the damage evolution, respectively. The developed semi-analytical model has been implemented in an efficient analysis tool, where Eq.(2) is solved through the Newton-Raphson numerical scheme.

To a better understanding of the matrices appearing in Eqs. (2) and (3) the reader is referred to Ref. [8].

Some remarks on the adaptive multi-domain approach are worthwhile [16]. The onset of the damage is monitored at specific sampling points corresponding to the domain integration points. When damage initiation is triggered, the adaptive multi-domain is activated to refine the discretization in the vicinity of the damaged zone. This refinement allows to represent the global response of the structure, in terms of strains, as a piecewise continuous function removing the Gibbs effect arising in single domain discretization. Some tests regarding the suppression of the Gibbs effect are presented in the next section.

Results

To assess the capability offered by the developed analysis tool, some preliminary results are herein reported and discussed.

Fig. 2 shows the obtained post-buckling results of different simply supported VAT composite plates subjected to compressive in-plane loading, using a single domain approach. The applied load and out-of-plane displacement are normalized with respect to the critical buckling load of a quasi-isotropic layup and the plate thickness, respectively. These results show that, for the layups considered, the laminate achieving the greatest buckling load is also the one that fails at the lowest out-of-plane displacement. This shows an example on how the developed tool may be used to efficiently investigate damage characteristics of VAT laminates and find trade-off design solutions.

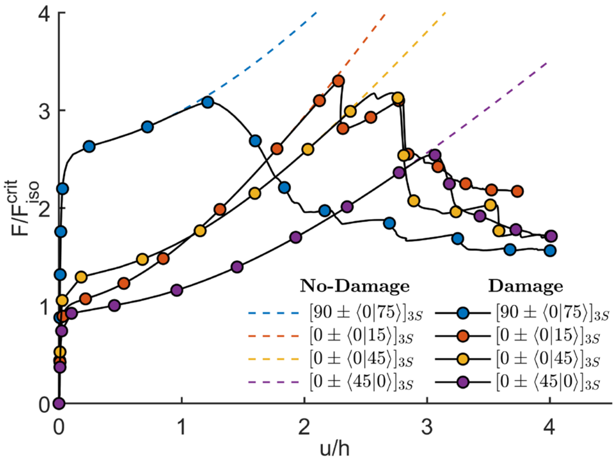

Figure 2. Post-buckling result for different VAT laminates under compression loading.

After considering a test case where the damage is distributed over a well-defined area, some tests of damage localization are reported, to assess the occurrence of the Gibbs phenomenon. To illustrate the problem, a rectangular unidirectional composite plate subjected to uniaxial tension is considered, Fig. 3. Moreover, a narrow band of material along the x_2 axis has strength lower than the rest of the bar, in order to artificially induce the onset of the damage.

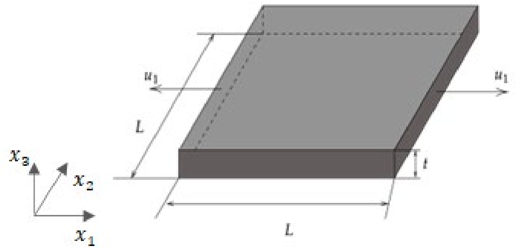

Figure 3. Schematic representation of rectangular plate under uniaxial tension.

Fig. 4 shows the contour plot of the damage state in fiber direction of the plate obtained with four different polynomial expansions, where the emergence of the Gibbs phenomenon is clearly highlighted.

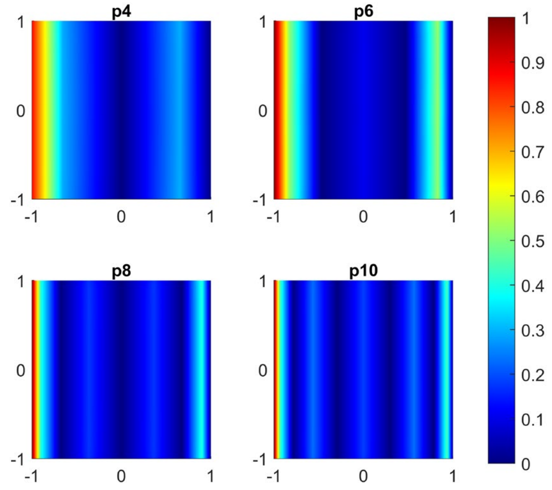

Figure 4. Gibbs effect on damage contour plots of the plate under uniaxial tension.

To address this issue, a possible approach involves discretizing the entire domain using a finer multi-patch representation, which is employed in this study, after having considered also alternative ideas, e.g. the use of filters to smooth out the Gibbs ripples.

