



Tapered drip laterals for sloping agricultural fields

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Abstract

Tapered drip laterals offer a promising solution to enhance hydraulic performance and reduce energy consumption in horizontal or sloping agricultural fields. However, their design typically relies on iterative numerical procedures, which limit their practical implementation. This study develops a fully analytical framework for the hydraulic design of dual-diameter tapered drip laterals installed on downhill slopes. The methodology is based on a normalised pressure head formulation that enables the explicit derivation of pressure head distribution and directly embeds pressure head tolerance criteria to limit emitter flow-rate variability across the entire lateral, without requiring iterative computations. A new pressure head tolerance is introduced at the diameter-change point, *DCP*, leading to three distinct pressure head distribution patterns and providing a systematic basis for selecting *DCP* and upstream diameter. The analytical procedure is validated against the exact step-by-step numerical method that relaxes the assumption of uniform emitter discharge. Relative errors in characteristic pressure heads remained below 0.81 %, while deviations in hydraulic performance indicators were negligible for practical design purposes. Application to representative case studies demonstrates that properly designed tapered laterals can reduce the required inlet pressure head by more than 60 % compared with equivalent one-diameter configurations, resulting in substantial energy savings and reduced pumping requirements. The analytical framework provides a practical and reliable tool for optimising the hydraulic and energetic performance of tapered drip laterals on sloping terrain.

Abbreviations

C	Pipe smoothness factor	$h_{*min,1}$	Normalised minimum pressure head of sub-lateral 1 ($j = j_{min,1}$)
D	Inside diameter of the one-diameter drip lateral (L)	$h_{*min,2}$	Normalised minimum pressure head of sub-lateral 2 ($j = j_{min,2}$)
D_1	Inside diameter of sub-lateral 1 (L)	h_{*n}	Normalised pressure head at the <i>DCP</i> ($j = N_1 + 1$)
D_2	Inside diameter of sub-lateral 2 (L)	h_d	Pressure head at the distal end of the lateral (L)
<i>DCP</i>	diameter-change point ($j = N_1 + 1$)	h_{in}	Inlet pressure head (L)
g	Acceleration of gravity ($L\ T^{-2}$)	$H_j^{(-r)}$	Generalised harmonic number of order $-r$ truncated at the emitter j
h_{*d}	Normalised pressure head at the distal end of the lateral ($j = 1$)	h_j	Pressure head at the emitter j (L)
h_{*in}	Normalised inlet pressure head ($j = N + 1$)	$h_{min,1}$	Minimum pressure head of sub-lateral 1 (L)
h_{*j}	Normalised pressure head at the emitter j	$h_{min,2}$	Minimum pressure head of sub-lateral 2 (L)
h_{*max}	Normalised maximum pressure head of the tapered lateral	h_n	Pressure head at the <i>DCP</i> (L)
		h_{var}	Pressure head variability
		j	Emitter position along the tapered lateral, counted from the distal end of the lateral
		$j_{min,1}$	Emitter position at $h_{min,1}$
		$j_{min,2}$	Emitter position at $h_{min,2}$
		K	Unit hydraulic gradient of the one-diameter drip lateral
		K_*	Ratio between K and S_0
		K_{*1}	Ratio between K_1 and S_0

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$K_{*1,th}$	K_{*1} threshold
K_{*2}	Ratio between K_2 and S_0
K_1	Unit hydraulic gradient associated with $q_{n,1}$ and D_1 for sub-lateral 1
K_2	Unit hydraulic gradient associated with $q_{n,2}$ and D_2 for sub-lateral 2
k	Flow resistance equation coefficient
$k_{e,1}$	Emitter characteristic of sub-lateral 1
$k_{e,2}$	Emitter characteristic of sub-lateral 2
L_1	Length of sub-lateral 1 (L)
L_2	Length of sub-lateral 2 (L)
N	Number of emitters of the tapered lateral
N_1	Number of emitters in sub-lateral 1
N_{*1}	Ratio between N_1 and N
N_2	Number of emitters in sub-lateral 2
q_n	Emitter flow rates of the one-diameter drip lateral ($L^3 T^{-1}$)
$q_{*n,1}$	Ratio between $q_{n,1}$ and $q_{n,2}$
Q_1	Circulating discharge that feeds sub-lateral 1 ($L^3 T^{-1}$)
$q_{n,1}$	Emitter flow rates of sub-lateral 1 ($L^3 T^{-1}$)
$q_{n,2}$	Emitter flow rates of sub-lateral 2 ($L^3 T^{-1}$)
r	Flow rate exponent of the flow resistance equation
s	Inside diameter exponent of the flow resistance equation
S_{*2}	Ratio between S_2 and S_1
S_0	Slope of the tapered lateral
S_1	Emitter spacing of sub-lateral 1 (L)
S_2	Emitter spacing of sub-lateral 2 (L)
x_1	Exponent of the emitter flow rate-pressure head relationship for sub-lateral 1
x_2	Exponent of the emitter flow rate-pressure head relationship for sub-lateral 2
δ	Tapered pressure head tolerance
δ_1	Pressure head tolerance of sub-lateral 1
δ_2	Pressure head tolerance of sub-lateral 2
δ_s	Pressure head tolerance at diameter-change point DCP
ζ	Riemann zeta function
λ	Amplification factor to account for local losses
ν	Water's kinematic viscosity ($L^2 T^{-1}$)

Introduction

Drip irrigation systems consist of low-pressure pipe networks equipped with emitters that deliver water directly to the root zone. By minimising evaporation, runoff, and deep percolation losses, these systems enhance water-use efficiency, reduce energy requirements, and frequently improve crop yield (Sokol et al. 2019; Singh and Su 2022;

Ravikumar 2023; Yang et al. 2023). To ensure adequate water distribution uniformity, a rigorous hydraulic design must be performed. In practice, the diameter of laterals and submain, or, equivalently, the maximum allowable length for a given diameter, are derived from the equations governing the system's energy state (Bralts and Segerlind 1985; Kang and Nishiyama 1996; Provenzano et al. 2005; Zhang et al. 2013; Baiamonte et al. 2015; Yurdem et al. 2015).

For a given lateral length, optimal hydraulic design may involve dividing the pipeline into segments of differing diameters, such that the pressure head variation along the lateral matches the maximum allowable pressure head difference (Valiantzas 2002). Traditionally, telescopic or tapered configurations have been adopted mainly for sub-mains, whereas their application to laterals has been limited, typically being confined to steep downhill fields where large elevation differences must be accommodated (Ravikumar 2023). More recently, tapering has been shown to improve the hydraulic performance of horizontal drip laterals by enhancing pressure and discharge uniformity along the pipeline and, consequently, application uniformity within the irrigated unit (Baiamonte et al. 2024).

Although drip irrigation systems have been widely investigated, the specific case of tapered laterals installed on sloping fields remains comparatively underexplored. In particular, limited attention has been devoted to developing rigorous analytical design methodologies and performance criteria tailored to laterals with variable diameters.

While hydraulic analysis can be carried out using classical numerical design procedures (Keller and Karmeli 1974; Wu and Gitlin 1975; Bralts et al. 1987), often involving laborious iterations, trial-and-error searches, or graphical techniques (Nakayama and Bucks 1986; Keller and Bliesner 1990), these approaches are frequently impractical for routine engineering practice. As a result, designers commonly favour simplified analytical methods that are easy to apply while still providing adequate accuracy for most applications. Among these, under the assumption of constant emitter discharge, Baiamonte (2018a) proposed straightforward design solutions for one-diameter drip laterals installed on flat or sloping fields, aiming to maximise water and energy savings. These solutions are particularly suitable for non-pressure-compensating (N-PC) emitters, which are less expensive than pressure-compensating (PC) emitters and typically offer a long service life because they lack the flexible pressure-regulating membrane (Melby 1995). The same framework was subsequently extended to rectangular irrigation units, leading to the development of the IRRILAB software (Baiamonte 2018b).

For tapered, as well as single and paired laterals, Jain et al. (2002) introduced a simplified design procedure based on a lateral-discharge formulation. The method describes

lateral hydraulics through a power-law relationship between inlet discharge and inlet pressure head, providing a practical basis for the design of drip laterals. However, the method still requires computationally intensive iterations and multiple trial-and-error adjustments, typically progressing from the lateral-manifold junction along the pipeline.

To reduce this burden, analytical formulations for tapered laterals have also been proposed. For example, Valiantzas (2002) developed an analytical method for the hydraulic analysis and cost-optimal design of horizontal and sloped two-diameter drip laterals that accounts for uniform outlet discharge, enabling direct pressure head calculations and yielding results close to numerical solutions.

Similarly, Yildirim and Araliolu (2004) proposed a simplified analytical solution for tapered microirrigation laterals that models spatially varying outflow, enabling straightforward prediction of pressure and discharge distributions under flat, uphill, and downhill slopes. However, this procedure is restricted to a linear discharge–pressure relationship, i.e., it assumes a unitary emitter exponent.

More recently, Baiamonte and Palermo (2025a) developed and experimentally and numerically validated a streamlined design analytical methodology for dual-diameter drip irrigation laterals equipped with N-PC emitters. The approach predicts hydraulic behaviour under a uniform emitter flow rate assumption while explicitly incorporating local head losses due to emitter insertions, and it provides straightforward design relationships (including pressure head tolerance criteria) to ensure prescribed pressure and discharge uniformity.

Building upon and further advancing this line of research, Baiamonte and Palermo (2025b) introduced an explicit analytical design procedure for drip irrigation systems with tapered laterals and manifolds on flat terrain. The method enables direct design calculations, employing a normalised pressure head formulation to control emitter flow rate variability and systematically optimise hydraulic performance, yielding energy savings of up to 55.6 % compared with traditional one-diameter configurations. Despite these advances, a normalised analytical design framework

specifically tailored to tapered drip laterals installed on sloping fields remains lacking.

The objective of this study is to develop a practical, fully analytical procedure for designing dual-diameter tapered drip laterals installed on downhill slopes. The proposed design procedure is based on a normalised pressure head tolerance-based approach and explicitly accounts for the hydraulic interaction between gravitational head gain and friction losses along the two pipe segments. A novel pressure head tolerance, δ_s , at the diameter-change point, *DCP*, is defined to distinguish three distinct pressure head distribution patterns, corresponding to $\delta_s > 0$, $\delta_s < 0$, and $\delta_s = 0$.

On this basis, explicit relationships are derived for the pressure head distribution, positions and values of the minimum pressure heads, admissible design domain, *DCP*, and upstream pipe diameter. These relationships are subsequently incorporated into a design procedure in which the allowable pressure head tolerance for the downstream pipe segment is prescribed a priori, while the required inlet pressure head and geometric and hydraulic design variables are analytically determined without iterative calculations. The hydraulic and energetic performance of the resulting tapered configurations is compared with that of an equivalent one-diameter drip lateral. Finally, the analytical predictions are assessed against the exact step-by-step (SBS) numerical method that accounts for variations in emitter discharge, in line with emitter flow relation.

Overall, the proposed formulation facilitates the practical selection of commercially available pipe diameters and suitable emitters type, characterised by different emitter flow relations, while ensuring hydraulic feasibility and performance constraints.

Materials and methods

Dual-diameter drip lateral geometry and design assumptions

Figure 1 illustrates a schematic representation of a generic tapered lateral installed on a field with a uniform slope, S_0 (positive downward). The lateral consists of two connected pipe segments: the downstream segment, denoted sub-lateral 1, and the upstream segment, denoted sub-lateral 2. In a conventional tapered configuration, sub-lateral 1 has a smaller inside diameter, D_1 (L), than the inside diameter of sub-lateral 2, D_2 (L). In addition, they are equipped with a number of emitters N_1 and N_2 , respectively. Therefore, the total number of emitters installed along the tapered lateral is $N = N_1 + N_2$.

Emitters' position is indexed by j and emitters are numbered sequentially from the downstream distal end toward

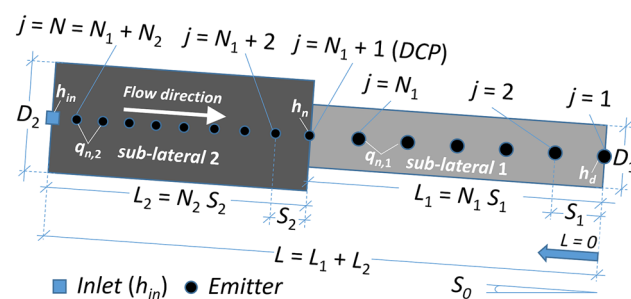


Fig. 1 Schematic representation of the tapered lateral on a sloping field, with slope S_0

the upstream inlet, i.e., in the opposite direction of the water flow. The emitters are located at $j = 1, 2, \dots, N$, whereas the inlet corresponds to the position $j = N + 1$. The diameter-change point, *DCP*, is located at $j = N_1 + 1$ (see Fig. 1). Accordingly, emitters belonging to sub-lateral 1 are located downstream of the *DCP* ($1 \leq j \leq N_1 + 1$), whereas those belonging to sub-lateral 2 are located between the *DCP* and the inlet ($N_1 + 1 \leq j \leq N + 1$).

The corresponding lengths of sub-lateral 1 and 2 are $L_1 = N_1 S_1$ (L) and $L_2 = N_2 S_2$ (L), respectively, where S_1 (L) and S_2 (L) are the corresponding emitter spacing.

The nominal emitter flow rates of the two sub-laterals, $q_{n,1}$ ($L^3 T^{-1}$) and $q_{n,2}$ ($L^3 T^{-1}$), respectively, as well as the emitter spacings, may differ between the two segments. This flexibility allows each sub-lateral to be equipped with a different emitter type and spacing, characterised by its own emitter flow relation, to satisfy crop water requirements and account for soil hydraulic characteristics.

Three characteristic pressure heads are identified in Fig. 1: h_d (L), at the distal end of the lateral ($j = 1$); h_n (L), at *DCP* ($j = N_1 + 1$); and the inlet pressure head, h_{in} (L), at the inlet ($j = N + 1$). Although the proposed design procedure can be extended to systems with more than two sub-laterals, the present study focuses on the two-segment configuration for the sake of analytical clarity and simplicity.

The idealized representation of a tapered lateral illustrated in Fig. 1 is based on the assembly of modular sub-laterals. In the corresponding analytical formulation, the diameter transition is assumed to occur exactly at diameter-change point, *DCP*, at $j = N_1 + 1$. This idealization greatly simplifies the analytical derivation while preserving the hydraulic characteristics of the system.

