Operational Methods in Fractional Dynamics

Nov. 6-10, 2016, Cracow, Poland

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	Sunday (6/11/2016)	**Monday (7/11/2016)**	**Tuesday (8/11/2016)**	**Wednesday (9/11/2016)**	**Thursday (10/11/2016)**
9:30-10:30		Registration	Registration		
10:00-10:30	Arriving day	Coffee break (9:45-10:15)	Coffee break	Coffee break	Departure day
10:30-11:00		Aleksei V. CHECHKIN	Rudolf HILFER	Paulius MISKINI	
11:00-11:30		Gianni PAGNINI	Svetlana MINCHEVA-KAMINSKA	Piotr WEBER	
11:30-12:00		Tomasz SROKOWSKI	Tibor K. POGANY	Silvia LICCIARDI	
12:00-14:30		Lunch	Lunch	Lunch	
14:30-15:00		Tadeusz KOSZTOŁOWICZ	Francesco MAINARDI	Dorota JAKUBCZYK	
15:00-15:30		Yuri POVSTENKO	Roberto GARRAPPA	Mirosłw ŁABUZ	
15:30-16:00		Coffee break	Coffee break		
16:00-16:30		Bartłomiej DYBIEC	Ralf METZLER		
16:30-17:00		Ewa GUDOWSKA-NOWAK	Ercılia SOUSA		
17:00-19:00		Dinner	Dinner		
19:00-21:00	Welcome Party			Conference Dinner	

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Natural and Modified Forms of Distributed Order Fractional Diffusion Equations

Abstract

We consider diffusion-like equations with time and space fractional derivatives of distributed-order for the kinetic description of anomalous diffusion and relaxation phenomena, whose mean squared displacement does not changes as a power law in time. Correspondingly, the underlying processes cannot be viewed as self-affine random processes possessing a unique Hurst exponent. We show that different forms of distributed-order equations, which we call "natural" and "modified" ones, serve as a useful tool to describe the processes which become more anomalous with time (retarding subdiffusion and accelerated superdiffusion) or less anomalous that demonstrate the transition from anomalous to normal diffusion (accelerated subdiffusion and truncated Levy flights). Fractional diffusion equation with the distributed-order time derivative also accounts for the logarithmic diffusion (strong anomaly). We also discuss the underlying random walk models leading to the natural and modified forms.

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Stationary states in systems driven by Levy noise

Abstract

Systems driven by Levy noises, in comparison to their Gaussian analogues, display unexpected properties. In particular, these intriguing properties are well visible with respect to stationary states. For $\alpha < 2$, i.e. in the non-Gaussian regime, stationary states are not of the Boltzmann-Gibbs type and they exist for potential wells which are steep enough. Otherwise, they do not exist. Here, we demonstrate, when stationary states for systems driven by Levy noises can be recorder. We will focus on 2D systems, nevertheless 1D examples will be also provided. Therefore, various version of (space) fractional Smoluchowski equation will be studied both analytically and numerically.

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On the Prabhakar derivative: theory, numerical treatment and applications

Abstract

In 1967 Havriliak and Negami [1] proposed a new empirical constitutive law to describe the anomalous dielectric behaviour of some glassy materials and amorphous polymers near the glass-liquid transition. The complex susceptibility in the Havriliak-Negami model

$$\hat{\chi}(\omega) = \frac{1}{\left[1 + \left(i\tau\omega\right)^{\alpha}\right]^{\gamma}},$$

is characterized by two real powers $0 < \alpha, \alpha\gamma < 1$, which take into account both the asymmetry and the broadness observed in the shape of the dielectric spectrum, and by a time-relaxation parameter $\tau > 0$.

Further investigations have highlighted the suitability of the Havriliak-Negami model also for fitting experimental data from a large extent of heterogeneous systems such as, for instance, biological tissue.

