



An application of fuzzy set theory and interval-based distances for expert judgment aggregation

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ABSTRACT

Judgment aggregation is essential whenever multiple experts provide subjective evaluations that must be combined into a single coherent decision. This work adopts the perspective of a meta-expert who forms its final judgment after consulting a group of experts, giving greater weight to opinions closer to its own. Expert assessments are modelled using trapezoidal fuzzy numbers for both criteria weights and ratings, while the aggregation process relies on t-norms and t-conorms to combine fuzzy information coherently. The influence of each expert is determined through a recently introduced distance measure for trapezoidal fuzzy numbers, suitably rescaled to quantify the divergence between the meta-expert's confidence index and those of the others. The proposed methodology is applied to a classical benchmark concerning the prediction of pressure increase in the containment building of the Sequoyah nuclear power plant, and the results are compared with those obtained through existing approaches.

1. Introduction

When faced with the prospect of making a decision, experts are often consulted anonymously for their personal opinions, which are based on certain pre-defined, not necessarily explicit, criteria. This contrasts with the fact that those judgements are collected in order to establish a consensus. One method of achieving this consensus would be to aggregate these individual judgments into a single one, thus ensuring that each consulted expert's opinion is somehow taken into account and evaluated on different criteria. These opinions are based on the knowledge of the individual agents (the experts), related to the problem's domain of interest. However, it is important to note that both the opinions and the knowledge the experts possess can be precise (crisp) or vague (fuzzy). Since the introduction of Zadeh's 1965 article [1], fuzzy set theory has had a significant impact on decision-making theories [2–9] and the aggregation of judgments [10–12].

The present study starts from the work of Moon and Kang [11]. The authors conducted a study of judgment aggregation using triangular fuzzy numbers. These are defined as fuzzy confidence indices related to each expert with respect to each decision criterion. Nonetheless, when considering the aggregation of fuzzy numbers, it is logical to contemplate the utilisation of t-norms and t-conorms as an alternative to conventional arithmetic operations, as previously outlined in [13]. Furthermore, to obtain these fuzzy confidence indices, Moon and Kang considered the total integral value introduced in [14]. This paradigm introduces the idea of a meta-expert, who forms a judgment by consulting other experts, giving greater weight to those whose opinions are more closely aligned with its own. The methodology employed in the present study is based on a distance measure between trapezoidal fuzzy

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numbers introduced in 2024 by He et al. [15]. Recent developments in uncertainty modelling and multisource information fusion have also explored the use of divergence-based measures to quantify discrepancies between heterogeneous sources of information [16,17]. These approaches highlight the importance of measuring disagreement among information sources, which is closely related to the distance-based weighting mechanism adopted in the present work.

Paper positioning and scope

Before discussing the methodology in detail, it is important to clarify the type of problem addressed in this work, in order to distinguish it from related lines of research in expert judgment aggregation. On the one hand, there exists a wide range of literature focused on the aggregation of *subjective expert opinions on quantitative parameters*, often under significant epistemic uncertainty. A representative example is [18]. In this work, the authors employ possibility theory to model and combine expert assessments on numerical quantities, capturing both the inherent imprecision of expert knowledge and the variability of judgments. On the other hand, a separate literature addresses the aggregation of *subjective preferences over alternatives or criteria*, where axiomatic properties of the aggregation operators, such as monotonicity, neutrality, or consensus preservation, play a central role [19].

In the present study, the technical application the assessment of containment pressure rise in the Sequoyah nuclear power plant primarily involves *quantitative expert opinions* on an objective parameter. However, the methodology also requires the representation of linguistic judgments and weights over multiple criteria, which introduces elements reminiscent of preference aggregation. Therefore, while the problem is essentially one of estimating a quantitative outcome from multiple, potentially conflicting expert judgments, it also borrows from the theory of preference aggregation to handle fuzzy linguistic evaluations, criterion weights, and the meta-expert framework.

By explicitly situating our work in this context, we clarify that our aggregation method is designed to combine *quantitative subjective assessments* under uncertainty, with additional mechanisms inspired by preference aggregation to accommodate fuzzy linguistic representations. This positioning justifies both the comparison with existing quantitative aggregation approaches and the use of AHP-like techniques to model the underlying structure of expert assessments, thereby bridging the two literatures in a coherent framework.

The novelty of the present study lies in the use of trapezoidal, rather than triangular, fuzzy numbers; the aggregation of expert judgments using t-norms and t-conorms; and the adoption of a new distance measure for trapezoidal fuzzy numbers.

The structure of the work is as follows: Section 2 provides a summary of the definitions required for the approach under discussion. The third section of this work (Section 3) is concerned with the exposition of the aggregation method, together with the results that are indispensable to its comprehension. In Section 4, the model is applied to a classic problem in expert judgment aggregation, and a benchmark is used to compare results obtained using the present model with those obtained using alternative methods. The section also includes a dedicated analysis of the sharpness and robustness of the proposed approach. Finally, Section 5 discusses the methodology, its limitations, and how it can be adapted to different application domains, along with directions for future research.

2. Preliminaries and theoretical backgrounds

This section is devoted to briefly collecting the concepts and notions necessary for a clear and complete reading of the paper. In particular, we recall some preliminary material on fuzzy numbers, t-norms and t-conorms, and distance measures between fuzzy numbers.

Trapezoidal Fuzzy Numbers. To begin, it is necessary to recall some definitions on fuzzy numbers and the interval-based distance for trapezoidal fuzzy numbers. These concepts were introduced and thoroughly studied by He et al. in [15]. In general, a *fuzzy numbers* is a particular fuzzy subset of the set of real numbers $\mathbb{R}$ that satisfies additional properties (see, e.g., [20,21]). Here we are going to use similar definitions and results presented by Van Laarhoven and Pedrycz [22] in 1983.

A *trapezoidal fuzzy number* (TpFN), denoted as $\tilde{F} = (a, b, c, d)$, with $a, b, c, d \in \mathbb{R}$ and $a \leq b \leq c \leq d$, is a fuzzy number whose membership function has a trapezoidal shape (as in Fig. 1), i.e., for every $x \in \mathbb{R}$,

$$\tilde{F}(x) = (a, b, c, d)(x) = \mu(x) = \begin{cases} \frac{x-a}{b-a}, & x \in [a, b], \\ 1, & x \in [b, c], \\ \frac{x-d}{c-d}, & x \in [c, d], \\ 0, & \text{otherwise.} \end{cases}$$

The symbol $\widetilde{\mathcal{P}}(\mathbb{R}^+)$ usually denotes the set of fuzzy subsets of the positive real numbers $\mathbb{R}^+$. Moreover, the reader can easily check that a triangular fuzzy number (TFN) can be considered as a particular case of TpFN, where $a \leq b = c \leq d$.

For TpFNs, there exist pseudo-arithmetic operations, that are addition, multiplication, subtraction, and division [23,24]. These operations are defined as follows for any TpFN $\tilde{F}_i = (a_i, b_i, c_i, d_i) \in \widetilde{\mathcal{P}}(\mathbb{R}^+)$, with $i = 1, 2$

$$\begin{aligned} \tilde{F}_1 * \tilde{F}_2 &= (a_1 * a_2, b_1 * b_2, c_1 * c_2, d_1 * d_2) \\ \tilde{F}_1 \circ \tilde{F}_2 &= (a_1 \circ a_2, b_1 \circ a_2, c_1 \circ b_2, d_1 \circ a_2) \end{aligned}$$

with $*$ $\in \{+, \cdot\}$, $\circ \in \{-, \div\}$, and $0 \notin \{a_2, b_2, c_2, d_2\}$ for $\circ = \div$.

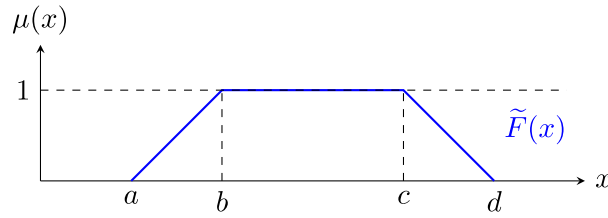


Fig. 1. Membership function of a trapezoidal fuzzy number $\tilde{F}$ defined by the parameters (a, b, c, d) .

Table 1

Table of Łukasiewicz, Gödel, and product logics.

	Łukasiewicz Logic	Gödel Logic	Product Logic
$x \otimes y$	$\max(x + y - 1, 0)$	$\min(x, y)$	$x \cdot y$
$x \oplus y$	$\min(x + y, 1)$	$\max(x, y)$	$x + y - x \cdot y$

Triangular Norms and Conorms. What follows briefly recalls the definitions of a t-norm and a t-conorm, and it presents the most well-known and commonly used examples found in the existing literature.

Definition 2.1 (cf.[25]). A function $\otimes : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is called *t-norm* if

1. $x \otimes y = y \otimes x$
2. $x \otimes (y \otimes z) = (x \otimes y) \otimes z$
3. $y \leq z \Rightarrow x \otimes y \leq x \otimes z$
4. $x \otimes 1 = x$,

for any $x, y, z \in [0, 1]$.

Altro

Definition 2.2 (cf.[25]). A function $\oplus : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is called *t-conorm* if satisfies conditions (1) – (3) of Definition 2.1 and

4. $x \oplus 0 = x$,

for any $x, y, z \in [0, 1]$.

In Table 1 there is a (non-exhaustive) list of some examples of t-norms and t-conorms [21]. One of the t-norm/t-conorm entailed from Łukasiewicz logic is the Zadeh's logic, that is $x \otimes_Z y = \min(x, y)$ and $x \oplus_Z y = \max(x, y)$.

For the sake of clarity, from now on, particularly in the Section 4 devoted to applications, we shall adopt the following notation: $\otimes_L$ and $\oplus_L$ will denote, respectively, the Łukasiewicz t-norm and t-conorm; the subscript G will indicate the Gödel operators, and P will stand for the Product ones.

A Distance for Fuzzy Numbers. Let us consider the definition of interval distance.

Definition 2.3. Let $I_{\mathbb{R}}$ be the set of all sub-intervals of $\mathbb{R}$. A mapping $d_I : I_{\mathbb{R}} \times I_{\mathbb{R}} \rightarrow \mathbb{R}$ is called a *distance between intervals* if and only if it satisfies the following properties, for any $[a_l, a_r], [b_l, b_r], [c_l, c_r] \in I_{\mathbb{R}}$

1. Non-negativity, i.e.,

$$d_I([a_l, a_r], [b_l, b_r]) \geq 0 \text{ and } d_I([a_l, a_r], [a_l, a_r]) = 0.$$

2. Symmetry, i.e.,

$$d_I([a_l, a_r], [b_l, b_r]) = d_I([b_l, b_r], [a_l, a_r]).$$

3. Triangle inequality, i.e.,

$$d_I([a_l, a_r], [b_l, b_r]) \leq d_I([a_l, a_r], [c_l, c_r]) + d_I([c_l, c_r], [b_l, b_r]).$$

An example of this distance is the following (see Theorem 1 in [15]). Let $[a_l, a_r]$ and $[b_l, b_r]$ be two numerical continuous intervals such that $a_l \leq a_r$ and $b_l \leq b_r$. Then, an example of distance between intervals is given by the following formula:

$$d_I([a_l, a_r], [b_l, b_r]) = \begin{cases} 0 & \text{if } [a_l, a_r] = [b_l, b_r]; \\ \int_0^1 \int_0^1 |a_l - b_l + (a_r - a_l)x - (b_r - b_l)y| dx dy & \text{otherwise.} \end{cases} \quad (1)$$

In [15], He et al. (2024) introduced a novel distance measure between fuzzy numbers. In the same paper, the authors also provided the definition of a distance between two fuzzy numbers, which we recall here for completeness.

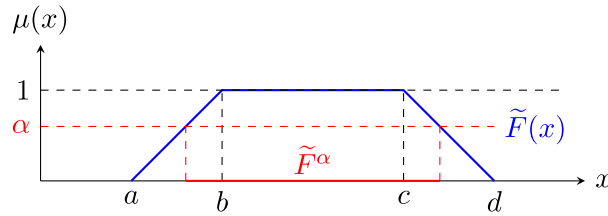


Fig. 2. Membership function of a trapezoidal fuzzy number $\tilde{F}$ with α -cut $\tilde{F}^\alpha = [a + \alpha(b - a), d - \alpha(d - c)]$ highlighted in red. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Definition 2.4. A function $d : \mathcal{P}(\mathbb{R}^+) \rightarrow \mathbb{R}$ is called a *distance for fuzzy numbers* if it satisfies the following properties for all $\tilde{F}_1, \tilde{F}_2, \tilde{F}_3 \in \mathcal{P}(\mathbb{R}^+)$:

1. Non-negativity, i.e.,

$$d(\tilde{F}_1, \tilde{F}_2) \geq 0, \quad \text{and} \quad d(\tilde{F}_1, \tilde{F}_1) = 0.$$

2. Symmetry, i.e.,

$$d(\tilde{F}_1, \tilde{F}_2) = d(\tilde{F}_2, \tilde{F}_1).$$

3. Triangle inequality, i.e.,

$$d(\tilde{F}_1, \tilde{F}_2) \leq d(\tilde{F}_1, \tilde{F}_3) + d(\tilde{F}_3, \tilde{F}_2).$$

The construction of this distance builds upon the interval distance d_I , previously introduced as an illustrative example. As a preliminary step, we recall that the α -cuts of a trapezoidal fuzzy number (TpFN) $\tilde{F} = (a, b, c, d)$ are the crisp intervals

$$\tilde{F}^\alpha = [a + \alpha(b - a), d - \alpha(d - c)], \quad (2)$$

for every $\alpha \in [0, 1]$.

