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Abstracts for

Joint Mathematics Meetings Denver, January 15–18, 2020	1
Abstracts of MAA Papers	351

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SUBMISSION INFORMATION

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* Indicates who will present the paper at the meeting.

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PAPERS PRESENTED AT MEETINGS

THIS CALENDAR lists meetings of the Society which have been approved by the Council at which papers may be presented. Programs of Annual Meetings appear in the *Notices* and on the AMS website; programs for sectional meetings appear on the AMS Web pages in the Meetings & Conferences section, and are electronically archived in the *Notices* section on the AMS website.

MEETING #	DATE	PLACE	ABSTRACT DEADLINE	ABSTRACT ISSUE
1155	March 13–15, 2020	Charlottesville, VA	EXPIRED	Vol 41, No. 2
1156	March 21–22, 2020	Medford, MA	EXPIRED	Vol 41, No. 2
1157	April 4–5, 2020	West Lafayette, IN	February 4	Vol 41, No. 2
1158	May 2–3, 2020	Fresno, CA	March 3	Vol 41, No. 2
1159	September 12–13, 2020	El Paso, TX	July 14	Vol 41, No. 3
1160	October 3–4, 2020	State College, PA	August 11	Vol 41, No. 3
1161	October 10–11, 2020	Chattanooga, TN	August 18	Vol 41, No. 4
1162	October 24–25, 2020	Salt Lake City, UT	September 1	Vol 41, No. 4
1163	January 6–9, 2021	Washington, DC	TBA	TBA

DENVER, CO, January 15–18, 2020

Abstracts of the 1154th Meeting.

00 ► General

1154-00-9 **Rajiv Maheswaran*** (rajiv@secondspectrum.com), Second Spectrum, Los Angeles, CA
90012. *The fantastic intersection of math and sports: Where no one is afraid of a decimal point.*

Stereotypes permeate the perceptions of people who are either athletic or analytical. The disciplines of mathematics and sports have been kept at a distance, yet they share a commonality of deep interest in numbers. In this talk, I'll detail how the emergence of new kinds of data and novel applications of it are building bridges across two worlds where no one is afraid of a decimal point. (Received May 16, 2019)

1154-00-18 **Michael C Fu*** (mfu@umd.edu), Smith School of Business, Van Munching Hall, College Park, MD 20742. *Escalator etiquette: Stand or walk? That is the question.* Preliminary report.

Users of escalators and moving walkways often follow an implied etiquette that there are two available lanes: one for walking and one for standing (in the US and many countries, the convention is “walk left, stand right”). When there is high volume, e.g., when exiting a subway or train station or athletic event, the escalators often experience bottleneck congestion that constitutes the primary source of delay. Recently, it has been suggested that in such cases it would actually be more efficient if everyone just stood, with empirical evidence used to support this counterintuitive finding. We use several mathematical models, from a very simple static “scheduling” model to more sophisticated flow/queueing models, to show under what condition this result holds. In particular, the biggest assumption is that no more than half of the users are walkers, which is likely the case for escalators (the longer the more likely) but not necessarily the case for moving walkways. (Received June 03, 2019)

1154-00-210 **Allison Henrich*** (henrich@seattleu.edu). *Six Fundamental Steps to Mentoring Student Research.*

To get started on research with students, there can be an overwhelming number of things to consider. How do I choose problems? How do I find students to work with? How can I set the tone for the research experience and manage tricky student issues as they arise? How can I help my students develop research and communication skills? In this talk, we'll discuss Six Fundamental Steps to mentoring student research. These steps can help new faculty members learn how to get started in mentoring student researchers while helping seasoned mentors think about how to improve their mentoring skills. We'll also share information about sources of funding for

research with students. This talk is based on advice given in *A Mathematician's Practical Guide to Mentoring Undergraduate Research*, a book that was jointly written with Michael Dorff and Lara Pudwell. (Received August 23, 2019)

1154-00-486 **Salim El Rouayheb*** (salim.elrouayheb@rutgers.edu), **Rafael G.L. D Oliveira** (rafno.fnord@gmail.com), **David Karpuk** and **Daniel Heinlein**. *How to distribute the multiplication of Secret Matrices?*

We consider the problem of Secure Distributed Matrix Multiplication (SDMM) in which a user wishes to compute the product of two matrices using the assistance of honest but curious workers. In our recent work, we linked code constructions for SDMM with low communication cost to a new combinatorial object that we call Additive Degree Table (ADT). By studying ADTs, we devise a new parametrized family of codes for SDMM that we call GASP (Gap Additive Secure Polynomial) Codes. We also derive lower bounds and prove that GASP codes are optimal in certain regimes. This is a joint work with Rafael D'Oliveira, Daniel Heinlein and David Karpuk. (Received September 05, 2019)

1154-00-743 **Seth Gerberding*** (seth.gerberding@coyotes.usd.edu), 414 E. Clark Street, Vermillion, SD 57069. *Preserving Identifiability in Linear Compartmental Models*.

In this talk, we address linear compartmental model identifiability, and prove several cases where changing a model preserves identifiability. We focus primarily on the *cycle model*, and prove several results. But first, we prove a conjecture from earlier work holds for certain models. Then, we prove that a cycle model, with one input, one output, and one leak, *in any compartment* is generically locally identifiable. Next, we prove that adding certain edges to cycle models also preserves identifiability. We do so by introducing two “hybrid” models, the Fin model and the Wing model, then prove that removing certain edges preserves identifiability. Our proofs are aided by results on elementary symmetric polynomials and the theory of input-output equations for linear compartmental models. (Received September 10, 2019)

1154-00-908 **Chad M. Topaz*** (cmt6@williams.edu). *Diversity through a data science lens*.

This talk presents two data science projects related to diversity. First, motivated by the grievous underrepresentation of women in mathematics, we conducted a crowd-funded and crowd-sourced study of 13,000 editorships on mathematical science journals. While women are known to comprise approximately 16% of tenure-stream faculty positions in doctoral-granting mathematical sciences departments in the United States, 8.9% of the editorships in our study are held by women. We also describe group variations within the editorships by identifying specific journals, subfields, publishers, and countries that significantly exceed or fall short of this average. Building on this study, a team of art scholars, statisticians, and mathematicians conducted the first large scale study of diversity amongst artists whose works are held in the collections of major U.S. museums. We estimate that 85% of these artists are white and 87% are men. The relationship between museum collection mission and artist diversity is weak, suggesting that a museum wishing to increase diversity might do so without changing its emphases on specific time periods and regions. We conclude with a brief description of a new endeavor called QSIDE, the Institute for the Quantitative Study of Inclusion, Diversity, and Equity. (Received September 11, 2019)

1154-00-953 **Mila Runnwerth*** (mila.runnwerth@tib.eu), Welfengarten 1B, 30167 Hannover, Germany. *The Neverending Story of a Holistic Research Infrastructure for Mathematics*.

Mathematical research has come far from ancient drawings to number crunching on supercomputers with both practices in use alongside of each other today. The digital era extends our research tool box: information, instruments, communication are not logistically and physically restricted but have become potentially available all over the world. Yet, a thorough technical, educational, and ethically sound framework has to be established not only to make effective use of ever improving technologies but to adapt to their volatility and unforeseen consequences. The German National Library of Science and Technology is dedicated to make a mathematician's life easier by establishing tools and services for the whole research process at every career stage. (Received September 12, 2019)

1154-00-957 **Ingrid Daubechies***, ingrid@math.duke.edu. *Towards a Global Digital Mathematics Library*.

This brief presentation will present an overview of the efforts of several national and international organizations, including the Committee on Electronic Information and Communication of the International Mathematical Union, to put together an organization that would help all interested parties to have access to and be able to get maximal use from digitized forms of mathematical knowledge. (Received September 12, 2019)

1154-00-1073 **Katja Berčić***, katja.bercic@fau.de, and **Michael Kohlhase, Florian Rabe and Tom Wiesing**. *Research data in mathematics: taking the high road*.

Mathematicians produce data and use them in their research to find patterns and test conjectures. This is evidenced by the success of several larger projects: the OEIS, the LMFDB, the Small Groups Library, and the House of Graphs, to name just a few. At the same time, mathematics remains largely unaffected by the recent trends of publication and management of research data. In particular, this holds true for the push to make research data FAIR (findable, accessible, interoperable, and reusable).

This talk will discuss the notions of mathematical data and put them into the context of doing mathematics. We will discuss data-related needs of the mathematical community, what could be done to address them, and what one may encounter while doing so.

There is a conceptual and technical tension between two kinds of representations of mathematical objects: those needed to store and index the objects in databases and those needed to communicate the mathematical meaning to humans and machines alike. Instead of treating this as a dilemma, we build a conceptual framework that reconciles the technical and semantic aspects. Last, but not least, we present a prototype for a unified data infrastructure that aims to improve the usability of mathematical data. (Received September 13, 2019)

1154-00-1116 **Richard Zanibbi*** (rxzvc@rit.edu) and **Anurag Agarwal** (axasma@rit.edu). *Math-Aware Search: A Progress Report from the MathSeer Project*.

This talk gives an overview of the MathSeer project. We want the general public to be able to quickly lookup unfamiliar math symbols and see how formulas and mathematical concepts are defined, used, and analyzed in online resources such as Wikipedia, Math StackExchange, and CiteSeerX. This talk will summarize our progress to date, including: (1) a novel search interface with a formula editor that supports images, handwriting, and LaTeX, (2) new techniques for indexing and retrieving mathematical formulas in symbolic representations and images, (3) a study of current math search behaviors in regular search engines from query logs, and (4) new tools for extracting symbols and formulas from documents in PDF format.

This is joint work by faculty and students at RIT, Penn State University, and the University of Maryland, College Park, supported by the National Science Foundation (USA) and the Alfred P. Sloan Foundation. MathSeer web page: <https://www.cs.rit.edu/dprl/mathseer> (Received September 13, 2019)

1154-00-1182 **Marilyn Vazquez*** (vazquezlandrove.1@osu.edu), **Jeova Farias and Pedro Felzenszwalb**. *Non-parametric Mixture Model for Unsupervised Image Segmentation*. Preliminary report.

Mixture models have many uses in data science, including clustering and image segmentation. We present an unsupervised image segmentation method applied to natural images. In our research, we model an image as a mixture of color distributions, each coming from a distinct segment in the image. The goal is to estimate the color distributions from all the regions in the image and use them to segment the image. To solve for the distributions, we developed a scheme that solves a subset of the equations in our model, uses least squares to find the missing entries, and then performs coordinate descent to refine the approximation. The parameters in our model are estimated by measuring how well they recover the observed data. We present results on natural images and materials. (Received September 13, 2019)

1154-00-1255 **Della Dumbaugh** (ddumbaugh@richmond.edu) and **Hannah Fenster*** (hafen001@mail.goucher.edu), Center for Contemporary and Creative Writing, Goucher College, 1021 Dulaney Valley Road, Baltimore, MD 21204. *Then and Now: Calculating Intersections of Math and Motherhood*.

In her memoir *Becoming*, Michelle Obama calls attention to two women she admired in the workforce who “didn’t try to compartmentalize work and home.” Obama’s voice lends credibility to a path some mothers in mathematics adopt. But what does this look like? And what are the outcomes of a life at the intersection of mathematics and motherhood? This talk by a mother-daughter duo explores how one mom unwittingly combined her personal life as a mother with her professional career in mathematics and how it played out in the life of her daughter. Looking further, we identify trends in the worldviews of some of these children-now-adults who grew up with “math moms.” (Received September 14, 2019)

1154-00-1503 **Mark Tomforde*** (tomforde@math.uh.edu). *Principles of Running an Outreach Program*. I will discuss my involvement in a mathematics outreach program for middle school and high school students in underserved neighborhoods of Houston. I wish to share some of the experiences from my six years of running this program, and describe how they have shaped my personal approach to outreach. (Received September 16, 2019)

1154-00-1523 **Alessandra Pantano*** (apantano@uci.edu), Department of Mathematics, University of California, Irvine, Irvine, CA 92697. *Supporting math proficiency and motivation through afterschool math enrichment: A focus on program design*. Preliminary report.

UC Irvine's Math CEO (Community Educational Outreach) is a transformative afterschool program that leverages UCI's unique social and cultural capital to inspire underserved youth to pursue college education and a career in STEM, while at the same time fostering math achievement and dispositions needed to persist in STEM pathways. Almost all of the participating students are Latinx, and come from low income families which lack familiarity with college.

In this talk we discuss some aspects of program organization which contributes to the success of Math CEO. In addition, we draw directly from participant interviews to highlight aspects of mentor-youth interactions and pedagogical choices which lead to positive and authentic mathematical experiences. (Received September 16, 2019)

1154-00-1557 **Zair Ibragimov***, 800 N State College Blvd., Fullerton, CA 92831. *International Research Experiences for Students in Uzbekistan*.

I will give an overview of IRES in Uzbekistan program from 2017-2019 and discuss the value of engaging internationally for undergraduate students. (Received September 16, 2019)

1154-00-1581 **Francesca Bernardi*** (fbernardi@fsu.edu) and **Katrina Morgan**. *Communication, Representation, and Community: Entry Points to Advanced Mathematics*.

Students from underrepresented groups face numerous barriers of entry in STEM. Fostering a community of young people who are interested in Mathematics, but do not necessarily see themselves as possible Mathematicians, can help them feel welcome in the field and help build their confidence. Outreach programs aligning Science communication with Mathematics can be a point of entry for such students, who often find these events to be less intimidating than those focused exclusively on technical activities.

Incorporating communication aspects in outreach serves several purposes: It encourages the participation of non-traditional STEM students by building a community of young people passionate about Mathematics and communicating it to the public; it helps educate these students to become better Mathematicians, teaching them communication skills as well as challenging Math topics; and it shows them the importance of focusing on both aspects.

Experiences will be reported from Girls Talk Math, a free day camp for female and gender non-conforming high schoolers focused on Mathematics and media. Founded in 2016 at the University of North Carolina at Chapel Hill, the camp has hosted more than 130 students. A sister camp at the University of Maryland at College Park was founded in 2018. (Received September 16, 2019)

1154-00-1638 **Roberto Carlos Pelayo*** (rcpelayo@uci.edu), University of California, Irvine, Mathematics Department, Irvine, CA 92697-3875. *Improving Equity and Access in Mathematics Curricula*.

For many students, access to rigorous and appropriately-paced mathematics curricula in high school and university settings is a barrier to graduation and the completion of a STEM degree. While this access is largely a function of school funding and demographics, several educational policies are attempting to provide more effective curricular pathways. This talk will discuss these efforts, including a re-crafting of the traditional mathematics course sequencing at the high school level and a decrease in developmental course offerings at the university level. We discuss the impacts of these policies and highlight ways that higher educational faculty can contribute to these efforts. (Received September 16, 2019)

1154-00-1884 **Alessandra Pantano*** (apantano@uci.edu), Department of Mathematics, University of California, Irvine, Irvine, CA 92697, and **Daniel R Bergman** (drbergma@uci.edu). *Increasing students' conceptual understanding of linear algebra through active discussion sessions*. Preliminary report.

After their first linear algebra course, many students master algorithms but lack conceptual understanding of the subject. As a result, they enter advanced classes knowing how to compute Gaussian elimination, determinants, eigenvalues and inverses of matrices, but with little understanding of the notions of basis vectors, linear independence and subspaces. This is not surprising, as these concepts require students to transition from computational algorithms to more sophisticated ways of reasoning. Inquiry-oriented teaching can ease this transition.

While active learning is positively associated with student achievement, most of the teaching at large R1 institutions remains in traditional lecture style. Providing TAs with (pre-tested) quality worksheets is an effective and low-stake approach to broaden the use of active learning in math classes.

In this talk, we present curricula for discussion sessions that give students a chance to practice the basic algorithms required in lower division linear algebra, but also to reflect upon the process and discuss what they did and why they did it. By asking students to reflect upon and communicate the relevant concepts in their solutions, we seek to strengthen their ability to understand and communicate mathematical concepts. (Received September 16, 2019)

1154-00-1964 **X Wang*** (xwang@tougalo.edu), **WA Pruett** and **J Wilson**. *Uncovering and Understanding Population Difference with Topological Method*. Preliminary report.

In this project, we use topological data analysis and our analytic tool suite to generate information for pathway analysis to better describe the mechanisms of disease in the study population. We begin with the hypothesis that obesity, hypertension, renal disease, and combinations of the three are the result of distinct etiologies within the study participants. Further, we hypothesize that by analyzing aptamer, metabolic, and clinical data alone or in tandem, then differences between populations will explain these etiologies. The primary output of TDA is a dissection of a study population into subpopulations that are internally consistent across all monitored variables. In diseases like hypertension and metabolic syndrome, the same "disease" may come from many different mechanisms. The mechanisms underlying the disease determine the best treatment. The goal of this analysis is to break populations with complex diseases into mechanistically consistent subpopulations and propose treatments based on the mechanism. The value of this technique is that it allows the structure of a space to be studied without reducing its dimensionality. This means that we can ostensibly study panels with tens of thousands of genes, metabolites, or other data points. (Received September 16, 2019)

1154-00-1996 **Emille Lawrence*** (edlawrence@usfca.edu). *Partisan Gerrymandering and the Efficiency Gap*. Preliminary report.

The United States Census Bureau will conduct its next census in 2020. But why is this decennial count so important? Aside from allocating funds for municipalities to receive certain social services, the census is used for the apportionment of Representatives in the House. Recent cases of partisan gerrymandering (redrawing district lines for ulterior political motives) have made it all the way to the Supreme Court and the efficiency gap formula, a formula which quantifies the amount of "wasted votes" by a party, played a pivotal in these cases. We will discuss the efficiency gap formula as it was originally defined by E. McGhee, as well as some competing new alternatives. (Received September 17, 2019)

1154-00-2034 **Timothy Jesse Redford*** (tredford@uncc.edu) and **Xingjie Helen Li** (xli47@uncc.edu). *Adaptive SVD Image Compression Framework For Modeling The Impact Of Visual Stimulus And Complexity in Digital Images*. Preliminary report.

Singular Value Decomposition (SVD) has been employed for processing a variety of data types and dimension reduction. Because all digital images can be represented as matrices, this method of matrix factorization is universally accessible to a variety of image processing applications including compression, encryption, analysis, and denoising. This paper proposes a benchmark investigation based on numerical experiments of how SVD can be used in conjunction with human visual system imitation and offers an adaptive image compression framework for optimizing the rank approximation of color images and their respective grayscale. Presented methodologies involve a heuristic argument for how adaptive sub-matrix approximations should be distributed, exploiting the fact that image deterioration is less obvious to the human eye when presented in areas of an image which do not receive the same level of visual attention. We discuss experimental evidence which appears to support this claim on this basis of compression ratio, time complexity, and semantic error types in comparison to visual inspection of the decompressed image(s) generated by our proposed methods of adaptive image compression. (Received September 17, 2019)

1154-00-2043 **Aliza Steurer*** (asteurer@dom.edu), Dominican University, 7900 W. Division, River Forest, IL 60305. *Using parenting to understand and manage inquiry-based learning and teaching*.

Inquiry-based learning and teaching (IBLT) is a student-centered form of instruction where students often work in small groups and explain ideas to peers. Instructor challenges can arise from these activities, and the challenges can be similar to ones a parent faces when caring for a child. For example, a parent must learn to understand and respond to a child's nonverbal cues, before the child uses formal language. In an IBLT classroom, an instructor may need to be sensitive to students who are quiet or withdrawn during small-group work and may need to carefully help such students become involved. Also, a child may ask a question that is difficult to understand or answer, and the parent must think carefully about how to respond; the same can be said of a student's question and an instructor's response in an IBLT classroom. We will explore the following questions. How can viewing the

challenges of IBLT through the lens of parenting help us understand and manage them? With respect to results, how has IBLT seemed to have affected student feedback in the presenter's courses? (Received September 17, 2019)

1154-00-2058 **Rana Parshad*** (rparshad@iastate.edu), Department of Mathematics, Ames, IA 50011.
Recent results for the Trojan Y Chromosome Model.

The Trojan Y Chromosome (TYC) model/strategy is a well known method for control of invasive species with an XY-XX chromosomal structure. Herein a modified YY male is introduced into an invasive population, to skew the sex ratio towards all males over time, yielding extinction. We show that the classical TYC model, can exhibit solutions that blow up in finite time, for various initial data and parameter regimes. This calls into suspect current modeling frameworks for such strategies, where it is attempted to manipulate the mating system as a means of population control. (Received September 17, 2019)

1154-00-2261 **Michelle K Schwalbe*** (mschwalbe@nas.edu), 500 5th St NW, K960, Washington, DC 20001, and **Mark L Green, Tamara Kolda and Russel Caffisch.** *Illustrating the Impact of the Mathematical Sciences: Opportunities in Mathematical Policy.*

The successes of the mathematical sciences research enterprise are numerous, significant, and ubiquitous and mathematical innovations build upon hundreds of years of research. Yet, simple stories designed for general audiences that trace mathematical discoveries to their impacts are few and far between. This session will explore some of these mathematical stories, discuss the value of communicating mathematical research to a broader audience, and explore the important role for mathematical policy.

The National Academies of Sciences, Engineering, and Medicine's Board on Mathematical Sciences and Analytics (BMSA) strives to provide mathematical advice to policy makers, strengthen connections between other domains and the mathematical sciences, support the health of the mathematical sciences ecosystem, and increase public awareness of the expanding role of the mathematical sciences. BMSA is conducting a study for the National Science Foundation to generate narratives and graphics for general audiences that demonstrate the fundamental role of the mathematical sciences in the U.S. economy, national security, health and medicine, and other science, engineering, and technology domains.

Learn more at: www.nas.edu/BMSA or www.nas.edu/IllustratingMath. (Received September 17, 2019)

1154-00-2269 **Yuliang Wang*** (wxwy1990402@gmail.com), **Zhe Xiong, Jiahao Qu, Nicholas Liskij, Nicholas Hanoian, Henry Sojico, Yuchen Guo, Zhexiao Lin and Zhenhong Zou.** *RNN and NMF Chatbot: Combining two machine-learning techniques to create a dynamic, topic-aware chatbot.* Preliminary report.

In this work we construct two models for a topic-aware chatbot by combining existing text-generation models with non-negative matrix factorization (NMF). In the first model, we augment a traditional encoder-decoder structure with message attention and topic attention layers. A topic representation of a given sentence is obtained using NMF and encoded into a topic attention vector. We train the model so that it prefers to sample relevant topic words as opposed to non-topic ones. In the second model, we utilize the pre-trained BERT architecture developed by Google and combine it with NMF to generate topical responses to questions. The relevant response to an input question is sampled from the joint distribution learned by the model using Markov Chain Monte Carlo. Lastly, we propose several approaches to improve both topic modeling and word generation. (Received September 17, 2019)

1154-00-2326 **Richard J Brown*** (brown@math.jhu.edu), Johns Hopkins University, 3400 North Charles Street, Baltimore, MD 21218. *A hybrid IBL-lecture design for a freshman honors single variable calculus course.* Preliminary report.

Inquiry-based learning, an active style of classroom learning where students develop classroom content in groups among themselves while guided by structured queries and mentored by a facilitator, has enjoyed success as an innovative model of mathematical study and classroom learning. However, in a freshman calculus class, we have found that the adaptation process needed to orient students to this new style of learning can take up to the first half of a semester course before real learning begins. During this phase, content progress slows to a crawl. And the lack of true exposure to a lecturer's mathematical thinking process can rob the students of a valuable role model for high-level and deep learning. At Hopkins, we have adapted the IBL model in an honors-level single variable calculus course to a hybrid lecture-IBL model, to help students not only develop the tools necessary to conduct meaningful mathematical inquiry among themselves, but also to have adequate exposure to a mentor's thinking processes through targeted lectures and lecture-based instruction. One side benefit of this

hybrid approach is its appeal to a larger differentiated-learning audience. In this talk, we describe the structure of the course and our experiences in its design and implementation. (Received September 17, 2019)

1154-00-2345 **Lola Thompson*** (lola.thompson@oberlin.edu). *"I did it my way": making undergraduate research work for you.*

For many new faculty members, the prospect of mentoring undergraduate research can be daunting. The words "undergraduate research" can trigger mental images of applying for large NSF grants, reviewing hundreds of REU applications, managing teams of students with widely varying backgrounds, feeling pressure to produce a paper in a limited amount of time, giving up conference travel and family vacations for three consecutive summers,...

In this talk, I will discuss how I have created meaningful, yet manageable, research opportunities for undergraduate students. For me, the key has been to recruit students early in their undergraduate careers so that we will have several years to work together. I treat my undergraduate researchers as "mini PhD students," working with them one-on-one and giving them a great deal of leeway in steering the direction of our work together. Topics will include: recruiting students early in their undergraduate careers, teaching research skills through IBL courses, finding appropriate research problems that might lead to other good questions, and strategies for keeping research students on track when they have many other demands on their time. (Received September 17, 2019)

1154-00-2483 **Nikhil Nagabandi*** (nn318@mynsu.nova.edu), 3308 Chase Jackson Branch, Lutz, FL 33559, and **Elizabeth Melville**. *Can we hear the shape of a fractal? Spectral Consideration in the context of self-similar sets.*

Analytic structures on fractals have been analyzed extensively in the past 50 years both because of their interesting mathematical properties and their potential applications in physics. One important question in this area is how the spectrum of a Laplacian on a fractal reflects its geometry; one version of the corresponding problem for domains in Euclidean space was famously described in Kac's question "Can you hear the shape of a drum?". Our interest is in more precise results that give the locations and multiplicities of eigenvalues explicitly. These are connected to a long strand of research in mathematical physics about the structure of spectra of Schrödinger operators and their relation to topological invariants of the underlying space (prominent results in this area are due to Landau, Peierls, Harper, Moser, Bellissard, and, recently, Avila and Jitomirskaya). One name for these results is gap-labeling theorems. For certain highly-symmetric self-similar sets the computation of the gap structure of the Laplacian spectrum is possible using spectral decimation. We use this method to explicitly compute the gap structure for the Laplacian on a particular two-point self-similar graph and its fractal limit, and for Sierpinski graphs and the Sierpinski gasket. (Received September 17, 2019)

1154-00-2549 **Long Nguyen***, nguye33@gmu.edu, and **Maria Emelianenko, Amy C and Barney Bishop**. *"Mathematical tools for automated detection of antimicrobial peptides".*

This project is focused on developing a set of mathematical tools for performing systematic analysis of peptide sequence data that can be used, in particular, for automatic screening and anomaly detection based on various types of experimental data. In the Bishop lab that we collaborate with, functionalized hydrogel particles are used to harvest peptides from biological samples with the ultimate objective of identifying novel cationic antimicrobial peptides (CAMPs). There is a need to evaluate meaningful similarities and differences between sets of peptides captured by different particle types and between different harvests. We explore the role of different types of distance metrics and physical characteristics of the peptide sequences in the statistical analysis that includes dimension reduction, classification and clustering. Observations based on the phylogenetic and demographic information are used to guide experiments as well as algorithmic development. (Received September 17, 2019)

1154-00-2661 **Heidi Berger*** (heidi.berger@simpson.edu), 701 N. C Street, Indianola, IA 50125. *The Upside of Down Syndrome: Math is My Superpower!*

My son Isaac has Down syndrome. He was born in 2015, within a year of me receiving tenure at Simpson College. The experience of being his mother has had a profound effect on me as a mathematician. Having been with him through eleven surgeries over sixteen hospitalizations, I wanted to learn about his medical complexities and, more generally, about coordinated health care for those with chronic illness. To accomplish these goals, I've looked to my teaching and research. In the spring of 2016, I designed a sophomore-level mathematical modeling course on the respiratory system. In the summer of 2016, I led a group of three undergraduates to utilize location analysis as a means of understanding patient access in the health care system. We used p-median and maximal covering models to investigate patient access to Down syndrome specialty care clinics in the United States. More recently, together with a graduate student in medical anthropology and a handful of undergraduates in mathematics, I've

initiated a project on family perspectives of attending a Down syndrome specialty care clinic. Through teaching and research, I've used math as my superpower to help understand and improve health care for individuals with Down syndrome. (Received September 17, 2019)

1154-00-2785 **Abigail T Small***, 34 Twin Brook Ln., Taunton, MA 02780, Taunton, MA 02780, and **Edward T Dougherty**. *Multi-scale Modeling of Neurostimulation with Application to Post-traumatic Stress Disorder*.

Post-traumatic stress disorder (PTSD) is a neurological condition which results from a traumatic experience caused by physiological shock or physical harm. Clinical results show success in combating the symptoms of PTSD with a neurostimulation treatment called transcranial direct current stimulation (tDCS). Though effective, the underlying mechanisms of the treatment and its success are not fully comprehended. To help elucidate reasons for its effectiveness, a partial differential equation based mathematical model of tDCS for PTSD has been implemented. Using the finite element method, numerical simulations generate results that predict and quantify the electrical energy distribution during tDCS sessions. Simulations utilize real-world electrode montages and treatment parameters for PTSD and a three-dimensional, MRI-derived cranial cavity with biologically-based tissue conductivities. Regions of the brain thought to be targeted by tDCS treatments are confirmed with in silico experiments, thus validating the model and approaches. Finally, a novel, partial differential equation based multiscale mathematical model of tDCS is presented, which uniquely adds the ability to quantify neural tissue response via tDCS-induced transmembrane voltage polarization. (Received September 17, 2019)

1154-00-2855 **Belin Tsinnajinnie***. *Moving from changing the faces of mathematics to changing who mathematics serves*.

Providing support for aspiring and current mathematicians from marginalized and underrepresented communities is often at the center of diversity and inclusion efforts. Conversations centered on the importance these efforts are often framed by the need for diversity and inclusion as a necessity to advance the fields of mathematics. However, there are few conversations taking place in diversity and inclusion in mathematics that center the needs and goals of marginalized and underrepresented communities. In this talk, I draw upon my own experiences of inclusion/exclusion and sense of inadequacy in both academic mathematics communities and my home communities to call for a shift in the ways we can frame conversations of diversity, equity, and inclusion in mathematics. I seek to shift our frames towards the question: How will diversity and inclusion in mathematics directly empower marginalized communities? (Received September 17, 2019)

01 ► *History and biography*

1154-01-100 **Jemma Lorenat***, jlorenat@pitzer.edu. *The case for mathematical models – in which Charlotte Angas Scott attempts to build a mathematical research school*.

In 1884 Charlotte Angas Scott had only recently finished her doctorate under the supervision of Arthur Cayley. She had no experience leading, or even fully participating in, a graduate-level research environment. Nevertheless, as the founding chair of the Bryn Mawr mathematics department, she aimed to engage students in “original work” in order “to advance Mathematics.” Such an endeavor required both intellectual and practical efforts. On the one hand, Scott introduced students to recent scholarship and advised on research topics. On the other hand, she developed institutional connections, wrote letters of recommendation for current students, and campaigned for funding a case to display the plaster models of geometric surfaces that were a necessary component of any serious mathematics department by the early-twentieth century. Scott successfully supervised eight PhD students, yet few found academic positions and none could sustain mathematical research after graduation. Through the lens of Scott at Bryn Mawr, this talk will consider the extent to which spaces for women in mathematics were possible, sustainable, and scalable at the turn of the twentieth century. (Received August 06, 2019)

1154-01-147 **Craig Fraser*** (craig.fraser@utoronto.ca) and **Michiyo Nakane**. *Canonical Transformations and Hamilton-Jacobi Theory 1866–1920*. Preliminary report.

In the second half of the nineteenth century, Hamilton-Jacobi theory provided important mathematical tools in celestial mechanics. One significant development concerned the use of canonical transformations—first introduced by Carl Jacobi—to integrate the equations of motion. In a canonical transformation of the variables of the system, Hamilton's equations remain valid in the transformed variables. One can obtain a canonical transformation from a generating function. If the generating function satisfies the Hamilton-Jacobi partial differential equation, one is led to a solution of the dynamical problem. The subject as it had coalesced by around 1910 resulted in analytical methods that were adopted by German quantum physicists in their investigation

of atomic phenomena. Canonical transformations were also explored on a more abstract level in the work of mathematicians. (Received August 15, 2019)

1154-01-196 **Johannes Familton*** (jfamilton@bmcc.cuny.edu), 199 Chambers Street, New York, NY 10007. *Olinde Rodrigues' contribution to Catalan numbers.*

Benjamin Olinde Rodrigues was born into an Iberian Jewish family in 1795 residing in France. Most mathematicians who heard of Rodrigues connect him with Rodrigues rotations, which were based in his PhD thesis, and a later paper that he wrote in 1840. Few mathematicians know that he also wrote three 'notes' on combinatorics. He specifically wrote two notes on Catalan numbers, and one on an elementary derivation, without using the Taylor series, of the expansion of binomial series. This presentation will focus on Rodrigues' 1838 contribution to Catalan numbers. This presentation will put this fascinating man's life in context with the two notes that he wrote about Catalan numbers. A brief history about what lead to the problem that Rodrigues tackled will be included. This will be followed by what Rodrigues pointed out and showed in his notes. This will include a summary of his first observation followed by a summary of a shorter proof' he later wrote of his original observation. (Received August 21, 2019)

1154-01-208 **Thomas Archibald*** (tarchi@sfu.ca). *Taking stock of the last 30 years : a project to survey the cultural history of mathematics.* Preliminary report.

In 1972, Morris Kline's *Mathematical Thought from Ancient to Modern Times* presented a 1200-page survey of the history of mathematics. In 1994, Ivor Grattan-Guinness's *Companion Encyclopedia*, followed three years later by his own synthesis of that collective work *The Rainbow of Mathematics* revised and extended Kline's picture, with different emphasis and some important use of work done up to the early nineties. Since then, trends in history of science and mathematics have markedly altered, many classic studies have been critically rethought, and a large amount of research production has enriched our picture of mathematics around the world. This talk will describe a work-in-progress, the *Bloomsbury Cultural History of Mathematics*, slated to appear in early 2021 under the general editorship of David Rowe and Joseph Dauben. This 6-volume collection aims at undergraduates above all, aiming to elucidate the role of mathematics in human cultures, and its often-key position in interactions between different cultures.

The purpose of this paper is to discuss this 50-author project from the point of view of a co-editor of two of the volumes, with an emphasis on the historiographic problems and approaches that we hope to highlight. (Received August 23, 2019)

1154-01-213 **Erik R. Tou*** (etou@uw.edu), **Christopher D. Goff** and **Michele Gibney**. *What's Happening with the Euler Archive?* Preliminary report.

In 2003, the Euler Archive (EA) was created by Dominic Klyve and Lee Stemkoski, both then at Dartmouth College, to raise awareness in the U.S. of the vast quantity of Euler's writings, and the small portion of which had been translated into English at that time. Some goals of the early project were to provide access to original versions of Euler's publications as well as to translations of those works into English or another modern language. The MAA took over web hosting duties from Dartmouth in 2011.

Now, the EA is undergoing a kind of renaissance. Many files have been moved to an academic repository, BePress's Scholarly Commons, hosted at University of the Pacific. In this presentation, current EA director Erik Tou (University of Washington Tacoma), Chris Goff (University of the Pacific), and Michele Gibney (University of the Pacific) will describe the details of their recent work to shore up the archive as well as their future plans for improving the generation of and access to modern translations of Euler's works. (Received August 24, 2019)

1154-01-222 **David E. Dunning***, Dickinson Hall #129, Princeton, NJ 08544. *Notational Norms in Charles Sanders Peirce's Circle of Logicians.*

From 1879 to 1884, Charles Sanders Peirce (1839–1914) taught logic at the recently founded Johns Hopkins University in Baltimore, where he and a small circle of graduate students collaboratively published influential research in mathematical logic. They formed a local community whose size and vibrancy was unprecedented in that fledgling field, which had so far developed through the published contributions of a few pioneers working at a distance from each other in Britain and Germany. I will explore the local culture of mathematical logic that Peirce and his students developed, with a focus on their approach to notation. We might expect a community working in close collaboration to share a single symbolic system; in fact the logicians of 1880s Baltimore tended to develop individual variations on existing notations. Whereas earlier mathematical logicians stubbornly defended their individual systems, Peirce and his circle of graduate students took a more flexible approach to symbolism, finding intellectual interest precisely in the range of notational possibilities presented by the new mathematical logic. (Received August 25, 2019)

1154-01-298 **Emelie A Kenney*** (kenney@siena.edu), Department of Mathematics, Siena College, Loudonville, NY 12211. *Polish Women Mathematicians in Secret Classes During World War II*. Preliminary report.

After the Nazis occupied Poland in WWII, they forbade Poles from learning many subjects, including mathematics beyond counting to 500. Nevertheless, because of a large and complex underground, Poles began learning and teaching in classes held clandestinely, despite the threat of death or imprisonment in a concentration camp. Among those students and lecturers were many well-known mathematicians, including Sierpiński, Borsuk, Lukaszewicz, and Kuratowski, among others. But lesser-known mathematicians participated in underground education, as well. Here, we focus on the future achievements and careers of some of the women who were involved, either as students, instructors, or both. (Received August 29, 2019)

1154-01-307 **Tinne Hoff Kjeldsen*** (thk@math.ku.dk), Department of Mathematical Sciences, University of Copenhagen, Universitetsparken 5, 2100 Copenhagen, Denmark. *From Mathematical Programming to Convex Analysis: Duality as a driving force in history of mathematics*.

The presentation will focus on the emergence of convex analysis in the 20th century in the context of mathematical programming with special attention to the significance of duality. More specific, we will look at duality in the history of mathematical programming from von Neumann's work in game theory to Fenchel's duality theorem in nonlinear programming and the role it played for the development of convex analysis. How did ideas of duality emerge in linear programming? What role did they play for the development of nonlinear programming? How did Fenchel introduce ideas of duality in nonlinear programming and how did his duality function as a driving force for the development of convex analysis? (Received August 30, 2019)

1154-01-356 **June Barrow-Green*** (june.barrow-green@open.ac.uk), School of Mathematics & Statistics, The Open University, Milton Keynes, MK7 6AA, United Kingdom. *"A Senior Wrangler among Senior Wranglers": the mathematical education of Robert Leslie Ellis*. Preliminary report.

Robert Leslie Ellis (1817–1851) was one of the most intriguing and wide-ranging intellectual figures of early Victorian Britain, his contributions ranging from advanced mathematical analysis to profound commentaries on philosophy and classics. His mathematical education began at home in Bath under the tutelage of Thomas Stephens Davies, later a mathematical master at the Royal Military Academy, Woolwich. At the age of 17, Ellis left home to prepare for Cambridge with the astronomer James Challis but his health broke down and his stay with Challis was cut short. After two further years at home he eventually went up to Cambridge in 1836 where he came under the guidance of George Peacock. Tutored in his third year by the famous coach William Hopkins, Ellis graduated as Senior Wrangler in the Mathematical Tripos of 1840. In this talk I shall examine Ellis's route to success in the Mathematical Tripos and consider to what extent he succeeded because of the Cambridge system or in spite of it. (Received September 02, 2019)

1154-01-371 **Glen R Van Brummelen*** (gvb@questu.ca), Quest University, 3200 University Boulevard, Squamish, BC V8B 0N8, Canada. *Who Invented the Decimal Point?* Preliminary report.

The question of the invention of the decimal point is vexed, with multiple interpretations and several different cultural claims. Within Europe, textbooks assert that decimal fractional notation was introduced in the late 16th century with Simon Stevin's *De Thiende*, and that the decimal point itself first appeared around the same time with Christopher Clavius or John Napier. These claims are mostly untrue. We shall explore the much older origin of the decimal point in European mathematical astronomy, and touch on some of the historiographic issues that arise when considering this issue. (Received September 02, 2019)

1154-01-374 **Glen R Van Brummelen*** (gvb@questu.ca), 3200 University Boulevard, Squamish, BC V8B 0N8, Canada. *Islamic Trigonometry: A Very Short Introduction*.

Trigonometry neither began nor ended in the Islamic world, but what was eventually transmitted to Europe (and especially what wasn't) was a powerful, systematic tool with a scope and a list of applications far richer than its state in the 8th century AD. Two flavours of the subject, one in the middle east and the other in al-Andalus, took different approaches and had very different fates. We shall discuss the origins of Islamic trigonometry, various ways in which the subject was transformed, and the curious path by which it found its way to the West. (Received September 02, 2019)

1154-01-400 **Mariya Boyko*** (mariya.boyko@mail.utoronto.ca), 6 - 469 Woodview Rd, Burlington, ON L7N 2X9, Canada. *Soviet Mathematics Curriculum Reforms (1958-1985): Redefining the Purpose of Mathematics Education*. Preliminary report.

The major mathematics education reforms that took place in the period of 1958-1985 in the USSR are referred to as Kolmogorov's reforms, after professor Andrei Kolmogorov. The reforms intended to bridge the gap between theoretical and practical skills and prepare students for entering postsecondary study and the workplace. Kolmogorov's view of practical mathematics education was different from the government's notion of practical. Kolmogorov believed the school mathematics needed to align with modern mathematics, while the government was interested in providing students with practical skills needed in the workplace. The Kolmogorov curriculum included the introduction of set theory, a deductive logical approach, and focused on the abstract character of mathematics. It resembled the new mathematics movement in American education in the 1960s. The Kolmogorov curriculum was fully introduced only in the 1970s. In the way in which it was implemented it turned out to be unsuitable for the broad range of students. Their grades declined. In the early 1980s the Kolmogorov reforms were replaced with counter-reforms, led by Ivan Vindogradov and Lev Pontryagin. The legacy of Kolmogorov's reforms endures in the form of a philosophical vision of mathematics education till this day. (Received September 03, 2019)

1154-01-437 **Brigitte Stenhouse*** (brigitte.stenhouse@open.ac.uk). *Mr Mary Somerville, Husband and Secretary*.

As a nineteenth-century British woman, Mary Somerville's engagement with learned academies and polite scientific society was neither consistent nor straightforward. Whilst she was 88 before being elected a full member of any institution (the American Philosophical Institution, 1869), Somerville benefited from the resources and social networks cultivated in such spaces from as early as 1812.

Dr William Somerville, her husband, was a key mediator between herself, her scientific contemporaries, and the institutions of which he was a member. Indeed William provided Somerville with vital access to both actors and knowledge. Using the extensive correspondence held in the Somerville Collection, at the Bodleian Library in Oxford, we will investigate how William took on the roles of chaperone, secretary, and later literary agent for his wife. Moreover, we will consider how Somerville actively used her husband to liberate knowledge from behind the closed doors of learned societies, and to pursue a successful career publishing mathematical and scientific books. (Received September 04, 2019)

1154-01-706 **Janet Heine Barnett*** (janet.barnett@csupueblo.edu). *Mathematics as Discourse: A Commognitive View of Late 19th-Century Algebra*. Preliminary report.

In *Thinking as Communicating* (2008), Sfard seeks to "change our thinking about thinking" by defining thinking as an intrapersonal form of communication, coining the term "commognition" to emphasize communication and cognition as "different manifestations of basically the same phenomenon" (p. 83). "Doing mathematics," at both the individual or community level, is then the act of participating in mathematical discourse, and "learning mathematics" that of becoming a full-fledged participant in a discourse community. Discourse here is an activity regulated by two types of rules: object-level rules reflecting regularities in the behavior of the discursive (e.g., mathematical) objects, and metadiscursive rules reflecting regularities in the activities of the discursants. In this framework, "studying the history of mathematical discourses and studying the evolving discourse of the child become different versions of the same endeavor" (Sfard 2008, p. 124). Thus, while motivated by quandaries about mathematical learning at an individual level, commognitive theory also offers historians a new historiographical lens. Drawing on works by Cayley, Dedekind and Hölder, this talk explores what this lens can reveal about the development of algebra in the late 19th century. (Received September 09, 2019)

1154-01-938 **Rosie Lev-Halutz***, rosielev@gmail.com. *Victorian London's Mathematical Culture — Non-Euclidean Geometry as a Case Study*. Preliminary report.

The history of nineteenth-century British mathematics has been a widely studied field within the history of mathematics. In the second decade of the century a novel mathematical center, alternative to the previously unrivaled mathematical center of Cambridge University, emerged at London and continuously gained prominence as the century progressed. My study pursues the hitherto disregarded story of this audacious mathematical community, the progressive set of values it advocated, and its impact on the broader British mathematical community. In this lecture, by the means of the illuminating case study of non-Euclidean geometry, I analyze how these values underlay the London-based mathematical activity, and inquire into the question of the impact this community had on the development of non-Euclidean research in Britain. (Received September 12, 2019)

1154-01-985 **Jacqueline Feke*** (jfeke@uwaterloo.ca), 200 University Avenue West, Hagey Hall, Room 362, Waterloo, Ontario N2L 3G1, Canada. *Ancient Greek Mathematics as Philosophy*.

Scholars tend to assume that, just as mathematics and philosophy are distinct disciplines today, so were they in antiquity. From the fourth century B.C.E. onward, mathematicians and philosophers did differentiate themselves. They criticized each other's work and, in some areas of the Greek world, strong rivalries developed between mathematicians and philosophers. I will argue, however, that the distinction between mathematicians and philosophers did not entail that their fields of inquiry were distinct. This talk re-examines the distinction between the mathematical sciences and philosophy from the perspective of ancient Greek mathematicians. I will argue that some mathematicians viewed the relationship between these fields of inquiry as more complex, where the mathematical sciences are not only in relationship to but, even stronger, forms of philosophy in the broadest sense. The mathematical sciences are types of the love of wisdom that seek to answer some of the most fundamental questions of philosophy: e.g., how to obtain knowledge, how to form a just society, and how to attain the good life. (Received September 12, 2019)

1154-01-1017 **Mitchel T. Keller*** (keller@mathgenealogy.org). *The Mathematics Genealogy Project as a Dataset*.

Harry B. Coonce founded the Mathematics Genealogy Project in the mid-1990s when he began trying to learn more about his PhD advisor's advisor. The original idea was the Project would be a success if a database of 10,000 individuals could be built. As of September 2019, there were over 245,000 individuals with records in the MGP database spanning over 700 years. The MGP data can naturally be considered as a (directed acyclic) graph, and some graph theoretic investigations have been conducted. However, the database contains considerable additional information about each vertex (e.g., year of degree, thesis title, and Mathematics Subject Classification). Some articles analyzing these aspects of the data have been published, but accessing the MGP data for research purposes has historically been cumbersome and involved a static snapshot of the data. As of September 2019, an API is in preparation to allow more widespread access to the MGP data. This will provide researchers interested in exploring the textual data of the MGP to look for trends or clusters or means of classification with broader access to conduct their investigations. We hope that this talk will facilitate dialogue with those interested in such research to allow the API to be as useful as possible. (Received September 12, 2019)

1154-01-1140 **Carrie Diaz Eaton*** (cdeaton@bates.edu). *Finding Luz*.

This talk reflects on the process of engaging in duoethnography through biography as part of a research article for the Journal of Humanistic Mathematics special issue on Mathematics and Motherhood. This article describes the life of Dr. Luz Antonia Mendizábal Gálvez de Rodríguez, a nationally recognized educator in Peru, her path to mathematics, her family, and the legacy she left in both Peru and the United States. In the process of conducting this biographical and ethnographic research, my co-author L. Marizza Bailey and I, chose to reframe the piece as a duoethnography. This was a direct result of our discussions as we undertook this writing, which manifested as a joint exploration of our identities at the intersection of ethnicity, gender, profession, and mother. While the story of Luz is itself incredible, there are other emergent storylines. One is the comparison of author storylines - Latinx belongingness, mathematics belongingness, and the influence of family decisions. The other is a call to move beyond the narrative to actions that reclaim, value, and build on our multiple identities. (Received September 13, 2019)

1154-01-1294 **Duncan J Melville*** (dmelville@stlawu.edu), 23 Romoda Drive, Canton, NY 13617. *Arithmetic in Sum*.

Alexander Malcolm's *New System of Arithmetick* appeared in 1730, with Edward Hatton's *Intire System of Arithmetic* in 1731. DeMorgan's reviews of these works in his *Arithmetic Books* considered Malcolm's volume 'unusable' and Hatton's 'A sound, elaborate, unreadable work'. DeMorgan's criticism is not unjust, but misses the point for the modern historian. While these books are largely unreadable and doubtless also languished unread (although Hatton's was a second edition), the comprehensive nature of these massive tomes provide insight into how arithmetic was conceptualized by the intended audience and allow us to trace its perceived breadth and nature in the early 18th century. (Received September 14, 2019)

1154-01-1482 **Maria R Zack*** (mzack@pointloma.edu), MICS Department, 3900 Lomaland Drive, San Diego, CA 92106. *Quadrature, Rectification and the Cycloid*. Preliminary report.

Many well-known mathematicians of the seventeenth and eighteenth centuries studied the cycloid. These include Roberval, Descartes, Pascal, Wallis, Huygens, Fermat, Newton and Leibniz and more than one Bernoulli. This

talk will consider the work done on the cycloid by a few of these individuals and examine how their work connects to the development calculus. (Received September 15, 2019)

1154-01-1489 **Brenda Davison*** (bdavison@sfu.ca), SFU, Mathematics, SSKC 10505, 8888 University Drive, Burnaby, B.C. V5A 1S6, Canada. *Divergent Series and Numeric Computation*. Preliminary report.

In a paper published in 1856, G.G. Stokes (1819-1903) used a divergent series to compute many values of the Airy integral. Some of these values had been previously computed via a convergent series but this method was too laborious to make all of the desired calculations. This talk will examine how Stokes numerically computed a class of definite integrals, including the Airy integral, using divergent infinite series. Emphasis will be placed on what lead Stokes to use this method, what types of physical problems required these solutions, how Stokes justified using his method, and how the results obtained were verified. How, when and for what purpose did other mathematicians and physicists use this method during the mid-19th century, before divergent series were given a rigorous treatment, will also be discussed. (Received September 15, 2019)

1154-01-1648 **Laura E. Turner*** (lturner@monmouth.edu), Department of Mathematics, Monmouth University, 400 Cedar Avenue, West Long Branch, NJ 07764. *“Peculiarly accessible”: Roles of postulate theory for different mathematical publics in E.V. Huntington’s work*. Preliminary report.

In addition to publishing postulate-theoretic results directed at his research-oriented contemporaries, Harvard mathematician Edward V. Huntington (1874–1952) wrote a number of pedagogical and expository works in which he outlined the principles and aims of postulate theory, and pointed to certain roles it might serve in mathematics and well beyond. In this talk we explore some of these roles, focusing on his arguments for the pedagogical and practical value of postulate theory, and the reasons for which he sought to present this material to non-research and even *non-mathematical* publics in the first place. (Received September 16, 2019)

1154-01-1778 **Thomas Drucker*** (druckert@uw.edu), Department of Mathematics, University of Wisconsin–Whitewater, Whitewater, WI 53190. *Standing on the Shoulders of Scripture*. Preliminary report.

What is seen as an odd mixture of religion and science in the mind of Isaac Newton has played a role in many recent biographical accounts. While this mixture may be a puzzlement for those looking at Newton as mathematician, the theological side has not usually been connected with Newton’s mathematical motivations. In Rob Iliffe’s recent volume on the religious worlds of Newton, he has gone through Newton’s theological writings in great detail to understand what led Newton to pursue science and mathematics. He argues that there is a theological basis for Newton’s scientific pursuits and documents the case scrupulously. In this talk the case will be made that mathematics also supplied an internal appeal to Newton that went beyond the motivation he had from religious sources. (Received September 16, 2019)

1154-01-1887 **Nuh Aydin** (aydinn@kenyon.edu), 201 N. College Rd., Gambier, OH 43022, and **Lakhdar Hammoudi*** (hammoudi@ohio.edu), 101 University Drive, Chillicothe, OH 45601. *Al-Kashi’s Miftah al-Hisab: a 21st century translation and reading*.

The mathematics book Miftah al-Hisab written in Arabic by al-Kashi in 1427 is considered as the synthesis and culmination of Islamic progress in mathematics at the time. The translation of the Miftah into English started by the authors is now in its final stages of publication by Birkhauser: volume 1 on Arithmetic has just been published in the summer of 2019, volume 2 on Geometry is being edited, and the upcoming volume 3 on Algebra is being completed. Volumes 2, and 3 are works of the authors with Ghada Bakbouk.

Luckey’s seminal work on this monumental book has mended the course of research in history of mathematics as a whole and that of Islamic mathematics in particular. In this talk, the authors report on their findings, mainly in Arithmetic and Geometry, that should add to the edifice of a more inclusive universal history of mathematics. (Received September 16, 2019)

1154-01-1936 **Kim Plofker*** (kim_plofker@alumni.brown.edu), Department of Mathematics, Union College, 807 Union Street, Schenectady, NY 12308. *Al-Bīrūnī and the mathematics of medieval Indian poetry*. Preliminary report.

