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Detection of spatio-temporal local structure on seismic data

Individuazione di strutture locali spazio-temporali su dati sismici

Marianna Siino, Francisco J. Rodríguez-Cortés, Jorge Mateu and Giada Adelfio

Abstract For the description of the seismicity of an area, the comparison between local features of background and induced events could be a new perspective of research. In spatio-temporal point process, local second-order statistics provide information on the relationships of each event and its nearby events. In this paper, we use a test based on local indicators of spatio-temporal association (LISTA functions) for identifying different local structures comparing the two previous sets of events. We present a simulation study on the test and show the main results of the application on Greece earthquake data.

Abstract *In questo lavoro si propone una nuova prospettiva di analisi per la descrizione della sismicità di un'area considerando il confronto delle caratteristiche locali degli evendi di fondo e quelli indotti. Nell'ambito dell'analisi dei processi puntuali di tipo spazio-temporale, le statistiche del secondo ordine locali descrivono la relazioni esistenti tra ciascun evento e i suoi più vicini. Per identificare differenze tra i due insiemi di eventi individuati in precedenza, utilizziamo un test basato sugli indicatori locali di associazione spazio-temporale, chimati funzioni LISTA. Presentiamo uno studio di simulazione e i principali risultati applicando la metodologia sui dati sismici della Grecia.*

Key words: earthquakes; local indicators of spatio-temporal association; second-order product density function

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1 Introduction

In an observed area, earthquake events can be considered as a realization of a marked space-time point process, where the magnitude is the mark, and a point is identified by its geographical coordinates and time of occurrence (Illian et al, 2008). Generally, the description of seismic events requires the definition of more complex models than stationary Poisson process since clustering structure characterises these events. Therefore, spatio-temporal cluster analysis has a relevant role in the comprehension of seismic processes.

Commonly, global spatio-temporal second-order summary statistics (such as the K- and pair-correlation functions) are used to detect deviations from the Poisson assumption (Gabriel and Diggle, 2009). These tools play a fundamental role in the phase of descriptive analysis, in model validation and for testing procedures giving global information of a given point pattern. An interesting question may concern if the same conclusions are valid locally, and thus, for example, in testing procedures if in subregions of the spatio-temporal window the pattern behaves differently identifying specific regions where the null hypothesis is not accepted.

Anselin (1995) proposed the idea of considering individual contributions of a global estimator as a measure of clustering under the name of Local Indicators of Spatial Association (LISA). In spatial point processes, Cressie and Collins (2001) propose a local product density function developing theoretical properties, namely first- and second-order moments, of these functions. Some applications of LISA functions are in Mateu et al (2007) and Moraga and Montes (2011). Rodríguez-Cortés (2014) and Siino et al (2016b) extend the concept of LISA function to the spatio-temporal point pattern context defining the LISTA functions. A brief summary on this methodology is in Section 2. Moreover, Siino et al (2016b) develop a testing procedure for the local structure comparing spatio-temporal point patterns based on Local indicators of spatio-temporal association (LISTA) functions described in Section 3. A simulation study is performed to illustrate that the test proposed has the prescribed size (Section 4). For the analysis in Section 5, we consider earthquakes occurred in the Hellenic area between 2005 and 2014. We aim to detect which triggered events have a significant different local cluster structure with respect to the underlying process, represented by the background events, linking the results with the geological information available in the study area.

2 Methodology

We consider a spatio-temporal point process with no multiple points as a random countable subset $\mathscr{X}$ of $\mathbb{R}^2 \times \mathbb{R}$, where for a point $(\mathbf{u},t) \in \mathscr{X}$, $\mathbf{u} \in \mathbb{R}^2$ is the spatial location and $t \in \mathbb{R}$ is the time of occurrence. In practice, an observed spatio-temporal pattern is a finite set $\{(\mathbf{u}_i,t_i)\}_{i=1}^n$ of distinct points within a bounded spatio-temporal region $W \times T \subset \mathbb{R}^2 \times \mathbb{R}$, where usually W is a polygon with area $|W| > 0$ and T a single closed interval with length $|T| > 0$. Considering a bounded spatio-temporal

region $A \subset W \times T$, $Y(A)$ denotes the number of the events of the process falling in A. The intensity of a process is defined as (Diggle, 2013)

$$\rho(\mathbf{u},t) = \lim_{|\mathrm{d}\mathbf{u}\times \mathrm{d}t|\to 0} \frac{\mathbb{E}[Y(\mathrm{d}\mathbf{u}\times \mathrm{d}t)]}{|\mathrm{d}\mathbf{u}\times \mathrm{d}t|}$$

where $\mathrm{d}\mathbf{u}\times \mathrm{d}t$ is a spatio-temporal region around the point $(\mathbf{u},t)$, $|\mathrm{d}\mathbf{u}|$ is the area of the spatial region, $|\mathrm{d}t|$ is the length of the time interval and, $\mathbb{E}(Y(\mathrm{d}\mathbf{u},\mathrm{d}t))$ denotes the expected number of events in the infinitesimal spatio-temporal region. The process is called homogeneous or stationary when the intensity is constant, $\rho(\mathbf{u},t) = \rho$ for all $(\mathbf{u},t) \in W \times T$.