Starting from the definition introduced in the seminal paper of Voxman [26], we now recall the notion of a reducing function (Fig. 2).

Definition 2.5. Let $s : [0, 1] \rightarrow [0, 1]$ be a function. We say that s is a *reducing function* if it satisfies:

- (i) $s(0) = 0$;
- (ii) $s(1) = 1$;
- (iii) s is increasing.

Moreover, a reducing function s is said to be *regular* if, in addition, it fulfills

$$(iv) \int_0^1 s(\alpha) d\alpha = \frac{1}{2}.$$

Example 2.1. The functions $s(\alpha) = \sin(\frac{\pi}{2}\alpha)$, $s(\alpha) = \frac{4}{\pi} \arctan(\alpha)$, and $s(\alpha) = \alpha^\kappa$, $\kappa > 0$, are reducing functions (Fig. 3).

For a deeper discussion on regular reducing functions, see the sub-Section 2.1 below. We are now ready for the following definition.

Definition 2.6. Let $\tilde{F}_1$ and $\tilde{F}_2$ be two arbitrary fuzzy numbers in $\mathcal{P}(\mathbb{R}^+)$, and let d_I be the distance between intervals defined by Eq. (1). The distance between the fuzzy numbers $\tilde{F}_1$ and $\tilde{F}_2$ with respect to a reducing function s is defined by

$$d_s(\tilde{F}_1, \tilde{F}_2) = \frac{\int_0^1 s(\alpha) \cdot d_I(\tilde{F}_1^\alpha, \tilde{F}_2^\alpha) d\alpha}{\int_0^1 s(\alpha) d\alpha} \quad (3)$$

where $\tilde{F}_1^\alpha$ and $\tilde{F}_2^\alpha$ are the α -cuts of $\tilde{F}_1$ and $\tilde{F}_2$, respectively, for all $\alpha \in [0, 1]$.

It can be shown that the above definition indeed defines a valid distance between fuzzy numbers (see Theorem 2 in [15]). Following the approach of the original authors, we set the reducing function as $s(\alpha) = \alpha^\kappa$ with the parameter $\kappa > 0$. With this choice, Eq. (3) takes the following form:

$$d_\kappa(\tilde{F}_1, \tilde{F}_2) = (\kappa + 1) \int_0^1 \alpha^\kappa d_I(\tilde{F}_1^\alpha, \tilde{F}_2^\alpha) d\alpha. \quad (4)$$

Before concluding this section, we take a closer look at the weighting function $f(\alpha, \kappa) = (\kappa + 1)\alpha^\kappa$ appearing in the definition of the distance between fuzzy numbers. Understanding its qualitative behavior provides further insight into how the parameter κ

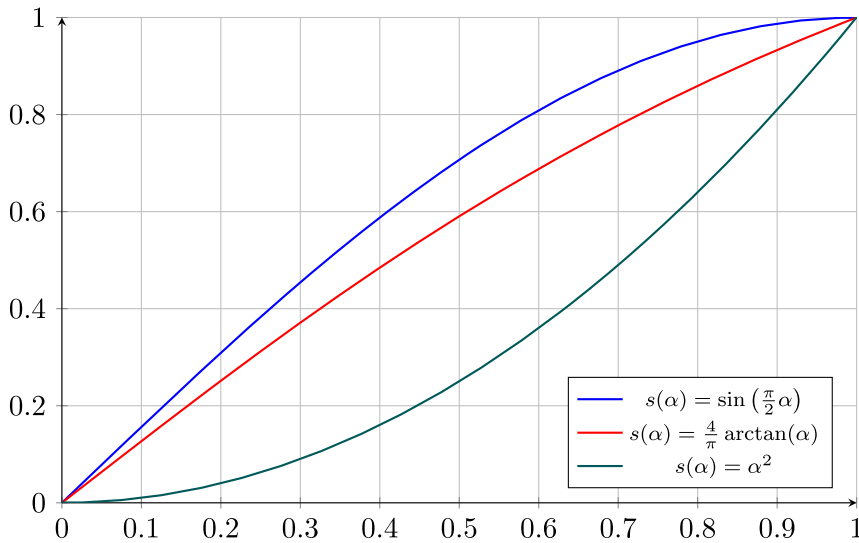
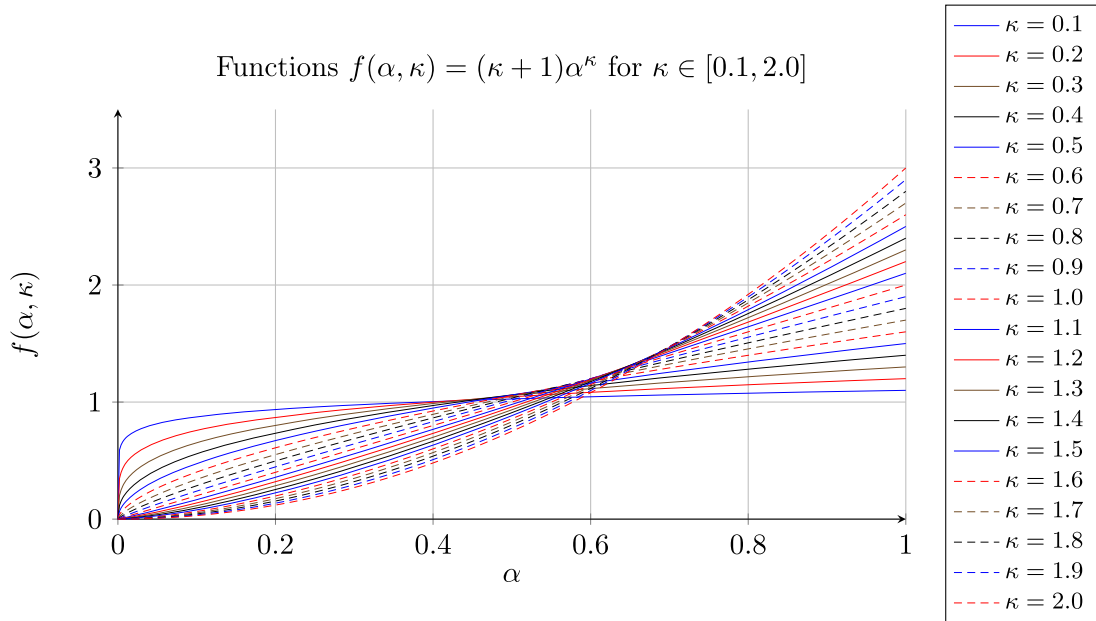


Fig. 3. Plots of functions considered in Example 2.1.

Fig. 4. The image of the function $f(\alpha, \kappa) = \frac{\alpha^\kappa}{\int_0^1 \alpha^\kappa d\alpha} = (\kappa + 1)\alpha^\kappa$ for each $\kappa \in \{0.1, 0.2, \dots, 2.0\}$.

influences the contribution of different α -levels in the computation of the distance. As illustrated in Fig. 4, the behavior of the function $f(\alpha, \kappa) = (\kappa + 1)\alpha^\kappa$ is consistent with intuition. As the parameter κ increases, the function becomes more concentrated near $\alpha = 1$, while for smaller values of κ the function places more weight near $\alpha = 0$. This reflects the role of $f(\alpha, \kappa)$ as a weighting function in the distance measure, emphasizing different regions of α depending on the choice of κ .

From Fig. 4, several qualitative observations can be made:

- For a fixed κ , the function $f(\alpha, \kappa)$ is strictly increasing with respect to α .
- At $\alpha = 1$, the maximum value of $f(\alpha, \kappa)$ is equal to $\kappa + 1$, which increases linearly with κ .
- When κ is close to zero, $f(\alpha, \kappa)$ approaches a constant value, meaning the corresponding integral behaves almost like an average aggregation.
- For larger values of κ , the contribution of higher α -cuts becomes increasingly dominant, making the weight distribution more focused near $\alpha = 1$.

2.1. Some remarks on regular reducing functions

We desire to conclude this section with some results on regular reducing functions. Clearly, as pointed out in He et al. [15], the function $s(\alpha) = \alpha$ is a regular reducing function. Now, we prove existence and nonexistence of non-polynomial regular reducing function. In what follows, properties (i), (ii), (iii), and (iv) refer to the corresponding items in Definition 2.5. First, we have the following.

Proposition 2.1. *Let B the unique positive solution of the equation*

$$\arctan(B) = \log(1 + B^2).$$

Then, the function $s(\alpha) = \frac{\arctan(B\alpha)}{\arctan(B)}$ is a regular reducing function.

Proof. We looking for regular reducing function of the form

$$s(\alpha) = A \arctan(B\alpha) \quad (5)$$

being $A, B > 0$. To this aim, we notice that, for these functions, the conditions (i) and (iii) are fulfilled for all $A, B > 0$. We focus on (ii) and (iv), a direct computation shows that it is equivalent to require that the following hold

$$\begin{cases} A \arctan(B) = 1, \\ A \arctan(B) - \frac{A}{2} \log(1 + B^2) = \frac{1}{2}. \end{cases} \quad (6)$$

Thus, equivalently, if B solves

$$\arctan(B) = \log(1 + B^2) \quad (7)$$

and $A = \frac{1}{\arctan(B)}$, we obtain our function.

It remains to prove that there exists a unique positive solution of Eq. (7). Let us define the function $g(B) = \arctan(B) - \log(1 + B^2)$. Clearly, g is a continuous function. Consider the interval $[1, \frac{6}{5}]$, a direct computation shows that $g(1) > 0$ and $g(\frac{6}{5}) < 0$. Thus, the classical Bolzano Theorem ensures the existence of at least one $B \in (1, \frac{6}{5})$ solution of Eq. (7). To prove the uniqueness, we consider again the function g , and its derivative $g'(B) = \frac{1-2B}{1+B^2}$. Due to the monotonicity of g , we infer the uniqueness of the positive solution for Eq. (7). $\square$

Finally, we prove two negative results, i.e., a nonexistence results for some classes of functions.

Theorem 2.1. *There are no regular reducing functions of the form*

$$s(\alpha) = A \sin(B\alpha) \quad (8)$$

for $A, B > 0$.

Proof. Let us consider function of type Eq. (8). Clearly, (i) is fulfilled for any $A, B > 0$. To guarantee (ii) and (iv), we need to require that

$$\begin{cases} \sin(B) = \frac{1}{A}, \\ \cos(B) = 1 - \frac{B}{2A}. \end{cases} \quad (9)$$

Thus, we can choose B among the (infinite) positive solutions of the equation

$$\cos(B) = 1 - \frac{B}{2} \sin(B), \quad (10)$$

and $A = \frac{1}{\sin(B)}$, i.e., we obtain functions of the form

$$s(\alpha) = \frac{\sin(B\alpha)}{\sin(B)}.$$

Finally, we prove that these functions do not satisfy (iii). Consider the derivative

$$s'(\alpha) = \frac{B}{\sin(B)} \cos(B\alpha).$$

To obtain increasing functions, one should have that for all $\alpha \in [0, 1]$,

$$\begin{cases} \sin(B) > 0, \\ \cos(B\alpha) \geq 0, \end{cases} \quad \vee \quad \begin{cases} \sin(B) < 0, \\ \cos(B\alpha) \leq 0. \end{cases} \quad (11)$$

A direct computation shows that both the above conditions cannot be satisfied for all $\alpha \in [0, 1]$, that is, at most α must be satisfied

$$\frac{2\eta}{2\eta+1} < \alpha < \frac{1}{2} + \frac{1}{4\eta} \vee \frac{1}{2} + \frac{1}{4\eta} < \alpha \leq 1$$

for all $\eta \geq 1, \eta \in \mathbb{Z}$, i.e., there exist some $\alpha = \alpha(B)$ such that the function $s(\alpha)$ is not increasing in $[0, 1]$. This completes the proof. $\square$

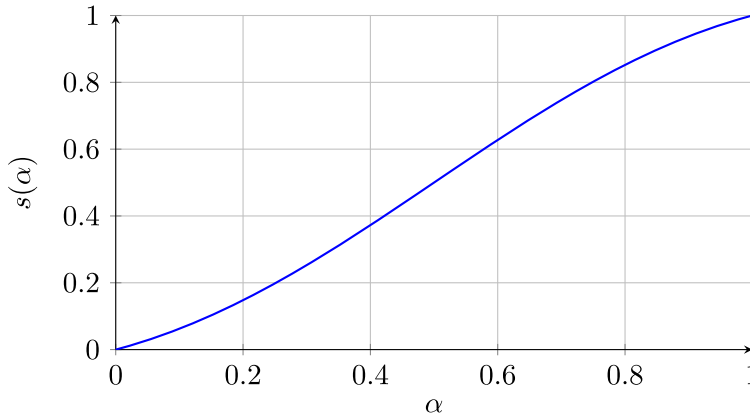


Fig. 5. Plot of the function $s(\alpha) = \frac{1}{2}(\alpha + \sin^2(\frac{\pi}{2}\alpha))$.