The combinatorial results of Classical Sanskrit poetic metrics have been traced from ancient texts containing early evidence of zero through some remarkable developments including binary computation and “Pascal’s triangle”. This talk will explore this subject through its Arabic presentation by the 11th-century Muslim scientist al-Bīrūnī in his treatise on India, and seek to shed light on some of its sources. (Received September 16, 2019)

1154-01-1943 **Kim Plofker*** (kim_plofker@alumni.brown.edu). *Indian mathematics and convergence of sequences and series*. Preliminary report.

Ancient Greece is well known for its seminal contributions to the study of infinite series in the work of Archimedes—and for its distrust of the subject as a source of “paradox” in the philosophy of Zeno. Here we examine a different set of approaches to the notion of a convergent sequence or series emerging in Indian exact sciences in the first millennium CE. This talk will explore sources from early medieval mathematical astronomy up through the infinite series of the second-millennium Kerala school and some later developments. (Received September 16, 2019)

1154-01-2171 **Helena Durnova*** (hdurnova@ped.muni.cz). *How differential geometry became (temporarily) obsolete in the life of Václav Hlavatý*.

In January 1950, Oswald Veblen indicated to Václav Hlavatý that the differential geometry he used to do before WWII was no longer in fashion among mathematicians. It was before the first ICM after WWII, the previous one having been held in Oslo, Norway in 1936. The wish of any mathematician of the time to participate at the congress was understandable: even mathematicians from behind the recently appearing Iron Curtain wished to participate. However, the comparison of programs of ICM in Oslo (1936) and at Harvard (1950) shows that the focus has indeed shifted. The main topic of 1950 ICM at Harvard was a new invention: the computer. Four years later, in an interview conducted in 1954, Hlavatý would still speak of pencil and paper being best for doing mathematics. However, the arrival of the compute was not the only reason for the temporary falling out of fashion of differential geometry. In my talk, I will explore how the developments in physics influenced the agenda in this field of mathematics. (Received September 17, 2019)

1154-01-2333 **Abram Kaplan*** (adkaplan@fas.harvard.edu), Society of Fellows, 78 Mt. Auburn St., Cambridge, MA 02138. *Erudition and Algebraic Practice at the End of the Sixteenth Century*.

The last two decades have seen new research on the role of erudition in early modern European science. In this talk I argue for a central role for erudition in the emergence of symbolic algebra at the end of the sixteenth century. This role is twofold. First, traditions of classical scholarship are shown to play an important role in François Viète’s epochal interpretation of Diophantus as an algebraist. I illustrate this role by comparing Viète’s interpretation of Diophantus to the pedagogical aims of sixteenth-century French algebra. Second, I show that Viète’s interpretation of his own mathematical practices owed much to traditions of erudite empiricism. I evidence this interpretation through examination of a particular theorem in Viète’s writings. Time permitting, I will indicate some seventeenth-century instances of mathematical erudition that my reading of Viète can help explain. (Received September 17, 2019)

1154-01-2571 **Julia C Tomasson*** (jct2182@columbia.edu). *How to “strain at a Gnat and swallow a Camel.” ‘The Analyst Controversy’ Reconsidered*.

Bishop George Berkeley’s (1685-1753) *The Analyst; Or, A Discourse Addressed to an Infidel Mathematician* (1734) lives on in the history of mathematics in infamy and ridicule. Berkeley’s incendiary tract was met with immediate, yet lasting, censure and is often glossed as follows: infinitesimals are nothing more than “Ghosts of departed Quantities” making the calculus incoherent at best and a metaphysical fraud at worst. In this paper I will not try to vindicate Berkeley’s critique of fluxions or metaphysical claims about infinitesimals. Nor will I exculpate him from his mathematical errors—which are as clear then as they are now. Instead, I ask a set of questions about the conditions which made the ensuing, and notably catalytic, controversy possible. I claim *The Analyst* as a text has been misunderstood; *The Analyst* was turned into a text about “Ghosts of departed Quantities” masking the true pretensions of the text in acceptable anti-Newtonian garb. I conclude that at the heart of this controversy is an image of mathematics in England post-Newton and pre-Cauchy in which mathematicians (experienced and amateur) are deeply concerned with if and when exactly they get to play by their own rules. (Received September 17, 2019)

03 ► *Mathematical logic and foundations*

1154-03-54 **Alexander S Kechris*** (kechris@caltech.edu), Department of Mathematics, California Institute of Technology, Pasadena, CA 91125. *Countable Borel equivalence relations*.

The theory of definable equivalence relations has been a very active area of research in descriptive set theory during the last three decades. It serves as a foundation of a theory of complexity of classification problems in mathematics. Another source of motivation for the theory of definable equivalence relations comes from the

study of group actions, in a descriptive, topological or measure-theoretic context, where one naturally studies the structure of the equivalence relation whose classes are the orbits of an action and the associated orbit space. An important part of this theory is concerned with the structure of countable Borel equivalence relations, i.e., those Borel equivalence relations all of whose classes are countable. It turns out that these are exactly the equivalence relations that are generated by Borel actions of countable discrete groups and this brings into this subject important connections with group theory, dynamical systems, and operator algebras. In this talk, I will give an introduction to some aspects of the theory of countable Borel equivalence relations. (Received September 02, 2019)

1154-03-227 **Russell Miller*** (russell.miller@qc.cuny.edu), Mathematics Dept. – Queens College, CUNY, 65-30 Kissena Blvd., Queens, NY 11367. *Lusin’s Theorem: How Computability Theory Proves a Real-Analysis Result.*

Lusin’s Theorem, from real analysis, states that for every Borel-measurable function f from \mathbb{R} to \mathbb{R} , and for every $\epsilon > 0$, there exists a continuous function g on \mathbb{R} such that $\{x \in \mathbb{R} : f(x) \neq g(x)\}$ has measure $< \epsilon$. This result appears in most introductory real analysis courses, and is often viewed as one of Littlewood’s Three Principles.

Here we will give a proof of Lusin’s Theorem using computability theory and computable analysis. In addition to the theorem itself, the proof will establish an effective way of producing g from f and ϵ , and will pick out, for each f , the specific measure-0 set of troublemakers x in \mathbb{R} that create all the discontinuities.

This talk will not assume any background in logic or computability theory. It will introduce a few standard computability results, bearing no obvious connection to real analysis, and by the end it will show how those results are precisely enough to establish Lusin’s Theorem. (Received August 25, 2019)

1154-03-317 **Andreas Blass*** (ablass@umich.edu), Mathematics Dept., University of Michigan, Ann Arbor, MI 48109. *Choice from finite sets — a topos view.* Preliminary report.

Work of Mostowski, Gauntt, and Truss provided finite group-theoretic characterizations for ZF-provability of implications of the form “For every set I , if every I -indexed family of sets with cardinalities in Z admits a choice function, then so does every I -indexed family of n -element sets.” Such an implication admits the topos-theoretic formulation “In any slice topos of a model of ZF, if every object of cardinality (in the internal sense) in Z has a global point, then so does every object of cardinality n .” I abbreviate this as “Slice topos of models of ZF satisfy $Z \rightarrow n$.” The group-theoretic equivalent turns out to be “Topoi of G -sets for arbitrary (finite) groups G satisfy $Z \rightarrow n$.” **Theorem:** These statements are also equivalent to “All topoi satisfy $Z \rightarrow n$.” (Received August 30, 2019)

1154-03-350 **Andrew S Marks*** (marks@math.ucla.edu). *The decomposability conjecture.*

We characterize which Borel functions are decomposable into a countable union of functions which are piecewise continuous on Π_n^0 domains. Our proof uses the consequence of determinacy that there are no definable ω_1 sequences of reals. This is joint work with Adam Day. (Received September 01, 2019)

1154-03-589 **William Chan*** (william.chan@unt.edu) and **Stephen Jackson.** *Definable Combinatorics at the First Uncountable Cardinal.*

Assume the axiom of determinacy. This talk will discuss the almost everywhere bounded sequence club uniformization for relations of the form $R \subseteq [\omega_1]^{<\omega} \times \text{club}_{\omega_1}$. This selection principle is used to investigate continuity properties for functions of the form $\Phi : [\omega_1]^{\omega_1} \rightarrow \omega_1$ and $\Phi : [\omega_1]^{\omega_1} \rightarrow \omega_1 \omega_1$. Some applications to definable cardinals below $\mathcal{P}(\omega_1)$ and to the stable theory of the partition measures will also be mentioned. (Received September 07, 2019)

1154-03-704 **Assaf Shani*** (assafshani@ucla.edu). *Friedman-Stanley jumps and Kinna-Wagner principles.*

We present a correspondence between Borel equivalence relations and symmetric models. We use a model developed by Monro (1973), in which the Kinna-Wagner principle fails, to study the second Friedman-Stanley jump. We characterize homomorphisms from the second Friedman-Stanley jump, generically, generalizing a result of Kanovei-Sabok-Zapletal about the first jump. In particular we conclude that the second Friedman-Stanley jump is in the spectrum of the meager ideal, that is, it retains its complexity on non meager sets. (Received September 09, 2019)

1154-03-768 **David Marker*** (marker@uic.edu), UIC Department of Mathematics, 322 SEO (MC 249), 851 S. Morgan St., Chicago, IL 60607. *The model theory of differential closures.*

Differentially closed fields play a role in differential algebra and differential algebraic geometry analogous to the role played by algebraically closed fields in algebraic geometry. Interestingly, the existence and uniqueness of

differential closures was first proved using model theoretic techniques and, while these proofs have been given algebraic translations they do not avoid the issues of the general model theoretic results. Differential closures also exhibit interesting phenomena not found in the classical case. I will survey these results. (Received September 10, 2019)

1154-03-774 **Victoria Gitman*** (vgitman@gmail.com), CUNY Graduate Center, Mathematics Department, 365 5th Avenue, New York, NY 10016. *Ground model definability in ZF.*

Laver, and independently Woodin, showed that a ground model is always definable in a set-forcing extension. This result gave rise to the very fruitful research topic of set theoretic geology initiated by Gunter Fuchs, Joel Hamkins, and Jonas Reitz. All known proofs of ground model definability rely heavily on the Axiom of Choice, and so it has been a long-standing open question whether ground model definability holds in ZF. With Johnstone, we adapted Laver's argument to the choiceless setting to show that a universe satisfying $ZF + DC_\delta$, for a cardinal δ , is definable in its set-forcing extensions by partial orders admitting a gap at δ - having the form $\mathbb{R} * \dot{\mathbb{Q}}$, where \mathbb{R} has size $< \delta$ and $\dot{\mathbb{Q}}$ is forced to be $\leq \delta$ -strategically closed. Recently, Usuba showed that if a ZF-universe has a proper class of Löwenheim-Skolem cardinals, a notion he introduces, then it is definable in its set-forcing extensions. Many naturally arising ZF-universes, such as symmetric models and universes over which choice is forceable, in particular $L(\mathbb{R})$, have a proper class of Löwenheim-Skolem cardinals. In the talk, I will survey these partial results. (Received September 12, 2019)

1154-03-825 **Gilles Dowek*** (gilles.dowek@ens-paris-saclay.fr). *Logipedia: towards a Wikipedia of formal proofs.*

Formal computerized proofs are now a central tool in computer science and in mathematics. But, each system – Coq, HOL Light, Isabelle/HOL, PVS... – implements its own language and its own theory, limiting the interoperability between systems and the sustainability of these proofs. Logipedia is an, in progress, encyclopedia of formal proofs, expressed in various theories. It is based on the idea to express these theories in a new logical framework allowing bound variables, explicit proof-terms, computation rules, and peaceful co-existence of constructive and non constructive proofs. (Received September 11, 2019)

1154-03-829 **Philipp Schlicht*** (philipp.schlicht@bristol.ac.uk), School of Mathematics, University of Bristol, Fry Building, Woodland Road, Bristol, BS81UG, United Kingdom. *Forcing over Cohen's symmetric model.*

Cohen constructed a choiceless symmetric model from a sequence of Cohen reals. In this model, there is an infinite Dedekind-finite set, i.e. no proper subset surjects onto it. We study new forcing phenomena that appear in this setting and use them to answer the following questions:

1. How large can an infinite Dedekind-finite set be in generic extensions?
2. Can an infinite Dedekind-finite set be collapsed by adding a Cohen subset?

By collapsing a set we mean adding a surjection from a subset where none existed previously. This is joint work with Asaf Karagila. (Received September 11, 2019)

1154-03-839 **Michael C. Laskowski*** (mc1@math.umd.edu), Department of Mathematics, University of Maryland, College Park, MD 20782. *Groundedness of infinitary sentences.* Preliminary report.

For a class $K = \text{Mod}(\Phi)$ for $\Phi \in L_{\omega_1, \omega}$, a *potential canonical Scott sentence* is a sentence $\sigma \in L_{\infty, \omega}$ such that, in some forcing extension $V[G]$ in which $\sigma \in L_{\omega_1, \omega}$, there is a countable $M \models \Phi$ whose canonical Scott sentence is σ .

Call such a class K *grounded* if every potential canonical Scott sentence σ for K is the canonical Scott sentence of some N in the ground universe V . Groundedness of K is desirable, as it is typically easier to count the number of canonical Scott sentences than the number of potential canonical Scott sentences, and by results in [URL], the latter number cannot decrease via a Borel embedding.

We discuss examples and non-examples of groundedness.

[URL] D. Ulrich, R. Rast, and M.C. Laskowski, Borel complexity and potential canonical Scott sentences. *Fund. Math.* **239** (2017), no. 2, 101–147. (Received September 11, 2019)

1154-03-878 **Victoria Gitman*** (vgitman@gmail.com), CUNY Graduate Center, Mathematics Department, 365 5th Avenue, New York, NY 10016. *Toy multiverses of set theory.*

Modern set theoretic research has produced a myriad of set-theoretic universes with fundamentally different properties and structures. Multiversists hold the philosophical position that none of these universes is the true universe of set theory - they all have equal ontological status and populate the set-theoretic multiverse. Hamkins,

one of the main proponents of this view, formulated his position via the heuristic Hamkins Multiverse Axioms, which include such radical relativity assertions as that any universe is ill-founded from the perspective of another universe in the multiverse. With Hamkins, we showed that the collection of all countable computably saturated models of ZFC satisfies his axioms. Countable computably saturated models form a unique natural class with a number of desirable model theoretic properties such as existence of truth predicates and automorphisms. Indeed, any collection of models satisfying the Hamkins Multiverse Axioms must be contained within this class. In a joint work with Toby Meadows, Michał Godziszewski, and Kameryn Williams, we explore which weaker versions of the multiverse axioms have ‘toy multiverses’ that are not made up entirely of computably saturated models. (Received September 11, 2019)

1154-03-897 **Ioannis Soudatos***, souldaio@udmercy.edu. *Local Hanf Numbers, Kurepa Trees, and Limit Characterizable Cardinals.*

About 30 years ago, Shelah conjectured that if $\aleph_{\omega_1} < 2^{\aleph_0}$, then any $\mathcal{L}_{\omega_1, \omega}$ -sentence with models of size \aleph_{ω_1} also has models of size 2^{\aleph_0} . He called \aleph_{ω_1} the *local Hanf number* below 2^{\aleph_0} .

His conjecture is equivalent to the statement “If $\aleph_{\omega_1} \leq \kappa < 2^{\aleph_0}$, then no $\mathcal{L}_{\omega_1, \omega}$ -sentence can have model-existence spectrum $[\aleph_0, \kappa]$ or $[\aleph_0, \kappa)$ ”.

Call κ a *characterizable* cardinal, if there exists an $\mathcal{L}_{\omega_1, \omega}$ -sentences with spectrum $[\aleph_0, \kappa]$, and *limit characterizable* cardinal, if κ is a limit cardinal and there exists an $\mathcal{L}_{\omega_1, \omega}$ -sentences with spectrum $[\aleph_0, \kappa)$. Although characterizable cardinals has been studied before, very little is known for limit characterizable cardinals.

In the lecture we will present some recent results about limit characterizable cardinals and their connection with the local Hanf numbers below 2^{\aleph_1} and $2^{2^{\aleph_0}}$. (Received September 11, 2019)

1154-03-911 **Alexandra Shlapentokh*** (shlapentokha@ecu.edu), Department of Mathematics, East Carolina University, Greenville, NC 27858. *What is the difference between sets of primes?* Preliminary report.

We will consider the following question: what are the interesting properties of infinite sets of primes, and what determines whether an infinite set of primes has a particular property. We will start with an obvious fact that each set of primes is Turing equivalent to a set of positive integers, and therefore sets of primes inherit the Turing degree structure of \mathbb{N} . We will next note that each set of primes also corresponds to a subring of \mathbb{Q} and is also Turing equivalent to the set of elements of the ring. Now these rings possess different arithmetic and logic properties that we can now attach to the sets of primes. Some of the questions we will discuss is whether two sets of primes differing by a finite number of elements, always have the same properties, and whether some logic properties of the sets of primes are equivalent to some arithmetic properties of these sets. (Received September 11, 2019)

1154-03-945 **Iian B. Smythe*** (i.smythe@rutgers.edu), Department of Mathematics, Rutgers University - New Brunswick, Piscataway, NJ 08854. *Parametrizing the Ramsey theory of block sequences.*

We will discuss recent work on parametrized versions of Ramsey-theoretic dichotomies for block sequences in infinite-dimensional vector spaces, and its connections to preservation of ultrafilter-like objects when adding Sacks reals. This aims to generalize versions of the Galvin-Prikry and Silver dichotomies parametrized by perfect sets, and the corresponding preservation results for selective ultrafilters. (Received September 12, 2019)

1154-03-1029 **Thomas Hales***, Math Department, 416 Thackeray Hall, University of Pittsburgh, Pittsburgh, PA 15260. *The Formalization of Mathematics and Controlled Natural Language.*

By a controlled natural language for mathematics, we mean an artificial language for the communication of mathematics that is deliberately designed with precise computer-readable syntax and semantics, is based on a single natural language (which for us will be English), and is broadly understood at least in an intuitive way by mathematically literate speakers of the natural language.

We have recently designed a controlled natural language for mathematics. Text written in this language may almost appear to be ordinary mathematical text, but the grammar is precisely specified and computer readable, eventually as expressions in the Lean theorem prover.

This work achieves a fusion of three linguistic traditions for representing mathematics: $\text{L}^{\text{A}}\text{T}_{\text{E}}\text{X}$, dependent type theory, and a long history of research into controlled natural language.

This talk will give an overview of research in computer-checked formal verification of mathematics, will present a controlled natural language for mathematics, and will discuss probable applications of this work. This project

is part of the Formal Abstracts initiative, which will produce an enormous collection of mathematical definitions and theorem statements in a form that is both human and computer readable. (Received September 12, 2019)

1154-03-1084 **Logan Crone*** (logancrone@my.unt.edu), **Lior Fishman** and **Stephen Jackson**.
Determinacy of Schmidt's Game and other Intersection Games.

Schmidt's game, and other similar intersection games have played an important role in recent years in applications to number theory, dynamics, and Diophantine approximation theory. These games are real games, that is, games in which the players make moves from a complete separable metric space. The determinacy of these games trivially follows from the axiom of determinacy for real games, $\text{AD}_{\mathbb{R}}$, which is a much stronger axiom than that asserting all integer games are determined, AD . We show that the determinacy of certain real intersection games that satisfy a strategy simplification hypothesis follows from AD alone, and show that Schmidt's game on \mathbb{R} satisfies the strategy simplification hypothesis. (Received September 13, 2019)

1154-03-1217 **Ruiyuan Chen*** (ruiyuan@illinois.edu). *Polish groupoids and $\mathcal{L}_{\omega_1\omega}$ -theories.*

It is well-known that every non-Archimedean Polish group is the automorphism group of a countable first-order structure. Analogously, every Polish group is the automorphism group of a separable metric structure. We prove multi-object generalizations of these results: every open non-Archimedean Polish groupoid is Borel equivalent to the groupoid of models on \mathbb{N} of some $\mathcal{L}_{\omega_1\omega}$ -sentence, while every open Polish groupoid is Borel equivalent to the groupoid of metric models on the Urysohn sphere \mathbb{U} of some $\mathcal{L}_{\omega_1\omega}$ -sentence in continuous logic. Moreover, in the discrete case, we can recover every theory from its groupoid of models, so that we have a complete correspondence between countable (discrete) $\mathcal{L}_{\omega_1\omega}$ -theories and open non-Archimedean Polish groupoids. (Received September 13, 2019)

1154-03-1223 **Cameron E. Freer*** (freer@mit.edu), 77 Massachusetts Ave. 46-5121, Cambridge, MA 02139. *Representation theorems for exchangeable structures: a computability-theoretic perspective.*

Exchangeability and related hypotheses describe the symmetries under which random sequences, arrays, and other structures are invariant. Classical theorems of probability theory due to de Finetti, Aldous, Hoover, Kallenberg, and others characterize the conditional independence that such structures must exhibit, and provide explicit ergodic decompositions. In this talk, we explore the computable content of these theorems, providing both positive and negative results. We also discuss some motivation from the theory of probabilistic programming languages. Joint work with Nathanael Ackerman, Jeremy Avigad, Daniel Roy, and Jason Rute. (Received September 14, 2019)

1154-03-1264 **Andrew Marks*** (marks@math.ucla.edu). *A constructive solution to Tarski's circle squaring problem.*

In 1925, Tarski posed the problem of whether a disc in \mathbb{R}^2 can be partitioned into finitely many pieces which can be rearranged by isometries to form a square of the same area. Unlike the Banach-Tarski paradox in \mathbb{R}^3 , it can be shown that two Lebesgue measurable sets in \mathbb{R}^2 cannot be equidecomposed by isometries unless they have the same measure. Hence, the disk and square must necessarily be of the same area.

In 1990, Laczkovich showed that Tarski's circle squaring problem has a positive answer using the axiom of choice. We give a completely constructive solution to the problem and describe an explicit (Borel) way to equidecompose a circle and a square. This answers a question of Wagon.

Our proof has three main ingredients. The first is work of Laczkovich in Diophantine approximation. The second is recent progress in a research program in descriptive set theory to understand how the complexity of a countable group is related to the complexity of the equivalence relations generated by its Borel actions. The third ingredient is ideas coming from the study of flows in networks.

This is joint work with Spencer Unger. (Received September 14, 2019)

1154-03-1280 **John Harrison*** (jrh013@gmail.com). *Automated Reasoning: retrospective and current progress.*

The subject of "automated reasoning" or "automated theorem proving" has been around since pioneering experiments in the 1950s on early computers. Since that time, it's undergone a few boom-and-bust cycles where unbridled optimism about checking mathematical proofs, proving correctness of computer systems and perhaps even finding new mathematical results has alternated with lulls and spells of disillusionment. Nevertheless there has been steady progress in the field, with some substantial recent achievements in the formalization of mathematics (e.g. the Flyspeck and Odd Order Theorem projects) and proofs of computer correctness (e.g. seL4

microkernel and CompCert compiler). In this talk I will give a broad survey of how the field has developed and talk about interesting recent developments. (Received September 14, 2019)

1154-03-1423 **Natasha Dobrinen*** (natasha.dobrinen@du.edu), University of Denver, Dept. of Mathematics, 2390 S. York St., C.M. Knudson Hall, Room 300, Denver, CO 80208. *Logic and Combinatorics*.

Combinatorics is a vast subject with connections to many areas of mathematics. Important problems in general mathematics often turn out to have combinatorial content as the core problem. Ramsey's Theorem was actually motivated by a problem in logic posed by Hilbert: Is there a way to decide exactly when a logic formula is valid? Currently, there are multiple facets of logic informing and providing methods for solving problems in combinatorics. Conversely, problems in logic are leading to progress in combinatorics. We will sample some highlights of the beautiful historical and modern interactions between Ramsey theory and logic, how progress in each field has motivated and helped solve problems in the other, as well as in other areas of mathematics. (Received September 15, 2019)

1154-03-1433 **Shay Allen Logan*** (salogan@ksu.edu). *Hyperdoctrines in Relevant Logic*.

An implicit assumption of much of the work in categorial logic in the half century since Lawvere's *Adjointness in Foundations* is that quantification, correctly done, should give rise to a type of structure known as a *hyperdoctrine*. From this perspective, Kit Fine's *stratified semantics* for quantified relevance logics looks like an outlier.

There are two purposes of this talk: first is to provide a brief but useful overview of the basics of hyperdoctrines. The second is to demonstrate a correspondence between hyperdoctrines and stratified models that can be used to prove completeness of a certain relevant logic with respect to a class of hyperdoctrines.

Concretely, we will show that corresponding to each stratified model S is a functor H_S mapping the category $\mathbf{FinSet}_{\subseteq}$ to a category of frames and frame morphisms. Quantification, as usual, is interpreted via adjoints to (images of) inclusions, and various logical properties are preserved by imposing versions of the Beck-Chevalley conditions. We will sketch how to prove that for all sentences ϕ , $S \models \phi$ iff $H_S \models \phi$. (Received September 15, 2019)

1154-03-1435 **Elliot Kaplan*** (eakapla2@illinois.edu), Department of Mathematics, University of Illinois at Urbana-Champaign, Urbana, IL 61801. *An introduction to HT-fields*.

In this talk, I will introduce the class of HT -fields. Let T be an o-minimal theory extending the theory of ordered fields and let K be a model of T which is also equipped with a nontrivial derivation $x \mapsto x'$ making it an H -field (a particularly nice type of ordered differential field). We require that this derivation interact nicely with the o-minimal structure on K . For example, if K is elementarily equivalent to the real exponential field, we require that $\exp(a)' = \exp(a)a'$ for all $a \in K$. If these conditions are met, we say that the expansion of K by this derivation is an HT -field. The class of H -fields has been thoroughly explored by Aschenbrenner, van den Dries, and van der Hoeven. I will establish some analogues of their results on H -fields for the class of HT -fields. (Received September 15, 2019)

1154-03-1436 **Alexi T Block Gorman*** (atb2@illinois.edu), 1409 W Green Street, Urbana, IL 61801. *Companionability Characterization for the Expansion of an O-minimal Theory by a Dense Subgroup*.

Let T be a complete o-minimal theory expanding the theory of groups, and let \mathcal{L} be the language in which T admits quantifier elimination. Let $T_{\mathcal{G}}$ be the expansion of T by a unary predicate \mathcal{G} that picks out a dense and codense subgroup. I provide a full characterization for when $T_{\mathcal{G}}$ has a model companion. This result is motivated by questions raised in recent works concerning preservation results in model companions for some neostability properties. I restrict my attention to the o-minimal setting because this permits a full and geometric characterization for companionability. I supply examples both in which the predicate is an additive subgroup, and where it is a multiplicative subgroup. I conclude with a brief discussion of neostability properties, and give examples that illustrate the lack of preservation for properties such as strong, NIP, and NTP_2 , though there are also examples for which the model companion preserves NTP_2 . (Received September 15, 2019)

1154-03-1514 **Linda Brown Westrick*** (westrick@psu.edu). *Computation and universality in multidimensional symbolic dynamics*.

In symbolic dynamics, the simplest systems to describe are the shifts of finite type (SFTs). However, a simple description does not guarantee simple behavior. It is well-known (but not currently well-exploited) that arbitrary computations can be embedded into the dynamics of \mathbb{Z}^2 -SFTs. As a result, these "simple" shifts often have behavior just as rich as unrestricted topological dynamical systems. We present a recent example of this

phenomenon: the property of topological completely positive entropy is no simpler in \mathbb{Z}^2 -SFTs than in general topological dynamical systems. (Received September 16, 2019)

1154-03-1517 **Linda Brown Westrick*** (westrick@psu.edu). *Borel sets in reverse mathematics.*

Reverse mathematics is a program that seeks to answer the question: which set-existence axioms are needed to prove theorems of ordinary mathematics? However, even the simplest theorems about Borel sets – that they have the property of Baire and are measurable – had not been analyzed in a satisfactory way. The standard definition of a Borel set in reverse mathematics has the axiom of arithmetic transfinite recursion (ATR) already baked into it. Thus no interesting reversals could be obtained for theorems such as the above, which are usually proved using only ATR. We propose a new definition for a Borel set in reverse mathematics, that of a *completely determined* Borel set. Using this definition, we find that “Every Borel set has the property of Baire” and “Every Borel set is measurable” are strictly weaker than ATR but do imply the existence of Δ_1^1 -generics and Δ_1^1 -randoms respectively. The same techniques permit a tighter analysis of the Borel dual Ramsey theorem for 3-partitions, and are beginning to populate a previously uncharted territory of the reverse mathematics universe. (Received September 16, 2019)

1154-03-1536 **Joachim Mueller-Theys*** (mueller-theys@gmx.de). *Metalogical Extensions.* Preliminary report.

Non-theoremhood—as with $x \circ y = y \circ x$ and group theory—cannot be formalised by logics like FOL. This has motivated adding new formulae $\Theta(\alpha)$ and defining a *metalogical extension* \mathcal{S} to be a conservative extension of the given consequence, having tautologies, modus ponens, replacement, and the specific properties:

$\Sigma \vdash \phi$ implies $\Sigma \mathcal{S} \Theta(\phi)$, and

$\Sigma \not\vdash \phi$ implies $\Sigma \mathcal{S} \neg \Theta(\phi)$,

reflecting the ways theorems and non-theorems are usually established.

We could realise \mathcal{S} deductively and semantically, and it turned out that there exists only one ME, whence, in particular, $\Sigma \vdash \alpha$ iff $\Sigma \models \alpha$.

Further information, especially regarding the relation to modal logic (where $\Box\alpha$ appears as a notational variant of $\Theta(\alpha)$) and to immanent attempts of self-representation (by formulae $\phi = \theta(\ulcorner \alpha \urcorner)$), can be found e. g. in the ASL-abstract and *The Bulletin of Symbolic Logic* **25** (2019), p. 272. (Received September 16, 2019)

1154-03-1565 **Natasha Dobrinen*** (natasha.dobrinen@du.edu), University of Denver, Dept. of Mathematics, 2390 S York St, C.M. Knudson Hall, Room 300, Denver, CO 80208, and **Daniel Hathaway** (daniel.hathaway@uvm.edu), University of Vermont, Department of Mathematics and Statistics, 787 Williams Hill Rd., Burlington, VT 05405. *Barren extensions.*

In their paper, “A barren extension,” Henle, Mathias, and Woodin showed that, assuming large cardinals, forcing with $([\omega]^\omega, \subseteq^*)$ over $L(\mathbb{R})$ adds no new sets of ordinals and preserves strong partition cardinals. Forcing with $([\omega]^\omega, \subseteq^*)$ produces an ultrafilter which is Ramsey: For each coloring of pairsets of ω into finitely many colors, there is a member of the ultrafilter which is homogeneous for the coloring. We asked, how special is the Ramsey-ness of the ultrafilter to these results? It turns out, not that much. The proofs really hinge on the availability of infinite dimensional Ramsey theory and certain other combinatorial properties of the forcing. We prove that a large class of forcings which add ultrafilters with weak partition properties yield barren extensions of $L(\mathbb{R})$. These include forcings of Baumgartner and Taylor adding ultrafilters with asymmetric partition relations, forcings of Laflamme adding weak hierarchies above weakly Ramsey ultrafilters, as well as many others. (Received September 16, 2019)

1154-03-1714 **Wesley H. Holliday*** (wesholliday@berkeley.edu). *Axiomatizing reasoning about sets: cardinality, mereology, and decisiveness.*

In this talk, I give three examples of axiomatizing reasoning about sets in special purpose languages. First, I consider reasoning about comparative cardinality: $A \geq B$ if there is an injection from B to A . I add principles to Boolean algebra to axiomatize reasoning not only about Boolean operations but also about \geq . Second, I consider reasoning about the subset relation (“set-theoretic mereology”) in a modal language: $\Diamond\varphi$ is true at a set A if there is a nonempty $B \subseteq A$ such that φ is true at B . I discuss the longstanding open problem of giving a recursive axiomatization of the set of validities for finite sets. Finally, I give an example outside of pure mathematics from voting theory: a set A of voters is decisive over candidates x, y if whenever all voters in A prefer x to y , society must rank x above y . I present an axiomatization of reasoning about decisive sets of voters for voting methods satisfying well-known axioms. These examples are meant to illustrate a methodology familiar to modal logicians: to better understand the core principles governing some mathematical concept, try

to axiomatize the validities of a lean language with dedicated operators whose semantics is given by the target concepts. (Received September 16, 2019)

1154-03-1732 **Karen M. Lange*** (karen.lange@wellesley.edu), Wellesley College, 106 Central St., Wellesley, MA 02481. *Different problems, common threads: Computing the difficulty of mathematical problems.*

Mathematics is filled with existence theorems such as “every vector space has a basis”. Such statements do not address how one goes about *finding* the known-to-exist object. The prior theorem naturally leads to the question “given a vector space, can we compute a basis for it?”. The answer to this “Basis Problem” is no, so we say that the problem is not “computable”.

We can further ask just how far from computable the Basis Problem is and what other problems have the same computational power. A natural way to compare the algorithmic difficulty of two problems is to determine whether having the ability to solve one allows for the solution of the other. Under this problem-reduction approach, two problems have the same computational power if each can be used to solve the other.

In this talk, we will explore the key ideas behind taking a “computable” perspective on mathematics and how this compares to an “existence” one. We will illustrate the problem-reduction approach using theorems from across mathematics. The overall structure of problem-difficulty is extremely rich and helps to illuminate what makes problems “tick”. Moreover, this approach has strong connections to calibrating exactly which axioms are needed to prove the original existence theorems. (Received September 16, 2019)

1154-03-1804 **William DeMeo*** (williamdemeo@gmail.com). *Formalizing Universal Algebra with Dependent and Inductive Types.* Preliminary report.

We introduce some practical aspects of our recently initiated project, “Formal Foundations for Informal Mathematics,” a goal of which is to formalize universal algebra in the Lean proof assistant.

We have implemented a small but growing collection of basic objects and theorems of universal algebra in Lean, exhibiting some of the power of inductive and dependent types. We review these recent developments and give a brief overview of the lean-ualib (our fledgling Lean library for universal algebra). (Received September 16, 2019)

1154-03-1819 **Margaret E. M. Thomas*** (memthomas@purdue.edu), Department of Mathematics, Purdue University, N. University St., West Lafayette, IN 47907-2067. *Parameterization, o-minimality and counting points.*

The geometric tool of smooth parameterization has important diophantine consequences, and was central to the proof of the Pila–Wilkie Theorem connecting o-minimality and diophantine geometry. It remains an important approach in the ongoing pursuit of refinements to this key theorem. In this talk, I will survey a number of results on different types of smooth parameterization in the o-minimal setting, focussing in particular on questions of definability and effectivity, as well as discuss some applications to point counting results. (Received September 16, 2019)

1154-03-2038 **John Temidayo Falade*** (jtf037@shsu.edu). *Completeness Theorem for Zero-Order Logic.* Preliminary report.

Zero-order logic consists of the fragment of first-order logic with no quantifier symbols. We prove the completeness theorem for zero-order languages that contain only the predicate symbol of “=” which we call the equality symbol. We conclude with a sketch of the completeness theorem for general first-order logic. (Received September 17, 2019)

1154-03-2040 **Rehana Patel*** (rpatel@aims-senegal.org). *What does a choiceless model theory look like?*

I will make some remarks suggesting that the search for a “choiceless model theory” leads to invariant measures, i.e., to probability distributions on a space of structures, invariant under the logic action. (Received September 17, 2019)

1154-03-2052 **Rehana Patel*** (rpatel@aims-senegal.org). *Exchangeable constructions via model theory.*

The Erdős–Rényi “coin flip” construction of the countable infinite random graph is *exchangeable*, in the sense that the resulting distribution does not depend on the order in which edges are decided. In a 2016 paper Ackerman, Freer and I characterised those countable infinite structures that have exchangeable constructions. Our method generalises one used by Petrov and Vershik (2010) to produce exchangeable constructions of Henson’s

homogeneous-universal K_n -free graphs; it involves building a “probabilistic Henkin structure” that, in the case of graphs, is what is known as a *graphon*.

In this talk I will discuss how the model-theoretic perspective enables such a generalisation, providing a broad view of the phenomenon of exchangeability. No prior knowledge of model theory will be assumed, and all definitions will be explained. The talk will also serve as a prelude, for the non-logician, to Freer’s AMS-ASL plenary, which will look at exchangeability from a computability-theoretic perspective. (Received September 17, 2019)

1154-03-2399 **Jack H. Lutz*** (lutz@iastate.edu). *Passing Hilbert’s Final Test.*

We review recent developments in which computability-theoretic *dimensions of individual points* have been used to answer open questions in *classical* geometric measure theory, questions whose statements do not involve computability theory or logic. (Received September 17, 2019)

1154-03-2467 **Sonia Navarro Flores***, sonian@matmor.unam.mx. *Borel ideals and topological Ramsey spaces.*

It is known that the partial order $P(\omega)/Fin$ forces a Ramsey ultrafilter. This quotient has several interesting properties, for example, $(P(\omega)/Fin, \subseteq)$ is forcing equivalent to the Ellentuck space with the almost inclusion order, which is the prototypical example of a topological Ramsey space. Also, Todorćević proved that under large cardinal assumptions every Ramsey ultrafilter is generic for $(P(\omega)/Fin, \subseteq)$. In this talk we present some advances done in order to classify those Borel ideals \mathcal{I} such that the quotient $P(\omega)/\mathcal{I}$ is forcing equivalent to a topological Ramsey space. This classification is interesting because the Ramsey structure inside those topological Ramsey spaces is crucial to understand combinatorial properties for such ideals and ultrafilters forced by those quotients. (Received September 17, 2019)

05 ► Combinatorics

1154-05-59 **Emma Farnsworth*** (efarnsworth3@fingerlakes.edu), Department of Mathematics, University of Rochester, Rochester, NY 14627, **Natalie Gomez** (12nat26@gmail.com), Department of Mathematics, Texas State University, San Marcos, TX, and **Herlandt Lino** (hx11249@g.rit.edu), School of Mathematical Sciences, Rochester Institute of Technology, Rochester, NY 14623. *Characterizing Nearly Asymmetric Graphs.*

A graph is asymmetric if its automorphism group of vertices is trivial. Asymmetric graphs were first studied by Erdős and Rényi in 1963. Following their paper, we define the asymmetric index of a graph, denoted $ai(G)$, to be the minimum value of $r + s$, where r is the number of edges removed from a graph and s is the number of edges added to a graph, so that the resulting graph is asymmetric. Furthermore, we investigate properties of asymmetric graphs and consider the larger problem of determining all non-isomorphic asymmetric graphs that can be created by adding and/or removing the minimum number of edges. (Received July 26, 2019)

1154-05-60 **Marina Jacobo*** (mjacob01@pride.hofstra.edu), Department of Mathematics, Hofstra University, Hempstead, NY 11549, and **David Shuster** (dshuster21@amherst.edu), Department of Mathematics and Statistics, Amherst College, Amherst, MA 01003. *Large Rank Numbers & $(K_s - e) \times P_n$.*

A k -ranking of a graph G is a function $f : V(G) \rightarrow \{1, 2, \dots, k\}$ such that if $f(u) = f(v)$ then every uv path contains a vertex w such that $f(w) > f(u)$. The rank number of G , denoted $\chi_r(G)$, is the minimum k such that a k -ranking exists for G . The rank number is a variant of graph colorings. It is known that given a graph G and a positive integer t the question of whether $\chi_r(G) \leq t$ is NP-complete. The characteristics of any n -vertex graph whose rank number is equal to $n - 1$ or $n - 2$ is known; in this talk we extend this question to $n - 3$. Also, we examine the extremal graphs such that their rank number is equal to n , $n - 1$, $n - 2$ and $n - 3$.

The ranking of $K_s \times P_n$ has been previously studied, and a recursive formula for $\chi_r(K_s \times P_n)$ has been established. In this talk, we study the ranking of $K_s - e \times P_n$. We establish the rank number of $K_s - e \times P_n$ for even $s \geq 4$ and provide a conjecture for $\chi_r(K_n - e \times P_n)$ for odd $s \geq 5$. (Received July 26, 2019)

1154-05-61 **Ryan Kannanaikal*** (rk635@cornell.edu), Department of Mathematics, Cornell University, Ithaca, NY 14853, and **Noah Krakoff** (nkrakoff@berkeley.edu), Department of Mathematics, University of California, Berkeley, Berkeley, CA 94704. *On the Ramsey Number $R(K_4 - e, K_8 - e)$.*

The Ramsey number $R(G, H)$ is the smallest integer n such that every graph on n vertices contains G as a subgraph or its complement contains H as a subgraph. We study (J_4, J_k) -graphs, where $J_k = K_k - e$ is the

complete graph on k vertices missing one edge. Note that J_4 is a pair of triangles sharing an edge. Thus, avoiding J_4 is less restrictive than the well studied case of avoiding triangles in Ramsey theory. Using a combination of theoretical and computational techniques we study properties of J_4 -free graphs, and thus the structure of lower bound witnesses for $R(J_4, J_k)$. We considered the first unknown case, namely $k = 8$. The previous bounds were $28 \leq R(J_4, J_8) \leq 38$, having been unchanged since 1998. Using our techniques we have improved the upper bound to 37 by exploiting the rich algebraic structure of the induced subgraphs a possible witness must contain. (Received July 27, 2019)

1154-05-69

Natasha Crepeau* (ncrepeau@hmc.edu), **Pamela E. Harris**, **Sean Hays**, **Marissa Loving**, **Joseph Rennie**, **Gordon Rojas Kirby** and **Alexandro Vasquez**. *On (t, r) broadcast domination of certain grid graphs.*

Let $G = (V(G), E(G))$ be a connected graph with vertex set $V(G)$ and edge set $E(G)$. We say a subset D of $V(G)$ dominates G if every vertex in $V \setminus D$ is adjacent to a vertex in D . A generalization of this concept is (t, r) broadcast domination. We designate certain vertices to be towers of signal strength t , which send out signal to neighboring vertices with signal strength decaying linearly as the signal traverses the edges of the graph. We let \mathbb{T} be the set of all towers, and we define the signal received by a vertex $v \in V(G)$ from a tower $w \in \mathbb{T}$ to be $f(v) = \sum_{w \in \mathbb{T}} \max(0, t - d(v, w))$. Blessing, Insko, Johnson, Mauretour (2014) defined a (t, r) broadcast dominating set, or a (t, r) broadcast, on G as a set $\mathbb{T} \subseteq V(G)$ such that $f(v) \geq r$ for all $v \in V(G)$. The minimal cardinality of a (t, r) broadcast on G is called the (t, r) broadcast domination number of G . In this talk, we present our research on the (t, r) broadcast domination number for certain graphs including paths, grid graphs, the slant lattice, and the king's lattice. (Received July 30, 2019)

1154-05-134

Mozhgan Mirzaei* (momirzae@ucsd.edu) and **Andrew Suk**. *Extremal Configurations in Point-Line Arrangements.* Preliminary report.

The famous Szemerédi-Trotter theorem states that any arrangement of n points and n lines in the plane determines $O(n^{4/3})$ incidences, and this bound is tight. In this talk, we present some Turán-type results for point-line incidences. Let \mathcal{L}_1 and \mathcal{L}_2 be two sets of t lines in the plane and let $P = \{\ell_1 \cap \ell_2 : \ell_1 \in \mathcal{L}_1, \ell_2 \in \mathcal{L}_2\}$ be the set of intersection points between \mathcal{L}_1 and \mathcal{L}_2 . We say that $(P, \mathcal{L}_1 \cup \mathcal{L}_2)$ forms a natural $t \times t$ grid if $|P| = t^2$, and $\text{conv}(P)$ does not contain the intersection point of some two lines in \mathcal{L}_i , for $i = 1, 2$. For fixed $t > 1$, we show that any arrangement of n points and n lines in the plane that does not contain a natural $t \times t$ grid determines $O(n^{\frac{4}{3}-\varepsilon})$ incidences, where $\varepsilon = \varepsilon(t)$. We also provide a construction of n points and n lines in the plane that does not contain a natural 2×2 grid and determines at least $\Omega(n^{1+\frac{1}{14}})$ incidences. This is joint work with Andrew Suk. (Received August 13, 2019)

1154-05-162

Colin Defant* (cdefant@princeton.edu). *Duluthian Developments in the Anti-Powerful Words World.*

Four years ago, Fici, Restivo, Silva, and Zamboni defined a k -anti-power to be a word of the form $w^{(1)} \dots w^{(k)}$, where $w^{(1)}, \dots, w^{(k)}$ are distinct words that have the same length. Since then, anti-powers have received vigorous attention, most of which has spawned from the Duluth REU Program. In this talk, we survey these Duluthian developments and highlight some unsolved mysteries. We begin with my own work on anti-power prefixes of the Thue-Morse word, which was subsequently extended by Narayanan and generalized to “ j -fixes” by Gaetz. Our journey continues with the work of Burcroff, who generalized the notion of an anti-power to that of a “block-pattern” and extended many of Fici, Restivo, Silva, and Zamboni’s results to that setting. Finally, we explore my work with Berger, in which we settle one of Fici, Restivo, Silva, and Zamboni’s open problems. Namely, we prove that 5 is the largest integer κ such that every aperiodic recurrent word contains a κ -anti-power. We pose a conjecture about the lengths of anti-powers appearing in morphic words, which we have proven for binary words generated by uniform morphisms. Garg has removed the hypothesis that these words are binary, and he has also proven our conjecture for the Fibonacci word. (Received August 16, 2019)

1154-05-169

Colin Defant* (cdefant@princeton.edu), **Michael Engen** and **Jordan A Miller**. *Uniquely Sorted Permutations and Fertility Numbers.*

The fertility of a permutation κ is $|s^{-1}(\kappa)|$, where s denotes West’s stack-sorting map. A permutation is uniquely sorted if its fertility is 1. It turns out that there are no uniquely sorted permutations of even length. We show that uniquely sorted permutations of odd length are counted by Lassalle’s sequence; this mysterious sequence emerged when Lassalle used algebraic methods to prove that its terms are positive and increasing, settling a conjecture of Zeilberger. Our enumeration of uniquely sorted permutations follows from a more general bijection that links new combinatorial objects called “valid hook configurations” with weighted set partitions and cumulants appearing in free probability theory. We also briefly discuss some bijective enumerations of pattern-avoiding

valid hook configurations and uniquely sorted permutations, including recent (separate) works of Mularczyk and Sankar. We then turn our attention to the set of nonnegative integers that arise as fertilities of permutations, revealing some unexpected patterns and several open problems. (Received August 17, 2019)

1154-05-171 **Alex Iosevich*** (iosevich@gmail.com), 145 Dunrovin Lane, Rochester, NY 14618.
Distance sets in discrete and continuous settings.

We shall discuss some recent advances in the study of distances sets in discrete and continuous cases, as well as connections between them. (Received August 17, 2019)

1154-05-173 **Jakob Ablinger*** (jablinge@risc.jku.at). *Proving Infinite Pochhammer Sum Identities.*
We consider nested sums involving the Pochhammer symbol at infinity and rewrite them in terms of a small set of constants, such as powers of π , $\log(2)$ or zeta values. In order to perform these simplifications, we view the series as specializations of generating series. For these generating series, we derive integral representations in terms of root-valued iterated integrals or directly in terms of cyclotomic harmonic polylogarithms. Using substitutions, we express the root-valued iterated integrals as cyclotomic harmonic polylogarithms. Finally, by applying known relations among the cyclotomic harmonic polylogarithms, we derive expressions in terms of several constants. We provide an algorithmic machinery to prove identities which so far could only be proved using classical hypergeometric approaches. These methods are implemented in the computer algebra package HarmonicSums. (Received August 19, 2019)

1154-05-184 **David Galvin*** (dgalvin1@nd.edu), Department of Mathematics, University of Notre Dame, Notre Dame, IN 46556. *Inverses and reciprocals of thinned exponential series.*

The compositional inverse of $e^x - 1$ is $\log(1+x)$, whose power series (about 0) is alternating. What about truncates of the power series of $e^x - 1$? The power series of the compositional inverse of the polynomial $\sum_{k=1}^r x^k/k!$ is alternating for some r , and not for others.

Somewhat surprisingly it seems easier to use a combinatorial (rather than an analytic) approach to pin down which r for which the inverse is alternating. The same combinatorial approach answers a 2006 question of Choi, Long, Ng and Smith, concerning the inverse of a matrix of certain restricted Stirling numbers.

In this talk I will attempt to make these vague statements more precise, and highlight some questions that remain in the area.

This is joint work with John Engbers (Marquette) and Cliff Smyth (UNC Greensboro). (Received August 20, 2019)

1154-05-195 **Tara C Davis*** (tdavis@hpu.edu), Hawaii Pacific University, Department of Mathematics, 500 Ala Moana Blvd., WFP 4-200Q, Honolulu, HI 96816, and **Alexxis De Lamere** (adelame1@my.hpu.edu) and **Gustavo Sopena**, California State University, Fullerton, Department of Mathematics, 800 North State College Boulevard, Fullerton, CA 92831, and **Roberto Soto** (rcsoto@fullerton.edu), California State University, Fullerton, Department of Mathematics, 800 North State College Boulevard, Fullerton, CA 92831. *Peg Solitaire in Multiple Colors on Graphs.* Preliminary report.