When the interest is in describing the spatio-temporal variability and correlations between points of a pattern, we have to consider second-order measures, such as the product density $\rho^{(2)}(\cdot,\cdot)$. This quantity provides an interpretable measure of the spatio-temporal dependence structure and it is defined as

$$\rho^{(2)}((\mathbf{u}_i,t_i);(\mathbf{u}_j,t_j)) = \lim_{|\mathrm{d}\mathbf{u}_i\times \mathrm{d}t_i||\mathrm{d}\mathbf{u}_j\times \mathrm{d}t_j|\to 0} \frac{\mathbb{E}[Y(\mathrm{d}\mathbf{u}_i\times \mathrm{d}t_i)Y(\mathrm{d}\mathbf{u}_j\times \mathrm{d}t_j)}{|\mathrm{d}\mathbf{u}_i\times \mathrm{d}t_i||\mathrm{d}\mathbf{u}_j\times \mathrm{d}t_j|} \tag{1}$$

where $\mathrm{d}\mathbf{u}_i\times \mathrm{d}t_i$ and $\mathrm{d}\mathbf{u}_j\times \mathrm{d}t_j$ are small cylinders around two distinct points $(\mathbf{u}_i,t_i)$ and $(\mathbf{u}_j,t_j)$.

Under the stationary case, and ignoring edge-effects, a global naive non-parametric kernel estimator for $\rho^{(2)}(r,h)$ in (1) (Rodríguez-Cortés, 2014) is given by

$$\widehat{\rho^{(2)}}_{\varepsilon,\delta}(r,h) = \frac{1}{4\pi r|B|}\sum_{i=1}^{n}\sum_{j\neq i}\kappa_{\varepsilon,\delta}(\|\mathbf{u}_i-\mathbf{u}_j\|-r,|t_i-t_j|-h), \tag{2}$$

where the sum is over all pairs $(\mathbf{u}_i,t_i) \neq (\mathbf{u}_j,t_j)$ of the data points, $B = W \times T$, $r > \varepsilon > 0$ and $h > \delta > 0$. The kernel function κ has a multiplicative form $\kappa_{\varepsilon\delta}(\|\mathbf{u}_i-\mathbf{u}_j\|-r,|t_i-t_j|-h) = \kappa_{1\varepsilon}(\|\mathbf{u}_i-\mathbf{u}_j\|-r)\,\kappa_{2\delta}(|t_i-t_j|-h)$, where $\kappa_{1\varepsilon}$ and $\kappa_{2\delta}$ are kernel functions with bandwidths ε and δ, respectively. For an approximately unbiased edge-corrected estimator for the spatio-temporal product density see Rodríguez-Cortés (2014). The R package `stpp` (Gabriel et al, 2013) implements the main code for the computation of the the estimator in (2).

Considering the spatio-temporal product density in (1), its local version is denoted by $\rho^{(2)i}(\cdot,\cdot)$. Rodríguez-Cortés (2014) extends the operational definition of local indicator introduced by Anselin (1995), for fixed r and h, it holds that

$$\widehat{\rho^{(2)}}_{\varepsilon,\delta}(r,h) = \frac{1}{n-1}\sum_{i=1}^{n}\widehat{\rho^{(2)i}}_{\varepsilon,\delta}(r,h), \tag{3}$$

An unbiased edge-corrected kernel-based estimator for $\widehat{\rho^{(2)i}}(r,h)$ is given by

$$\widehat{\rho^{(2)i}}_{\varepsilon,\delta}(r,h) = \frac{n-1}{4\pi r|B|}\sum_{j\neq i}\frac{\kappa_{\varepsilon,\delta}(\|\mathbf{u}_i-\mathbf{u}_j\|-r,|t_i-t_j|-h)}{w(\mathbf{u}_i,\mathbf{u}_j)w(t_i,t_j)}, \tag{4}$$

with $r > \varepsilon > 0$, $h > \delta > 0$, for $(\mathbf{u}_i, t_i) \in W \times T$, $i = 1, \ldots, n$, $w(\mathbf{u}_i, \mathbf{u}_j)$ and $w(t_i, t_j)$ are the edge-effect factors. For formal theoretical details on the LISTA functions see Rodríguez-Cortés (2014) and for the implementation the function `LISTAfunct` in the GitHub repository `pdLISTA` (Rodríguez-Cortés, 2016).

