Non-Markovian dynamics of a qubit due to accelerated light in a lattice

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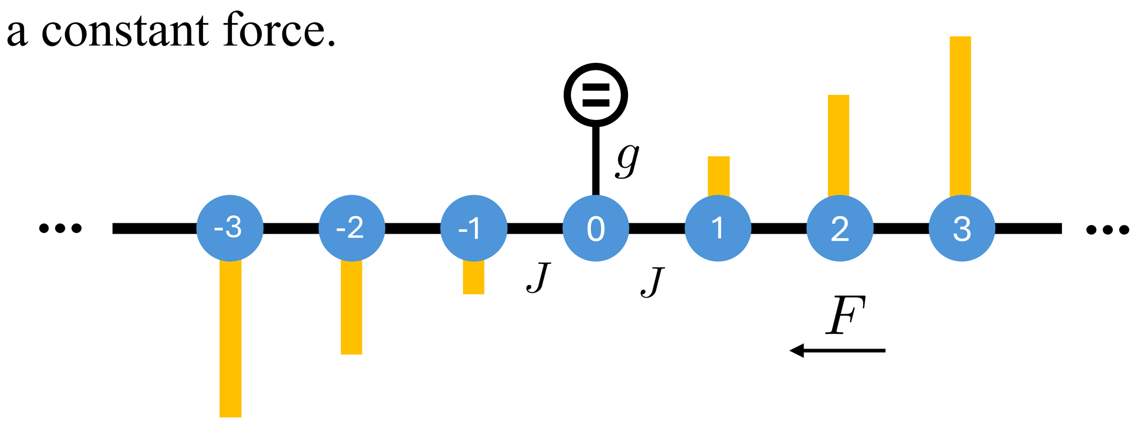
ABSTRACT

We study the emission dynamics of a qubit weakly coupled to a one-dimensional coupled-cavity array with a synthetic frequency gradient that mimics a uniform force on photons. This engineered force induces Bloch oscillations, leading to distinct emission regimes. For strong gradients, the qubit exhibits reversible, chiral emission resembling a Jaynes-Cummings model, selectively exciting the array to one side depending on the qubit frequency. For weaker gradients, non-Markovian decay with revivals emerges, analogous to mirror-induced effects in conventional waveguides despite the absence of boundaries. In this regime, the qubit dynamics are captured by a delay differential equation akin to that of an atom in a multi-mode cavity, with Bloch oscillation parameters playing the role of cavity.

MODEL

Consider a qubit initially in the excited state, coupled to a 1D photonic lattice whose cavities have a frequency gradient. As a result, the emitted photon is subjected to a constant force.

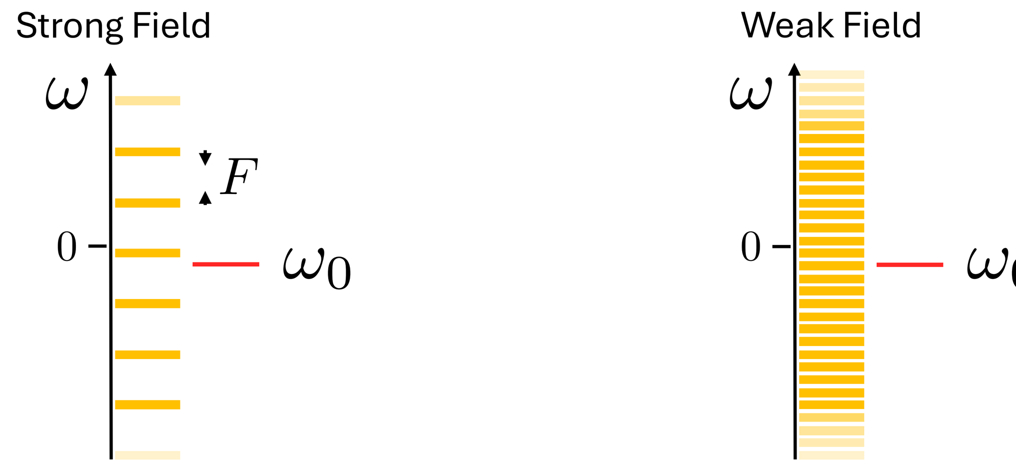


The Hamiltonian for the total system can be written as

$$H = \omega_0 \sigma_+ \sigma_- + \sum_n F n a_n^\dagger a_n - J \sum_n \left(a_{n+1}^\dagger a_n + \text{H.c.} \right) + g \left(\sigma_- a_0^\dagger + \text{H.c.} \right)$$

TWO TYPES OF REGIME

Since the field strength F gives us the spacing between the levels in the Wannier-Stark ladder, when we couple the qubit to it we will have two distinct regimes.