Peg solitaire is a classical one-person game that has been played in various countries on different types of boards. Numerous studies have focused on the solvability of the games on these traditional boards and more recently in two colors on mathematical graphs. In this presentation, we go beyond traditional peg solitaire and explore the solvability on graphs with multiple colored pegs and arrive at results that differ from previous works on the subject. The paper focuses on classifying the solvability of peg solitaire in n colors on several different types of common mathematical graphs, including the path, cycle, and complete bipartite graphs. (Received August 21, 2019)

1154-05-198 **Thomas Morrill*** (t.morrill@adfa.edu.au), Northcott Drive, Campbell, ACT 2600, Australia. *Combinatorial Proof of Jackson's Transformation via Overpartitions.*

We give a combinatorial proof of Jackson's ${}_6\Phi_5$ q -hypergeometric transformation in terms of quadruples of overpartitions. (Received August 22, 2019)

1154-05-200 **Kathryn Beck*** (beckk@dickinson.edu), **Lisa Cenek** (lcenek21@amherst.edu) and **Brittany Gelb** (begeb1@muhlenberg.edu). *Chorded Pancyclic Properties in Claw-Free Graphs.*

A graph G is (doubly) chorded pancyclic if G contains a (doubly) chorded cycle of every possible length m for $4 \leq m \leq |V(G)|$. In 2018, Cream, Gould, and Larsen completely characterized the pairs of forbidden subgraphs that guarantee chorded pancyclicity in 2-connected graphs. In our work, we show that the same

pairs also imply doubly chorded pancyclicity. We further characterize conditions for the stronger property of doubly chorded (k, m) -pancyclicity where every set of k vertices in G is contained in a doubly chorded m -cycle for all $4 \leq m \leq |V(G)|$. In particular, we examine forbidden pairs and degree sum conditions that guarantee this recently defined cycle property. This work was completed as a part of the 2019 Lafayette College REU and supported by the NSF under grant number 1560222. (Received August 22, 2019)

1154-05-263 **Andrzej Ehrenfeucht*** (andrzeje@cs.colorado.edu), 2736 Cheyenne Drive, Las Cruces, NM 88011. *Syntactic Tokens.*

Definition: The set of strings S over an alphabet A is a set of syntactic tokens if and only if (1) S is finite. (2) No two strings in S are substrings of each other. (3) S is closed under all permutations of letters in A . (4) Every string in A^* is covered by instances of tokens from S , with the possible exception of a prefix and suffix, each of which has a bounded length. (5) Let $t(w)$, where w is in A , be the sequence of uncovered prefix and suffix and all tokens from S in between. For every u, v in A^* , if $t(u) = t(v)$, then $u = v$. Definitions: A string is simple if and only if all its characters are different, and it is otherwise structured. A string awb , where a, b are in A , and w is in A^+ , is bounded if and only if w is simple and both aw and wb are structured. Theorem: The set of all bounded strings over alphabet A is a set of syntactic tokens. In the talk we introduce the concept of syntactic tokens, and describe combinatorial properties of sets of bounded strings over an alphabet of 20 letters. And we will suggest that sequences of tokens $t(w)$ may be used as tools for comparing and classifying primary structures of proteins, when A is the set of amino-acids. (Received August 27, 2019)

1154-05-274 **Steven Simon*** (ssimon@bard.edu) and **Leah Leiner.** *Regular Polygonal Partitions of a Tverberg Type.*

A seminal theorem of Tverberg states that any $T(r, d) = (r - 1)(d + 1) + 1$ points in \mathbb{R}^d can be partitioned into r subsets whose convex hulls have non-empty r -fold intersection. Almost any set of fewer points in \mathbb{R}^d cannot be so divided, and in these cases we ask whether the set can nonetheless be $P(r, d)$ -partitioned, i.e., divided into r subsets so that there exist r points, one from each resulting convex hull, which form the vertex set of a prescribed convex d -polytope $P(r, d)$. Our main result shows that this is the case for any generic $T(r, 2) - 2$ points in the plane and any $r \geq 3$ when $P(r, 2) = P_r$ is a regular r -gon. For $r = r_1 \cdots r_k$, $r_i \geq 3$, this generalizes to generic sets of $T(r, 2k) - 2k$ points and orthogonal products of regular polygons $P(r, 2k) = P_{r_1} \times \cdots \times P_{r_k}$ in \mathbb{R}^{2k} , and likewise to $T(2r, 2k + 1) - (2k + 1)$ points and the product polytopes $P(2r, 2k + 1) = P_{r_1} \times \cdots \times P_{r_k} \times P_2$ in \mathbb{R}^{2k+1} . As with Tverberg's original theorem, these have topological extensions when r is a prime power, and, using the "constraint method" of Blagojević, Frick, and Ziegler, can be made to satisfy additional conditions such as those of a van Kampen–Flores type. (Received August 27, 2019)

1154-05-280 **Harry Crane, Stephen DeSalvo and Sergi Elizalde*** (sergi.elizalde@dartmouth.edu), Department of Mathematics, 6188 Kemeny Hall, Hanover, NH 03755. *The probability of avoiding consecutive patterns in the Mallows distribution.*

We use combinatorial and probabilistic techniques to study growth rates for the probability that a random permutation from the Mallows distribution avoids consecutive patterns. The Mallows distribution is a q -analogue of the uniform distribution weighting each permutation by $q^{\#\text{inversions}}$. We prove that the growth rate exists for all patterns and all positive real values of q , and we generalize Goulden and Jackson's cluster method to keep track of the number of inversions in permutations avoiding a given consecutive pattern. Using singularity analysis, we approximate the growth rates for length-3 patterns, monotone patterns, and some non-overlapping patterns. We also show that, under certain assumptions, the number of occurrences of a given pattern is well approximated by the normal distribution. (Received August 28, 2019)

1154-05-290 **Raihana Mokhlissi*** (mokhlissiraihana@gmail.com), Phd Student in Computer Sciences and Telecomm, Rabat IT Center, Faculty of Sciences, Mohammed V University, B.P 1014, Rabat, Morocco, and **Joyati Debnath** (jdebmath@winona.edu), Dept. of Mathematics and Statistics, Winona State University, Winona, MN 55987. *The analysis of complex networks and the evaluation of their complexity.*

Many real world networks are modeled as complex networks by applying theories and methods developed in Graph theory. Structural properties like the average path length, diameter, clustering coefficient, degree distribution, average degree, etc. are used to understand them. One of the crucial invariants to characterize their structures and understand dynamical processes is the spanning tree of a network, a powerful tool to model and analyze the structure of connected networks and defined as a connected and acyclic subgraph of a network having all its vertices and some or all its edges. The main objective is to find efficient methods to obtain exact expressions of

the number of spanning trees for complex networks. The calculation of the number of spanning trees of a network also known as the complexity of a network, provides a measure to predict the reliability and the robustness of a network. The primary interest of this study is to create new models for each category of complex networks based on real-networks that grow by the gradual addition of vertices and edges. Then, analyze them by finding and highlighting their structural properties to predict their mechanisms and their functions. Furthermore, evaluate their complexity using combinatorial methods and geometric approaches. (Received August 29, 2019)

1154-05-376 **Gweneth McKinley*** (gweneth@mit.edu). *Super-logarithmic cliques in dense inhomogeneous random graphs.*

In the theory of dense graph limits, a graphon is a symmetric measurable function $W : [0, 1]^2 \rightarrow [0, 1]$. Each graphon gives rise naturally to a random graph distribution, denoted $\mathbb{G}(n, W)$, that can be viewed as a generalization of the Erdős-Rényi random graph. Recently, Doležal, Hladký, and Máthé gave an asymptotic formula of order $\log(n)$ for the size of the largest clique in $\mathbb{G}(n, W)$ when W is bounded away from 0 and 1. We show that if W is allowed to approach 1 at a finite number of points, and displays a moderate rate of growth near these points, then the clique number of $\mathbb{G}(n, W)$ will be $\Theta(\sqrt{n})$ almost surely. We also give a family of examples with clique number of order $\Theta(n^\alpha)$ for any $\alpha \in (0, 1)$, and some conditions under which the clique number of $\mathbb{G}(n, W)$ will be $o(\sqrt{n})$ or $\omega(\sqrt{n})$. This talk assumes no previous knowledge of graphons. (Received September 02, 2019)

1154-05-377 **Houston S Schuerger*** (houstonsschuerger@my.unt.edu), TX 76210, and **Andrew Schwartz**. *Vertex Pasted Skeletal Graph Pairs and Zero Forcing Numbers.*

We continue our work with skeletal graph pairs (G, \mathcal{G}) , more specifically the new concept of vertex pasted skeletal graph pairs and explore how this concept can be used to determine $Z(G)$ by considering the vertex induced subgraphs $\mathcal{G} = \{G_i\}_{i=1}^n$ given by a partition of the graph's vertices. Viewing each vertex induced subgraph as a single vertex we construct a new graph called the skeleton \mathcal{S} . We then examine which properties of \mathcal{S} and \mathcal{G} can be used to determine bounds and even precise values for $Z(G)$, compare these results to those of the more restrictive edge adjoined skeletal graph pairs, and finally introduce some new classes of graphs for which the developed theorems yield immediate results.

(Received September 02, 2019)

1154-05-378 **Steve Butler*** (butler@iastate.edu). *Undergraduates count.*

One approach to working with early undergraduate students is to engage them in counting problems with objects that they can directly relate with (e.g. tilings). There are a variety of combinatorial tools that can be used to do this (recursion is often seen); we give particular emphasis on the transfer matrix method which can cover many tiling problems by focusing on what happens “between the columns”. A variety of problems and tools will be discussed as well as ways to look for more problems. (Received September 02, 2019)

1154-05-382 **JD Nir*** (jnir@huskers.unl.edu) and **Xavier Pérez Giménez**. *The Chromatic Number of Random Lifts of Regular Graphs.*

Graph coloring is one of several constraint satisfaction problems that are studied on random structures. The problem at the heart of this talk is to identify the chromatic number of a random d -regular graph. However, inspired by an open question of Linial, rather than choose our regular graph uniformly we take a random lift of a smaller regular graph. When the host graph is K_{d+1} , our method resolves the chromatic number exactly for roughly half of the choices of d and in the other cases give a window of size two. Furthermore, our proof contains several topics of independent interest, including a second moment that's easier to analyze than the first moment and a novel application of Kirchoff's Matrix Tree Theorem. (Received September 03, 2019)

1154-05-383 **Emma Christensen, Ryan DeMuse, Jeremy Martin and Mei Yin***, Department of Mathematics, University of Denver, Denver, CO 80208. *Interval parking functions.*

The topic of parking functions has wide applications in probability, combinatorics, group theory, and computer science. One generalization of the classic parking function is the interval parking function, where each car has an interval rather than a single spot of preference. We classify features of interval parking functions, and in particular develop the link between an interval parking function and the Bruhat order on permutations. Based on joint work with Emma Christensen, Ryan DeMuse, and Jeremy Martin. (Received September 02, 2019)

1154-05-387 **Soichi Okada*** (okada@math.nagoya-u.ac.jp). *Positivity for symplectic Q -functions.*

Symplectic Q -functions are a family of Weyl group invariant Laurent polynomials, which are obtained by putting $t = -1$ in the Hall-Littlewood polynomials associated to the root system of type C . They are a type C analogue of Schur Q -functions originally introduced by Schur in his study of projective representations of symmetric

groups. In this talk, we establish several Pfaffian formulas, a tableau description (Hamel–King conjecture) and a Pieri-type rule for symplectic Q -functions. Also we discuss some conjectures including the positivity conjecture of structure constants. (Received September 03, 2019)

1154-05-413 **Boris Brimkov, Jesse Geneson, Alatheia Jensen*** (jensena@susqu.edu), **Jordan Miller and Pouria Salehi Nowbandegani.** *Intersections, circuits, and colorability of line segments.*

We derive sharp upper and lower bounds on the number of intersection points and closed regions that can occur in sets of line segments with certain structure, in terms of the number of segments. We consider sets of segments whose underlying planar graphs are Halin graphs, cactus graphs, maximal planar graphs, and triangle-free planar graphs, as well as randomly produced segment sets. We also apply these results to a variant of the Erdős–Faber–Lovász (EFL) Conjecture stating that the intersection points of m segments can be colored with m colors so that no segment contains points with the same color. We investigate an optimization problem related to the EFL Conjecture for line segments, determine its complexity, and provide some computational approaches. (Received September 03, 2019)

1154-05-415 **John Asplund*** (jasplund@daltonstate.edu), 81 High St., Ringgold, GA 30736. *The Quick Introduction to the Mathematics of Redistricting.*

Gerrymandering occurs when the boundaries of a region (e.g. U.S. districts) are manipulated to favor one or more political parties. This is a wide-spread problem that has occurred in the U.S. since its inception.

Currently, courts in several U.S. states are addressing cases involving gerrymandering. So why should mathematicians get involved in gerrymandering and redistricting? It is solely because computational redistricting is not a solved problem and a computationally difficult problem that mathematicians should get involved. In this talk, I will go through the impact mathematics has had on redistricting research and future research directions. (Received September 03, 2019)

1154-05-423 **Donglei Yang, Joshua Carlson, Andrew Owens, K E Perry, Inne Singgih*** (isinggih@math.sc.edu), **Zi-Xia Song, Fangfang Zhang and Xiaohong Zhang.** *Antimagic orientations of graphs with large maximum degree.*

Given a digraph D with m arcs, a bijection $\tau : A(D) \rightarrow \{1, 2, \dots, m\}$ is an antimagic labeling of D if no two vertices in D have the same vertex-sum, where the vertex-sum of a vertex u in D under τ is the sum of labels of all arcs entering u minus the sum of labels of all arcs leaving u . We say (D, τ) is an antimagic orientation of a graph G if D is an orientation of G and τ is an antimagic labeling of D . Motivated by the conjecture of Hartsfield and Ringel from 1990 on antimagic labelings of graphs, Hefetz, Mütze, and Schwartz in 2010 initiated the study of antimagic orientations of graphs, and conjectured that every connected graph admits an antimagic orientation. This conjecture seems hard, and few related results are known. However, it has been verified to be true for regular graphs and biregular bipartite graphs. In this paper, we prove that every connected graph G on $n \geq 9$ vertices with maximum degree at least $n - 5$ admits an antimagic orientation. (Received September 03, 2019)

1154-05-431 **Christina Eubanks-Turner** (ceturner@lmu.edu) and **Aihua Li*** (lia@montclair.edu), 1 Normal Avenue, Montclair, NJ 07044. *Interlace Polynomials of Friendship Graphs.*

In this presentation, properties of the interlace polynomials of friendship graphs and related butterfly graphs are provided. Friendship graphs are graphs that satisfy the Friendship Theorem given by Erdos, Renyi and Sos. An application of such polynomials in analyzing the adjacency matrixes of the ground graphs is given. (Received September 04, 2019)

1154-05-441 **Kate Lorenzen*** (lorenkj@iastate.edu), **Boris Brimkov, Ken Duna, Leslie Hogben, Carolyn Rienhart, Sung-Yell Song and Mark Yarrow.** *Constructions of distance Laplacian cospectral graphs.*

Graphs are mathematical objects that can be embedded into matrices. Two graphs are cospectral if they have the same set of eigenvalues with respect to a matrix. In this talk, we discuss two constructions of cospectral graphs for the distance Laplacian matrix. The first uses vertex twins which have predictable eigenvectors and eigenvalues in the distance Laplacian. The second develops a relaxation of twins called vertex cousins. This second construction produces the only pair of bipartite distance Laplacian cospectral graphs on eight vertices. (Received September 04, 2019)

1154-05-442 **Christopher O'Neill***, 5500 Campanile Dr, San Diego, CA 92182. *Numerical semigroups - a sales pitch.*

A numerical semigroup is a subset of the non-negative integers that is closed under addition. The field of numerical semigroups has a knack for producing problems that are easy to state but very difficult to answer in general. In this talk, we will touch on two such problems that remain open despite considerable effort, along the way surveying results spanning numerous undergraduate research projects. (Received September 04, 2019)

1154-05-444 **James D. Currie, Lucas Mol*** (l.mol@uwinnipeg.ca) and **Narad Rampersad**. *The repetition threshold for binary rich words.*

A word of length n is rich if it contains n nonempty palindromic factors. An infinite word is rich if all of its finite factors are rich. Baranwal and Shallit produced an infinite binary rich word with critical exponent $2 + \sqrt{2}/2$ and conjectured that this was the least possible critical exponent for infinite binary rich words (i.e., that the repetition threshold for binary rich words is $2 + \sqrt{2}/2$). In this article, we give a structure theorem for infinite binary rich words that avoid $14/5$ -powers. As a consequence, we deduce that the repetition threshold for binary rich words is $2 + \sqrt{2}/2$, as conjectured by Baranwal and Shallit. This resolves an open problem of Vesti for the binary alphabet; the problem remains open for larger alphabets. (Received September 04, 2019)

1154-05-451 **Katie Anders*** (kanders@uttyler.edu), **Daniel Arreola, Luisa Asencio, Chloe Ireland and Luke Smith**. *Graph Splines and the Universal Difference Property.* Preliminary report.

We study the generalized graph splines introduced by Gilbert, Tymoczko, and Viel and focus on an attribute known as the Universal Difference Property (UDP). We prove that paths, trees, cycles, and certain theta graphs all satisfy UDP. We also show that UDP is a structural property of edge labeled isomorphic graphs. We use Prüfer domains to illustrate that not every edge labeled graph satisfies UDP. (Received September 04, 2019)

1154-05-464 **Ayomikun Adeniran*** (ayojeng@math.tamu.edu), **S. Butler, C. Defant, Y. Gao, P. E. Harris, C. Hettle, Q. Liang, H. Nam and A. Volk**. *On the Genus of a quotient of a numerical semigroup.*

A numerical semigroup is a subset of \mathbb{N}_0 that is closed under addition, contains 0, and has finite complement in \mathbb{N}_0 . There are several fundamental invariants of a numerical semigroup S among which are the *Frobenius number* and *genus* of S , denoted $F(S)$ and $g(S)$, respectively. The *quotient* of a numerical semigroup S by a positive integer d is the set $S/d = \{x \in \mathbb{N}_0 : dx \in S\}$ which is also a numerical semigroup. In this talk, we present a recent result showing the relation between the genus of S/d and the genus of S . Also, we will show certain identities relating the Frobenius numbers and the genus of quotients of numerical semigroups that are generated by certain types of arithmetic progressions. This is joint work with S. Butler, C. Defant, Y. Gao, P. E. Harris, C. Hettle, Q. Liang, H. Nam, and A. Volk. (Received September 04, 2019)

1154-05-469 **Katie V. Johnson*** (kjohnson@fgcu.edu), 10501 FGCU Blvd. S., Fort Myers, FL 33965. *Early Explorations: A Variation on the Traditional Undergraduate Research Project.* Preliminary report.

The standard model for an undergraduate research project is often faculty-driven; we have questions that intrigue us, and we share these with students who would like to work with us, usually starting by assigning a few papers to read as background material. However, as undergraduate research projects become more common and students begin working on problems earlier and earlier, it is worth considering whether the standard model for a project is the best choice for every student. In this talk I will share an alternative process aimed at younger students where the focus is on explorations: asking questions, writing mathematically precise conjectures, and doing literature reviews. I will also include a few examples of graph theory problems that were developed by a student of mine who experienced this process as part of an honors contract. (Received September 04, 2019)

1154-05-491 **Kenneth Barrese*** (k.barrese@bucknell.edu). *The Roots of Rook Polynomials and Connected Graphs.*

The rook polynomial, as defined and factored by Goldman, Joichi, and White, allows for easy classification of the rook equivalence classes of Ferrers boards. For a given rook equivalence class, we define a graph where the vertices are identified with individual Ferrers boards in the class. We then use the multiset of roots of the rook polynomial to determine if the graph in question is connected, show that every complete graph, K_n , is a rook equivalence graph for some equivalence class of Ferrers boards, and, time permitting, show that no complete bipartite graph other than $K_{1,1}$ will occur as a rook equivalence graph. (Received September 05, 2019)

1154-05-498 **Anthony Bonato, Jane Breen, Boris Brimkov, Joshua Carlson, Sean English*** (senglish@illinois.edu), **Jesse Geneson, Leslie Hogben, K E Perry and Carolyn Reinhart.** *Catching Robbers Quickly and Efficiently.*

The game of cops and robbers is a 2-player game played on a graph in which a team of cops try to catch a moving robber. The minimum number of cops necessary to catch a robber on the graph G is the *cop number*, denoted $c(G)$. In this talk we will discuss *cop-throttling*, in which we are concerned with catching the robber quickly. More precisely, the *capture time* with k cops, denoted $\text{capt}_k(G)$, is the length of the longest game of cops and robbers possible, assuming the cops play optimally. The *cop-throttling number* is given by

$$\text{th}_c(G) := \min_{c(G) \leq k \leq |V(G)|} \{k + \text{capt}_k(G)\}.$$

We will briefly give background on the game of cops and robbers, and then we will show that the cop throttling number grows sublinearly with the number of vertices of G . (Received September 05, 2019)

1154-05-528 **Oliver Clarke, Kevin Grace*** (kevin.grace@bristol.ac.uk), **Fatemeh Mohammadi and Leonid Monin.** *Minimally dependent matroids.* Preliminary report.

Let \mathbb{F} be a field, and denote by $\mathcal{D}(M)$ the collection of dependent sets of a matroid M . Consider a collection \mathcal{D} of subsets of a finite set E , and consider the problem of determining all \mathbb{F} -representable matroids M such that no \mathbb{F} -representable matroid $N \neq M$ exists with $\mathcal{D} \subseteq \mathcal{D}(N) \subseteq \mathcal{D}(M)$. We describe work on this problem for some specific cases. This work is motivated by problems in algebraic statistics. (Received September 16, 2019)

1154-05-545 **Amanda M Harsy*** (harsyram@lewisu.edu), Lewis University, One University Parkway, Unit #298, Department of Computer and Mathematical Scien, Romeoville, IL 60446, and **Cory Johnson, Eric Redmon, Paul Buldak, James Sparks and Miles Mena.** *Using Graph Theoretical Designs of Self Assembling DNA to Motivate Undergraduate Research.* Preliminary report.

Motivated by the recent advancements in nanotechnology and the discovery of new laboratory techniques using the Watson-Crick complementary properties of DNA strands, formal graph theory has recently become useful in the study of self-assembling DNA complexes. Construction methods developed with concepts from undergraduate level graph theory have resulted in significantly increased efficiency for laboratory processes. One recent focus in DNA nanotechnology is the formation of nanotubes using lattice structures. These nanotubes are thought to have wide-ranging potential, serving as containers for nano-cargos and as drug-delivery vehicles. Rules governing the self-assembly of these nanotubes are not yet well understood, and this naturally creates open problems in applied graph theory. In this talk, we give an overview of lattice-based construction of nanostructures and explore related design strategy problems. We highlight some of our undergraduate student projects and outcomes. (Received September 06, 2019)

1154-05-553 **Nathan Chenette, Lara Pudwell*** (lara.pudwell@valpo.edu) and **Manda Riehl.** *Statistics on hypercube orientations.* Preliminary report.

In mathematical biology, acyclic orientations of the n -dimensional hypercube graph provide a model for fitness landscapes after sequences of gene mutations. Given such an orientation, we say vertex v is a peak if all edges containing v are directed towards v . We also say four vertices a, b, c , and d form a reverse sign epistasis (RSE) if there are edges directed from a to b , a to d , c to b , and c to d . In this talk, we consider the combinations of (number of peaks, number of RSEs) that are possible in such an orientation. (Received September 06, 2019)

1154-05-561 **Shyam Narayanan and Alec Sun*** (suna@college.harvard.edu), 1 Oxford St., Cambridge, MA 02138. *Bounds on expected propagation time of probabilistic zero forcing.*

Probabilistic zero forcing is a coloring game played on a graph where the goal is to color every vertex blue starting with an initial blue vertex set. As long as the graph is connected, if at least one vertex is blue then eventually all of the vertices will be colored blue. The most studied parameter in probabilistic zero forcing is the expected propagation time starting from a given vertex of G . In this work we improve on known upper bounds for the expected propagation time in terms of a graph's order and radius. In particular, for a connected graph G of order n and radius r , we prove the bound $\text{ept}(G) = O(r \log(n/r))$. We also show using Doob's Optional Stopping Theorem and a combinatorial object known as a cornerstone that $\text{ept}(G) \leq n/2 + O(\log n)$. Finally, we approximate the expected propagation time of the Erdős-Renyi graph $G(n, p)$. (Received September 06, 2019)

1154-05-571 **Eva Czabarka*** (czabarka@math.sc.edu). *Some crossing number results using probabilistic techniques.*

The crossing number of a graph is the minimum number of crossings it can be drawn in a plane. Let $\kappa(n, m)$ be the minimum crossing number of graphs with n vertices and m edges. Erdős and Guy conjectured and Pach,

Spencer and Tóth proved that for any $m = m(n)$ satisfying $n \ll m \ll n^2$ $\lim_{n \rightarrow \infty} \frac{\kappa(n,m)n^2}{m^3}$ exists and is positive. The k -planar crossing number is the minimum crossing number obtained when we partition the edges of the graph into k sub graphs and draw them in k planes. Using designs and a probabilistic algorithm, the guaranteed factor of improvement α_k between the k -planar and regular crossing number is $\frac{1}{k^2}(1 + o(1))$, while if we restrict our attention to biplanar graphs, this constant is $\beta_k = \frac{1}{k^2}$ exactly. The lower bound proofs require the existence of a midrange crossing constant. Motivated by this, we show that the midrange crossing constant exists for all graph classes (including bipartite graphs) that satisfy certain conditions. The regular midrange crossing constant was shown to be at most $\frac{8}{9\pi^2}$; we present a probabilistic construction that also shows this bound. (Received September 07, 2019)

1154-05-624 **Jesse Geneson*** (geneson@iastate.edu) and **Leslie Hogben**. *Propagation time for probabilistic zero forcing.*

Zero forcing is a coloring game played on a graph that was introduced more than ten years ago in several different applications. The goal is to color all the vertices blue by repeated use of a (deterministic) color change rule. Probabilistic zero forcing was introduced by Kang and Yi in [Probabilistic zero forcing in graphs, Bull. Inst. Combin. Appl. 67 (2013), 9–16] and yields a discrete dynamical system, which is a better model for some applications. Since in a connected graph any one vertex can eventually color the entire graph blue using probabilistic zero forcing, the expected time to do this is a natural parameter to study. We determine expected propagation time exactly for paths and cycles, establish the asymptotic value for stars, and present asymptotic upper and lower bounds for any graph in terms of its radius and order. We apply these results to obtain values and bounds on l -round probabilistic zero forcing, throttling number for probabilistic zero forcing, and confidence levels for propagation time. (Received September 08, 2019)

1154-05-635 **Justin M. Troyka*** (jmtroyka@yorku.ca). *Split graphs: Combinatorial species and asymptotics.*

A *split graph* is a graph whose vertices can be partitioned into a clique (complete graph) and a stable set (independent set). How many split graphs on n vertices are there? Approximately how many are there, as n goes to infinity? Collins and Trenk (2018) have worked on these questions, and in this talk I give a generalization of their results in the setting of species theory, a powerful framework for counting combinatorial objects acted on by isomorphisms. This generalization leads to a result relating split graphs and bicolored graphs, allowing me to prove the conjecture of Cheng, Collins, and Trenk (2016) that almost all split graphs are “balanced”. The proof makes use of an asymptotic result about bicolored graphs: namely, that the number of unlabeled bicolored graphs is asymptotically the number of labeled ones divided by $n!$. (Received September 09, 2019)

1154-05-638 **Julianne Vega*** (julianne.vega@uky.edu). *k -matching sequences of simplicial complexes.*

The homotopy type of $M_1(G)$ the matching complex for a graph G has been studied for paths, cycles, and trees. In this talk we will generalize to k -matching complexes, denoted $M_k(G)$ and consider the sequence $(M_1(G), M_2(G), \dots, M_n(G))$ up to homotopy for perfect caterpillar graphs. (Received September 09, 2019)

1154-05-641 **Galen Dorpalen-Barry, Cyrus Hettle, David Livingston, Jeremy Martin, George Nasr, Julianne Vega*** (julianne.vega@uky.edu) and **Hays Whitlatch**. *A positivity phenomenon in Elser’s Gaussian-cluster percolation model.*

Veit Elser proposed a random graph percolation model in which physical dimension appears as a parameter. From this model, numerical graph invariants $els_k(G)$, called Elser numbers, naturally arise and Viet Elser conjectured that $els_k(G) \geq 0$ for all graph G and integers $k \geq 2$. In this talk we will interpret the Elser numbers as Euler characteristics of nucleus (simplicial) complexes and prove Elser’s conjecture. (Received September 09, 2019)

1154-05-647 **Florian Kohl, McCabe Olsen*** (colsen.149@osu.edu) and **Raman Sanyal**. *Unconditional Reflexive Polytopes.*

A convex body is unconditional if it is symmetric with respect to reflections in all coordinate hyperplanes. In this paper, we investigate unconditional lattice polytopes with respect to geometric, combinatorial, and algebraic properties. In particular, we characterize unconditional reflexive polytopes in terms of perfect graphs. As a prime example, we study the signed Birkhoff polytope. Moreover, we derive constructions for Gale-dual pairs of polytopes and we explicitly describe Gröbner bases for unconditional reflexive polytopes coming from partially ordered sets. (Received September 09, 2019)

- 1154-05-657 **Ilkyoo Choi, Felix Christian Clemen, Michael Ferrara, Paul Horn, Fuhong Ma and Tomáš Masařík*** (masarik@kam.mff.cuni.cz), KAM, MFF, Malostranské náměstí 25, 11800 Praha 1, Czech Rep. *Flexibility of Planar Graphs—Sharpening the Tools to Get Lists of Size Four*. Preliminary report.
- Proper graph coloring assigns different colors to adjacent vertices of the graph while keeping the number of colors small. In some applications, vertices have certain preferences for specific colors. This led to study of the precoloring extension problem. However, this problem is usually too difficult to make use of it. Recently, Dvořák, Norin, and Postle relaxed this notion in a way that not all preferences needs to be satisfied. Formally, given a graph with lists of allowed colors for each vertex and given the preferred color for some of the vertices, the task is to return a proper list coloring of the graph satisfying at least a constant fraction of all the preferences.
- Subsequently, a bit of an attention was given to study of this problem for several subclasses of planar graphs, e.g., triangle-free, girth 6, without C_4 . We give a stronger version of the main tool that was used in the proofs of the results mentioned above. We prove that there is an absolute constant such that if G is a planar graph without diamonds and all lists have size at least five, then there exists an L -coloring respecting at least a constant fraction of the preferences. Moreover, we showed that lists of size 4 suffice for some subclasses of planar graphs without 4-cycles. (Received September 09, 2019)
- 1154-05-687 **Joshua Carlson*** (jc31@williams.edu) and **Juergen Kritschgau** (jkritsch@iastate.edu). *Extreme Throttling Numbers*.
- Zero forcing is a process on graphs that uses a color change rule to force the color of the vertices in a graph to become blue. The throttling number of a graph optimizes the balance between the number of blue vertices at the start of the process and time taken for all vertices in the graph to become blue. Positive semidefinite (PSD) zero forcing is a variant that allows forcing to occur in separate components of a graph. There are also minor monotone floor variants that allow forcing across large distances by "hopping". Each variant gives rise to a different type of throttling number. This talk will present structural and forbidden subgraph characterizations of throttling numbers. (Received September 09, 2019)
- 1154-05-695 **Benjamin Peet*** (bpeet@stmartin.edu), St. Martin's University, 5000 Abbey Way, Lacey, WA 98503. *k-Configurations and Superconfigurations*. Preliminary report.
- This paper begins by extending the notion of a configuration of points and lines to a configuration of points and planes. We then proceed to investigate a further extension to the notion of points and k -planes (k -dimensional planes) which we refer to as k -configurations. Then, the notion of a superconfiguration of points, lines, and planes is introduced, where the points and lines form a (1-)configuration; the lines and planes form a (1-)configuration; and the points and planes form a 2-configuration. We present a number of examples as well as computing the number of possible symmetric 2-configurations when each plane contains 3 points. (Received September 09, 2019)
- 1154-05-697 **Paul M Terwilliger*** (terwilli@math.wisc.edu), Math Department, U. Wisconsin, 480 Lincoln Drive, Madison, WI 53706. *The alternating central extension for the positive part of the quantum group $U_q(\widehat{\mathfrak{sl}}_2)$* .
- In the theory of Q -polynomial distance-regular graphs, a role is played by the positive part U_q^+ of the quantum group $U_q(\widehat{\mathfrak{sl}}_2)$. The algebra U_q^+ has a presentation with two generators A, B that satisfy the cubic q -Serre relations. Recently we introduced a type of element in U_q^+ , said to be alternating. Each alternating element commutes with exactly one of $A, B, qBA - q^{-1}AB, qAB - q^{-1}BA$; this gives four types of alternating elements. There are infinitely many alternating elements of each type, and these mutually commute. We use the alternating elements to obtain a central extension \mathcal{U}_q^+ of U_q^+ . We then use the alternating elements to obtain a PBW basis for \mathcal{U}_q^+ . (Received September 09, 2019)
- 1154-05-720 **Sinan Aksoy, Emily Heath, Brett Jefferson, Cliff Joslyn, Jason McDermott, Hugh Mitchell, Brenda Praggastis and Emilie Purvine*** (emilie.purvine@pnnl.gov). *Topology and Hypergraphs for Biological Data*.
- Biological data like measured response of protein levels to presence of virus or disease, or experimentally identified protein complexes and pathways, contain many complex interactions. Hypergraphs are a natural mathematical structure to model and explore complex multi-relations and topology helps discover higher dimensional features within data. This talk will explore our recent work to understand how analogs of traditional network science concepts, like centrality and spectral clustering, can be used in the context of hypergraphs for discovery of central biological pathways, characterization of unknown transcription factors, and comparison of diseases with different pathogenesis. (Received September 10, 2019)

1154-05-722 **Amelie Trotignon*** (amelie.trotignon@idpoisson.fr). *Walks in the three-quarter plane.* Enumeration of lattice walks in cones has many applications in combinatorics and probability theory. These objects are amenable to treatment by many techniques: combinatorics, complex analysis, probability theory, computer algebra and Galois theory of difference equations. While walks restricted to the first quadrant have been well studied, the case of non-convex cones has been approached recently. In this talk, we extend the analytic method of the study of walks in the quarter plane to the three-quarter plane applying the strategy of splitting the domain into two symmetric convex cones. This method is composed of three main steps: write a system of functional equations, which may be simplified into one simple equation under symmetry conditions; transform the functional equation into a boundary value problem; and solve this problem using conformal mapping. The result is a contour integral expression for the generating function. The advantage of this method is the uniform treatment of models corresponding to different step sets. (Received September 10, 2019)

1154-05-731 **Amelia Cantwell*** (cantwellamelia@gmail.com) and **Juliann Geraci.** *Graph Universal Cycles of Combinatorial Objects.* Preliminary report.

In 1992, Chung et al. showed that while the set of permutations on $[n]$ could not be placed into a universal cycle, there are ways to circumvent this issue, by enhancing the set of symbols. In the same paper, and in 1994 by Hurlbert, the question of universal cycles of k -subsets of $[n]$ is addressed, with definitive results only for $k = 3, 4, 6$. Finally, Chung et. al prove the existence of universal cycles for partitions of an n -element set but with the number of symbols being unspecified.

We will describe a new method of producing universal cycles of these objects. Inspired by Brockman et al. (2010), where *graph universal cycles* are introduced, we use these in our various contexts. For S_n , we will show that the set of *permutation graphs* of elements of S_n can be placed in a graph universal cycle. Moreover, the set of *transposition graphs* of all involutions on n elements can also be placed in a graph universal cycle. For k -subsets of $[n]$, we use the complete graph and the n -cycle as our parent graphs to exhibit graph universal cycles. Finally, disjoint unions of complete graphs are used as starting points for graph universal cycles of set partitions. (Received September 10, 2019)

1154-05-744 **Carolyn Reinhart*** (reinh196@iastate.edu). *The normalized distance Laplacian.*

The distance matrix $\mathcal{D}(G)$ of a graph G is the matrix containing the pairwise distances between vertices. The transmission of a vertex v_i in G is the sum of the distances from v_i to all other vertices and we let $T(G)$ be the diagonal matrix of transmissions of the vertices of the graph. The new matrix the normalized distance Laplacian, denoted $\mathcal{D}^{\mathcal{L}}(G)$, is defined such that $\mathcal{D}^{\mathcal{L}}(G) = I - T(G)^{-1/2} \mathcal{D}(G) T(G)^{-1/2}$. This is analogous to the normalized Laplacian matrix, defined such that $\mathcal{L}(G) = I - D(G)^{-1/2} A(G) D(G)^{-1/2}$ where $D(G)$ is the diagonal matrix of degrees of the vertices of the graph and $A(G)$ is the adjacency matrix. Two non-isomorphic graphs G and H are M -cospectral if $M(G)$ and $M(H)$ have the same multiset of eigenvalues. New results to be presented include bounds on the spectral radius of $\mathcal{D}^{\mathcal{L}}$ and connections with the normalized Laplacian matrix. Methods for determining eigenvalues of $\mathcal{D}^{\mathcal{L}}$ will be discussed, including the use of twin vertices. Finally, examples of $\mathcal{D}^{\mathcal{L}}$ -cospectrality will be presented and compared to instances of cospectrality for other well known matrices. (Received September 10, 2019)

1154-05-751 **Neal Madras*** (madras@mathstat.yorku.ca), Dept of Mathematics & Statistics, York University, 4700 Keele Street, Toronto, Ontario M3J 1P3, Canada, and **Justin M. Troyka** (jmtroyka@yorku.ca), Dept of Mathematics & Statistics, York University, 4700 Keele Street, Toronto, Ontario M3J 1P3, Canada. *Periodic Pattern-Avoiding Permutations.* Preliminary report.

To gain insight into the structure of pattern-avoiding permutations, and motivated by the idea of periodic boundary conditions in physics, we propose a new “boundedness” condition for affine permutations. An *affine permutation of period N* is a bijection ω of \mathbb{Z} satisfying

$$\omega(i + N) = \omega(i) + N \quad \forall i \in \mathbb{Z}$$

as well as the centering condition

$$\sum_{i=1}^N \omega(i) = \sum_{i=1}^N i,$$

and we say it is *bounded* if

$$|\omega(i) - i| < N \quad \forall i \in \mathbb{Z}.$$

Let BA_N be the set of bounded affine permutations of period N . Note that for any (ordinary) permutation σ on $\{1, \dots, N\}$, the periodic extension of σ via $\sigma(i + kN) = \sigma(i) + kN$ ($k \in \mathbb{Z}$) is in BA_N .

For a fixed short permutation τ , let $\text{AvBA}_N(\tau)$ be the set of $\omega \in \text{BA}_N$ that avoid the pattern τ (i.e., as a sequence, ω has no subsequence with the same relative order as τ).

We focus on the decreasing pattern $\text{Decr}_k := \mathbf{k(k-1)} \cdots \mathbf{321}$ for fixed $k \geq 3$. We obtain the exact asymptotic behaviour of $|\text{AvBA}_N(\text{Decr}_k)|$ as $N \rightarrow \infty$. We also describe a corresponding permuton-like result for $\text{AvBA}_N(\text{Decr}_k)$. (Received September 10, 2019)

1154-05-779 **Chaim Even Zohar*** (chaim@ucdavis.edu). *Some Random Words.*

Consider a uniformly random word over some alphabet A, B, \dots , where the letter A appears exactly a times, B exactly b times, and so on. This talk regards subwords of this word, not necessarily of consecutive letters, but always taken in order of appearance. Counting the occurrences of various subwords in such a random word leads to interesting limit distributions, which are typical to generalized U-statistics. We demonstrate this in several cases that have been considered in the combinatorial literature. (Received September 10, 2019)

1154-05-780 **Margaret Bayer, Bennet Goeckner*** (goeckner@uw.edu) and **Marija Jelić Milutinović.** *Manifold Matching Complexes.*

Given a graph, a matching is a collection of its edges such that no two of these edges share a common endpoint. The set of all possible matchings of a graph is called the matching complex of the graph. Much research has been conducted on the topology of matching complexes for various graph families, in particular for complete graphs and complete bipartite graphs. We instead ask the opposite question: Given a combinatorial manifold, when is it a matching complex? We completely characterize all graphs and manifolds that arise in this way. In particular, we show that, outside of dimension two, all manifold matching complexes are either spheres or balls. (Received September 10, 2019)

1154-05-783 **Torin Greenwood*** (torin.greenwood@ndsu.edu). *Analyzing the outputs of grammars.*

Stochastic context-free grammars (SCFGs) use recursive rules to model families of objects, including languages, Catalan objects, and RNA secondary structures. Grammars can be converted into algebraic multivariate generating functions. Then, analytic combinatorics can yield insights into the distributions of objects grammars produce. However, slight modifications of grammatical rules can greatly alter the distribution of objects produced by a grammar. In this talk, we discuss examples and challenges of analyzing SCFGs at large. (Received September 10, 2019)

1154-05-790 **Tova Brown*** (tova.brown@wlc.edu) and **Nicholas M. Ercolani.** *Integrable Mappings from a Unified Perspective.*

The combinatorics in this talk include rather well-known problems and results in map enumeration, fruit of Shaeffer's bijective methods which opened up opportunities to study geodesic distances on maps. Our work studies recurrence relations for these generating functions from the perspective of integrable systems. The integrability of the recurrence relations, $x_n = 1 + gx_n(x_{n+1} + cx_n + x_{n-1})$ for $c \in \{0, 1\}$, is known (one of them is the discrete Painlevé-I equation). Our work uses this integrability to establish global closed-forms, which contain elegant formulas for the generating functions of the map enumeration problems. (Received September 10, 2019)

1154-05-809 **G Eric Moorhouse*** (moorhous@uwyo.edu), Department of Mathematics, University of Wyoming, Laramie, WY 82071. *Cubical arrays of projective planes.* Preliminary report.

We discuss $N \times N \times N$ cubical arrays of zeroes and ones ($N = n^2 + n + 1$) in which each of the $3N$ 'slices' is the incidence matrix of a projective plane of order n . It seems that essentially one construction is known, using only classical planes. Are there others? (Received September 10, 2019)

1154-05-813 **Maria Monks Gillespie*** (maria.gillespie@colostate.edu), Department of Mathematics, Colorado State University, Fort Collins, CO 80523, and **Kenneth G. Monks** (monks@scranton.edu) and **Kenneth M. Monks** (kenneth.monks@frontrange.edu). *Enumerating Anchored Permutations with Bounded Gaps.*

Say that a permutation of $1, 2, \dots, n$ is k -bounded if every pair of consecutive entries in the permutation differs by no more than k . Such a permutation is *anchored* if the first entry is 1 and the last entry is n . We show that the generating function for the enumeration of k -bounded anchored permutations is always rational, mirroring the known result on (non-anchored) k -bounded permutations due to Avgustinovich and Kitaev. We then explicitly determine the recursive formulas of minimal depth for the number of anchored k -bounded permutations of n for $k = 2$ and $k = 3$, resolving a conjecture listed on the Online Encyclopedia of Integer Sequences (entry A249665). We additionally show that the number of anchored k -bounded permutations of n is asymptotically $O(k^n)$ as a function of n for a given k . (Received September 10, 2019)

1154-05-842 **Sung Y Song*** (sysong@iastate.edu), 442 Carver, Department of Mathematics, Iowa State University, Ames, IA 50014-2104. *Character tables of the association schemes coming from the action of the groups $GU_n(q)$ on n -dimensional vectors over $GF(q^2)$* . Preliminary report.

When a finite group G acts transitively on a finite set Ω , we obtain an association scheme, so called Schurian association scheme. In this case, there is a natural one-to-one correspondence between the set Ω and the set of cosets of a point stabilizer for any element in Ω . In this talk we discuss the Schurian association schemes obtained from the action of the general unitary groups on the transitive sets of vectors over the finite fields. We will construct their character tables, and see that some of these schemes have non-Schurian fusion schemes. If time permits, we will also discuss some examples and methods to obtain a set of vectors at few angles on the unit sphere with respect to a non-singular Hermitian form. This work is based on joint work with Robert Lazar. (Received September 11, 2019)

1154-05-843 **Jeff Kahn and Jinyoung Park*** (jp1324@math.rutgers.edu). *Stability for maximal independent sets and its applications*.

We will introduce “stability” versions of upper bounds on maximal independent set counts in graphs under various restrictions. Roughly these say that being close to the maximum implies existence of a large induced matching or triangle matching (depending on assumptions). We will also discuss applications and related open problems in various directions. (Received September 11, 2019)

1154-05-851 **Asaf Ferber, Vishesh Jain*** (visheshj@mit.edu) and **Yufei Zhao**. *Hadamard matrices are even harder to find*.

A square matrix H of order n whose entries are $\{\pm 1\}$ -valued is called a Hadamard matrix of order n if its rows are pairwise orthogonal. The famous Hadamard conjecture asserts that there exists a Hadamard matrix for every n which is a multiple of 4. We consider the complimentary problem of bounding $H(n)$ – the number of Hadamard matrices of order n – from above.

It is easily established that $H(n)$ is at most $2^{\binom{n+1}{2}}$. Using a novel approach to the so-called Littlewood-Offord problem for vector sums, we show that there exists some absolute constant $c > 0$ such that for all sufficiently large n , $H(n)$ is at most $2^{(1-c)n^2/2}$, thereby providing the only known non-trivial upper bound on the number of Hadamard matrices of order n . (Received September 11, 2019)

1154-05-856 **Cosmin Pohoata*** (apohoata@caltech.edu), 1200 E California Blvd, Pasadena, CA 91125. *On the Erdős-Gyárfás distinct distances problem with local constraints*.

In 1946 Erdős asked to determine or estimate the minimum number of distinct distances determined by an n -element planar point set V . He showed that a square integer lattice determines $\Theta(n/\sqrt{\log n})$ distinct distances, and conjectured that any n -element point set determines at least $n^{1-o(1)}$ distinct distances. In 2010, Guth and Katz answered Erdős’s question by proving that every n -element planar point set determines $\Omega(n/\log n)$ distinct distances. In this talk, we will discuss a variant of this problem by Erdős and Gyárfás. For integers n, p, q with $p \geq q \geq 2$, let $D(n, p, q)$ denote the minimum number of distinct distances determined by a planar n -element point set V which has the property that every subset of p points from V spans at least q distinct distances. In a recent paper, Fox, Pach and Suk prove that, when $q = \binom{p}{2} - p + 6$, $D(n, p, q)$ is always at least $n^{8/7-o(1)}$. We will discuss an improvement of their result and some recent nearly sharp bounds for a related (more general) graph Ramsey problem of Erdős and Shelah which arise. (Received September 11, 2019)

1154-05-918 **Yuki Irie*** (yirie@tohoku.ac.jp), 6-3 Aoba, Aramaki, Aoba-ku, Sendai, Miyagi, Japan. *Representations of Symmetric Groups and Welter’s Game*.

In the 1970s, Mikio Sato conjectured that Welter’s game, a game played with a Young diagram, is related to representations of symmetric groups. In support of this conjecture, he pointed out that its Sprague-Grundy function, which gives the winning way of the game, can be expressed in a form similar to the hook-length formula.

In this talk, we present a relation between representations of symmetric groups and Welter’s game. Irreducible representations with degree prime to p play an important role in this context, where p is a prime. For a Young diagram Y , let R^Y denote the irreducible representation of $\text{Sym}(n)$ corresponding to Y . We give a function $\psi_p(Y)$ such that the restriction of R^Y to $\text{Sym}(\psi_p(Y))$ has an irreducible component with degree prime to p . From this, we prove that $\psi_p(Y)$ is equal to the Sprague-Grundy function for a p -saturated Welter’s game, where Welter’s game is a 2-saturated Welter’s game. (Received September 11, 2019)

1154-05-927 **Wing Hong Tony Wong***, 15200 Kutztown Road, Kutztown, PA 19530, and **Joshua Harrington**. *Super totient numbers and the super totient index of graphs.*

For any integer n , let $R(n)$ be the set of positive integers less than n and relatively prime to n . If $R(n)$ can be partitioned into two subsets with the same sum, then we say n is super totient. We completely characterize all super totient numbers. Furthermore, we define and study two new concepts in graph labeling: the restricted super totient labeling and the super totient index of graphs. (Received September 12, 2019)

1154-05-932 **Takayuki Nozaki*** (tnozaki@yamaguchi-u.ac.jp), 1677-1, Yoshida, Yamaguchi, Yamaguchi 753-0056, Japan. *Weight Enumerators and Cardinalities for Non-binary VT Codes.*

Non-binary VT codes are number-theoretic codes which correct a single insertion or deletion. In this talk, we provide the Hamming weight enumerators and cardinalities of the non-binary VT codes. To derive those, we define the simultaneous congruences (SC) code, which is a general class of number-theoretic codes, and derive a formula for the extended weight enumerator of SC codes. (Received September 12, 2019)

1154-05-933 **Émilie Charlier, Manon Philibert and Manon Stipulanti*** (m.stipulanti@uliege.be), m.stipulanti@uliege.be. *Nyldon words.*

The Chen-Fox-Lyndon theorem states that every finite word over a fixed alphabet can be uniquely factorized as a lexicographically nonincreasing sequence of Lyndon words. This theorem can be used to define the family of Lyndon words in a recursive way. In a Mathoverflow post dating from November 2014, Darij Grinberg defines a variant of Lyndon words, which he calls Nyldon words, by reversing the lexicographic order. In a recent collaboration with Émilie Charlier (University of Liège) and Manon Philibert (Aix-Marseille University), we show that every finite word can be uniquely factorized into a lexicographically nondecreasing sequence of Nyldon words. Otherwise stated, Nyldon words form a complete factorization of the free monoid with respect to the decreasing lexicographic order. In our paper, we investigate this new family of words by presenting some of their properties. (Received September 12, 2019)

1154-05-937 **Matthew Beaudouin-Lafon, Margaret Crawford, Serena Chen and Nathaniel Karst*** (nkarst@babson.edu), 316 Babson Hall, Babson College, Wellesley, MA 02457, and **Louise Nielsen and Denise Sakai Troxell**. *Minimal Zero Blocking Sets of Rectangular, Cylindrical, and Mobius Grids.*

In a zero forcing process, an initial vertex labeling of a graph using labels 0 and 1 is updated iteratively according to the following conversion rule: change the label of a vertex from 1 to 0 if this vertex is the only neighbor labeled 1 of some vertex labeled 0. In this process, a zero forcing set is a set of vertices initially labeled 0 such that all the remaining vertices will ultimately have their labels changed from 1 to 0, and the zero forcing number is the minimum cardinality of a zero forcing set. We investigate two related concepts: a zero blocking set is the complement of a set which is not a zero forcing set, and the zero blocking number is the minimum cardinality of a zero blocking set. We provide upper and lower bounds for the zero blocking number of rectangular grids and discuss conditions under which these bounds coincide. We go on to use the same machinery to provide similar results for certain cylindrical and Mobius grids. (Received September 12, 2019)

1154-05-940 **Kento Nakada*** (nakada@okayama-u.ac.jp), 1-1-1 Tsushima-naka, Kita-ku, Okayama-shi, 700-0016, Japan. *Order structure of infinite d-complete posets.*

Shapes of infinite predominant integral weights are regarded as infinite d -complete posets. In general, we can define two different order relations over shapes of infinite predominant integral weights. One is the ordinary order of coroots, the other is the heap order. In this talk, we give necessary and sufficient conditions for both orders to coincide. (Received September 12, 2019)

1154-05-942 **Kento Nakada*** (nakada@okayama-u.ac.jp), 1-1-1 Tsushima-naka, Kita-ku, Okayama-shi, 00-0017, Japan. *A certain code correcting two deletions with specified position.*

In this work, we give a certain class of binary codes which correct two deleted bits with specified positions. One position is beforehand known, and the other is unknown. If time is permitted, we give analogous codes which correct three deleted bits with specified positions with a certain parity condition. (Received September 12, 2019)

1154-05-969 **Alyssa N Adams and Bonnie C Jacob*** (bcjntm@rit.edu). *Failed zero forcing on oriented and directed graphs: results, applications, and open questions.*

Let Γ be a simple directed graph with vertex set V and edge set E where $n = |V|$, and each edge is assigned one or more orientations, $e = (u, v)$ or $e = (v, u)$. The closed out-neighborhood of $u \in V$ is $N^+(u) = \{u\} \cup \{v :$

$v \in V$ and $(u, v) \in E$. For $S \subseteq V$, $B^0(S) := S$ and for $i = \{0, 1, \dots\}$, $B^{i+1}(S) := B^i \cup \{w : \{w\} = N^+[v] \setminus B^i(S) \text{ for some } v \in B^i(S)\}$. If $B^i(S) \neq V$ for any i , then S is a *failed zero forcing set*. In this talk, we introduce the *failed zero forcing number* $F(\Gamma)$ on a directed graph, which is the maximum cardinality of any failed zero forcing set in Γ . We characterize oriented graphs that have $F(\Gamma) < Z(\Gamma)$ and directed graphs with $F(\Gamma) = n - 1$, $F(\Gamma) = n - 2$, and $F(\Gamma) = 0$. We also show that for any integer $n \geq 3$ and any non-negative integer k with $k < n$, there exists a directed graph Γ whose underlying undirected graph is a cycle with $F(\Gamma) = k$. Finally, we discuss potential applications of the failed zero forcing number and some open questions. (Received September 12, 2019)

1154-05-973

Ravi Cho* (ravi.cho@sjsu.edu), Department of Mathematics and Statistics, San José State University, San José, CA 95192-0103, and **Tim Hsu** and **Ardak Kapbasov**. *Balanced spiders*. Preliminary report.

In the two-player combinatorial game of (finite) Blue-Red Hackenbush, a position is a finite graph of blue and red edges attached to the “ground”; the players L and R alternate taking blue and red edges respectively, with edges also being deleted if they no longer connect to the ground; and the first player unable to move loses. A Hackenbush *string* is a string (path) of edges connected to the ground at one end. Berlekamp first solved (gave a recipe for optimal play for) strings, and van Roode solved *arches* (two strings attached at their tops).

A *balanced spider with k legs* is a game formed by taking k strings of the same height n and attaching them at their tops, generalizing both strings and arches. (For technical reasons, we also assume that the bottom two edges of each leg are different colors.) We present a solution for balanced spiders with 3 legs and discuss further results and conjectures for balanced spiders with more legs. (Received September 12, 2019)

1154-05-982

Samuel F Hopkins* (shopkins@umn.edu). *Cyclic sieving for plane partitions and symmetry*.

The cyclic sieving phenomenon of Reiner, Stanton, and White says that we can often count fixed points for a cyclic group acting on a combinatorial set by plugging roots of unity into a polynomial related to this set. One of the most impressive instances of the cyclic sieving phenomenon is a theorem of Rhoades asserting that the set of plane partitions in a rectangular box under the action of promotion exhibits cyclic sieving. In Rhoades’s result the sieving polynomial is the size generating function for these plane partitions, which has a well-known product formula due to MacMahon. We extend Rhoades’s result by studying the interaction of promotion with symmetries of plane partitions. We obtain cyclic sieving-like formulas in this context where the relevant polynomial is the size generating function for symmetric plane partitions, whose product formula was conjectured by MacMahon and proved by Andrews. We then go on to consider the way the symmetries interact with rowmotion, another operator acting on plane partitions which is closely related to promotion. We end by explaining the connection of our work to some earlier conjectures we made concerning rowmotion acting on the P -partitions of various “triangular” posets P . (Received September 12, 2019)

1154-05-995

Niklas Affolter, **Max Glick*** (glick.107@osu.edu), **Pavlo Pylyavskyy** and **Sanjay Ramassamy**. *Vector-relation configurations and plabic graphs*.

We study a simple geometric model for local transformations of bipartite graphs. The state consists of a choice of a vector at each white vertex made in such a way that the vectors neighboring each black vertex satisfy a linear relation. Evolution for different choices of the graph coincides with many notable dynamical systems including the pentagram map, Q -nets, and discrete Darboux maps. On the other hand, for plabic graphs we prove unique extendability of a configuration from the boundary to the interior, an elegant illustration of the fact that Postnikov’s boundary measurement map is invertible. In all cases there is a cluster algebra operating in the background, resolving the open question for Q -nets of whether such a structure exists. (Received September 12, 2019)

1154-05-996

Robert W Bell* (bellro@msu.edu). *Cops and Robbers and the SURIEM REU Program*. Preliminary report.