Notice that a slight perturbation of the previous function can realize a regular reducing function, see the next example.

Example 2.2. The function $s : [0, 1] \rightarrow [0, 1]$ defined as

$$s(\alpha) = \frac{1}{2} \left(\alpha + \sin^2 \left(\frac{\pi}{2} \alpha \right) \right)$$

is a regular reducing function (see Fig. 5). Clearly (i)-(ii)-(iii) are satisfied, we prove (iv):

$$\frac{1}{2} \int_0^1 \left(\alpha + \sin^2 \left(\frac{\pi}{2} \alpha \right) \right) d\alpha = \frac{1}{2} \left[\frac{1}{2} + \int_0^1 \sin^2 \left(\frac{\pi}{2} \alpha \right) d\alpha \right] = \frac{1}{2}.$$

Theorem 2.2. There are no regular reducing functions of the form

$$s(\alpha) = A \sinh(B\alpha) \quad (12)$$

for $A, B > 0$.

Proof. Arguing as in the previous results, to guarantee (i) is free. We check the assumptions (ii) and (iv), i.e.,

$$\begin{cases} \sinh(B) = \frac{1}{A}, \\ \cosh(B) = \frac{B}{2A} + 1. \end{cases} \quad (13)$$

Thus, we looking for positive solutions of the equation

$$\cosh(B) = 1 + \frac{B}{2} \sinh(B). \quad (14)$$

However, Eq. (14) admits only the trivial solution $B = 0$. $\square$

3. Method

Here we present our method for aggregating judgments, beginning with the model setup and the notation. Next, we establish some theoretical results that guarantee the correctness and coherence of the proposed algorithm. Finally, we describe in detail the precise steps of the algorithm.

3.1. The meta-expert

This subsection clarifies the role of the meta-expert in the judgment aggregation process and articulates the theoretical motivations behind the choice of our meta-expert-driven paradigm. In general, we think of the meta-expert as a super partes judge (not limited in a juridical sense) that has to judge or give an opinion on a problem or an issue. The initial stance of the meta-expert is grounded in a knowledge base $\mathcal{K}$, which provides the epistemic foundation upon which its judgments are modelled. In practice, this foundation is operationalised through the specification of weights assigned to each evaluation criterion (see Step 2 of the aggregation procedure described below). Weighting schemes for expert judgments across multiple criteria have been extensively studied in the literature in different directions [27–32] and are orthogonal to the primary methodological contribution of the present framework. Nonetheless, it is useful to outline a few practical examples of how the knowledge base $\mathcal{K}$ may be constructed and subsequently operationalized for weighting purposes. In general, $\mathcal{K}$ can be grounded in one of the following approaches:

- *Domain-specific gold standards*: The knowledge base can be linked to a set of reference benchmarks that are separately validated in the particular domain of interest, or to a set of procedures that estimate a latent truth, as in the case of standard models of reliability (e.g. [33,34]). In this case, the level of confidence in each of the experts reflects how well their opinions align with this standard, whether directly observed or statistically modeled.
- *Aggregated prior expert knowledge*: $\mathcal{K}$ can be obtained by pre-eliciting or aggregating the opinions of experts using formal opinion aggregation techniques like linear or logarithmic opinion pooling [35–38]. In this case, the confidence weights represent the epistemic aggregation of the prior.
- *Data-driven baselines*: Alternatively, the knowledge base may be derived from empirical data by aggregating expert outputs through ensemble techniques [39–41]. In this setting, confidence weights can be calibrated on predictive performance using proper scoring rules [28,42], so that influence reflects statistically assessed reliability.

The choice to weight experts proportionally to their alignment with the meta-expert's baseline is not intended as a form of epistemic partiality, but rather reflects a well-established debate in the epistemology of disagreement. In particular, the literature on higher-order evidence emphasizes that disagreement with a sufficiently reliable evaluator constitutes evidence about the quality of one's own belief-forming process [43]. Under this perspective, divergence from the meta-expert is not merely first-order disagreement, but higher-order evidence bearing on the reliability of the dissenting judgment. Moreover, discussions of peer disagreement highlight that the rational response to disagreement depends crucially on background assumptions about reliability and the structure of the evidential situation [44]. If one endorses some form of the Uniqueness Thesis according to which a given body of evidence rationally supports only one attitude, then persistent disagreement implies that at least one party is mis-evaluating the shared evidence, thereby justifying differential weighting [45]. Even conciliatory views, such as Elga's Equal Weight View [46], presuppose symmetry of epistemic standing; once such symmetry is broken by the introduction of a meta-expert grounded in a structured knowledge base, alignment can legitimately function as an indicator of epistemic reliability. In this sense, prioritising proximity to the meta-expert's judgment is not an arbitrary design choice, but a principled implementation of higher-order updating in contexts of structured disagreement.

However, we do not claim that this method is necessarily superior to non-meta-expert-driven approaches. Rather, it is intended to capture a different epistemic configuration. In addition to modelling a structurally distinct setting from standard symmetric aggregation frameworks, this approach may be considered more ecologically grounded, insofar as many real-world decision contexts involve hierarchical expertise, supervisory evaluation, or asymmetric informational structures.

3.2. Model setup and notation

Suppose we have n experts, namely $E_1, E_2, \dots, E_n$, and a meta-expert E who acts in a supervisory, impartial role above the individual experts, basing their judgment on a knowledge base $\mathcal{K}$. Let $C_1, C_2, \dots, C_m$ be m criteria. We collect these experts and criteria into two distinguished sets, that is, the set of experts $\mathcal{E} = \{E_i \mid 1 \leq i \leq n\}$ and the set of criteria $\mathcal{C} = \{C_j \mid 1 \leq j \leq m\}$. Let $\tilde{R}_j^i$ be the preference ratings of expert E_i to the criterion C_j , $\tilde{W}_j$ is the fuzzy weight of the criterion C_j , that is, the rating of its importance in the construction of the final judgment. We note here that, to ensure regularity results later on, we assume that the trapezoidal fuzzy numbers expressing the criteria's preference ratings and weights are defined by four real numbers within the unit interval $[0, 1]$. Finally, let $\tilde{F}_i$ be the fuzzy confidence index for the expert E_i .

This last index represents the degree of confidence in the expert, and it is obtained by aggregating $\tilde{R}_j^i$ and $\tilde{W}_j$ via the formula

$$\tilde{F}_i = \bigoplus_{j=1}^m (\tilde{R}_j^i \otimes \tilde{W}_j). \quad (15)$$

More precisely, since $\tilde{R}_j^i$ and $\tilde{W}_j$ are TpFNs, say $\tilde{R}_j^i = (\alpha_j^i, \beta_j^i, \gamma_j^i, \delta_j^i)$ and $\tilde{W}_j = (X_j, Y_j, Z_j, T_j)$, the fuzzy indices are also TpFNs. Specifically, $\tilde{F}_i = (a_i, b_i, c_i, d_i)$, where

$$\begin{aligned} a_i &= \bigoplus_{j=1}^m (\alpha_j^i \otimes X_j), & b_i &= \bigoplus_{j=1}^m (\beta_j^i \otimes Y_j), \\ c_i &= \bigoplus_{j=1}^m (\gamma_j^i \otimes Z_j), & d_i &= \bigoplus_{j=1}^m (\delta_j^i \otimes T_j), \end{aligned}$$

for every $1 \leq i \leq n$ and $1 \leq j \leq m$. The same can be done for meta-expert E , which has its own fuzzy confidence index

$$\tilde{F} = \bigoplus_{j=1}^m (\tilde{R}_j \otimes \tilde{W}_j), \quad (16)$$

where $\tilde{R}_j$ is the preference rating of the meta-expert E to the criterion C_j , for every $1 \leq j \leq m$.

Remark 3.1. A more natural framework for describing the computation of the fuzzy confidence indices $\tilde{F}_i$ involves the use of aggregation operators. To be precise, given the nature of the trapezoidal fuzzy numbers under consideration, it is necessary to refer to the concept of fuzzy aggregation operators, which inherently involve t-norms and t-conorms. That is, the indices $\tilde{F}_i$ should ideally

be presented as the result of triangular fuzzy aggregation operators (as in [13]). However, this approach has not been adopted for a fundamental reason. In Section 4, the methodology outlined herein will be implemented, employing the identical dataset utilized in [11,47,48], thereby facilitating a balanced and precise comparative assessment of the outcomes. In order to facilitate a comprehensive discussion regarding aggregation operators, it is imperative to ascertain that the axioms defining such operators are satisfied. In this case, if we do not consider a weight system $\tilde{W}_j$ normalized with respect to the t-conorm, one of these axioms is not fulfilled. Specifically, the axiom that is not a priori satisfied is the boundary condition axiom (for further details see, for instance, [3,5]).

3.3. Theoretical results

Theorem 3.1. Let $0 \leq a_i \leq b_i \leq c_i \leq d_i \leq 1$, for $i \in \{1, 2\}$, let $\tilde{F}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{F}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers. Set $\tilde{F}_i^\alpha = [a_i + \alpha(b_i - a_i), d_i - \alpha(d_i - c_i)]$, for all $\alpha \in [0, 1]$ and for $i \in \{1, 2\}$. Then,

$$\frac{(\kappa + 1)}{6} \int_0^1 \alpha^\kappa d_I(\tilde{F}_1^\alpha, \tilde{F}_2^\alpha) d\alpha \leq 1, \quad \forall \kappa > 0.$$

Proof. Clearly, it is enough to prove that $d_I(\tilde{F}_1^\alpha, \tilde{F}_2^\alpha) \leq 6$. In fact, it follows from this claim that

$$\frac{(\kappa + 1)}{6} \int_0^1 \alpha^\kappa d_I(\tilde{F}_1^\alpha, \tilde{F}_2^\alpha) d\alpha \leq (\kappa + 1) \int_0^1 \alpha^\kappa d\alpha = 1.$$

Without loss of generality, we can assume that $\tilde{F}_1^\alpha \neq \tilde{F}_2^\alpha$. Otherwise, the thesis is true since $d_\kappa(\tilde{F}_1^\alpha, \tilde{F}_2^\alpha) = 0$ (see Eq. (4)). Then, by triangular inequality, we infer

$$\begin{aligned} \frac{1}{6} d_I(\tilde{F}_1^\alpha, \tilde{F}_2^\alpha) &= \frac{1}{6} \int_0^1 \int_0^1 |a_1 + \alpha(b_1 - a_1) - a_2 - \alpha(b_2 - a_2) \\ &\quad + (d_1 - \alpha(d_1 - c_1) - a_1 - \alpha(b_1 - a_1))x \\ &\quad - (d_2 - \alpha(d_2 - c_2) - a_2 - \alpha(b_2 - a_2))y| dx dy \\ &= \frac{1}{6} \int_0^1 \int_0^1 [|a_1 - a_2| + \alpha(|b_1 - a_1| + |b_2 - a_2|)] dx dy \\ &\quad + \frac{1}{6} \int_0^1 \int_0^1 [|d_1 - a_1| + \alpha(|d_1 - c_1| + |b_1 - a_1|)] x dx dy \\ &\quad + \frac{1}{6} \int_0^1 \int_0^1 [|d_2 - a_2| + \alpha(|d_2 - c_2| + |b_2 - a_2|)] y dx dy \\ &\leq \frac{1}{2} \left(\int_0^1 \int_0^1 dx dy + \int_0^1 \int_0^1 x dx dy + \int_0^1 \int_0^1 y dx dy \right) = 1. \end{aligned}$$

Thus, the result follows from the previous considerations. $\square$

Remark 3.2. We emphasise that, once $\tilde{F}_1$ and $\tilde{F}_2$ are fixed, it is possible to reason as in the previous proof and obtain a sharper coefficient for the normalised distance d_κ , see Corollary 4.1 below. In any case 6 is sufficient for all date choices.