When moving from the frequency domain to the time domain the Havriliak-Negami model involves a fractional pseudo-differential operator

$$\left({}_0D_t^{\alpha} + \lambda\right)^{\gamma}$$

which is strictly related to the integral operator studied by Prabhakar in 1971 [5] and based on a generalization to three parameters of the Mittag-Leffler function

$$E_{\alpha,\beta}^{\gamma}(z) = \frac{1}{\Gamma(\gamma)} \sum_{k=0}^{\infty} \frac{\Gamma(\gamma + k) z^k}{k! \Gamma(\alpha k + \beta)}.$$

also known as the Prabhakar function. For this reason $\left({}_0D_t^{\alpha} + \lambda\right)^{\gamma}$ is often refereed to as the Prabhakar derivative.

The problem of characterizing and formally defining the Prabhakar derivative $\left({}_0D_t^{\alpha} + \lambda\right)^{\gamma}$ has been the subject of recent investigation. In this talk we discuss the main approaches introduced to characterize the Prabhakar derivative and, in particular, we focus on recent results [2, 3, 4] which allow to open new possibilities for the numerical treatment of the Prabhakar derivative, especially in view of the application in computational electromagnetism.

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Linear response, fluctuation-dissipation relations and detailed balance breaking with Lévy noises

Abstract

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Mathematical Analysis of Time Flow

Abstract

The mathematical analysis of time flow in physical many body systems leads to the study of long time limits. This work discusses the interdisciplinary problem of local stationarity, how stationary solutions can remain slowly time dependent after a long time limit. A mathematical definition of almost invariant and nearly indistinguishable states on $C^{\star}$-algebras is introduced. It is based on functions of bounded mean oscillation [1]. Rescaling of time yields generalized time flows of almost invariant and macroscopically indistinguishable states, that are mathematically related to stable convolution semigroups and fractional calculus. The infinitesimal generator is a fractional derivative of order less than or equal to unity [2]. Applications of the analysis are given to irreversibility and experiment.

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Entanglement of one-excitation states in one-dimensional spin and electron systems

Abstract

We show that entanglement in one-dimensional spin and electron systems, with one excitation, depends only on the system size and has very simple form in both multipartite and bipartite case. We consider the exact solutions given by the Fourier transform known in literature as the basis of wavelets. Regarding the multipartite case, we present very simple expressions for global entanglement and N-concurrence, and show that they are mutually related. In the bipartite case, we give expressions for I-concurrence and negativity, and show that they are also depend on each other. Presented formulas allow one to calculate entanglement for an arbitrary size N, while the original definitions practically work only for the systems consisting of a few qubits. We expect that the size dependence of the entanglement of states with elementary excitation may help to understand entanglement in the systems with greater number of elementary excitations.

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Normal diffusion, subdiffusion, and slow subdiffusion in a membrane system

Abstract

We consider various kinds of diffusion in a system with a thin membrane. The membrane is treated as a partially permeable wall. Using the random walk model with discrete time and space variables the probabilities describing a particle?s random walk are found. Probabilities are transformed to the system in which the variables are continuous. From the obtained probabilities we derive boundary conditions at the membrane. One of the condition demands the continuity of flux at the membrane, but the other one is unexpected and contains the Riemann-Liouville fractional time derivative. The additional memory effect, represented by the fractional derivative, is created by the membrane even for normal diffusion case. In the presented model a kind of diffusion is defined by a probability density of time which is needed for the particle to take its next step.

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Theory of Hermite Calculus

Abstract

The use of umbral methods and of other concepts borrowed from algebraic theory of operators is a powerful tool to treat problems concerning the theory of special functions and the relevant applications in physical problems. The Hermite Calculus allows tremendous simplifications in a variety of problems involving Hermite polynomials and functions. It yields the possibility of providing explicit integration of families of function hardly achievable with conventional means and open new scenarios for the definition of new integral transforms of noticeable interest in application in optics.