The cop number of a discrete graph is a deceptively simple invariant to define: the least number of cops required to always win a pursuit-evasion game versus one robber on the given graph, where both players have complete information and alternate turns by moving any number of their cops/robber to an adjacent vertex. Here a win occurs if a cop moves to the same vertex as the robber. The richness of this invariant is more subtle and is the subject of active research. Here I will share some results obtained over the past ten years at the Summer Undergraduate Research Institute in Experimental Mathematics (SURIEM) REU program hosted at Michigan State University. In particular, I will define and discuss the weak cop number of an infinite graph and will discuss recent work on graphs that are edge critical with respect to the cop number or weak cop number. I will

also share my experiences in collaborating with and mentoring many REU students who are at an early stage of their study of mathematics. (Received September 12, 2019)

1154-05-1028 **Elizabeth Drellich***, edrellich@haverford.edu, and **Heather Smith**. *Folding words around trees: Combinatorics inspired by biology.*

At the intersection of mathematics and biology we find mathematical models built to address biological questions as well as new mathematical theories inspired by biology. This talk will present a combinatorial model for folding words around plane trees, inspired by the bond formed between the nucleotides of a single-stranded RNA molecule. Valid plane trees, consisting of a word in a complementary alphabet wrapped around a plane tree, are fairly new constructions, and much of the work on these combinatorial structures has been joint with undergraduates. This talk will serve as an introduction and present several open questions that are appropriate for undergraduate research. (Received September 12, 2019)

1154-05-1035 **Dana C. Ernst*** (dana.ernst@nau.edu). *Architecture of braid classes in Coxeter systems.*

Any two reduced expressions for the same Coxeter group element are related by a sequence of commutations and so-called braid moves. We say that two reduced expressions are braid equivalent if they are related via a sequence of braid moves, and the corresponding equivalence classes are called braid classes. Each braid class can be encoded in terms of a braid graph, where each vertex is an element of the braid class and two vertices are connected by an edge whenever the corresponding reduced expressions are related via a single braid move. In this talk, we will discuss the structure of braid graphs for several families of Coxeter systems, including types A , B , and D . (Received September 12, 2019)

1154-05-1049 **Jae-Ho Lee*** (jaeho.lee@unf.edu), 1 UNF Drive, Jacksonville, 32224. *Distance-regular graphs and degenerate DAHAs.*

By a theorem of Leonard (1982), the duality property of Q -polynomial distance-regular graphs characterizes the terminating branch of the Askey scheme of (basic) hypergeometric orthogonal polynomials, at the top level of which are the q -Racah polynomials. In this talk we consider the degeneration of the q -Racah polynomials until two arrows down in the Askey scheme. For each level, we discuss the associated distance-regular graphs, (degenerate) DAHAs, and the non-symmetric (or Laurent) polynomials and their orthogonal relations. (Received September 12, 2019)

1154-05-1055 **Tingting Ou*** (tou2@jhu.edu) and **Michelle Shu** (mshu1@jhu.edu). *Probabilistic Counting-Out Game on a Line.* Preliminary report.

Our research studies a novel problem posted on a question-and-answer website. There are n people in a line at positions $1, 2, \dots, n$. For each round, we randomly select a person at position k , where k is odd, to leave the line, and shift the people at position i such that $i > k$ to position $i - 1$. We continue to select people until there is only one person left, who then becomes the winner. We are interested in two questions: which initial position has the greatest chance to win and which has the longest expected time to stay in the line. We have derived recursions to solve for exact values of the winning probabilities and expected time, the exact formula for the winning probabilities, and the asymptotic behaviors of the expected survival time. We will also present a conjecture on the winning probabilities formula of a variation of the problem, where people are grouped into triples, quadruples, etc. and the first person in each group is at the risk of being selected, as well as other possible extensions and related findings concerning this problem. (Received September 12, 2019)

1154-05-1058 **Michael C. Strayer*** (mstrayer@hsc.edu). *Classifications of minuscule Kac–Moody representations built from colored posets.*

The finite colored “minuscule” and “ d -complete” posets of R.A. Proctor, the finite “dominant minuscule heaps” of J.R. Stembridge, and the infinite “full heaps” of R.M. Green are classes of colored posets that have been used in various combinatorial, representation theoretic, and geometric settings. For example, the minuscule posets were used by Thomas and Yong to compute the cohomology of minuscule flag varieties, and Green’s Cambridge monograph *Combinatorics of Minuscule Representations* contains several applications of full heaps to representation theory and geometry. The posets in these classes each correspond to a Dynkin diagram and can be used to build “minuscule” representations of the corresponding Kac–Moody algebra or Borel subalgebra. We present new sets of cardinality-independent poset coloring axioms that characterize when such representations can be built. Consequently, these axioms unify the above settings; they also provide the first definition of infinite colored d -complete posets. We give the classifications of the resulting new classes of posets, which includes those of Proctor, Stembridge, and Green as special cases. Our classifications produce a complete list of the minuscule representations that can be built from colored posets. (Received September 12, 2019)

1154-05-1060 **Melissa Gutiérrez González*** (mgutierrezgo@oxy.edu), 1600 Campus Road #948, Los Angeles, CA 90041, and **Jerrell Cockerham, Amaury Minino and Pamela E Harris (Mentor)**. *Weight q -Multiplicities for Representations of the Exceptional Lie Algebra \mathfrak{g}_2* . Preliminary report.

In combinatorial representation theory, Kostant's partition function counts the number of ways a given weight (vector) of a Lie algebra \mathfrak{g} can be written as a nonnegative integral linear combination of the positive roots of \mathfrak{g} . Although it is very difficult to give a general closed formula for this vector partition function for Lie algebras of arbitrary rank, we are motivated by previous successes in low rank cases and present a closed formula of Kostant's partition function for the exceptional Lie algebra \mathfrak{g}_2 . We then use this result to present a new closed formula for the q -multiplicity formula in this Lie algebra. (Received September 12, 2019)

1154-05-1061 **Sarah K. Salmon*** (sarah.salmon@colorado.edu). *Full heaps and their convex subheaps*. Preliminary report.

A full heap is an infinite partially ordered set equipped with a labeling taken from an underlying Dynkin diagram and satisfying certain conditions. It is known that convex subheaps of full heaps give rise to fully commutative elements in the associated Coxeter group. In general, not every fully commutative element can be associated to a convex subheap of a full heap, but we will show that we can accomplish this association in type affine A. (Received September 16, 2019)

1154-05-1071 **Jadyn V. Breland*** (jbreland@ucsc.edu). *Braid shadows in simply-laced Coxeter systems*. Preliminary report.

Any two reduced expressions for the same element of a Coxeter group are related by a sequence of commutations and so-called braid moves. We say that two reduced expressions are braid equivalent if they are related via a sequence of only braid moves. The arrangement of generators in a reduced expression may provide several opportunities for a braid move. We refer to the location of the generators involved in a potential braid move as a braid shadow. In this talk, we will describe the combinatorial structure of braid shadows, and their support, for braid equivalent reduced expressions in simply-laced Coxeter systems. (Received September 13, 2019)

1154-05-1085 **Adam Blumenthal*** (ablument@iastate.edu), **Bernard Lidicky, Ryan Martin, Sergey Norin, Florian Pfender and Jan Volec**. *Counterexamples to a conjecture of Harris on Hall ratio*.

The Hall ratio of a graph G is the maximum value of $v(H)/\alpha(H)$ taken over all non-null subgraphs H of G . For any graph, the Hall ratio is a lower-bound on its fractional chromatic number. In this note, we present various constructions of graphs whose fractional chromatic number grows much faster than their Hall ratio. This refutes a conjecture of Harris. (Received September 13, 2019)

1154-05-1113 **Mary K Flagg** (flaggm@stthom.edu), 3800 Montrose, Houston, TX 77006, and **Phylicia Tran*** (tranpk@stthom.edu). *Power Domination in ciclo and estrella graphs*. Preliminary report.

The PMU placement problem in electrical engineering is the challenge of determining the optimal location of PMUs (phasor measurement units) in an electric power network to monitor the current and voltage in the network. Power domination is a coloring game on a simple graph which models the PMU placement problem. A power dominating set is a set of vertices (corresponding to the location of the PMUs) which color the whole graph according to the color change rule basic on the physics. The power domination number is the minimum cardinality of a power dominating set. To model the possibility that the data may not be available from one PMU, a 1-vertex fault-tolerant power dominating set is one in which the set minus any one vertex is also a power dominating set. The minimum cardinality of a 1-vertex fault-tolerant power dominating set is called the 1-vertex fault-tolerant power domination number. In this talk, we will present our results on the power domination and 1-vertex fault-tolerant power domination numbers of various ciclo and estrella graphs. (Received September 13, 2019)

1154-05-1117 **Ae Ja Yee*** (yee@psu.edu). *Index of seaweed algebras and integer partitions*.

The index of a Lie algebra is an important algebraic invariant. In 2000, Dergachev and Killilov defined seaweed subalgebras of \mathfrak{gl}_n (or \mathfrak{sl}_n) and provided a formula for the index of a seaweed algebra using a certain graph, so called a meander.

In a recent paper, Vincent Coll, Andrew Mayers, and Nick Mayers defined a new statistic for partitions, namely the index of a partition, which arises from seaweed Lie algebras of type A. At the end of their paper, they presented an interesting conjecture, which involves integer partitions. In this talk, I will discuss their

conjecture. This is joint work with Seunghyun Seo from Kangwon National University. (Received September 13, 2019)

1154-05-1138 **Sean English, Pamela Gordom, Nathan Graber, Abhishek Methuku and Eric C Sullivan*** (eric.2.sullivan@ucdenver.edu). *Saturation of Berge Hypergraphs.*

Abstract: Let H be a hypergraph, and G be a simple graph on the same vertex set. We say H is *Berge- G* if there exists a bijection $f : E(G) \rightarrow E(H)$ such that for each $e \in E(G)$, we have $e \subset f(e)$. If there exists a subhypergraph of H that is *Berge- G* we say that H contains G , otherwise H is said to be *G -free*. A hypergraph, H is *Berge- G -saturated* if H does not contain G but $H + e$ contains G for every $e \notin E(H)$. The *Berge-saturation number*, denoted $\text{B-sat}(H, G)$, is the minimum number of edges in a hypergraph H such that H is G -saturated.

In this talk we will discuss the Berge-saturation number for several classes of graphs and draw comparisons between Berge-saturation and saturation in the traditional graph sense. (Received September 13, 2019)

1154-05-1164 **Joshua Harrington and Eugene Henninger-Voss***, eugene.henninger_voss@tufts.edu, and **Kedar Karhadkar, Emily Robinson and Tony W. H. Wong.** *Sum index and difference index of graphs.* Preliminary report.

Let G be a nonempty simple graph with a vertex set V and an edge set E . For every injective vertex labeling $f : V \rightarrow \mathbb{Z}$, there are two induced edge labelings, namely $f^+ : E \rightarrow \mathbb{Z}$ defined by $f^+(uv) = f(u) + f(v)$, and $f^- : E \rightarrow \mathbb{Z}$ defined by $f^-(uv) = |f(u) - f(v)|$. The sum index and the difference index are the minimum cardinalities of the ranges of f^+ and f^- , respectively. We give the upper and lower bounds on the sum index and difference index, and determine the sum index and difference index of various families of graphs. (Received September 13, 2019)

1154-05-1181 **Mélodie Lapointe***, lapointe.melodie@courrier.uqam.ca. *Automorphisms of the Free Group and Perfectly Clustering Words.*

Perfectly clustering words are a generalization of Christoffel words, introduced by Simpson and Puglisi in 2009. A word on an ordered alphabet $a_1 < a_2 < \dots < a_n$ is *perfectly clustering* if its Burrows-Wheeler transform has the following form: $a_n^{i_n} a_{n-1}^{i_{n-1}} \dots a_1^{i_1}$. Simpson and Puglisi describe the perfectly clustering words on a three letters alphabet. We consider certain automorphisms of the free group to generate all perfectly clustering words on an arbitrary alphabet. Our automorphisms are not morphisms of the free monoid, but the result is a generalization of the tree of Christoffel words. To achieve the main result we also use the bijection between clustering words and discrete interval exchange shown by Ferenczi and Zamboni. (Received September 13, 2019)

1154-05-1191 **Daniela Ferrero*** (dferrero@txstate.edu), Texas State University, San Marcos, TX 78666. *Redundant Power Dominating Sets.* Preliminary report.

A Phasor Measurement Unit (PMU) is a device placed on a bus of an electrical power system to obtain time-stamped readings of the voltage and current phasors of the electromagnetic wave at the bus. In addition, if PMUs are placed at strategically selected buses, their synchronized readings yield the electromagnetic wave at any bus without a PMU. The PMU placement problem asks for the minimum number of PMUs needed to monitor a network, and the buses where they must be placed. When an electrical power network is modeled by a graph, a solution to the PMU placement problem for the network corresponds to a minimum power dominating set for the graph.

Since 2010, multiple measurement systems based on PMUs have been implemented, and they have shown that minimizing the number of PMUs alone yields unsatisfactory results, due to frequent loss of PMU readings caused by network failures. While higher levels of redundancy require larger number of PMUs, implying increased cost, the addition of a few redundant PMUs has been proven to yield multiple advantages, resulting in a cost-effective system upgrade. In this talk, we present results on the relationship between the level of added redundancy and the fault-tolerance capability of the system obtained, from the power domination viewpoint. (Received September 13, 2019)

1154-05-1202 **Nathan Vallapureddy*** (nvallap1@jhu.edu) and **Lyle Paskowitz** (lyle@jhu.edu). *Efficient and Non-efficient Domination of \mathbb{Z} -stacked Archimedean Lattices.* Preliminary report.

On a graph, a vertex v dominates another vertex v' if $v = v'$ or v is adjacent to v' . An efficient dominating set is a subset of vertices D such that every vertex in the graph is dominated by exactly one vertex in D . We developed general results about the efficient domination problem on graphs. In particular, we investigated the existence of efficient dominating sets on the stacked versions of each of the eleven Archimedean Lattices. We

constructed efficient dominating sets on eight of the stacked lattices, and used integer programming to prove that no such sets exist on the other three stacked lattices. (Received September 13, 2019)

1154-05-1221 **Joel Louwsma*** (jlouwsma@niagara.edu), Department of Mathematics, Niagara University, P.O. Box 2044, Niagara University, NY 14109. *Arithmetical structures on graphs.*

An arithmetical structure on a finite, connected, simple graph is a labeling of the vertices with positive integers such that, at each vertex, the number there is a divisor of the sum of the numbers at adjacent vertices, and where the numbers used have no common factor. We discuss results, joint with Archer, Bishop, Diaz-Lopez, García Puente, and Glass, about counting arithmetical structures and classifying their critical groups on graphs such as bidents. We also discuss results, joint with Harris, about possible largest values of arithmetical structures on complete graphs. These projects stem from the 2017 REUF program. (Received September 14, 2019)

1154-05-1254 **Hannah E Burson*** (hburso2@illinois.edu) and **Dennis Eichhorn.** *A Novel Generalization of Partitions with Parts Separated by Parity.* Preliminary report.

In a study of the third order mock theta functions, George Andrews introduced the function $\mathcal{EO}^*(2n)$, which counts the number of partitions where each even part is less than each odd part and only the largest even part appears an odd number of times. In this talk, we introduce a new class of partition theoretic objects, called coaugmented partitions, that generalize the partitions counted by $\mathcal{EO}^*(2n)$. Coaugmented partitions have remarkable symmetry that is not obvious from the definition of $\mathcal{EO}^*(2n)$ and provide a natural combinatorial framework for studying $\mathcal{EO}^*(2n)$. (Received September 14, 2019)

1154-05-1275 **Linyuan Lu** and **Zhiyu Wang*** (zhiyuw@math.sc.edu). *Find Berge hypergraphs by looking at the shadow.*

For a fixed set of positive integers R , we say \mathcal{H} is an R -uniform hypergraph, or R -graph, if the cardinality of each edge belongs to R . For a graph $G = (V, E)$, a hypergraph \mathcal{H} is called a *Berge- G* , denoted by BG , if there is an injection $i: V(G) \rightarrow V(\mathcal{H})$ and a bijection $f: E(G) \rightarrow E(\mathcal{H})$ such that for all $e = uv \in E(G)$, we have $\{i(u), i(v)\} \subseteq f(e)$. We present some recent results about extremal problems on Berge hypergraphs from the perspectives of the shadow graph. In particular, we define variants of the Ramsey number and Turán number in Berge hypergraphs, namely the *cover Ramsey number* and *cover Turán number*, and show some general lower and upper bounds on these variants. We also determine the cover Turán density of all graphs when the uniformity of the host hypergraph equals to 3. These results are joint work with Linyuan Lu. (Received September 14, 2019)

1154-05-1278 **Harish Vemuri*** (harish.vemuri@yale.edu). *Domination in direct products of complete graphs.*

Let X_n denote the unitary Cayley graph of $\mathbb{Z}/n\mathbb{Z}$. We continue the study of cases in which the inequality $\gamma_t(X_n) \leq g(n)$ is strict, where γ_t denotes the total domination number, and g is the arithmetic function known as Jacobsthal's function. The best known result in this direction is a construction of Burcroff in 2018 which gives a family of n with arbitrarily many prime factors that satisfy $\gamma_t(X_n) \leq g(n) - 2$. We present a new interpretation of the problem which allows us to use recent results on the computation of Jacobsthal's function to construct n with arbitrarily many prime factors that satisfy $\gamma_t(X_n) \leq g(n) - 16$. We also present new lower bounds on the domination numbers of direct products of complete graphs, which in turn allow us to derive new asymptotic lower bounds on $\gamma(X_n)$, where γ denotes the domination number. Finally, resolving a question of Defant and Iyer from 2017, we completely classify all graphs $G = \prod_{i=1}^t K_{n_i}$ satisfying $\gamma(G) = t + 2$. (Received September 14, 2019)

1154-05-1298 **Greg Churchill*** (gregory.churchill@oswego.edu) and **Brendan Nagle** (bnagle@usf.edu). *Extending Hansel's Theorem to Hypergraphs.*

For integers $n \geq k \geq 2$, let V be an n -element set, and let $\binom{V}{k}$ denote the set of all k -element subsets of V . Let \mathcal{C} be a collection of pairs $\{A, B\} \in \mathcal{C}$ of disjoint subsets $A, B \subseteq V$. We say that \mathcal{C} covers $\binom{V}{k}$ if, for every $K \in \binom{V}{k}$, there exists $\{A, B\} \in \mathcal{C}$ so that $K \subseteq A \cup B$ and $K \cap A \neq \emptyset \neq K \cap B$. When $k = 2$, such a family \mathcal{C} is called a separating system of V , where this concept was introduced by Rényi and studied by many authors.

Let $h(n, k)$ denote the minimum value of $\sum_{\{A, B\} \in \mathcal{C}} (|A| + |B|)$ over all covers \mathcal{C} of $\binom{V}{k}$. For $k = 2$, Hansel determined the sharp bounds $\lceil n \log_2 n \rceil \leq h(n, 2) \leq n \lceil \log_2 n \rceil$, and Bollobás and Scott sharpened these bounds to an exact formula for $h(n, 2)$ for all integers $n \geq 2$. Here, we extend these results by determining an exact formula for $h(n, k)$ for all integers $n \geq k \geq 2$. Also, we present some results regarding d -partite covers of $\binom{V}{k}$, which mirror Hansel's result. (Received September 14, 2019)

1154-05-1299 **Svetlana Puzyrmina*** (spuzyr@spbu.ru), Saint Petersburg State University, 7-9 Universitetskaya Emb., Saint Petersburg, 199034, Russia. *Abelian subshifts*.

Two finite words u and v are called abelian equivalent if each letter of the alphabet occurs the same number of times in both u and v . The abelian subshift \mathcal{A}_x of an infinite word x is the set of words y such that, for each factor u of y , there exists a factor v of x such that u and v are abelian equivalent. The notion of an abelian subshift gives a characterization of Sturmian words: among binary uniformly recurrent words, Sturmian words are exactly those words for which \mathcal{A}_x equals the shift orbit closure Ω_x . On the other hand, the abelian subshift of the Thue-Morse word contains uncountably many minimal subshifts. In this talk we discuss general properties of abelian subshifts. In particular, we consider the abelian subshifts of binary words, non-binary balanced words, and characterize abelian subshifts of aperiodic words of minimal complexity over an alphabet of cardinality k for each $k \geq 2$. (Received September 14, 2019)

1154-05-1322 **Sami Assaf*** (shassaf@usc.edu) and **Dominic Searles**. *Kohnert polynomials as characters of flagged Schur modules*. Preliminary report.

To each diagram of cells in the first quadrant, one may associate a Specht module, Schur module, and flagged Schur module. Reiner and Shimozono gave formulas for the irreducible decomposition of Specht and Schur modules when the diagram is "percent-avoiding" using the flagged Schur modules. In this talk, we give a direct method for computing the characters. (Received September 14, 2019)

1154-05-1343 **Ali K Uncu*** (akuncu@risc.jku.at). *The method of weighted words re-revisited*. Preliminary report.

Alladi and Gordon introduced the method of weighted words in 1993 to prove a refinement and generalization of Schur's partition identity. Together with Andrews, they later used it to refine Capparelli's and Goellnitz' identities. In 2017, Dousse published the article "The method of weighted words revisited", where she explained her variant on this method. In a series of papers, she demonstrated the strength and applicability of her straightforward method. Recently, joint with Jakob Ablinger, the author automated many steps of the weighted words approach of Dousse using symbolic computation.

In this talk, we are planning to introduce the method, introduce the weighted words portion of the soon-to-be-public Mathematica package qFunctions, and demonstrate this implementation while we prove some partition identities. (Received September 15, 2019)

1154-05-1344 **Orli Herscovici*** (orli.herscovici@gmail.com). *Generalized permutations*. Preliminary report.

The Eulerian polynomials are closely related to permutations. In 1954 Carlitz defined a generalization, q -Eulerian polynomials. The connection between these polynomials and permutations was only discovered about 20 years later. Another generalization of the Eulerian polynomials, the so-called degenerate Eulerian polynomials, was proposed by Carlitz in 1956. In this work we will define a new class of generalized permutations and demonstrate a relation between them and the degenerate Eulerian polynomials. (Received September 15, 2019)

1154-05-1358 **Rachel Zhang*** (rachelyz@mit.edu). *C - (k, ℓ) -Sum-Free Sets*. Preliminary report.

The Minkowski sum of two subsets A and B of a finite abelian group G is defined as all pairwise sums of elements of A and B : $A + B = \{a + b : a \in A, b \in B\}$. The largest size of a (k, ℓ) -sum-free set in G has been of interest for many years and in the case $G = \mathbb{Z}/n\mathbb{Z}$ has recently been computed by Bajnok. Motivated by sum-free sets of the torus, Kravitz introduces the noisy Minkowski sum of two sets, which can be thought of as discrete evaluations of these continuous sumsets. That is, given a noise set C , the noisy Minkowski sum is defined as $A +_C B = A + B + C$. We give bounds on the maximum size of a (k, ℓ) -sum-free subset of $\mathbb{Z}/n\mathbb{Z}$ under this new sum, for C equal to an arithmetic progression with common difference relatively prime to n and for any two element set C . (Received September 15, 2019)

1154-05-1363 **Walter M. Bridges*** (wbridg6@su.edu). *Limit Shapes for Unimodal Sequences*.

A limit shape for a type of unimodal sequence of integers is a certain 0-1 Law satisfied by their diagrams. The diagrams of a unimodal sequence are stacks of boxes in the plane. One can ask whether 100% of boundaries of diagrams of size n approach some limiting curve as $n \rightarrow \infty$. This type of question has been well-studied for partitions. Using a method of F. Petrov, we obtain limit shapes for a variety of unimodal sequences. (Received September 15, 2019)

1154-05-1404 **Mark Ellingham*** (mark.ellingham@vanderbilt.edu), **J. Zachary Gaslowitz** and **Ryan Solava**. *The structure of 4-connected $K_{2,t}$ -minor-free graphs.*

Guoli Ding has provided a rough structure theorem for $K_{2,t}$ -minor free graphs for all t . As a special case of his theorem, 4-connected $K_{2,t}$ -minor-free graphs are obtained by attaching *strips*, consisting of two paths joined by edges with restricted crossings, to a finite set of base graphs. The first value of t where this applies in a nontrivial way is $t = 5$. We give a characterization of 4-connected $K_{2,5}$ -minor-free graphs that shows that they can be obtained from a cyclic sequence of four types of subgraph. Consequently, we can derive a generating function and asymptotic estimate for the number of nonisomorphic 4-connected $K_{2,5}$ -minor-free graphs of a given order. Our work extends to general t by providing a more precise description of the strips in Ding's result, suggesting a general asymptotic counting conjecture. (Received September 15, 2019)

1154-05-1417 **Oleg R Musin*** (oleg.musin@utrgv.edu), One West University Boulevard, Mathematics, UTRGV, Brownsville, TX 78520. *Majorization and energy on spheres.*

Let S be an arbitrary set and $\rho : S \times S \rightarrow \mathbb{R}$ be any symmetric function. Then for a given real function f and for any finite subset X of S can be defined the potential energy $E_f(X) := \sum_{x \neq y \in X} f(\rho(x, y))$.

Let $R_\rho(X)$ denote the set of all values $\rho(x, y)$, where $x \neq y$. The *majorization theorem* (Karamata's inequality) yields that if $R_\rho(X)$ majorizes $R_\rho(Y)$, $|X| = |Y|$, then $E_f(X) \leq E_f(Y)$ for any continuous convex monotonically non-increasing function f . Therefore, if $M(S, \rho, n)$ denote the set of all $X \subset S$ with $|X| = n$ such that for any $Y \subset S$, $|Y| = n$, either $R_\rho(X)$ majorizes $R_\rho(Y)$ or they are incomparable, then $M(S, \rho, n)$ consists of all X that give minimum energy of E_f for certain f .

In this paper we discuss $M = M(S, \rho, n)$ with $S = \mathbb{S}^d$. In particular, we describe M with $n \leq 4$, consider ρ such that $M(\mathbb{S}^d, \rho, d + 2)$ consists of regular simplices and their relations to universally optimal spherical configurations. (Received September 15, 2019)

1154-05-1443 **Tianyuan Xu*** (tixu6187@colorado.edu), Department of Mathematics, University of Colorado Boulder, Campus Box 395, Boulder, CO 80309, and **Richard M. Green** (rmg@colorado.edu), Department of Mathematics, University of Colorado Boulder, Campus Box 395, Boulder, CO 80309. *On elements of \mathbf{a} -value 2 in Coxeter groups.*

The \mathbf{a} -function on a Coxeter group W is a function $\mathbf{a} : W \rightarrow \mathbb{N}$ defined by Lusztig which has important connections with the cell representation theory of W and its Hecke algebra. It is known that the identity element of W is the only element with \mathbf{a} -value 0, while a non-identity element has \mathbf{a} -value 1 if and only if it has a unique reduced word. However, as the definition of the \mathbf{a} -function involves products of the Kazhdan-Lusztig basis element in the Hecke algebra, \mathbf{a} -values of elements are often difficult to compute in general.

In this talk, we present some recent progress on elements of \mathbf{a} -value 2. We show that elements of \mathbf{a} -value 2 must be fully commutative in the sense of Stembridge, which allows us to associate certain posets called *heaps* to such elements. Using heaps, we conjecture a combinatorial characterization of elements of \mathbf{a} -value 2, classify Coxeter groups with finitely many elements of \mathbf{a} -value 2, and enumerate such elements in all groups from the classification. (Joint work with Richard Green.) (Received September 15, 2019)

1154-05-1445 **Hanmeng Zhan***, h3zhan@yorku.ca. *New advances in quantum walks.*

I will talk about some recent progress in quantum walks on graphs. (Received September 15, 2019)

1154-05-1448 **Ben Nassau*** (bcnassau@udel.edu), **Felix Lazebnik** and **Andrew J Woldar**. *On q -lifts: Finite Geometries and Algebraically Defined Graphs.* Preliminary report.

Let q be a prime power and \mathbb{F}_q be the field of q elements. For $i = 2, 3$, let $P_i = L_i = \mathbb{F}_q^i$, and $f_2 \in \mathbb{F}_q[X_1, Y_1]$, $f_3 \in \mathbb{F}_q[X_1, Y_1, X_2, Y_2]$ be polynomials over \mathbb{F}_q . We define $\Gamma_2 = \Gamma(q; f_2)$ to be the bipartite graph with partition sets P_2 and L_2 such that $p = (p_1, p_2) \in P_2$ and $l = [l_1, l_2] \in L_2$ are adjacent if and only if

$$p_2 + l_2 = f_2(p_1, l_1).$$

Similarly, we define $\Gamma_3 = \Gamma(q; f_2, f_3)$ to be the bipartite graph with partition sets P_3 and L_3 such that $p = (p_1, p_2, p_3) \in P_3$ and $l = [l_1, l_2, l_3] \in L_3$ are adjacent if and only if

$$\begin{aligned} p_2 + l_2 &= f_2(p_1, l_1) \\ p_3 + l_3 &= f_3(p_1, l_1, p_2, l_2). \end{aligned}$$

The canonical projection $\overline{\Phi} : \mathbb{F}_q^3 \rightarrow \mathbb{F}_q^2$, $\langle v_1, v_2, v_3 \rangle \mapsto \langle v_1, v_2 \rangle$ induces a surjective q -to-1 map $\Phi : V(\Gamma_3) \rightarrow V(\Gamma_2)$ by $(p_1, p_2, p_3) \mapsto (p_1, p_2)$ and $[l_1, l_2, l_3] \mapsto [l_1, l_2]$. This map Φ is a graph homomorphism, and so we call Γ_3 a q -lift of Γ_2 .

We present some properties of q -lifts and explain how their specializations relate to finite geometries. (Received September 15, 2019)

1154-05-1457 **Eric Stucky*** (stuck127@umn.edu), 1920 S 1st St., #506, Minneapolis, MN 55454. *Cyclic Sieving, Necklaces, and Bracelets.*

In their celebrated 2004 paper, Reiner, Stanton, and White define and collate examples of the “cyclic sieving phenomenon” (CSP). Notably, the classical analogue of the q -binomial coefficient exhibits a CSP with respect to the long cycle $(1\ 2\ \dots\ n)$ acting on subsets of $[n]$.

In this talk, we interpret these q -binomial coefficients as exhibiting a “secondary” CSP. For instance, for $\binom{n}{k}$ with $(k, n) = 1$, the action of the long cycle is free. This immediately yields a proof that the (Mahonian) rational q -Catalan numbers are polynomials in q . Then, using necklaces with specified bead-counts as our model for rational Catalan objects, we observe a $q = -1$ phenomenon and generalize it in some circumstances to cyclic sieving phenomena of higher order. (Received September 15, 2019)

1154-05-1461 **Margaret Bayer, Bennet Goeckner, Su Ji Hong, Tyrrell McAllister, McCabe Olsen, Casey Pinckney*** (pinckney@math.colostate.edu), **Julianne Vega, Martha Yip** and **Semin Yoo**. *Lattice polytopes from Schur polynomials.*

Our focus is to study lattice polytopes that arise from a classical family found in algebraic combinatorics called Schur polynomials. In particular, we are interested in determining when such a polytope has the Integer Decomposition Property and when it is reflexive. A polytope P has the Integer Decomposition Property if every lattice point in the t^{th} dilation of P can be written as a sum of t lattice points in P . A polytope P whose interior contains the origin is reflexive if for every facet of P , there are no integer lattice points between the affine hyperplane spanned by P and the parallel hyperplane passing through the origin. In other words, reflexive polytopes contain exactly one interior lattice point. We show that every polytope arising from Schur polynomials in the way we are considering has the Integer Decomposition Property, and we give a complete characterization of those which are reflexive. (Received September 15, 2019)

1154-05-1463 **Daryl R. DeFord*** (ddeford@mit.edu), 32 Vassar St., D475A, Cambridge, MA 02139. *Markov chain sampling for connected graph partitions.*

Applications to political redistricting have led to increased interest in methods that generate large collections of maps for evaluating potential gerrymanders. A natural approach is to use Markov chains defined on the space of connected graph partitions to generate these plans. In this talk, I will discuss several recently designed proposal methods, based on spanning trees, that move efficiently through this space and address related questions of computational complexity. As with many similar problems where it is difficult to prove bounds on mixing times directly, this research has been driven by computational approaches, with well-designed experiments motivating and leading towards the theoretical results. (Received September 16, 2019)

1154-05-1466 **Mingjia Yang*** (my237@math.rutgers.edu) and **Doron Zeilberger** (doronzeil@gmail.com). *Increasing consecutive patterns in words.*

We will discuss how we used experimental math methods to conjecture the generating function for words in the alphabet $\{1, 2, \dots, n\}$ ($n \geq 1$) avoiding the consecutive pattern $12 \cdots r$ for any $r \geq 2$, and how to tweak the Goulden-Jackson cluster method to prove this result. Time permitting, we will also discuss extension to words with a certain number of the consecutive pattern $12 \cdots r$ (not just avoiding) and recurrences we came up with, which lead to efficient computations. (Received September 15, 2019)

1154-05-1470 **K. A. S. Factor, L. J. Langley** and **S. K. Merz***, Department of Mathematics, University of the Pacific, Stockton, CA 95211. *Tight Bounds for the Split Domination Number of Regular and Nearly Regular Tournaments.*

A set of vertices, S , in a strongly connected digraph D , is split dominating provided it is: 1) dominating and 2) $D[V(D) \setminus S]$ is trivial or not strongly connected. The split domination number of a strongly connected digraph is the minimum cardinality of a split dominating set for that digraph. We show that for any k -regular tournament, the split domination number is at least $\lceil \frac{2k+3}{3} \rceil$. Furthermore, we show that when T is a nearly regular $2k$ -tournament, then the split domination number of T is at least $\lceil \frac{2k}{3} \rceil$. Both bounds are tight. (Received September 15, 2019)

1154-05-1516 **Zewei Li*** (zli128@jhu.edu) and **Hongkang Yang** (hy1194@nyu.edu). *On the ℓ_∞ Distance between Two Random Permutations.*

Consider two random permutations X_1, X_2, \dots, X_n and Y_1, Y_2, \dots, Y_n that are uniformly chosen from any of the permutation of $[n]$. We investigate the difference among them under multiple commonly used metrics. We

derive that the expected value of the distance to the power p is approximate $\frac{2n^{p+1}}{(p+1)(p+2)}$ under the general ℓ_p metric, and we also get the approximate expression of the variance of the difference. Using a specific version of the Central Limit Theorem, we prove that the distribution of the difference converges to a normal distribution for each $p \in [1, \infty)$. For specific interest, let D_∞ be the distance of the two random permutation under the ℓ_∞ metric, we prove that $\frac{E[D_\infty]}{n}$ converges to 1. We also extract precise estimates of the rate of convergence to 1. (Received September 16, 2019)

1154-05-1520 **Anders Buch***, 110 Frelinghuysen Rd, Piscataway Township, NJ 08854, and **Chengxi Wang**. *Positivity determines quantum cohomology.*

I will show that the small quantum cohomology ring of a Grassmannian is, up to rescaling the deformation parameter q , the only graded q -deformation of the singular cohomology ring with non-negative Schubert structure constants. This implies that the (three point, genus zero) Gromov-Witten invariants are uniquely determined by Witten's presentation of the quantum ring and the fact that they are non-negative. A similar statement appears to be true for any flag variety of simply laced Lie type. For the variety of complete flags, this statement is equivalent to Fomin, Gelfand, and Postnikov's conjecture that the quantum Schubert polynomials are uniquely determined by positivity properties. The proof for Grassmannians answers a question of Fulton. (Received September 16, 2019)

1154-05-1534 **Kateřina Medková, Edita Pelantova** and **Elise Vandomme***, e.vandomme@alumni.uliege.be. *On non-repetitive complexity of Arnoux-Rauzy words.*

Variability of an infinite word $\mathbf{u} = u_0u_1u_2 \cdots$ over a finite alphabet can be judged from distinct points of view. For instance, Moothathu introduced in 2012 the non-repetitive complexity $nrC_{\mathbf{u}}$ which reflects the structure of \mathbf{u} with respect to the repetitions of factors of a given length. The value $nrC_{\mathbf{u}}(n)$ is the maximal m such that for some $i \in \mathbb{N}$ any factor of \mathbf{u} of length n occurs at most once in $u_iu_{i+1}u_{i+2} \cdots u_{i+m+n-2}$. He also considered a prefix variant of this function called the initial non-repetitive complexity function $inrC_{\mathbf{u}}$. In 2016, Nicholson and Rampersad described some general properties of $inrC_{\mathbf{u}}$ and evaluated $inrC_{\mathbf{u}}$ for the Fibonacci and Tribonacci words. Recently, Bugeaud and Kim studied $inrC_{\mathbf{u}}$ for Sturmian sequences. All these words belong to the class of Arnoux-Rauzy words, which are one of the generalizations of Sturmian words to multi-letter alphabets. In this talk, we determine $nrC_{\mathbf{u}}$ for the Arnoux-Rauzy words and $inrC_{\mathbf{u}}$ for the standard Arnoux-Rauzy words. Our main tools are S -adic representation of Arnoux-Rauzy words and description of return words to their factors. The formulas we obtain are then used to evaluate $nrC_{\mathbf{u}}$ and $inrC_{\mathbf{u}}$ for the d -bonacci word. (Received September 16, 2019)

1154-05-1559 **Jonathan Ramalheira-Tsu*** (jramalheiratsu@math.arizona.edu), Department of Mathematics, University of Arizona, 617 N. Santa Rita Ave., Tucson, AZ 85721-0089, and **Nicholas M Ercolani**. *Combinatorial Dynamics: From Patience Sorting to the Discrete-Time Toda Lattice.*

Tropicalisation provides a mechanism for relating combinatorial/algorithmic processes to dynamic/integrable ones, often of a stochastic character. One of the classical examples of this was the algorithmic (in terms of "patience sorting") determination of the asymptotic expected value of the longest increasing sequence in a permutation which is related to a Poissonised process that can be effectively analysed by integrable methods of Riemann-Hilbert analysis. However, in this example, the nature of the underlying dynamics is somewhat obscured. A model of great current interest (which in fact generalises the patience sorting example) starts with the Robinson-Schensted-Knuth (RSK) algorithm and relates it by a "tropical" correspondence to the dynamics of the discrete Toda lattice. The novelty of our approach is to provide deeper insight into all this in terms of factorisations of one-dimensional discrete Schrodinger-type operators and related Backlund transformations. (Received September 16, 2019)

1154-05-1580 **J. W. Moon** and **Laura L.M. Yang*** (laura.yang@ucf.edu), Department of Mathematics, University of Central Florida, Orlando, FL 32816. *Asymptotic Results on Weighted Ordered Trees*. Preliminary report.

Binomial trees are defined as a sub-family of simply-generated trees, which have been studied by Meir and Moon and others. The paper is to investigate four statistics on simply-generated families: leaves, non-rightmost leaves, proper edges and proper vertices. We use Darboux's theorem to obtain asymptotic results on their expectations $\mu_i(n)$ and variances, where n denotes the number of vertices in the trees being considered. For any constant $0 < c < 1$ (for proper edges, $0 \leq c \leq 1/2$), we prove that there exists a simply-generated family such that $\mu_i(n)/n$ tends to c when $n \rightarrow \infty$. For binomial trees, we derive explicit expressions for $\mu_i(n)$. Using Lyapunov's

condition, we prove the the distribution of the number of proper vertices is asymptotically normal. (Received September 16, 2019)

1154-05-1628 **Atsuo Kuniba** (atsuo.s.kuniba@gmail.com), Tokyo, 153-8902, Japan, **Hanbaek Lyu*** (colourgraph@gmail.com), Los Angeles, CA 90025, and **Masato Okado** (okado@sci.osaka-cu.ac.jp), Osaka, 558-8585, Japan. *Large deviations and one-sided scaling limit of randomized box-ball system.*

The box-ball system (BBS) system is an integrable cellular automaton on one dimensional lattice, where each of the first n sites is occupied with a semistandard tableau of rectangular shape with fillings $\{0, 1, \dots, \kappa\}$, and the time evolution is given by successive application of the combinatorial R . We analyze the limiting shape of the invariant Young diagrams when the initial configuration is randomized. In the large n limit, we show that the ‘equilibrium shape’ of the invariant Young diagrams are given by Schur polynomials, using two different methods of Markov chains and Thermodynamic Bethe Ansatz (TBA). Furthermore, in the special case where each site is occupied by a single ball of color in $\{0, 1, \dots, \kappa\}$, we establish a large deviations principle for the row lengths of the invariant Young diagrams. As a corollary, they are shown to converge almost surely to the equilibrium shape at an exponential rate. (Received September 16, 2019)

1154-05-1632 **Bennet Goeckner, Corbin Groothuis, Cyrus Hettle, Brian Kell, Pamela Kirkpatrick, Rachel Kirsch*** (r.kirsch1@lse.ac.uk) and **Ryan Solava**. *Universal partial words.*

Chen, Kitaev, Mütze, and Sun recently introduced the notion of universal partial words, a generalization of universal words and de Bruijn sequences. Universal partial words allow for a wild-card character \diamond , which is a placeholder for any letter in the alphabet. We extend results from the original paper and develop additional proof techniques to study these objects. For non-binary alphabets, we show that universal partial words have periodic \diamond structure and are cyclic, and we give number-theoretic conditions on the existence of universal partial words. In addition, we provide an explicit construction for an infinite family of universal partial words over non-binary alphabets. (Received September 16, 2019)

1154-05-1634 **Lina Li*** (linali2@illinois.edu). *Rainbow variants of the Erdős-Rothchild problem.*

In 1974, Erdős and Rothchild conjectured that $K_{\lfloor \frac{n}{2} \rfloor, \lceil \frac{n}{2} \rceil}$ has the maximum number of two-edge-colorings without monochromatic triangles among all n -vertex graphs. Since then, this new class of colored extremal problems has been extensively studied by many researchers on various discrete structures, such as graphs, hypergraphs, boolean lattices, and sets.

In this talk, we investigate the rainbow variants of the Erdős-Rothchild problem. Our first main result, confirming conjectures of Benevides, Hoppen, and Sampaio, and Hoppen, Lefmann, and Odermann, completes the characterization of the extremal graphs for the edge-colorings without rainbow triangles. We also studied a similar question on sum-free sets, in which we describe the extremal configurations for the colorings of integers without rainbow sums. (Received September 16, 2019)

1154-05-1643 **Zachary Gershkoff*** (zgersh2@lsu.edu), Baton Rouge, LA , and **James Oxley**. *Cubic binary matroids*. Preliminary report.

A cubic graph on n vertices has a cycle matroid where every cocircuit is generated by a collection of $n - 1$ cocircuits, each of size three, where the symmetric difference of these cocircuits is also a 3-element cocircuit. This talk considers the class of binary matroids that have this property. (Received September 17, 2019)

1154-05-1654 **Calum MacRury*** (cmacrury@cs.toronto.edu), **Tomáš Masařík** (tarken@kam.mff.cuni.cz), **Xavier Pérez-Giménez** (xperez@unl.edu) and **Leilani Pai** (lpai@huskers.unl.edu). *Lower Bounds on Discrepancy of Random Hypergraphs*. Preliminary report.

Motivated by the Beck-Fiala conjecture, we study the discrepancy problem in two random hypergraph models, each of which has n vertices and m edges. In the first model, \mathcal{G}_1 is constructed by fixing a parameter p and allowing its vertices to join each of its m edges independently with probability p . In the parameter range for which $pn \rightarrow \infty$ and $pm \rightarrow \infty$, we show that \mathcal{G}_1 has discrepancy $\Omega(2^{-n/m} \sqrt{np})$ with high probability.

In the second model, d is fixed and each vertex of \mathcal{G}_2 independently joins exactly d edges uniformly at random. We explore a number of coupling techniques relating \mathcal{G}_1 to \mathcal{G}_2 for the special case when p is set to d/m . Using an argument based around these techniques, we explore how to extend the lower bound of $\Omega(2^{-n/m} \sqrt{nd/m})$ to \mathcal{G}_2 for the parameter range when $d \rightarrow \infty$ and $dn/m \rightarrow \infty$. (Received September 16, 2019)

1154-05-1659 **Stephen M Gagola, Jr*** (gagola@math.kent.edu), Department of Mathematics, 1300 University Esplanade, Kent State University, Kent, OH 44242. *Latin Squares from Polygonal Tiles*. Preliminary report.

It is possible to find a polygonal tile having no symmetry so that 64 congruent copies of it may be fitted together snugly in the style of M. C. Escher to form a Latin square (the “elements” of the square being the 8 distinct orientations of the original tile). The artist Peter Raedschelders has produced two such examples of Latin squares having distinct autotopy groups (and hence are not isotopic). We produce a third example of a polygonal shape that yields a Latin square that is not isotopic to either of these two earlier examples. Up to an obvious mathematical equivalence, there are no further polygonal shapes having this property. (Received September 16, 2019)

1154-05-1713 **Mehtaab Sawhney*** (msawhney@mit.edu), **Ashwin Sah, David Stoner and Yufei Zhao**. *A Reverse Sidorenko Inequality*.

We prove a number of tight graph homomorphism inequalities, where, for a fixed H , we wish to maximize the number of homomorphism from G to H (after exponentially normalizing by the size of G) under certain degree constraints on G (e.g., d -regular). A highlight of our results is that, among d -regular graphs of the same size, a disjoint complete bipartite graphs has the most number of proper q -colorings. Our results also extend to irregular graphs and list colorings. These results settle a number of conjectures by Kahn, Galvin-Tetali, Galvin, and Cohen-Csikvári-Perkins-Tetali. (Received September 16, 2019)

1154-05-1730 **Alex W Nowak*** (anowak@iastate.edu), 396 Carver Hall, 411 Morrill Road, Ames, IA 50011. *Distributive Mendelsohn triple systems and the Eisenstein integers*.

We study Mendelsohn triple systems whose quasigroup multiplication is self-distributive; that is, $x(yz) = (xy)(xz)$, $(zy)x = (zx)(yx)$. Building on the work of Donovan et al. we give a direct product decomposition for any such system whose point set has order coprime with 3. This allows for a complete description and enumeration of isomorphism classes. We present the complications related to the case in which the point set is divisible by 3. (Received September 16, 2019)

1154-05-1735 **Isaac Brown*** (isaac.l.brown@wsu.edu), **Brendan Miller, Tyler Russell and Christina Graves**. *Most Reliable Two-Terminal Graphs With Node Failures*.

A two-terminal graph is an undirected graph with two specified target vertices. If each vertex of such a graph fails independently with the same fixed probability (and edges are perfectly reliable), the two-terminal node reliability is the probability that the target vertices are in the same connected component in the induced subgraph of all operational nodes. A two-terminal graph is uniformly most reliable if its node reliability polynomial is greater than or equal to that of all other two-terminal graphs with the same fixed number of vertices, n , and edges, m . In this paper, we show that there is always a uniformly most reliable two-terminal graph. Furthermore, when the distance between the target vertices is restricted to be at least three, we completely classify which values of n and m produce a uniformly most reliable graph and which do not.

This work was done as part of an REU at the University of Texas at Tyler under the guidance of our advisor, Dr. Christina Graves. (Received September 16, 2019)

1154-05-1737 **Jakub Byrzejewski*** (jakub.byrzejewski@gmail.com). *Substitutive systems and a finitary version of Cobham’s theorem*.

We study substitutive systems generated by nonprimitive substitutions and show that transitive subsystems of substitutive systems are substitutive. As an application we obtain a complete characterisation of the sets of words that can appear as common factors of two automatic sequences defined over multiplicatively independent bases. This generalises the famous theorem of Cobham and answers a question posed by Jeffrey Shallit.

The talk is based on joint work with Jakub Konieczny and Elżbieta Krawczyk. (Received September 16, 2019)

1154-05-1741 **Matthew D Welz*** (mwelz@fortlewis.edu), 1000 Rim Drive, Durango, CO 81301, and **Samuel Fulton** (sjfulton@fortlewis.edu), 1000 Rim Drive, Durango, CO 81301. *An Index-Preserving Bijection Between Marked Tableaux and $P_{n,2}$ -Tableaux*.

In his work with marked tableaux, Stembridge shows that the number of admissible marked tableaux of shape $\lambda \vdash n$ and index i is equal to the multiplicity of the irreducible Specht module S^λ in a certain representation of S_n . Through their seemingly unrelated work with chromatic quasisymmetric functions, Shareshian and Wachs establish that this multiplicity is also equal to the number of $P_{n,2}$ -tableaux of shape λ and index i . This equality established by Shareshian and Wachs is indirect and relies on q -Eulerian polynomials, chromatic quasisymmetric functions, and Smirnov words. Therefore, they ask for a direct, index-preserving combinatorial

bijection between marked tableaux and $P_{n,2}$ -tableaux. We present such a bijection. In particular, we develop an index-preserving bijection from the set of all marked tableaux of shape λ to the set of all $P_{n,2}$ -tableaux of shape λ . (Received September 16, 2019)

1154-05-1745 **Stefaan De Winter*** (sgdewint@mtu.edu), Michigan Technological University, Dept. Mathematical Sciences, Houghton, MI 49931. *Maximal arcs and special point sets in partial geometries*. Preliminary report.

Maximal arcs of Mathon type in $\text{PG}(2, 2^h)$ form the largest class of known maximal arcs. These arcs are constructed by gluing disjoint conics on a common nucleus together. In this talk I will show how these maximal arcs can be interpreted as point sets with nice combinatorial properties in a certain partial geometry, each conic of the arc corresponding to a point in the special point set. This model is interesting in its own right, as it allows an alternative way to study Mathon maximal arcs, but is also interesting as it provides some hope for generalizing Mathon's construction. (Received September 16, 2019)

1154-05-1753 **Iwan Duursma*** (duursma@illinois.edu), **Xiao Li** and **Hsin-Po Wang**. *Graph based codes for distributed storage*.

We use Johnson graphs and Cayley graphs to construct regenerating codes for use in distributed storage. Such codes guarantee that (1) data stored on n servers can be downloaded from any chosen k -subset of servers and (2) any failed server can be reconstructed using small amounts of repair data from any chosen d -subset of the remaining servers. (Received September 16, 2019)

1154-05-1758 **Michael Tait*** (michael.tait@villanova.edu). *The Zarankiewicz problem in 3-partite graphs*.

Let F be a graph, $k \geq 2$ be an integer, and write $\text{ex}_{\chi \leq k}(n, F)$ for the maximum number of edges in an n -vertex graph that is k -partite and has no subgraph isomorphic to F . The function $\text{ex}_{\chi \leq 2}(n, F)$ has been studied by many researchers. Finding $\text{ex}_{\chi \leq 2}(n, K_{s,t})$ is a special case of the Zarankiewicz problem. We prove an analogue of the Kővári-Sós-Turán Theorem by showing

$$\text{ex}_{\chi \leq 3}(n, K_{s,t}) \leq \left(\frac{1}{3}\right)^{1-1/s} \left(\frac{t-1}{2} + o(1)\right)^{1/s} n^{2-1/s}$$

for $2 \leq s \leq t$. Using Sidon sets constructed by Bose and Chowla, we prove that this upper bound is asymptotically best possible in the case that $s = 2$ and $t \geq 3$ is odd, i.e., $\text{ex}_{\chi \leq 3}(n, K_{2,2t+1}) = \sqrt{\frac{t}{3}} n^{3/2} + o(n^{3/2})$ for $t \geq 1$.

This is joint work with Craig Timmons (Received September 16, 2019)

1154-05-1759 **Tara Fife*** (tfife2@lsu.edu), Mathematics Department, Louisiana State University, Baton Rouge, LA 70803-4918, and **James Oxley**. *Laminar Matroids and their Generalizations*.

Matroids were introduced to provide an abstract generalization of the notion of linear dependence. This talk begins by introducing matroids, nested matroids, and laminar matroids. One characterization of laminar matroids is that, for all circuits $C_1 \cap C_2 \neq \emptyset$, either C_1 is in the closure of C_2 , or C_2 is in the closure of C_1 . We use this characterization to define two infinite families of generalized laminar matroids and give structural results for these classes. (Received September 16, 2019)

1154-05-1816 **Zhenchao Ge*** (zge@olemiss.edu) and **Thái Hoàng Lê**. *Essential components in vector spaces over finite fields*.

A subset H of non-negative integers is called an essential component, if $\underline{d}(A + H) > \underline{d}(A)$ for all $A \subset \mathbb{N}$ with $0 < \underline{d}(A) < 1$, where $\underline{d}(A)$ is the lower asymptotic density of A . How sparse can an essential component be? This problem was solved completely by Ruzsa. Here, we generalize the problem to the additive group $(\mathbb{F}_p[t], +)$, where p is prime. Our result is analogous to but more precise than Ruzsa's result in the integers. Like Ruzsa's, our method is probabilistic. We also construct an explicit example of an essential component in $\mathbb{F}_p[t]$ with a small counting function, based on an argument of Wirsing. This is joint work with Thái Hoàng Lê. (Received September 16, 2019)

1154-05-1827 **Sawyer Jack Robertson*** (sawyerjack@ou.edu), Norman, OK, and **Javier Alejandro Chávez-Domínguez**. *Kantorovich Duality and Signature Balance for Generalized Signed Graphs*.