In practice, the diameter transition is achieved by means of a connector that cannot be placed at the same location as the emitter $j = N_1 + 1$, but only in its immediate vicinity. Since the connector displacement is very small compared with the emitter spacing, its effect on pressure distribution and flow is negligible, and the results obtained with the

simplified formulation that will be presented later remain essentially unchanged.

According to Baiamonte et al. (2015) and Baiamonte (2018a), designing drip irrigation laterals such that the pressure head at the distal end of the lateral, h_d , equals the inlet pressure head, h_{in} , can promote high emission uniformity and reduce the required operating energy. More recently, Baiamonte et al. (2026) demonstrated the energy benefits of adopting this design criterion for paired lateral lines installed on sloping fields.

Pressure head distribution patterns and tolerances

Although the assumption $h_d \equiv h_{in}$ imposes equal pressure heads at the two ends of the lateral, different pressure head distribution, PHD, patterns may develop within the two sub-laterals. These patterns result from the balance between the gravitational pressure head gain associated with the downward field slope and the friction losses occurring along the pipe.

Figure 2a-c illustrates the three normalised PHD regimes that may occur in a tapered lateral. In addition to the distal end, *DCP*, and inlet pressure heads, h_d , h_n , and h_{in} , respectively, introduced in Fig. 1, the figure identifies the minimum pressure heads in sub-laterals 1 and 2, denoted $h_{min,1}$ and $h_{min,2}$. These minima occur at the corresponding emitter positions $j_{min,1}$ and $j_{min,2}$.

In Fig. 2a-c, the pressure head variables are denoted by an asterisk, namely h_{*d} , $h_{*min,1}$, h_{*n} , $h_{*min,2}$, and h_{*in} , indicating that they are expressed in normalised form with respect to the uniform slope field, S_0 , and the emitter spacing of sub-lateral 1, S_1 , according to the normalisation procedure introduced in Section "Analytical formulation".

To describe the cumulative balance between gravitational head gain and friction losses from the inlet to the *DCP*, a pressure head tolerance referred to the *DCP*, δ_s , is introduced. Because $h_{*d} \equiv h_{*in}$, the same value of δ_s is obtained whether it is referred to h_{*in} ($j = N + 1$) or to h_{*d} ($j = 1$). In dimensionless terms, δ_s is equal to:

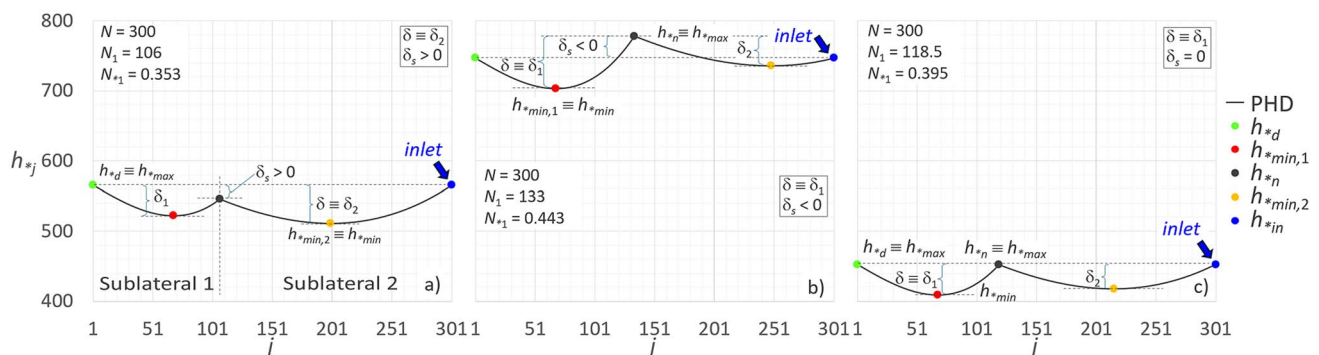


Fig. 2 Normalised pressure head distributions of sloped tapered laterals for **a** $\delta_s > 0$, **b** $\delta_s < 0$, and **c** $\delta_s = 0$

$$\delta_s = \frac{h_{*in} - h_{*n}}{h_{*in} + h_{*n}} = \frac{h_{*d} - h_{*n}}{h_{*d} + h_{*n}} \quad (1)$$

Depending on the sign of δ_s , three PHD patterns can be defined:

$$\begin{aligned} \delta_s > 0 & \quad h_{*n} < h_{*d} = h_{*in} \\ \delta_s < 0 & \quad h_{*n} > h_{*d} = h_{*in} \\ \delta_s = 0 & \quad h_{*n} = h_{*d} = h_{*in} \end{aligned} \quad (2)$$

When $\delta_s > 0$, the friction losses accumulated between the inlet and the DCP exceed the gravitational pressure head gain, so that $h_{*n} < h_{*in}$, as shown in Fig. 2a. Conversely, when $\delta_s < 0$, the gravitational contribution exceeds friction losses and the pressure head at the DCP becomes greater than that at the inlet, i.e., $h_{*n} > h_{*in}$, as illustrated in Fig. 2b. The condition $\delta_s = 0$ represents the regime in which gravitational head gain and friction losses are exactly balanced between the inlet and the DCP, resulting in $h_{*n} = h_{*in}$, as shown in Fig. 2c.

For the three considered cases, the tapered pressure head tolerance, δ , is defined as:

$$\delta = \frac{h_{*max} - h_{*min}}{h_{*max} + h_{*min}} = \frac{\max\{h_{*in}, h_{*n}\} - \min\{h_{*min,1}, h_{*min,2}\}}{\max\{h_{*in}, h_{*n}\} + \min\{h_{*min,1}, h_{*min,2}\}} \quad (3)$$

where h_{*max} and h_{*min} are the absolute maximum and minimum normalised pressure heads of the tapered lateral. When $\delta_s > 0$, the maximum pressure head is $h_{*max} = h_{*in}$, whereas for $\delta_s < 0$, it is $h_{*max} = h_{*n}$. Under the condition $\delta_s = 0$, $h_{*max} = h_{*n} = h_{*in}$.

The pressure head tolerances of sub-lateral 1 and 2, denoted by δ_1 and δ_2 , respectively, are defined as:

$$\delta_1 = \frac{\max\{h_{*in}, h_{*n}\} - h_{*min,1}}{\max\{h_{*in}, h_{*n}\} + h_{*min,1}} \quad (4)$$

$$\delta_2 = \frac{\max\{h_{*in}, h_{*n}\} - h_{*min,2}}{\max\{h_{*in}, h_{*n}\} + h_{*min,2}} \quad (5)$$

Accordingly, the tapered pressure head tolerance, δ , is governed by the sub-lateral exhibiting the largest pressure head tolerance, $\delta = \max\{\delta_1, \delta_2\}$ (Eq. 3).

For the PHDs patterns shown in Fig. 2, $\delta \equiv \delta_2$ in Fig. 2a, whereas $\delta \equiv \delta_1$ in Fig. 2b and c. Contrary to δ_s , which may be positive, negative or equal to zero (Eq. 2), δ , δ_1 , and δ_2 quantify the magnitude of the pressure head tolerance and therefore are positive.

The tapered pressure head tolerance, δ , used in this study (Eq. 3) follows the formulation proposed by Baiamonte (2018a). This formulation differs from the pressure head variation, h_{var} , proposed by Wu (1992), who defined h_{var}

$= (h_{in} - h_{min})/h_{in}$, despite describing the same concept. Wu (1992) reported that a proper hydraulic design should be based on a maximum value of $h_{var} = 20\%$.

The present analysis, however, relies on δ (and not h_{var}) since it is an extension of the work conducted by Baiamonte (2018a) and Baiamonte and Palermo (2025a). Nevertheless, conversion from h_{var} to δ is straightforward through the following linking relationship (Baiamonte et al. 2024):

$$h_{var} = \frac{2\delta}{1 + \delta} \vee \delta = \frac{h_{var}}{2 - h_{var}} \quad (6)$$

that provides $\delta = 11.1\%$, for $h_{var} = 20\%$.

Based on this framework, Section "Analytical formulation" develops the analytical pressure head distributions of the two sub-laterals and derives the corresponding feasibility constraints, diameter relationships, and minimum pressure heads required to evaluate the tapered pressure head tolerance. The analytical formulation assumes uniform emitter flow rate within the two sub-laterals. This simplifying assumption is subsequently relaxed during validation against the exact step-by-step (SBS) numerical method (Section "Comparison with the exact SBS method and uniformity performance"), which is considered a reference as it is based on the repetitive application of the basic hydraulic equations (laws of conservation of mass, motion, and continuity equations).

SBS has the advantage of being applied for non-pressure compensating emitters, i.e., when the exponent x of the emitter flow relation differs from zero, thus helping detect the errors in the design procedure, under the simplified assumption of uniform flow rate ($x = 0$). On the other hand, SBS requires trial-and-error iterations and time-consuming calculations.

Analytical formulation

Pressure head distribution in sub-lateral 1

For sub-lateral 1 ($1 \leq j \leq N_1 + 1$), and under the assumption of uniform emitter flow rate, PHD can be expressed as (Baiamonte 2024):

$$h_j = h_d + K_1 S_1 \sum_{j=1}^{j-1} j^r - (j-1) S_1 S_0 \quad \text{for } 1 \leq j \leq N_1 + 1 \quad (7)$$

where h_j (L) denotes the pressure head at the emitter j , and K_1 represents the unit hydraulic gradient associated with the emitter discharge, $q_{n,1}$ and inside pipe diameter, D_1 :

$$K_1 = k \frac{q_{n,1}^r}{D_1^s} \quad (8)$$

Equation (8) highlights the strong influence of pipe diameter on hydraulic resistance, since K_1 is inversely proportional to D_1^s , due to the high s value. Consequently, even a moderate decrease in pipe diameter can produce a substantial increase in the unit friction loss. This dependence provides the physical basis for the tapered configuration: the larger diameter of sub-lateral 2, D_2 , reduces friction losses in the segment pipe conveying the highest discharge, whereas the smaller diameter of sub-lateral 1, D_1 , is used where the discharge has progressively decreased because of emitters' outflow.

The variation in diameter therefore redistributes frictional losses between the two sub-laterals and modifies the balance between friction losses and slope-induced pressure head gain, thereby affecting the resulting pressure head distribution along the lateral (Fig. 2a-c).

The coefficients k , r , and s depend on the selected hydraulic resistance equation. For smooth-walled polyethylene pipes, the Blasius and Hazen-Williams equations are commonly adopted (Keller and Bliesner 1990):

$$k = \frac{0.316 \cdot 4 \cdot \sqrt{2}}{g\pi^r} v^{0.25} \lambda, \quad r = 1.75, \quad s = 4.75 \quad (\text{Blasius}) \quad (9)$$

$$k = \frac{10.675}{C^r} \lambda, \quad r = 1.852, \quad s = 4.871 \quad (\text{Hazen - Williams}) \quad (10)$$

where g (L T^{-2}) is the acceleration of gravity, v is water's kinematic viscosity ($v = 1.008 \cdot 10^{-6} \text{ m}^2 \text{ s}^{-1}$ at the standard water temperature of 20°C), C is pipe smoothness factor, which is a function of the pipe material's characteristics, and λ is an amplification factor to account for local losses based on the method of equivalent length, L_e (L) (Juana et al. 2004):

$$\lambda = 1 + \frac{L_e}{S} \quad (11)$$

where S (L) is the emitter spacing of sub-lateral 1 or 2. Although local losses and different pipe materials could be selected in the suggested methodology, applications will be performed for $C = 135$ (polyethylene pipes), and $\lambda = 1$ (i.e. neglecting local losses, $L_e = 0$).

The Hazen-Williams resistance equation is adopted in later applications; thus, regardless of the flow regime that is generally turbulent in the upstream section. The flow regime still may be turbulent or enter the transition regime in the middle section, and may reach transition or even laminar flow in the last few meters of long laterals, especially with low-discharge emitters (e.g., $1\text{--}2 \text{ l h}^{-1}$).

However, any other resistance equation can also be incorporated into the proposed procedure. For example, the Blasius equation, which was developed for hydraulically smooth pipes and explicitly depends on the Reynolds

number, can be readily adopted by simply replacing the expression for k given in Eq. (8) with that of Eq. (9).

Dividing Eq. (7) by S_1 and S_0 , and using the generalised harmonic number of order $-r$ truncated at the emitter $j-1$ (Abramowitz and Stegun 1972):

$$H_{j-1}^{(-r)} = \sum_{j=1}^{j-1} j^r /; j \in \mathbb{N} \quad (12)$$

the pressure head distribution can be rewritten in dimensionless terms, highly simplifying the design procedure. Accordingly, Eq. (7) can be expressed as:

$$h_{*j} = h_{*d} + K_{*1} H_{j-1}^{(-r)} - (j-1) \quad \text{for } 1 \leq j \leq N_1 + 1 \quad (13)$$

where the normalised pressure heads h_{*j} and h_{*d} and K_{*1} are equal to:

$$h_{*j} = \frac{h_j}{S_1 S_0} \quad (14)$$

$$h_{*d} = \frac{h_d}{S_1 S_0} \quad (15)$$

$$K_{*1} = \frac{K_1}{S_0} \quad (16)$$

For $j = 1$, Eq. (13) yields $h_{*j} = h_{*d}$, while by imposing $j = N_1 + 1$, the normalised pressure head, h_{*n} , at the diameter-change point, DCP, is obtained (see Fig. 1):

$$h_{*n} = h_{*in} + K_{*1} H_{N_1}^{(-r)} - N_1 \quad (17)$$

where $h_{*d} \equiv h_{*in}$ was replaced by h_{*in} .