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Magnetic pentagonal ring - Galois extensions and crystallography

Abstract

The Galois symmetry of exact Bethe Ansatz eigenstates for magnetic pentagonal ring is shown to bear a close analogy to some crystallographic constructions. Automorphisms of number field extensions associated with these eigenstates prove to be related to choices of Bravais cells in the finite crystal lattice $\mathbb{Z}_2 \times \mathbb{Z}_2$, responsible for extension of the cyclotomic field by Bethe parameters.

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Complete monotonicity for fractional relaxation processes

Abstract

We revisit some models of anomalous relaxation based on evolutions equations of fractional order. Our attention is on the complete monotonicty of the functions characterizing the relaxation processes, both in viscoelastic and dielectric media. The monotonicity requirement is known to provide a sufficient condition for the physical feasibility of the corresponding models.

Let us recall that a real function $f(t)$ defined for $t \in \mathbb{R}^+$ is said to be completely monotonic (CM) if it possesses derivatives $f^{(n)}(t)$ for all $n = 0, 1, 2, 3, ..$ and if $(-1)^n f^{(n)}(t) \geq 0$ for all $t > 0$. The limit $f^{(n)}(0^+) = \lim_{t \to 0^+} f^{(n)}(t)$ finite or infinite exists. It is known from the Bernstein theorem that a necessary and sufficient condition that $f(t)$ be CM is that

$$f(t) = \int_0^\infty \mathrm{e}^{-rt}\, \mathrm{d}\mu(r),$$

where $\mu(t)$ is non-decreasing and the integral converges for $0 < t < \infty$. In other words $f(t)$ is required to be the real Laplace transform of a non negative measure, in particular

$$f(t) = \int_0^\infty \mathrm{e}^{-rt}\, K(r)\, \mathrm{d}r\,, \quad K(r) \geq 0,$$

where $K(r)$ is a standard or generalized function known as spectral density of the CM function.

As discussed by several authors, the CM is an essential property for the physical acceptability and realizability of the models since it ensures that in isolated systems the energy decays monotonically as expected from physical considerations. Moreover, in view of the spectral density, the relaxation process can be seen as a linear (discrete or continuous) superposition of elementary exponential relaxation processes. The resulting process, however, can decay in a non-exponential way, so

giving raise to the so-called anomalous relaxation processes. Studying the conditions under which the response function of a system is CM is therefore of fundamental importance. In particular, it is found that the governing equations are expressed in terms of derivatives and integrals of fractional order.

In the enclosed bibliography we report only the papers by the author and his colleagues published in recent years related to this field. The fundamental contributions by other authors will be referred in the lecture.

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Anomalous diffusion in membranes and cytoplasm of biological cells

Abstract

A surging amount of experimental and simulations studies reveals persistent anomalous diffusion in both cellular membranes and the cytoplasm [1, 2]. The anomalous diffusion is observed for micron-sized objects down to labelled single molecules such as green fluorescent proteins [3].

This talk will first present results from large scale computer simulations and stochastic analysis of the motion of lipids and embedded proteins in lipid bilayer model membranes [4], indicating that increased disorder leads to longer and longer lasting anomalous diffusion. In particular, the motion of lipids and proteins can become non-Gaussian [4]. In the membranes of living cells anomalous diffusion of embedded protein channels can last over several hundreds of seconds [5]. In particular,

this anomalous diffusion can become non-ergodic and exhibit ageing, two topics explained and discussed in this talk [6].

The findings of anomalous diffusion in membranes will be complemented by a discussion of anomalous diffusion in the framework of the continuous time random walk and the fractional Langevin equation [6].

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Convolutional approach to fractional calculus for distributions of several variables

Abstract

We extend the semigroup $\{\Phi_\lambda\}_{\lambda\in\mathbb{C}}$ of Gel?fand and Shilov to the semigroup $\{\Phi_\lambda\}_{\lambda\in\mathbb{C}^n}$ and the one-dimensional fractional calculus of distributions to the case of distributions on $\mathbb{R}^n$ with supports in the nonnegative cone $\overline{\mathbb{R}}^n_+$. We use distributional mixed derivatives of an arbitrary complex order to solve Abel's multidimensional integral equation and a distributional fractional differential equation of wave type.