A twofold generalization of results from the theory of signed graphs to the case of signatures taking values in arbitrary groups which then act on a normed space. The first concerns the notion of signature balance, establishing a three-part characterization of the trait demonstrated by some signatures. The second is a Kantorovich-type duality between Lipschitz- and Arens-Eells-type function spaces whose norm and generator structures, respectively, incorporate the signature. (Received September 16, 2019)

1154-05-1848 **Patrick Bennett, Andrzej Dudek and Shira Zerbib***, Iowa State University, Ames, IA. *Large triangle packings and Tuza's conjecture in sparse random graphs.*

The triangle packing number $\nu(G)$ of a graph G is the maximum size of a set of edge-disjoint triangles in G . Tuza conjectured in 1981 that in any graph G there exists a set of at most $2\nu(G)$ edges intersecting every triangle in G . We show that Tuza's conjecture holds in the random graph $G = G(n, m)$, when $m \leq 0.2403n^{3/2}$ or $m \geq 2.1243n^{3/2}$. This is done by analyzing a greedy algorithm for finding large triangle packings in random graphs. (Received September 16, 2019)

1154-05-1866 **Joshua Harrington, Kedar Karhadkar*** (kjk5800@psu.edu), **Madeline Kohutka, Tessa Stevens and Tony Wong.** *Two dependent probabilistic chip-collecting games.*

Alice and Bob take turns to collect chips in the following manner. In each turn, Alice tosses a fair coin, which decides whether she collects a or b chips, where a and b are positive integers. If Alice collects a chips, then Bob collects b chips, and vice versa. We consider two variants of game play that have different rules in determining the winner. Namely, the winner of Game 1 is the first player to collect at least n chips, while the winner of Game 2 is the first player to collect a positive number of chips congruent to 0 modulo n . We fully determine the formula for the winning probabilities of each player in Game 1, and determine the best and worst case scenarios in terms of winning probabilities in Game 2. (Received September 16, 2019)

1154-05-1881 **Hayan Nam***, 443 Carver, 411 Morrill Rd. Ames IA, Ames, IA 50011, and **Nathan Kaplan.** *Counting the number of numerical semigroups using polytopes.*

Numerical semigroups are additive monoids of the nonnegative integers that have finite complements. In 2008, Bras-Amorós conjectured that the number of numerical semigroups with genus g is increasing in g . In this talk, we discuss partial results for this conjecture. We will show how some families of numerical semigroups are in bijection with integer points in certain polytopes and explain how geometric techniques lead to enumerative results. (Received September 16, 2019)

1154-05-1914 **Daniel P Johnston*** (djohnst1@skidmore.edu) and **Puck Rombach.** *On Rainbow Turán Numbers of Paths and Other Trees.*

For a fixed graph F , we consider the maximum number of edges in a properly edge-colored graph on n vertices which does not contain a rainbow copy of F , that is, a copy of F all of whose edges receive a different color. This maximum, denoted by $ex^*(n; F)$, is the rainbow Turán number of F , and its systematic study was initiated by Keevash, Mubayi, Sudakov and Verstráte [*Combinatorics, Probability and Computing* **16** (2007)]. In this talk, we look at previous results and explore the rainbow Turán number when F is a path or another tree. This is joint work with Puck Rombach. (Received September 16, 2019)

1154-05-1928 **Hayan Nam***, 443 Carver, 411 Morrill Rd, Ames, IA 50011, and **Gilyoung Cheong and Myungjun Yu.** *Some convergence of random characteristic polynomials to random permutations and its applications.*

We show that the distribution of degree d irreducible factors of the characteristic polynomial of a random $n \times n$ matrix over \mathbb{F}_q converges to the distribution of length d cycles of a random permutation of n letters, as q goes to infinity, while d is fixed. This convergence will be used for the following three different applications: a matrix version of theorems of Jordan and Landau, a relationship between Cohen-Lenstra and Poisson distributions, and the distribution of the cokernel of a Haar $n \times n$ random \mathbb{Z}_p -matrix when p goes to infinity. (Received September 16, 2019)

1154-05-1948 **Ashleigh Adams*** (adams869@umn.edu), 2929 University Ave. SE, #910, Minneapolis, MN 55414. *Group actions on Stanley-Reisner rings and Stanley rings.*

A group G acting on a simplicial complex Δ or, more generally a simplicial poset P gives rise to an action on its Stanley-Reisner ring $\mathbb{C}[\Delta]$ or its Stanley ring $\mathbb{C}[P]$, respectively. Then, these rings being both graded, have \mathfrak{S}_n -representations on each graded piece of their respective rings. We are concerned with describing all of these \mathfrak{S}_n -representations compactly. We will use a result of DeConcini, Eisenbud, and Procesi to produce a canonical homogeneous system of parameters, $\theta = (\theta_1, \dots, \theta_m)$ for these rings that is always G -invariant. If Δ or P is Cohen-Macaulay, their corresponding ring is a free module over the subalgebra $\mathbb{C}[\theta]$ and one can describe the whole ring by describing the finite dimensional quotient $\mathbb{C}[\Delta]/(\theta)$ or $\mathbb{C}[P]/(\theta)$. In this talk we will give examples, such as the boundaries of simplices and the complex of injective words. (Received September 16, 2019)

1154-05-1983 **Christopher Eur*** (chrisweur@berkeley.edu). *Towards a tropical geometry of Coxeter matroids*. Preliminary report.

Matroids are combinatorial objects that capture the essence of linear independence, whose theory has recently seen a fruitful interaction with tropical and toric geometry. As matroids can be considered as “type A” objects, we ask whether similar interaction occurs for Coxeter matroids, which are generalizations of matroids to arbitrary Coxeter types. We discuss some progress in the study of Coxeter matroids from the perspective of tropical and toric geometry, and lay out some of the challenges ahead. (Received September 17, 2019)

1154-05-1998 **Ruth Luo***, Department of Mathematics, University of California, San Diego (UCSD), 9500 Gilman Drive # 0112, La Jolla, CA 92093, and **Zoltan Furedi**. *Induced Turán problems for hypergraphs*.

Let F be a graph. We say that a hypergraph \mathcal{H} is an induced Berge F if there exists a bijective mapping f from the edges of F to the hyperedges of \mathcal{H} such that for all $xy \in E(F)$, $f(xy) \cap V(F) = \{x, y\}$.

In this talk, we show asymptotics for the maximum number of edges in r -uniform hypergraphs with no induced Berge F . In particular, this function is strongly related to the generalized Turán function $ex(n, K_r, F)$, i.e., the maximum number of cliques of size r in n -vertex, F -free graphs. (Received September 17, 2019)

1154-05-2008 **David Jordan** and **Monica Vazirani***, One Shields Ave, Davis, CA 95616. *The rectangular representation of the double affine Hecke algebra via elliptic Schur-Weyl duality*.

Building on the work of Calaque-Enriquez-Etingof, Lyubashenko-Majid, and Arakawa-Suzuki, Jordan constructed a functor from quantum D -modules on general linear groups to representations of the double affine Hecke algebra (DAHA) in type A . When we input quantum functions on $GL(N)$ the output is $L(k^N)$, the irreducible DAHA representation indexed by an $N \times k$ rectangle. For the specified parameters $L(k^N)$ is Y -semisimple, i.e. one can diagonalize the Dunkl operators. We give an explicit combinatorial description of this module via its Y -weight basis. This is joint work with David Jordan. (Received September 17, 2019)

1154-05-2023 **Yibo Gao** and **Kaarel Hänni***, tahvend@mit.edu. *Counting Signed Vexillary Permutations*.

We show that the number of signed permutations avoiding 1234 equals the number of signed permutations avoiding 2143 (also called vexillary signed permutations), resolving a conjecture by Anderson and Fulton. The main tool that we use is the generating tree developed by West. Our methods can be further generalized. (Received September 17, 2019)

1154-05-2031 **Fatmanur Gursoy** (fnurgursoy@gmail.com), Istanbul, Turkey, and **Elif Segah Oztas*** (esoztas@kmu.edu.tr), Karamanoglu Mehmetbey University, Science Faculty, Department of Mathematics, Karaman, Turkey. *Cosine distance on lifted polynomials*.

New studies of different areas have focused on artificial intelligence in new trend. One of these areas is algebraic coding theory. It is well known that a linear code over F_q with minimum Hamming distance 3, can correct 1 error. A question is “can this code correct more than 1 error by changing decoding algorithm?”. One of the tools to measure distance in artificial intelligence systems (e.g. word2vec) is cosine distance. According to the question, we study on cosine distance on lifted polynomials that a special type of polynomials to generate optimal codes over F_q . We present a method to find the best angle to correct more error for optimal codes obtained by lifted polynomials. (Received September 17, 2019)

1154-05-2035 **Darren A. Narayan***, School of Mathematical Sciences, Rochester Institute of Technology, Rochester, NY 14623, and **Brendan Rooney**, School of Mathematical Sciences, Rochester Institute of Technology, Rochester, NY 14623. *Minimally Non-Asymmetric Graphs*.

A graph is asymmetric if the automorphism group of its vertices is trivial. A graph G is minimally non-asymmetric if G is non-asymmetric, but $G - e$ is asymmetric for any $e \in E(G)$. We investigate necessary conditions for a graph to be minimally non-asymmetric. We present infinite families of graph with this property, including some that arise from combinatorial designs. Finally we present a surprising result: there exist vertex transitive graphs that are minimally non-asymmetric. This shows the existence of graphs G where the automorphism group of vertices is the symmetric group, but the removal of any edge results in a graph where the automorphism group of vertices is trivial. (Received September 17, 2019)

1154-05-2071 **Ben Clark, Kevin Grace*** (kevin.grace@bristol.ac.uk), **James Oxley** and **Stefan H.M. van Zwam**. *Dyadic matroids with spanning cliques.*

The Matroid Minors Project of Geelen, Gerards, and Whittle describes the structure of minor-closed classes of matroids representable over a fixed finite field. In order to use these results to study specific classes, it turns out to be important to study the matroids in the class containing spanning cliques. A spanning clique of a matroid M is a complete-graphic restriction of M with the same rank as M .

In this talk, we will describe the structure of dyadic matroids with spanning cliques. The dyadic matroids are those matroids that can be represented by a real matrix A where every nonzero subdeterminant is $\pm 2^i$ for some integer i . A subclass of the dyadic matroids is the signed-graphic matroids. In the class of signed-graphic matroids, the entries of the matrix A are determined by a signed graph. Our result is that dyadic matroids with spanning cliques are signed-graphic matroids and a few exceptional cases. (Received September 17, 2019)

1154-05-2078 **Catherine Lee*** (catherine.lee@yale.edu). *Minimum coprime graph labelings.*

A coprime labeling of a graph G is a labeling of the vertices of G with distinct integers from 1 to k such that adjacent vertices have coprime labels. The minimum coprime number of G is the least k for which such a labeling exists. In this talk, we discuss the minimum coprime number for several well-studied classes of graphs, including the coronas of complete graphs with empty graphs, the joins of two paths, and prisms. In particular, we resolve a conjecture of Seoud, El Sonbaty, and Mahran and three conjectures of Asplund and Fox. We also provide bounds on the minimum coprime number of a random subgraph. (Received September 17, 2019)

1154-05-2100 **Michael Young***, 411 Morrill Rd, Ames, IA 50011. *Rainbow Problems in Groups.*

A set is considered rainbow if each element of the set is assigned a distinct color. In this talk, we will discuss problems and techniques about determining the minimum number of colors needed in order to guarantee that every coloring of the integers contains a rainbow solution to a given equation. Open problems that are accessible to any undergraduate that has completed an introductory proof course will also be presented. (Received September 17, 2019)

1154-05-2128 **Beth Bjorkman*** (bjorkman@iastate.edu). *Infectious Power Domination for Hypergraphs.*

The power domination problem seeks to find the placement of the minimum number of sensors needed to monitor an electric power network. We generalize the power domination problem to hypergraphs using the infection rule from Bergen et al (2018): given an initial set of observed vertices, S_0 , a set $A \subseteq S_0$ may infect an edge e if $A \subseteq e$ and for any unobserved vertex v , if $A \cup \{v\}$ is contained in an edge, then $v \in e$. We combine a domination step with this infection rule to create *infectious power domination*. We compare this new parameter to the previous generalization by Chang and Roussel (2015). We provide general bounds and determine the impact of some hypergraph operations. (Received September 17, 2019)

1154-05-2136 **Trajan Hammonds*** (thammond@andrew.cmu.edu). *Modified Erdős-Ginzburg-Ziv Constants for $(\mathbb{Z}/n\mathbb{Z} \times \mathbb{Z}/m\mathbb{Z})$.*

For an abelian group G and an integer $t > 0$, the *modified Erdős-Ginzburg-Ziv constant* $s'_t(G)$ is the smallest integer ℓ such that any zero-sum sequence of length at least ℓ with elements in G contains a zero-sum subsequence (not necessarily consecutive) of length t . We compute bounds for $s'_t(G)$ for $G = (\mathbb{Z}/n\mathbb{Z})^2$ and $G = (\mathbb{Z}/n_1\mathbb{Z} \times \mathbb{Z}/n_2\mathbb{Z})$. We also compute bounds for $G = (\mathbb{Z}/p\mathbb{Z})^d$ where the subsequence can be any length in $\{p, \dots, (d-1)p\}$. Lastly, we investigate the Erdős-Ginzburg-Ziv constant for $G = (\mathbb{Z}/n\mathbb{Z})^2$ and subsequences of length tn . (Received September 17, 2019)

1154-05-2157 **Opel Jones*** (ojones@towson.edu), Towson University, Department of Mathematics, 7800 York Road, Room 321, Towson, MD 21252. *Enumerations of Restricted Dumont Permutations.*

In 1974 Dumont found two types of permutations are counted the same sequence. The first type is a permutation in which each even entry is followed by a smaller entry, and each odd entry is followed by a larger entry, or ends the permutation. The second type is a permutation wherein if an entry is a deficiency, it must be even, and if an entry is an excedance or a fixed point, it must be odd. These are now known as Dumont permutations of the first and second kinds. There are two other types of permutations which are also counted by the same sequence, known as Dumont permutations of the third and fourth kinds. In this talk we will discuss several enumerations of restricted Dumont permutations, that is Dumont permutations avoiding certain patterns. We will also briefly discuss their proofs which involve methods using induction, block decomposition, Dyck paths, and generating

functions. We will conclude with a conjecture that the patterns 2143 and 3421 are indeed Wilf-equivalent on Dumont permutations of the first kind. (Received September 17, 2019)

1154-05-2167 **Swapnil Garg and Alan Peng*** (apeng1@mit.edu). *Pattern Avoidance in Rooted Forests*. Following a definition made by Anders and Archer, say that an unordered rooted labeled forest avoids the pattern $\sigma \in S_k$ if each sequence of labels along the shortest path from a root to a vertex does not contain a subsequence with the same relative order as σ . For a set S of patterns, let $F_n(S)$ be the set of forests on n vertices that avoid each pattern in S . Anders and Archer give a bijection between $F_n(123)$ and $F_n(132)$. Here we use techniques by Anders-Archer and West to give a bijection between $F_n(\sigma_1 \dots \sigma_{k-2}(k-1)k)$ and $F_n(\sigma_1 \dots \sigma_{k-2}k(k-1))$ for all permutations $\sigma_1 \dots \sigma_{k-2} \in S_{k-2}$. We then generalize the notion of shape-Wilf equivalence to forests, proving an analog of this result for sets of multiple patterns. (Received September 17, 2019)

1154-05-2177 **K. Daly, C. Gavin, G. Montes de Oca, D. Ochoa, E. Stanhope*** (stanhope@clark.edu) and **S. Stewart**. *Orbigraphs: a directed graph analog of a Riemannian orbifold*.

A Riemannian orbifold is a mildly singular generalization of a Riemannian manifold that is locally modeled on R^n modulo the action of a finite group. Orbifolds have proven interesting in a variety of settings. Spectral geometers have examined the link between the Laplace spectrum of an orbifold and the singularities of the orbifold. One open question in this field is whether or not a singular orbifold and a manifold can be Laplace isospectral. Motivated by the connection between spectral geometry and spectral graph theory, we define a directed graph analog of an orbifold called an orbigraph. We obtain results about the relationship between an orbigraph and the spectrum of its adjacency matrix. We prove that the number of singular vertices present in an orbigraph is bounded above and below by spectrally determined quantities, and show that an orbigraph with a singular point and a regular graph cannot be cospectral. We also provide a lower bound on the Cheeger constant of an orbigraph. (Received September 17, 2019)

1154-05-2183 **Axel Brandt, Michael Ferrara, Nathan Graber*** (nathan.graber@ucdenver.edu), **Stephen Hartke and Sarah Loeb**. *Plane Graphs with Maximum Degree 7 and no Adjacent Triangles are Entirely 10-Colorable*.

Let G be a plane graph with maximum degree Δ . If all vertices, edges, and faces of G can be colored with k colors so that any two adjacent or incident elements have distinct colors, then G is said to be entirely k -colorable. In 2011, Wang and Zhu asked if every simple plane graph except K_4 is entirely $(\Delta+3)$ -colorable. In 2012, Wang, Mao, and Miao answered in the affirmative for simple plane graphs with $\Delta \geq 8$. In this paper, we show that every plane multigraph with $\Delta = 7$, no loops, no 2-faces, and no 3-faces sharing an edge is entirely $(\Delta+3)$ -colorable. (Received September 17, 2019)

1154-05-2197 **Cathy Erbes, Michael Ferrara, Nathan Graber*** (nathan.graber@ucdenver.edu) and **Paul Wenger**. *Realization Problems for Hypergraphic Sequences*. Preliminary report.

A nonnegative integer sequence is *graphic* if it is the degree sequence of some graph. Given a graph H a graphic sequence π is *potentially H -graphic* if there is some realization of π that contains H as a subgraph. Let $\sigma(\pi)$ denote the sum of a graphic sequence π . The *potential number*, $\sigma(H, n)$, is the minimum even integer such that every n -term graphic sequence π with $\sigma(\pi) \geq \sigma(H, n)$ is potentially H -graphic.

For $r \geq 2$, a sequence is *r -graphic* if it is the degree sequence of an r -uniform hypergraph. While several efficient characterizations exist for determining if a given sequence is graphic, determining if a given sequence is r -graphic for any $r \geq 3$ was recently shown to be *NP*-complete by Deza, Levin, Meesum, and Onn [Optimization Over Degree Sequences. *SIAM Journal on Discrete Mathematics*, 32(3), 2067-2079.].

In this talk, we consider an extension of the potential problem to the setting of r -graphic sequences. In particular, we determine the potential number for complete hypergraphs. We additionally present some results on the stability of the potential function for r -graphs that highlight an important distinction between the $r = 2$ and $r \geq 3$ cases. (Received September 17, 2019)

1154-05-2203 **Maya R Sankar*** (mayars@mit.edu). *Enumerating a Certain Class of Valid Hook Configurations*.

Valid hook configurations are combinatorial objects used to understand West's stack-sorting map. We extend existing bijections corresponding valid hook configurations to intervals in partial orders on Motzkin paths. To enumerate valid hook configurations on 312-avoiding permutations, we build off of an existing bijection into a Motzkin poset and construct a bijection to certain well-studied closed lattice walks in the first quadrant. We use existing results about these lattice paths to show that valid hook configurations on 312-avoiding permutations are

not counted by a D -finite generating function, resolving a question of Defant's, and to compute asymptotics for the number of such configurations. We additionally extend a bijection of Defant's to a correspondence between valid hook configurations on 132-avoiding permutations and intervals in the Motzkin-Tamari posets, providing a more elegant proof of Defant's enumeration thereof. To investigate this bijection, we present a number of lemmas about valid hook configurations that are generally applicable, as well as further studying the bijections of Defant's. (Received September 17, 2019)

1154-05-2208 **Vincent N. Marani*** (vincent.marani@usc.edu), **Lucy Chikamai W** and **Shem O. Aywa**. *Some Codes and Designs Related to Mathieu Group M_{24}* .

Let G be a primitive group M_{24} . We determine the 1st and 2nd non trivial binary code from each representation of degree 24, 276, 759, 1288, 1771 and 2024 respectively and determine its properties. We determine designs defined by the support of codewords of minimum weight and establish their primitivity.

(Received September 17, 2019)

1154-05-2232 **Greta Panova*** (gpanova@usc.edu), Los Angeles, CA 90089. *Lozenge tilings in Algebraic Combinatorics*.

Lozenge tilings (dimer covers of the hexagonal lattice) are naturally related to objects from Algebraic Combinatorics like plane partitions and Semi-Standard Young Tableaux. We will show how their limiting behavior can be studied via algebraic tools like the Schur functions, and derive limit shapes (surfaces) in the case of global symmetries. We will also show how lozenge tilings arise in connection with the enumeration of Standard Young Tableaux. This talk will touch on many different results with a variety of coauthors (V. Gorin, A. Morales, I. Pak etc). (Received September 17, 2019)

1154-05-2279 **Alex Cameron*** (alexander.cameron@vanderbilt.edu). *Extremal problems for directed graphs and hypergraphs*.

For a fixed graph F , let $\text{ex}(n, F)$ denote the maximum number of edges than an F -free graph G can have if G has n vertices. This definition extends naturally to digraphs and to directed hypergraphs (the latter has many possible definitions). In this talk, we will discuss some recent results in extremal numbers for directed graphs and hypergraphs as well as directions for future research. (Received September 17, 2019)

1154-05-2310 **Tom Shlomi*** (tshlomi1836@eagle.fgcu.edu). *Discrete Balls and New Bounds on (t, r) Broadcast Domination of n -Dimensional Grids*.

A well-studied problem in graph theory is graph domination, or the guard problem. If each guard can protect the vertex it is placed at and all adjacent vertices, what is the minimum number of guards needed to guard the graphs? In the past decade, study has begun in (t, r) broadcast domination. In the 21st century, cellular reception is needed in the graph. Each broadcast provides $t - d$ reception to each vertex a distance $d < t$ away, and r reception is needed for the cell signal to work. Again, we are trying to minimize the number of broadcasts needed. In this talk, I'll explore this problem on infinite grids of arbitrary dimension, and some counterintuitive facts I discovered during my research. (Received September 17, 2019)

1154-05-2344 **Hanna Mularczyk*** (hmularczyk@college.harvard.edu). *Lattice Paths and Pattern-Avoiding Uniquely Sorted Permutations*.

This talk will dive into a fascinating natural analog between a foundational set of combinatorial objects—lattice paths—and a newer class of permutations that arise from West's stack-sorting map—uniquely sorted permutations. Using this analog as a basis for bijections, we will enumerate several classes of pattern-avoiding uniquely sorted permutations, allowing us to prove several conjectures of Defant. (Received September 17, 2019)

1154-05-2347 **Swapnil Garg*** (swapnilg@mit.edu). *New Results on Nyldon Words Derived Using an Algorithm from Hall Set Theory*.

Grinberg defined Nyldon words as those words which cannot be factorized into a sequence of lexicographically nondecreasing smaller Nyldon words. He was inspired by Lyndon words, defined the same way except with "nondecreasing" replaced by "nonincreasing." Charlier, Philibert, and Stipulanti proved that, like Lyndon words, any word has a unique nondecreasing factorization into Nyldon words. They also show that the Nyldon words form a right Lazard set, and equivalently, a right Hall set. We provide a new proof of unique factorization into Nyldon words related to Hall set theory and resolve several questions of Charlier et al. In particular, we prove that Nyldon words of a fixed length form a circular code, we prove a result on factorizing powers of words into Nyldon words, and we investigate the Lazard procedure for generating Nyldon words. (Received September 17, 2019)

1154-05-2359 **Malgorzata Aneta Marciniak***, 31-10 Thomson Avenue room E223N, Long Island City, NY 11101, and **Ryan Alvarado, Maia Averett,, Benjamin Gaines, Christopher Jackson, Mary Leah Karker, Francis Edward Su and Shanise Walker.** *Using matrices for analyzing the Game of Cycles.* Preliminary report.

Playing the game of cycles (as introduced in Su's 2020 book *Mathematics for Human Flourishing*) can be interpreted as converting a planar non-directed graph into a planar directed graph. Thus, every stage of the game can be represented as a graph of mixed type with a suitable adjacency matrix. The rules of the game and the winning moves have interpretations in terms of that matrix, which suggests another way to analyze the game and its strategies. This is a preliminary report describing work in progress. (Received September 17, 2019)

1154-05-2360 **Ji Young Choi*** (jychoi@ship.edu), 1871 Old Maind Dr., Shippensburg, PA 17257. *Generating functions of the b -nomial numbers.* Preliminary report.

Let b be an integer greater than 1. We define four different types of b -nomial numbers, by sorting and counting the strings of digits in $\{0, 1, 2, \dots, b-1\}$ using the length, the digit sum, and the number of indispensable digits in each string. This talk will present an ordinary generating function for the b -nomial numbers of each type. (Received September 17, 2019)

1154-05-2364 **Maia Averett*** (maverett@mills.edu), Mills College MCS Dept, 5000 MacArthur Blvd, Oakland, CA 94613, and **Ryan Alvarado, Benjamin Gaines, Christopher Jackson, Mary Leah Karker, Malgorzata Aneta Marciniak, Francis Su and Shanise Walker.** *The Game of Cycles: Winning Strategies and Open Problems.*

The Game of Cycles is a new game introduced in Su's 2020 book *Mathematics for Human Flourishing*. The game is played on a planar graph or a simplicial complex, and players take turns marking edges with arrows according to certain rules about 'sources' and 'sinks', which give the game a distinct topological flavor. In this talk, we analyze the two-player game for various classes of graphs and introduce many accessible open questions and conjectures appropriate for undergraduate research. (Received September 17, 2019)

1154-05-2388 **Alvin Chiu*** (88alvinchiu88@gmail.com), **William Hoganson** (whogans1@swarthmore.edu), **Thomas C. Hull** (thull@wne.edu) and **Sylvia Wu** (sylviauw@g.clemson.edu). *Counting locally-flat-foldable origami configurations via 3-coloring graphs.*

One fundamental, and generally open, question in the mathematics of origami (paper folding) is counting the number of ways that a given crease pattern can be folded flat (i.e., can be pressed in a book without crumpling). Each way a crease pattern can be folded results in a mountain-valley (MV) assignment, denoting which creases bend convexly (mountains) or concavely (valleys) when looking at one side of the paper. Currently, this problem is only known in general for single-vertex crease patterns, which have been thoroughly studied and understood. We built off that understanding to create a new method counting MV assignments of a given crease pattern C that are locally valid (where each vertex folds flat, but globally there could still be self-intersection problems). That is, we find a graph C^* , called a SAW graph of C , whose proper 3-vertex colorings (with one vertex pre-colored) are in one-to-one correspondence with the locally-valid MV assignments of C . Our results show that SAW graphs can be made by tiling SAW graphs of flat-foldable vertices, and begin generating a library of SAW graphs for wide range of single vertices. This provides significant evidence that the combinatorial structure underlying locally-valid MV assignments is that of 3-coloring graphs. (Received September 17, 2019)

1154-05-2404 **Thomas C. Hull** (thull@wne.edu), **Manuel Morales** (mamora22@asu.edu), **Sarah Nash*** (sarah.nash18@ncf.edu) and **Natasha Ter-Saakov** (natalyat@mit.edu). *Connectivity in origami flip graphs for flat-foldable vertices.*

Given a flat-foldable origami crease pattern C , we can describe how we fold it flat with a mountain-valley (MV) assignment, where each crease is described as bending convexly (mountain) or concavely (valley) when viewed from one side of the folding material. A MV assignment is called valid if it can be used to fold the crease pattern flat along all of its creases (i.e., it can be pressed in a book without crumpling or self-intersecting). We construct the *origami flip graph*, $OFG(C)$, from C as follows: the vertices are all valid MV assignments of C , and two vertices u and v are connected by an edge if and only if the MV assignment u can be turned into that of v by "flipping" one face F of C (reversing the MV parity of the creases bordering F). In this talk we examine origami flip graphs of single-vertex crease patterns. We describe, for a single-vertex crease pattern C , when $OFG(C)$ is connected and prove that if the number of valid MV assignments of C is 2^n then $OFG(C)$ is a subgraph of the n -cube. (Received September 17, 2019)

1154-05-2411 **Thomas C. Hull** (thull@wne.edu), **Manuel Morales*** (mamora22@asu.edu), **Sarah Nash** (sarah.nash18@ncf.edu) and **Natasha Ter-Saakov** (natalyat@mit.edu). *Graph properties of and algorithms on origami flip graphs for flat-foldable vertices with equal sector angles.*

The *origami flip graph* of a flat-foldable origami crease pattern C is a graph that represents the structure of valid mountain-valley (MV) assignments of C under *face flips*. That is, a MV assignment is a mapping $\mu : E(C) \rightarrow \{-1, 1\}$ where $\mu(c) = 1$ means that the crease c is a mountain (convex) and $\mu(c) = -1$ means c is a valley (concave). A face flip of a MV assignment μ is where we choose a face F of the crease pattern C and “flip” all the creases that border F (turning mountains to valleys and vice versa). Then the origami flip graph of C , denoted $OFG(C)$, is the graph whose vertices are all valid MV assignments of C and where two MV assignments μ_1 and μ_2 of C are joined by an edge if and only if we can turn μ_1 into μ_2 by flipping exactly one face. In this talk we examine the surprisingly complex case of a single-vertex crease pattern C where the sector angles of C are all equal. We construct algorithms to count the number of edges in $OFG(C)$ and to describe a path in $OFG(C)$ between any two vertices. We use the latter to prove that if the degree of the vertex in C is $2n$ then the diameter of $OFG(C)$ is n . If time permits, we will discuss other graph properties of $OFG(C)$ for this case. (Received September 17, 2019)

1154-05-2413 **Jean A Guillaume*** (guillaumej@sacredheart.edu) and **Michael D. Barrus** (barrus@uri.edu), 5 Lippitt Road, Math Department, Kingston, RI 02881. *The NGD-Circulant Graphs $C_n(1, k)$.*

Nordhaus and Gaddum showed, for any given graph G , that $\chi(G) + \chi(\overline{G}) \leq n + 1$, where χ denotes the chromatic number and n the number of vertices. Collins and Trenk established an analogous result for the distinguishing chromatic number. They proved, for any graph G , that $\chi_D(G) + \chi_D(\overline{G}) \leq n + D(G)$, where $D(G)$ is the distinguishing number of the graph. They called the class of graphs that satisfy equality in this bound NGD-graphs after Nordhaus and Gaddum. In this talk, we present our investigation of the distinguishing chromatic number for the complements of circulant graphs $G = C_n(1, k)$. We also discuss our characterization of the NGD-graphs for this particular class of graphs as well as our improvement of the bound of Collins and Trenk for this family. Lastly, we show how to extend our investigation of the distinguishing chromatic number to a larger class of circulant graphs $G = C_n(1, k_1, \dots, k_m)$. (Received September 17, 2019)

1154-05-2440 **Emina Soljanin*** (emina.soljanin@rutgers.edu), Rutgers University, Piscataway, NJ 08854. *Load Balancing Properties of Codes and Matching Problems on Graphs.* Preliminary report.

There are k files to be stored (redundantly) across n servers. We assume that $n \geq k$ and that files are mathematically elements of some finite field, e.g., bits. Each server can store a certain number of linear combinations of files (storage constraint), and can serve a limited number of users (computing jobs) simultaneously (bandwidth constraint). We are interested in 1) designing redundancy schemes that maximize the number of users that can be concurrently served by the system as their requests for the files vary, and 2) evaluating the the number of users that can be concurrently served by the system implementing a particular coding scheme. These problems, in several ways, generalize the batch coding problem, where the goal is to minimize the worst case maximal load on any of the n servers, where the load on a server is measured by the number of bits read from it, while also minimizing the total amount of storage used. The talk will describe equivalence between the posed problems and various matching questions in graph theory. (Received September 17, 2019)

1154-05-2482 **Sean T. Griffin***, stgriff@uw.edu. *Ordered set partitions, Garsia-Procesi modules, and rank varieties.* Preliminary report.

We introduce a family of ideals $I_{n,\lambda}$ in $\mathit{mathbb{Q}}[x_1, \dots, x_n]$ for λ a partition of $k \leq n$. This family contains both the Tanisaki ideals I_λ and the ideals $I_{n,k}$ of Haglund-Rhoades-Shimozono as special cases. We study the corresponding quotient rings $R_{n,\lambda}$ as symmetric group modules. When $n = k$, we recover the Garsia-Procesi modules, and when $\lambda = (1^k)$, we recover the generalized coinvariant algebras of Haglund-Rhoades-Shimozono.

We will present a monomial basis for $R_{n,\lambda}$ in terms of (n, λ) -staircases, unifying the monomial bases studied by Garsia-Procesi and Haglund-Rhoades-Shimozono. Furthermore, we realize the S_n -module structure of $R_{n,\lambda}$ in terms of an action on (n, λ) -ordered set partitions. We will then show that the graded Frobenius characteristic of $R_{n,\lambda}$ has a positive expansion in terms of dual Hall-Littlewood functions. We also show that the rings $R_{n,\lambda}$ have connections to the rank varieties of Eisenbud-Saltman. We then generalize results of De Concini-Procesi and Tanisaki on “nilpotent” diagonal matrices. (Received September 17, 2019)

1154-05-2491 **Rachelle Bouchat*** (rbouchat@iup.edu), Indiana University of Pennsylvania, 210 South Tenth Street, Stright Hall, Room 233, Indiana, PA 15705, and **Patrick Cone** and **Williem Rizer**. *A Tale of Two Graph Theory Projects: From Graceful Trees to Domino Tilings*. Preliminary report.

In this talk, two different research projects will be discussed, both of them are joint works with students at Indiana University of Pennsylvania. The first project is joint work with an undergraduate student, Patrick Cone, on the Graceful Tree Conjecture. This conjecture is a graph labeling problem for trees that was first posed by Kotzig, Ringel, and Rosa in 1967 and remains open today (despite hundreds of publications on the topic). The results presented will showcase our progress on the problem through our various approaches.

The second project that will be discussed is joint work with a master's student, Williem Rizer, on domino tilings of rectangles. The result that will be discussed is that placing dominoes on the doubly even positions of the rectangle, will result in either no tiling or a unique tiling. The proof will involve the use of forests with colored vertices, showcasing a nice connection of a tiling problem to graph theory. (Received September 17, 2019)

1154-05-2494 **Pedro H Sacramento de Oliveira*** (pedrohso@seas.upenn.edu), 3901 Locust Walk, Philadelphia, PA 19104. *Implementing an efficient algorithm to 4-color planar graphs*.

It is a well-known theorem that one can color a planar graph G with 4 distinct colors. Nevertheless, usual implementations of this rely on brute force which takes an exponential amount of time. This talk will introduce the ideas behind Robertson, Sanders, Seymour and Thomas's proof of the Four Color Theorem and how we used them to implement a quadratic algorithm for the 4-color problem. This talk is based on joint work with Robin Thomas and Haidar Jamal. (Received September 17, 2019)

1154-05-2515 **Eric S. Piato*** (esp6@geneseo.edu), Department of Mathematics, SUNY Geneseo, Geneseo, NY 14454, and **Josh Ducey**, **David Duncan**, **Liz Engelbrecht**, **Jawahar Madan**, **Christina Shatford** and **Angela Vichitbandha**. *Critical groups of strongly regular graphs*.

Let $G = (V, E)$ be a simple graph. The *critical group* (also called the *sandpile group*), denoted $K(G)$, is a finite abelian group associated with G . Concretely, viewing the Laplacian matrix L as a linear mapping $\mathbb{Z}^{|V|} \rightarrow \mathbb{Z}^{|V|}$, it turns out that $\mathbb{Z}^V / \text{Im}(L) \cong \mathbb{Z} \oplus K(G)$. In this talk, we discuss our results regarding the critical groups of strongly regular graphs Γ . In particular, we provide a complete characterization of $K(\Gamma)$ under certain assumptions regarding the associated eigenvalues of Γ . In other cases, when the eigenvalues of Γ satisfy different (weaker) conditions, we are able to provide constraints on the form of the critical group. We conclude with a brief discussion regarding the question of existence of a strongly regular graph with given parameters, and explore how our work could be used to resolve this inquiry. (Received September 17, 2019)

1154-05-2554 **Suhyung An** and **JiYoon Jung***, jungj@marshall.edu, and **Sangwook Kim**. *Enumeration of Fuss-Catalan paths*.

In this paper we enumerate the number of k -Fuss-Catalan paths with fixed type and fixed number of blocks. We provide two proofs of this result. Also, we give a conjecture generalizing this result to the family of (k, r) -Fuss-Schröder paths. (Received September 17, 2019)

1154-05-2628 **Jonad Pulaj*** (jopulaj@davidson.edu), Dept. of Mathematics and Computer Science, Davidson College, Davidson, NC 28035. *Automated tree strategy selection for graph pebbling numbers*.

Given a distribution of pebbles to the vertices of a graph, a pebbling move across an edge removes two pebbles from one endpoint and places exactly one at the other endpoint. The pebbling number $\pi(G)$ is the smallest number such that for any distribution of $\pi(G)$ pebbles on a graph G there is a sequence of pebbling moves that places one pebble on any chosen vertex. Computing $\pi(G)$ is difficult in theory and practice, although approaches with linear optimization techniques have yielded promising results. One such approach uses a combination of appropriately chosen trees in G that yield an upper bound on $\pi(G)$. Using integer programming we automate this tree selection strategy by casting it as a variant of the classical Steiner tree problem with hop constraints. This yields improved bounds for pebbling numbers of specific graphs of interest related to Graham's conjecture, a central open question in graph pebbling. (Received September 17, 2019)

1154-05-2733 **Craig Timmons*** (craig.timmons@csus.edu), **Benjamin Cole**, **Albert Curry** and **David Davini**. *Triangles in K_s -saturated graphs with minimum degree t* .

Answering a question of Bollobás, Day proved that the minimum number of edges in an n -vertex K_s -saturated graph with minimum degree t is at least $tn - c_t$ where c_t is a constant depending only on t . Motivated by this

result, as well as the recent work of Kritschgau, Methuku, Tait, and the presenter, we discuss some new results on counting K_r 's in K_s -saturated graphs with minimum degree t . An exact answer is known in the case of counting K_3 's in a K_4 -saturated graph with minimum degree 4, however, many other questions are open. This is joint work with Benjamin Cole, Albert Curry, and David Davini. (Received September 17, 2019)

1154-05-2744 **Erin Meger*** (erin.k.meger@ryerson.ca), **Sam Spiro**, **Sean English**, **Tomas Masarik**, **Mike Ross**, **Grace McCourt** and **Cedar Wiseman**. *Odd Cycle Saturation Games*.

A graph, G , is \mathcal{F} -saturated, if no subgraph of G is isomorphic to any graph $F \in \mathcal{F}$, but for any edge e not in G there is some subgraph of $G + e$ in \mathcal{F} . Whenever G does not contain such a subgraph we say G is \mathcal{F} -free. Originally proposed by Hajnal, the saturation game involves two players, Mini and Maxi, who take turns adding single edges to a graph with n vertices and no edges. The game ends when the graph is \mathcal{F} -saturated, and neither player can ever add an edge that creates a copy of any graph $F \in \mathcal{F}$. For each edge in the resulting graph, Maxi gains a point, and Mini loses a point. Thus each player is trying to maximize or minimize the number of edges at the end of the game. The game saturation number of the graph is denoted $\text{sat}_g(n; \mathcal{F})$. In this talk we consider the particular game where we forbid odd cycles. Specifically, we consider forbidding all odd cycles other than triangles and denote $\mathcal{F} = \mathcal{C}_\infty \setminus \{C_3\}$. We will discuss particular strategies of each player, and bound the game saturation number in a variety of cases. (Received September 17, 2019)

1154-05-2753 **Erin Meger*** (erin.k.meger@ryerson.ca), **Anthony Bonato**, **Sean English**, **Bill Kay** and **Huda Chuangpishit**. *The Iterated Local Model for Social Networks*.

Complex networks are said to exhibit four key properties: large scale, evolving over time, small world properties, and power law degree distribution. The Preferential Attachment Model (Barabási-Albert, 1999) and the ACL Preferential Attachment Model (Aiello, Chung, Lu, 2001) for random networks, evolve over time and rely on the structure of the graph at the previous time step. Further models of complex networks include: the Iterated Local Transitivity Model (Bonato, Hadi, Horn, Pralat, Wang, 2011) and the Iterated Local Anti-Transitivity Model (Bonato, Infeld, Pokhrel, Pralat, 2017). In this talk, we will define and discuss the Iterated Local Model. This is a generalization of the ILT and ILAT models, where at each time step edges are added deterministically according to the structure of the graph at the previous time step. (Received September 17, 2019)

1154-05-2759 **Talia Goldwasser*** (tgoldwasser@smith.edu), Department of Mathematics and Statistics, Smith College, Northampton, MA 01060, **Meera Nadeem**, Smith College, and **Garcia Sun**, Smith College. *Matchings and Springer fibers*.

(n, n) Springer fibers have remarkable and unusual geometric features: they have a Catalan number's worth of components, and each component is not only smooth but an iterated tower of \mathbb{P}^1 -bundles. One combinatorial index set for the components is the collection of noncrossing matchings with n arcs. We show an explicit bijection between the cells in a paving of the (n, n) Springer fibers and the combinatorial index set of *standard dotted* noncrossing matchings, and describe some combinatorial results describing the closures of these cells. (Received September 17, 2019)

1154-05-2760 **Julian A Marquez*** (jam197@shsu.edu), 18403 Fairwood Meadow Ct., Houston, TX 77084. *Lower Bounds on Rectangular Ramsey Numbers*. Preliminary report.

The probabilistic method pioneered by Paul Erdős provides lower bounds for Ramsey numbers. In a similar manner the probabilistic method is utilized for the game of $\text{Rec}(j, l_1, l_2, \dots, l_d)$ to find lower bounds on rectangular Ramsey numbers. Applying the probabilistic method a random variable is defined for the coloring of the positions of an d -dimensional combinatorial hyper-rectangle. Similarly, a different random variable is defined to count the number of monochromatic d -dimensional combinatorial hyper-rectangle after the game has been played randomly. Relevant theorems are proven for $\text{Rec}(j, l_1, l_2, \dots, l_d)$ regarding rectangular Ramsey numbers. Lower bounds are generated given the dimension of the game board and size of the combinatorial hyper-rectangles. The game has interesting results in combinatorics, and further provides questions regarding rectangular Ramsey numbers and various winning conditions. (Received September 17, 2019)

1154-05-2769 **Laetitia Huang**, Smith College, **Valeri Jean-Pierre*** (vjeanpierre@smith.edu), Department of Mathematics and Statistics, Smith College, Northampton, MA 01060, **Karen Reed**, Smith College, and **Isabella Zhu**, Smith College. *Dimensions of generalized spline spaces*.

Suppose you have an edge-labeled graph. A spline is a way of labeling the vertices so that each pair of adjacent vertices differ by a multiple of the corresponding edge. Splines come up naturally in many different applications, including numerical analysis, data interpolation, computer graphics, and engineering; the edge-labels roughly correspond to the slopes between two observed points. We describe computational and theoretical results about

dimensions of spline spaces research in cases that relate to the longstanding upper bound conjecture. (Received September 17, 2019)

1154-05-2776 **Kassie Archer*** (karcher@cuttyler.edu), Virginia Germany, Catherine Marin King and Lindsey-Kay Lauderdale. *λ -unimodal involutions.*

For a composition $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_k)$ of n , we say a permutation on n letters is λ -unimodal if there are k contiguous unimodal segments, the i -th of which is length λ_i . We enumerate λ -unimodal involutions (those permutations that are equal to their own algebraic inverse) by descent number. As a corollary, we obtain a generating function for the Gelfand character for the symmetric group. (Received September 17, 2019)

1154-05-2787 **Janet Fierson*** (fierson@lasalle.edu), Dept. of Mathematics and Computer Science, 1900 W. Olney Ave., Philadelphia, PA 19046, and Tyler Knappenberger and Ricky Whitmoyer. *Vertex differentiation in reconfiguration graphs.* Preliminary report.

For any problem with multiple solutions, we can construct an associated reconfiguration graph. This reconfiguration graph has a vertex for each solution of the problem. Two vertices are connected by an edge if the solution associated with one may be transformed into the solution associated with the other by a single application of a specified reconfiguration rule.

Reconfiguration graphs have been applied to a wide variety of problems. However, when properties of these graphs are investigated, vertices are typically treated equally, without regard for differences among solutions. When we return to these problems but allow for differentiation of vertices in reconfiguration graphs based on the associated solutions, some interesting questions naturally arise. We present results obtained from considering this alternative view of vertices and relative goodness of solutions. (Received September 17, 2019)

1154-05-2797 **Arthur L Gershon*** (arthur.gershon@case.edu). *On the Number of Restricted Strip Arrangements on Square Chessboards.*

We are interested in counting the number $T(m, n)$ of ways to place $1 \times k$ strips on a rectangular lattice or chessboard of dimension $m \times n$ so that there is at most one horizontal strip in each row and at most one vertical strip in each column. Previous work have analyzed the case when one side length (say, m) is held fixed and the other (say, n) tends to infinity. Another case of interest, however, is when both side lengths $m = n$ tend to infinity together in equal measure. By taking logarithms in the latter case, we can use convexity and other analytic properties to deduce $\log(T(n, n)) = 4n \log n - 2n \log 8 + O(n^{2/3})$. (Received September 18, 2019)

1154-05-2827 **Lydia K. Rukaria***, P.O.Box 1699, Bungoma 50200, Kenya, Lucy Chikamai W, P.O.Box 1699, Bungoma, 50200, Kenya, and Ileri . Kamuti, P.O.Box 1699, Bungoma, 50200, Kenya. *Binary Codes from the Projective Symplectic Group $S_8(2)$.*

We find all of the binary codes constructed from the primitive permutation representation of the projective symplectic group $S_8(2)$ of degree 255. It is shown that in total we have 76 non-trivial and non-isomorphic codes. The properties of the codes with small dimension are given and links with modular representation theory established. Further from the support of the codewords, we construct the 1 and 2 - designs associated to the code and the graphs of the designs.

(Received September 18, 2019)

06 ► Order, lattices, ordered algebraic structures

1154-06-634 **Marco Bonatto***, marco.bonatto.87@gmail.com. *The classification of connected quandles of size p to the cube.*

We classify non-affine connected quandles of size p to the cube for p odd. As a byproduct we obtain a classification of Bruck loops of the same size and that they are in one-to-one correspondence with commutative automorphic loops. (Received September 09, 2019)

1154-06-1056 **Ryan R. Martin, Heather C. Smith and Shanise Walker*** (walkersg@uwec.edu). *Lower bounds for induced poset saturation.*

Given a finite poset \mathcal{P} , a family \mathcal{F} of elements in the Boolean lattice is induced- \mathcal{P} -saturated if \mathcal{F} contains no copy of \mathcal{P} as an induced subposet but every proper superset of \mathcal{F} contains a copy of \mathcal{P} as an induced subposet. The minimum size of an induced- \mathcal{P} -saturated family in the n -dimensional Boolean lattice, denoted $\text{sat}^*(n, \mathcal{P})$, was first studied by Ferrara et al. (2017).

We are interested in improving the lower bounds for $\text{sat}^*(n, \mathcal{P})$. In particular, for the 4-point poset known as the diamond, we improve upon a logarithmic lower bound. For the antichain with $k + 1$ elements, we improve upon a previously known lower bound. (Received September 17, 2019)

1154-06-1231 **Jinha Kim, Ryan R Martin, Tomáš Masařík, Warren Shull, Heather C Smith, Andrew Uzzell and Zhiyu Wang*** (zhiyuw@math.sc.edu). *On difference graphs and the local dimension of posets.*

The dimension of a partially-ordered set (poset), introduced by Dushnik and Miller (1941), has been studied extensively in the literature. Recently, Ueckerdt (2016) proposed a variation called local dimension which makes use of partial linear extensions. While local dimension is bounded above by dimension, they can be arbitrarily far apart as the dimension of the standard example is n while its local dimension is only 3. Hiraguchi (1955) proved that the maximum dimension of a poset of order n is $n/2$. However, we find a very different result for local dimension, proving a bound of $\Theta(n/\log n)$. This follows from connections with covering graphs using difference graphs which are bipartite graphs whose vertices in a single class have nested neighborhoods. We also prove that the local dimension of the n -dimensional Boolean lattice is $\Omega(n/\log n)$. (Received September 14, 2019)

1154-06-1307 **Richard Ehrenborg and Alex Happ*** (ahapp@cbu.edu). *The antipode of the noncrossing partition lattice.*

The antipode of the reduced incidence Hopf algebra of posets is an extension of the Möbius function of a poset. However, very few posets have had their antipode determined in an explicit form. Haiman and Schmitt computed the antipode of the partition lattice, and Einziger in her dissertation computed the antipode of the noncrossing partition lattice in terms of polygon dissections where each region has an even number of sides. These dissections are in bijective correspondence with noncrossing hypertrees.

We present here a different approach to the antipode of the noncrossing partition lattice directly in terms of noncrossing hypertrees. The proof is based on a map from chains in the noncrossing partition lattice to noncrossing hypertrees and expressing the alternating sum over these fibers as an Euler characteristic. (Received September 14, 2019)

1154-06-1386 **Nick Galatos*** (ngalatos@du.edu), Department of Mathematics, Univ. of Denver, C.M. Knudson Hall, Room 300 2390 S. York St., Denver, CO 80210, and **George Metcalfe and Alमुदना Colacito**. *Inverse-free reducts of lattice-ordered groups.*

Lattice-ordered groups (ℓ -groups) have inverse-free reducts that are distributive as lattices and multiplication distributes over both meet and join (totally distributive ℓ -monoids). It is known that in the abelian case, the inverse-free subreducts satisfy more equations than the above and they are actually not finitely based. We prove that in the general case of all ℓ -groups, the inverse-free subreducts are exactly the totally distributive ℓ -monoids. Also, for the intermediate case of subreducts of semilinear ℓ -groups (subdirect products of chains) we show that special equations hold. We further provide an equational axiomatization for the variety of all semilinear totally distributive ℓ -monoids.

A proof theory for ℓ -groups exist but it is complicated, relying on hypersequents. On the other hand a tame proof-theoretic calculus exists for totally-ordered ℓ -monoids. We provide a syntactic transformation from an ℓ -group identity to an inverse-free identity so that they are both equi-valid in ℓ -groups; so the existing calculus for totally distributive ℓ -monoids can be used for deciding ℓ -group equations. The translation can be seen as an application of the density rule in proof theory. (Received September 15, 2019)

1154-06-1495 **Bryan R. Gillespie***, brg008@gmail.com. *Discrete Convexity and the Active Matroid Orders.*

In this talk we will discuss the properties of the *external order*, a poset on the independent sets of an ordered matroid which refines the geometric lattice of flats. If M is an ordered matroid, then the *external active closure function* is defined in terms of external matroid activity. We show that this closure function is anti-exchange, and thus defines a discrete abstraction of the notion of convexity, known as a *convex geometry*. The inclusion relation on its closed sets induces a lattice structure on the independent sets of M which is meet-distributive and supersolvable. We will additionally give a complete characterization of the lattices which arise from an ordered matroid by this construction. (Received September 15, 2019)

1154-06-2013 **Nikolaos Galatos** (ngalatos@du.edu), University of Denver, **Peter Jipsen*** (jipsen@chapman.edu), Chapman University, **Olim Tuyt** (olim.tuyt@math.unibe.ch), University of Bern, and **Diego Valota** (valota@di.unimi.it), University of Milan. *The structure of idempotent involutive residuated lattices and weakening relation algebras.*

In joint research with Olim Tuyt and Diego Valota we have obtained a full description of all finite commutative idempotent involutive residuated lattices. The multiplicative order of these algebras is a distributive semilattice that is a disjoint union of Boolean algebras, with involution as complementation within each Boolean algebra. The top elements of the Boolean algebras form a distributive lattice, and given this family of Boolean algebras indexed by the distributive lattice, there is an algorithm for reconstructing the original residuated lattice.

With Nick Galatos we prove a characterization of the congruence filters of generalized bunched implication algebras (GBI-algebras = residuated Brouwerian algebras). Weakening relation algebras are cyclic involutive residuated Heyting algebras in which the congruence filters have a simpler description. We show that all representable weakening relation algebras are subalgebras of double-division conucleus images of representable relation algebras, and that this construction preserves a certain discriminator term. As a result, the class of representable weakening relation algebras forms a discriminator variety of cyclic involutive GBI-algebras, and we also obtain a discriminator variety of abstract weakening relation algebras. (Received September 17, 2019)

1154-06-2234 **Natanael Alpay*** (alpay100@mail.chapman.edu), Chapman University, 1 University Drive, Orange, CA 92866, and **Peter Jipsen** (jipsen@chapman.edu), Chapman University. *Commutative doubly-idempotent semirings determined by chains and by preorder forests.*

A commutative doubly-idempotent semiring (cdi-semiring) $(A, \vee, \cdot, 0, 1)$ is a semilattice $(A, \vee, 0)$ with $x \vee 0 = 0$ and a semilattices $(A, \cdot, 1)$ with identity 1 such that $x0 = 0$, and $x(y \vee z) = xy \vee xz$ holds for all $x, y, z \in A$. Bounded distributive lattices are cdi-semirings that satisfy $xy = x \wedge y$, and the variety of cdi-semirings covers the variety of distributive lattices.