Pressure head distribution in sub-lateral 2

For sub-lateral 2 ($N_1 + 1 \leq j \leq N + 1$), and again assuming uniform emitter discharge, the pressure head distribution must account for the possibly different emitter flow rate, $q_{n,2}$, and spacing, S_2 , relative to sub-lateral 1 ($q_{n,1}$ and S_1). Under these conditions, the PHD is expressed as (Baiaumont and Palermo 2025a):

$$h_{*j} = h_{*n} + K_{*2} \frac{S_2}{S_1} \sum_{j=N_1+1}^{j-1} (Q_1 + q_{n,2} (j - N_1))^r - S_{*2} (j - N_1 - 1) \quad \text{for } N_1 + 1 \leq j \leq N + 1 \quad (18)$$

where K_{*2} is derived by Eq. (8) and Eq. (16), after replacing the subscript 1 by 2, $Q_1 = N_1 q_{n,1}$ is the circulating discharge that feeds the sub-lateral 1. The summation term in Eq. (18) can be expressed using the generalised Riemann

zeta function, $\zeta(-r, q)$ (Abramowitz and Stegun 1972), as shown by Baiamonte and Palermo (2025a):

$$\sum_{j=N_1+1}^{j-1} (Q_1 + q_{n,2}j)^r = \zeta\left(-r, \frac{Q_1 + q_{n,2}}{q_{n,2}}\right) - \zeta\left(-r, \frac{Q_1 + q_{n,2}}{q_{n,2}} + j - N_1 - 1\right) \quad (19)$$

Substituting Eq. (19) into Eq. (18) and denoting the ratio of the emitter flow rate, $q_{*n,1}$, and emitter spacing, S_{*2} , as:

$$q_{*n,1} = \frac{q_{n,1}}{q_{n,2}} \quad (20)$$

$$S_{*2} = \frac{S_2}{S_1} \quad (21)$$

the PHD for the sub-lateral 2 can be expressed as:

$$h_{*j} = h_{*n} + K_{*2} S_{*2} \left[\frac{\zeta(-r, 1 + N_1 q_{*n,1}) - \zeta(-r, N_1 q_{*n,1} + j - N_1)}{\zeta(-r, 1 + N_1 q_{*n,1} + N - N_1)} \right] - S_{*2} (j - N_1 - 1) \text{ for } N_1 + 1 \leq j \leq N + 1 \quad (22)$$

The normalised inlet pressure head, h_{*in} , can be obtained by imposing $j = N + 1$ into Eq. (22) (see Fig. 1):

$$h_{*in} = h_{*n} + K_{*2} S_{*2} \left[\frac{\zeta(-r, 1 + N_1 q_{*n,1}) - \zeta(-r, 1 + N_1 q_{*n,1} + N - N_1)}{\zeta(-r, 1 + N_1 q_{*n,1} + N - N_1)} \right] - S_{*2} (N - N_1) \quad (23)$$

By analytic number theory, using the known identity relating the generalised Riemann zeta function and the generalised harmonic number (Abramowitz and Stegun 1972):

$$\zeta(-r, j) = \zeta(-r) - H_{j-1}^{(-r)} \quad (24)$$

Eqs. (22) and (23) can be rewritten as a function of the generalised harmonic number, similarly to sub-lateral 1:

$$h_{*j} = h_{*n} + K_{*2} S_{*2} \left(H_{j-1+N_1(q_{*n,1}-1)}^{(-r)} - H_{N_1 q_{*n,1}}^{(-r)} \right) - S_{*2} (j - N_1 - 1) \text{ for } N_1 + 1 \leq j \leq N + 1 \quad (25)$$

$$h_{*in} = h_{*n} + K_{*2} S_{*2} \left(H_{N+N_1(q_{*n,1}-1)}^{(-r)} - H_{N_1 q_{*n,1}}^{(-r)} \right) - S_{*2} (N - N_1) \quad (26)$$

Eq. (26) is worth considering, since it makes it possible to further simplify the same relationship by using an approximate relationship of the generalised harmonic number that fits the exact values well, with relative errors less than 0.3 % (Baiamonte 2024):

$$H_j^{(-r)} = \frac{1}{1+r} \left(\frac{2j+1}{2} \right)^{1+r} \quad (27)$$

By applying Eq. (27) to Eqs. (17) and (26), h_{*n} and h_{*in} can be rewritten:

$$h_{*n} = h_{*d} + \frac{K_{*1}}{1+r} \left(\frac{1}{2} + N_1 \right)^{1+r} - N_1 \quad (28)$$

$$h_{*in} = h_{*n} + \frac{\left[(1+2(N-N_1(1-q_{*n,1})))^{1+r} - (1+2N_1 q_{*n,1})^{1+r} \right] K_{*2} S_{*2}}{2^{1+r}(1+r)} - S_{*2}(N-N_1) \quad (29)$$

that do not require special functions and are easy to apply in practice.

Hydraulic and pressure head tolerance constraints

The pressure head relationships derived in Sections "Pressure head distribution in sub-lateral 1" and "Pressure head distribution in sub-lateral 2" are used to derive the pressure head tolerance at DCP, δ_s . Using Eq. (17), the numerator and denominator of δ_s (Eq. 1) can be expressed as:

$$h_{*in} - h_{*n} = N_1 - K_{*1} H_{N_1}^{(-r)} \quad (30)$$

$$h_{*in} + h_{*n} = 2h_{*d} + K_{*1} H_{N_1}^{(-r)} - N_1 \quad (31)$$

Substituting Eqs. (30) and (31) into Eq. (1), yields:

$$\delta_s = \frac{N_{*1}N - K_{*1}H_{N_{*1}N}^{(-r)}}{2h_{*in} + K_{*1}H_{N_{*1}N}^{(-r)} - N_{*1}N} \quad (32)$$

where $N_{*1} = N_1/N$ and, as discussed in Section "Materials and methods", h_{*d} was replaced by h_{*in} . Eq. (32) shows that for a fixed sub-lateral 1 (i.e., N_{*1} , N and K_{*1}), a prescribed value of δ_s can be achieved by appropriately selecting h_{*in} .

If $q_{n,1}$ and D_1 are treated as calculated variables, which is generally not recommended, Eq. (32) can alternatively be expressed by explicitly defining K_{*1} :

$$K_{*1} = \frac{N_{*1}N - 2\frac{\delta_s}{1+\delta_s}h_{*in}}{H_{N_{*1}N}^{(-r)}} \quad (33)$$

For the three cases illustrated in Fig. 2a-c, with $N = 300$, and $h_{*in} = 565.52$ (Fig. 2a), 746.77 (Fig. 2b), and 452.46 (Fig. 2c), Eq. (32) is illustrated in Fig. 3a-c using the Hazen-Williams resistance equation (Eq. (10), with $C = 135$, and $\lambda = 1$). The contour plot shows the variation of N_{*1} within the K_{*1} - δ_s parameter space. Across the entire range $N_{*1} \in [0, 1]$ (see colour bar), as K_{*1} decreases, δ_s increases, achieving a constant value. The increase in δ_s appears more pronounced for lower values of h_{*in} (Fig. 3c). Moreover, by varying N_{*1} , when K_{*1} is large, the curve exhibits a power-law behaviour which can be attributed to the dominance of pressure

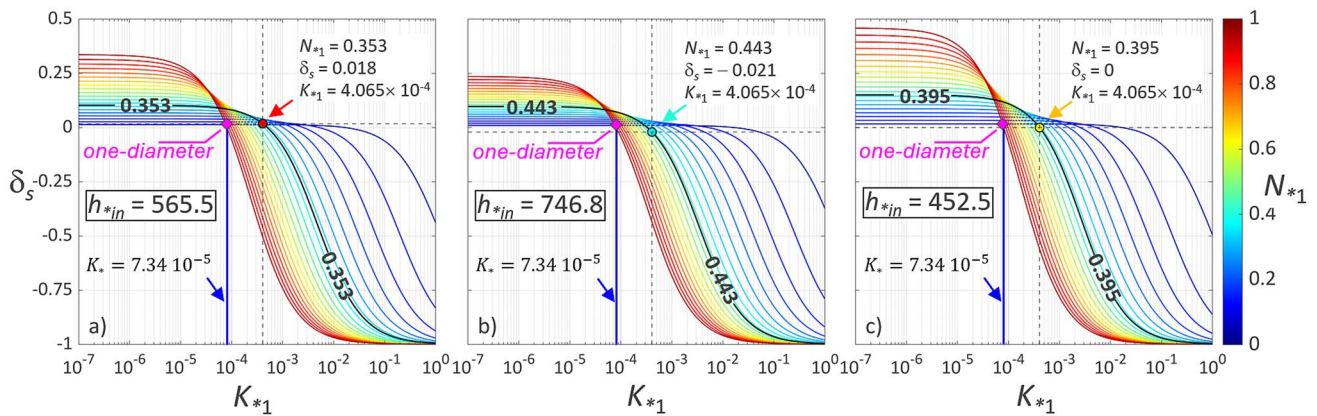


Fig. 3 Under the Hazen-Williams resistance equation, contour plot of N_{*1} in the K_{*1} - δ_s space for $N = 300$ and **a** $h_{*in} = 565.52$, **b** $h_{*in} = 746.77$, and **c** $h_{*in} = 452.46$. The colour bar refers to N_{*1} , dots refer to Fig. 2a–c, while the magenta diamond denotes the one-diameter configuration ($K_{*1} = K_*$)

variations due to friction losses. In contrast, for small values of K_{*1} , the curve tends toward a linear trend, which can be attributed to the dominance of pressure variations induced by the slope.

From Eq. (32), the sign of δ_s determines threshold conditions for K_{*1} :

$$\begin{aligned} \delta_s > 0 \quad K_{*1} &< \frac{N_{*1}N}{H_{N_{*1}N}^{(-r)}} \\ \delta_s < 0 \quad K_{*1} &> \frac{N_{*1}N}{H_{N_{*1}N}^{(-r)}} \\ \delta_s = 0 \quad K_{*1} &= \frac{N_{*1}N}{H_{N_{*1}N}^{(-r)}} \end{aligned} \quad (34)$$

Constraints in K_{*1} described by Eq. (34) establish whether the pressure head at the DCP, h_{*n} , is higher, lower, or equal to the normalised pressure at the inlet, h_{*in} , thereby determining the overall PHD pattern along the tapered lateral. Note that for $\delta_s = 0$, $N_{*1} = 1$, and $N = 300$, the value of K_{*1} , associated with one-diameter lateral, denoted as K_* , as illustrated in Fig. 3a–c, is obtained:

$$K_* = \frac{N}{H_N^{(-r)}} = 7.34 \times 10^{-5} \quad (35)$$

In Fig. 3a–c, the triplets $(K_{*1}, \delta_s, N_{*1})$ associated with PHDs of Fig. 2a–c ($\delta_s = 0.018$, $\delta_s = -0.021$, $\delta_s = 0$, respectively) are indicated.

Substituting Eq. (17) into Eq. (26), the relationship between K_{*2} and K_{*1} , regardless of h_{*in} , is provided:

$$K_{*2} = \frac{N_{*1}N - K_{*1}H_{N_{*1}N}^{(-r)} + S_{*2}N(1 - N_{*1})}{S_{*2} \left(H_{N(1-N_{*1})+N_{*1}Nq_{*n,1}}^{(-r)} - H_{N_{*1}Nq_{*n,1}}^{(-r)} \right)} \quad (36)$$

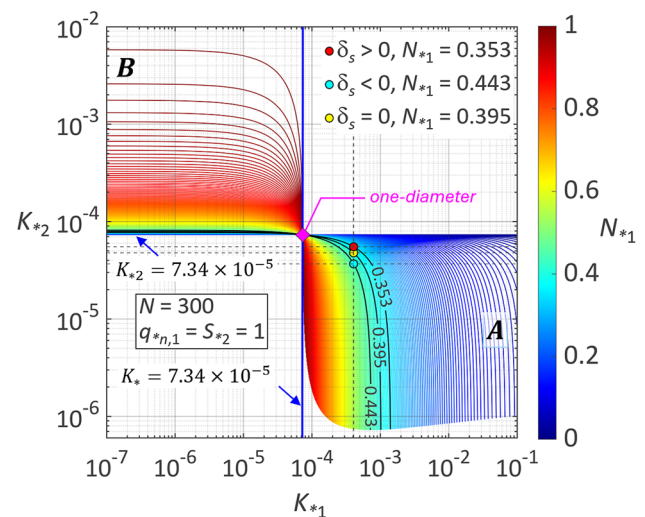


Fig. 4 Under the Hazen-Williams resistance equation, contour plot of N_{*1} within the K_{*1} - K_{*2} parameter space for $N = 300$, $q_{*n,1} = 1$, and $S_{*2} = 1$. The colour scale represents N_{*1} , the dots refer to the cases associated with the PHDs illustrated in Fig. 2a–c, while the magenta diamond denotes the one-diameter configuration ($K_{*1} = K_{*2} = K_*$)

where N_1 was replaced by N_{*1} . Eq. (36) implies that, to ensure physically meaningful solutions (i.e., $K_{*2} > 0$), K_{*1} must remain below the threshold value, $K_{*1,th}$:

$$K_{*1,th} = \frac{N_{*1}N + S_{*2}N(1 - N_{*1})}{H_{N_{*1}N}^{(-r)}} \quad (37)$$

This constraint defines the admissible design domain for sub-lateral 1. Only combinations of N_{*1} and K_{*1} that satisfy $K_{*1} < K_{*1,th}$ produce feasible dual-diameter configurations. Moreover, for one-diameter laterals ($N_1 = N$, $N_{*1} = 1$, $S_{*2} = 1$) with $N = 300$ emitters (Fig. 3), Eq. (37) agrees with Eq. (35).

Figure 4 shows K_{*1} versus K_{*2} relationship from Eq. (36), for $N = 300$, $q_{*n,1} = S_{*2} = 1$, by varying N_{*1} . Two distinct

main regions, denoted A and B , are identified by the vertical and horizontal reference blue lines ($K_{*1} = K_{*2} = 7.34 \times 10^{-5}$). In region A , where $K_{*1} > K_*$ and $K_{*2} < K_*$, N_{*1} decreases as K_{*1} increases. Conversely, in region B , where $K_{*1} < K_*$ and $K_{*2} > K_*$, N_{*1} increases with increasing K_{*2} .

Moreover, for small values of K_{*2} and large values of K_{*1} (large D_2 and small D_1 values, respectively), as expected, N_{*1} becomes nearly constant beyond a certain threshold (zone A).

Based on Eqs. (8) and (16) applied to sub-laterals 1 and 2, regions A and B can also be defined in terms of inside diameters. Particularly, region A corresponds to $D_1 < q_{*n,1}^{r/s} D_2$, whereas region B corresponds to $D_1 > q_{*n,1}^{r/s} D_2$. The coloured markers identify the three tapered-lateral configurations associated with PHDs shown in Fig. 2a–c, while the magenta diamond indicates the one-diameter configuration ($K_{*1} = K_{*2} = 7.34 \times 10^{-5}$), as in Fig. 3a–c.