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Some aspects of the fractional dynamical systems

Abstract

The possible generalization of dynamical systems with a nonlocal evolution operator is considered. For this purpose, the concepts of fractional phase semiflow, fractional autonomous system and the generalized exponent of the vector field are introduced. Two examples of their application are explained in detail. As an example, the fractional generalization of the Lorenz system is introduced and its two-dimensional reduction is analyzed. Depending on the type of reduction, the fractional Lorenz system is shown to be closely related to the one-dimensional Richard's or Gierer-Meinhardt models.

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Fractional diffusion from Gaussian processes

Abstract

Following the results presented in [1, 2, 3], it is shown the emergence of space-, time- and space-time fractional diffusions from Gaussian processes when the medium where the diffusion takes place is characterised by a population of length-scales and/or time-scales. Different Gaussian processes, even based on different physical models, are considered: the Continuous Time Random Walk (CTRW) model; the fractional Brownian motion (fBm) and the Langevin equation. Moreover, the proposed approach establishes a relation between the emergence of fractional diffusion and the ergodicity breaking [4], and the resulting stochastic processes allow for reproducing diffusion with mixed characteristics as observed for example in single particle tracking experiments in living cells. Hence, the proposed approach is a promising tool to formulate stochastic processes for biological and physical systems showing complex dynamics characterised by anomalous diffusion, ergodicity breaking and aging. Furthermore, it is shown that for the same, and supposed experimentally

observed, macroscopic fractional diffusion equation, the characterisation of the medium by the population of its length-scales and/or time-scales allows to infer the correct underlying microscopic Gaussian process, which is eventually not experimentally observable. At the same time, for a given and expected microscopic Gaussian process the characteristics of the medium can be inferred when indeed a fractional diffusion process is observed.

References

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On the three parameter Mittag-Leffler function

Abstract

Prabhakar [1] was introduced the three–parameter Mittag–Leffler type function

$$E_{\alpha,\beta}^{\gamma}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n}{\Gamma(\alpha\, n+\beta)}\, \frac{z^n}{n!}\,, \qquad \big(\Re(\alpha), \gamma > 0,\, \beta \in \mathbb{C}\big)\,. \tag{1}$$

Deriving a Laplace–integral expression for the function [2]

$$e_{\alpha,\beta}^{\gamma}(t;\lambda) = t^{\beta-1}\, E_{\alpha,\beta}^{\gamma}(-\lambda t^{\alpha})\,,$$

one extends certain findings by Mainardi [3]. As a consequence of the obtained representation the complete monotonicity (CM) of $e_{\alpha,\beta}^{\gamma}(t;\lambda)$ is studied and related positivity results and certain uniform upper bounds are established upon $e_{\alpha,\beta}^{\gamma}(t;\lambda)$, see [2].

The second part of the talk consists from a study [4] of a probability distribution associated with the Prabhakar function and consequent related Turán and Laguerre type inequalities will be presented, pointing out the fresh research paper [5].

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Fractional Thermoelasticity

Abstract

The classical theory of thermoelasticity is governed by the equation of motion in terms of displacements

$$\mu\Delta\mathbf{u} + (\lambda+\mu)\mathrm{grad}\,\mathrm{div}\mathbf{u} - \rho\frac{\partial^2\mathbf{u}}{\partial t^2} = \beta_T K_T\,\mathrm{grad}\,T, \tag{2}$$

the stress–strain–temperature relation

$$\boldsymbol{\sigma} = 2\mu\mathbf{e} + (\lambda\,\mathrm{tr}\,\mathbf{e} - \beta_T K_T T)\mathbf{I}, \tag{3}$$

and the standard heat conduction equation

$$\frac{\partial T}{\partial t} = a\,\Delta T. \tag{4}$$

Nonclassical theories of heat conduction and generalized theories of thermoelasticty, in which the Fourier law and the standard heat conduction equation are replaced by more general equations, constantly attract the attention of the researchers. For example, in materials with complex internal structure described by memory and long-range interaction, the parabolic heat conduction equation is substituted the equation with differential operators of fractional order

$$\frac{\partial^\alpha T}{\partial t^\alpha} = -a(-\Delta)^{\beta/2}T, \qquad 0<\alpha\le 2, \qquad 0<\beta\le 2\,. \tag{5}$$