We construct all cdi-semirings for which their multiplicative semilattice is a chain with n elements, and we show that up to isomorphism the number of such algebras is the $(n-1)^{\text{th}}$ Catalan number $C_{n-1} = \frac{1}{n} \binom{2n-2}{n-1}$. We also show that cdi-semirings with a complete atomic Boolean \vee -semilattice on the set of atoms A are determined by rooted preorder forests on the set A .

The first author did this research while an undergraduate at Chapman University. (Received September 17, 2019)

08 ► General algebraic systems

1154-08-659 **Matt Valeriote*** (matt@math.mcmaster.ca), Department of Mathematics & Statistics, McMaster University, Hamilton, Ontario L8S 4K1, Canada. *Finite algebras and regular tree languages.*

By regarding deterministic finite state automata as finite multi-unary algebras, and vice versa, one can define the notion of a regular language in algebraic terms. From this perspective, a multi-unary algebra is a string processor, where strings correspond to unary terms, and the algebra will accept or reject a string/term, depending on the value the term produces when applied to a designated initial element of the algebra. In a similar fashion, a finite algebra of arbitrary type can be regarded as a tree processor, with trees corresponding to the terms of the algebra. In this way, the notion of a regular language of strings can be extended to that of a regular tree language.

In my talk I will discuss several definability questions and results for regular tree languages and connect them with some decision problems for classes of finite algebras that are closed under operations, that from a computational perspective, seem quite natural. One of the operations of interest is the matrix power construction and another is the wreath product. This is joint work with Mikołaj Bojańczyk. (Received September 09, 2019)

1154-08-718 **Ralph S Freese*** (ralph@math.hawaii.edu). *A class of modular lattices embedded into congruence lattices of algebras in almost all varieties.*

There is a large class of modular lattices such that, for most varieties, each of these lattices can be embedded into $\text{Con } \mathbf{A}$ for some \mathbf{A} in the variety. For a large subclass of these lattices, the embedding is cover-preserving. There are other modular lattices that essentially can never be embedded. This talk will make these ideas precise and indicate how they are proved. (Received September 16, 2019)

1154-08-931 **Dmitriy Zhuk*** (zhuk@intsys.msu.ru), Saratovskaya 22, 503, Moscow, 109125, Russia.
Four types of subuniverses and the complexity of the constraint satisfaction problem.
 Preliminary report.

The Constraint Satisfaction Problem (CSP) is the problem of deciding whether there is an assignment to a set of variables subject to some specified constraints. In general, this problem is NP-complete but if we restrict the constraint language it can be solved in polynomial time. In 2007 it was conjectured that CSP over a constraint language Γ is tractable if and only if Γ is preserved by a weak near-unanimity operation. After many efforts and partial results, this conjecture was independently proved by Andrei Bulatov and the author in 2017.

In my talk we will discuss the main ingredient of my proof, that is, four types of subuniverses and their algebraic properties. We will consider finite idempotent algebras $(A; w)$, where w is a WNU operation, and will show that on every algebra of size greater than 1 there exists a nontrivial subuniverse C (not empty, not full) of one of four types: binary absorbing, central, PC, and linear. Moreover, these subuniverse have a lot of nice properties such as: two nonempty subuniverses of different types cannot have an empty intersection. We will discuss these properties as well as their implications for the complexity of the constraint satisfaction problem. (Received September 12, 2019)

1154-08-1102 **Keith Kearnes, Benoit Larose*** (blarose@lacim.ca), **Barnaby Martin** and **Ágnes Szendrei.** *QCSPs on finite groups.* Preliminary report.

We investigate the computational complexity of deciding, for a fixed finite group \mathbf{G} , the validity of sentences of the form

$$\forall y_1 \exists x_1 \dots \Phi$$

where Φ is a system of equations over \mathbf{G} (with or without constants). As a byproduct of our study, we present various structure results on group homomorphisms. (Work in progress) (Received September 13, 2019)

1154-08-1128 **Clifford Bergman*** (cbergman@iastate.edu), **T. Penza** and **A. B. Romanowska.**
Varieties of Semilattice Sums. Preliminary report.

Let \mathcal{V} be a variety of algebras, and let \mathcal{S} be term-equivalent to the variety of semilattices. We show that if \mathcal{V} satisfies a strongly irregular identity then the Maltsev product $\mathcal{V} \circ \mathcal{S}$ will again be a variety. (In particular, it will be closed under homomorphic images.) By contrast, the class $\mathcal{S} \circ \mathcal{S}$ is not closed under homomorphic images. Members of $\mathcal{V} \circ \mathcal{S}$ are called *semilattice sums of \mathcal{V} -algebras*. We provide some examples. (Received September 13, 2019)

1154-08-1304 **Emily Frances Leiter Olson*** (olson13@stolaf.edu) and **Abror Khudoyberdiyev.**
Classification of Leibniz Algebras Corresponding to Three Dimensional Solvable Lie Algebras.

A Leibniz algebra, named after Gottfried Wilhelm Leibniz is defined as a module L over a commutative ring R , with a bilinear product, denoted by $[-, -]$, such that the following Leibniz identity is satisfied: $[x, [y, z]] = [[x, y], z] - [[x, z], y]$.

Right (respectively, left) multiplication is then a derivation. If in addition to the Leibniz identity, the bracket on L is alternating, $[x, x] = 0$, and anticommutativity, $[x, y] = -[y, x]$, then we say that L is a Lie algebra.

In this presentation we will examine a special type of Leibniz algebras, those associated with two and three dimensional solvable Lie algebras. In our examination, we adopt methods from representation theory, module theory, and rigorous parameter manipulation. Through basis changes and substitution, we are able to define some non-isomorphic Leibniz algebras that share similarities with the known Lie algebra. (Received September 14, 2019)

1154-08-1375 **George F McNulty*** (mcnulty@math.sc.edu), University of South Carolina, Department of Mathematics, Columbia, SC 29208. *What I would still like to know about equational logic.* Preliminary report.

I began thinking about equational logic in 1968. There were a couple of theorems that I just couldn't understand properly. Since then several more problems and theorems have been proven that still trouble me. In this talk, I will present three of these problems, giving the context for each of them. (Received September 15, 2019)

1154-08-1529 **Alexandr Kazda*** (alex.kazda@gmail.com), Katedra algebrы MFF UK, Sokolovska 83, Praha 8, 18675 Prague, Czech Rep, and **Michael Kompatscher**. *Maltsev conditions invariant under permutation group actions.*

Maltsev conditions are essentially functional equations to be solved in a given algebra. Classically, Maltsev conditions helped investigate the properties of congruence lattices in varieties. More recently, certain Maltsev conditions turned out to describe the hardness of the Constraint Satisfaction Problem with a fixed target structure.

In our talk, we will study the problem of deciding, for a fixed Maltsev condition C , if an input finite idempotent algebra satisfies the condition C . The complexity of this problem depends on C in a way that is far from understood. We essentially only have one polynomial time algorithm, the “local to global” algorithm, and we lack tools for proving hardness.

In this talk, we will investigate a class of conditions (suggested to us by Matt Valeriote) that have the shape

$$f(x_1, \dots, x_n) \approx f(x_{\sigma(1)}, \dots, x_{\sigma(n)}),$$

where f is an operation symbol and σ ranges over a fixed group of permutations \mathbf{G} ; denote this system of identities by $I(\mathbf{G})$. We will sketch why the “local to global” algorithm fails for $I(\mathbf{S}_n)$ when $n \geq 3$ and speculate about the properties of \mathbf{G} that lead to tractability or hardness of deciding $I(\mathbf{G})$ for idempotent algebras. (Received September 16, 2019)

1154-08-1687 **Athena Sparks*** (athena.sparks@colorado.edu). *Clonoids into finite target algebras without cube terms.* Preliminary report.

A clonoid is a set of finitary functions from a source set A to a target algebra \mathbf{B} that is closed under taking minors and the operations of \mathbf{B} . Clonoids are generalizations of clones and have been studied in connection to classifying the complexity of Promise Constraint Satisfaction Problems. We have shown that the number of clonoids with a finite source set and finite idempotent algebra of sizes at least two is continuum if and only if the target algebra has no cube term. We investigate the number of clonoids into finite non-idempotent algebras without cube terms. (Received September 16, 2019)

1154-08-1993 **Josh T. Grice*** (jtgrice@email.sc.edu). *Axiomatisability of subdirectly irreducible members of congruence modular varieties generated by certain finite nilpotent algebras.*

If \mathbf{A} is a finite nilpotent algebra of finite signature which is a product of algebras of prime power order and \mathbf{A} generates a congruence modular variety \mathcal{V} , then the subdirectly irreducible members of \mathcal{V} can be axiomatised by a finite set of elementary sentences. (Received September 17, 2019)

1154-08-2030 **Michael Kompatscher*** (michael@logic.at). *Topology is relevant (in discussing the complexity of ω -categorical CSPs).*

It is well-known that the complexity of the CSP of a finite structure A is determined by its polymorphism clone $\text{Pol}(A)$. This universal algebraic approach resulted in the proofs of the dichotomy conjecture by Bulatov and Zhuk, showing that $\text{CSP}(A)$ is NP-complete if there is a minion homomorphism from $\text{Pol}(A)$ to the projection clone and in P otherwise.

In general, omega-categorical structures are the biggest class where the universal algebraic approach is applicable. However then also the topology on $\text{Pol}(A)$ is relevant, as NP-hardness of $\text{CSP}(A)$ follows from uniformly continuous minion homomorphism to the projection clone. And by a result of Bodirsky, Mottet, Olšák, Opršal, Pinsker and Willard, there is an omega-categorical structure A , such that $\text{Pol}(A)$ has a minion homomorphism to the projections, but no uniformly continuous one.

However their example required an infinite signature. In this talk I would like to discuss the construction of such A in finite relational language. In particular, this allows a discussion of the complexity of the resulting CSPs. As it turns out, we can obtain such A with coNP-complete, k-EXP-time and undecidable CSPs.

This is joint work with Pierre Gillibert, Julius Jonušas, Antoine Mottet and Michael Pinsker. (Received September 17, 2019)

1154-08-2473 **Matthew Moore*** (matthew.moore@ku.edu) and **Andrew Moorhead**. *Supernilpotence Need Not Imply Nilpotence.*

Supernilpotence is a generalization of nilpotence using a recently developed theory of higher-arity commutators for universal algebras. Many important structural properties have been shown to be associated with supernilpotence, and the exact relationship between nilpotence and supernilpotence has been the subject of investigation. We construct an algebra which is not solvable (and hence not nilpotent) but which is supernilpotent, thereby showing that in general supernilpotence does not imply nilpotence. (Received September 17, 2019)

11 ► *Number theory*

1154-11-24 **Kenneth A. Ribet*** (ribet@berkeley.edu), Math Department, M/C 3840, 970 Evans Hall, Berkeley, CA 94720-3840. *A 2020 view of Fermat's Last Theorem.*

Fermat's Last Theorem was formulated in the seventeenth century and proved a little over 25 years ago. I will recall the statement of the theorem and present a skeletal summary of the proof as it was viewed in 1993 or 1994. (I gave an invited hour address at the 1994 JMM in Cincinnati on this topic but concluded my address with a discussion of the "gap" that Nick Katz had identified in the proof that Andrew Wiles announced in June, 1993. The gap was repaired in late 1994 by an article that was co-authored by Richard Taylor and Andrew Wiles.)

I will describe (or at least allude to) some the advances in this general subject that were made possible by the new techniques that were introduced in the proof: for example, the proofs of Serre's conjecture about mod p Galois representations and the Fontaine–Mazur conjecture about p -adic Galois representations should be viewed as outgrowths of the Taylor–Wiles method that was introduced in 1994.

The question is whether or not recent progress in the study of Galois representations and modular forms has streamlined or simplified the proof of Fermat's Last Theorem to a significant extent. The short answer is: maybe yes, and maybe no. For a longer answer... come to the talk. (Received June 11, 2019)

1154-11-39 **George E. Andrews*** (gea1@psu.edu), Department of Mathematics, McAllister Bldg., The Pennsylvania State University, University Park, PA 16802. *Separable Integer Partition (SIP) Classes.*

Three of the most classical and well-known identities in the theory of partitions concern: (1) the generating function for $p(n)$ (Euler); (2) the generating function for partitions into distinct parts (Euler), and (3) the generating function for partitions in which parts differ by at least 2 (Rogers-Ramanujan). The lovely, simple argument used to produce the relevant generating functions is mostly never seen again. Actually, there is a very general theorem here which we shall present. We then apply it to prove two familiar theorems; (1) Goellnitz-Gordon, and (2) Schur 1926. We also consider an example where the series representation for the partitions in question is new. We close with an application to "partitions with n copies of n ." (Received July 10, 2019)

1154-11-121 **Adriana Salerno**, Department of Mathematics, Bates College, Lewiston, ME 04240, and **Joseph H Silverman***, Department of Mathematics, Box 1917, Brown University, Providence, RI 02912. *p -adic Properties of Böttcher Coordinates.* Preliminary report.

Let $\varphi(x) \in x^m + x^{m+1}\mathbb{Z}_p[[x]]$ be a power series. Then there is a unique power series $f_\varphi(x) \in x + x^2\mathbb{Q}_p[[x]]$, called the Böttcher coordinate of φ , satisfying the functional equation $\varphi \circ f_\varphi(x) = f_\varphi(x^m)$. We investigate how p -divisibility of the coefficients of φ influences p -adic integrality properties of the coefficients of f_φ (Received August 10, 2019)

1154-11-123 **Jordan Ellenberg**, **Matthew Satriano*** (msatrian@uwaterloo.ca) and **David Zureick-Brown**. *New types of heights with connections to the Batyrev-Manin and Malle Conjectures.*

The Batyrev-Manin conjecture gives a prediction for the asymptotic growth rate of rational points on varieties over number fields when we order the points by height. The Malle conjecture predicts the asymptotic growth rate for number fields of degree d when they are ordered by discriminant. The two conjectures have the same form and it is natural to ask if they are, in fact, one and the same. We develop a theory heights on stacks, and give a conjecture for the growth rate of points on stacks which specializes to the two aforementioned conjectures. (Received August 11, 2019)

1154-11-130 **Wade Hindes*** (wah33@txstate.edu). *Dynamical height growth: left, right, and total orbits.* Preliminary report.

Let S be a set of dominant rational self-maps on \mathbb{P}^N . We study the arithmetic and dynamical degrees of infinite sequences of S obtained by sequentially composing elements of S on the right and left. (Received August 12, 2019)

1154-11-137 **Keping Huang*** (keping.huang@rochester.edu), 915 Hylan Building, Rochester, NY 14620. *Uniform Bounds for Periods of Endomorphisms of Varieties.* Preliminary report.

Suppose X is a projective variety defined over a finite extension K of \mathbb{Q}_p and suppose X admits a model \mathcal{X} defined over the ring of integers R of K . Let $f : X \rightarrow X$ be an endomorphism of X defined over K that can be extended to an endomorphism of \mathcal{X} defined over R . We prove an upper bound for the primitive period of periodic points defined over R . (Received August 14, 2019)

1154-11-168 **Alexander J Dunn*** (ajdunn2@illinois.edu). *Maass forms and the mock theta function $f(q)$.*

In 1964, George Andrews proved an asymptotic formula (finite sum of terms) involving generalized Kloosterman sums and the I -Bessel function for the coefficients of Ramanujan's famous third order mock theta function. Andrews conjectured that these series converge when extended to infinity, and that they do not converge absolutely. Bringmann and Ono proved the first of these conjectures in 2006. Here we obtain a power savings bound for the error in Andrews' formula, and we also prove the second of these conjectures.

Our methods depend on the spectral theory of Maass forms of half-integral weight, and in particular on a new estimate which we derive for the Fourier coefficients of such forms. This talk aims to be a blend of number theory, combinatorics and analysis. This is a joint work with Scott Ahlgren. (Received August 17, 2019)

1154-11-172 **Vefa Goksel*** (goksel@math.wisc.edu). *Misiurewicz polynomials and irreducibility.*

Let $f_{c,d}(x) = x^d + c \in \mathbb{C}[x]$. The c_0 values for which $f_{c_0,d}$ has a strictly pre-periodic finite critical orbit are called Misiurewicz points. Any Misiurewicz point lies in \mathbb{Q} . Suppose that the Misiurewicz points $c_0, c_1 \in \bar{\mathbb{Q}}$ are such that the polynomials $f_{c_0,d}$ and $f_{c_1,d}$ have the same orbit type. One classical question is whether c_0 and c_1 need to be Galois conjugates or not. I will talk about some partial results I have recently obtained related to this question. (Received September 03, 2019)

1154-11-190 **Keshav Aggarwal*** (keshav.aggarwal@maine.edu), 5752 Neville Hall, Room 334, Orono, ME 04469. *A new subconvex bound for $GL(3)$ L -functions in the t -aspect.*

Let π be a Hecke cusp form for $SL(3, \mathbb{Z})$. We revisit Munshi's proof of the t -aspect subconvex bound for $SL(3)$ L -functions, and we are able to remove the 'conductor lowering' trick. This simplification along with a more careful stationary phase analysis allows us to improve Munshi's bound to,

$$L(1/2 + it, \pi) \ll_{\pi, \epsilon} (1 + |t|)^{3/4 - 3/40 + \epsilon}$$

for any $\epsilon > 0$. (Received August 30, 2019)

1154-11-192 **Peter Christopher Leslie Humphries*** (pclhumphries@gmail.com). *Small Scale Equidistribution of Lattice Points on the Sphere.*

Consider the projection onto the unit sphere in \mathbb{R}^3 of the set lattice points $(x_1, x_2, x_3) \in \mathbb{Z}^3$ lying on the sphere of radius \sqrt{n} . Duke and Schulze-Pillot showed in 1990 that these points equidistribute on the sphere as $n \rightarrow \infty$. We study a small scale refinement of this theorem, where one asks whether these points equidistribute in subsets of the sphere whose surface area shrinks as n grows. A particular case of this is a conjecture of Linnik, which states that for all $\epsilon > 0$, the equation $x_1^2 + x_2^2 + x_3^2 = n$ has a solution with $|x_3| < n^\epsilon$ for all sufficiently large n . We make nontrivial progress towards this and also prove an averaged form of this conjecture. This is joint work with Maksym Radziwiłł. (Received August 21, 2019)

1154-11-219 **Kelly Isham*** (ishamk@uci.edu), **Nathan Kaplan** and **Max Weinreich**. *Counting n -arcs in projective planes.*

An n -arc in $\mathbb{P}^2(\mathbb{F}_q)$ is a collection of n distinct points such that no three lie on a line. In 1988, Glynn gave an algorithm for counting the number of n -arcs in a projective plane of order q . He found that for $n \leq 6$, the formula for the number of n -arcs is polynomial in q and that for $n = 7$ and 8 , the formula is quasipolynomial in q . In 1995, Iampolskaia et al showed that the formula for $n = 9$ is also quasipolynomial. Recent work by Kaplan et al extended this result to arbitrary projective planes of order q . This leads to the question - will the number of n -arcs over $\mathbb{P}^2(\mathbb{F}_q)$ continue to be quasipolynomial in q ? In this talk, we discuss a modification of Glynn's algorithm that makes computation simpler and we explain how the problem of counting the number of n -arcs in the projective plane over \mathbb{F}_q is equivalent to counting the number of rational points on certain varieties over \mathbb{F}_q . We use this new approach to prove that the number of 10-arcs in a projective plane over \mathbb{F}_q is not quasipolynomial. Lastly, we discuss analogous results for larger n and we relate this counting problem to the study of \mathbb{F}_q -points on Grassmannians and to MDS codes. (Received September 16, 2019)

1154-11-221 **Robert L. Benedetto*** (rlbenedetto@amherst.edu) and **Su-Ion Ih** (ih@colorado.edu). *Isolation of postcritically finite parameters in p -adic dynamical moduli spaces.*

Fix a prime number p , and let $f_c(z) = f(c, z) \in \mathbb{C}_p[[c]](z)$ be a one-parameter analytic family of rational functions of degree $d \geq 2$, for c in some open disk D . Suppose that all $2d - 2$ critical points of f_c are also analytic functions of c . A parameter c is postcritically finite, or PCF, if all of the critical points of f_c have finite forward orbit under the iteration of f_c . Under mild conditions, including that all critical points lie in the Fatou set, we show that any proper subdisk of D contains only finitely many PCF parameters, even though D itself may contain

infinitely many. In particular, all PCF parameters of $f_c(z) = z^d + c$ are isolated in p -adic dynamical moduli space. (Received August 24, 2019)

1154-11-246 **Mohammad K. Azarian*** (azarian@evansville.edu), Department of Mathematics, University of Evansville, 1800 Lincoln Avenue, Evansville, IN 47722. *On Convergent of Some Infinite Series Involving Fibonacci Numbers and Identities*. Preliminary report.

In this talk we present some convergent infinite series as well as some convergent infinite products involving Fibonacci numbers and identities. We use modifications of some known Fibonacci identities, including Cassini's, Catalan's, and Vajda's identities. Also, we utilize known results from analysis, including some properties of convergent infinite series. The talk is suitable for students. (Received August 26, 2019)

1154-11-286 **Kiran S. Kedlaya***, Department of Mathematics, 9500 Gilman Drive #0112, La Jolla, CA 92093. *Computing Coleman integrals on modular curves*. Preliminary report.

In computational applications of the (abelian and nonabelian) Chabauty methods to finding rational points on curves, a key point is the computation of (single and double) p -adic Coleman integrals on these curves. The usual method of doing this uses the change of variables formula for a Frobenius lift, as in Coleman's original definition. We describe a modified approach in the case of modular curves, which uses the p -th Hecke operator and the Eichler-Shimura congruence instead. (Received August 28, 2019)

1154-11-295 **Naser Talebizadeh Sardari*** (nasertalebi1367@gmail.com), 813 Eagle heights, Apt F, Madison, WI 53705. *The least prime number represented by a binary quadratic form*.

Let $D < 0$ be a fundamental discriminant and $h(D)$ be the class number of $\mathbb{Q}(\sqrt{D})$. Let $R(X, D)$ be the number of classes of the binary quadratic forms of discriminant D which represent a prime number in the interval $[X, 2X]$. Moreover, assume that $\pi_D(X)$ is the number of primes, which split in $\mathbb{Q}(\sqrt{D})$ with norm in the interval $[X, 2X]$. We prove that

$$\left(\frac{\pi_D(X)}{\pi(X)}\right)^2 \ll \frac{R(X, D)}{h(D)} \left(1 + \frac{h(D)}{\pi(X)}\right),$$

where $\pi(X)$ is the number of primes in the interval $[X, 2X]$ and the implicit constant in \ll is independent of D and X . (Received August 29, 2019)

1154-11-296 **Chantal David** and **Alexandra Florea***, aflorea@math.columbia.edu, and **Matilde Lalin**. *Moments of cubic L -functions over function fields*.

I will focus on the mean value of L -functions associated to cubic characters over $\mathbb{F}_q[t]$ when $q \equiv 1 \pmod{3}$. I will explain how to obtain an asymptotic formula which relies on obtaining cancellation in averages of cubic Gauss sums over function fields. I will also talk about the corresponding non-Kummer case when $q \equiv 2 \pmod{3}$ and I will explain why this setting is somewhat easier to handle than the Kummer case, which allows us to prove some better results. This is joint work with Chantal David and Matilde Lalin. (Received August 29, 2019)

1154-11-315 **Shashank Kanade*** (shashank.kanade@du.edu), CO, and **Matthew C Russell** (russell12@math.rutgers.edu). *On certain partition identities related to $A_9^{(2)}$* . Preliminary report.

I will give an overview of the partition identities related to certain level 2 standard modules for the affine Lie algebra $A_9^{(2)}$ that were found in a joint work with Matthew C. Russell. Ideas related to these identities (staircases and jagged partitions) lead us to analytic sum-sides and further companions to several other identities. Many of these identities have been proven by Bringmann, Jennings-Shaffer and Mahlburg. (Received August 30, 2019)

1154-11-331 **Freydoon Shahidi*** (shahidi@math.purdue.edu), Department of Mathematics, 150 n. University Street, West Lafayette, IN 47907, and **William Sokurski** (wsokursk@purdue.edu), 150 n. University Street, West Lafayette, IN 47907. *On multiplicativity of gamma factors via Braverman-Kazhdan program*.

After a short survey of the Braverman-Kazhdan/Ngo/Lafforgue program, extending the work of Godement-Jacquet on principal L -functions for $GL(n)$ to any reductive group and any finite dimensional representation of its L -group, we sketch a proof of the multiplicativity of gamma factors via parabolic induction in general, an important question in any theory of L -functions. The proof relies on suitable assumptions on corresponding Fourier transforms. (Received August 31, 2019)

1154-11-349 **Renee Bell, Jeremy Booher*** (jeremy.booher@canterbury.ac.nz), **William Y Chen** and **Yuan Liu**. *Tamely Ramified 3-Pointed Covers with Alternating and Symmetric Monodromy.*

Let k be an algebraically closed field of characteristic p , and U be the projective line over k with three points removed. We investigate which groups G can arise as the monodromy group of tamely ramified covers (étale covers of U with *tame ramification* at the three removed points). This provides new information about the tame fundamental group of U . In particular, we show that for fixed p , there are families of tamely ramified covers with monodromy the symmetric group S_n for infinitely many n , and similarly for alternating groups. The standard approach is to reduce covers from characteristic zero, imposing restrictions on the monodromy group in order to apply criteria of Raynaud and Obus to guarantee good reduction. We overcome this restriction to produce symmetric and alternating monodromy groups by constructing our covers using the moduli spaces of elliptic curves with $\mathrm{PSL}_2(\mathbf{F}_\ell)$ -structure, which has a natural map to \mathbb{P}^1 and where the primes of good reduction are readily apparent. We use work of Bourgain, Gamburd, and Sarnak, and adapt work of Meiri and Puder, to understand the monodromy action for this moduli space; reducing an appropriate component modulo p gives the desired tamely ramified covers. (Received September 01, 2019)

1154-11-360 **Yuan Liu*** (yyliu@umich.edu), **Melanie Matchett Wood** and **David Zureick-Brown**. *Heuristics on distributions of Galois groups of unramified extensions.*

We will first review several heuristics on the distributions of Galois groups of unramified extensions of global fields, which include the Cohen-Lenstra Heuristics regarding the class groups of quadratic fields and the Boston-Bush-Hajir Heuristics regarding the p -class tower groups of quadratic fields. We will then discuss how these heuristics relate to reasonable random group models, and then explain a new conjecture on the distributions of the Galois groups of the maximal unramified extensions of Galois Γ number fields or function fields for a large family of finite groups Γ . Finally, we will give theorems in the function field case to support this new conjecture. This work is joint with Melanie Matchett Wood and David Zureick-Brown. (Received September 02, 2019)

1154-11-362 **Andrew V Sills*** (asills@georgiasouthern.edu). *Finite Rogers-Ramanujan type identities.* Preliminary report.

In the 1980's, George Andrews introduced a heuristic by which a given Rogers-Ramanujan type identity could be generalized to an infinite family of polynomial identities. This "finitization" process remained popular, particularly with physicists (including Berkovich, McCoy, Warnaar, and others), through the 1990's and into the early 2000's. In my 2002 PhD thesis, I took Andrews' heuristic and refined it into an algorithm that I implemented in Maple. As a result, I was able to produce at least one finite form for each of the 130 identities on Slater's list of identities of Rogers-Ramanujan type. Now, 18 years later, computers have improved significantly, and perhaps it is time to revisit and build on this earlier work. We will look at some new results, and suggest some possibilities for new algorithms which would be useful to advance the subject. (Received September 02, 2019)

1154-11-366 **Charles F. Doran** (doran@math.ualberta.edu), Department of Mathematics, Edmonton, AB, Canada, **Tyler L. Kelly** (t.kelly.1@bham.ac.uk), School of Mathematics, University of Birmingham, Edgbaston, Birmingham, B15 2TT, United Kingdom, **Adriana Salerno** (asalerno@bates.edu), Department of Mathematics, 3 Andrews Rd., Lewiston, ME 04240, **Steven Sperber** (sperber@umn.edu), School of Mathematics, 206 Church Street SE, Minneapolis, MN 55455, **John Voight*** (jvoight@gmail.com), Department of Mathematics, 6188 Kemeny Hall, Hanover, NH 03755, and **Ursula Whitcher** (uaw@umich.edu), 416 Fourth St, Ann Arbor, MI 48103. *Zeta functions of alternate mirror Calabi-Yau families.*

We prove that if two Calabi-Yau invertible pencils have the same dual weights, then they share a common factor in their zeta functions. By using Dwork cohomology, we demonstrate that this common factor is related to a hypergeometric Picard-Fuchs differential equation. The factor in the zeta function is defined over the rationals and has degree at least the order of the Picard-Fuchs equation. As an application, we relate several pencils of K3 surfaces to the Dwork pencil, obtaining new cases of arithmetic mirror symmetry. (Received September 02, 2019)

1154-11-379 **Chris Jennings-Shaffer*** (christopher.jennings-shaffer@du.edu), **Kathrin Bringmann** (kbringma@math.uni-koeln.de) and **Karl Mahlburg** (mahlburg@math.lsu.edu). *Some q -series conjectures related to Rogers-Ramanujan type identities of Kanade and Russell.*

In 2014, and in follow-up work in 2018, Kanade and Russell conjectured a large number of identities of the Rogers-Ramanujan type. Recently, in joint work with Kathrin Bringmann and Karl Mahlburg, we proved seven of their conjectures. Rather than focus on the proven identities, here we discuss some conjectural identities for certain q -hypergeometric series that came out of a reduction of other conjectures of Kanade and Russell. In these cases, the series can be expressed as ${}_2\phi_1$ series, rather than multi-sums, and the result should be a simple infinite product. While this sounds quite simple, somehow a full proof has eluded us. (Received September 02, 2019)

1154-11-393 **Xander Faber*** (awfaber@super.org) and **Clayton Petsche**. *Totally T -adic functions of small height.* Preliminary report.

A nonzero algebraic number α is *totally p -adic* if its minimal polynomial (over \mathbb{Q}) splits completely over \mathbb{Q}_p . If α is not a $(p-1)st$ root of unity, then the naive logarithmic height of such an element is uniformly bounded away from zero by an equidistribution result of Bombieri/Zannier or an elementary inequality of Pottmeyer.

In this work, we introduce a geometric analogue. Fix a finite field \mathbb{F}_q , and consider the rational function field $\mathbb{F}_q(T)$. An algebraic function f that generates a separable extension of $\mathbb{F}_q(T)$ is *totally T -adic* if its minimal polynomial (over $\mathbb{F}_q(T)$) splits completely in the field of Laurent series $\mathbb{F}_q((T))$. We will discuss a lower bound for the height of any nonconstant totally T -adic function, and we will show that functions achieving the lower bound give rise to curious algebraic curves over \mathbb{F}_q with many rational points. We also investigate the limit-infinum of the heights of totally T -adic functions using a dynamical construction. (Received September 03, 2019)

1154-11-395 **Andrea Ferraguti*** (and.ferraguti@gmail.com), Madrid, Spain, **Carlo Pagano**, Bonn, Germany, and **Daniele Casazza**, Madrid, Spain. *The inverse problem for arboreal Galois representations of index two.*

Let F be a field of characteristic not 2 and f be a monic and quadratic polynomial with coefficients in F . Let Ω_∞ be the automorphism group of the infinite tree associated to f , and let $M \leq \Omega_\infty$ be a given maximal subgroup. In this talk, I will first explain how to produce necessary and sufficient conditions, depending exclusively on the post-critical orbit of f , for the arboreal representation ρ_f to have image equal to M . Our way of thinking allows to quickly recover classical results, such as Stoll's criterion for surjectivity and infinite index image for PCF quadratic polynomial, from a structural point of view that does not involve the use of ramification theory. Next, I will provide infinite families of examples for polynomials of the form $x^2 + a$ over the rationals. Finally, I will briefly show how to prove that any two closed subgroups of Ω_∞ of index at most 2 are non-isomorphic as topological groups. This involves the use of a new invariant for topological groups endowed with a system of topological generators named *graph of commutativity*. (Received September 03, 2019)

1154-11-406 **John Greene*** (jgreene@d.umn.edu), Department of Mathematics and Statistics, 1117 University Drive, Duluth, MN 55812, and **Mark Broderius** (brode076@d.umn.edu), Department of Mathematics and Statistics, 1117 University Drive, Duluth, MN 55812. *Primes in Lucas sequences.* Preliminary report.

A Lucas sequences is a sequence $U_n(P, Q)$ satisfying the recurrence

$$U_0 = 0, U_1 = 1, U_n = PU_{n-1} - QU_{n-2}, \text{ for } n \geq 2.$$

Special cases include the Mersenne numbers, $2^n - 1 = U_n(3, 2)$ and the Fibonacci numbers, $F_n = U_n(1, -1)$. A problem beyond current techniques is how often $U_n(P, Q)$ should be prime, for relatively prime integers P and Q . Heuristically, when n is prime, one might expect U_n to be prime with probability

$$\frac{1.8 \log(n)}{\log |U_n|},$$

and this estimate appears to be relatively accurate for $n \leq 1000$.

par However, some Lucas sequences appear to contain very few primes. For example, $U_n(5, 4)$ is prime only when $n = 2$, for rather obvious reasons. Here, we give families of P and Q for which we can prove that there are only finitely many primes in the corresponding Lucas sequence, and other families for which we conjecture that there are only finitely many primes. Some proofs seem surprisingly difficult. (Received September 03, 2019)

1154-11-422 **Caroline Matson*** (caroline.matson@colorado.edu). *Formal Groups with Complex Multiplication.*

In their 1965 paper Lubin and Tate showed the existence of one-dimensional formal group laws with large endomorphism rings, or complex multiplication. They used these to generate abelian extensions, culminating a lovely development of local class field theory. In this talk we examine the associated theory for higher-dimensional formal group laws. In particular, we show that given a formal group law \tilde{F} over a field of characteristic p there exists a lift F to a p -adic ring and we prove that this lift has complex multiplication. (Received September 03, 2019)

1154-11-427 **Jordan S Ellenberg, Wanlin Li*** (wanlinli@mit.edu) and **Mark Shusterman.** *Nonvanishing of hyperelliptic zeta functions over finite fields.*

Fixing $t \in \mathbb{R}$ and a finite field \mathbb{F}_q of odd characteristic, we give an upper bound on the proportion of genus g hyperelliptic curves over \mathbb{F}_q whose zeta function vanishes at $\frac{1}{2} + it$. Our upper bound is independent of g and tends to 0 as q grows. This result is obtained by studying rational points on twisted Hurwitz spaces over finite fields. (Received September 03, 2019)

1154-11-458 **Emily Eckels, Steven Jin*** (sjin6816@umd.edu), **Andrew Ledoan** and **Brian Tobin.** *Linnik's large sieve and the L^1 norm of exponential sums.*

The method of proof of Balog and Ruzsa and the large sieve of Linnik are used to investigate the behaviour of the L^1 norm of a wide class of exponential sums over the square-free integers and the primes. Further, a new proof of the lower bound due to Vaughan for the L^1 norm of an exponential sum with the von Mangoldt Λ function over the primes is furnished. Ramanujan's sum arises naturally in the proof, which also employs Linnik's large sieve. (Received September 04, 2019)

1154-11-468 **D. A. Goldston** and **C. L. Turnage-Butterbaugh*** (cturnageb@carleton.edu). *Small gaps between zeros of the Riemann zeta-function.*

I will discuss recent joint work with Dan Goldston concerning the vertical distribution of zeros of the Riemann zeta-function, denoted $\zeta(s)$. In particular, we improve on previous results by proving, under the assumption of the Riemann Hypothesis, that there are infinitely many zeros of $\zeta(s)$ whose differences are smaller than 0.50345 times the average spacing. (Received September 17, 2019)

1154-11-473 **Adela Gherga*** (ghergaa@math.ubc.ca). *Computing elliptic curves over \mathbb{Q} .* Preliminary report.

Let S be a set of rational primes and consider the set of all elliptic curves over \mathbb{Q} having good reduction outside S and bounded conductor N . Currently, using modular forms, all such curves have been determined for $N \leq 500000$; the bulk of this work is attributed to Cremona.

Early attempts to tabulate all such curves often relied on reducing the problem to one of solving a number of certain integral binary forms called Thue-Mahler equations. These are Diophantine equations of the form $F(x, y) = u$, where F is a given binary form of degree at least 3 and u is an S -unit.

A theorem of Bennett-Rechnitzer shows that the problem of computing all elliptic curves \mathbb{Q} of conductor N reduces to solving a number of Thue-Mahler equations. To compute all such equations, there exists a practical method of Tzanakis-de Weger using bounds for linear forms in p -adic logarithms and various reduction techniques. In this talk, we describe our refined implementation of this method and discuss the key refinements used in our algorithm. (Received September 04, 2019)

1154-11-479 **Jason P Bell*** (jpbell@uwaterloo.ca), **Dragos Ghioca** and **Matthew Satriano.** *A gap conjecture for heights of iterates.*

Let X be a quasi-projective variety defined over a field K of characteristic 0, endowed with the action of an étale endomorphism Φ , and $f: X \rightarrow Y$ is a morphism with Y a quasi-projective variety defined over K . Then we show that a uniform result of the following type holds: if for a given $x \in X(K)$, for each $y \in Y(K)$ the set $S_y := \{n \in \mathbb{N}: f(\Phi^n(x)) = y\}$ is finite, then there exists a positive integer N such that $\#S_y \leq N$ for each $y \in Y(K)$. We use this to prove that a "gap" theorem holds for étale endomorphisms, which we now describe. Let K be a number field, $f: X \rightarrow \mathbb{P}^1$ a rational map, and Φ be an étale endomorphism of X . If \mathcal{O} denotes the forward orbit of x under the action of Φ , then either $f(\mathcal{O})$ is finite, or $\limsup_{n \rightarrow \infty} h(f(\Phi^n(x)))/\log(n) > 0$, where $h(\cdot)$ represents the usual logarithmic Weil height for algebraic points. We conjecture that this dichotomy should hold for general endomorphisms of quasi-projective varieties when everything is defined over a number field. (Received September 05, 2019)

- 1154-11-499 **Emerald T Stacy*** (estacy2@washco11.edu), 300 Washington Ave, Chestertown, MD 21620, and **Ozlem Ejder, Jaime Juul and Borys Kadets**. *Periodic Points of Polynomials in Finite Fields*.

Let $\mathcal{F}(d, q)$ denote the set of monic, degree d , polynomials over the finite field \mathbb{F}_q . For $f \in \mathcal{F}(d, q)$, let $\text{Per}(f)$ denote the number of periodic points of f over \mathbb{F}_q . Fixing d , as $q \rightarrow \infty$, what happens to the average number of periodic points under each function in $\mathcal{F}(d, q)$? In this talk, we will explore a heuristic to estimate this average, and compare the heuristic to data collected for some d and q . (Received September 05, 2019)

- 1154-11-500 **Machiel van Frankenhuijsen*** (vanframa@uvu.edu), Utah Valley University, Mathematics, Orem, UT 84058. *Geometric Understanding of a Zero Free Region*.

That the Riemann zeta function does not vanish on the line $1 + i\mathfrak{p}$ is equivalent to the prime number theorem. It follows from combining the Euler product with a positive function, and can be strengthened to yield a zero free region $\zeta(D + i\mathfrak{p}) = 0$ implies $\frac{1}{1-D} \leq A + B \log \mathfrak{p}$. Fractal geometry yields a deeper understanding inside the critical region $0 < D < 1$. We will discuss the latest work and possibilities for further research. (Received September 05, 2019)

- 1154-11-503 **Sarah Arpin, Richard Griffon, Libby Taylor*** (lt691@stanford.edu) and **Nicholas Triantafyllou**. *The arithmetic of an explicit family of superelliptic curves over function fields*. Preliminary report.

We consider the family of superelliptic curves with Weierstrass equation $y^b = x^a + t^q - t$ over a function field of characteristic p . We compute the L-functions of the Jacobians of such curves and compute various arithmetic invariants of the Jacobian, such as the regulator, local Tamagawa numbers, and the Tate-Shafarevich group (which we prove is finite). The behavior of such invariants is strongly dependent on the congruence class of \mathfrak{p} mod ab ; for example, the analytic rank is zero when $p \equiv 1 \pmod{a}$ and $p \equiv 1 \pmod{b}$, but is unbounded as $q \rightarrow \infty$ when $p \equiv -1 \pmod{a}$. Our approach relies on the BSD conjecture for such Jacobians and an explicit computation of the L-function. (Received September 05, 2019)

- 1154-11-510 **Hester Graves and Lindsey-Kay Lauderdale*** (llauderdale@towson.edu). *The Minimal Euclidean function on the Eisenstein Integers*. Preliminary report.

Zariski first inquired about the relationship between different Euclidean functions in a fixed Euclidean ring R . Motzkin and Samuel studied this relationship and determined an algorithm that established the so-called minimal Euclidean function on R . Their algorithm involved the construction of recursively defined subsets of R . However, determining these subsets is complicated in most cases, and it remains difficult to compute the value of the minimal Euclidean function on a given element of R without an exhaustive search. In this talk, we will focus on the ring of Eisenstein integers and discuss a geometric description of its minimal Euclidean function. We then use this description to establish an efficient method to compute the value of the minimal Euclidean function on a given Eisenstein integer. (Received September 05, 2019)

- 1154-11-531 **Seoyoung Kim*** (sk206@queensu.ca), Department of Mathematics and Statistics, Queen's University, 48 University Ave, Jeffery Hall, Kingston, Ontario K7L 3N6, Canada, and **NicoleLooper**. *On the square-freeness of elliptic divisibility sequences over function fields*.

For elliptic divisibility sequences defined over a one-variable function field K of characteristic zero, we prove a uniform bound (depending on K) on the number of terms that lack an odd multiplicity primitive prime divisor are not square-free. This improves a result of Ingram and Silverman on Zsigmondy sets of elliptic divisibility sequences. (Received September 06, 2019)

- 1154-11-534 **Maria Fox** (mariafox@uoregon.edu), **Vlad Matei** (vmatei@uci.edu) and **Soumya Sankar*** (ssankar3@wisc.edu). *Hecke Orbits and Unlikely Intersections*. Preliminary report.

Let $X(1)$ denote the modular curve over the finite field \mathbb{F}_p . The Hecke orbit of a point $(j(E_1), j(E_2) \cdots j(E_n)) \in X(1)^n(\mathbb{F}_p)$ is the set of $(j(E'_1), \cdots j(E'_n)) \in X(1)^n(\mathbb{F}_p)$ such that E_i is isogenous to E'_i for each i . One can ask when a subvariety V of $X(1)^n_{\mathbb{F}_p}$ satisfies the condition that $V(\overline{\mathbb{F}_p})$ intersects every ordinary Hecke orbit. We provide a heuristic for the expected size of the intersection of $V(\overline{\mathbb{F}_p})$ and a general ordinary Hecke orbit. (Received September 06, 2019)

1154-11-538 **Alejandra Alvarado, Angelos Koutsianas, Beth Malmskog*** (bmalmskog@coloradocollege.edu), **Chris Rasmussen, Christelle Vincent and Mckenzie West.** *Solving the S -unit equation in Sage: Methods, Applications, and Next Steps.*

Many finiteness and enumerative problems in number theory rely on the finiteness/enumeration of the set of solutions to the equation $x+y=1$ over the group of S -units in a number field, where S is a finite set of primes. Unfortunately, this is a very difficult computational problem that has challenged mathematicians for decades. In the last several years, a group consisting of Alejandra Alvarado, Angelos Koutsianas, Beth Malmskog, Christopher Rasmussen, Christelle Vincent, and Mckenzie West created functions for Sage to solve this problem, the first publicly released implementation for general number fields. Their functions were incorporated into released versions of Sage in 2019. This talk will give a brief overview of the theory, methods, the capabilities of our implementation, applications in algebraic curves including cases of asymptotic Fermat's Last Theorem, and discuss the next steps in this ongoing project. (Received September 06, 2019)

1154-11-550 **Alexander Berkovich*** (alex@ufl.edu), University of Florida, Dept of Mathematics, Little Hall 496, Gainesville, FL 32601. *On the refined q -trinomial coefficients.*

Doubly bounded refinement of the q -trinomial coefficients of Andrews were first introduced by Warnaar in 2003 in his work on the generalized Borwein conjecture. In this talk I discuss new transformation properties of these coefficients. These properties can be used to derive many new doubly bounded polynomial identities of the Rogers-Ramanujan type. In particular, doubly bounded refinement of Berkovich, McCoy and Orrick (1996) identities were obtained this way. This talk is based on my recent joint work with Toh and Warnaar.

(Received September 06, 2019)

1154-11-552 **Dean Bisogno*** (bisogno@math.colostate.edu), **Wanlin Li, Daniel Litt and Padmavathi Srinivasan.** *A non-hyperelliptic curve with torsion l -adic Ceresa class.*

Let X/K be a smooth curve over a field K . The Ceresa class $c(X)$ is a Galois cohomology class which controls the action of the absolute Galois group G_K on the second term of the lower central series of the pro- l étale fundamental group of X based at a rational point $x \in X$. If X is hyperelliptic, then $c(X) = 0$. We provide an example of a non-hyperelliptic curve with torsion Ceresa class. For every genus $g \geq 3$ we also provide infinitely many examples of curves over number fields with genus g and non-torsion Ceresa class. (Received September 06, 2019)

1154-11-562 **Edgar Costa** (edgarc@mit.edu), **Ravi Donepudi*** (donepud2@illinois.edu), **Ravi Fernando** (fernando@berkeley.edu), **Valentijn Karemaker** (vkarem@math.upenn.edu), **Caleb Springer** (cks5320@psu.edu) and **Mckenzie West** (westmr@uwec.edu). *Abelian varieties not isogenous to a hyperelliptic jacobian.*

Let W_g be the set of \mathbb{F}_q -isogeny classes of abelian varieties of dimension g defined over \mathbb{F}_q . By Honda-Tate theory, W_g is identified with the set of q -Weil polynomials of degree $2g$. We show that certain congruence conditions on the coefficients of a q -Weil polynomial preclude the corresponding isogeny class from containing a hyperelliptic jacobian. In particular, as $q \rightarrow \infty$ this result implies that asymptotically at least 25% of q -isogeny classes of abelian threefolds over \mathbb{F}_q do not contain the jacobian of a smooth hyperelliptic curve defined over \mathbb{F}_q . (Received September 12, 2019)

1154-11-578 **Allison Arnold-Roksandich*** (allisonarnoldrok@boisestate.edu). *Creating Several Infinite Classes of Mock and Quantum Modular Forms.*

In 2013, Lemke Oliver created a list of all eta-quotients which are theta functions. In 2016, Folsom, Garthwaite, Kang, Swisher and Treeneer utilized this list of "eta-theta" functions along with Zwegers's construction of mock theta functions to create a set of mock modular forms which are also quantum modular forms. Later in 2016, Diaz, Ellefsen and Swisher generalized a subset of these quantum modular forms to a single general form which included the every element of this subset. This talk will discuss the work done to extend this generalization to a larger general form which includes all functions made by Folsom et al. (Received September 07, 2019)

1154-11-640 **Daniel Corey, Jordan S Ellenberg and Wanlin Li*** (wanlinli@mit.edu). *Computations on Ceresa class.*

The Ceresa cycle is an algebraic cycle attached to a smooth algebraic curve, which is trivial when the curve is hyperelliptic, and which has not been proven non-trivial for any non-hyperelliptic curve over a number field. Its image under a certain cycle class map provides a class in étale cohomology called the Ceresa class. This class encodes information of the action of the absolute Galois group on the second term of the lower central series of the pro- l étale fundamental group of the curve. Using an l -adic version of the Johnson morphism, via explicit

computation, we determine whether the Ceresa class for curves over $C((t))$ is rationally trivial. Moreover, we define a Ceresa invariant for tropical curves corresponding to the special fiber of curves over $C[[t]]$ and show it is always torsion when all vertices have weight 0. (Received September 09, 2019)

1154-11-653 **Patrick Ingram*** (pigram@yorku.ca), Department of Mathematics and Statistics, York University, 4700 Keele St., Toronto, Ontario M3J 1P3, Canada. *A dynamical Néron symbol*. Preliminary report.

We will discuss a local Néron symbol for arithmetic dynamics, which pairs points and divisors on projective space relative to a given endomorphism, with applications to computing the canonical and critical heights. (Received September 09, 2019)

1154-11-656 **Liyang Zhang***, Department of Mathematics, Maloney Hall, Fifth Floor, Boston College, Chestnut Hill, MA 02467. *Quantum Unique Ergodicity of Eisenstein Series on $GL(n)$* .

In the area of quantum chaos, it is of great interest to investigate the distribution of the L^2 - mass of the eigenfunctions of the Laplacian as the eigenvalues tend to infinity. Luo and Sarnak first formulated and proved quantum unique ergodicity of Eisenstein series on $SL(2, Z)\backslash\mathbb{H}$. In this talk, we extend the result of Luo and Sarnak and prove quantum unique ergodicity for a subspace of the continuous spectrum spanned by the degenerate Eisenstein Series on $GL(n)$. (Received September 09, 2019)

1154-11-696 **Maggie Wieczorek*** (mwieczor@vols.utk.edu). *Andrews-Bressoud Series and Wronskians*. Preliminary report.

The Andrews-Bressoud identities are one of many families of q -series identities relating an infinite sum to an infinite product. While the original motivation for studying these series relates to partitions, they can also be viewed in relation to irreducible characters of minimal models in the theory of vertex operator algebras. Furthermore, considering certain Wronskians of the Andrews-Bressoud series for a given k produce additional q -series, which themselves exhibit interesting and predictable modularity properties. In this paper, we connect the Andrews-Bressoud series to modular forms and prove results about the modularity of their associated Wronskians. (Received September 09, 2019)

1154-11-719 **Olivia Beckwith***, obeck@illinois.edu, and **Gene Kopp**, gene.kopp@bristol.ac.uk. *Polyharmonic Maass forms and ray class zeta functions for real quadratic fields*.

We construct a basis for the vector space of polyharmonic Maass forms for $\Gamma(N)$ with bounded polyharmonic depth, generalizing a result of Lagarias and Rhoades for $N = 1$. We show that twisted traces of geodesic integrals of certain polyharmonic Maass forms are central values of Hecke L-series for real quadratic fields. (Received September 10, 2019)

1154-11-729 **Byungchul Cha*** (cha@muhlenberg.edu), 2400 W. Chew st., Allentown, PA 18104, and **Dong Han Kim**. *Lagrange spectra and intrinsic Diophantine approximation of spheres*. Preliminary report.

Let S^1 be the unit circle in \mathbb{R}^2 centered at the origin and let Z be a countable dense subset of S^1 , for instance, the set $Z = S^1(\mathbb{Q})$ of all rational points in S^1 . We give a complete description of an initial discrete part of the Lagrange spectrum of S^1 , in the sense of intrinsic Diophantine approximation. This is an analogue of the classical result of Markoff, which first appeared in 1879. In addition, we present similar results for a few different choices of Z . Finally, we give some partial results of similar type for S^2 . (Received September 10, 2019)

1154-11-735 **Vishal Arul*** (varul.math@gmail.com). *Torsion points on curves of the form $y^n = x^d + 1$* . Preliminary report.

We classify torsion points on the curve $y^n = x^d + 1$ over \mathbf{C} , where n and d are coprime and satisfy $n, d \geq 2$. When $n + d \geq 8$, we show that the only torsion points on this curve are: (i) those whose x -coordinate is zero, (ii) those whose y -coordinate is zero, (iii) the point at infinity. When $n + d = 7$, there are more torsion points and we classify them all. (Received September 10, 2019)

1154-11-763 **Borys Kadets*** (bkadets@mit.edu). *Sectional monodromy groups of projective curves*.