For $q_{*n,1} = S_{*2} = 1$, meaning equal emitters' spacing and flow rate in the two sub-laterals, Eq. (36) reads:

$$K_{*2} = \frac{N - K_{*1} H_{N_{*1}N}^{(-r)}}{H_N^{(-r)} - H_{N_{*1}N}^{(-r)}} \quad (38)$$

Of course, if K_{*1} is imposed, only Eq. (36) or Eq. (38) are required to calculate K_{*2} .

To inspect the role of K_* , K_{*1} , K_{*2} variation, and the corresponding inside diameters, using Eq. (35), Eq. (38) can be rewritten as:

$$\frac{H_{N_{*1}N}^{(-r)}}{H_N^{(-r)}} = \frac{K_* - K_{*2}}{K_{*1} - K_{*2}} \quad (39)$$

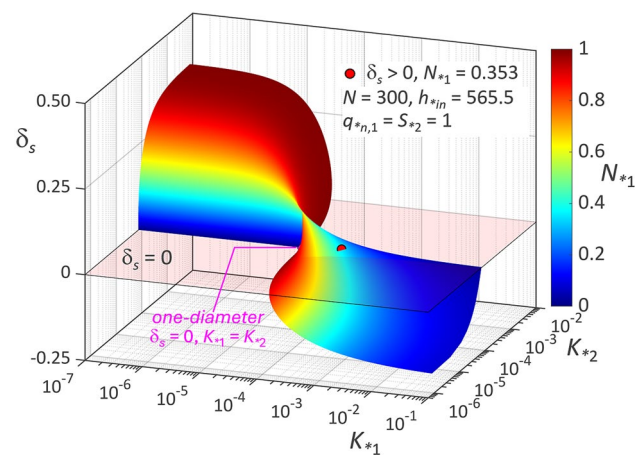


Fig. 5 Under the Hazen-Williams resistance equation, 3D plot K_{*1} – K_{*2} – δ_s parameters, and N_{*1} as a colour map, for $N = 300$, $h_{*in} = 0565.5$, and $q_{*n,1} = S_{*2} = 1$. The dot refers to the case associated with the PHD illustrated in Fig. 2a, while the magenta diamond denotes (not clearly shown) the one-diameter configuration ($K_{*1} = K_{*2} = K_*$)

or in terms of inside diameters:

$$\frac{H_{N_{*1}N}^{(-r)}}{H_N^{(-r)}} = \frac{\left(\frac{D_1}{D}\right)^s - D_{*1}^s}{1 - D_{*1}^s} \quad (40)$$

where $D_{*1} = D_1/D_2$ is the inside diameter ratio (Baiamonte and Palermo 2025a). Eqs. (39) and (40) elucidate the asymptotic behaviour observed in Fig. 4. In region A , $K_{*1} > K_* > K_{*2}$, and an increase in friction loss of sub-lateral 2 (lower D_2 , Eq. (40)) causes the right-hand side to approach zero; consequently, N_{*1} approaches zero. Physically, the highly hydraulic resistive downstream segment must occupy a progressively smaller fraction of the lateral to satisfy the condition $h_{*d} = h_{*in}$.

Conversely, in region B , $K_{*2} > K_* > K_{*1}$, and the harmonic-number ratio approaches unity as K_{*2} increases; therefore, N_{*1} approaches unity, and the highly resistive upstream sub-lateral 2 becomes progressively shorter. Thus, the nearly uniform values of N_{*1} observed in the outer portions of regions A (conventional tapered configuration, $D_1 < D_2$) and B (reverse tapered configuration, $D_1 > D_2$) represent asymptotic saturation toward zero and unity, respectively, rather than exact constant values.

At the point ($K_{*1} = K_{*2} = K_*$), representing the single-diameter configuration, the location of the DCP loses its hydraulic significance because the two sub-laterals have the same hydraulic resistance.

To improve overall readability of Figs. 3a and 4, a unified approach is attempted in Fig. 5 by using a 3D plot with K_{*1} and K_{*2} , and δ_s as variables, and N_{*1} as a colour map. This figure evidences that reverse tapered ($K_{*2} > K_{*1}$, $D_1 > D_2$) can be obtained only for $\delta_s > 0$, whereas for conventional tapered ($K_{*2} < K_{*1}$, $D_1 < D_2$), no δ_s sign constraints occur.

Admissible solutions of Eqs. (33) and (36) (positive values of K_{*1} and K_{*2}) can also be expressed in terms of N_{*1} . Depending on δ_s , admissible values of N_{*1} are delimited by the bounds, $N_{*1,min}$ and $N_{*1,max}$, respectively:

$$K_{*1} > 0 \quad N_{*1,min} > 2 \frac{\delta_s}{1+\delta_s} \frac{h_{*in}}{N} \quad (41)$$

$$K_{*2} > 0 \quad N_{*1,max} < 1 + \frac{2\delta_s}{S_{*2}(1+\delta_s)} \frac{h_{*in}}{N} \quad (42)$$

The limiting N_{*1} values are illustrated in Fig. 6. For the same parameters adopted in Figs. 3 and 4 ($q_{*n,1} = 1$ and $S_{*2} = 1$), and for the three considered values of δ_s (0.018, −0.021, 0) and h_{*in} (565.52, 746.77, 452.46), Eqs. (33) and (36) are plotted vs. N_{*1} . The figure shows that for $\delta_s = -0.021 < 0$, no N_{*1} constraint exists for $K_{*1} > 0$, since $N_{*1} \in [0, 1]$, whereas for K_{*2} , Eq. (42) yields $N_{*1,max} = 0.895$ (Fig. 6).

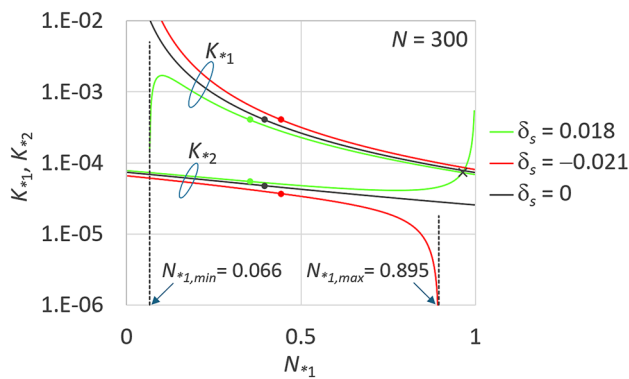


Fig. 6 Under the Hazen-Williams resistance equation, relationship between K_{*1} and K_{*2} versus N_{*1} , for different values of δ_s . Filled dots identify the three cases illustrated in Fig. 2a-c, whereas cross mark indicates the value of N_{*1} for which $K_{*1} = K_{*2}$ (one-diameter)

Conversely, for $\delta_s = 0.018 > 0$, the opposite behaviour is observed; the N_{*1} constraint applies to K_{*1} , providing $N_{*1,min} = 0.066$ (Eq. (41)), while no constraint is found for K_{*2} . Finally, no N_{*1} constraints arise for $\delta_s = 0$ (black lines), since $N_{*1} \in [0,1]$. The dots illustrated in Fig. 6 also depict the values of N_{*1} , K_{*1} , and K_{*2} corresponding to Fig. 2a-c, while the cross symbol indicates the value of N_{*1} for which $K_{*1} = K_{*2}$ (one-diameter).

Minimum pressure heads

To evaluate the pressure head tolerance of the tapered lateral, δ (Eq. (3)), both h_{*max} and h_{*min} must be determined. While h_{*max} depends on δ_s , h_{*min} may occur in sub-lateral 1 or 2, since $h_{*min} = \min\{h_{*min,1}, h_{*min,2}\}$, and its determination requires derivation.

By differentiating Eqs. (13) and (25) with respect to j and setting the derivative equal to zero, the locations of the minimum pressure heads in the two sub-laterals, $j_{min,1}$ and $j_{min,2}$, can be obtained:

$$\frac{dh_{*j}}{dj} = 1 + rK_{*1} \left(\zeta(1-r) - H_{j_{min,1}-1}^{(1-r)} \right) = 0 \quad (43)$$

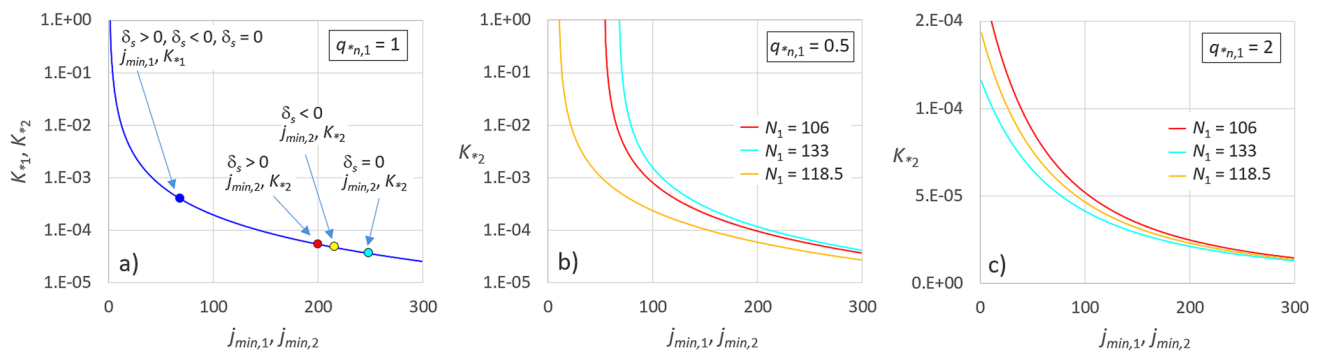


Fig. 7 Under the Hazen-Williams resistance equation, relationship between K_{*1} vs $j_{min,1}$ and K_{*2} versus $j_{min,2}$, for $aq_{*n,1} = 1$, $bq_{*n,1} = 0.5$, and $cq_{*n,1} = 2$, and different N_1 values. Filled dots identify the three cases illustrated in Fig. 2a-c

$$\frac{dh_{*j}}{dj} = 1 + rK_{*2} \left(\zeta(1-r) - H_{j_{min,2}-1+N_1(q_{*n,1}-1)}^{(1-r)} \right) = 0 \quad (44)$$

where the Riemann zeta function $\zeta(1-r)$, which equals -0.110779 for $r = 1.852$, derives from the Harmonic Number property $\frac{\partial H_{j-1}^{(-r)}}{\partial j} = -r \left(\zeta(1-r) - H_{j-1}^{(1-r)} \right)$. From Eqs. (43) and (44), the emitters at the minimum pressure head, $j_{min,1}$ and $j_{min,2}$, can be expressed as:

$$K_{*1} = \frac{1}{r \left(H_{j_{min,1}-1}^{(1-r)} - \zeta(1-r) \right)} \quad (45)$$

$$K_{*2} = \frac{1}{r \left(H_{j_{min,2}-1+N_1(q_{*n,1}-1)}^{(1-r)} - \zeta(1-r) \right)} \quad (46)$$

Figure 7 plots K_{*1} and K_{*2} versus $j_{min,1}$ and $j_{min,2}$, respectively, for different $q_{*n,1}$ and N_1 values, showing that in both sub-laterals, to increase $j_{min,1}$ and $j_{min,2}$, K_{*1} and K_{*2} decrease.

Eqs. (45) and (46) imply that the inside diameters ratio, D_1/D_2 , can be expressed as (see Eqs. 8 and 16):

$$\frac{D_1}{D_2} = \left(q_{*n,1}^r \frac{K_{*2}}{K_{*1}} \right)^{\frac{1}{s}} = \left(\frac{\zeta(1-r) - H_{j_{min,1}-1}^{(1-r)}}{\zeta(1-r) - H_{j_{min,2}-1+N_1(q_{*n,1}-1)}^{(1-r)}} \right)^{\frac{1}{s}} \quad (47)$$

The latter establishes a direct relationship between the diameter ratio D_1/D_2 and the emitters where the minimum pressure heads occur. Thus, hydraulic behaviour and geometric design parameters are explicitly linked.

For $q_{*n,1} = q_{*n,2}$, i.e. $q_{*n,1} = 1$, Eq. (47) becomes:

$$\frac{D_1}{D_2} = \left(\frac{\zeta(1-r) - H_{j_{min,1}-1}^{(1-r)}}{\zeta(1-r) - H_{j_{min,2}-1}^{(1-r)}} \right)^{\frac{1}{s}} \quad (48)$$

Eq. (48) is plotted in Fig. 8, showing that when $D_1 = D_2$, the condition of one-diameter lateral, $j_{min,1} = j_{min,2}$, occurs (red

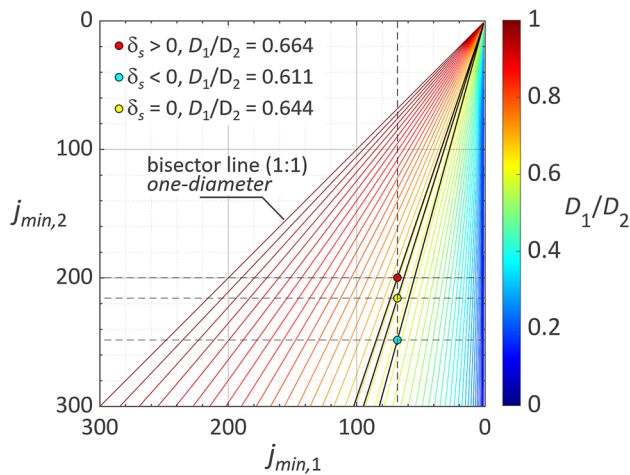


Fig. 8 Under the Hazen-Williams resistance equation, contour plot of the inside diameter ratio, D_1/D_2 , in the $j_{min,1}$ and $j_{min,2}$ space, for $N = 300$. The colour scale represents D_1/D_2 , while the dots identify the three configurations corresponding to $\delta_s > 0$, $\delta_s < 0$ and $\delta_s = 0$ illustrated in Fig. 2a–c

bisector line). In Fig. 8, the dots identify the three configurations corresponding to $\delta_s > 0$, $\delta_s < 0$, and $\delta_s = 0$ (Fig. 2a–c) are also illustrated.