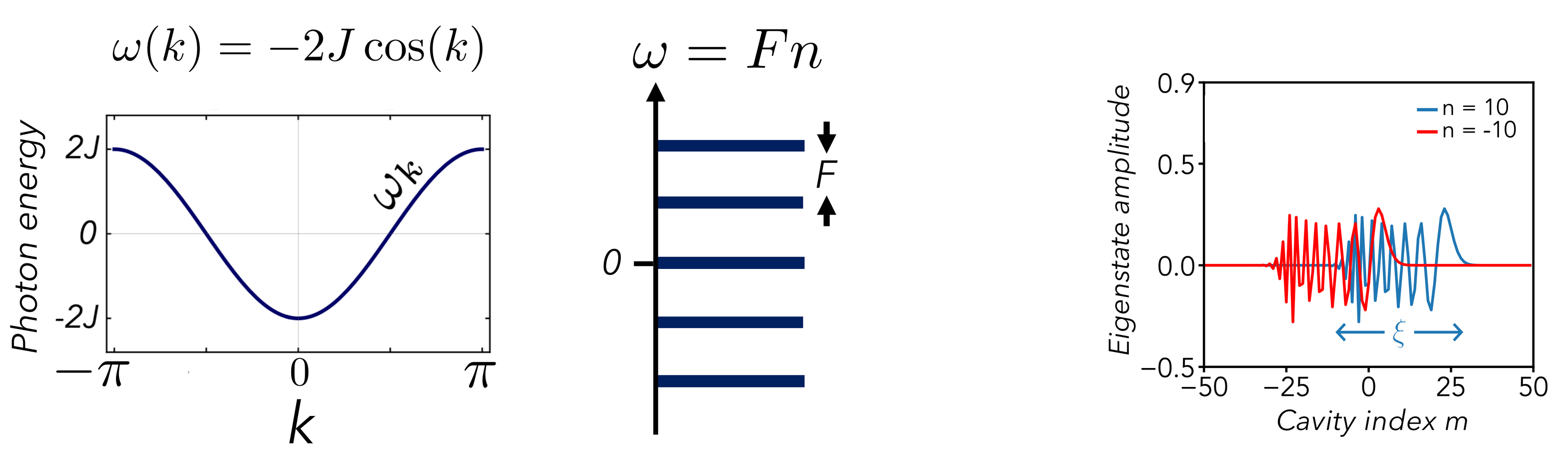


The qubit "senses" levels' decreteness

The qubit "sees" a quasi-continuum

LATTICE EIGENSTATES AND SPECTRUM

As we introduce the force into the lattice, the spectrum is no longer continuous. Now it features discrete equidistant levels, the so called Wannier-Stark ladder.



F=0 F>0

$$|\phi_n\rangle = \sum_m J_{m-n}(\xi)|m\rangle$$

The eigenstates are a combination of Bessel functions of the first kind and are localized in a portion of the lattice. We can define the localization length as

$$\xi = \frac{2J}{F}$$

BLOCH OSCILLATIONS

It is possible to show that in the presence of a constant force we can describe the evolution of the quasi-momentum k as

