Dynamic measurement of the elastic constant of an helicoidal spring by a smartphone

Aurelio Agliolo Gallitto, Onofrio Rosario Battaglia, and Claudio Fazio

Department of Physics and Chemistry - Emilio Segrè, University of Palermo, Palermo, Italy

Abstract. We describe an educational activity that can be done by using smartphones to collect data in physics experiments aimed to measure the oscillating period of a spring-mass system and the elastic constant of the helicoidal spring by the dynamic method. Results for the oscillating period and for the elastic constant of the spring agree very well with measurements obtained by different methods. We also discuss the error analysis that can be done in an introductory physics laboratory at undergraduate level.

1. Introduction

In the recent literature, an increasing number of papers are devoted to the use of smartphones as laboratory tools for school physics experiments [1, 2, 3, 4, 5], since they are usually equipped with several sensors that, controlled by appropriate software (app), allows one to perform accurate measurements [6, 7, 8, 9, 10, 11, 12, 13]. In this article, we describe an educational activity that can be carried out by using smartphones to collect data in physics experiments aimed to measure the oscillating period of a spring-mass system and the elastic constant of the helicoidal spring by the dynamic method. For the estimation of these parameters, we perform a graphical analysis of the experimental data [14]. This allows one to discuss a simple error analysis that can be carried out in an introductory physics laboratory at undergraduate level [14, 15].

2. Use of smartphones in physics experiments

To explore physical phenomena, considering the difficulties to perform experiments in physics laboratories, one may resort to the new technologies and in particular to the use of smartphones, which are equipped with several different sensors and then can be used to perform different measurements of physical quantities [6, 7, 8, 9, 10, 11, 12, 13]. This opens the possibility of designing and developing versatile low-cost laboratories, where expensive devices can be replaced by smartphones.

The measurement of the spring constant by using a vertical spring-mass oscillator is a standard school-laboratory activity. Traditionally, the period of oscillations of the spring-mass system is measured with a stopwatch by hand. In the present work, we use the smartphone accelerometer sensor (accelerometer) controlled by the free Phyphox app [16, 17].

The accelerometer housed in the smartphone is usually a micro-electro-mechanical system (MEMS), which processes the mechanical system into electrical information. A more detailed description of the accelerometer, from an educational point of view, is given in Ref. [8]. The accelerometer measures the gravitational acceleration as a force acting on a sample mass, therefore it will report the values of the gravity acceleration g while the smartphone is at rest. Actually, the acceleration is zero when the smartphone is at rest (or moving at a constant speed). To report an acceleration of zero value while the smartphone is not accelerated, the Phyphox app has a specific function called "Acceleration without g", which uses a virtual sensor that subtracts the constant acceleration g (usually by taking into account the data from other sensors as well). Furthermore, this app allows one also to remotely control and observe real-time experimental data from any second network-connected device [17].

3. Experimental setup

In the experiments we propose here, to collect experimental data we use a smartphone with the Phyphox app for the measurement of the period of oscillation of the spring-mass system with different masses, from which we determine the elastic constant of the helicoidal spring. A photo of the experimental setup is shown in Figure 1. The smartphone is hanged to a helicoidal spring that can oscillate vertically almost freely. The smartphone is fixed to home-made expanded-PVC support, having a total mass of 175 g. The mass of the oscillator can be changed by adding weights, thus changing the frequency of oscillation.

The Phyphox app reports the three components of the smartphone's acceleration at regularly spaced time intervals of about 0.01 s. As shown in Figure 1, the oscillations take place along the vertical y-axis. Moreover, the values of the acceleration along the x- and z-axis remains very close to zero. The app also allows one to save the output raw data into a file, from which the data can be used for further analysis. The technical characteristics of the smartphone used in the experiment provide us with an estimation of the uncertainty of measurements of the acceleration and time $\delta_a = 0.010$ m/s^2 and $\delta_t = 0.02$ s, respectively.