Once $j_{min,1}$ and $j_{min,2}$ are known, $h_{*min,1}$ and $h_{*min,2}$ can be derived by Eqs. (13) and (25), respectively:

$$h_{*min,1} = h_{*in} + K_{*1} H_{j_{min,1}-1}^{(-r)} - (j_{min,1} - 1) \quad (49)$$

$$h_{*min,2} = h_{*n} + K_{*2} S_{*2} \left(H_{j_{*min,2}-1+N_1(q_{*n,1}-1)}^{(-r)} - H_{N_1 q_{*n,1}}^{(-r)} \right) - S_{*2} (j_{*min,2} - N_1 - 1) \quad (50)$$

where in Eq. (49), h_{*d} was substituted by h_{*in} , and in Eq. (50), h_{*n} is expressed by Eq. (17). Finally, for $q_{*n,1} = S_{*2} = 1$, substituting Eq. (17) into Eq. (50), $h_{*min,2}$ can also be written as:

$$h_{*min,2} = h_{*in} + K_{*1} H_{N_1}^{(-r)} - N_1 + K_{*2} \left(H_{j_{*min,2}-1}^{(-r)} - H_{N_1}^{(-r)} \right) - (j_{*min,2} - N_1 - 1) \quad (51)$$

Pressure head tolerance of the sub-lateral 2

The pressure head tolerance of the tapered lateral, δ , agrees with that of sub-lateral 1, δ_1 , only if $\delta_1 > \delta_2$, because δ was defined as $\max(\delta_1, \delta_2)$, which occurs when $h_{*min,1} > h_{*min,2}$. If $h_{*min,1} < h_{*min,2}$ (e.g., Fig. 2a), $\delta \equiv \delta_2$ and pressure head tolerance of sub-lateral 2, δ_2 , can be obtained by combining Eqs. (4) and (5):

$$\delta_2 = \frac{(1 + \delta_1) - (1 - \delta_1) \frac{h_{*min,2}}{h_{*min,1}}}{(1 + \delta_1) + (1 - \delta_1) \frac{h_{*min,2}}{h_{*min,1}}} \quad (52)$$

that is valid for both $\delta_s > 0$ and $\delta_s < 0$. Eq. (52) is plotted in Fig. 9 for several δ_1 values. Of course, for $h_{*min,2} = 0$, and any δ_1 , Eq. (52) gives $\delta_2 = h_{*in}/h_{*in} = h_{*n}/h_{*n} = 1$, corresponding to a one-diameter lateral. For a fixed δ_1 , as the ratio $h_{*min,2}/h_{*min,1}$ increases, δ_2 decreases, as expected. The dots in Fig. 9 represent the δ_2 values associated with Fig. 2a–c, which lie on the corresponding curves ($\delta_1 = 0.04$ for Fig. 2a, and $\delta_1 = 0.05$ for Fig. 2bc).

Procedure for designing tapered drip laterals

In Section "Analytical formulation", analytical relationships to determine the pressure head distributions and minimum pressure heads of the two sub-laterals, under general conditions, have been derived. In that formulation, hydraulic parameters K_{*1} and K_{*2} are prescribed for facilitating the illustration of the derived relationships. However, the corresponding pressure head tolerances, δ_1 and δ_2 , are not directly controlled. From a practical design perspective, pressure head tolerances should be treated as imposed design parameters rather than passive outcomes of the calculation, since they determine the emission uniformity of the tapered lateral and, more generally, of drip laterals (Baiamonte 2018a).

For sloped tapered laterals, a pressure head tolerance for sub-lateral 1, δ_1 , denoted as δ_1^{in} can be prescribed *a priori* with respect to h_{*in} from Eq. (4). This prescribed parameter is used to determine the required normalised inlet pressure head, h_{*in} . However, it coincides with the actual δ_1 only when h_{*in} is its maximum normalised pressure head, namely for $\delta_s > 0$, or when $h_{*in} = h_{*n}$ as occurs for $\delta_s = 0$. For $\delta_s < 0$, the pressure head at the DCP exceeds that at the inlet, $h_{*n} > h_{*in}$ (Fig. 2b), and the actual δ_1 must therefore be recalculated referring to h_{*n} (Eq. 4).

For practical design purposes, the analytical relationships derived in Section "Analytical formulation" are reformulated below as an inverse design procedure for determining the inlet pressure head, DCP, and inside diameter of sub-lateral 2, D_2 , for the three regime scenarios ($\delta_s > 0$, $\delta_s < 0$, and $\delta_s = 0$). Moreover, in Section "One-diameter lateral ($K_{*1} = K_{*2} = K_{*}$)", the simple case of one-diameter lateral and equal flow rate ($K_{*1} = K_{*2} = K_{*}$) is addressed, which will be considered for comparison with the tapered lateral in Section "Comparison between tapered laterals and one-diameter lateral".

In the following, the design procedure is applied for a prescribed set of input parameters characterising the tapered

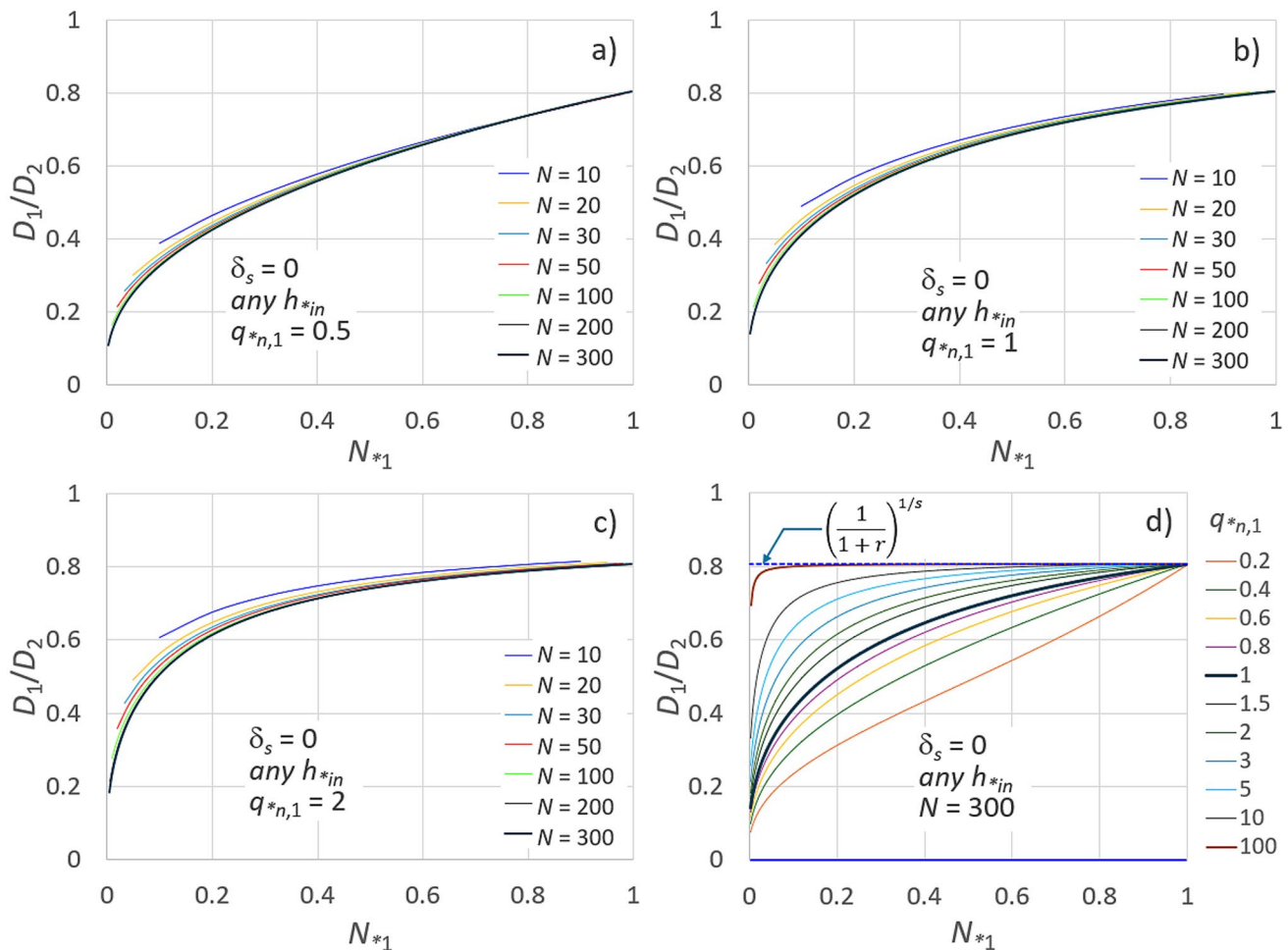


Fig. 9 Under the Hazen-Williams resistance equation, relationship between δ_2 and the ratio $h_{*min,2}/h_{*min,1}$ for different values of δ_1 . Dots refer to the PHD cases illustrated in Fig. 2a–c

lateral, namely N , S_0 , S_1 , S_2 , D_1 , $q_{n,1}$, $q_{n,2}$, δ_1 , and the emitter flow exponents x_1 and x_2 , for sub-laterals 1 and 2, respectively. The emitter exponents are used to determine the corresponding emitter coefficients, $k_{e,1}$ and $k_{e,2}$, in the emitter flow relation, thereby supporting the selection of commercially available emitters. On this basis, the three PHD regimes corresponding to $\delta_s > 0$, $\delta_s < 0$, and $\delta_s = 0$ are examined separately in a logical sequence, using bullet points.

Positive pressure head tolerance at DCP ($\delta_s > 0$)

- 1) For positive pressure head tolerance at diameter-change point (DCP, $\delta_s > 0$, Fig. 2a), after selecting the resistance equation (i.e., r , s and k , Eq. (8)), for an assigned inside pipe diameter, D_1 , and emitter flow rate, $q_{n,1}$, K_{*1} can be calculated using Eq. (16).
- 2) Once K_{*1} is known, the emitter at the minimum pressure head in sub-lateral 1, $j_{min,1}$, is determined by Eq. (45).

- 3) For the prescribed value of δ_1^{in} , which for $\delta_s > 0$ agrees with the actual δ_1 ($h_{*in} > h_{*n}$), substituting the expression for the minimum pressure head of sub-lateral 1, $h_{*min,1}$ (Eq. (49)), into Eq. (4) and solving for the normalised inlet pressure head, $h_{*in} = h_{*d}$, yields:

$$h_{*in} = \frac{1 + \delta_1}{2\delta_1} \left(j_{min,1} - 1 - K_{*1} H_{j_{min,1}-1}^{(-r)} \right) \quad (53)$$

- 4) Once h_{*in} and $j_{min,1}$ are calculated, the corresponding $h_{*min,1}$ is provided by Eq. (49).
- 5) For an assigned number of emitters, N , using Eqs. (32) and (36), an admissible integer value of the number of emitters in sub-lateral 1, N_1 , i.e. $N_{*1} = N_1/N$, can be selected, satisfying both $\delta_s > 0$ and $K_{*2} > 0$.
- 6) For the resulting K_{*2} , the inside diameter of sub-lateral 2, D_2 , can be obtained by inverting Eq. (8), using Eq. (16), and replacing the subscript 1 by 2:

$$D_2 = k_2^{\frac{1}{s}} \frac{q_{n,2}^{\frac{r}{s}}}{(K_{*2} S_0)^{\frac{1}{s}}} \quad (54)$$

In practical applications, N_1 must be chosen so that the calculated D_2 matches a commercially available pipe diameter.

- 7) Once N_1 is established, the normalised pressure head at the DCP ($j = N_1 + 1$), h_{*n} , can be determined by Eq. (17).
- 8) The emitter position, $j_{min,2}$, and the corresponding minimum pressure head in sub-lateral 2, $h_{min,2}$, are determined from Eq. (46) and (51), respectively. Both quantities depend on previously defined parameters ($q_{*n,1}$, S_{*2} , h_{*in} , K_{*1} , N_1).
- 9) To determine the emitter characteristics of sub-lateral 1, $k_{e,1}$, and sub-lateral 2, $k_{e,2}$, from the emitter flow relation, the average pressure heads of the corresponding sub-laterals, $h_{*n,1}$ and $h_{*n,2}$, can be calculated as:

$$h_{*n,1} = \frac{h_{*in} + h_{*min,1}}{2} \quad (55)$$

$$h_{*n,2} = \frac{h_{*in} + h_{*min,2}}{2} \quad (56)$$

- 10) For a fixed exponent of the emitter flow relations x_1 and x_2 , Eqs. (55) and (56) make it possible to calculate $k_{e,1}$ and $k_{e,2}$:

$$k_{e,1} = \frac{q_{n,1}}{(h_{*n,1} S_0 S_1)^{\frac{1}{x_1}}} \quad (57)$$

$$k_{e,2} = \frac{q_{n,2}}{(h_{*n,2} S_0 S_1)^{\frac{1}{x_2}}} \quad (58)$$

The emitter coefficients, x_1 , x_2 , $k_{e,1}$, $k_{e,2}$, allow each sub-lateral to be equipped with a different emitter type, characterised by its own emitter flow relation.

- 11) Finally, the tapered pressure head tolerance, δ , is equal to the imposed δ_1 , if $h_{*min,1} < h_{*min,2}$, otherwise coincides with δ_2 ($h_{*min,1} > h_{*min,2}$), which can be determined from Eq. (5) or from Eq. (52). Of course, dimensional pressure heads are obtained multiplying dimensionless pressure heads by the imposed slope, S_0 , and emitter spacing of sub-lateral 1, S_1 .

Negative pressure head tolerance at DCP ($\delta_s < 0$)

For negative pressure head tolerance at diameter-change point (DCP, $\delta_s < 0$, Fig. 2b), the design sequence described

in Section "Positive pressure head tolerance at DCP ($\delta_s > 0$)" remains applicable, except that the pressure head at the DCP exceeds that at the inlet, i.e., $h_{*n} > h_{*in}$. The imposed pressure head tolerance of sub-lateral 1, δ_1^{in} , calculated from Eq. (4), makes it possible to determine the required value of h_{*in} from Eq. (53). However, because $h_{*n} > h_{*in}$, the imposed $\delta_1^{in} \neq \delta_1$; therefore, the pressure head tolerance of sub-lateral 1 needs to be recalculated with respect to h_{*n} (Eq. (4)).