In order to weigh the opinion of each expert based on a meta-expert, we adopt the following paradigm based: if an expert E_i has a confidence level significantly different from that of the meta-expert E , then their weight should be adjusted accordingly (for further details, we refer to the subsection above devoted to the meta-expert). To this end, we employ a rescaled version of the novel fuzzy number distance proposed by He et al. in [15] to compute the distance $D_{i,\kappa}$ between each expert's fuzzy confidence index $\tilde{F}_i$ and the meta-expert's index $\tilde{F}$, that is

$$D_{i,\kappa} = \frac{1}{6} d_\kappa(\tilde{F}, \tilde{F}_i). \quad (17)$$

By Theorem 3.1 and assumptions made before on the TpFNs involved, we are sure that the distances $D_{i,\kappa} \leq 1$, for any $i \in \{1, \dots, n\}$. Therefore, in accordance with the previously defined meta-expert paradigm, and in order to penalise (i.e., assign decreasing weights to) judgments that are further from those of the meta-expert, we consider a weight equal to $1 - D_{i,\kappa}$. Thus, by normalising them, we obtain a system of weights w_k defined as

$$w_i = \text{norm}\{1 - D_{i,\kappa}\}, \quad (18)$$

with $w_i \geq 0$ and $\sum_{i=1}^n w_i = 1$. Finally, if we denote by $f_i(\theta)$ the estimate provided by expert E_i regarding a given parameter θ , the aggregated judgment of the n experts is computed as:

$$f(\theta) = \sum_{i=1}^n w_i f_i(\theta). \quad (19)$$

3.4. Procedure of aggregating experts' judgements

The following outlines the procedure used for the present study. We have summarised the steps to ensure clarity and reproducibility for the reader and to facilitate its application by any interested user. A schematic diagram summarizing the main steps of this procedure can be found in Fig. 6.

STEP 1. Problem definition.

1. Identify the meta-expert E .
2. The meta-expert E , drawing on their knowledge base $\mathcal{K}$, identifies n experts and m evaluation criteria for the problem under consideration, that is, defines the sets $\mathcal{E} = \{E_i \mid 1 \leq i \leq n\}$ and $\mathcal{C} = \{C_j \mid 1 \leq j \leq m\}$.
3. The meta-expert E sets a t-norm $\otimes$ and a t-conorm $\oplus$.
4. The meta-expert E establishes the preference rating sets associated with both the experts and the evaluation criteria. Each set includes a linguistic variable x , along with its corresponding term set: $W(x)$ for importance ratings, representing the possible importance values, and $S(x)$ for confidence ratings, representing the possible confidence levels. Each term set is associated with its respective membership functions.

STEP 2. Evaluate the fuzzy degree of confidence attributed to each expert and the meta-expert across the set of evaluation criteria.

1. The meta-expert E identifies his preference ratings $\tilde{R}_j$ to the criterion C_j , for every $1 \leq j \leq m$.
2. The weight $\tilde{W}_j$ of the criterion C_j is assigned by the meta-expert E , for every $1 \leq j \leq m$.
3. Each expert E_i assigns the preference rating $\tilde{R}_j^i$ to the criterion C_j , for every $1 \leq j \leq m$.
4. Compute the fuzzy confidence index $\tilde{F}$ by using Eq. (16).
5. For each expert E_i , compute the fuzzy confidence indices $\tilde{F}_i$ by using Eq. (15).

STEP 3. Computation of the weight assigned to each expert to be employed in the aggregation of experts' judgments.

1. Choose a value for $\kappa > 0$.
2. Compute the distance $D_{i,\kappa}$ between $\tilde{F}$ and $\tilde{F}_i$ using Eqs. (4) and (17).
3. Compute $1 - D_{i,\kappa}$, for each distance $D_{i,\kappa}$.
4. Normalise the values of the previous step to obtain the experts' weights by Eq. (18).

STEP 4. Aggregation of judgments using experts' weights.

1. Compute the group judgment as a weighted sum of the n expert opinions, using their respective weights.

$$f(\theta) = \sum_{i=1}^n w_i f_i(\theta). \quad (20)$$

3.5. Evaluating metrics

We conclude this section by introducing three quantitative criteria used to evaluate the performance of the proposed framework: consistency, sharpness, and robustness.

- **Consistency:** In the framework of aggregation processes, consistency has played a prominent role in scholarly debates across multiple disciplines, with important developments emerging in the economic literature, particularly in index number theory [49–51]. Owing to their transversal nature in any setting involving aggregation procedures, such consistency principles may be fruitfully examined together with stability-related properties, as considered in [52,53]. Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ be an aggregation operator. The operator f is said to satisfy *consistency-in-aggregation* (CA) if, for any finite partition $\{I_1, \dots, I_k\}$ of the index set $\{1, \dots, n\}$, the value obtained by aggregating all inputs in a single step coincides with the value obtained by first aggregating within each subset and then aggregating the resulting intermediate values using the same operator f . Formally, f satisfies CA if

$$f(x_1, \dots, x_n) = f(f(x_{I_1}), \dots, f(x_{I_k})),$$

whenever the aggregation procedure is coherently defined across stages. A canonical example of an operator satisfying CA is the weighted arithmetic mean. Indeed, the weighted arithmetic mean coincides with the operator defined in Eq. (20). Accordingly, the judgment aggregation algorithm presented above satisfies the CA property.

- **Sharpness:** In the context of judgment aggregation, the absence of an objective ground truth is inherent. Because the objects under consideration are expert judgments (e.g., forecasts), no empirical reference value is available against which absolute errors can be computed. Consequently, accuracy must be interpreted in distributional rather than absolute terms. Accordingly, we assess aggregation accuracy through the notion of *sharpness*, in the spirit of [28]. Sharpness refers to the concentration of the predictive distribution and is a property of the forecast itself. For interval forecasts, this notion naturally translates into the width of the prediction interval, as explicitly reflected in the interval score introduced in Section 6.2 of [28].

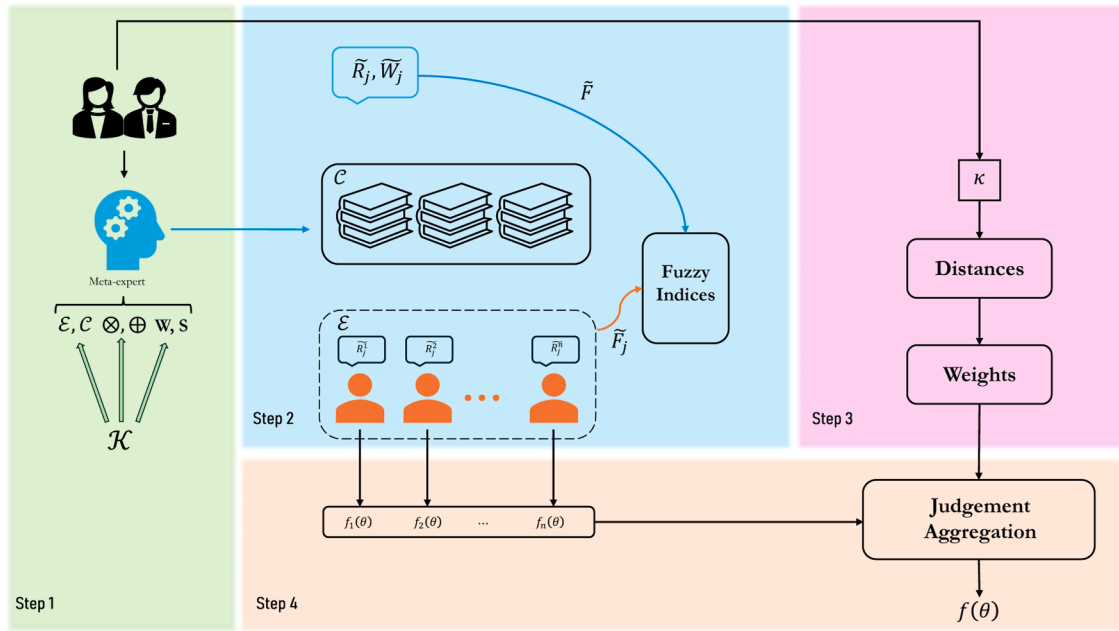


Fig. 6. Schematic representation of the proposed judgment aggregation framework. The process starts from the meta-expert, whose knowledge base $\mathcal{K}$ influences the selection of the sets of experts $\mathcal{E}$ and criteria $\mathcal{C}$, as well as the choice of the t-norm $\otimes$, t-conorm $\oplus$, and the linguistic scales W and S . Fuzzy ratings $\tilde{R}_j$ and weights $\tilde{W}_j$ are used to compute the fuzzy confidence indices $\tilde{F}_j$, which are then compared with the meta-expert's index $\tilde{F}$ to obtain distances and corresponding weights through the parameter κ . The final aggregated judgment $f(\theta)$ is obtained by combining the expert outputs in the Step 4.

- **Robustness** Let us consider f as in Eq. (20). We prove that f is a Lipschitz function w.r.t. the weight w_i . In fact, let choose $f_i(\theta)$ and consider the weights $W = (w_1, \dots, w_n)$ and $V = (v_1, \dots, v_n)$ with $W, V \in [0, 1]^n$ and $f_i(\theta) \in [a, b]$, one has

$$\begin{aligned} \left| \sum_{i=1}^N w_i f_i(\theta) - \sum_{i=1}^N v_i f_i(\theta) \right| &= \left| \sum_{i=1}^N (w_i - v_i) f_i(\theta) \right| \\ &\leq \sum_{i=1}^N |w_i - v_i| |f_i(\theta)| \\ &\leq b \sum_{i=1}^N |w_i - v_i| = bd(W, V) \end{aligned}$$

Hence, w.l.o.g., we may rescale the interval where fall the output of each experts and then infer $b = 1$. However, to meaningfully discuss and compare robustness within our framework, classical sensitivity techniques must be adapted to explicitly account for the structural parameters of the aggregation procedure, namely the choice of the distance parameter κ , as well as the selected t-norm $\otimes$, and t-conorm $\oplus$. Let s denote a generic aggregation setting, determined by a triple $s = (\kappa, \otimes, \oplus)$, and denote the set of these settings by S . For each $s \in S$, the aggregation algorithm produces an output value $f_s(\theta)$ (Eq. (20)). We quantify the robustness in our framework by measuring the variability of $f_s(\theta)$ across all configurations $s \in S$, e.g.

$$R(\theta) = \max_{s \in S} |f_s(\theta) - \bar{f}(\theta)|, \quad \bar{f}(\theta) = \frac{1}{|S|} \sum_{s \in S} f_s(\theta).$$

This quantity measures, in the same physical unit as θ , the maximal deviation induced purely by methodological choices. Smaller values of $R(\theta)$ indicate that the aggregation outcome is structurally stable with respect to the underlying fuzzy operators and distance parameter κ .

4. Application

4.1. Description of the accident scenario and expert judgement framework

The judgement aggregation problem considered in this work is based on a well-established benchmark scenario from nuclear engineering, originally analysed in [47] and later employed in several studies on expert elicitation and aggregation [11,48]. The scenario concerns the evaluation of the pressure rise inside the containment building of the Sequoyah Nuclear Power Plant following

Table 2
List of criteria.

Criterion	Abbreviation
Personal Knowledge	PK
Sources of Information	SI
Degree of Unbiasedness	UNB
Level of Independence from other Experts	IND
Personal Interest in the Study	PI
Past Experience	PE
Performance Measures	PM

Table 3
Trapezoidal fuzzy numbers (TpFNs) for the corresponding pairs of linguistic labels W and S .

Labels		TpFNs
W	S	
VL	VP	(0, 0, 0.1, 0.2)
L	P	(0.1, 0.2, 0.3, 0.4)
M	F	(0.3, 0.4, 0.5, 0.6)
H	G	(0.5, 0.6, 0.7, 0.8)
VH	VG	(0.7, 0.8, 0.9, 1)

a hypothetical Reactor Vessel Rupture (RVR). Because the dynamic response of the containment is strongly affected by the thermodynamic conditions at the moment of failure, expert assessments are required to estimate the expected pressure increase under a prescribed set of operating and environmental conditions.

The accident sequence is initiated by a postulated rupture of the reactor pressure vessel while the Reactor Cooling System is operating at a pressure of 2500 psia. The scenario assumes a dry reactor cavity at the time of the rupture, non-operational spray systems, and the presence of residual ice in the ice condenser. These assumptions reflect a high-stress condition for the containment response: the unavailability of sprays eliminates an important heat sink, while the residual ice provides only a transient mitigation of the steam release. As a consequence, the containment pressure rise becomes highly sensitive to uncertainties in heat transfer mechanisms, the amount of flashing, the blowdown rate, and the performance of passive heat removal features.

The Sequoyah plant employs an ice-condenser containment system designed to limit peak pressure by directing the blowdown flow through arrays of ice compartments. The amount of residual ice, its distribution, and its melting rate introduce significant uncertainty into the thermodynamic evolution of the containment atmosphere. In addition, the complex geometry of the compartments and the presence of structural obstructions affect both flow resistance and the effective heat-transfer area. These features make detailed first principles modelling of the accident progression challenging, thereby motivating the use of expert judgement to estimate the expected pressure rise.

In the original elicitation work [47], several experts provided probabilistic assessments of the containment peak pressure under the specified conditions. Their estimates reflected both modelling uncertainties and epistemic uncertainties related to limited experimental data and the difficulty of extrapolating containment behaviour to severe accident conditions. The resulting distributions exhibit substantial variability, which makes the scenario particularly suitable as a benchmark for aggregation methods.