4. Harmonic oscillations and data analysis

Figure 2 shows a detail of the screenshot of the Phyphox app during a nearly-free harmonic motion, with negligible friction. The value of the oscillation period, T, is determined by measuring the time of ten oscillations and dividing it by ten, in order to

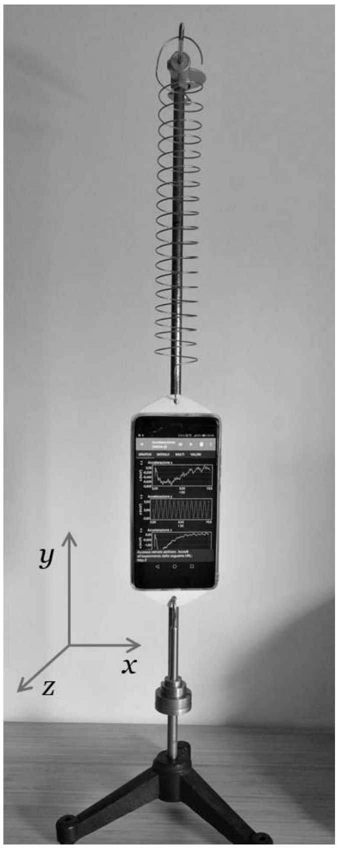

Figure 1. Experimental setup: the smartphone is hanged to an helicoidal spring on a vertical shaft. In the figure, the orientation of axes with respect to the smartphone is shown. The total mass of the system can be changed by adding weights.

reduce the instrumental uncertainty. One could repeat the measurements several times in order to collect enough data to conduct a reliable statistical analysis.

In the free harmonic motion of a spring-mass system of elastic constant k and mass M, the position of the mass varies as a function of time as a sinusoidal function [18]

$$y(t) = A\sin(\omega t + \phi) \tag{1}$$

where A is the amplitude of the oscillations, ϕ the phase constant, and ω the angular frequency that depends on k and M as

$$\omega = \sqrt{k/M} \tag{2}$$

From Eq. (1), one obtains the acceleration as

$$a(t) = \frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} = -\omega^2 A\sin(\omega t + \phi) \tag{3}$$

where $\omega^2 A$ describes the acceleration amplitude.
Since the acceleration, in the harmonic motion, varies periodically with time, one can

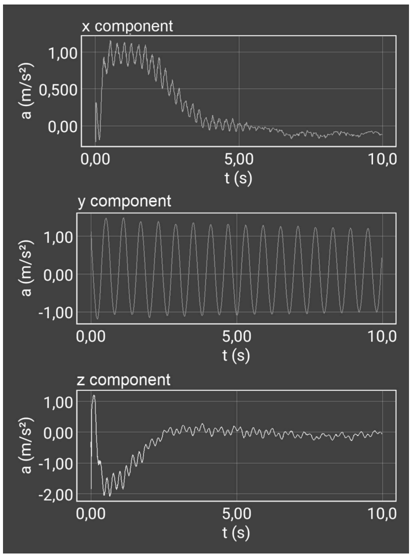

Figure 2. A detail of the screenshot of the Phyphox app, which shows the acceleration of the mass against time, during the harmonic oscillations. Since the oscillations take place along the y-axis, the values of the acceleration along the x- and z-axis, during the experiment, remains very close to zero.

measure the period of oscillation T from $a(t)$. Once the values of T and ω have been obtained by using different masses, one can easily determine the spring constant by the dynamic method. One can linearize Eq. (2) by plotting ω^2 against $1/M$, obtaining a linear graph with a slope equal to the spring constant that can be determined by numerically fitting the experimental data. At an undergraduate level, it may be also interesting to plot ω against M in a log-log graph, in which both axes use logarithmic scales. When a variable changes as $y = Cx^n$, a log-log graph shows this relationship as a straight line, such that $\log(C)$ is the constant and n is the slope. Equivalently, the linear function is $\log(y) = \log(C) + n\log(x)$. Therefore, from the graph one can determined both C and n. Figure 3 shows the values of ω against M in a log-log graph. It follows a power law with $n = -0.5$ as it is given by Eq. (2).

The values of the elastic constant of the spring are determined by drawing two straight lines, with a fixed slope of -0.5, through the data points, as illustrated in Figure 3. The best value of k and its uncertainty are estimated by considering extremes of maximum and minimum constant values, respectively k_{max} and k_{min}, that conceivably fit the experimental data. The continuous red line in Figure 3 has a fixed slope of -0.5 and $k_{min} = 19.15$ N/m. The dashed blue line has a fixed slope of -0.5 and $k_{max} = 19.55$ N/m. From these values, one obtains $k = (19.4 \pm 0.2)$ N/m, with a relative uncertainty of about 1%.