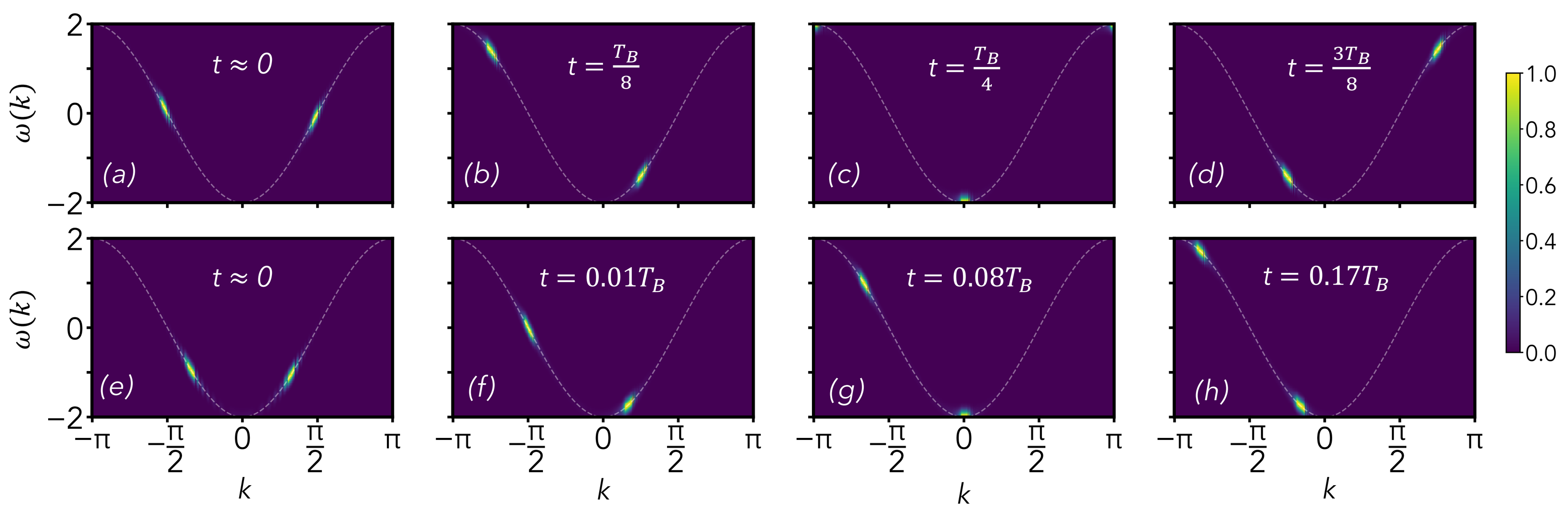
$$k(t) = k_0 - Ft$$

Therefore we can write the time dependent dispersion relation as

$$\omega(t) = -2J\cos(k_0 - Ft)$$

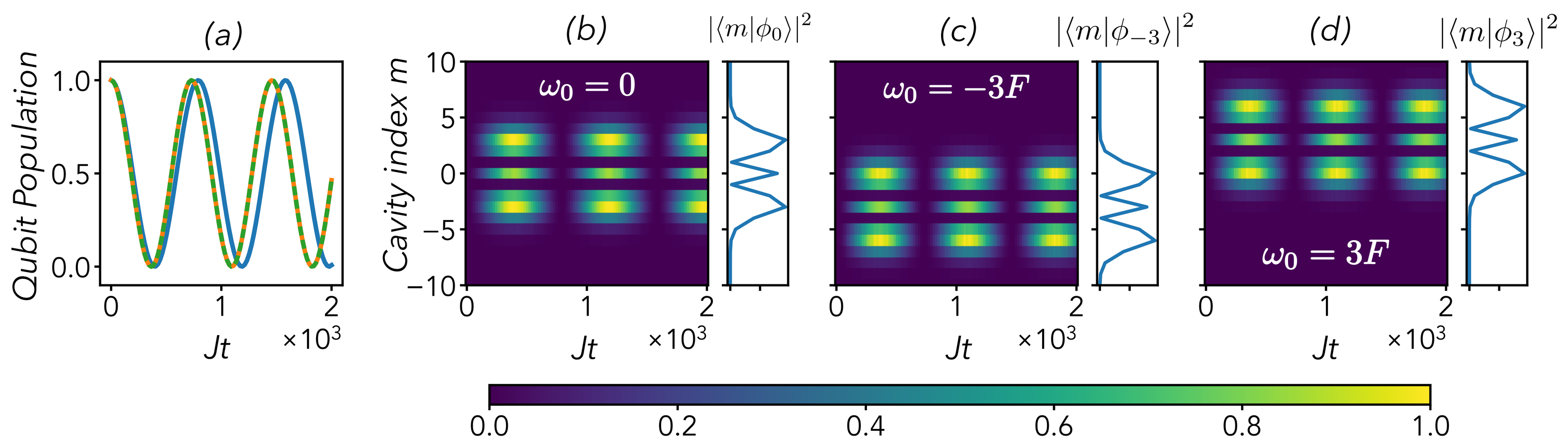
The energy of the photon will oscillate with a period given by the Bloch period.