In this work we adopt the same technical assumptions reactor operating conditions, system unavailabilities, and containment configuration to allow direct comparison with the results reported in [11,48]. Building on the detailed documentation available from the Nuclear Regulatory Commission [47], we slightly extend the scenario description to emphasise the thermal hydraulic and structural factors that shape expert assessments. This refined formulation highlights the inherent complexity of the problem and supports the evaluation of advanced aggregation techniques capable of integrating heterogeneous and potentially conflicting opinions.

4.2. Results

In this application, the meta-expert is denoted by E , while the panel of experts consists of E_1 , E_2 , and E_3 . The evaluation is carried out with respect to the set of criteria $C = \{PK, SI, UNB, IND, PI, PE, PM\}$ (see Table 2).

Furthermore, the term sets $W(x)$ and $S(x)$ are

$$W = \{\text{Very Low (VL), Low (L), Medium (M), High (H), Very High (H)}\}$$

$$S = \{\text{Very Poor (VP), Poor (P), Fair (F), Good (G), Very Good (VG)}\}.$$

Each item of these term sets is represented by a trapezoidal fuzzy number as shown in Table 3 and Fig. 7.

Lastly, Tables 4 and 5 show, respectively, preference rating for each criterion, as well as the confidence ratings given by each expert for each criterion. In this context, where the role of a meta-expert is recognised, it is reasonable to hypothesise that the preference ratings are provided by the meta-expert.

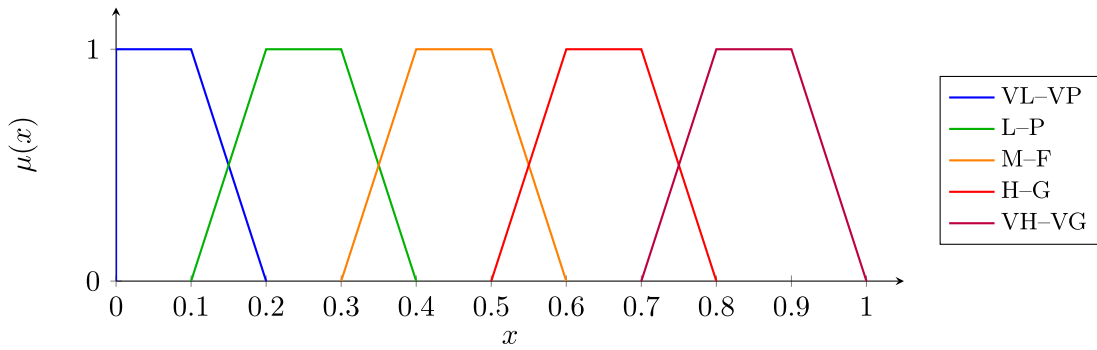


Fig. 7. Trapezoidal fuzzy numbers corresponding to the linguistic label pairs as in Table 3.

Table 4

Criteria and their associated ratings.

Criteria	Rating
Expert's personal knowledge (PK)	Very High
Expert's sources of information (SI)	High
Expert's degree of unbiasedness (UNB)	High
Expert's level of independence from other experts (IND)	Low
Expert's personal interest in the study (PI)	Medium
Expert's past experience (PE)	Low
Expert's performance measures (PM)	Medium

Table 5

Confidence ratings of the meta-expert and three experts for criteria.

Experts	PK	SI	UNB	IND	PI	PE	PM
E	Poor	Fair	Poor	Good	Very Poor	Very Good	Fair
E ₁	Good	Fair	Good	Very Good	Good	Good	Good
E ₂	Fair	Poor	Fair	Fair	Fair	Good	Fair
E ₃	Very Good	Good	Good	Good	Fair	Good	Good

Following the procedure described in Steps 1 to 3 of Section 3.4, we obtain the fuzzy confidence indices summarised in Table 6, using the nine possible combinations of t-norms and t-conorms listed in Table 1.

Here, before continuing, we include a brief remark on the choice of the parameter κ in Step 3, which appears in Eq. (4).

Remark 4.1. For each pair $(\tilde{F}, \tilde{F}_i)$, where $1 \leq i \leq 3$, we computed the distances $d_{\kappa}(\tilde{F}, \tilde{F}_i)$ for every $\kappa \in [0.1, 2]$, with steps of 0.1. More precisely, let $N = 20$ and let $\kappa_N = \kappa_{N-1} + 0.1 \cdot N$, where $\kappa_1 = 0.1$. The mean value μ of each distance $d_{\kappa_N}(\tilde{F}, \tilde{F}_i)$, with $1 \leq N \leq 20$ and $1 \leq i \leq 3$, for a chosen t-norm $\otimes$ and t-conorm $\oplus$, will be denoted by $\mu_i(\otimes, \oplus)$, that is

$$\mu_i(\otimes, \oplus) = \sum_{j=1}^N \frac{d_{\kappa_j}(\tilde{F}, \tilde{F}_i)}{N}.$$

With this notation, and according to the previous tables, Table 7 summarises the obtained average values.¹

Based on these values, we computed the root mean square deviation as

$$\sigma_i(\otimes, \oplus) = \sqrt{\frac{1}{N} \sum_{j=1}^N \left(d_{\kappa_j}(\tilde{F}, \tilde{F}_i) - \mu_i(\otimes, \oplus) \right)^2},$$

for $1 \leq i \leq 3$ (Table 8). Now, having both the mean value of the distances for the different t-norms and t-conorms considered, and thus also the standard deviation, we can compute the coefficient of variation $CV = \frac{\sigma}{|\mu|}$ for each distance. Since all coefficients of variation are of the order of 10^{-10} , their contribution is not relevant. Consequently, the mean values $\mu_i(\otimes, \oplus)$ can be regarded as accurate and representative. To conclude this remark, we observe that, given the small values of the coefficient of variation, the mean of the distances computed for varying values of κ can be considered reliable. Therefore, from now on, the distance between the fuzzy indices $\tilde{F}$ and $\tilde{F}_i$ will be taken as the mean value reported in Table 7, depending on the chosen t-norm and t-conorm, that is, $d_{\kappa}(\tilde{F}, \tilde{F}_i) = \mu_i(\otimes, \oplus)$ for every $1 \leq i \leq 3$.

¹ The datasets used to compute these values are available at the following [link](#).

Table 6

Fuzzy confidence indices of the meta-expert and of experts E_1 , E_2 , and E_3 for the different t-norm and t-conorm choices considered.

T-norm	T-conorm	$\tilde{F}$	$\tilde{F}_i$
Łukasiewicz	Łukasiewicz	(0.20, 0.40, 1.00, 1.00)	(0.20, 0.60, 1.00, 1.00)
Łukasiewicz	Gödel	(0.20, 0.40, 0.60, 0.80)	(0.20, 0.40, 0.60, 0.80)
Łukasiewicz	Product	(0.20, 0.40, 0.74, 0.95)	(0.20, 0.52, 0.90, 0.99)
Gödel	Łukasiewicz	(1.00, 1.00, 1.00, 1.00)	(1.00, 1.00, 1.00, 1.00)
Gödel	Gödel	(0.50, 0.60, 0.70, 0.80)	(0.50, 0.60, 0.70, 0.80)
Gödel	Product	(0.87, 0.96, 0.99, 1.00)	(0.93, 0.98, 0.99, 1.00)
Product	Łukasiewicz	(0.85, 1.00, 1.00, 1.00)	(1.00, 1.00, 1.00, 1.00)
Product	Gödel	(0.35, 0.48, 0.63, 0.80)	(0.35, 0.48, 0.63, 0.80)
Product	Product	(0.62, 0.81, 0.93, 0.99)	(0.73, 0.89, 0.97, 0.99)
-	-	$\tilde{F}_2$	$\tilde{F}_3$
Łukasiewicz	Łukasiewicz	(0.00, 0.20, 0.60, 1.00)	(0.40, 1.00, 1.00, 1.00)
Łukasiewicz	Gödel	(0.00, 0.20, 0.40, 0.60)	(0.40, 0.40, 0.80, 1.00)
Łukasiewicz	Product	(0.00, 0.20, 0.52, 0.90)	(0.40, 0.74, 0.94, 1.00)
Gödel	Łukasiewicz	(1.00, 1.00, 1.00, 1.00)	(1.00, 1.00, 1.00, 1.00)
Gödel	Gödel	(0.30, 0.40, 0.50, 0.60)	(0.70, 0.80, 0.90, 1.00)
Gödel	Product	(0.82, 0.93, 0.98, 0.99)	(0.97, 0.99, 1.00, 1.00)
Product	Łukasiewicz	(0.67, 1.00, 1.00, 1.00)	(1.00, 1.00, 1.00, 1.00)
Product	Gödel	(0.21, 0.32, 0.45, 0.60)	(0.45, 0.64, 0.81, 1.00)
Product	Product	(0.51, 0.74, 0.89, 0.97)	(0.80, 0.93, 0.98, 1.00)

Table 7

Average values of the distances $D_{i,k}$, denoted by μ_i for $i = 1, 2, 3$, for the different t-norm and t-conorm choices considered.

T-norm	T-conorm	μ_1	μ_2	μ_3
Łukasiewicz	Łukasiewicz	0.143	0.000	0.286
Łukasiewicz	Gödel	0.143	0.000	0.286
Łukasiewicz	Product	0.143	0.000	0.286
Gödel	Łukasiewicz	0	0	0
Gödel	Gödel	0.143	0	0.286
Gödel	Product	0.180	0.105	0.208
Product	Łukasiewicz	0.371	0.136	0.371
Product	Gödel	0.143	0.043	0.243
Product	Product	0.242	0.084	0.288

Table 8

Root mean squared deviations σ_i for $i = 1, 2, 3$ computed under the different t-norm and t-conorm configurations considered.

T-norm	T-conorm	σ_1	σ_2	σ_3
Łukasiewicz	Łukasiewicz	1.1330×10^{-10}	1.1295×10^{-15}	2.2660×10^{-10}
Łukasiewicz	Gödel	1.1330×10^{-10}	1.1295×10^{-15}	2.2660×10^{-10}
Łukasiewicz	Product	1.1330×10^{-10}	1.1295×10^{-15}	2.2660×10^{-10}
Gödel	Łukasiewicz	0	0	0
Gödel	Gödel	1.1330×10^{-10}	0	2.2660×10^{-10}
Gödel	Product	1.4277×10^{-10}	8.2965×10^{-11}	1.6526×10^{-10}
Product	Łukasiewicz	2.9457×10^{-10}	1.0763×10^{-10}	2.9457×10^{-10}
Product	Gödel	1.1330×10^{-10}	3.3989×10^{-11}	1.9261×10^{-10}
Product	Product	1.9219×10^{-10}	6.6265×10^{-11}	2.2858×10^{-10}

In this application, the specific TpFN values selected in Table 3 allow for a refinement of Eq. (17) in Step 3. More precisely, the following corollary guarantees that, under our setting, the distance d_k can be further rescaled.

Corollary 4.1. Let $0 \leq a_i \leq b_i \leq c_i \leq d_i \leq 1$, for $i \in \{1, 2\}$, be as in Table 3. Then,

$$\frac{5(\kappa+1)}{7} \int_0^1 \alpha^\kappa d_I(\tilde{F}_1^\alpha, \tilde{F}_2^\alpha) d\alpha \leq 1, \quad \forall \kappa > 0.$$

Proof. Arguing as in Theorem 3.1, it is sufficient to prove that

$$\frac{5}{7} d_I(\tilde{F}_1, \tilde{F}_2) \leq 1.$$

Table 9

Expert weights for each combination of T-norm and T-conorm. Left: weights w_1 , w_2 , w_3 for all three experts. Right: normalised weights w_1 and w_3 considering only E_1 and E_3 .

T-norm	T-conorm	All experts			Only E_1 and E_3	
		w_1	w_2	w_3	w_1	w_3
Łukasiewicz	Łukasiewicz	0.333	0.389	0.278	0.545	0.455
Łukasiewicz	Gödel	0.333	0.389	0.278	0.545	0.455
Łukasiewicz	Product	0.333	0.389	0.278	0.545	0.455
Gödel	Łukasiewicz	0.333	0.333	0.333	0.500	0.500
Gödel	Gödel	0.333	0.389	0.278	0.545	0.455
Gödel	Product	0.327	0.357	0.316	0.509	0.491
Product	Łukasiewicz	0.296	0.407	0.296	0.500	0.500
Product	Gödel	0.333	0.372	0.294	0.531	0.469
Product	Product	0.318	0.384	0.298	0.516	0.484

Table 10

Aggregated pressure values (in bar) corresponding to each probability level for all the aggregation methods considered.