We also carried out separate measurements of the elastic constant of the spring, with the static method, by hanging different masses on the spring and measuring its equilibrium length as in Hooke's law. The applied force has been determined by multiplying the sample mass times the gravity acceleration g (the local value of g is

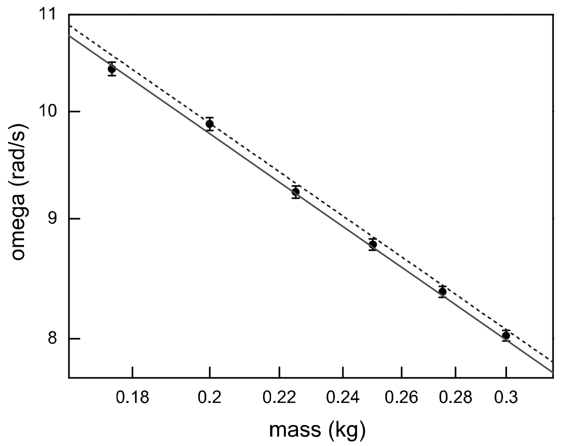

Figure 3. log-log graph of ω vs. M. The continuous red line is a graph of Eq. (2) with a fixed slope of -0.5 and $k = k_{min} = 19.15$ N/m. The dashed blue line is a graph of Eq. (2) with a fixed slope of -0.5 and $k = k_{max} = 19.55$ N/m. From these values one obtains $k = (19.4 \pm 0.2)$ N/m.

9.801 m/s^2) [19]. The spring length has been measured as the distance from the fixed end of the spring on the shaft and the other end of the spring, with an uncertainty of about 2 mm. In this case, the unstretched length of spring, l_0, is unknown and it is determined by data analysis. Figure 4 shows the values of force against spring length in a linear graph. From the slope of the straight lines plotted in the graph one obtains $k_{min} = 18.8$ N/m and $k_{max} = 20.2$ N/m, which give $k = (19.5 \pm 0.7)$ N/m, with a relative uncertainty of about 3.6%. In the graph, the length of the unstretched spring corresponds to the length value at which the external force is zero and it results $l_0 = (128 \pm 4)$ mm.

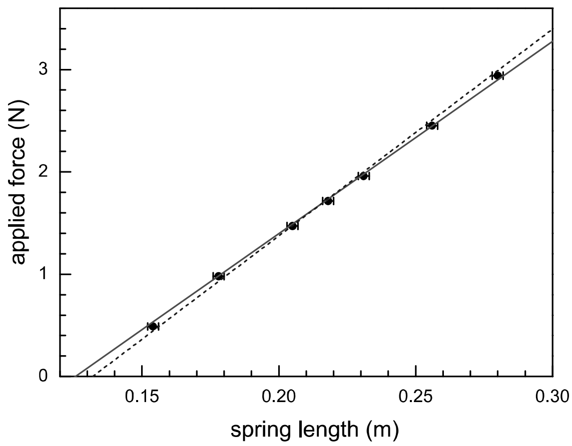

Figure 4. Force against spring length. The continuous red lines is a straight line of slope $k_{min} = 18.8$ N/m. The dashed blue line is a straight line of slope $k_{max} = 20.2$ N/m. From these values one obtains $k = (19.5 \pm 0.7)$ N/m.

From a comparison of the two obtained results for k, we may infer that the measurements done by the dynamic method give more accurate results than those performed with the static method. It is worth nothing that, with the simple graphical method of error analysis we followed, the uncertainty of the measured quantities is quickly obtained by the straight-line graphs, without long mathematical treatments. This activity allows students to discuss about a simple error analysis and acquire the ability to perform some elementary error analysis as part of a laboratory activity at an undergraduate level.

5. Discussion

For a more appropriate analysis of the experimental data, the mass of the spring, m, should be taken into account [20, 21]. This can be done by considering an effective mass, m_0, as the mass that must be added to the mass of the object M to correctly predict the behavior of the system. During the oscillations, when the object M passes from the equilibrium position, in which the spring is not deformed, M will have a velocity v_M. On the other hand, spring segments will have a velocity that depends on their position: the segment attached to M will have the same velocity v_M as the mass M, the segment attached to the wall will be at rest. In the simplest case, assuming a linear distribution of the velocities of the spring segments along the spring itself, it results $m_0 = m/3$. This result allows us to neglect the effective mass of the spring in the analysis of experimental data, since $m_0 = m/3 \approx 5.5$ g. However, by taking into account the mass of the spring, the value of k would vary within the experimental uncertainty.