Here $\frac{\partial^\alpha T}{\partial t^\alpha}$ is the Caputo fractional derivative, $(-\Delta)^{\beta/2}$ is the Riesz fractional operator being the fractional generalization of the standard Laplace operator and having the Fourier transform

$$\mathcal{F}\left\{(-\Delta)^{\beta/2}f(\mathbf{x})\right\} = |\boldsymbol{\xi}|^\beta\mathcal{F}\left\{f(\mathbf{x})\right\}\,, \tag{6}$$

where $\mathbf{x}$ is a vector of variables, $\boldsymbol{\xi}$ is a vector of transform variables.

Each generalization of the heat conduction equation leads to the corresponding generalized theory of thermal stresses. Thermoelasticity based on the fractional heat conduction equation (fractional thermoelasticity) was proposed in [1, 2]; the book [3] sums up investigations in this field (see also [4]). We present several problems solved in the framework of fractional thermoelasticity and reflecting the typical features of the solutions.

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Numerical solutions of fractional advection diffusion equations

Abstract

This study looks at a governing equation of Lévy motion. This governing equation can be seen as a generalization of the classical, deterministic advection diffusion equation. It can be represented by fractional derivatives, which are integrodifferential operators, characterizing a spatially nonlocal process. We describe how to determine a numerical solution for advection dominated problems in the presence of the anomalous diffusion. Some numerical experiments will be presented to show the performance of the method and to observe the anomalous diffusion.

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Fractional equations with variable diffusion coefficient: transport in nonhomogeneous media

Abstract

The fractional equation with a variable diffusion coefficient is discussed. It comprises the derivatives over both time and position. The equation results from a random walk in a nonhomogeneous medium and from a Langevin formalism in the framework of a subordination method. Techniques of solving the fractional equation for both Gaussian and general stable case are presented. The density distributions for a few simple cases - a free particle, an harmonic and anharmonic oscillator - serve as an illustration. All kinds of the anomalous diffusion are indicated. The influence of the nonhomogeneity effects on the pattern of a relaxation to stationary states and the relaxation time is discussed.

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Chaotic Hamiltonian systems and their statistical properties

Abstract

Nonlinear chaotic Hamiltonian system theory shows that even simple models can exhibit very complex dynamics [1, 2]. Considering trajectories of such Hamiltonian systems in phase space, we encounter very complicated and not completely known topology of that space. The lecture presents a family of models, which are modification of the Chirikov standard map [3, 6].

For certain range of parameters in the phase space of this model, there are area of regular motions.called accelerator modes [3, 4]. In the center of that mode trajectory has a constant position variable while momentum increases monotonically with time. Rest of trajectories from accelerated modes follow this central trajectory. However, even in the case when no orbits had initial phase space coordinates lying within stable accelerator-mode area, the anomalous diffusion of angular momentum was observed. It is due to the intermittent transition of the system from the chaotic orbits to the unstable accelerator orbits i.e. trajectories lying close to the accelerator modes. These trajectories will follow the stable accelerator orbits for some period of time and this effect is sometimes called "the stickiness" of modes [5].

The lecture also presents a way of building fractional stochastic process, that macroscopic probability density is governed by parametrized fractional partial differential equation. This density preserves some features of angular momentum density for analyzed family of chaotic Hamiltonian systems where accelerated modes emerge [6].

This work has been done in colaboration with Piotr Pepłowski from Nicolaus Copernicus University.

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