As a consequence, the average normalised pressure head of the sub-lateral 1, $h_{*n,1}$, and sub-lateral 2, $h_{*n,2}$, that are required to determine the emitter characteristics of the sub-laterals, $k_{e,1}$ and $k_{e,2}$, are equal to, respectively:

$$h_{*n,1} = \frac{h_{*n} + h_{*min,1}}{2} \quad (59)$$

$$h_{*n,2} = \frac{h_{*n} + h_{*min,2}}{2} \quad (60)$$

With this modification, the bullet-point procedure outlined in Section "Positive pressure head tolerance at DCP ($\delta_s > 0$)" applies equally to the case $\delta_s < 0$.

Zero pressure head tolerance at DCP ($\delta_s = 0$)

Under zero pressure head tolerance at diameter-change point (DCP, $\delta_s = 0$, Fig. 2c), Eq. (33) reads:

$$K_{*1} = \frac{N_{*1} N}{H_{N_{*1} N}^{(-r)}} \quad (61)$$

Substituting Eq. (61) into Eq. (36), yields:

$$K_{*2} = \frac{N(1 - N_{*1})}{H_{N - N_{*1} N(1 - q_{*n,1})}^{(-r)} - H_{N_{*1} N q_{*n,1}}^{(-r)}} \quad (62)$$

Note that, contrary to the case $\delta_s \neq 0$, K_{*2} does not depend on S_{*2} , but only on $q_{*n,1}$. For $q_{*n,1} = 1$, Eq. (62) becomes:

$$K_{*2} = \frac{N(1 - N_{*1})}{H_N^{(-r)} - H_{N_{*1} N}^{(-r)}} \quad (63)$$

The case $\delta_s = 0$ is very easy to apply, as it only requires using Eq. (63), independently on h_{*in} , which can be freely chosen. This is because, under $\delta_s = 0$, h_{*in} acts merely as a PHD scaling factor, provided K_{*1} satisfies Eq. (61). For given values of K_{*1} and N , this condition directly determines the corresponding N_{*1} , which therefore cannot be arbitrarily chosen as for $\delta_s \neq 0$.

For $\delta_s = 0$ and any $q_{*n,1}$, the inside diameters ratio D_1/D_2 , which is related to K_{*2}/K_{*1} (see Eqs. 8 and 16), can be obtained by using Eqs. (61) and (62), yielding:

$$\frac{D_1}{D_2} = \left(q_{*n,1}^r \frac{K_{*2}}{K_{*1}} \right)^{\frac{1}{s}} = \left[q_{*n,1}^r \left(\frac{1 - N_{*1}}{N_{*1}} \right) \frac{H_{N_{*1}N}^{(-r)}}{H_{N_{*1}N}^{(-r)} q_{*n,1} + (1 - N_{*1}) N - H_{N_{*1}N}^{(-r)} q_{*n,1}} \right]^{\frac{1}{s}} \quad (64)$$

that once again highlights the dependence solely on $q_{*n,1}$. For $q_{*n,1} = 1$, Eq. (64) simplifies as:

$$\frac{D_1}{D_2} = \left(\frac{K_{*2}}{K_{*1}} \right)^{\frac{1}{s}} = \left[\left(\frac{1 - N_{*1}}{N_{*1}} \right) \left(\frac{H_{N_{*1}N}^{(-r)}}{H_N^{(-r)} - H_{N_{*1}N}^{(-r)}} \right) \right]^{\frac{1}{s}} \quad (65)$$

Eq. (64) is plotted in Fig. 10a-c, vs. N_{*1} , for different N values, and for $q_{*n,1} = 0.5$ (Fig. 10a), 1 (Fig. 10b, Eq. 65), and 2 (Fig. 10c), while in Fig. 10d, the same D_1/D_2 ratio is graphed for a fixed $N = 300$, by varying $q_{*n,1}$.

Figure 10a-c show that, for any N_{*1} and $q_{*n,1}$, the ratio D_1/D_2 decreases as N increases. This effect is particularly pronounced at low N_{*1} , when sub-lateral 2 dominates the tapered lateral. D_2 must increase to accommodate the longer lateral length and higher circulating discharge.

For a fixed $N = 300$, Fig. 10d shows that for $q_{*n,1} = 0$ (i.e., $q_{n,1} = 0$), $D_1/D_2 = 0$, for any N_{*1} , corresponding to a one-diameter lateral. As $q_{*n,1}$ increases, D_1/D_2 also increases, approaching to $(1/(1+r))^{1+s}$ for $N_{*1} = 1$, as indicated in Fig. 10d.

One-diameter lateral ($K_{*1} = K_{*2} = K_*$)

According to the normalised approach adopted in this study, one-diameter drip laterals on slopes were previously

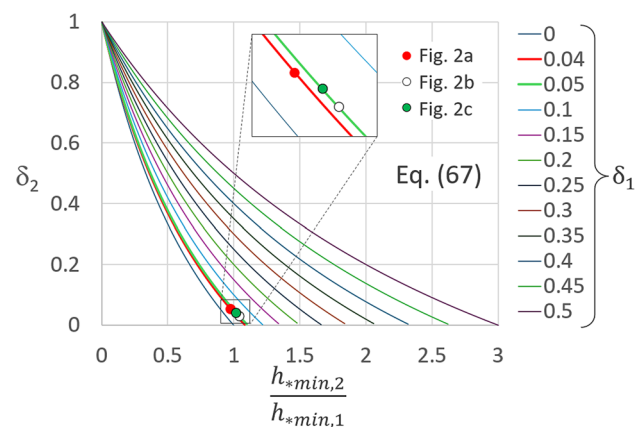


Fig. 10 Under the Hazen-Williams resistance equation, relationship between the inside diameter ratio, D_1/D_2 , vs N_{*1} , for $\delta_s = 0$ and different N values, **a** for $q_{*n,1} = 0.5$, **b** $q_{*n,1} = 1$, and **c** $q_{*n,1} = 2$. In **d**, the same relationship, for $N = 300$, with $q_{*n,1}$ as a parameter

analysed by Baiamonte (2018a). This configuration represents a limiting case of the general tapered layout developed herein. For an imposed number of emitters, N , the corresponding K_* is calculated from Eq. (35), where $K_* = K/S_0$ with K to be referred to the emitter flow rate, q_n , and inside diameter, D , of the one-diameter lateral (Eq. 6, with $q_{n,1} = q_n$ and $D_1 = D$). Similarly, the emitter where the minimum pressure head occurs, j_{min} , is expressed by Eq. (45), if imposing in it $K_{*1} = K_*$ and $j_{min,1} = j_{min}$ (Baiamonte 2018a):

$$K_* = \frac{1}{r \left(H_{j_{min}-1}^{(1-r)} - \zeta (1-r) \right)} \quad (66)$$

The same approach applies to h_{*min} (Eq. 49) and to the normalised inlet pressure head $h_{*in} \equiv h_{*d}$, which can be obtained by Eq. (53) if imposing $\delta_1 = \delta$ and $j_{min,1} = j_{min}$:

$$\frac{2\delta}{1+\delta} h_{*in} = j_{min} - 1 - K_* H_{j_{min}-1}^{(-r)} \quad (67)$$

where j_{min} is defined by Eq. (66), which corresponds to the same condition imposed in Baiamonte (2018a, see Eq. 5), for one-diameter drip laterals on slope. The average pressure head is given by $h_{*n} = h_{*in}/(1+\delta)$, while k_e can be obtained by Eq. (57), by setting $q_{n,1} = q_n$, $h_{*n,1} = h_{*n}$, $S_1 = S$, and $x_1 = x$.

In conclusion, the design relationships for one-diameter laterals are obtained directly by setting $K_{*2} = K_{*1}$ within the tapered-lateral design procedure presented herein. This demonstrates that the proposed tapered-lateral design procedure is an extension of the previously developed procedure for designing one-diameter laterals on sloping fields.

In the main axis of Fig. 11a, Eqs. (66) and (67) are graphed, together with Eq. (35) (secondary axis). This figure allows the design parameters of any drip lateral (for any δ) on a slope to be described, with the slope S_0 and S not appearing explicitly, as they serve as a scaling factor for the pressure heads.

For example, for a fixed $\delta = 5\%$ and $N = 100, 200$ and 300 , Fig. 11b illustrates the PHDs corresponding to the dots indicated in Fig. 11a, which behave as expected. A further application will be performed in Section "One-diameter lateral" for a comparison with the tapered laterals addressed in this work.

In summary, Section "Procedure for designing tapered drip laterals" converts the analytical relationships derived in Section "Analytical formulation" into a fully explicit design procedure that allows the inlet pressure head, the diameter-change point location, and the required pipe diameter, D_2 , to be determined by the designer without iterative numerical schemes.

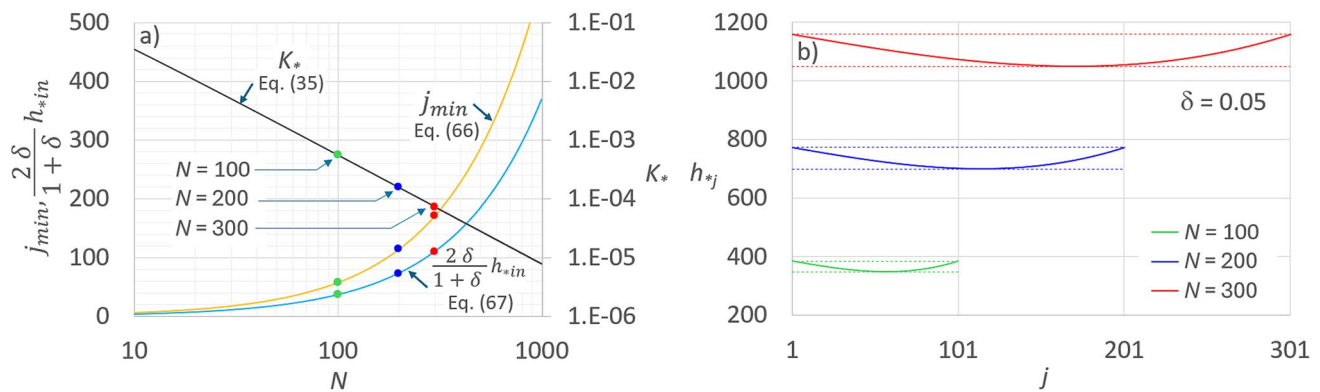


Fig. 11 Under the Hazen-Williams resistance equation, relationship between $j_{min,1}$ and $2\delta/(1+\delta) h_{*in}$ (main axis), and K_* (secondary axis) versus N , for one-diameter laterals, and **b** PHDs corresponding to the dots indicated in Fig. 11a

Table 1 Under the Hazen-Williams resistance equation, input parameters of the three considered cases illustrated in Figs. 2a–c and 13a–c

Parameter	$\delta_s > 0$	$\delta_s < 0$	$\delta_s = 0$	Equations
r		1.852		Eq. (8)
s		4.871		Eq. (8)
C		135		Eq. (10)
k		1.211×10^{-3}		Eq. (8)
N		300		–
S_0		0.04		–
S_1 (m)		0.75		–
S_2 (m)		0.75		–
S_{*2}		1		S_2/S_1
L (m)		225		$N_1 S_1 + N_2 S_2$
D_1 (mm)		13.8		–
$q_{n,1}$ (l h ⁻¹)		4.50		–
x_1		0.48		–
$q_{n,2}$ (l h ⁻¹)		4.50		$q_{n,2}/q_{n,1}$
x_2		0.48		–
$q_{*n,1}$		1		$q_{n,1}/q_{n,2}$
δ^{in}	0.04	0.03	0.0505	–

Applications and discussions

To validate the analytical design framework, applications were performed using prescribed pressure tolerances of sub-lateral 1, for the three examined scenarios: $\delta_s > 0$, $\delta_s < 0$, and $\delta_s = 0$.

For each case, the results obtained from the analytical procedure were compared with those derived from the exact step-by-step (SBS) numerical method, in which the assumption of uniform emitter discharge is relaxed. The SBS method is based on the repetitive application of the basic hydraulic equations (momentum and continuity) to consecutive emitters, thus providing the most exact method for estimating the pressure heads and actual flow rates of the emitters along the lateral line, thus it is considered here as a reference.

Finally, the performance of the tapered configuration is compared with that of an equivalent one-diameter lateral

(Section "Comparison between tapered laterals and one-diameter lateral").

Tapered laterals

For a tapered lateral characterised by $N = 300$ emitters spaced at 0.75 m, on a slope of $S_0 = 4\%$, and an emitter flow rate $q_{n,1} = q_{n,2} = 4.5 \text{ l h}^{-1}$, and for the three possible PHD patterns ($\delta_s > 0$, $\delta_s < 0$, and $\delta_s = 0$), Table 1 reports the input parameters (see notation list) selected to apply the design procedure, which are identical for $\delta_s > 0$, $\delta_s < 0$, and $\delta_s = 0$ cases, except for δ_1^{in} .

Following the bullet points described in Section "Positive pressure head tolerance at DCP ($\delta_s > 0$)", Table 2 reports the output design parameters of sub-laterals 1 and 2. For the fixed set of input parameters of the three scenarios ($\delta_s > 0$, $\delta_s < 0$, and $\delta_s = 0$), including δ_1^{in} , as reported in Table 1, different values of the normalised inlet pressure head h_{*in} have been determined (Eq. (53), Table 2). This confirms that δ_1^{in} primarily governs the required inlet pressure head of sub-lateral 1 (for fixed D_1 , $q_{n,1}$, S_1 and S_0), while the sign of δ_s influences the PHD along sub-lateral 2.

Selection of δ_s was performed according to Fig. 12a, where the δ_s relationship (Eq. 32) is plotted by varying N_1 for $K_{*1} = 4.065 \cdot 10^{-4}$, and $S_{*2} = q_{*n,1} = 1$. Fig. 12a shows that selecting for example $N_1 = 106$, 133 and 118.5, corresponding to $N_{*1} = 0.353$, 0.443, and 0.395, produce the three cases to be tested, $\delta_s > 0$, $\delta_s < 0$, and $\delta_s = 0$, respectively.