The apparatus we used in the experiments allows one also to qualitatively study the oscillations of coupled modes of the system, since a spring of proper length makes it also a pendulum that can oscillate with a proper frequency [22, 23, 6, 24]. This system can oscillate along the vertical axis (spring mode) and in the horizontal plane (pendulum mode). A resonance phenomenon occurs when the elastic frequency (spring-type oscillation) is about two times greater than the pendular frequency (pendulum-type oscillation) [22, 23], that is

$$\sqrt{k/M_{eq}} = 2\sqrt{g/l_{eq}} \tag{4}$$

where M_{eq} is the equivalent mass of the spring-mass system and l_{eq} is the equivalent length of the pendulum.

In this case, one can observe both modes of oscillation. As in slightly coupled oscillators, energy initially stored in the compressed spring, as time elapses, trades back and forth between the vertical and horizontal oscillations. In Figure 5, an example of such oscillations is reported. These results have been obtained with a spring of elastic constant $k \approx 20$ N/m, at which is hanged a total mass of about 225 g, composing a pendulum of an effective length of about 0.45 m. Figure 6 shows the oscillations in a narrower time interval, from about 23 s to 27 s. From the graph, it is possible to obtain qualitatively the values of periods and phases of the three oscillating modes: $T_x \approx T_z \approx 2T_y \approx 1.3$ s, at which corresponds an angular frequency $\omega_x \approx \omega_z \approx 0.5\omega_y \approx 4.8$ rad/s. Here, we will not discuss about spring pendulum, or coupled oscillators, because it goes beyond the aim of the present paper.

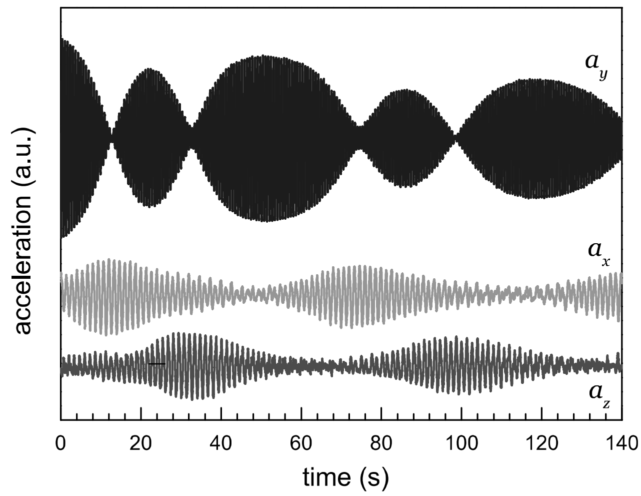

Figure 5. Acceleration against time. The y-axis indicates the vertical direction, whereas the x-axis and the z-axis lie on the horizontal plane. The curves $a_x(t)$ and $a_y(t)$ have been shifted vertically for clarity.

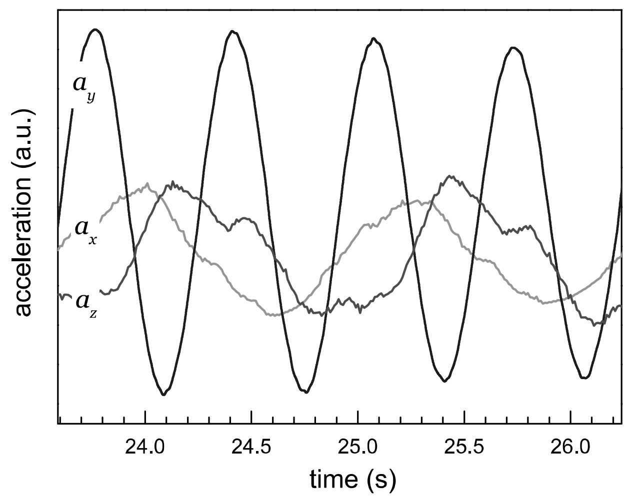

Figure 6. Oscillations in the time interval from about 23 s to 27 s. From the graph it is possible to obtain qualitatively the values of periods and phases of the three oscillating modes: $T_x \approx T_z \approx 2T_y \approx 1.3$ s, at which corresponds an angular frequency $\omega_x \approx \omega_z \approx 0.5\omega_y \approx 4.8$ rad/s.