$$T_B = \frac{2\pi}{F}$$



STRONG FORCE REGIME

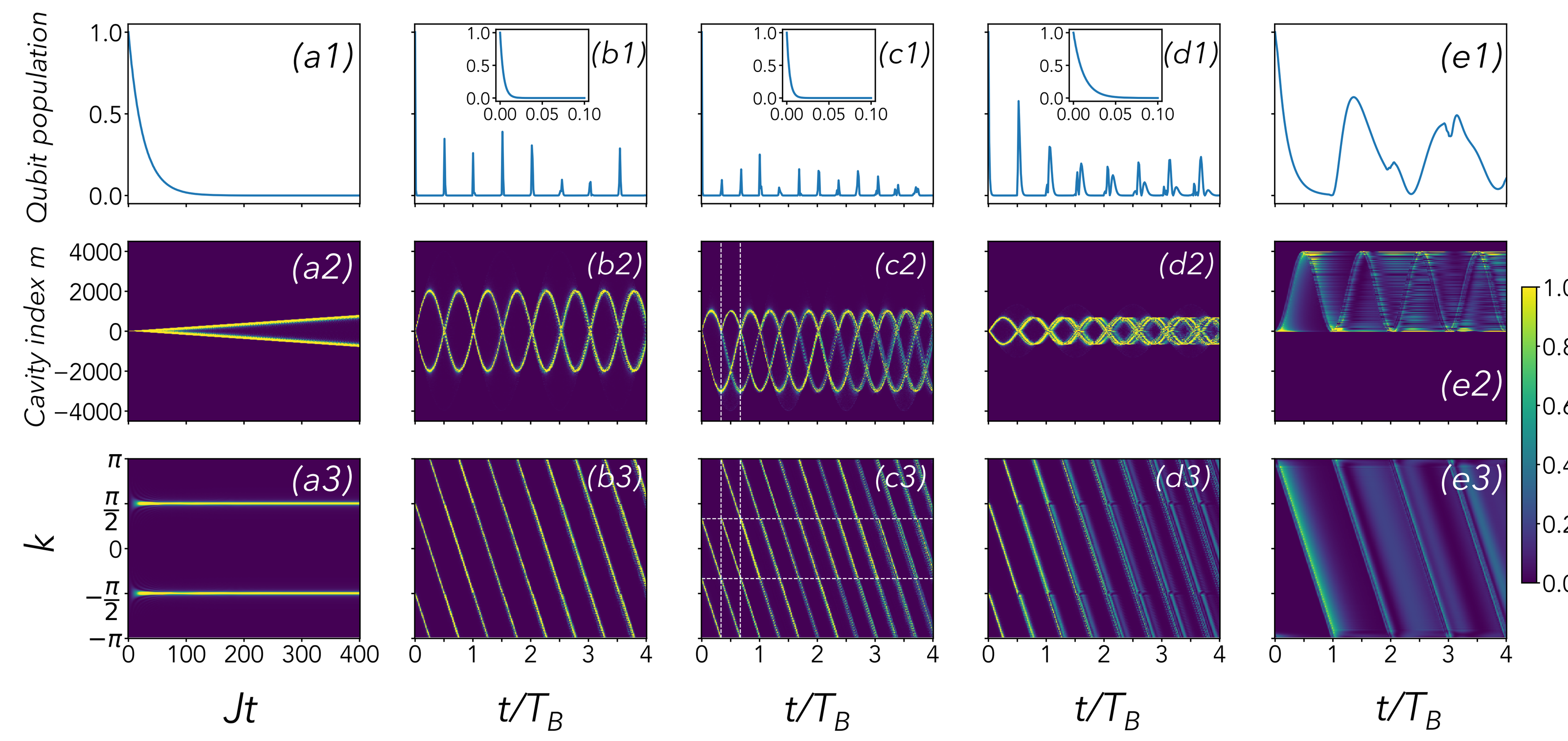
In this regime we have Jaynes-Cummings like interaction with a set of cavities involved in the dynamics.



By tuning the qubit frequency, lit is possible to move the bound state from the qubit position at the center of the lattice.

WEAK FORCE REGIME

In the weak force regime the photon propagates in a confined region of the lattice. When the photon reaches back the qubit we see the revivals in the qubit population.



Considering the atomic frequency at the center of the spectrum. It is possible to describe the non-Markovian behavior of the qubit population by the following equation,

$$\dot{\alpha_e}(t) = -\frac{\Gamma}{2}\alpha_e(t) - \Gamma \sum_{n=1}^{\infty} \alpha_e(t - nT_B)\Theta(t - nT_B) - \Gamma \sin(2\xi) \sum_{n=0}^{\infty} \alpha\left[t - \left(n + \frac{1}{2}\right)T_B\right] \Theta\left[t - \left(n + \frac{1}{2}\right)T_B\right]$$

where,

$$\Gamma = \frac{g^2}{J} \qquad T_B = \frac{2\pi}{F} \qquad \xi = \frac{2J}{F}$$

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