Figs. (5a–c) show three different multi-domain discretization of the plate subjected to uniaxial tension. The generalized displacement of the patches in the narrow band, where the damage spread, are approximated using first order polynomials, whilst in the bigger patch a higher number of polynomial degree is maintained to avoid losing accuracy. The corresponding damage plots are reported in Figs. (5d–f), where it is shown how the Gibbs effect is completely removed. Moreover, Fig. 5(g) shows the results in term of force-vs-displacements, which confirms the independence from the level of discretization used, i.e. the objectivity of the response.

Finally, the solution of the present method is compared with FE results obtained with the ABAQUS built-in CDM model, Fig. (5h). The results comparison shows excellent agreement with established FE analysis methods, with the great advantage of a noticeable reduction of computational effort in terms of degrees of freedom.

Conclusions

In this work, a Ritz approach for the analysis of progressive failure of composite plates has been developed.

The present study adopts the first order shear deformation theory with non-linear von Karman strains assumptions for representing geometrically non-linear deformations and Continuum Damage Mechanics framework for capturing the initiation and evolution of damage. The method, its capabilities and limits have been presented and discussed. A multi-domain approach has been seen as a good candidate for removing the Gibbs effect that arises when the damage tends to localize in narrows band in single domain discretization.

The developed analysis tool may be useful for identifying the operational limits and providing valuable insights to the designer for the analysis of VAT layups that, with standard FE analysis, require a high number of elements to properly describe the fiber angle variation of a VAT lamina.

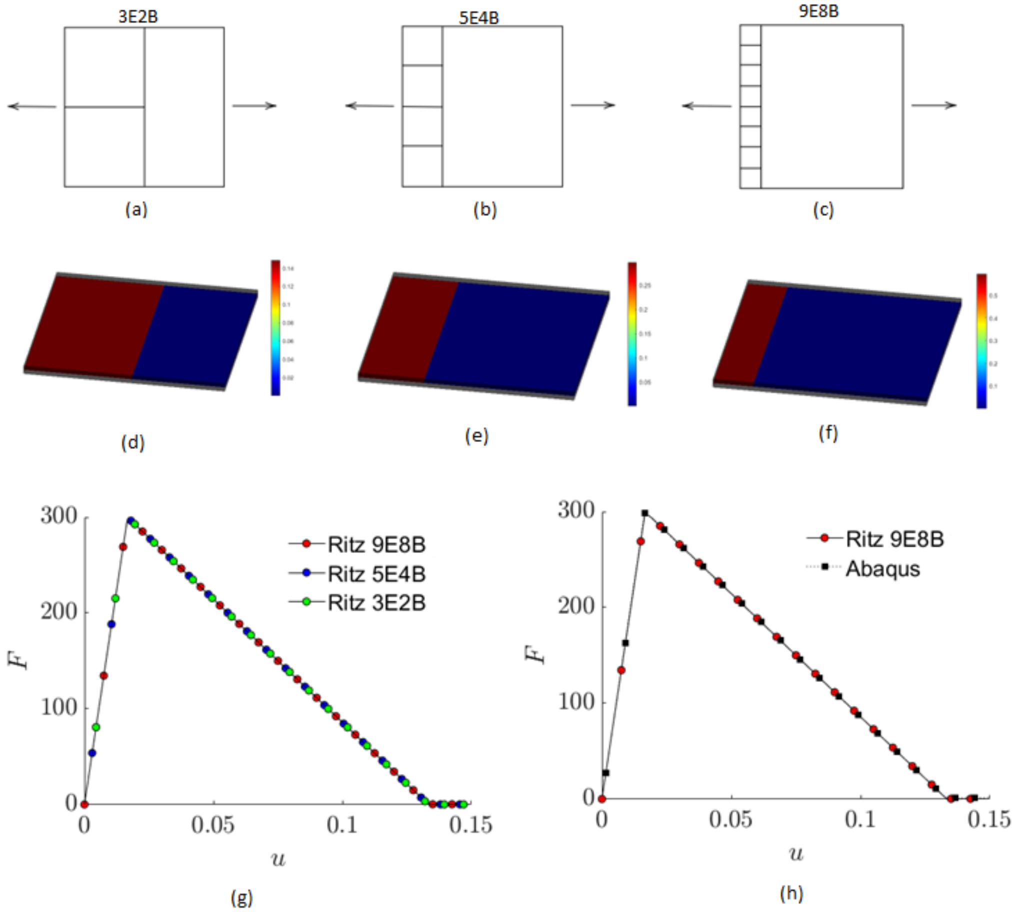

Figure 5. (a)-(c) Multi-patch discretization used. (d)-(f) damage plot of each discretization. (g) Comparison of result for different discretizations. (h) Comparison of results with Abaqus.

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