Fix a field K . Let $X \subset \mathbb{P}^n$ be a projective curve of degree d . Let $H \in (\mathbb{P}^n)_{K(t_1, \dots, t_n)}^*$ denote the generic hyperplane, i.e. the hyperplane given by the equation $x_0 + t_1 x_1 + \dots + t_n x_n$ over the field $L = K(t_1, \dots, t_n)$. A point of $H \cap X$ has residue field M/L of degree d . The Galois group G_X of the Galois closure of M/L is the *sectional monodromy group* of X , it is naturally a permutation group on d letters. Geometrically, G_X is the monodromy group of a hyperplane section $H \cap X$ as H varies. When K has characteristic 0, the sectional monodromy group is always the full symmetric group S_d ; this fact is important in studying the degree-genus

problem for projective curves. In characteristic p sectional monodromy groups can be much smaller than S_d . I will talk about a method for computing sectional monodromy groups. I will show two applications of the method: determining sectional monodromy groups of nonstrange curves in \mathbb{P}^n , $n \geq 3$ and determining sectional monodromy groups of the plane monomial curves $x^n = y^m$. The possibilities for the latter include Mathieu groups M_{11}, M_{23}, M_{24} and linear groups $\text{AGL}_1(q), \text{PGL}_d(q)$. (Received September 10, 2019)

1154-11-794 **Naser Talebizadeh Sardari*** (nasertalebi1367@gmail.com). *Complexity of strong approximation on the sphere.*

By assuming some widely-believed arithmetic conjectures, we show that the task of accepting a number that is representable as a sum of $d \geq 2$ squares subjected to given congruence conditions is NP-complete. On the other hand, we develop and implement a deterministic polynomial-time algorithm that represents a number as a sum of 4 squares with some restricted congruence conditions, by assuming a polynomial-time algorithm for factoring integers and Conjecture ???. As an application, we develop and implement a deterministic polynomial-time algorithm for navigating LPS Ramanujan graphs, under the same assumptions. (Received September 10, 2019)

1154-11-824 **Michael Harris*** (harris@math.columbia.edu). *Local Langlands parametrization for G_2 .* This is a report on joint work with C. Khare and J. Thorne. We construct a Langlands parameterization of supercuspidal representations of G_2 over a p -adic field. More precisely, for any finite extension K/\mathbb{Q}_p we will construct a bijection

$$\mathcal{L}_g : \mathcal{A}_g^0(G_2, K) \rightarrow \mathcal{G}^0(G_2, K)$$

from the set of generic supercuspidal representations of $G_2(K)$ to the set of irreducible continuous homomorphisms $\rho : W_K \rightarrow G_2(\mathbb{C})$ with W_K the Weil group of K . The construction of the map is simply a matter of assembling arguments that are already in the literature, plus an unpublished result of Savin (included as an appendix in our article) on the global genericity of an exceptional theta lift. The proof of surjectivity is an application of a recent result of Hundley and Liu on automorphic descent from $GL(7)$ to G_2 . This allows us to carry out a strategy, based on automorphy lifting theorems, that was initially developed in our joint work with G. Böckle on potential automorphy over function fields. The proof of injectivity also uses global arithmetic methods. (Received September 14, 2019)

1154-11-830 **Madeline Locus Dawsey, Ken Ono and Ian Wagner*** (ian.c.wagner@vanderbilt.edu). *Partitions and a conjecture of John Thompson.*

For a finite group G , let $K(G)$ denote the field generated over \mathbb{Q} by its character values. For alternating groups, G. R. Robinson and J. G. Thompson determined $K(A_n)$ as an explicit multiquadratic field. Confirming a speculation of Thompson, we show that arbitrary suitable multiquadratic fields are similarly generated by the values of A_n -characters restricted to elements whose orders are only divisible by ramified primes. We also extend this result to suitable linear groups and show that cyclotomic fields and subfields are generated by the values of characters restricted to elements with prime power order. (Received September 11, 2019)

1154-11-832 **Hanson Smith*** (hanson.smith@colorado.edu), Department of Mathematics, University of Colorado Boulder, Campus Box 395, Boulder, CO 80309. *Non-monogenic Division Fields of Elliptic Curves.*

We will outline recent work studying monogeneity and the division fields of elliptic curves. For a variety of positive integers n , we are able to show the existence of infinite families of elliptic curves over \mathbb{Q} with n -division fields, $\mathbb{Q}(E[n])$, that are non-monogenic, i.e., the ring of integers does not admit a power integral basis. Moreover, we can parametrize some of these families explicitly. Our main technique combines a global description of the Frobenius by Duke and Tóth with a simple algorithm based on ideas of Dedekind. If time permits, we will also describe recent progress in generalizing these ideas to abelian surfaces. (Received September 11, 2019)

1154-11-833 **Amanda Folsom*** (afolsom@amherst.edu), **Min-Joo Jang** (min-joo.jang@hku.hk), **Sam Kimport** (skimport@stanford.edu) and **Holly Swisher** (swisherh@math.oregonstate.edu). *Ranks for n -marked Durfee symbols and quantum modular forms.*

In 2007, G.E. Andrews introduced notion of n -marked Durfee symbols, and n associated notions of rank. When $n = 1$, the symbols reduce to classical partitions and the rank is exactly Dyson's rank. Andrews offered an $n + 1$ -variable generating function R_n for ranks of n -marked Durfee symbols, an $(n + 1)$ -multisum generalizing the ordinary two-variable partition rank generating function. Since then, it has been a problem of interest to understand its automorphic properties. In certain cases, when viewed as a function on the upper half-plane, the function has been shown to possess modular, quasimodular, and mock modular properties, by work of authors

including Bringmann, Garvan, Mahlburg, Ono, and the authors, in a series of papers from 2009-2013. Quantum modular forms, defined by Zagier in 2010, are similar to (mock) modular forms but are defined on the rational numbers as opposed to the upper half-plane, and exhibit modular transformations there up to suitably analytic error functions in the real line. Here, we establish quantum modular properties of R_n . This is joint work with M-J Jang (U. Hong Kong), S. Kimport (Stanford U.), and H. Swisher (Oregon State U.), and was partly established during the Women in Numbers 4 Workshop at Banff International Research Station (Received September 11, 2019)

1154-11-871 **Savana Ammons, Young Jin Kim, Laura Seaberg and Holly Swisher*** (swisherh@math.oregonstate.edu). *An analogue of k -marked Durfee symbols for strongly unimodal sequences.* Preliminary report.

Andrews' k -marked Durfee symbols, which generalize partitions, have been widely studied from both a combinatorial and automorphic forms perspective. In particular, there have been many interesting and motivating results establishing modularity properties for multivariate rank generating functions for these objects. Here, we define an analog of k -marked Durfee symbols which generalize strongly unimodal sequences. We establish a multivariate rank generating function for these objects, and prove some partition-theoretic results which mirror Andrews' original work with k -marked Durfee symbols. We conclude with some questions about potential modularity properties.

This work is joint with Savana Ammons, Young Jin Kim, and Laura Seaberg. It was initiated during the 2019 Oregon State University summer REU program, funded by the NSF grant DMS-1757995 and Oregon State University. (Received September 11, 2019)

1154-11-877 **Raymond van Bommel***, 77 Massachusetts Avenue, Cambridge, MA 02139. *Explicit arithmetic intersection theory and computation of Néron-Tate heights.*

We describe a general algorithm for computing intersection pairings on arithmetic surfaces. We have implemented our algorithm for curves over \mathbb{Q} , and we show how to use it to compute regulators for a number of Jacobians of smooth plane quartics, and to numerically verify the conjecture of Birch and Swinnerton-Dyer for the Jacobian of the split Cartan curve of level 13, up to squares. This is joint work with David Holmes and J. Steffen Müller. (Received September 11, 2019)

1154-11-929 **Alex Cowan***, cowan@math.harvard.edu. *Non-random behaviour in sums of modular symbols.*

We give explicit expressions for the Fourier coefficients of Eisenstein series twisted by Dirichlet characters and modular symbols on $\Gamma_0(N)$ in the case where N is prime and equal to the conductor of the Dirichlet character. We obtain these expressions by computing the spectral decomposition of automorphic functions closely related to these Eisenstein series. As an application, we then evaluate certain sums of modular symbols in a way which parallels past work of Goldfeld, O'Sullivan, Petridis, and Risager. In one case we find less cancellation in this sum than would be predicted by the common phenomenon of "square root cancellation", while in another case we find more cancellation. (Received September 12, 2019)

1154-11-990 **Adriana Salerno*** (asalerno@bates.edu) and **Ursula Whitcher**. *Hasse-Witt matrices and mirror toric pencils.*

Mirror symmetry predicts unexpected relationships between arithmetic properties of distinct families of algebraic varieties. For example, Wan and others have shown that for some mirror pairs, the number of rational points over a finite field matches modulo the order of the field. In this talk, we obtain a similar result for certain mirror pairs of toric varieties. We use recent results by Huang, Lian, Yau and Yu describing the relationship between the Picard-Fuchs equations and the Hasse-Witt matrix of these varieties, which encapsulates information about the number of points. The result allows us to compute the number of points modulo the order of the field explicitly, and we illustrate this by computing K3 surface examples related to hypergeometric functions. (Received September 12, 2019)

1154-11-1089 **Nicolas Allen Smoot*** (nsmoot@risc.uni-linz.ac.at) and **Cristian-Silviu Radu**. *Verifying Partition Congruences with Symbolic Computation.* Preliminary report.

A key problem in studying a conjectured infinite family of partition congruences is the difficulty of checking all but the smallest individual cases. Generally, such families necessitate the examination of exponentially large integers, which are input into arithmetic functions that already exhibit subexponential growth. Recent computational tools developed at the Research Institute for Symbolic Computation permit a substantial amount of evidence to be collected for a conjectured infinite family of congruences by more efficiently examining a large

number of individual cases. These tools also have utility in constructing some of the functions necessary for a complete proof, particularly when the associated modular curve has genus 1. We will present these results and demonstrate their usefulness for checking future families of congruences. (Received September 13, 2019)

1154-11-1093 **Max Alekseyev*** (maxal@gwu.edu) and **Neil Sloane** (njasloane@gmail.com). *Combinatorics of Kaprekar's Junction Numbers*. Preliminary report.

A base b junction number u has the property that there are at least two ways to write it as $u = v + s(v)$, where $s(v)$ is the sum of the digits in the expansion of the number v in base b . For the base 10 case, Kaprekar in the 1950's and 1960's studied the problem of finding $K(n)$, the smallest u such that the equation $u = v + s(v)$ has exactly n solutions. He gave the values $K(2) = 101$, $K(3) = 10^{13} + 1$, and conjectured that $K(4) = 10^{24} + 102$. In 1966 Narasinga Rao gave the upper bound $10^{11111111111124} + 102$ for $K(5)$, as well as upper bounds for $K(6)$, $K(7)$, $K(8)$, and $K(16)$. We will present a set of recurrences which determine $K(n)$ for any base b , and in particular we will show that these conjectured values of $K(n)$ are correct. Rather surprisingly, the solution to the base 2 problem is determined by the classical Thue-Morse sequence. For fixed b , it appears that $K(n)$ grows as a tower of height about $\log_2(n)$. (Received September 13, 2019)

1154-11-1100 **Dean Bisogno**, Fort Collins, CO 80523, **Wanlin Li** (wanleen@gmail.com), Cambridge, MA 02139, **Daniel Litt** (dlit@uga.edu), Athens, GA 30602, and **Padmavathi Srinivasan*** (padmavathi.srinivasan@uga.edu), Athens, GA 30602. *Obstructions to rational points from Galois actions on fundamental groups*. Preliminary report.

We study obstructions to rational points on curves coming from the Galois action on the fundamental group of the curve, with the goal of producing curves with no π_1 -section, and hence no rational points. In particular, such curves would give examples where the section conjecture holds. This is joint work in progress with Dean Bisogno, Wanlin Li and Daniel Litt. (Received September 16, 2019)

1154-11-1123 **David Grant*** (grant@colorado.edu). *Resultants of division polynomials and their derivatives, and singular torsion on elliptic curves*.

If ψ_m is the m^{th} division polynomial of an elliptic curve E , calculating the resultant of ψ_m and ψ_n for general E is tantamount to the theory of elliptic units, and calculating the resultant of ψ_m and its derivative gives information on the ramification in the m^{th} division field of E . We will discuss the arithmetic significance of the resultant of ψ_m and its second derivative with respect to an invariant differential, by relating it to the location on E of singular torsion points, a class of torsion points developed with John Boxall, which give an analogue of the Manin-Mumford conjecture for elliptic curves. Namely, if J is the generalized jacobian of E with modulus twice the origin on E (a \mathbb{G}_a -extension of E), then there is a section $\phi : E \rightarrow J$ such that the singular torsion points are the intersection $\phi(E) \cap J_{\text{tors}}$. If time permits we will discuss the genus 2 analogue. (Received September 13, 2019)

1154-11-1127 **Minsik Han*** (minsik_han@brown.edu). *Gleason-type polynomials for rational maps*. Preliminary report.

For the family of polynomial maps $\phi_c(z) = z^d + c$ on \mathbb{P}^1 parametrized by the variable c , the values of c such that ϕ_c is post-critically finite with a fixed dynamical portrait are the roots of a polynomial, which is called the Gleason polynomial. The irreducibility of Gleason polynomials in $\mathbb{Q}[c]$ has been much studied, but is still open in general. In this talk, we consider instead a 1-parameter family of rational maps. We construct and study the associated Gleason-type polynomials, including proving irreducibility in some cases. (Received September 13, 2019)

1154-11-1158 **Matthew Dolan Jobrack*** (matthew.jobrack@wsu.edu), 317 NW True Street, Pullman, WA 99163. *Non-vanishing of the Derivative of L-Functions at the Central Point*.

We show that a positive proportion of the derivatives of L-functions associated to holomorphic cusp forms of large weight do not vanish at the central point. Specifically, by optimizing the choice of quadratic form in the mollified moments of the L-function, we prove that this proportion is at least 39%. (Received September 13, 2019)

1154-11-1168 **Andrew Simoson*** (ajsimoson@king.edu), King University, 1350 King College Road, Bristol, TN 37620. *A gold version of Dirichlet's bronze approximation theorem*.

Call a reduced fraction $\frac{p}{q}$ *bronze*, *silver*, or *gold* with respect to an irrational number ω if $|\omega - \frac{p}{q}|$ is less than $\frac{1}{q^2}$, $\frac{1}{2q^2}$, and $\frac{1}{\sqrt{5}q^2}$, respectively. Dirichlet proved that there are an infinite number of bronze fractions for ω , and Hurwitz proved the same result with respect to gold. More recently, W. Bosma discovered a recursion generating

only silver or gold fractions. We present a similar recursion generating only gold fractions. (Received September 13, 2019)

1154-11-1180 **Michel L. Lapidus, Machiel van Frankenhuijsen and Edward K. Voskian*** (voskani@tcnj.edu). *Towards the Development of a Diffraction Measure for the Sets of Complex Dimensions of Nonlattice Self-Similar Fractal Strings via Lattice Approximation*. Preliminary report.

Lapidus and van Frankenhuijsen have established an intersection between their theory of complex dimensions, and the theory of mathematical quasicrystals in several books and papers. Following the work of Lagarias on diffraction by ideal crystals under a measure theoretic idealization of kinematic diffraction developed by Hof, a diffraction measure for the sets of complex dimensions of regular lattice self-similar fractal strings was developed in Voskian's Ph.D. thesis. In this talk, the latter result along with a conjecture by Lapidus towards an extension to the nonlattice case via lattice approximation is presented. (Received September 13, 2019)

1154-11-1196 **Giacomo Micheli***, Department of Mathematics, University of South Florida, Tampa, FL. *Optimal Locally Recoverable Codes via Galois Theory*.

We provide a Galois theoretical framework which allows to produce good polynomials for the Tamo and Barg construction of optimal locally recoverable codes (LRC). Our approach allows to prove existence results and to construct new good polynomials, which in turn allows to build LRCs with new parameters. The existing theory of good polynomials fits in our framework. (Received September 13, 2019)

1154-11-1197 **Heidi Goodson*** (heidi.goodson@brooklyn.cuny.edu). *Sato-Tate Groups of Trinomial Hyperelliptic Curves*.

Let $C_m : y^2 = x^m + c$ be a smooth projective curve defined over \mathbb{Q} . We would like to study the limiting distributions of the coefficients of the normalized L-polynomial for C_m . To determine the distributions, we study the Sato-Tate groups of the Jacobians of the curves. In this talk we give both general results and explicit examples of Sato-Tate groups for certain curves C_m . We will use these groups to determine the limiting distributions of the coefficients of the normalized L-polynomial. (Received September 13, 2019)

1154-11-1205 **Ben Kane and Jingbo Liu*** (jliu@tamusa.edu). *Universal sums of generalized m -gonal numbers*.

Conway–Schneeberger Fifteen Theorem states that a given positive definite integral quadratic form is universal (i.e., represents every positive integer) if and only if it represents all the positive integers up to 15. We are interested in generalizing this question to sums of generalized m -gonal numbers with positive coefficients:

$$f(x) = \sum_{j=1}^n a_j P_m(x_j)$$

where

$$P_m(x) := \frac{(m-2)x^2 - (m-4)x}{2}, \quad x \in \mathbb{Z}.$$

Let $\gamma(m)$ be the smallest positive integer such that f is universal if and only if every positive integer less than or equal to $\gamma(m)$ is represented by f . We have known that $\gamma(3) = \gamma(6) = 8$ and $\gamma(4) = 15$. Recently Ju and Oh have proven that $\gamma(8) = 60$. In this talk, we will approach this problem from both algebraic and analytic sides and determine an asymptotic upper bound, as a function of m , for $\gamma(m)$.

This is a joint work with Ben Kane. (Received September 13, 2019)

1154-11-1208 **Amita Malik*** (amita.malik@rutgers.edu). *Partitions and characters of symmetric groups*. Preliminary report.

In this talk, we focus on a certain problem concerning the positivity of characters of symmetric groups. Using partition theoretic interpretation, we discuss a question raised by R. Stanley. (Received September 13, 2019)

1154-11-1210 **Amita Malik*** (amita.malik@rutgers.edu). *Zeros of derivatives of the completed Riemann zeta function*.

In this talk, we discuss the distribution of zeros of derivatives of completed Riemann zeta function. Along the way, we prove a zero density result and an explicit formula. This is joint work with Arindam Roy. (Received September 18, 2019)

1154-11-1214 **Levent Alpoge*** (levent.alpoge@gmail.com), Fine Hall, Princeton, NJ 08540. *Rational points on hyperelliptic curves*. Preliminary report.

We discuss modularity of fake elliptic curves and (theoretical) computational implications for hyperelliptic curves. (Received September 13, 2019)

1154-11-1235 **Brendan Creutz**, Christchurch, New Zealand, and **Bianca Viray*** (bviray@uw.edu), Seattle, WA. *Quadratic points on quartic del Pezzo surfaces*. Preliminary report.

Results of Colliot-Thélène–Coray and Colliot-Thélène–Poonen together imply that a locally soluble quartic del Pezzo surface has *no* Brauer–Manin obstruction to the existence of points over every quadratic extension. Motivated by this, we explore constructions of quadratic points on such surfaces. This is joint work with Brendan Creutz. (Received September 14, 2019)

1154-11-1236 **Bianca Viray*** (bviray@uw.edu), Seattle, WA. *Local integral models of cubic del Pezzo surfaces with a nontrivial Brauer class*. Preliminary report.

It is an open question whether a cubic surface X with a 0-cycle of degree 1 has a rational point. Over a global field k , this is closely related to the question of whether a Brauer–Manin obstruction to the existence of k -points on X persists over any extension L/k of degree coprime to 3. By work of Bloch, Colliot-Thélène, Saito and Sato, persistence of the Brauer–Manin obstruction can be determined from the reduction type of cubic surfaces over local fields. We report on preliminary classification in this direction. (Received September 14, 2019)

1154-11-1242 **Max A Alekseyev*** (maxal@gwu.edu). *On partitions into squares of distinct integers whose reciprocals sum to 1*.

In 1963, Ron Graham proved that all integers greater than 77 (but not 77 itself) can be partitioned into distinct positive integers whose reciprocals sum to 1. He further conjectured that for any sufficiently large integer, it can be partitioned into squares of distinct positive integers whose reciprocals sum to 1. In this study, we present an algorithm for finding such partitions and establish the exact bound for their existence by proving that 8542 is the largest integer that does not have such a representation. (Received September 14, 2019)

1154-11-1256 **Michael J. Mossinghoff*** (mossinghoff@alumni.stanford.edu), Center for Communications Research, 805 Bunn Dr., Princeton, NJ 08540, and **Timothy S. Trudgian** (t.trudgian@adfa.edu.au), School of Science, UNSW Canberra at ADFA, Northcott Dr., Campbell, ACT 2610, Australia. *A Tale of Two Omegas*.

On average, would you expect there to be more positive integers $n \leq x$ with an even number of prime divisors, counting multiplicities, or with an odd number? What if instead one counted the number of *distinct* prime divisors of each integer? We investigate the two functions $L(x) = \sum_{n \leq x} (-1)^{\Omega(n)}$ and $H(x) = \sum_{n \leq x} (-1)^{\omega(n)}$, where $\Omega(n)$ counts the total number of prime divisors of n , including multiplicity, and $\omega(n)$ counts the number of distinct prime divisors of n . Studies of $L(x)$ date to work of Pólya from a century ago; the behavior of its oscillations is connected to the Riemann Hypothesis and related problems. The function $H(x)$ is less well studied. We describe some experimental investigations of these functions, and some surprising ways in which $H(x)$ and $L(x)$ qualitatively differ. In particular, we describe a method involving substantial computation that produces lower bounds on the oscillations exhibited by $L(x)$ and $H(x)$, as well as some other functions arising in number theory. (Received September 14, 2019)

1154-11-1318 **Nickolas Andersen** (nick@math.byu.edu) and **Kyle Pratt*** (kvpratt@gmail.com). *Open Problems in Analytic Number Theory I*.

We present several open problems in analytic number theory of wide interest to the audience of this special session, together with brief expositions of necessary background material. One goal of this presentation is to help bring researchers together in different sub-disciplines as they discover problems that interest them outside their usual fare. Some of these problems are explicit conjectures, while others are more of an open-ended nature. Both N. Andersen and K. Pratt will present during this talk. (Received September 17, 2019)

1154-11-1324 **Geoffrey Smith** and **Isabel Vogt***, Stanford University, Department of Mathematics, 450 Serra Mall, Building 380, Stanford, CA 94305. *Low degree points on curves*.

In this talk we will discuss an arithmetic analogue of the gonality of a curve over a number field: the smallest positive integer e such that the points of degree bounded by e are infinite. By work of Faltings, Harris–Silverman and Abramovich–Harris, it is well-understood when this invariant is 1, 2, or 3; by work of Debarre–Fahlaoui these criteria do not generalize to e at least 4. We will study this invariant using the auxiliary geometry of a surface containing the curve and devote particular attention to scenarios under which we can guarantee that this invariant is actually equal to the gonality. (Received September 14, 2019)

- 1154-11-1325 **Isabel Vogt***, Stanford University, Department of Mathematics, 450 Serra Mall, Building 380, Stanford, CA 94305. *A local-global principle for isogenies of elliptic curves of composite degree.*

Let E be an elliptic curve defined over a number field. In this talk, we will address the following question: if the reduction of E modulo almost all primes has a rational cyclic isogeny of degree N , is E forced to as well? Building on the work of Sutherland, Anni, and Banwait–Cremona when N is prime, we will focus on the case that N is composite. (Received September 14, 2019)

- 1154-11-1340 **Bjorn Poonen*** (poonen@math.mit.edu), MIT Department of Mathematics, 77 Massachusetts Ave., Bldg. 2-243, Cambridge, MA 02139-4307. *A p -adic approach to rational points on curves.*

In 1922, Mordell conjectured the striking statement that for a polynomial equation $f(x, y) = 0$, if the topology of the set of complex number solutions is complicated enough, then the set of rational number solutions is finite. This was proved by Faltings in 1983, and again by a different method by Vojta in 1991, but neither proof provided a way to provably find all the rational solutions, so the search for other proofs has continued. Recently, Lawrence and Venkatesh found a third proof, relying on variation in families of p -adic Galois representations; this is the subject of the lecture. (Received September 15, 2019)

- 1154-11-1362 **Jeff Achter***, Department of Mathematics, Colorado State University, Fort Collins, CO 80523-1874, and **S. Ali Altug**, **Luis Garcia** and **Julia Gordon**. *Equidistribution counts abelian varieties.*

An isogeny class of elliptic curves over a finite field is determined by a quadratic Weil polynomial. For each rational prime ℓ , one could compute how likely it is that a random ℓ -adic matrix has the specified characteristic polynomial, and compare this to the average among all characteristic polynomials. An irrationally exuberant interpretation of equidistribution might lead one to believe that the product, over all ℓ , of this quantity might somehow compute the size of the isogeny class. Gekeler actually proved that this relation holds.

I will explain a new, transparent proof of this formula, as well as its generalization to principally polarized abelian varieties of arbitrary dimension. (Received September 15, 2019)

- 1154-11-1371 **Kathrin Bringmann**, **Chris Jennings-Shaffer** and **Karl Mahlburg*** (mahlburg@math.lsu.edu), 320 Lockett Hall, Baton Rouge, LA 70802. *Statistics for unimodal sequences of integers.*

A unimodal sequence of integers is a sequence of positive integers that increase to a peak, and then decrease; there are many natural variants/subclasses: for example, the increasing parts might be required to increase strictly. I will discuss recent work on the combinatorial and asymptotic properties of such sequences, particularly focusing on the asymptotic behavior and asymptotic probabilistic distribution of the "rank" statistic. (Received September 15, 2019)

- 1154-11-1372 **Alina Bucur** (alina@math.ucsd.edu), **Francesc Fité*** (ffite@mit.edu) and **Kiran Kedlaya** (kedlaya@ucsd.edu). *Effective Sato-Tate conjecture for abelian varieties and applications.*

I will present an effective version of the Sato–Tate conjecture for an abelian variety A defined over a number field with connected Sato–Tate group that is derived from the generalized Riemann hypothesis. The "effectivity" refers to the obtaintion of an upper bound on the error term in the count predicted by the Sato-Tate measure that only depends on certain invariants of the Lie algebra of the Sato-Tate group of A . I will discuss three applications of this conditional result: an interval variant of Linnik's problem for an abelian variety, a sign variant of Linnik's problem for a pair of abelian varieties, and the determination (up to multiplication by a nonzero constant) of the asymptotic number of primes whose Frobenius trace attains the integral part of the Hasse–Weil bound when A is an elliptic curve with complex multiplication. This is a joint work with Alina Bucur and Kiran Kedlaya. (Received September 15, 2019)

- 1154-11-1407 **Shashika Nuwan Chamara Petta Mestri*** (pchama1@lsu.edu), 4728, Y.A. Tittle Ave, APT 32, Baton Rouge, LA 70820. *Ramanujan congruences for a class of eta-quotients.* Preliminary report.

The partition function $p_{[1^c \ell^d]}(n)$ can be defined using the generating function,

$$\sum_{n=0}^{\infty} p_{[1^c \ell^d]}(n) q^n = \prod_{n=1}^{\infty} \frac{1}{(1 - q^n)^c (1 - q^{\ell n})^d}.$$

In this talk, we prove infinite families of congruences for the partition function $p_{[1^c \ell^d]}(n)$ modulo powers of ℓ where $\ell = 5, 7$ for any integers c and d . We use Hecke operators, explicit basis of the vector space of modular functions of the congruence subgroup $\Gamma_0(\ell)$ and work of Atkin and Gordon on proving congruences for the partition function $p_{-k}(n)$. (Received September 15, 2019)

1154-11-1418 **Ae Ja Yee*** (yee@psu.edu). *Parity considerations in Rogers–Ramanujan–Gordon type overpartitions.*

In 2010, George Andrews investigated a variety of parity questions in partition identities. Some of his results are related to the Rogers-Ramanujan-Gordon identities. At the end of his paper, he raised a question if his work could be extended to overpartitions.

In 2013, Chen, Sang and Shi derived an overpartition analogue of the Rogers–Ramanujan–Gordon theorem. In this talk, I will discuss these Roger-Ramanujan-Gordon type overpartitions with additional parity restrictions. This is joint work with Dorian Sang and Diane Shi. (Received September 15, 2019)

1154-11-1437 **Dandan Chen** (ddchen@stu.ecnu.edu.cn), School of Mathematical Sciences, East China Normal University, Shanghai, Peoples Rep of China, **Rong Chen** (rchen@stu.ecnu.edu.cn), School of Mathematical Sciences, East China Normal University, Shanghai, Peoples Rep of China, and **Frank Garvan*** (fgarvan@ufl.edu). *Congruence mod powers of 5 and 7 for the rank and crank parity functions.* Preliminary report.

It is well known that Ramanujan conjectured congruences mod powers of 5, 7 and 11 for the partition function. These were subsequently proved by Watson (1938) and Atkin (1967). In 2009 Choi, Kang, and Lovejoy proved congruences mod powers of 5 for the crank parity function. The generating function for rank parity function is $f(q)$, the first function Ramanujan mentioned in his last letter as being a mock theta function. We prove congruences mod powers of 5 and 7 for the rank parity function as well as new congruences mod powers of 7 for the crank parity function. (Received September 15, 2019)

1154-11-1439 **Caleb J. Springer*** (cks5320@psu.edu). *Computing the endomorphism ring of an ordinary abelian surface over a finite field.*

The endomorphism ring of an abelian variety is an important object which is useful in many contexts, including understanding isogeny graphs, computing class polynomials, and other cryptographic applications. If A is a simple ordinary abelian variety over the finite field \mathbb{F}_q , then the endomorphism ring $\text{End}(A)$ is isomorphic to an order in a CM field K . In this talk, we will first review an algorithm of Bisson and Sutherland for computing the endomorphism ring of an ordinary elliptic curve, then present a generalization for certain simple ordinary abelian surfaces which have maximal real multiplication. Both algorithms are subexponential in $\log q$. The main idea is to probe the ideal class group of $\text{End}(A)$ by computing certain isogenies. By comparing the class groups of $\text{End}(A)$ and various known orders $\mathcal{O} \subseteq K$, one can then deduce $\text{End}(A)$. (Received September 15, 2019)

1154-11-1440 **Bernardo Bianco Prado***, bianc072@umn.edu, and **Kim Klinger-Logan**, kklingerlogan@ksu.edu. *Differential equations of infinite order and the zeta function.*

In Hilbert's 1900 address at the International Congress of Mathematicians, he claimed that the Riemann zeta function is not the solution of any algebraic ordinary differential equation on its region of analyticity. In a 2015 paper, Van Gorder addresses the question of whether the Riemann zeta function satisfies a non-algebraic differential equation. Van Gorder constructs a differential equation of infinite order that that Riemann zeta function satisfies. However, as he notes in the paper, this representation is clearly formal and Van Gorder does not attempt to claim a region or type of convergence. In our work we see that this operator applied to the Riemann zeta function does not in fact converge at any point and investigate what this means about Van Gorder's result. (Received September 15, 2019)

1154-11-1459 **Jennifer Berg*** (jsb047@bucknell.edu) and **Masahiro Nakahara** (mn634@bath.ac.uk). *Conic bundles over positive rank elliptic curves.* Preliminary report.

In this talk, we will explore the arithmetic of conic bundles $X \rightarrow E$ over elliptic curves of positive Mordell-Weil rank over a number field k . We will consider questions regarding the distribution of the rational points of X by examining the image of $X(k)$ inside of the rational points of the base elliptic curve E . In the process, we will mention a result on a local-to-global principle for torsion points on elliptic curves over the rational numbers. This is joint work with Masahiro Nakahara. (Received September 15, 2019)

1154-11-1491 **Marie Jameson*** (mjameso2@utk.edu) and **Sharon Garthwaite**. *Incongruences for modular forms and applications to partition functions.*

The study of arithmetic properties of coefficients of modular forms enjoys a rich history, including deep results regarding congruences in arithmetic progressions. Recently, work of S. Ahlgren, B. Kim, N. Andersen, and S. Löblich have employed the q -expansion principle of P. Deligne and M. Rapoport in order to determine more about where these congruences can occur. Here, we extend the method to give additional results in a large class of modular forms, and investigate the consequences of that result. (Received September 15, 2019)

1154-11-1498 **Edgar Costa*** (edgarc@mit.edu). *Variation of the rank of the group of algebraic cycles under reduction.*

We study the behavior of the rank of the group of algebraic cycles of abelian varieties and Calabi-Yau manifolds over \mathbb{Q} under reduction modulo primes. We compute these ranks for reductions of representative examples and investigate the resulting statistics. (Received September 15, 2019)

1154-11-1500 **Edgar Costa*** (edgarc@mit.edu). *Computing central endomorphisms of an abelian variety via reductions modulo p .*

We show that the center of the endomorphism ring of an abelian variety defined over a number field can be recovered from an appropriate intersection of the fields obtained from its Frobenius endomorphisms, under the Mumford-Tate conjecture. We then apply this result to exhibit a practical algorithm to compute this center. Jointly work with Davide Lombardo and John Voight. (Received September 15, 2019)

1154-11-1512 **Lian Duan*** (l.duanzww@gmail.com). *Comparison of 3-dimensional Galois representations by computational method.*

The Faltings-Serre method is an effective method used in comparison of Galois representations. It is widely used to verify the equivalence between Galois representations induced by elliptic curves or modular forms. However, besides of examples of GL_2 representations, there are not many applications of this method for higher dimensional representations due to the increase of complexity and the limit of current hardware. In this talk, we will discuss an improvement of this method for GL_3 Galois representations over a family of quadratic extensions. As an application, we give a computational proof of one specific case of the conjecture of van Geemen and Top. (Received September 16, 2019)

1154-11-1549 **Scott Ahlgren*** (sahlgren@illinois.edu), Department of Mathematics, 1409 W. Green St., Urbana, IL 61801, and **Byungchan Kim**. *The smallest parts function.*

Many of the interesting properties of the ordinary partition function arise from the fact that its generating function is a modular form on the full modular group. I will discuss the analogous “smallest parts function,” whose generating function is a mock modular form on the same group. I will describe some recent results on the smallest parts function; in particular I will discuss a result which shows that its congruences are rare in a certain precise sense. (Received September 16, 2019)

1154-11-1552 **Kubra Nari*** (nari15@itu.edu.tr), **Enver Ozdemir** (ozdemiren@itu.edu.tr), **Neslihan Aysen Ozkirci** (aozk@yildiz.edu.tr) and **Elif Segah Oztas** (esoztas@kmu.edu.tr). *Computing Roots in Finite Fields*. Preliminary report.

Let \mathbb{F} be a finite field and p be a prime integer. For an element a in \mathbb{F} finding a p^{th} root of a (if exists) is an active research topic in computational number theory. The problem has been long investigated for $p = 2$ and several theoretical and practical solutions were presented. For $p = 3$ the well known Lucas sequences were employed. In this research, we present a method of finding p^{th} root of elements in \mathbb{F} finite field for $p = 3, 5, 7$. We exploit elliptic curves and their corresponding torsion subgroups to construct an efficient algorithm in practice. (Received September 16, 2019)

1154-11-1600 **Jiakun Pan*** (jpan@math.tamu.edu), 227 Blocker Building, Ireland St., College Station, TX 77843-3368, and **Matthew P. Young**. *Quantum Unique Equidistribution conjecture for Eisenstein series in the level aspect.*

We study Eisenstein series on growing levels with general central characters, and find an asymptotic formula for their mass distribution, which we call Quantum Unique Ergodicity (**QUE**). In addition, as an analogy of the t -aspect small scale QUE for Eisenstein series by Young and Humphries separately, we consider above formula under weaker assumptions for the test functions. As a result, complications occur, to our surprise...

As a variation of the QUE conjecture raised by Rudnick and Sarnak, our research extends previous work of Kowalski, Michel, and VanderKam, Holowinsky and Soundararajan, Nelson, Pitale, and Saha, and Koyama,

among many other authors. Our estimation for the error terms applies the subconvexity bound of twisted L -functions by Blomer and Harcos. (Received September 16, 2019)

1154-11-1619 **Bella Tobin*** (bella.tobin@okstate.edu). *Post-Critically Finite Bicritical Polynomials over Number Fields*. Preliminary report.

Unicritical polynomials have been studied extensively in dynamics. A natural next step is to consider polynomials with two finite critical points, namely bicritical polynomials. I present results on the family of bicritical polynomials over a number field K , including results on post-critically finite bicritical polynomials over \mathbb{Q} . (Received September 16, 2019)

1154-11-1650 **Jeremy Lovejoy*** (lovejoy@math.berkeley.edu) and **Robert Osburn**. *The colored Jones polynomial of the odd double twist knots*.

Let $K_{(m,p)}$ be the family of double twist knots where $2m - 1$ and $2p$ are non-zero integers denoting the number of half-twists in each region. We find two types of q -hypergeometric formulas for the colored Jones polynomial of $K_{(m,p)}$. The case $m = 0$ recovers a result of Hikami for torus knots. The main tools are work of Takata and Walsh combined with the Bailey chain. As applications we find “quantum” q -series identities; that is, identities which hold only at roots of unity. If time permits, we will discuss the possibility of using our formulas to compute the tails of the colored Jones polynomials, leading to q -series identities in the classical sense. (Received September 16, 2019)

1154-11-1653 **Peter Illig, Rafe Jones*** (rfjones@carleton.edu), **Eli Orvis, Yukihiro Segawa** and **Nick Spinale**. *Newly Reducible Polynomial Iterates*.

A curious property of the minimal polynomial $f(x) = x^2 - x - 1$ of the golden ratio is that its second iterate is irreducible over \mathbb{Q} but its third iterate is not; we say that f has newly reducible third iterate over \mathbb{Q} . Given integers d, n that are at least two, it is interesting to ask whether there exists $f \in \mathbb{Q}[x]$ of degree d with newly reducible n th iterate, or equivalently whether the absolute Galois group of \mathbb{Q} acts transitively on the roots of the $(n - 1)$ st iterate of f but not on those of the n th iterate. We give results on this question, and study the collection of fields K for which infinitely many $f \in K[x]$ of degree d have newly reducible n th iterate. We pose several conjectures and questions. (Received September 16, 2019)

1154-11-1672 **Jordan Ellenberg*** (ellenber@math.wisc.edu), **Matthew Satriano** and **David Zureick-Brown**. *Heights of rational points on stacks*.

The theory of rational points on stacks over global fields is just as interesting as the theory of rational points on varieties, and handles many examples the classical theory does not; for instance, a finite extension of \mathbb{Q} can be seen as a rational point on the classifying stack of a finite group. But one crucial element is missing – a notion of height. We define such a notion, explain how it recovers many classical notions of “complexity” in arithmetic, and speculate about the asymptotics of counting points on stacks of bounded height. (Received September 16, 2019)

1154-11-1722 **Daniel Rayor Hast*** (drhast@bu.edu). *Rational points on varieties and the unipotent Albanese map*. Preliminary report.

By Faltings’ theorem, any curve of genus at least 2 over a number field has only finitely many rational points. Provably computing the set of all rational points remains a major open problem. We will survey recent progress and ongoing work using the Chabauty–Kim method, which uses the fundamental group to construct p -adic analytic functions that vanish on the set of rational points. In particular, we present a new proof of Faltings’ theorem for superelliptic curves over the rational numbers (due to joint work with Jordan Ellenberg), and a conditional generalization of the Chabauty–Kim method to number fields and higher dimensions. (Received September 16, 2019)

1154-11-1743 **Alexander D Smith*** (adsmith@math.harvard.edu). *2^k -Selmer groups and Goldfeld’s conjecture*.

Take E to be an elliptic curve over a number field whose four torsion obeys certain technical conditions. In this talk, we will outline a proof that 100% of the quadratic twists of E have rank at most one. To do this, we will find the distribution of 2^k -Selmer ranks in this family for every $k > 1$. Using this framework, we will also find the distribution of the 2^k -class ranks of the imaginary quadratic fields for all $k > 1$. (Received September 16, 2019)

1154-11-1760 **Andrew G Earnest** and **Lakshika Gunawardana*** (laksh.gg@siu.edu). *A Primitive Counterpart to the Fifteen Theorem.*

In 1993, J.H. Conway and W.A. Schneeberger presented the Fifteen Theorem, which provides simple criteria to determine whether a positive definite classically integral quadratic form in any number of variables is universal. Later in 2000, M. Bhargava provided a refinement of the Fifteen Theorem and showed that there are exactly 204 positive definite classically integral quaternary quadratic forms, up to equivalence, which are universal. We try to determine which of the forms in the 204 list are primitively universal, and try to determine whether there exists a finite set S of integers such that every positive definite integral quadratic form that primitively represents the integers in S , primitively represents all positive integers. In this talk, we introduce a conjecture which could be a primitive counterpart to the Fifteen Theorem.

(Received September 16, 2019)

1154-11-1771 **Anthony Várilly-Alvarado*** (av15@rice.edu), Rice University MS 136, 6100 S. Main St, Houston, TX 77005. *The geometric disposition of Diophantine Equations.*

Which integers can be expressed as a sum of three cubes? Is there a box such that the distance between any two of its corners is a rational number? Is there a 3×3 magic square of squares? These and many other questions can be phrased mathematically as diophantine problems. The properties of the geometric avatars corresponding to these diophantine problems help understand why these problems are so difficult, and why there almost certainly isn't an elementary way to answer the original questions.

In this talk I will survey the idea that geometric classification theorems provide a good lens for use in the study of diophantine equations, and explain concrete instances in my own research where geometric insights in the theory of algebraic surfaces have led a better understanding of certain diophantine equations. (Received September 16, 2019)

1154-11-1780 **Elena Fuchs, Katherine E Stange*** (kstange@math.colorado.edu) and **Xin Zhang**. *Local-global principles in circle packings.*

We generalize work of Bourgain-Kontorovich and Zhang, proving an almost local-to-global property for the curvatures of certain circle packings, to a large class of Kleinian groups. Specifically, we associate in a natural way an infinite family of integral packings of circles to any Kleinian group $A \leq \mathrm{PSL}_2(K)$ satisfying certain conditions, where K is an imaginary quadratic field, and show that the curvatures of the circles in any such packing satisfy an almost local-to-global principle. A key ingredient in the proof of this is that A possesses a spectral gap property, which we prove for any infinite-covolume, geometrically finite, Zariski dense Kleinian group in $\mathrm{PSL}_2(\mathcal{O}_K)$ containing a Zariski dense subgroup of $\mathrm{PSL}_2(\mathbb{Z})$. (Received September 16, 2019)

1154-11-1783 **Alexander D Smith*** (adsmith@math.harvard.edu). *Distributions of 2-Selmer groups in quadratic twist families.*

Given an elliptic curve over a number field obeying certain technical conditions, we prove that 100% of the quadratic twists of the curve have rank at most one. To do this, we find the distribution of 2^k -Selmer ranks in this family for $k \geq 1$. Previously, our approach relied on Kane's work on the distribution of 2-Selmer groups as a base case, and this meant that our theorems only applied to elliptic curves over \mathbb{Q} with full rational 2-torsion. In this talk, we will give a generalization of Kane's work to more general classes of elliptic curves. We will especially focus on the roles of the Erdős-Kac theorem and the large sieve in the proofs of these results. (Received September 16, 2019)

1154-11-1784 **Renee Hyunjeong Bell*** (rhbell@math.upenn.edu), 209 S 33rd St, Philadelphia, PA 19104. *Local-to-Global Extensions for Wildly Ramified Covers of Curves.*

Given a Galois cover of curves $X \rightarrow Y$ with Galois group G which is totally ramified at a point x and unramified elsewhere, restriction to the punctured formal neighborhood of x induces a Galois extension of Laurent series rings $k((u))/k((t))$. If we fix a base curve Y , we can ask when a Galois extension of Laurent series rings comes from a global cover of Y in this way. Harbater proved that over a separably closed field, every Laurent series extension comes from a global cover for any base curve if G is a p -group, and he gave a condition for the uniqueness of such an extension. Using a generalization of Artin-Schreier theory to non-abelian p -groups, we fully characterize the curves Y for which this extension property holds and for which it is unique up to isomorphism, but over a more general ground field. We also use our explicit characterization of Galois covers of curves to work toward a characterization of ramification filtrations of p -group extensions. (Received September 16, 2019)

1154-11-1790 **Katherine E Stange*** (kstange@math.colorado.edu). *Numberscope*. Preliminary report. I will describe and demonstrate Numberscope, a new tool in development for exploring and interacting with integer sequences visually. Mathematicians and citizen scientists can explore their favourite sequences by utilizing interactive online tools that provide visualizations highlighting various properties such as growth, factorizations, residues, self-similarity, etc. The project has had contributions from many authors, including undergraduate students, as part of the University of Colorado, Boulder, Experimental Mathematics Lab. (Received September 16, 2019)

1154-11-1831 **Robin Zhang*** (rzhang@math.columbia.edu). *The Galois-dynamics correspondence for unicritical polynomials*.

We study a correspondence between Galois actions and dynamical actions on periodic points of the polynomial $\phi(z) = z^d + c$ with d an integer greater than 1 and c a rational number. In particular, this correspondence exists for almost all rational c by a form of Hilbert's irreducibility theorem. When K is a quadratic number field and $d = 2$, this correspondence gives a criterion for the nonexistence of K -rational 5-cycles of $z^2 + c$ and for the complete determination of K -rational 6-cycles of $z^2 + c$. (Received September 16, 2019)

1154-11-1840 **Andrew Bridy*** (andrew.bridy@yale.edu), Yale University, PO Box 208301, New Haven, CT 06520. *The Arakelov-Zhang pairing and Julia sets*.

The Arakelov-Zhang pairing $\langle \psi, \phi \rangle$ is a measure of the dynamical distance between two rational maps ψ and ϕ over a number field K , defined in terms of local integrals on Berkovich space at each completion of K . We obtain a simple expression for the important case of the pairing with a power map, which may be interpreted as a limiting height of generic preimages. The expression is in terms of integrals over Julia sets; under certain disjointness conditions on Julia sets, it simplifies to a single canonical height term (in general, this term is a lower bound). This is joint work with Matt Larson. (Received September 16, 2019)

1154-11-1841 **David Pollack*** (dpollack@wesleyan.edu) and **Avner Ash**. *Computations of $GL(3)$ eigencurves*. Preliminary report.

Let f be a Hecke eigenclass in the cohomology of a congruence subgroup of SL_3 with classical weight λ coefficients, and let Φ be a lift of f to a cohomology class with coefficients in a certain module \mathbf{D}_λ of p -adic distributions. Assuming Φ is ordinary at p , or numerically non-critical at p and appearing in a 1-dimensional generalized eigenspace, Φ is known to deform along a 1-dimensional (mod twisting) eigencurve. We will discuss ongoing calculations to study the projection of that curve to weight space, and in particular to compute the mod- p reduction of its tangent line. (Received September 16, 2019)

1154-11-1844 **Robert J. Lemke Oliver, Sunrose Shrestha and Frank Thorne***, thorne@math.sc.edu. *Asymptotic identities for additive convolutions of sums of divisors*. Preliminary report.

In a 1916 paper, Ramanujan defined an "additive convolution sum" $S_{r,s}(n)$, proved an asymptotic formula for it for all positive integers $r, s > 1$, and obtained exact formulas in several cases – connected to the theory of modular forms. He also conjectured that his formula should hold for all real $r, s > 1$.

Ramanujan's conjecture was proved in 1927 by Ingham. We will sketch two more proofs of his conjecture. The first proof is quite simple, and improves Ingham's error term to a power savings. The second proof involves the machinery of L-functions, further improves the error terms, and obtains a secondary main term for some ranges of the parameters. (Received September 16, 2019)

1154-11-1860 **Travis Scholl*** (traviswscholl@gmail.com). *Super-Isolated Abelian Varieties*.

We call an abelian variety over a finite field *super-isolated* if its (rational) isogeny class contains a single isomorphism class. In this talk, we will search for super-isolated abelian varieties over finite fields using Honda-Tate theory. This reduces to looking for algebraic integers with certain properties. Counting such varieties relates to classical conjectures on prime values of polynomials. (Received September 16, 2019)

1154-11-1874 **Andrew R. Booker and Andrew V. Sutherland*** (drew@math.mit.edu). *Life, the universe, and everything*.

A 1992 conjecture of Heath-Brown states that every integer not congruent to 4 or 5 modulo 9 can be expressed as a sum of three cubes in infinitely many different ways. But finding even one representation of such an integer as a sum of three cubes can be very difficult, and there are several small integers for which no such representation is known, despite extensive searching. In this talk I will describe the computational methods we used to find new representations of some small integers as sums of three cubes, including the integer 42. (Received September 16, 2019)

1154-11-1898 **Soumya Sankar*** (ssankar3@wisc.edu). *Proportion of ordinary curves in some families.* Let k be a perfect field of characteristic p . An abelian variety A over k is said to be ordinary if its p -torsion, $A[p](\bar{k})$ is the largest possible. We ask the following question: what is the probability that a curve has ordinary Jacobian? We answer this for certain families of curves, including Artin-Schreier and superelliptic curves. (Received September 16, 2019)

1154-11-1900 **Andrew Bridy*** (andrew.bridy@yale.edu), Yale University, PO Box 208301, New Haven, CT 06520. *Automatic sequences and curves over finite fields.*

An amazing theorem of Christol states that a power series over a finite field is an algebraic function if and only if its coefficient sequence can be produced by a finite automaton. The proof uses combinatorics and linear algebra, but hidden in the theorem there is geometric information about a curve that contains the series in its function field. I make this explicit by demonstrating a precise link between the complexity of the automaton and the geometry of the curve. (Received September 16, 2019)

1154-11-1906 **Daniel Garbin*** (dgarbin@qcc.cuny.edu), Dept. of Mathematics and Computer Science, Queensborough Community College, CUNY, 222-05 56th Avenue, Bayside, NY 11364. *Eisenstein series at marked points approaching a cusp.*

In this research project we investigate the behavior of the non-holomorphic real-analytic Eisenstein series associated to marked points on a non-compact hyperbolic Riemann surface of finite volume. Namely, we show that if a marked point approaches a cusp, then a multiple of the elliptic Eisenstein series associated to the marked point converges to the parabolic Eisenstein series associated to a cusp. (Received September 16, 2019)

1154-11-1924 **Robert L Benedetto, Dragos Ghioca, Jamie Juul*** (jjjuul@math.ubc.ca) and **Thomas J Tucker**. *Arboreal Galois representations of PCF quadratic polynomials.* Preliminary report.

Let K be a number field and $f(x) \in K[x]$ be a quadratic post-critically finite polynomial. We study the Galois groups $\text{Gal}(K(f^{-n}(t))/K(t))$. The case t is transcendental over K was studied in previous unpublished work of Pink; we give new proofs of his results. We also examine the case $t \in K$. This is joint work with Rob Benedetto, Dragos Ghioca, and Tom Tucker. (Received September 16, 2019)

1154-11-1941 **Sayok Chakravarty*** (s7chakra@ucsd.edu) and **Aitzin Cornejo-Reynoso**. *Extremal and Probabilistic Results for Sum-Difference Sidon Sets.*

Let $\mathcal{A} \subset [N]$. For $1 \leq i \leq n$, define

$$\sigma(i) := |\{(a, b) \in \mathcal{A}^2 : a \leq b, a + b = i\}|$$

and

$$\delta(i) := |\{(a, b) \in \mathcal{A}^2 : a \leq b, b - a = i\}|$$

\mathcal{A} is called Sidon if $\sigma(i) \leq 1$ for all i . We say a set \mathcal{A} is $\text{SD}_{2,g}$ if $\sigma(i) + \delta(i) \leq g$ for all i . In 1932 Hungarian mathematician Simon Sidon posed the following question: how dense can a $\text{SD}_{2,g}$ set be? We have shown if $\mathcal{A} \subset [N]$ is $\text{SD}_{2,g}$, then

$$|\mathcal{A}| \leq \sqrt{gN} + \left(\frac{1}{2}\sqrt{g} + \frac{1}{2}\right) \sqrt[4]{N} + 1$$

We also consider the related probabilistic problem for $\text{SD}_{2,2}$ sets. Let $A \subset [N]$ be a random subset in which each element of $[N]$ is chosen for membership in A with probability p . We find a threshold for a random set being $\text{SD}_{2,2}$ with probability that tends to 1/0 as $N \rightarrow \infty$.

We have found similar results for random sets $\mathcal{A} \subset [N]$ such that any $j \in \left[\frac{N}{2}, \frac{5N}{6}\right]$ can be represented as a sum and difference of two elements from \mathcal{A} in at least 1 way. (Received September 16, 2019)

1154-11-1946 **Nickolas Andersen*** (nick@math.byu.edu) and **Kyle Pratt** (kvpratt@gmail.com). *Open Problems in Analytic Number Theory II.*

We present several open problems in analytic number theory of wide interest to the audience of this special session, together with brief expositions of necessary background material. One goal of this presentation is to help bring researchers together in different sub-disciplines as they discover problems that interest them outside their usual fare. Some of these problems are explicit conjectures, while others are more of an open-ended nature. Both N. Andersen and K. Pratt will present during this talk. (Received September 16, 2019)

1154-11-1951 **Jen Paulhus*** (paulhus@math.grinnell.edu). *A database of group actions on Riemann surfaces.*

The automorphism group G of a Riemann surface X and the monodromy of the corresponding branched cover $X \rightarrow X/G$ are useful in studying subgroups of the mapping class group, attacking inverse Galois theory questions, and decomposing Jacobian varieties. Various mathematicians have created code to generate such groups and monodromy for relatively low genus. We discuss work done to unify this data, add additional information, and make it broadly accessible on the L-functions and Modular Forms Database. (Received September 16, 2019)

1154-11-1956 **Keerthi S Madapusi Pera*** (madapusi@bc.edu), Maloney Hall, Dept of Mathematics, 140 Commonwealth Ave, Boston College, Chestnut Hill, MA 02467. *Modularity of generating series of cycles on orthogonal Shimura varieties.* Preliminary report.