Figure 12a illustrates that the sign of δ_s is determined by the location of N_1 . Small variations in N_1 around intermediate values can reverse the sign of δ_s , demonstrating the sensitivity of the pressure redistribution mechanism. Figure 12b further shows that the corresponding K_{*2} values, and thus D_2 , vary smoothly with N_1 , allowing the practical selection of commercially available diameters without compromising hydraulic feasibility.

Table 2 Under the Hazen-Williams resistance equation, output design parameters of sub-lateral 1 and sub-lateral 2

Lateral	Parameter	$\delta_s > 0$	$\delta_s < 0$	$\delta_s = 0$	Equations
Sub-lateral 1	K_1		1.626×10^{-5}		Eq. (8)
	K_{*1}		4.065×10^{-4}		Eq. (16)
	$j_{min,1}$		68.3		Eq. (45)
	$h_{*min,1}$	522.02	703.27	408.96	Eq. (49)
	$h_{min,1}$ (m)	15.66	21.10	12.27	$h_{*min,1} S_0 S_1$
	h_{*d}	565.52	746.77	452.46	Eq. (53)
	h_d (m)	16.97	22.40	13.57	$h_{*d} S_0 S_1$
	N_1	106	133	118.55	—
	N_{*1}	0.353	0.443	0.395	N_1/N
	h_{*n}	545.80	778.12	452.46	Eq. (17)
	h_n (m)	16.37	23.34	13.57	$h_n S_0 S_1$
	δ_1	0.04	0.505	0.0505	Eq. (4)
	L_1 (m)	79.50	99.75	88.91	$N_1 S_1$
	$h_{*n,1}$	543.77	740.70	430.71	Eq. (55)
	$h_{n,1}$ (m)	16.31	22.22	12.92	$h_{*n,1} S_0 S_1$
	$k_{e,1}$	1.18	1.02	1.32	Eq. (57)
Sub-lateral 2	δ_s	0.018	-0.021	0	Eq. (32)
	DCP	107	134	119.55	$N_1 + 1$
	K_{*2}	5.512×10^{-5}	3.681×10^{-5}	4.778×10^{-5}	Eq. (38)
	K_2	2.205×10^{-6}	1.472×10^{-6}	1.911×10^{-6}	$K_{*2} S_0$
	N_2	194	167	181.45	$N - N_1$
	L_2 (m)	145.50	125.25	136.09	$N_2 S_2$
	D_2 (mm)	20.80	22.60	21.42	Eq. (54)
	$j_{min,2}$	199.8	248.3	215.8	Eq. (46)
	$h_{*min,2}$	511.18	735.80	417.77	Eq. (51)
	$h_{min,2}$ (m)	15.34	22.07	12.53	$h_{*min,2} S_0 S_1$
	h_{*min}	511.18	703.27	408.96	$\min\{h_{*min,1}, h_{*min,2}\}$
	h_{min} (m)	15.34	21.10	12.27	$h_{*min,1} S_0 S_1$
	δ_2	0.0505	0.0074	0.0399	Eq. (52)
	h_{*in}	565.52	746.77	452.46	Eqs. (53)
	h_{in} (m)	16.97	22.40	13.57	$h_{*in} S_0 S_1$
	$h_{*n,2}$	538.35	756.96	435.11	Eq. (56)
	$h_{n,2}$ (m)	16.15	22.71	13.05	$h_{*n,2} S_0 S_1$
	$k_{e,2}$	1.18	1.01	1.31	Eq. (58)
Tapered	δ	0.0505	0.0505	0.0505	$\max\{\delta_1, \delta_2\}$

For the same N_1 values, the K_{*2} relationship (Eq. 38) is plotted in Fig. 12b (main axis), making it possible to determine the corresponding K_{*2} and D_2 (secondary axis), as described in Section "Positive pressure head tolerance at DCP ($\delta_s > 0$)", point 6). Therefore, the inside diameter of the sub-lateral 2, D_2 , was obtained from Eq. (54).

The remaining output parameters of sub-lateral 2 are reported in Table 2, in a logical sequence, according to the bullet points illustrated in Section "Positive pressure head tolerance at DCP ($\delta_s > 0$)". In particular, for the three considered cases, once δ_s , N_1 (or $DCP = N_1 + 1$), and K_{*2} (or D_2 , Eq. 54) are known, the normalised characteristic pressure heads, $h_{*n,1}$, h_{*n} , $h_{*min,2}$, and $h_{*n,2}$, are calculated. The emitters' characteristics, $k_{e,2}$, can also be determined (Eq. 58). In the last column, the corresponding equation is provided for each parameter.

The results, together with Eqs. (13) and (25), were used to draw the corresponding PHDs, which are shown in Fig. 13a–c.

To explore the influence of the resistance equation, the same design procedure was replicated under the Blasius resistance equation. With reference to input data, compared to Hazen-Williams (Table 1), only the parameters r , s , and k (Eq. 8) needed to be modified by considering those reported in Eq. (9). The water temperature of 20 °C was assumed giving the kinematic viscosity, ν , and the corresponding k values equal to $1.008 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$ and 7.788×10^{-4} , respectively. Similar to Table 2, for Hazen-Williams resistance equation, Table 3 reports the corresponding output data, where can be observed that marginal differences with respect to Table 2 can be observed. For $\delta_s < 0$, the comparison was carried out using the same value of N_1 ; as a consequence, a slight difference arose in pressure head tolerance of sub-lateral 1, δ_1 (and δ) (0.0471 and 0.0505). These differences result from

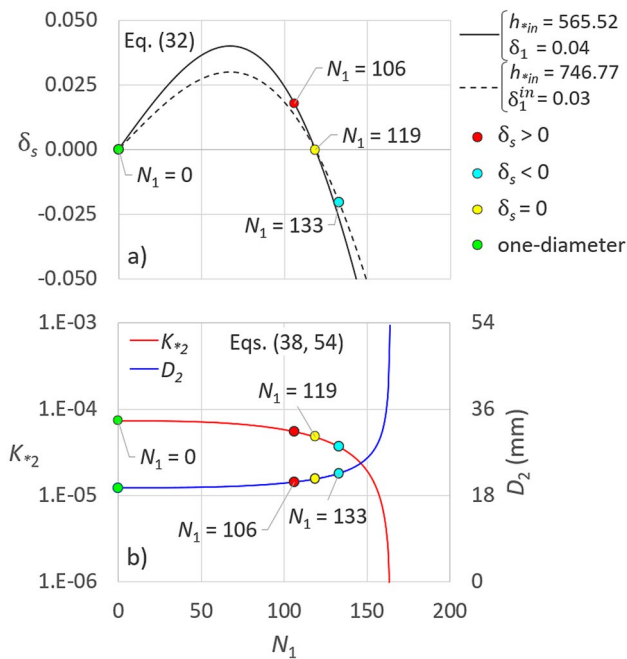


Fig. 12 Under the Hazen-Williams resistance equation, relationships between a) the pressure head tolerance at DCP, δ_s , and b) K_{*2} (main axis) and D_2 (secondary axis) versus N_1 . Dots refer to the N_1 values associated with Fig. 2a–c, and to one-diameter lateral ($N_1 = 0$)

the corresponding changes in h_{*n} and $h_{*min,1}$ (Eq. 4). Overall, the results from the Blasius equation are highly consistent and directly comparable with those from Hazen-Williams equation, as expected.

The PHD corresponding to the output data (Table 3) are shown in Fig. 13ab (dashed lines). As can be observed, with the Blasius resistance equation, slightly lower pressure heads than the Hazen-Williams equation are predicted. In addition, the diameter of sub-lateral 2 also results in slightly lower values: 20.45, 22.08, and 21.12 mm, for Blasius, and 20.80, 22.60, and 21.42 mm, for Hazen-Williams equation, for $\delta_s > 0$, $\delta_s < 0$, and $\delta_s = 0$, respectively.

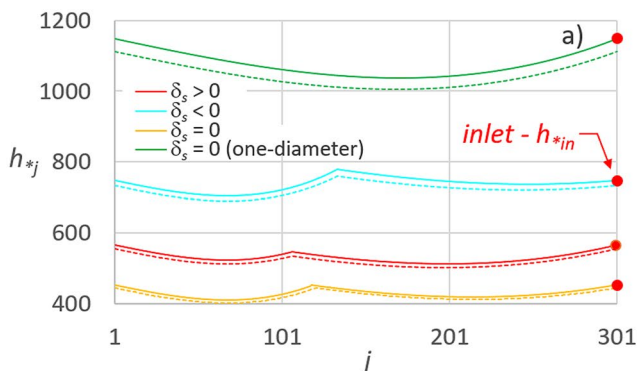


Fig. 13 Under the Hazen-Williams and Blasius resistance equations, comparison between the PHDs determined by the proposed analytical design procedure for tapered laterals and those obtained for the one-

One-diameter lateral

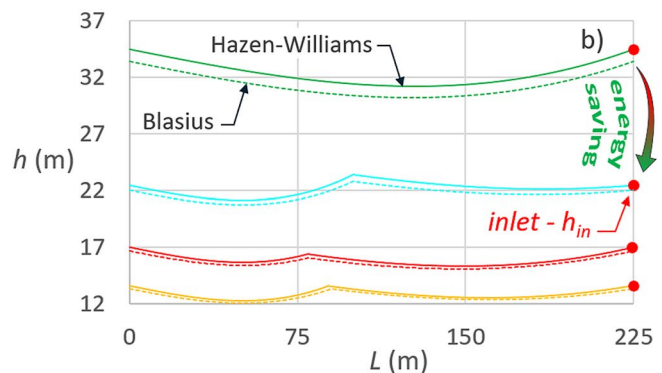
For the purpose of comparing the design of the tapered lateral on a sloping field with the one-diameter lateral, the procedure summarised in Section "One-diameter lateral ($K_{*1} = K_{*2} = K_{*}$)" (Baiaumont 2018a) was also applied. For the same length, emitter flow rate and slope of the tapered drip lateral considered in Fig. 13 (Table 1), Table 4 reports the input and output parameters, and the corresponding equation associated with each calculated parameter, for both the Blasius and Hazen-Williams resistance equations.

For tapered laterals (Table 3), the results in Table 4 indicate that adopting the Blasius resistance equation instead of the Hazen-Williams equation produces only negligible differences in the output parameters. Specifically, using the Blasius equation reduces the required inlet pressure head from 34.45 to 33.35 m and the lateral diameter from 19.6 to 19.3 mm.

Comparison between tapered laterals and one-diameter lateral

To compare the energy-saving potential of tapered and one-diameter laterals, as reflected by the required inlet pressure head, it is essential to consider hydraulically equivalent design conditions (Baiaumont et al. 2025c). Accordingly, the results presented in the previous sections are directly comparable, since all applications were carried out using the same allowable pressure head tolerance ($\delta = 5.05\%$), for the tapered and one-diameter laterals. This ensures that any differences in the required inlet pressure head can be attributed solely to the lateral configuration rather than to variations in the design criteria.

Furthermore, adopting the allowable pressure head tolerance, δ , as the design criterion is consistent to ensure comparable emission performance among the examined layouts. For uniform-diameter laterals, δ has been shown to exhibit a



diameter lateral, under the same field slope, emitter flow rate, emitter spacing, and total lateral length, in terms of a dimensionless and b dimensional pressure heads

Table 3 Under the Blasius resistance equation, output design parameters of sub-lateral 1 and sub-lateral 2

Lateral	Parameter	$\delta_s > 0$	$\delta_s < 0$	$\delta_s = 0$	Equations
Sub-lateral 1	K_1		2.492×10^{-5}		Eq. (8)
	K_{*1}		6.231×10^{-4}		Eq. (16)
	$j_{min,1}$		68.4		Eq. (45)
	$h_{*min,1}$	512.30	690.18	401.34	Eq. (49)
	$h_{min,1}$ (m)	15.37	20.71	12.04	$h_{*min,1} S_0 S_1$
	h_{*d}	554.99	732.87	444.04	Eq. (53)
	h_d (m)	16.65	21.99	13.32	$h_{*d} S_0 S_1$
	N_1	106	133	120.20	—
	N_{*1}	0.353	0.443	0.401	N_1/N
	h_{*n}	534.19	758.47	444.04	Eq. (17)
	h_n (m)	16.03	22.75	13.32	$h_n S_0 S_1$
	δ_1	0.04	0.0471	0.0505	Eq. (4)
	L_1 (m)	79.50	99.75	90.15	$N_1 S_1$
	$h_{*n,1}$	533.65	724.33	422.69	Eq. (55)
	$h_{n,1}$ (m)	16.01	21.73	12.68	$h_{*n,1} S_0 S_1$
	$k_{e,1}$	1.19	1.03	1.33	Eq. (57)
Sub-lateral 2	δ_s	0.019	0.019	0	Eq. (32)
	DCP	107	134	121.20	$N_1 + 1$
	K_{*2}	9.619×10^{-5}	6.685×10^{-5}	8.259×10^{-5}	Eq. (38)
	K_2	3.847×10^{-6}	2.674×10^{-6}	3.304×10^{-6}	$K_{*2} S_0$
	N_2	194	167	179.80	$N - N_1$
	L_2 (m)	145.50	125.25	134.85	$N_2 S_2$
	D_2 (mm)	20.45	22.08	21.12	Eq. (54)
	$j_{min,2}$	197.9	243.5	215.9	Eq. (46)
	$h_{*min,2}$	501.91	720.29	411.75	Eq. (51)
	$h_{min,2}$ (m)	15.06	21.61	12.35	$h_{*min,2} S_0 S_1$
	h_{*min}	501.91	690.18	401.34	$\min\{h_{*min,1}, h_{*min,2}\}$
	h_{min} (m)	15.06	20.71	12.04	$h_{*min,1} S_0 S_1$
	δ_2	0.0502	0.0258	0.0377	Eq. (52)
	h_{*in}	554.99	732.87	444.04	Eqs. (53)
	h_{in} (m)	16.65	21.99	13.32	$h_{*in} S_0 S_1$
	$h_{*n,2}$	528.45	739.38	427.89	Eq. (56)
	$h_{n,2}$ (m)	15.85	22.18	12.84	$h_{*n,2} S_0 S_1$
	$k_{e,2}$	1.19	1.02	1.32	Eq. (58)
Tapered	δ	0.0502	0.0471	0.0505	$\max\{\delta_1, \delta_2\}$

simple linear relationship with both the coefficient of variation, CV , and the emission uniformity, EU , indicating that designs based on the same pressure head tolerance achieve similar levels of irrigation uniformity (Baiaumont 2018a, Fig. 9).