3 Testing procedure

The test procedure is an extension of the test proposed in Moraga and Montes (2011) into the spatio-temporal context. It detects differences in the local structure of two given point spatio-temporal patterns X and Z. We test the null hypothesis of no difference in the spatio-temporal local structure of X and Z with respect to the i-th point $(\mathbf{u}_i, t_i) \in X$, where the number of points in the two patterns are respectively $N(X) = n$ and $N(Z) = m$. The steps of the testing procedure are the following.

1. For each point $(\mathbf{u}_i, t_i) \in X$, for $i = 1, \ldots, n$ the LISTA function $\widehat{\rho^{(2)i}}_{\varepsilon,\delta}(r,h)$ is estimated.
2. Secondly, for each fixed point $(\mathbf{u}_i, t_i) \in X$, k point patterns are generated under the null hypothesis. For each fixed point $(\mathbf{u}_i, t_i)$, k local spatio-temporal product density surfaces are estimated, $\widehat{\rho^{(2)i_q}}_{\varepsilon,\delta}(r,h)$ for $q = 1, \ldots, k$. They are summarised in terms of the average surface, denoted by $\bar{\rho}^i_{H_0}(r,h)$.
3. Based on the previous quantities, the following statistic is considered

$$T^i = \int_0^{h_0} \int_0^{r_0} \left(\widehat{\rho^{(2)i}}_{\varepsilon,\delta}(r,h) - \bar{\rho}^i_{H_0}(r,h) \right)^2 \mathrm{d}r\,\mathrm{d}h, \tag{5}$$

   were r_0 and h_0 are chosen using the Diggle's rule (Diggle, 2013).
4. The theoretical distribution of our statistics under the null hypothesis is not known, so we rely on simulation-based empirical distributions. Fixing a point $(\mathbf{u}_i, t_i) \in X$, the estimated value of the statistic, is compared with the empirical distribution of the k values of $T^{i_q}_{H_0}$ with $q = 1, \ldots, k$ that are obtained computing the test between the q-th generated LISTA surfaces under the null hypothesis and their sample mean function. The p-value of T^i is the following ratio $p^i = \sum_{q=1}^{k} \mathbf{I}(T^{i_q}_{H_0} \geq T^i)/k$. The null hypothesis is rejected if $p^i \leq \alpha$, where α is the type I error.

4 Simulation study

A simulation study with some scenarios is carried out to assess the performances in terms of type I error of the test introduced in the previous section.

The patterns are generated in the unit cube, $W \times T = [0,1]^2 \times [0,1]$ and varying the type of process (Poisson, Poisson cluster). We also consider $\mathbb{E}[N(W \times T)] = n+m = \{150, 300\}$ for $X \cup Z$. Under the null hypothesis, a pattern is generated with expected number of points equal to $n+m$, and the points are randomly associated to the pattern X or Z such that the number of points for the two sets is equal, and the test is computed for all the points belonging in X. For each point, the number of permutations is equal to $k = 99$.

The spatio-temporal Poisson point patterns are generated using the function `rpp` in the package `stpp` of R. The Poisson cluster processes are simulated using `rpcp` in `stpp`. Given the values of $n+m$ and the dispersion parameters, we control the degree of clustering by changing the expected number of parents ($n_p = \{5, 10\}$) and the number of offspring points with respect to each parent.

For each of the resulting scenarios under the null hypothesis, 100 pairs of patterns of (X, Z) are generated, and the type I error probability is defined as the proportion of points belonging to X for which the null hypothesis is rejected considering a fixed nominal value of α. Table 1 presents the average and the variance of the p-values under H_0 with the rejection rate for $\alpha = 0.05$. The statistical test exhibits acceptable empirical rejection rates for the several scenarios. There are no remarkable differences in the results when changing the intensity, the type of the process and the degree of clustering.

Scenarios		Dispersion	$n+m$	n_p	ε	δ	T_1		
X	Z	parameters					Rej.	Mean	Var
P	P	-	150	-	0.134	0.080	0.057	0.490	0.087
		-	300	-	0.111	0.069	0.055	0.487	0.086
PC	PC	$\{h,r\} = (0.26; 0.13)$	150	5	0.094	0.069	0.053	0.500	0.086
				15	0.114	0.074	0.048	0.493	0.084
			300	5	0.082	0.061	0.047	0.500	0.085
				15	0.095	0.065	0.047	0.511	0.085

Table 1: Rejection rates (Rej.) at $\alpha = 0.05$, the mean and the variance (Var.) of the p-values for the statistic. The spatio-temporal models considered are homogenous Poisson point processes (P)and Poisson cluster point processes (PC). Dispersion parameters for the PC model are given in the table, $n+m$ is the total expected number of points for $X \cup Z$, and n_p is the expected number of parents for the PC model. For each scenario, 100 simulations are considered, ε and δ are the bandwidths in space and time, respectively.