	5%	25%	50%	75%	95%
E_1	3.60	4.00	4.50	5.460	6.22
E_2	1.60	–	4.00	–	10.80
E_3	3.00	4.00	5.00	6.50	8.00
Simple	2.73	4.00	4.50	5.98	8.34
AHP	2.868	4.000	4.560	5.976	8.034
Fuzzy	2.870	4.000	4.567	5.992	8.049
$(\otimes_L, \oplus_L)$	2.656	4.000	4.444	5.933	8.496
$(\otimes_L, \oplus_G)$	2.656	4.000	4.444	5.933	8.496
$(\otimes_L, \oplus_P)$	2.656	4.000	4.444	5.933	8.496
$(\otimes_G, \oplus_L)$	2.733	4.000	4.500	5.980	8.340
$(\otimes_G, \oplus_G)$	2.656	4.000	4.444	5.933	8.496
$(\otimes_G, \oplus_P)$	2.696	4.000	4.479	5.971	8.418
$(\otimes_P, \oplus_L)$	2.607	4.000	4.444	5.980	8.613
$(\otimes_P, \oplus_G)$	2.679	4.000	4.461	5.948	8.449
$(\otimes_P, \oplus_P)$	2.653	4.000	4.457	5.964	8.510

By a direct computation on the values in Table 3, one has

$$d_I(\tilde{F}_1, \tilde{F}_2) \leq \int_0^1 \int_0^1 \frac{9}{10} dx dy + \int_0^1 \int_0^1 \frac{x}{2} dx dy + \int_0^1 \int_0^1 \frac{y}{2} dx dy = \frac{7}{5}.$$

□

Remark 4.2. Notice that, in the previous corollary, once the values in Table 3 have been fixed, we have an estimate that is more than four times better than that of Theorem 3.1 ($\frac{7}{5}$ compared to 6).

Therefore, for this application, we obtain the simplified expression

$$D_{i,k} = \frac{5}{7} d_k(\tilde{F}, \tilde{F}_i).$$

First, the distances between the fuzzy indices were computed (see Remark 4.1 and Table 7). Then, the weights for each expert were calculated as in point 3 of Step 3. A clarification is necessary: there are probability values (such as 25% and 75%) for which expert E_2 , involved in the study [47], did not provide an opinion in terms of pressure values. Therefore, following standard practice, for the aggregation of these two values, the weights and normalisation were computed considering only experts E_1 and E_3 . For this reason, Table 9 reports the normalised weights both with respect to all three experts and with respect to E_1 and E_3 only.

Lastly, we aggregate the percentile estimates of the pressure rise (in bar) provided by each expert and aggregated using different methods (Simple [47], AHP [48], and Fuzzy [11]). Depending on the t-norm and t-conorm employed in the computations, we use the corresponding symbols to indicate how the percentage estimates have been generated. For example, the notation $(\otimes_L, \oplus_G)$ denotes that the Łukasiewicz t-norm and the Gödel t-conorm were used. This convention will also be adopted in the legend of Fig. A.1. After completing Step 4 in Section 3.4 and applying Eq. (20), we obtain the values reported in Table 10.

Some remarks are in order concerning the output of our aggregation algorithm, especially when compared with the results already available in the literature. To this end, we refer to Fig. A.1, which allows us to visualise more clearly the observations discussed below.

- A first point to highlight concerns the 25% percentile column. Since both experts E_1 and E_3 assign a pressure value of 4.00 bar (and expert E_2 provides no judgement for this percentile), all aggregation methods yield exactly 4.00 bar. This behaviour is

Table 11
Sharpness of the aggregated pressure values from expert judgments.

M	W_{90}	W_{50}
Simple	5.610	1.980
AHP	5.166	1.976
Fuzzy	5.179	1.992
$(\otimes_L, \oplus_L)$	5.840	1.933
$(\otimes_L, \oplus_G)$	5.840	1.933
$(\otimes_L, \oplus_P)$	5.840	1.933
$(\otimes_G, \oplus_L)$	5.607	1.980
$(\otimes_G, \oplus_G)$	5.840	1.933
$(\otimes_G, \oplus_P)$	5.722	1.971
$(\otimes_P, \oplus_L)$	6.006	1.980
$(\otimes_P, \oplus_G)$	5.770	1.948
$(\otimes_P, \oplus_P)$	5.857	1.964

fully consistent with the agreement between the two contributing experts and is observed not only for the classical aggregation techniques reported in the literature, but also for our method, regardless of the chosen t-norm or t-conorm. This is clearly visible in the plot, where all fifteen curves intersect at the point with coordinates (4, 0.25).

- The choice $(\otimes_G, \oplus_L)$ produces outputs that are identical to those obtained with the Simple method. This behaviour can be explained as follows: given the specific representation of the linguistic labels used to express opinions for both experts and criteria, applying first the Gödel t-norm and then the Łukasiewicz t-conorm always yields an output equal to 1. Indeed, in each case the Gödel t-norm produces four values whose sum exceeds 1, resulting in fuzzy indices of (1.0, 1.0, 1.0, 1.0), as reported in Table 6. Consequently, all distances are zero, all weights are equal to 1, and after normalisation across the three experts, the aggregated weights are uniformly $\frac{1}{3}$.
- Regarding the rows corresponding to the choices of the Łukasiewicz t-norm $\otimes_L$, one observes that the output pressure values are identical. This occurs because the weights, both when expert E_2 is considered and when it is not (see Table 9), turn out to be the same. This, in turn, follows from the (averaged) distances reported in Table 7. In principle, since the fuzzy indices in Table 6 associated with the t-norm $\otimes_L$ differ from one another, one would not expect the resulting distances to coincide. However, in this specific setting, the coincidence arises due to the numerical approximation adopted: distances were truncated to the third significant digit, and after this truncation, the values become identical. For this reason, in Fig. A.1 the three curves corresponding to the aggregated judgements obtained using the Łukasiewicz t-norm have been omitted and replaced by a single representative curve.

Finally, to complete this experimental analysis, we considered two additional meta-experts (Meta-Expert 2 and Meta-Expert 3), whose confidence ratings are reported in Table A.1. By applying the same aggregation algorithm described above, under the same settings of this specific application, we obtain the aggregated pressure values shown in Tables A.2 and A.3. The corresponding probability distributions are illustrated in Figs. A.2 and A.3. These tables and figures are provided in the Appendix.

It is worth noting that the observations made above for the first meta-expert apply verbatim to the other two meta-experts as well. This is because none of the three observations depends on the specific confidence ratings assigned by the meta-expert. Consequently, even when the meta-expert changes, the aggregated judgements produced by our method retain the same qualitative features.

4.2.1. Evaluation

Sharpness. Lacking any data on an explosion to validate against, the quantiles collectively define an implicit predictive distribution over the critical pressure threshold. Accordingly, sharpness measures the concentration of collective expert judgment rather than predictive accuracy. Let $q_\alpha^{(M)}$ denote the aggregated pressure quantile at probability level $\alpha \in \{5, 25, 50, 75, 95\}$ produced by aggregation method M , with

$$M = \{\text{Simple, AHP, Fuzzy, L-L, L-G, L-P, G-L, G-G, G-P, P-L, P-G, P-P}\}.$$

Accordingly to Section 3.5, in our setting, we quantify sharpness through

$$W_{90}^M = q_{95}^M - q_5^M, \quad W_{50}^M = q_{75}^M - q_{25}^M,$$

which measure the dispersion of the aggregated pressure distribution at the 90% and 50% credibility levels.

The results in Table 12 reveal a fairly consistent pattern across the different choices of t-norms and t-conorms (see also Fig. 8). Combinations involving the Łukasiewicz t-norm $(\otimes_L, \oplus)$ produce identical outcomes regardless of the t-conorm, suggesting that once $\otimes_L$ is fixed, the choice of t-conorm becomes essentially irrelevant. The combination $(\otimes_G, \oplus_L)$ is the only one that slightly improves sharpness over the Simple method (negative variation in W_{90}), behaving much like a classical averaging scheme. The Product t-norm $(\otimes_P, \oplus)$, on the other hand, tends to widen the tails of the aggregated distribution (larger positive variations in W_{90}), while leaving the central region largely unaffected (small or slightly negative variations in W_{50}). In other words, Product-based combinations increase dispersion in the extremes without significantly altering the core of the distribution.

Table 12

Relative percentage change in sharpness with respect to the baseline methods (Simple, AHP, and Fuzzy). Negative values indicate sharper (more concentrated) aggregated outputs, while positive values indicate increased dispersion.

M	$\Delta\%W_{90}^{\text{Simple}}$	$\Delta\%W_{90}^{\text{AHP}}$	$\Delta\%W_{90}^{\text{Fuzzy}}$	$\Delta\%W_{50}^{\text{Simple}}$	$\Delta\%W_{50}^{\text{AHP}}$	$\Delta\%W_{50}^{\text{Fuzzy}}$
$(\otimes_L, \oplus_L)$	4.100	13.047	12.763	-2.388	-2.190	-2.976
$(\otimes_L, \oplus_G)$	4.100	13.047	12.763	-2.388	-2.190	-2.976
$(\otimes_L, \oplus_P)$	4.100	13.047	12.763	-2.388	-2.190	-2.976
$(\otimes_G, \oplus_L)$	-0.059	8.530	8.258	0.000	0.202	-0.602
$(\otimes_G, \oplus_G)$	4.100	13.047	12.763	-2.388	-2.190	-2.976
$(\otimes_G, \oplus_P)$	1.990	10.755	10.477	-0.462	-0.260	-1.062
$(\otimes_P, \oplus_L)$	7.057	16.259	15.967	0.000	0.202	-0.602
$(\otimes_P, \oplus_G)$	2.852	11.692	11.411	-1.627	-1.428	-2.220
$(\otimes_P, \oplus_P)$	4.411	13.385	13.100	-0.820	-0.619	-1.418

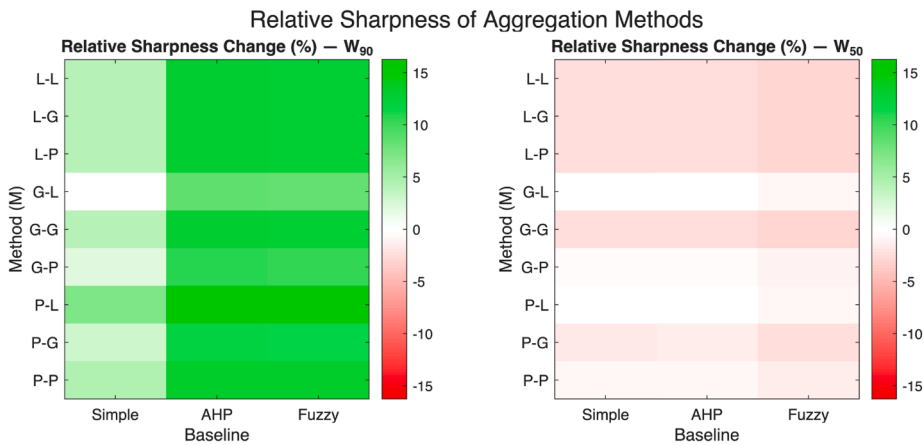


Fig. 8. Relative percentage variation in sharpness with respect to the baseline methods (Simple, AHP, Fuzzy) for W_{90} and W_{50} .

Table 13

Design-wise robustness and relative variability of the aggregated pressure values.

%	R (bar)	R%	Min (%)	Max (%)	Mean (%)	Median (%)	Std (%)
5	0.186	5.990	-5.990	2.953	-1.77×10^{-14}	0.701	3.218
25	0.000	0.000	0.000	0.000	-1.25×10^{-15}	0.000	3.73×10^{-15}
50	0.141	2.978	-2.978	1.828	-9.35×10^{-15}	0.774	1.967
75	0.114	1.869	-1.869	0.976	-2.34×10^{-14}	0.659	1.135
95	0.367	4.846	-2.248	4.846	9.82×10^{-15}	-0.454	2.494

Overall, the relative variations remain small across all configurations, which suggests that the aggregation framework is quite robust to the choice of $(\otimes, \oplus)$: different fuzzy operators tend to produce second-order adjustments rather than substantial differences in the outcomes.

The analysis above focuses on Meta-Expert 1 as the reference configuration. To assess how sensitive the framework is to this choice, we also considered two alternative meta-experts; the corresponding results are reported in the Appendix (see [Tables A.4–A.6](#) and [Figs. A.4, A.5](#)).

The main qualitative patterns carry over unchanged. Combinations involving the Łukasiewicz t-norm $(\otimes_L, \oplus_L)$ continue to produce nearly identical outcomes, and the configuration $(\otimes_G, \oplus_L)$ consistently stays close to the Simple method, confirming that neither depends meaningfully on the choice of meta-expert. Product-based combinations $(\otimes_P, \oplus_P)$ remain the most sensitive to this choice. The tail-widening effect observed for Meta-Expert 1 is attenuated for Meta-Expert 2 and, in some configurations, slightly amplified for Meta-Expert 3, reflecting the greater responsiveness of these operators to changes in the underlying weighting structure. Across all meta-experts, variations in W_{50} stay small and predominantly negative, confirming that the central region of the aggregated distribution is stable and that differences are confined to the tails.

In short, while numerical values naturally vary with the meta-expert, the qualitative behaviour of the different $(\otimes, \oplus)$ combinations remains consistent throughout, which speaks to the overall robustness of the proposed framework.