Although differences in lateral diameter should also be evaluated from an economic perspective, such an analysis requires consideration of material costs, manufacturing, and installation aspects. Since the present study is focused exclusively on hydraulic performance and energy requirements, an economic assessment is beyond its scope.

The comparison between the PHDs obtained by the procedure suggested to design tapered laterals on a sloping field, and those for a one-diameter lateral, for the same slope, emitter flow rate, emitter spacing, and length, is illustrated in Fig. 13, in terms of dimensionless (Fig. 13a) and dimensional pressure heads (Fig. 13b).

The advantage of selecting tapered laterals instead of one-diameter laterals becomes immediately evident, particularly from an energy-saving perspective. Specifically, tapered laterals require a significantly lower inlet pressure head compared to one-diameter laterals, resulting in reduced energy consumption.

For the one-diameter lateral, $h_{in} = 34.45$ m, whereas the tapered configuration requires $h_{in} = 16.97$, 22.40, and 13.57 m, for $\delta_s > 0$, $\delta_s < 0$, and $\delta_s = 0$ configurations, respectively. This corresponds to a 50.7, 35.0, and 60.6 % reduction in the required inlet pressure head. Therefore, as illustrated in Fig. 13b, such a decrease directly translates into lower pumping energy demand, also reducing operational costs, underscoring the substantial energy-saving potential of properly designed tapered laterals.

Moreover, the one-diameter lateral requires $D = 19.6$ mm. In contrast, the tapered configurations use $D_1 = 13.8$ mm in the downstream section, and a D_2 slightly higher than

Table 4 Input and output parameters of the one-diameter lateral, under the Hazen-Williams and Blasius resistance equations

Input/output	Parameter	Value (Hazen-Williams)	Value (Blasius)	Equa- tion
Input parameters	r	1.852	1.75	Eq. (9, 10)
	s	4.871	4.75	Eq. (9,10)
	C/v	135	1.008×10^{-6}	Eq. (9,10)
	k	1.211×10^{-3}	7.788×10^{-4}	Eq. (9,10)
	N	300	—	—
	S_0	0.04	—	—
	S (m)	0.75	—	—
	L (m)	225.00	—	NS
	D_1	19.6	19.3	—
	x	0.48	—	—
Output parameters	q_n (l h ⁻¹)	4.50	—	—
	K_*	7.336×10^{-5}	1.266×10^{-4}	Eq. (35)
	K	2.934×10^{-6}	5.063×10^{-6}	Eq. (8)
	δ	0.0505	—	—
	j_{min}	171.3	169.24	Eq. (66)
	h_{*min}	1037.95	1004.76	Eq. (49)
	h_{min} (m)	31.14	30.14	h_{*min} $S_0 S$
	h_{*in}	1148.36	1111.64	Eq. (67)
	h_{in} (m)	34.45	33.35	h_{*in} $S_0 S$
	h_{*n}	1093.16	1058.20	$h_{*in}/$ (1+ δ)
	h_n (m)	32.79	31.75	h_{*n} $S_0 S$
	k_e	0.84	0.86	Eq. (57)

D , equal to 20.80 ($N_{*1} = 0.353$), 22.60 ($N_{*1} = 0.443$), and 21.42 mm ($N_{*1} = 0.395$), in the upstream section, for $\delta_s > 0$, $\delta_s < 0$, and $\delta_s = 0$, respectively.

Because a substantial portion of the lateral length is characterised by a smaller diameter (D_1), material usage is reduced, indicating potential savings in investment cost. Depending on the selected N_1 , the tapered configurations result in effective reductions in equivalent pipe diameter of approximately 6.56 %, 4.67 % and 6.15 % than the one-diameter lateral.

In conclusion, although the analysis presented here refers to a representative case study, the results clearly demonstrate the hydraulic and energetic advantages of tapered laterals on sloping fields. When accurately designed using the proposed analytical framework, tapered configurations

offer substantial reductions in required inlet pressure head and material usage compared to traditional one-diameter laterals. A comprehensive economic analysis, including additional installation costs, would further quantify these benefits; however, such an analysis is beyond the scope of the present study.

Comparison with the exact SBS method and uniformity performance

A comparison between the pressure head distribution, PHD, obtained by the described procedure and that derived by the exact step-by-step (SBS) method, which relaxes the assumption of uniform emitter flow rate, was performed.

For the three cases, the results of the comparison are illustrated in Fig. 14a–f, where the corresponding PHD, in both dimensionless and dimensional pressure heads and for both the analytical and numerical solutions, are shown for $\delta_s > 0$ (Fig. 14a–b), $\delta_s < 0$ (Fig. 14c–d), and $\delta_s = 0$ (Fig. 14e–f). Results indicate that slight differences between the two methods occur, and the highest discrepancies appear for $\delta_s < 0$ (Fig. 14c–d).

The performance of the proposed procedure to design tapered laterals on slopes is quantitatively represented in Table 5, where the relative errors, REs , of the characteristic pressure heads, at the distal end of the drip lateral, h_{*d} ($j = 1$), at the minimum of sub-lateral 1, $h_{*min,1}$ ($j = j_{min,1}$), at the changing point, h_{*n} ($j = N_1 + 1$), at the minimum of sub-lateral 2, $h_{*min,2}$ ($j = j_{min,2}$), and at the inlet, h_{*in} ($j = N + 1$) are reported. REs were less than 0.81 %, with the highest values for $\delta_s > 0$, as observed in Fig. 14ab and, in any case, sufficiently low to consider the procedure presented here suitable for practical application.

Table 5 also reports the uniformity parameters of the tapered laterals, the coefficient of variation of the pressure heads:

$$CV = \frac{\sigma}{\mu} = \frac{\sqrt{\frac{1}{N} \sum_{j=1}^N \left(h_{*j} - \frac{1}{N} \sum_{j=1}^N h_{*j} \right)^2}}{\frac{1}{N} \sum_{j=1}^N h_{*j}} \quad (68)$$

with σ the standard deviation and μ the mean pressure head, and the emission uniformity coefficient of Keller and Karmeli (1975):

$$EU = 100 \left(1 - 1.25 CVT \right) \frac{q_{min}}{q_{max}} \quad (69)$$

which is commonly used for evaluating the performance of drip irrigation systems. In Eq. (69), CVT is the coefficient of the manufacturing variation, which was neglected in the present study, without affecting the results relevant to the

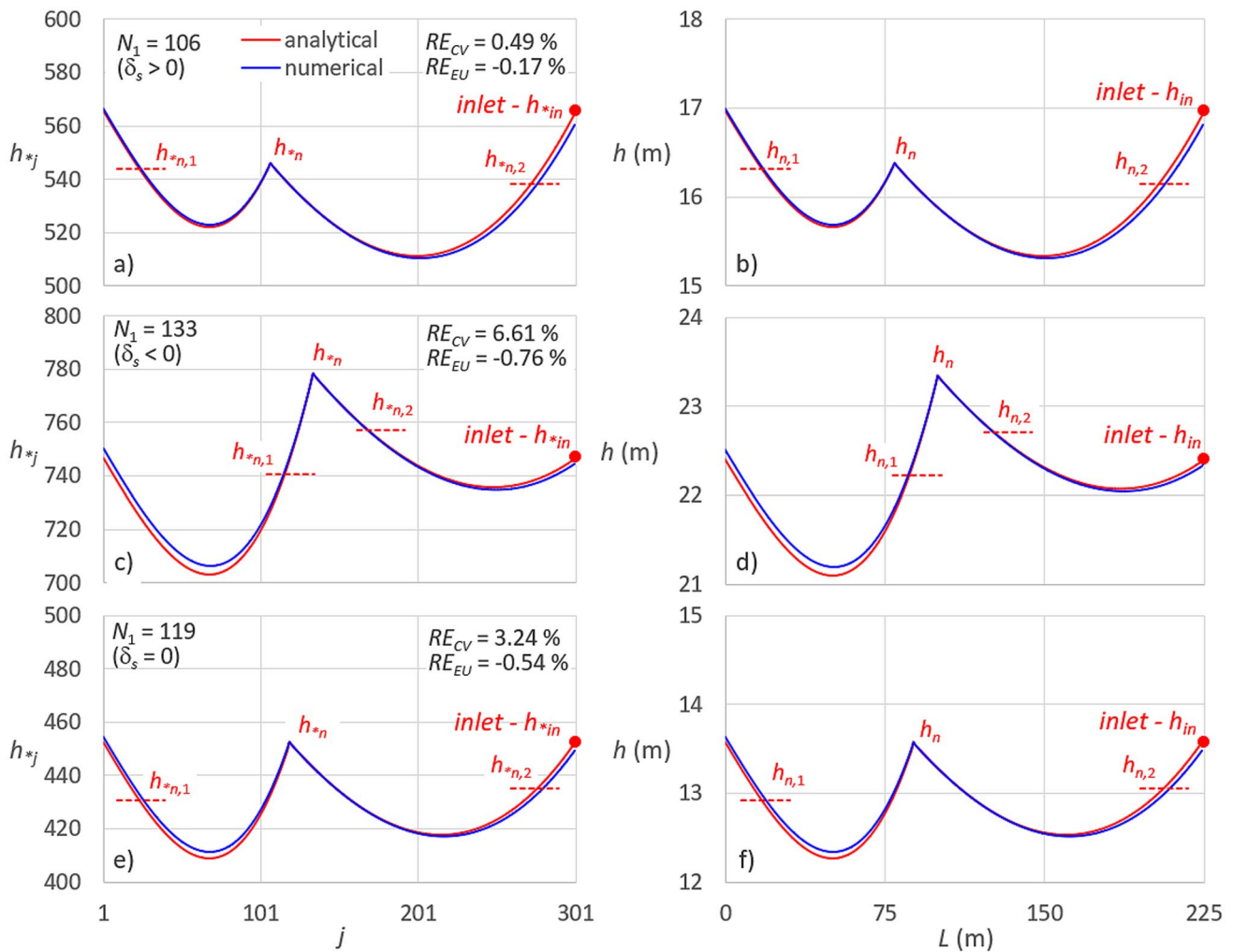


Fig. 14 Under the Hazen-Williams resistance equation, comparison between the PHDs obtained using the proposed analytical procedure for designing tapered laterals on sloping fields and those computed

using the exact step-by-step, SBS, procedure, in terms of dimensionless and dimensional pressure heads, for $\delta_s > 0$ (a,b), $\delta_s < 0$ (c,d), and $\delta_s = 0$ (e,f)

Table 5 Relative errors of the characteristic pressure heads and parameters of tapered lateral performance, under the Hazen-Williams resistance equation

Parameter	$\delta_s > 0$	$\delta_s < 0$	$\delta_s = 0$	Equation
$RE(h_{*d})$ (%)	-0.13	-0.48	-0.49	—
$RE(h_{*min,1})$ (%)	-0.15	-0.46	-0.57	—
$RE(h_{*n})$ (%)	0.00	0.00	0.00	—
$RE(h_{*min,2})$ (%)	0.17	0.13	0.15	—
$RE(h_{*in})$ (%)	0.81	0.24	0.49	—
CV_{an}	0.0265	0.0262	0.0277	Eq. (68)
EU_{an}	98.35	97.82	97.92	Eq. (69)
CV_{num}	0.0270	0.0422	0.0267	Eq. (68)
EU_{num}	98.53	97.62	98.45	Eq. (69)
$RE(CV)$ (%)	0.49	6.61	3.24	—
$RE(EU)$ (%)	-0.17	-0.76	-0.54	—

objective of this study. The corresponding parameters associated with the SBS method and the REs are also indicated.

Table 5 reports the coefficient of variation, CV , and the emission uniformity coefficient of Keller and Karmeli (1975), EU , calculated under the assumption of uniform and varying emitter flow rate (SBS method), showing negligible errors, as already demonstrated, especially for low values of the exponent x of the emitter flow relation (Baiamonte and Palermo 2025b).

The agreement between analytical and numerical solutions is further confirmed through relative errors in the performance indicators. Except for $RE(CV)$ in the case of $\delta_s < 0$, which reaches 6.61 %, differences in CV and EU remain, in absolute value, below 3.24 % and 0.76 %, respectively. These results demonstrate that the simplified analytical procedure preserves hydraulic performance predictions with good reliability, especially if we consider the potential

PHD variability due to *CVT*, which in this study was not considered.

Conclusions

In this study, a fully analytical design framework for dual-diameter tapered drip laterals installed on sloping agricultural fields, is developed. The proposed methodology is based on a normalised pressure head formulation, which enables explicit and analytical derivation of pressure head distributions, including the pressure head tolerance criteria to limit emitter flow-rate variability across the entire lateral, without the need for iterative numerical calculations and numerous attempts.

A novel pressure head tolerance (δ_s) defined at the diameter-change point (*DCP*) was introduced, identifying three distinct pressure head distribution patterns and establishing a rigorous design criterion for determining both *DCP* and the upstream diameter. This formulation facilitates the practical selection of commercially available pipe diameters while preserving hydraulic feasibility and performance constraints.

The proposed method was validated against the exact step-by-step (SBS) numerical method that relaxes the uniform emitter discharge assumption. Relative errors in characteristic pressure heads remained below 0.81 %, and deviations in performance indicators (*CV* and *EU*) were negligible for practical design purposes. These results confirm the reliability and robustness of the simplified analytical approach.

Application of the methodology to representative case studies demonstrated that tapered laterals can significantly reduce the required inlet pressure head compared with traditional one-diameter configurations. In the examined scenario, the tapered design reduced the required inlet pressure head by more than 60% (for $\delta_s = 0$), implying substantial energy savings and lower pumping requirements. In addition, because a significant portion of the lateral can operate effectively with a smaller pipe diameter than in the one-diameter configuration, total material consumption decreases, offering meaningful reductions in both investment and installation costs.

No specific limitations were identified for the proposed approach. However, its application is particularly recommended for long sloping laterals with a large number of emitters. In such cases, the energy savings achieved are more likely to justify the additional cost of installing connection adapters at the *DCP*. Conversely, for short laterals, the installation cost of the connection adapters may outweigh the potential energy savings, making the approach less economically advantageous.

As a final remark, the results of this study demonstrate that tapered laterals should be adopted more widely than is currently the case in practice, owing to their clear hydraulic and energy-efficiency advantages.

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Declarations

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