The use of smartphones in physics experiment is considerably increasing, providing a new way of teaching and learning, reinforced by the ample availability of smartphones among students. The development of smartphone apps aims to complement the traditional learning and to help students to learn anytime and anywhere. Smartphones make this goal easy to achieve since they combine multiple features and at the same time give more mobility to the students for distance learning [5, 25, 26]. Students' perceptions regarding the capabilities of smartphones and apps for improving learning processes in university subjects indicate that the use of apps is highly valued by students as a new format which both supports and enhances learning practices, also fostering collaborative work among students and teachers [25]. On the other hand, the use of smartphones as measurement devices in physics experiments opens the possibility of designing and developing versatile low-cost laboratories, also for distance learning. This different approach certainly contributes to students' learning of science concepts and stimulates the interest of students towards physics [4]. Furthermore, it is worth noting that performing physics experiments by using smartphones, as a laboratory tool, may foster students' active learning [27], enhancing engagement and interaction during lessons [28, 26] and communication in lectures [29]. Indeed, the interest of students in smartphones, as well as their expertise to use such devices, can result in a powerful tool to bolster active learning and reinforce their interest in learning scientific issues [28, 4, 30]. It is worth noting that these very same experiments could easily be used for distance/remote learning, lecture experiments or traditional school laboratory activities, at a very low cost. Furthermore, we believe that allowing students to perform experiments in a different context than in a school laboratory, related to real-time situations, can be a powerful way of contextualizing physics concepts.

6. Conclusion

We have described an educational activity that can be done by using smartphones to collect data in physics experiments aimed to measure the oscillating period of a spring-mass system and the elastic constant of the helicoidal spring by the dynamic method. Results for the oscillating period and elastic constant of the spring agree very well with measurements obtained by different methods. Furthermore, with the simple graphical method of error analysis we followed, the uncertainty of the measured quantities is simply obtained by straight-line graphs. This simple error analysis can be done in an introductory physics laboratory, also allowing undergraduate students to acquire the ability to perform some elementary error analysis. The success of these measurements demonstrates the feasibility of using a smartphone acceleration sensor in introductory physics laboratory at an undergraduate level.

References

[1] Hammond E C and Assefa M (2007). Cell phones in the classroom. Phys Teach **45**, 312–312. DOI: 10.1119/1.2731283

[2] Kuhn J and Vogt P (2013). Applications and Examples of Experiments with Mobile Phones and Smartphones in Physics Lessons. Frontiers in Sensors **1**, 67–73

[3] Oprea M and Miron C (2014). Mobile phones in the modern teaching of physics. Romanian Reports in Physics **66**, 1236–1252.

[4] Gonzalez M A, Gonzalez Manuel A, Martin M E, Llamas C, Martinez O, Vegas J, Herguedas M, and Hernandez C (2015). Teaching and Learning Physics with Smartphones. Journal of Cases on Information Technology **17**, 31–50. DOI: 10.4018/JCIT.2015010103

[5] Wright K (2020). Smartphone Physics on the Rise. Physics **13**, 68–68. URL: physics.aps.org/articles/v13/68.

[6] Kuhn J and Vogt P (2012). Analyzing spring pendulum phenomena with a smart-phone acceleration sensor. Phys Teach **50**, 504–505. DOI: 10.1119/1.4758162

[7] Castro-Palacio J C, Velázquez-Abad L, Giménez M H, and Monsoriu J A (2013). Using a mobile phone acceleration sensor in physics experiments on free and damped harmonic oscillations. Am J Phys **81**, 472–475. DOI: 10.1119/1.4793438

[8] Agliolo Gallitto A and Lupo L (2015). A mechanical model of the smartphone's accelerometer. Phys Educ **50**, 646–547. DOI: 10.1088/0031-9120/50/6/646

[9] Pili U (2018). A dynamic-based measurement of a spring constant with a smartphone light sensor. Phys Educ **53**, 033002–3. DOI: 10.1088/1361-6552/aaa927