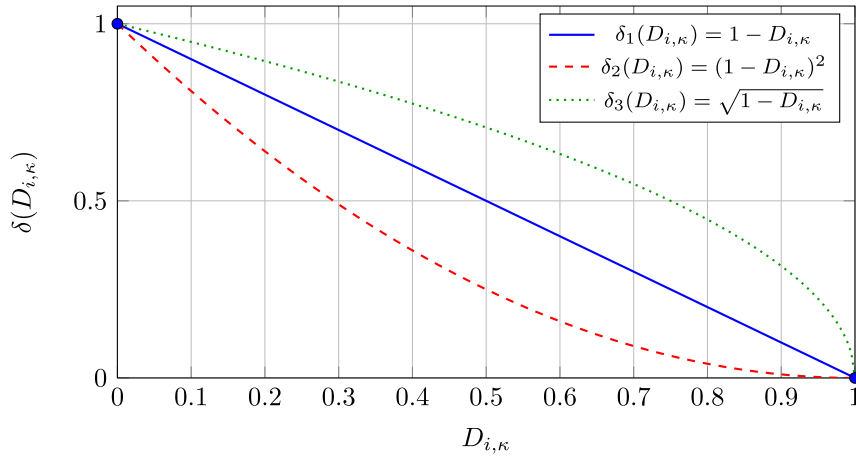


Fig. 9. Three decreasing functions on $[0, 1]$ with $\delta(0) = 1$ and $\delta(1) = 0$, which may be used by the meta-expert to penalise opinions that are farther from their own.

Robustness. In addition to the quantity $R(\theta)$ introduced in Section 3, Table 13 (and similarly Tables A.7 and A.8) also reports its relative version

$$R_{\text{design}}^{\%}(\theta) = 100 \cdot \frac{R(\theta)}{\bar{f}(\theta)},$$

which expresses the maximum deviation as a percentage of the mean aggregated value. For each percentile, we further summarise the distribution of the relative deviations

$$100 \cdot \frac{f_s(\theta) - \bar{f}(\theta)}{\bar{f}(\theta)}$$

across all configurations $s \in S$ by reporting their minimum, maximum, median, and standard deviation. These statistics give a more complete picture of the spread and symmetry of the variability induced by the choice of κ and $(\otimes, \oplus)$.

The results confirm that the framework is generally robust. For Meta-Expert 1 (Table 13), deviations remain small across all percentiles, both in absolute and relative terms. Meta-Expert 2 (Table A.7) tells essentially the same story, with results that are nearly identical, pointing to a high degree of stability with respect to the choice of meta-expert. Meta-Expert 3 (Table A.8) is the exception: it shows larger variability, particularly in the lower and upper percentiles, suggesting that the weighting structure induced by the meta-expert can influence the magnitude of the deviations, even if the overall pattern stays consistent.

In all three cases, the central percentile (25%) varies very little, while the largest deviations appear in the tails. The method is thus stable where it matters most, with sensitivity concentrated at the extremes.

5. Discussion and conclusions

In this paper, we proposed a judgment aggregation framework that combines expert assessments under epistemic uncertainty using trapezoidal fuzzy numbers and interval-based distances. The key idea is to adopt the viewpoint of a meta-expert, who weights each specialist according to how close their fuzzy confidence index is to their own, and then aggregates the individual judgments through a weighted scheme. Compared to earlier approaches built on triangular fuzzy numbers, our framework gains flexibility on several fronts: it uses trapezoidal representations, incorporates t-norms and t-conorms in the construction of the confidence indices, and relies on a recently introduced distance measure for fuzzy numbers. We tested the methodology on the well-known Sequoyah nuclear power plant benchmark, comparing our results with those of existing aggregation procedures. The application showed that the proposed approach reproduces the main qualitative features of the benchmark while offering a richer set of tools for understanding how different fuzzy operators and weighting structures shape the final aggregated judgment.

The proposed framework can be directly compared with the aggregation approaches used in the same benchmark, namely the Simple, AHP, and fuzzy methods of [11,47,48]. All methods produce qualitatively consistent results, but ours offers a more flexible structure for handling both epistemic uncertainty and heterogeneity among experts. Trapezoidal fuzzy numbers allow for a richer representation of uncertainty, and the distance-based weighting provides a principled way to account for disagreement with the meta-expert.

That said, the framework has some limitations worth acknowledging. The final outcome depends on the choice of the meta-expert E and on the parameter κ , which controls the sensitivity of the distance measure. The robustness analysis shows that these choices do not affect the qualitative behaviour of the aggregation, but they can influence the magnitude of the deviations, particularly in the tails. Similarly, the choice of t-norm and t-conorm matters: Łukasiewicz-based operators tend to yield stable (Tables 11 and A.4),

nearly invariant results (Tables 13, A.7, and A.8), while Product-based ones introduce larger dispersion at the extremes. In practice, the choice of aggregation operators should therefore be guided by the specific needs and priorities of the decision context at hand.

Before concluding, a few remarks on the method itself are in order. The decision to penalise opinions that differ from the meta-expert's own is, of course, just one possible modelling choice. We placed ourselves in a situation where a neutral, *super partes* expert is asked to evaluate and aggregate the judgments of a panel, and naturally decides to give more weight to opinions closer to their own and less to those further away. Importantly, this does not mean ignoring dissenting voices: the algorithm includes no threshold below which a judgment would be discarded. Such a threshold might seem appealing in theory, but in practice it would be hard to justify, since all expert opinions should be part of the decision-making process.

Even within our setting, there is room for variation. In Step 3 of the algorithm, one could replace the specific penalisation scheme with any decreasing function δ on $[0, 1]$ satisfying $\delta(0) = 1$ and $\delta(1) = 0$, opening the door to a whole family of weighting mechanisms (see, for instance, Fig. 9).

More broadly, the proposed framework is not tied to the nuclear engineering context considered here. Its core components, namely fuzzy representations of uncertainty, distance-based weighting, and weighted aggregation, can be naturally adapted to other multi-attribute decision-making contexts by calibrating the membership functions, the distance scaling, and the choice of aggregation operators. In particular, the parameter κ and the scaling of the distance measure can be tuned to reflect the nature of the uncertainty in the problem at hand. In safety-critical contexts such as nuclear engineering or healthcare, larger values of κ may be preferred to emphasise the upper α -cuts and adopt a more conservative stance, whereas in exploratory settings such as environmental policy or economic analysis a more balanced weighting may be more appropriate. Similarly, different t-norm and t-conorm choices control the sensitivity of the aggregation to extreme opinions, while alternative mappings from distance to weights (through functions δ) allow for different penalisation schemes. In this respect, our approach is related to evidential fuzzy MCDM frameworks such as [54–56], which also combine uncertainty modelling and multi-criteria aggregation, but rely on belief-based fusion mechanisms. Here, instead, the influence of each expert is directly tied to its distance from a reference meta-expert, offering an alternative and complementary perspective on the aggregation problem.

CRediT authorship contribution statement

Giuseppe Failla: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Resources, Methodology, Investigation, Formal analysis; **Gianmarco La Rosa:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization; **Marco Elio Tabacchi:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Resources, Project administration, Methodology, Investigation, Formal analysis, Data curation.

Ethical approval

Not applicable.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

All datasets used in the study were generated directly by the authors and are fully reported within the manuscript.

Appendix A.

Tables

Table A.1

Confidence ratings of meta-expert 2 and meta-expert 3 for the criteria.

Meta-Experts	PK	SI	UNB	IND	PI	PE	PM
ME ₂	Very Poor	Very Good	Good	Poor	Very Good	Fair	Good
ME ₃	Good	Fair	Fair	Fair	Fair	Good	Poor

Table A.2

Aggregated pressure values (in bar) corresponding to each probability level for all the aggregation methods considered, obtained using Meta-Expert 2.

	5%	25%	50%	75%	95%
E ₁	3.60	4.00	4.50	5.460	6.22
E ₂	1.60	–	4.00	–	10.80
E ₃	3.00	4.00	5.00	6.50	8.00
Simple	2.73	4.00	4.50	5.98	8.34
AHP	2.868	4.000	4.560	5.976	8.034
Fuzzy	2.870	4.000	4.567	5.992	8.049
($\otimes_L, \oplus_L$)	2.779	4.000	4.500	5.940	8.228
($\otimes_L, \oplus_G$)	2.779	4.000	4.500	5.940	8.228
($\otimes_L, \oplus_P$)	2.779	4.000	4.500	5.940	8.228
($\otimes_G, \oplus_L$)	2.733	4.000	4.500	5.980	8.340
($\otimes_G, \oplus_G$)	2.779	4.000	4.500	5.940	8.228
($\otimes_G, \oplus_P$)	2.744	4.000	4.501	5.972	8.315
($\otimes_P, \oplus_L$)	2.830	4.000	4.543	5.980	8.130
($\otimes_P, \oplus_G$)	2.764	4.000	4.500	5.953	8.264
($\otimes_P, \oplus_P$)	2.765	4.000	4.508	5.967	8.268

Table A.3

Aggregated pressure values (in bar) corresponding to each probability level for all the aggregation methods considered, obtained using Meta-Expert 3.

	5%	25%	50%	75%	95%
E ₁	3.60	4.00	4.50	5.460	6.22
E ₂	1.60	–	4.00	–	10.80
E ₃	3.00	4.00	5.00	6.50	8.00
Simple	2.73	4.00	4.50	5.98	8.34
AHP	2.868	4.000	4.560	5.976	8.034
Fuzzy	2.870	4.000	4.567	5.992	8.049
($\otimes_L, \oplus_L$)	2.779	4.000	4.500	5.940	8.228
($\otimes_L, \oplus_G$)	2.779	4.000	4.500	5.940	8.228
($\otimes_L, \oplus_P$)	2.779	4.000	4.500	5.940	8.228
($\otimes_G, \oplus_L$)	2.733	4.000	4.500	5.980	8.340
($\otimes_G, \oplus_G$)	2.779	4.000	4.500	5.940	8.228
($\otimes_G, \oplus_P$)	2.729	4.000	4.494	5.972	8.347
($\otimes_P, \oplus_L$)	2.742	4.000	4.504	5.980	8.320
($\otimes_P, \oplus_G$)	2.764	4.000	4.500	5.953	8.264
($\otimes_P, \oplus_P$)	2.725	4.000	4.490	5.967	8.354

Table A.4

Sharpness of the aggregated pressure values for meta-experts 2 and 3.

(a) Meta-Expert 2		
M	W_{90}	W_{50}
Simple	5.610	1.980
AHP	5.166	1.976
Fuzzy	5.179	1.992
$(\otimes_L, \oplus_L)$	5.449	1.940
$(\otimes_L, \oplus_G)$	5.449	1.940
$(\otimes_L, \oplus_P)$	5.449	1.940
$(\otimes_G, \oplus_L)$	5.607	1.980
$(\otimes_G, \oplus_G)$	5.449	1.940
$(\otimes_G, \oplus_P)$	5.572	1.972
$(\otimes_P, \oplus_L)$	5.300	1.980
$(\otimes_P, \oplus_G)$	5.500	1.953
$(\otimes_P, \oplus_P)$	5.503	1.967

(b) Meta-Expert 3		
M	W_{90}	W_{50}
Simple	5.610	1.980
AHP	5.166	1.976
Fuzzy	5.179	1.992
$(\otimes_L, \oplus_L)$	5.449	1.940
$(\otimes_L, \oplus_G)$	5.449	1.940
$(\otimes_L, \oplus_P)$	5.449	1.940
$(\otimes_G, \oplus_L)$	5.607	1.980
$(\otimes_G, \oplus_G)$	5.449	1.940
$(\otimes_G, \oplus_P)$	5.618	1.972
$(\otimes_P, \oplus_L)$	5.578	1.980
$(\otimes_P, \oplus_G)$	5.500	1.953
$(\otimes_P, \oplus_P)$	5.629	1.967

Table A.5

Relative percentage change in sharpness with respect to the baseline methods for Meta-Expert 2.

M	$\Delta\% W_{90}^{\text{Simple}}$	$\Delta\% W_{90}^{\text{AHP}}$	$\Delta\% W_{90}^{\text{Fuzzy}}$	$\Delta\% W_{50}^{\text{Simple}}$	$\Delta\% W_{50}^{\text{AHP}}$	$\Delta\% W_{50}^{\text{Fuzzy}}$
$(\otimes_L, \oplus_L)$	-2.861	5.487	5.222	-2.020	-1.822	-2.610
$(\otimes_L, \oplus_G)$	-2.861	5.487	5.223	-2.020	-1.822	-2.610
$(\otimes_L, \oplus_P)$	-2.861	5.487	5.223	-2.020	-1.822	-2.610
$(\otimes_G, \oplus_L)$	-0.059	8.530	8.258	0.000	0.202	-0.602
$(\otimes_G, \oplus_G)$	-2.861	5.487	5.223	-2.020	-1.822	-2.610
$(\otimes_G, \oplus_P)$	-0.685	7.850	7.580	-0.386	-0.184	-0.986
$(\otimes_P, \oplus_L)$	-5.521	2.599	2.341	0.000	0.202	-0.602
$(\otimes_P, \oplus_G)$	-1.961	6.465	6.198	-1.382	-1.183	-1.976
$(\otimes_P, \oplus_P)$	-1.912	6.519	6.251	-0.640	-0.439	-1.239

Table A.6

Relative percentage change in sharpness with respect to the baseline methods for meta-expert 3.