5 Application

In the seismological context, a background event refers to an earthquake that has not been triggered by another and that might be related to changes in the tectonic field. On the other hand, triggered events are thought to have been caused by a pre-

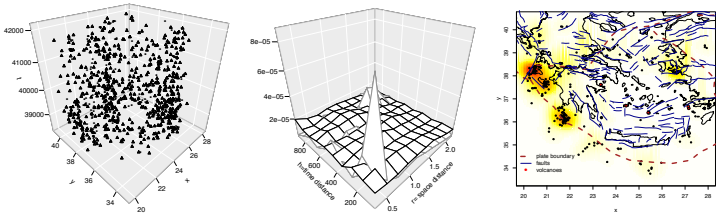
vious earthquake. Globally, these two set of events present different spatio-temporal global interaction structure, however it can be of interest to compare them focusing on a local scale.

In this application, we consider earthquake events occurred in the Greek area between 2005 and 2014 with a magnitude greater than 4 (Figure 1a), for a total number of 1105 events. Its complex spatial multiscale structure has been analysed in Siino et al (2016a).

The earthquakes are classified into background and induced events using a declustering procedure: a probability of being independent events is assigned to each one and it comes from an algorithm for the estimation of Epidemic Type Aftershocks-Sequences (ETAS) model (Ogata, 1988). We fitted the model using the R package `etasFLP` (Chiodi and Adelfio, 2014) based on the method developed in Adelfio and Chiodi (2015). We use the final probabilities provided in the last step of the iteration procedure to classify the events with a magnitude greater than 4 into the two groups, obtaining 580 background events and 525 triggered events (Figure 1a).

Considering the two clusters of independent and induced earthquakes, we would answer the following research questions: Is there a different global structure between the two point patterns? Which triggered events have a significant different local structure with respect to the underlying process (background events)? Is there any geological justification for the identified clusters?

As expected, the estimated spatio-temporal product density of the spontaneous events does not show any particular behaviour. On the other hand, for the induced seismicity, there is a spatio-temporal clustering around at $t < 300$ days and $r < 65$ kilometres, in terms of temporal and spatial distances, respectively (Figure 1b). However, we further aim to detect if we can obtain different conclusions focusing on a local scale detecting the spatio-temporal clusters.



(a) Spatio-temporal background (△) and induced (●) events

(b) Estimated global product density of background and induced events

(c) Spatial density of the significant induced events and the seismic sources

Fig. 1: (1a) Scatterplot of the spatio-temporal earthquake data classified in background events (triangles) and induced events (points) according to the procedure of declustering using the ETAS model. (1b)Estimated global product density for the background events (black surface), and induced ones (grey surface) with bandwidths $\varepsilon = 30.44$ km and $\delta = 44.14$ days. (1c)Image plot in space of the significant induced events and seismogenetic sources.

We apply the testing procedure of Section 3. The point pattern Z is represented by the background events and the events in X are the triggered ones. The representation of the results of the significant points in space allows to interpret them in relation to the geological information available in the study area (Figure 1c). We can identify some areas in which the induced events (with a magnitude greater than 4) are different in terms of spatio-temporal local structure than the background seismicity: islands of Kefalonia and Zakynthos and the Samos area (East Aegean Sea). The different behaviour is due to their specific geological characteristics and, in particular, to a higher fracturaction degree of their seismogenetic volumes. These results confirm our idea that the observed seismicity is generated by a complex model, characterised by spatial-temporal interaction, with events happening at several scales, and with spatial inhomogeneity related to the geological information available in the study area.

6 Final remarks

We deal with a non-parametric testing approach for spatio-temporal point processes, in order to compare the local structure of two spatio-temporal point patterns (say X and Z). The used statistic leads to approximately valid test and the results in terms of type I error are reasonably good. Using the aforementioned test, we compare background and induced seismicity with a magnitude greater than 4 in the Greek area. It seems that the sequences of events that are strongly different to the underlying process, are placed in specific regions of the study window.

As a possible future development, we may consider further simulation scenarios to assess the power of the test. Moreover, it could be interesting to define other local tests based on the LISTA surfaces, changing what is postulated under the null hypothesis. With the analysis of the LISTA surfaces for a given point pattern, we can explore how individual points are related to their neighbouring events, clustering surfaces in order to classify points with similar spatio-temporal local structure. Moreover, we could develop a diagnostic tool based on the LISTA functions computing a weighted version of them by the inverse of the intensity function, looking for points with a more relevant contribution to the global summary statistics.

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