[10] Agliolo Gallitto A, Battaglia O R, Cavallaro G, Lazzara G, Lisuzzo L, and Fazio C (2020). Exploring historical scientific instruments by using mobile media devices. Phys Teach, submitted

[11] Pierratos T and Polatoglou H M (2020). Utilizing the phyphox app for measuring kinematics variables with a smartphone. Phys Educ **55**, 025019–6. DOI: 10.1088/1361-6552/ab6951

[12] Listiaji P, Darmawan M S, Fitri Daeni and Karmuji (2021). Comparison between the use of acceleration sensor and video tracker on smartphone for spring oscillation experiment. Phys Educ **56**, 013001–5. DOI: 10.1088/1361-6552/abb88b

[13] Organtini G (2021). *Physics Experiments with Arduino and Smartphones.* Springer. ISBN: 978-3-030-65140-4

[14] Agliolo Gallitto A (2016). *Introduzione al Laboratorio di Fisica: gli errori nelle misure sperimentali.* University of Palermo. ISBN: 978-88-941245-9-0

[15] Deacon C G (1992). Error analysis in the introductory physics laboratory. Phys Teach **30**, 368–371. DOI: 10.1119/1.2343578

[16] The Phyphox app was developed by Sebastian Staacks in 2016 at the RWTH Aachen University, Germany. Phyphox Web Site URL: www.phyphox.com

[17] Staacks S, Hütz S, Heinke H, and Stampfer C (2018). Advanced tools for smartphone-based experiments: phyphox. Phys Educ **53**, 045009–6. DOI: 10.1088/1361-6552/aac05e

[18] Serway R A and Jewett J W (2018). *Physics for Scientists and Engineers with Modern Physics.* Cengage Learning, 10th ed. ISBN: 978-1337553292

[19] Chamber of Commerce, Industry, Crafts and Agricolture of Bolzano (Italy) Web Site, DM n. 1326557 19/05/1999, URL: www.handelskammer.bz.it/en/node/5314

[20] E Galloni and M Kohen (1979). Influence of the mass of the spring on its static and dynamic effects. Am J Phys **47**, 1076–1078. DOI: 10.1119/1.11978

[21] Rodriguez X E R and Gesnouin G A (2007). Effective mass of an oscillating spring. Phys Teach **45**, 100–103. DOI: 10.1119/1.2432087

[22] Lai H M (1984). On the recurrence phenomenon of a resonant spring pendulum. Am J Phys **52**(3), 219–223. DOI: 10.1119/1.13696

[23] Lynch P (2002). Resonant motions of the three-dimensional elastic pendulum. International Journal of Non-Linear Mechanics **37**, 345–367. DOI: 10.1016/S0020-7462(00)00121-9

[24] Ostrander J (2015). Wave function modelling of a pendulum-spring system exhibiting simple harmonic motion. Phys Educ **51**, 015016–6. DOI: 10.1088/0031-9120/51/1/015016

[25] Vázquez-Cano E (2014). Mobile Distance Learning with Smartphones and Apps in Higher Education. Educational Sciences: Theory & Practice **14**, 1505–1520. DOI: 10.12738/estp.2014.4.2012

[26] Shi W Z, Sun J, Xu C, and Huan W (2016). Assessing the Use of Smartphone in the University General Physics Laboratory. Eurasia Journal of Mathematics, Science & Technology Education

12(1), 125–132

[27] Lindquist D, Denning T, Kelly M, Malani R, Griswold WG, and Simon B (2007). Exploring the potential of mobile phones for active learning in the classroom. ACM SIGCSE Bulletin **39**, 384–388

[28] Ayu M A, Taylor K, and Mantoro T (2009). Active learning: Engaging students in the classroom using mobile phones. Industrial Electronics & Applications, ISIEA 2009. IEEE Symposium, pp. 711–715.

[29] Scornavacca E, Huff S, and Marshall S (2009). Mobile phones in the classroom: If you can't beat them, join them: Communications of the ACM **52**, 142–146

[30] Hochberg K, Kuhn J, and Müller A (2018). Using Smartphones as Experimental Tools - Effects on Interest, Curiosity, and Learning in Physics Education. Journal of Science Education and Technology **27**, 385–403. DOI: 10.1007/s10956-018-9731-7