M	$\Delta\%W_{90}^{\text{Simple}}$	$\Delta\%W_{90}^{\text{AHP}}$	$\Delta\%W_{90}^{\text{Fuzzy}}$	$\Delta\%W_{50}^{\text{Simple}}$	$\Delta\%W_{50}^{\text{AHP}}$	$\Delta\%W_{50}^{\text{Fuzzy}}$
$(\otimes_L, \oplus_L)$	-2.861	5.487	5.223	-2.020	-1.822	-2.610
$(\otimes_L, \oplus_G)$	-2.861	5.487	5.223	-2.020	-1.822	-2.610
$(\otimes_L, \oplus_P)$	-2.861	5.487	5.223	-2.020	-1.822	-2.610
$(\otimes_G, \oplus_L)$	-0.059	8.530	8.258	0.000	0.202	-0.602
$(\otimes_G, \oplus_G)$	-2.861	5.487	5.223	-2.020	-1.822	-2.610
$(\otimes_G, \oplus_P)$	0.137	8.743	8.470	-0.393	-0.192	-0.994
$(\otimes_P, \oplus_L)$	-0.576	7.969	7.698	0.000	0.202	-0.602
$(\otimes_P, \oplus_G)$	-1.961	6.465	6.198	-1.382	-1.183	-1.976
$(\otimes_P, \oplus_P)$	0.338	8.961	8.688	-0.675	-0.474	-1.273

Table A.7

Design-wise robustness and relative variability of the aggregated pressure values for meta-expert 2.

%	R (bar)	$R^{\%}$	Min (%)	Max (%)	Mean (%)	Median (%)	Std (%)
5	0.646	28.758	-28.758	11.124	2.86×10^{-14}	2.410	11.277
25	0.000	0.000	-2.22×10^{-14}	0.000	-1.19×10^{-15}	0.000	3.93×10^{-15}
50	0.424	9.592	-9.592	1.709	-3.96×10^{-15}	1.709	3.692
75	0.226	3.533	-3.533	1.400	-1.42×10^{-14}	1.400	2.222
95	1.315	13.862	-6.064	13.862	2.04×10^{-14}	-0.898	5.539

Table A.8

Design-wise robustness and relative variability of the aggregated pressure values for meta-expert 3.

%	R (bar)	$R^{\%}$	Min (%)	Max (%)	Mean (%)	Median (%)	Std (%)
5	0.343	13.884	-7.030	13.884	5.26×10^{-14}	-7.029	9.276
25	0.000	0.000	-1.11×10^{-14}	0.000	-3.47×10^{-16}	0.000	1.94×10^{-15}
50	0.091	2.003	-1.179	2.003	2.21×10^{-14}	-0.488	1.076
75	0.356	5.620	-5.620	2.587	1.94×10^{-14}	2.587	3.400
95	0.784	8.718	-8.718	4.579	1.59×10^{-14}	4.579	6.002

Figures

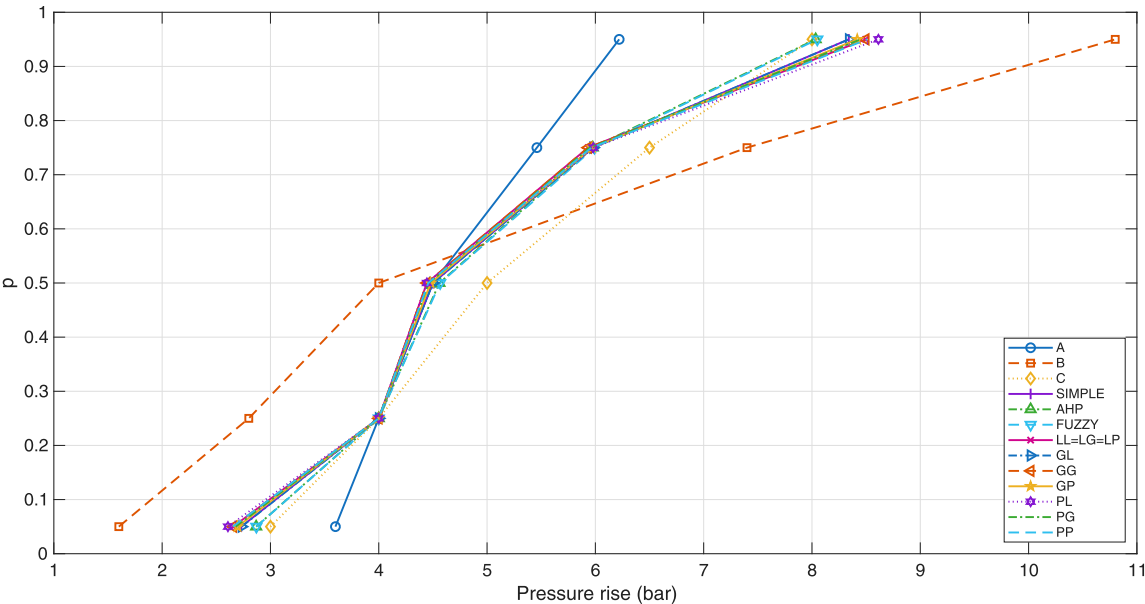


Fig. A.1. Estimates of pressure rise in the Sequoyah containment building (Table 10).

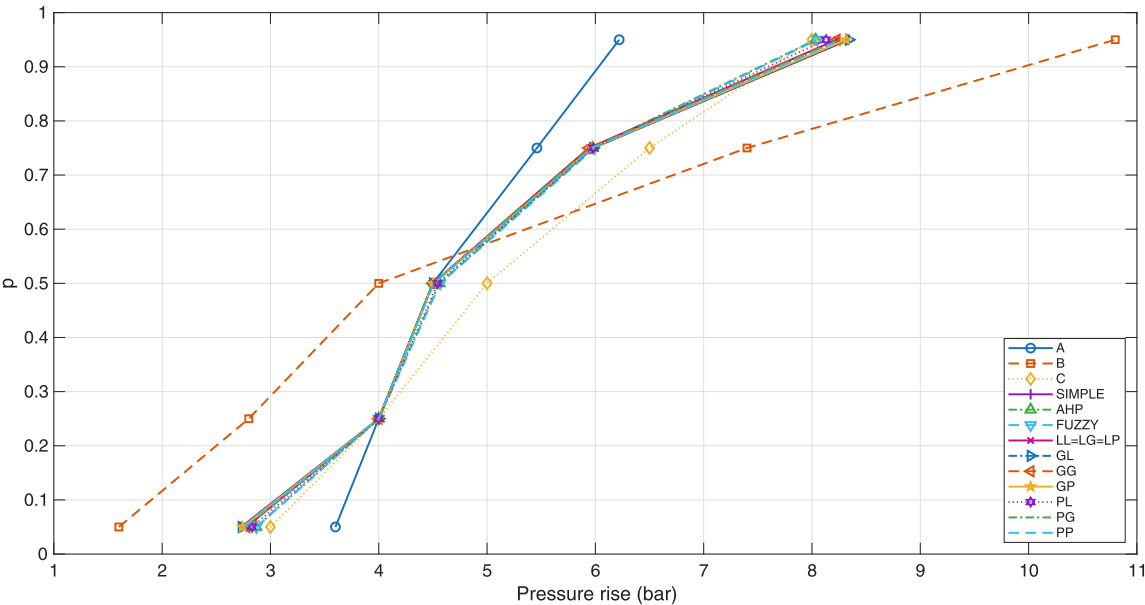


Fig. A.2. Aggregated estimates of pressure rise in the Sequoyah containment building (Table A.2), obtained using Meta-Expert 2.

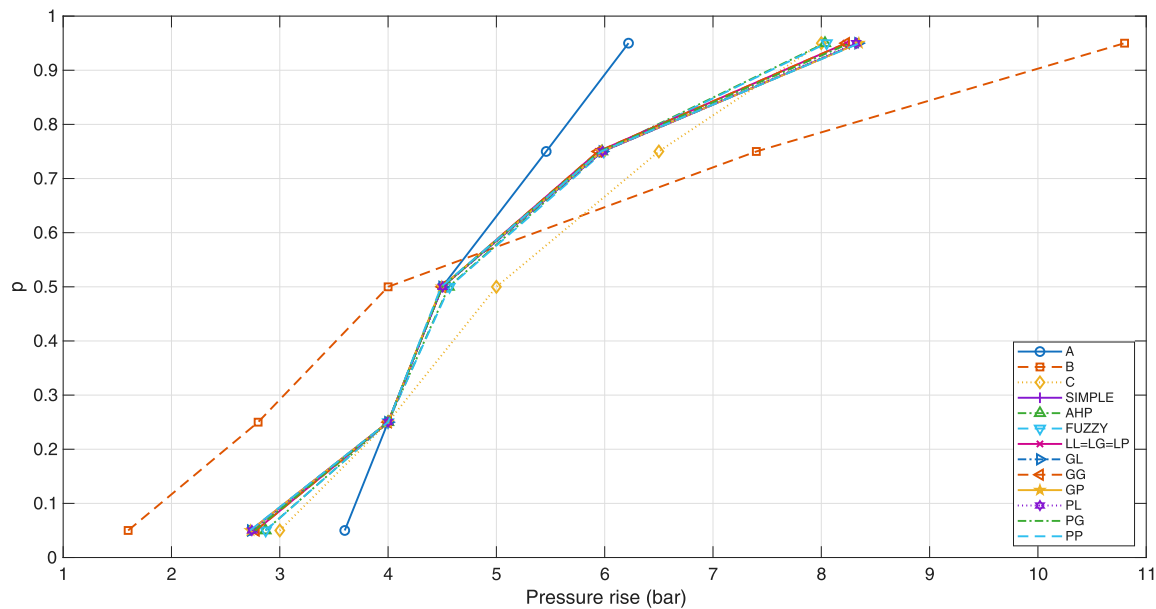


Fig. A.3. Aggregated estimates of pressure rise in the Sequoyah containment building (Table A.3), obtained using Meta-Expert 3.

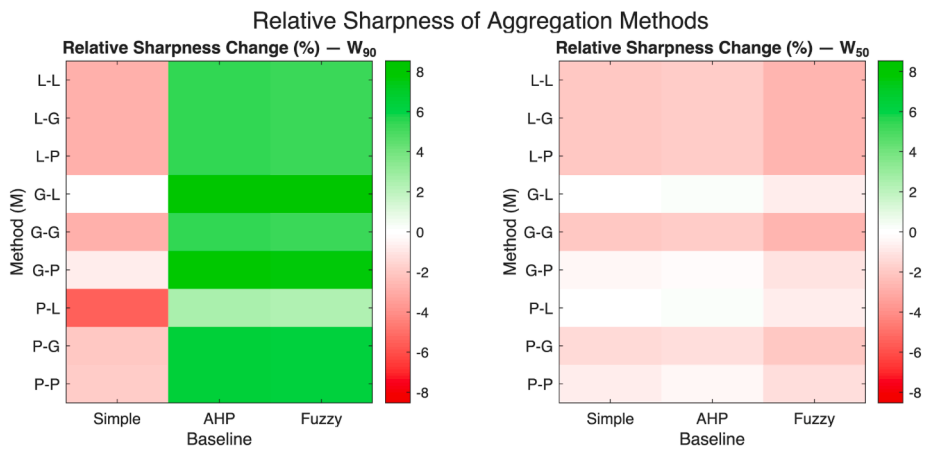


Fig. A.4. Relative percentage variation in sharpness with respect to the baseline methods for W_{90} and W_{50} (Meta-Expert 2).

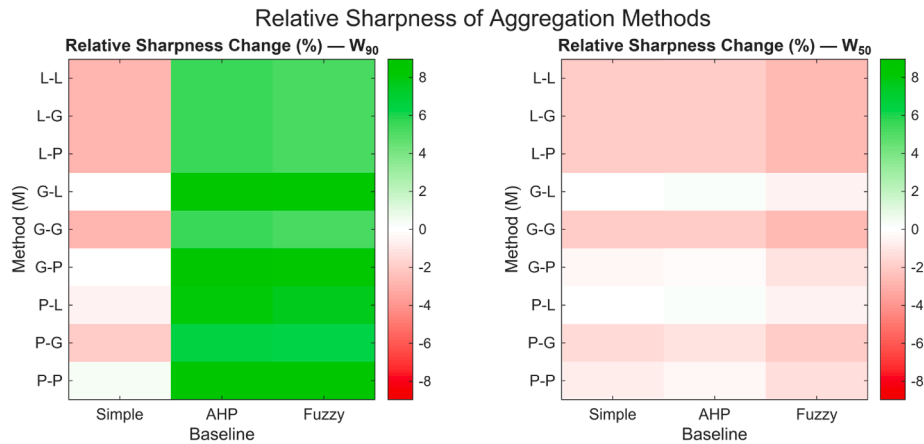


Fig. A.5. Relative percentage variation in sharpness with respect to the baseline methods for W_{90} and W_{50} (Meta-Expert 3).

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