Starting from the seminal work of Hirzebruch-Zagier and Gross-Kohnen-Zagier, it has been observed that special cycles formed by sub-Shimura varieties can sometimes be organized into generating series that turn out to be modular forms. This area now forms part of S. Kudla's wideranging program on special cycles on Shimura varieties of orthogonal type. At this point, the modularity is essentially known in the generic fiber due to work of Borcherds, Bruinier-Raum and others.

However, for applications to arithmetic intersections, it is important to also know this for the Arakelov version of this cycles, which, among other things, involves knowing something about the integral models of Shimura varieties, and the behavior of these cycles with respect to these models.

In this talk, we will present some recent results, joint with B. Howard, on the definition and modularity of these generating series on the integral models of Shimura varieties of orthogonal type (Received September 16, 2019)

1154-11-2027 **Abbey Bourdon, Ozlem Ejder*** (ejder@math.colostate.edu), **Yuan Liu, Frances Odumodu and Bianca Viray.** *Sporadic Points on Modular Curves.*

We call a closed point x on $X_1(n)$ sporadic if there are only finitely many closed points of degree at most $\deg(x)$; hence classifying sporadic points on $X_1(n)$ is closely related to determining the torsion subgroups of elliptic curves over a degree d field. When $d = 1$ or 2 , Mazur and Kamienny's work show that there are no sporadic points of degree d on $X_1(n)$. In this talk, I will focus on the sporadic points with rational j -invariants. This is joint with A. Bourdon, Y. Liu, F. Odumodu and B. Viray. (Received September 17, 2019)

1154-11-2083 **Michelle Manes*** (mmanes@math.hawaii.edu), Department of Mathematics, 2565 McCarthy Mall, Keller 401A, Honolulu, HI 96822, and **Bella Tobin.** *Post-critically finite cubic polynomials.* Preliminary report.

We find all cubic post-critically finite (PCF) polynomials defined over \mathbb{Q} , up to conjugacy over $\mathrm{PGL}_2(\bar{\mathbb{Q}})$. The techniques involve finding normal forms for cubic polynomials that respect field of definition, adapting some techniques of Ingram on coefficient bounds, and a bit of Sage computation. The same ingredients allow us to tackle questions of potential good reduction for these functions. (Received September 17, 2019)

1154-11-2124 **Christelle Vincent*** (christelle.vincent@uvm.edu), 82 University Place, Burlington, VT 05405. *Computing hyperelliptic modular invariants from period matrices.* Preliminary report.

We define the modular invariants of a hyperelliptic curve to be the value of certain Siegel modular functions that correspond to classical invariants of hyperelliptic curves, evaluated at a period matrix of the Jacobian of the curve. In this talk, we discuss this correspondence between modular functions and invariants of curves, as well as certain computational considerations that arise when recognizing the invariants as algebraic numbers from their floating point approximation. This is joint work with Ionica, Kilicer, Lauter, Lorenzo Garcia, Massierer and Manzateanu. (Received September 17, 2019)

1154-11-2159 **Junehyuk Jung*** (junehyuk@gmail.com), Department of Mathematics, College Station, TX 77843. *On the sparsity of positive-definite automorphic forms.*

Roughly speaking, we say that an automorphic form is positive-definite if the corresponding automorphic L-function when restricted to the critical line is a positive-definite function. It is known that most of automorphic forms of small conductor are positive-definite, including the Riemann zeta function. In fact there are infinitely many positive-definite automorphic forms. Together with Sug Woo Shin, we however proved that in any reasonable family of automorphic forms, their natural density is in fact 0. I'm going to explain briefly what the idea of proof is. (Received September 17, 2019)

1154-11-2172 **Lydia Eldredge*** (laldredg@math.fsu.edu) and **Kate Petersen**. *Small Mahler Measure in Cubic Fields*. Preliminary report.

The Mahler measure of a polynomial in $\mathbb{Z}[x]$, is the product, taken over all roots r , of $\max\{1, |r|\}$ and the absolute value of its leading coefficient. We'll talk about the problem of determining the smallest Mahler measure of a primitive element in a cubic field. This smallest value is known to depend on the discriminant of the field by work of Silverman and Ruppert. We'll discuss an algorithm to determine this minimum value, our numerical findings, and bounds for this minimum for some families of cubic fields. (Received September 17, 2019)

1154-11-2231 **Susil Kumar Jena*** (susil_kumar@yahoo.co.uk), Plot No-170, Sector-5, Niladri Vihar, Chandrasekharapur, Bhubaneswar, Odisha 751021, India. *The Diophantine equation*
 $mA^2 + nB^4 = C^3$.

If m , n and k are three non-zero integers which are pair-wise co-prime such that $(m+n) = k^3$, then the Diophantine equation $mA^2 + nB^4 = C^3$ has an infinite number of integral solutions for (A, B, C) . In this paper, we apply a technique to generate these solutions. Again, we give the conditions when mA , nB and C are pair-wise co-prime. (Received September 17, 2019)

1154-11-2247 **Breanne Baker Swart, Kristen Beck, Susan Crook, Christina Eubanks-Turner, Helen G. Grundman*** (grundman@brynmawr.edu), **Laura Hall-Seelig, May Mei** and **Laurie Zack**. *Augmented Generalized Happy Functions: Deserts and Oases*.

For integers $c \geq 0$ and $b \geq 2$, the augmented generalized happy function, $S_{c,b} : \mathbf{Z}^+ \rightarrow \mathbf{Z}^+$, is defined by

$$S_{c,b} \left(\sum_{i=0}^n a_i b^i \right) = c + \sum_{i=0}^n a_i^2,$$

where $0 \leq a_i \leq b-1$ and $a_n \neq 0$. For a fixed b , a *desert* is a sequence of consecutive values of c for which $S_{c,b}$ has no fixed point. Similarly, an *oasis* is a sequence of consecutive values of c for which $S_{c,b}$ has at least one fixed point.

In this talk, I will discuss properties of the fixed points of these functions and results concerning the possible lengths of deserts and oases. (Received September 17, 2019)

1154-11-2289 **Melissa Dennison*** (mdenniso@bw.edu). *Bow Sequences*.

The bow sequences are a 2-parameter family of recursive sequences defined similarly to the Stern sequence. The recursions for the bow and Stern sequences are opposite, and thus the bow sequences are named for the opposite side of the boat. Many of the properties of the bow sequences are related to known properties of the Stern sequence. We will discuss sums and divisors of terms, and consider a couple of interesting cases in more detail. (Received September 17, 2019)

1154-11-2315 **Simone Sisneros-Thiry*** (thiry2@illinois.edu). *Quotients of integers with restricted digit sets mod b*.

We discuss some conjectures and results about integers which can be written as a quotient of elements with a restricted set of digits in base b :

$$\sum_{j=0}^n a_j b^j, a_j \in A; \quad A = \{0, a_1, \dots, a_r\}$$

For example, if $b = 3$ and $A = \{0, 1\}$, then every integer quotient can be written as $3^m \cdot n$, where $n \in \cup_{k=0}^{\infty} (\frac{2}{3}, \frac{3}{2}) 3^k$ and $n \equiv 1 \pmod{3}$. The smallest such integer which satisfies these conditions and does *not* have such a representation is 529. The proof involves the construction of digraphs (representing finite state automata). We will discuss other values of b and digit sets $\{0, a_1, \dots, a_r\}$. (Received September 17, 2019)

1154-11-2325 **Mckenzie West*** (westmr@uwec.edu), Mathematics, Hibbard Humanities Hall 508, 124 Garfield Avenue, Eau Claire, WI 54701. *Computing curves on surfaces*. Preliminary report.

The study of rational points on varieties is intrinsically linked to the geometry of the variety. In the case of surfaces, we wish to find the curves on the surface and their algebraic relationships. We will discuss the strategies and challenges to computing curves on surfaces using existing computational software, including a recent effort to implement a general cubic surface curve-finding algorithm in Sage. (Received September 17, 2019)

1154-11-2358 **Lola Thompson*** (lola.thompson@oberlin.edu) and **Harald Andres Helfgott**. *Summing $\mu(n)$: a faster elementary algorithm*.

Consider either of two related problems: determining the precise number $\pi(x)$ of prime numbers $p \leq x$, and computing the Mertens function $M(x) = \sum_{n \leq x} \mu(n)$, where μ is the Möbius function.

The two best algorithms known are the following:

- (1) An analytic algorithm (Lagarias-Odlyzko, 1987), with computations based on integrals of $\zeta(s)$; its running time is $O(x^{1/2+\epsilon})$.
- (2) A more elementary algorithm (Meissel-Lehmer, 1959 and Lagarias-Miller-Odlyzko, 1985; refined by Deléglise-Rivat, 1996), with running time about $O(x^{2/3})$.

The analytic algorithm had to wait for almost 30 years to receive its first rigorous, unconditional implementation (Platt), which concerns only the computation of $\pi(x)$. Moreover, in the range explored to date ($x \leq 10^{24}$), the elementary algorithm is faster in practice.

We present a new elementary algorithm with running time $O(x^{3/5} \log x)$ for computing $M(x) = \sum_{n \leq x} \mu(n)$. The algorithm should be adaptable to computing $\pi(x)$ and other related problems. This talk is based on joint work with Harald Andrés Helfgott. (Received September 17, 2019)

1154-11-2381 **Brandon Alberts*** (brandon.alberts@uconn.edu). *Counting Towers of Number Fields.*

Fix a number field K and a finite transitive subgroup $G \leq S_n$. Malle's conjecture proposes asymptotics for counting the number of G -extensions of number fields F/K with discriminant bounded above by X . A recent and fruitful approach to this problem introduced by Lemke Oliver, Wang, and Wood is to count inductively. Fix a normal subgroup $T \trianglelefteq G$. Step one: for each G/T -extension L/K , first count the number of towers of fields $F/L/K$ with $\text{Gal}(F/L) \cong T$ and $\text{Gal}(F/K) \cong G$ with discriminant bounded above by X . Step two: sum over all choices for the G/T -extension L/K . In this talk we discuss the close relationship between step one of this method and the first Galois cohomology group. This approach suggests a refinement of Malle's conjecture which gives new insight into the problem. We give the solution to step one when T is an abelian normal subgroup of G , and convert this into nontrivial lower bounds for Malle's conjecture whenever G has an abelian normal subgroup. (Received September 17, 2019)

1154-11-2435 **Vlad Matei*** (vmatei@uci.edu). *The average size of the automorphism group of a smooth projective hypersurface in the large degree limit.*

We will show that the average size of the automorphism group defined over \mathbb{F}_q of a smooth projective degree d hypersurface in $\mathbb{P}_{\mathbb{F}_q}^n$ is 1 in the large degree limit. We will also discuss some consequences for the cohomology of the moduli spaces of smooth projective hypersurfaces. (Received September 17, 2019)

1154-11-2459 **Holley Friedlander, Elena Fuchs, Piper H, Catherine Hsu, Katherine Sanden, Damaris Schindler and Katherine E Stange*** (kstange@math.colorado.edu). *Primes in Apollonian circle packings.* Preliminary report.

An Apollonian circle packing is a fractal arrangement of circles with disjoint interiors, generated by a simple geometric recursion. It is often the case that the curvatures of the circles are all integers, and it is natural to ask about this collection. It is conjectured that, subject to a congruence obstruction, the collection contains every sufficiently large admissible integer. We extend such investigations to certain subsets controlled by the circles of prime curvature. (Received September 17, 2019)

1154-11-2560 **Murilo Zanarella*** (muriloz@mit.edu), 151 Elm Street, Unit 1, Somerville, MA 02144. *On Howard's main conjecture and the Heegner point Kolyvagin system.*

Since Kolyvagin's groundbreaking work on the proof of the Birch and Swinnerton-Dyer conjecture for analytic ranks 0 and 1, the theory of Euler systems and Kolyvagin systems has been used extensively in the study of the arithmetic of elliptic curves. For instance, they play an important role in Iwasawa theory, where extensions of Kolyvagin's arguments have been used to prove a divisibility for several main conjectures.

This talk will focus on the relation between the Kolyvagin system of Heegner points of an elliptic curve and its anticyclotomic Iwasawa main conjecture. We will discuss how one can improve Howard's treatment of such Kolyvagin system to upgrade his divisibility for the main conjecture to the full equality. We will also deduce as a consequence of such improvement that the primitivity of the Heegner point Kolyvagin system is in fact equivalent to the anticyclotomic main conjecture and the p -indivisibility of certain Tamagawa factors. (Received September 17, 2019)

1154-11-2606 **Robert Schneider*** (robert.schneider@uga.edu), Athens, GA 30602, and **Andrew V. Sills.** *Analysis and combinatorics of partition zeta functions.*

We examine "partition zeta functions" analogous to the Riemann zeta function but summed over subsets of integer partitions. We prove an explicit formula for a family of partition zeta functions already shown to have nice properties — those summed over partitions of fixed length — which yields complete information about analytic continuation, poles and trivial roots of the zeta functions in the family. We then present a combinatorial proof of

the explicit formula, which shows it to be a zeta function analog of MacMahon's partial fraction decomposition of the generating function for partitions of fixed length. (Received September 17, 2019)

1154-11-2639 **Nathan G McNew*** (nmcnew@towson.edu), Department of Mathematics, 8000 York Road, Towson, MD 21286. *Counting primitive subsets and other statistics of the divisor graph of $\{1, 2, \dots, n\}$.*

A set of integers is primitive if no element of the set divides another. Let $Q(n)$ denote the count of the primitive subsets of the integers $\{1, 2, \dots, n\}$. Erdős and Cameron conjectured in 1990 that $Q(n) = c^{n+o(n)}$ for some constant c . This conjecture was proven in 2018 by Angelo, however his proof was not effective in that it gave no information as to the value of the constant c . We give a new proof of the fact that $Q(n) = c^{n+o(n)}$ which provides an algorithm to compute the value of this constant c to arbitrary precision, and also gives a much stronger error term than $o(n)$ in the exponent. The method developed can be applied to various related problems which can be stated in terms of the divisor graph. In particular it dramatically improves the error term in the estimates of Mazet and Chadozeau for the path cover number of the divisor graph of the integers from 1 to n . (Received September 17, 2019)

1154-11-2696 **Sara Chari*** (schari@bates.edu), ME, and **Angelica Babei**. *Metacommutation and ideals of prime norm in locally Eichler orders*. Preliminary report.

We study the metacommutation problem in locally Eichler orders. We first give a description of the set of principal left ideals of a given prime reduced norm and relate this set to the analogous set for each of the two maximal orders containing the given Eichler order. We then study the permutation of this set that arises from metacommutation and extend results on the cycle structure of this permutation to locally Eichler orders. (Received September 17, 2019)

1154-11-2708 **Paul Baginski*** (pbaginski@fairfield.edu), Fairfield University, Dept. of Mathematics, 1073 North Benson Rd., Fairfield, CT 06824, and **Jill Stifano**. *Abundant Numbers and Nonunique Factorization*.

An integer n is abundant if the sum of its divisors, $\sigma(n) = \sum_{d|n} d$, is greater than $2n$; n is perfect if $\sigma(n) = 2n$; and otherwise n is deficient. Both the set H of abundant numbers and the set H^* of non-deficient numbers are closed under multiplication, making them submonoids of (\mathbb{N}, \times) . As a result, we can consider how elements of H^* (or H) factor into irreducible elements of H^* (resp. H), a concept related to the previously studied idea of primitive non-deficient numbers. As it turns out, non-deficient numbers (or abundant numbers) do not factor uniquely into products of irreducible non-deficient numbers (resp. irreducible abundant numbers). We describe the factorization theory of these two monoids, demonstrating them to be particular cases of a wider class of extremal submonoids of (\mathbb{N}, \times) . Lastly, we consider questions about other arithmetic functions that will produce submonoids of (\mathbb{N}, \times) in this class. (Received September 17, 2019)

1154-11-2712 **Alexander J. Barríos*** (abarríos@carleton.edu), Carleton College, Department of Mathematics and Statistics, One North College St, Northfield, MN 55057. *Explicit Classification of Isogeny Classes of Rational Elliptic Curves*.

Let E be a rational elliptic curve and let \mathcal{I}_E be the isogeny class of E . Nitaj explicitly calculated \mathcal{I}_E for an elliptic curve E with torsion subgroup isomorphic to $\mathbb{Z}/9\mathbb{Z}$, $\mathbb{Z}/10\mathbb{Z}$, $\mathbb{Z}/12\mathbb{Z}$, or $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/8\mathbb{Z}$. In this talk, we extend Nitaj's work by explicitly computing \mathcal{I}_E for an elliptic curve E with a non-trivial cyclic isogeny defined over \mathbb{Q} . In particular, we explicitly find all possible isogeny graphs over \mathbb{Q} and provide an explicit Weierstrass model for each vertex in the graph. (Received September 17, 2019)

1154-11-2747 **Alvaro Cornejo*** (alvaro.j.cornejo@gmail.com), University of California, Santa Barbara, Department of Mathematics, South Hall, Room 6607, Santa Barbara, CA 93106, **Owen Ekblad** (oekblad@umich.edu), University of Michigan at Dearborn, Department of Mathematics and Statistics, 4901 Evergreen Rd 2002 CASL Building, Dearborn, MI 48128, **Marietta Geist** (geistm@carleton.edu), Carleton College, Department of Mathematics and Statistics, One North College St., Northfield, MN 55057, **Kayla Harrison** (kaylah98@yahoo.com), Mathematics, Physics, Computer Science, 4200 54th Avenue South, St. Petersburg, FL 33711, and **Abigail Loe** (loea@carleton.edu), Carleton College, Department of Mathematics and Statistics, One North College St., Northfield, MN 55057. *Minimal Discriminants of Rational Elliptic Curves with Specified Isogeny*.

By a rational elliptic curve, we mean a projective variety of genus 1 that admits a Weierstrass model of the form $y^2 = x^3 + Ax + B$ where A and B are integers. For a rational elliptic curve E , there is a unique quantity known as the minimal discriminant which has the property that it is the smallest integer (in absolute value) occurring

in the \mathbb{Q} -isomorphism class of E . In 1975, Hellegouarch showed that the elliptic curve $y^2 = x(x+a)(x-b)$ for relatively prime integers a and b comes equipped with an easily computable minimal discriminant. Recently, Barrios extended this result to all rational elliptic curves with non-trivial torsion subgroup. This project gives a classification of minimal discriminant for rational elliptic curves that admit a cyclic isogeny of degree $N = 5, 6, 7, 8, 9, 10, 13$. This work is part of PRiME (Pomona Research in Mathematics Experience) with assistance by Alex Barrios and Timothy McEldowney. This work was sponsored by the National Science Foundation (DMS-1560394). (Received September 17, 2019)

1154-11-2764 **Angelica Babei*** (angelica.babei@vanderbilt.edu), Nashville, TN 37212, and **Sara Chari**. *Metacommutation in quaternion orders and actions on the Bruhat-Tits tree*. Preliminary report.

The metacommutation problem in a quaternion order \mathcal{O} gives rise to a permutation of the left ideals of a given prime norm \mathfrak{p} . When \mathcal{O} is Eichler, we can interpret each permutation locally as an action on the Bruhat-Tits tree associated with the prime. We give a correspondence between the ideals of local norm \mathfrak{p} and a certain set of segments in the tree, and use this interpretation to describe the cycle structure of the metacommutation permutation. (Received September 17, 2019)

1154-11-2833 **Jing-Jing Huang*** (jingjingh@unr.edu). *Rational points close to a hypersurface and the dimension growth conjecture*.

This talk mainly concerns problems related to counting rational points in a thin neighborhood of a manifold. We have recently solved this problem in the cases of hypersurfaces and affine subspaces, and made some progress in the case of space curves. As an immediate application of the counting results for hypersurfaces, the dimension growth conjecture is established for smooth manifolds which are not flat. There are also significant applications of our results to diophantine inequalities and metric diophantine approximation on manifolds. (Received September 18, 2019)

12 ► Field theory and polynomials

1154-12-873 **Sarah Cobb, Michelle Knox, Marcos Lopez*** (marcos.lopez@msutexas.edu), **Terry McDonald** and **Patrick Mitchell**. *Distribution of Constant Terms of Irreducible Polynomials in $\mathbb{Z}_p[x]$* .

There is a bias in the number of irreducible polynomials over a finite field with a given constant term. In this talk, we will show explicit formulas for the number of monic irreducible polynomials of degree q^k over a finite field and prescribed constant term. We show that the number such polynomials depends only on whether the constant term is a residue in the underlying field. We further show that as k becomes large, the proportion of irreducible polynomials having each constant term is asymptotically equal. (Received September 11, 2019)

1154-12-1297 **Marc Chamberland*** (chamberl@grinnell.edu), Department of Mathematics and Statistics, Gri, Grinnell, IA 50112. *When are a Polynomial's Zeros all Real and Distinct?*

This talk gives necessary and sufficient conditions for all the zeros of a single-variable polynomial to be real and distinct. (Received September 14, 2019)

1154-12-1306 **Eduardo A Sala-Ramirez*** (eduardoabnel@gmail.com), Urbanización Santa Elvira, Calle Santa María G-18, Caguas, PR 00725, and **Dylan G Cruz-Fonseca**. *Binary Goppa Codes from Trace Polynomials*. Preliminary report.

Linear Codes are algebraic objects which are used in the digital communications. In particular, linear codes introduce a particular kind of redundancy in order to make communications reliable and resistant to errors. Several codes are constructed with powerful algebraic techniques. Binary Goppa codes are linear codes made from univariate polynomial evaluations. They have promising applications in Forward Error Correction and Cryptography. In this talk we study a class of Binary Goppa codes where the defining polynomial is a trace function of degree three. We show some improvements over previously known bounds on the dimension of the codes. (Received September 14, 2019)

1154-12-2647 **Gretchen L. Matthews*** (gmatthews@vt.edu), Department of Mathematics, Virginia Tech, Blacksburg, VA 24060. *Repairing codes and applications*.

In order to recover missing symbols in a codeword of an $[n, k]$ Reed-Solomon code, one can use any k available symbols together with polynomial interpolation. In doing so, the entire codeword can be determined. Recently, Guruswami and Wootters introduced repairing Reed-Solomon codes which allow for recovery of a single symbol

using traces, a method which may circumvent the required k symbols mentioned above. This work has been extended to other families of codes. In this talk, we consider applications of this technique. (Received September 17, 2019)

1154-12-2837 **Lucian Miti Ionescu*** (lmiones@ilstu.edu), Illinois State University, Fell Street, Bloomington, IL 61790-4520. *Arithmetic Galois Theory*. Preliminary report.

Arithmetic Galois Theory (AGT) is a Galois connection on cyclic groups which corresponds to classical Galois correspondents between subfields and subgroups in the cyclotomic case (Abelian Class Field Theory).

It is instructive to see how \mathbb{Z} -module theory point of view in AGT explains the decomposition of primes in Abelian Galois extensions.

This arithmetic-algebraic correspondence has an interpretation in Grothendieck's Anabelian Geometry, beyond the abelian case of algebraic de Rham cohomology. (Received September 18, 2019)

13 ► Commutative rings and algebras

1154-13-17 **Mohammad Obiedat*** (mohammad.obiedat@gallaudet.edu), 800 Florida Avenue NE, Washington, DC 20002. *Gradations of Cyclic Rings and Homogeneity of their Nil and Jacobson Radicals*.

A formula for the number of gradations, up to equivalence, of cyclic rings by cancellative monoids is given. As an application, the nil and Jacobson radicals of cyclic rings are shown to be homogeneous. (Received May 29, 2019)

1154-13-194 **Kuei-Nuan Lin*** (kul20@psu.edu) and **Yi-Huang Shen**. *On the conjecture of Vasconcelos for Artinian almost complete intersection monomial ideals*.

In this talk, we confirm a conjecture of Vasconcelos which states that the Rees algebra of any Artinian almost complete intersection monomial ideal is almost Cohen–Macaulay. This is joint work with Yi-Huang Shen. (Received August 21, 2019)

1154-13-322 **Irina Iliaea*** (iiliaea1@gsu.edu), 3200 Lenox Rd Ne, Apt F414, Atlanta, GA 30324. *On the Frobenius Complexity Sequence of Stanley-Reisner Rings*. Preliminary report.

The Frobenius complexity of a local ring R measures asymptotically the abundance of Frobenius actions of order e on the injective hull of the residue field of R . It is known that, for Stanley-Reisner rings, the Frobenius complexity is either $-\infty$ or 0. We will give a complete description of the Frobenius complexity sequence $c_e(R)$ for all values of e , when R is Stanley-Reisner, generalizing work of Álvarez Montaner, Boix and Zarzuela. Our result settles an open question mentioned by Álvarez Montaner in one of his papers. (Received August 31, 2019)

1154-13-403 **Monica Ann Lewis*** (malewi@umich.edu). *The local cohomology of a parameter ideal with respect to an arbitrary ideal*.

Let S be a complete intersection presented as R/J for R a regular ring and J a parameter ideal in R . Let I be an ideal containing J . It is well known that the set of associated primes of $H_I^i(S)$ can be infinite, but far less is known about the set of minimal primes. In 2017, Hochster and Núñez-Betancourt showed that if R has prime characteristic $p > 0$, then the finiteness of $\text{Ass } H_I^i(J)$ implies the finiteness of $\text{Min } H_I^{i-1}(S)$, raising the following question: is $\text{Ass } H_I^i(J)$ always finite? We give a positive answer when $i = 2$ but provide a counterexample when $i = 3$. The counterexample crucially requires $\text{Ass } H_I^2(S)$ to be infinite. The following question, to the best of our knowledge, is open: (under suitable hypotheses on R) does the finiteness of $\text{Ass } H_I^{i-1}(S)$ imply the finiteness of $\text{Ass } H_I^i(J)$? When S is a domain, we give a positive answer when $i = 3$. When S is locally factorial, we extend this to $i = 4$. Finally, if R has prime characteristic $p > 0$ and S is regular, we give a complete answer by showing that $\text{Ass } H_I^i(J)$ is finite for all $i \geq 0$. (Received September 03, 2019)

1154-13-426 **Aida Maraj*** (aida.maraj@uky.edu) and **Uwe Nagel**. *Filtrations of Ideals Arising from Hierarchical Models*. Preliminary report.

One can realize hierarchical models via a simplicial complex that describes the dependency relationships among random variables and a vector that records the number of states for each random variable. Moreover, Diaconis and Sturmfels have constructed toric ideals that provide useful information about the model. The goal of this talk is to study the invariance properties of families of ideals arising from hierarchical models with the same dependency relations among variables and varying number of states. We will see that these families of ideals

form a filtration under some group action, and we will compute an equivariant Hilbert series for some classes of such filtrations. (Received September 03, 2019)

1154-13-443 **Youngsu Kim*** (yk009@uark.edu), Fayetteville, AR 72701, and **Lance Edward Miller** and **Wenbo Niu**. *Generic Links of Determinantal Varieties.*

Linkage is a classical topic in algebraic geometry and commutative algebra. Fix an affine space A . We say two subschemes X, Y of A are linked if their union is a complete intersection in A and X and Y do not have a common component. Two linked subschemes share several properties in common. Linkage has been studied by various people, Artin-Nagata, Peskine-Szprio, Huneke-Ulrich, to name a few.

In 2014, Niu showed that if Y is a generic link of a variety X , then $\text{LCT}(A, X) \leq \text{LCT}(A, Y)$, where LCT stands for log canonical threshold. In this talk, we show that if Y is a generic link of a determinantal variety X , then X and Y have the same log canonical threshold. This is joint work with Lance E. Miller and Wenbo Niu. (Received September 04, 2019)

1154-13-591 **Chelsea Drescher*** (chelseadrescher@my.unt.edu) and **Anne V. Shepler**. *Invariants of modular reflection groups and (q, t) -binomial coefficients.*

Generalizations of Catalan numbers connect the Hilbert series of certain invariant spaces with the representation theory of rational Cherednik algebras for Coxeter and complex reflection groups. In 2017, Lewis, Reiner, and Stanton conjectured an analogous connection between the modular general linear group and (q, t) -binomial coefficients. We will discuss a solution to a local case of this conjecture. When the characteristic of the underlying field divides the order of the group, the subgroup fixing a reflecting hyperplane is a semi-direct product of diagonalizable reflections and transvections. We will describe the invariant ring for these Landweber-Stong groups reflecting about a fixed hyperplane acting on a polynomial ring modulo Frobenius powers. The resulting Hilbert series counts the number of orbits of the group acting on a vector space. (Received September 07, 2019)

1154-13-631 **Rimpa Nandi*** (rimpanandi0610@gmail.com), Nivedita Hall of Residence, IIT Kharagpur, Kharagpur, WestBengal 721302, India, and **Ramakrishna Nanduri**. *On Betti numbers of toric algebras of certain bipartite graphs.*

Suppose G be a finite simple graph with vertex set $V = \{x_1, x_2, \dots, x_n\}$ and edge set $E = \{e_1, e_2, \dots, e_q\}$. Let $S = K[x_1, \dots, x_n]$ and $R = K[T_1, \dots, T_q]$ be polynomial rings, where k is a field. Define a homomorphism $\phi : S \rightarrow R$, $\phi(T_i) = x_{i_1}x_{i_2}$ where $e_i = \{x_{i_1}, x_{i_2}\} \in E$. Then the $\ker(\phi)$ is known as the *toric ideal* of G and denoted by I_G and $k[G] = R/I_G$ is known as the toric algebra of G . In this talk, we compute the graded Betti numbers of toric algebras of certain bipartite graphs $G_{r,s,d}$. We achieve this by showing that the successive colon ideals of the initial ideal of $I_{G_{r,s,d}}$, $\text{in}_{<}(I_{G_{r,s,d}})$ with respect to reverse lex order are complete intersection ideals generated by d elements with at least $(d-1)$ elements of degree 1 and at most one element of degree $r-1$. Using this property we determine explicitly the graded Betti numbers of $\text{in}_{<}(I_{G_{r,s,d}})$. Also we show that $I_{G_{r,s,d}}$ and $\text{in}_{<}(I_{G_{r,s,d}})$ have the same graded Betti numbers. As a consequence, we explicitly determine the Castelnuovo-Mumford regularity, Hilbert series and multiplicity of $k[G_{r,s,d}]$. (Received September 09, 2019)

1154-13-648 **Zach Greif** and **Jason McCullough*** (jmccullo@iastate.edu). *Green-Lazarsfeld condition for toric edge ideals of bipartite graphs.*

Previously, Ohsugi and Hibi gave a combinatorial description of bipartite graphs G whose toric edge ideal I_G is generated by quadrics, showing that every cycle of G of length at least 6 must have a chord. This corresponds to the Green-Lazarsfeld condition \mathbf{N}_1 . In this paper, we investigate the higher syzygies of I_G and give combinatorial descriptions of the Green-Lazarsfeld conditions \mathbf{N}_p of toric edge ideals of bipartite graphs for all $p \geq 1$. In particular, we show that I_G is linearly presented (i.e. satisfies condition \mathbf{N}_2) if and only if the bipartite complement of G is a tree of diameter at most 3. We also investigate the regularity of linearly presented toric edge ideals and give criteria for polyomino ideals to satisfy the Green-Lazarsfeld conditions. (Received September 09, 2019)

1154-13-827 **Bidwan Chakraborty*** (bidwanc@gmail.com), B. R. Ambedkar Hall of Residence, IIT Kharagpur, Kharagpur, WestBengal 721302, India, and **Mousumi Mandal**. *Invariants of the symbolic powers of edge ideals.*

Let G be a graph and $I = I(G)$ be its edge ideal. When G is the clique sum of two different length odd cycles joined at single vertex then we give an explicit description of the symbolic powers of I and compute the Waldschmidt constant. When G is complete graph then we describe the generators of the symbolic powers of I and compute the Waldschmidt constant and the resurgence of I . Moreover for complete graph we prove that the Castelnuovo-Mumford regularity of the symbolic powers and ordinary powers of the edge ideal coincide. (Received September 11, 2019)

1154-13-834 **Erica Barrett, Emil Graf and Susan Loepp*** (sloepp@williams.edu), 33 Stetson Ct., Williams College, Williamstown, MA 01267, and **Kimball Strong and Sharon Zhang.** *The Prime Spectra of Precompletions.*

Let T be a complete local ring and X a partially ordered set. Under what conditions is there a local ring A whose completion is T and whose spectrum, when viewed as a partially ordered set, is X ? In this talk, we will discuss recent progress on this open question. (Received September 11, 2019)

1154-13-840 **Michael DiPasquale*** (michael.dipasquale@colostate.edu), Colorado State University, Department of Mathematics, 1874 Campus Delivery, Fort Collins, CO 80523. *A generalization of Wilf's Conjecture.*

Wilf's conjecture is a long standing open problem about the density of holes in a numerical semigroup, which is a submonoid of the natural numbers \mathbb{N} with finite complement. We propose a generalization of Wilf's conjecture for submonoids of \mathbb{N}^d with finite complement (called generalized numerical semigroups). We prove this conjecture for several large classes of generalized numerical semigroups, including irreducible, symmetric, and monomial classes.

We also discuss the relationship of our conjecture to a different generalization of Wilf's conjecture proposed by García-García, Marín-Aragón, and Vigneron-Tenorio. This is joint work with C. Cisto, G. Failla, Z. Flores, C. Peterson, and Rosanna Utano. (Received September 11, 2019)

1154-13-864 **Taran H Funk*** (taran.funk@huskers.unl.edu), 203 Avery hall, Lincoln, NE 69599, and **Thomas Marley.** *Frobenius and Homological Dimensions of Complexes.*

It is proved that a module M over a Noetherian local ring R of prime characteristic and positive dimension has finite flat dimension if $\mathrm{Tor}_i^R({}^e R, M) = 0$ for $\dim R$ consecutive positive values of i and infinitely many e . Here ${}^e R$ denotes the ring R viewed as an R -module via the e th iteration of the Frobenius endomorphism. In the case R is Cohen-Macaulay, it suffices that the Tor vanishing above holds for a single $e \geq \log_p e(R)$, where $e(R)$ is the multiplicity of the ring. This improves a result of D. Dailey, S. Iyengar, and the second author, as well as generalizing a theorem due to C. Miller from finitely generated modules to arbitrary modules. We also show that if R is a complete intersection ring then the vanishing of $\mathrm{Tor}_i^R({}^e R, M)$ for single positive values of i and e is sufficient to imply M has finite flat dimension. This extends a result of L. Avramov and C. Miller. (Received September 11, 2019)

1154-13-988 **Juliette Bruce***, 480 Lincoln Dr., Van Vleck Hall, Department of Mathematics, Madison, WI 53706. *Semi-Ample Asymptotic Syzygies.*

I will discuss the asymptotic non-vanishing of syzygies for products of projective spaces, generalizing the monomial methods of Ein-Erman-Lazarsfeld. This provides the first example of how the asymptotic syzygies of a smooth projective variety whose embedding line bundle grows in a semi-ample fashion behave in nuanced and previously unseen ways. (Received September 12, 2019)

1154-13-1122 **Linquan Ma*** (ma326@purdue.edu), 150 N University Street, West Lafayette, IN 47906. *Homological conjectures, perfectoid spaces, and singularities in all characteristic.*

The homological conjectures have been a focus of research in commutative algebra since 1960s. They concern a number of interrelated conjectures concerning homological properties of commutative rings to their internal ring structures. These conjectures had largely been resolved for rings that contain a field, but several remained open in mixed characteristic—until in 2016 Yves Andre proved Hochster's direct summand conjecture and the existence of big Cohen-Macaulay algebras, which lie in the heart of the homological conjectures. The main new ingredient in the solution is to systematically use the theory of perfectoid spaces, which leads to some further developments on related topics. Using integral perfectoid big Cohen-Macaulay algebras, we define mixed characteristic analogs of rational/F-rational and log terminal/F-regular singularities from equal characteristic, and they have applications to the study of singularities when the characteristic varies. We define analogs of multiplier ideals and adjoint ideals and prove adjunction type formulas, which lead to new forms of the Briançon-Skoda theorem in mixed characteristics (this is based on recent work with Karl Schwede, Kevin Tucker, Joe Waldron, and Jakub Witaszek). We give a brief survey on these results and methods. (Received September 13, 2019)

1154-13-1190 **Laura Felicia Matusevich** (laura@math.tamu.edu), Department of Mathematics, Mailstop 3368, Texas A&M University, College Station, TX 77843-3368, and **Aleksandra Sobieska*** (ola@math.tamu.edu), Department of Mathematics, Mailstop 3368, Texas A&M University, College Station, TX 77843-3368. *Toward Free Resolutions Over Scrolls.*

Free resolutions over the polynomial ring have a storied and active record of study. However, resolutions over other rings are much more mysterious; even simple examples can be infinite! In these cases, we look to any combinatorics of the ring to glean information. This talk will present a minimal free resolution of the ground field over the semigroup ring arising from rational normal 2-scrolls, and (if time permits) a computation of the Betti numbers of the ground field for all rational normal k -scrolls. (Received September 13, 2019)

1154-13-1276 **Rachel N. Diethorn*** (rngettin@syr.edu). *Generators of Koszul homology.*

One approach to understanding Koszul homology is to find, explicitly, its generators. In the first part of this talk, I will provide explicit formulas for the generators of Koszul homology on the minimal generators of an ideal J with coefficients in what we call a J -weak complete intersection module. This generalizes work of Herzog and of Corso, Goto, Huneke, Polini, and Ulrich. In the second part of this talk, I will demonstrate the utility of having such an explicit understanding of Koszul homology. In particular, I will discuss an application to the Koszul homology algebra of quotients by certain edge ideals. (Received September 14, 2019)

1154-13-1401 **Majed Zailaee*** (mz576413@ohio.edu), Ohio University, Morton Hall, 321, Athens, OH 45701, **Sergio Lopez** (lopez@ohio.edu), Ohio University, Morton Hall, 321, Athens, OH 45701, and **Rebin Muhammad** (rm775311@ohio.edu), Ohio University, Morton Hall, 321, Athens, OH 45701. *Two-option magmas.* Preliminary report.

Binary operations on the natural numbers spanned by their addition and multiplication. Let S be any set and $*$ and \circ be two arbitrary operations on S . An operation \star on S is said to be a two-option operation spanned by $*$ and \circ if for all $a, b \in S$, $a \star b \in \{a * b, a \circ b\}$. Any two-option operation may be represented by a graph having the elements of S as vertices and such that there is an edge between a and b precisely when $a \star b = a * b$. Two-option operations were motivated by graph magmas and two-valued magmas studied earlier in other projects. We are interested in learning what associative operations may be spanned by two given operations $*$ and \circ . Interestingly, $*$ and \circ need not be associative themselves to yield \star associative. As an initial experiment, we aim to produce an exhaustive list of associative two option operations on the set of natural numbers for $*$ and \circ being, respectively, the usual addition and multiplication of natural numbers.

(This is a preliminary report on work in progress with S. López-Permouth and R. Muhammad.) (Received September 15, 2019)

1154-13-1542 **Petter Andreas Bergh** and **Peder Thompson*** (peder.thompson@ntnu.no). *Relating matrix factorizations and totally acyclic complexes.*

There is a classic correspondence, due to Eisenbud and Buchweitz, between matrix factorizations of a nonzero element f in a regular local ring Q and totally acyclic complexes of projective $Q/(f)$ -modules. Over any commutative ring, we show how this correspondence can be realized more generally as an embedding of the homotopy category of matrix factorizations belonging to a self-orthogonal subcategory of modules into a corresponding homotopy category of totally acyclic complexes. Some cases of interest will include when the self-orthogonal subcategory is that of projective or flat cotorsion modules. (Received September 16, 2019)

1154-13-1752 **Michael C. Loper***, 206 Church St. SE, Minneapolis, MN 55455. *What Makes a Complex Virtual.*

Let S be the Cox ring of a smooth projective toric variety and B be the irrelevant ideal. In 2017, Berkesch, Erman, and Smith introduced virtual resolutions for toric varieties as an analogue of minimal free resolutions for projective varieties. Virtual resolutions are complexes of free S -modules that allow B -torsion homology. I will name two algebraic conditions that determine whether a bounded chain complex of free S -modules is a virtual resolution. This theorem is similar to the depth criterion of exactness that Buchsbaum and Eisenbud published in 1973. (Received September 16, 2019)

1154-13-1773 **Jesse Keyton*** (jskeyton@uark.edu). *Homogeneous Liaison and the Sequentially Bounded Licci Property.*

In CI-Liaison, much effort has been made to understand ideals in the liaison class of a complete intersection, called licci ideals. We consider zero-dimensional licci ideals in a polynomial ring and focus on the degrees of the forms generating the regular sequences. Using a sequentially bounded condition on these degrees, E. Chong discovered a large class of licci ideals satisfying the EGH conjecture (among them, grade 3 Gorenstein ideals). He raised the question of whether such links were possible for all homogeneous licci ideals. We answer his question

in the negative, and in doing so answer a question of C. Huneke and B. Ulrich about strongly licci ideals. The structure of certain Betti tables plays a central role in our proof. (Received September 16, 2019)

1154-13-1873 **Ben Blum-Smith*** (ben@cims.nyu.edu) and **Sophie Marques**. *When are permutation invariants Cohen-Macaulay over all fields?*

We show that the polynomial invariants of a permutation group are Cohen–Macaulay for any choice of coefficient field if and only if the group is generated by transpositions, double transpositions, and 3-cycles, using a construction based on strict henselization and methods from combinatorial commutative algebra. This unites and generalizes several previously known results. (Received September 16, 2019)

1154-13-1912 **Reeve M. Garrett*** (garrett.425@osu.edu), 231 W 18th Ave, Columbus, OH 43210. *On the inductive construction of rank 1, 2, and 3 valuations on $K(x, y)$ and their associated structures.*

In a 1935 paper, Saunders MacLane constructed all rank one and some rank two valuations on the field of rational functions $K(x)$ extending a discrete valuation v_0 on a field K via inductively defined sequences of key polynomials in $K[x]$, and in 2009, K.A. Loper and F. Tartarone introduced the notion of an "upside down" valuation to construct the remaining rank two valuations on $K(x)$ extending v_0 on K and used both MacLane's construction and their own to represent integrally closed rings between $\mathbb{Z}_p[x]$ and $\mathbb{Q}[x]$ as a minimal intersection of valuation overrings and classify such rings (when they are Noetherian, Prüfer, Mori, PvMD, etc.) by the form of the valuations corresponding to the valuation overrings in that minimal intersection. In a series of 3 papers published in 1969, 1971, and 1973, H. Inoue attempted to generalize MacLane's approach to $K(x, y)$ but was not able to fully accomplish this goal. This talk will present fixes to the problems in Inoue's approach and some results on the construction of rank two and rank 3 valuations on $K(x, y)$ and integrally closed rings between $\mathbb{Z}_p[x, y]$ and $\mathbb{Q}[x, y]$. (Received September 16, 2019)

1154-13-1979 **Roswitha Rissner** and **Irena Swanson*** (iswanson@reed.edu). *Numbers of associated prime ideals of powers of monomial ideals.* Preliminary report.

What are the allowed sequences $\{a_n\}$ of positive integers for which there exists a monomial ideal I (in a polynomial ring over a field in finitely many variables) such that the number of associated prime ideals of the n th power of I equals a_n ? By a result of Brodmann the sequence must be eventually constant. A result of the second author and Sarah Weinstein says that all non-increasing sequence are realized in this way. Here we examine other classes of realizable sequences. (Received September 16, 2019)

1154-13-1991 **Paige G. Beidelman*** (pbeidelm@mail.umw.edu) and **Kimberly Nicole Hancock** (khancock@bowdoin.edu), 16 Deerfield Road, Brookfield, CT 06804, and **Kaiwen Lu** (kailu@umich.edu) and **Pedro D Morales Vega** (pedromv9616@gmail.com), 903 Lambertson DR, Silver Spring, MD 20902, and **Nathan J Akerhielm** (nakerhielm@haverford.edu), 540 Crescent Avenue, Greenville, SC 29601. *Solvability of Multiple Unicast Networks Over Finite Fields.*

At Clemson University, we researched communication across interference networks called multiple unicast networks. Our goal was to determine the achievable capacity of some classes of networks when one is only using linear operations. After an introduction to the problem, we translated the problem in commutative algebra language by showing that achievability corresponded to the existence of a solution of an ideal defined by the network. We then showed that some transformations on the adjacency matrix of networks preserved achievability allowing us to work with a smaller class of representative networks. Finally we discovered a lower bound of messages that can be sent from one source to its corresponding terminal given the entire network in reduced form. This project was a joint work with Felice Manganiello and Kristen Savary. (Received September 17, 2019)

1154-13-1992 **Javier Carvajal-Rojas**, **Linquan Ma**, **Thomas Polstra**, **Karl Schwede*** (schwede@math.utah.edu) and **Kevin Tucker**. *Covers of rational double points in mixed characteristic.*

We further the classification of rational surface singularities. Suppose $(S, \mathfrak{n}, \|\cdot\|)$ is a strictly Henselian regular local ring of mixed characteristic $(0, p > 5)$. We classify functions f for which $S/(f)$ has an isolated rational singularity at the maximal ideal \mathfrak{n} . The classification of such functions are used to show that if (R, \mathfrak{m}) is an excellent, strictly Henselian, Gorenstein rational singularity of dimension 2 and mixed characteristic $(0, p > 5)$, then there exists a split finite cover of $\text{Spec}(R)$ by a regular scheme. We give an application of our result to the study of 2-dimensional BCM-regular singularities in mixed characteristic. (Received September 17, 2019)

1154-13-2037 **Sema Gunturkun*** (sgunturkun@amherst.edu) and **Mel Hochster**. *A bound on the Hilbert function of a quadratic ideal*. Preliminary report.

We define a defect δ quadratic ideal as a homogeneous ideal generated by $n + \delta$ quadrics containing a regular sequence of quadratic forms. A bound for the Hilbert function of such quadratic ideals is conjectured by Eisenbud, Green and Harris using a corresponding monomial defect δ quadratic ideal. I will discuss this bound especially in degree three for the defect three quadratic ideals. This is a joint work with Mel Hochster. (Received September 17, 2019)

1154-13-2116 **Patricia Klein, Linquan Ma, Pham Hung Quy, Ilya Smirnov and Yongwei Yao*** (yyao@gsu.edu). *Lech's inequality, the Stückrad-Vogel conjecture, and uniform behavior of Koszul homology*.

Let (R, \mathfrak{m}) be a Noetherian local ring, and let M be a finitely generated R -module of dimension d . We prove that the set $\left\{ \frac{e(M/IM)}{e(I, M)} \right\}_{\sqrt{I}=\mathfrak{m}}$ is bounded below by $1/d!e(\overline{R})$ where $\overline{R} = R/\text{Ann}(M)$. Moreover, when \widehat{M} is equidimensional, this set is bounded above by a finite constant depending only on M . The lower bound extends a classical inequality of Lech, and the upper bound answers a question of Stückrad–Vogel in the affirmative. As an application, we obtain results on uniform behavior of the lengths of Koszul homology modules. (Received September 17, 2019)

1154-13-2151 **Neil E Steinburg*** (neil.steinburg@drake.edu). *How to tell the torsion a tensor takes*.

While tensor products are quite prolific in commutative algebra, even some of their most basic properties remain relatively unknown. We explore one of these properties, namely a tensor's torsion. Given any finitely generated modules, M and N over a ring R , the tensor product $M \otimes_R N$ almost always has nonzero torsion unless one of the modules M or N is free. We seek to determine which rings guarantee nonzero torsion in a tensor product of non-free modules over the ring. In particular, if R is a Gorenstein one-dimensional domain, let E be the endomorphism ring of the maximal ideal of R , viewed as a subring of the integral closure, \overline{R} . If $M \otimes_R N$ is torsion-free with neither M nor N free, we show that E must be local with the same residue field as R . This is joint work with Roger Wiegand. (Received September 17, 2019)

1154-13-2550 **Jay White*** (jay.white@uky.edu). *Maximum Betti Numbers when fixing partial data about Hilbert functions*.

Fixing a family of ideals in a polynomial ring, consider the problem of finding a single ideal that has Betti numbers greater than or equal to the other ideals. Or, decide if such an ideal even exists. Bigatti and Hulett showed that if we take the ideals with a fixed Hilbert function, there is such an ideal: the lexsegment ideal. Caviglia and Murai showed that if we take the saturated ideals with a fixed Hilbert polynomial, there is again such an ideal. We will talk about a generalization of these two situations, an algorithm for finding the special ideals and when they exist, and some cases where we guarantee existence or find nonexistence. (Received September 17, 2019)

1154-13-2603 **Claudia Miller, Hamid Rahmati*** (hrahmati2@unl.edu) and **Rebecca R.G.**. *Free resolutions of the Frobenius powers of the maximal ideal over a general hypersurface in 3 variables*. Preliminary report.

We study the asymptotic behavior of free resolutions of the Frobenius powers of the maximal ideal of hypersurface rings $R = k[x, y, z]/(f)$, where k is a field of positive characteristic. For general choices of f , we discuss the structure of free resolutions of the Frobenius powers of the maximal ideal and show that high enough powers have identical graded Betti numbers up to explicit shifts. We also compute the Hilbert-Kunz function of such rings. (Received September 17, 2019)

1154-13-2693 **Lance Edward Miller***, Department of Mathematical Sciences, 309 SCEN, University of Arkansas, Fayetteville, AR 72701, and **William D. Taylor**, 3500 John A. Merritt Boulevard, Nashville, TN 37209. *Lower bounds on s -multiplicity*.

W. D. Taylor introduced an interpolation between Hilbert-Samuel and Hilbert-Kunz multiplicity of a local positive characteristic ring by giving a continuous family of multiplicity functions based on a parameter $s > 0$. In this talk, we discuss this an extension of the Watanabe-Yoshida minimality conjecture to the s -multiplicity setting as well as the s -multiplicity function for 2 minors of a generic matrix. (Received September 17, 2019)

1154-13-2808 **Benjamin J Drabkin*** (benjamin.drabkin@huskers.unl.edu) and **Alexandra Seceleanu** (aseceleanu@unl.edu). *Containment-Tight Ideals from Singular Loci of Reflection Arrangements*.

Given an ideal I in a commutative Noetherian ring R , the m -th symbolic power of I is defined to be $I^{(m)} = \bigcap_{\mathfrak{p} \in \text{Ass}(I)} (I_{\mathfrak{p}}^m \cap R)$. By results of Ein-Lazarsfeld-Smith, Hochster-Huneke, and Ma-Schewede every ideal I of

codimension e in a regular ring satisfies the containment $I^{(er)} \subseteq I^r$. In many cases, this containment can be improved upon; however, in recent years a number of ideals have been found for which this containment is tight.

All known ideals exhibiting tight containments are codimension 2 and satisfy $I^{(3)} \not\subseteq I^2$. Furthermore these ideals define the singular loci of the hyperplane arrangements for some complex reflection groups. This talk will aim to classify which complex reflection groups give rise to hyperplane arrangements whose singular loci exhibit the noncontainment $I^{(3)} \not\subseteq I^2$. (Received September 18, 2019)

14 ► Algebraic geometry

1154-14-146 **Ben Wormleighton*** (b.wormleighton@berkeley.edu), 970 Evans Hall, Department of Mathematics, Berkeley, CA 94720. *Walls for G -Hilb via Reid's recipe.*

The three-dimensional McKay correspondence seeks to relate the geometry of crepant resolutions of Gorenstein 3-fold quotient singularities \mathbb{C}^3/G with the representation theory of the group G . The first crepant resolution to be studied in depth was the G -Hilbert scheme G -Hilb, which is also a moduli space of stable representations of the McKay quiver associated to G . As the stability parameter varies, we obtain many other crepant resolutions. We focus on the case where G is abelian, and compute explicit inequalities defining the chamber of the stability space for G -Hilb in terms of a marking of exceptional subvarieties of G -Hilb called Reid's recipe. We further show which of these inequalities define walls and describe their wall-crossing behaviour using results of Craw-Ishii. (Received August 15, 2019)

1154-14-333 **Campbell Hewett*** (chewett@mit.edu). *Computability of rational points on curves over function fields in characteristic p .*

Let k be a perfect field of characteristic p , and let K be a field finitely generated over k . This talk is concerned with regular nonsmooth curves X over K , also known as genus-changing curves. Assuming k has finite transcendence degree over its prime field, we give an algorithm to compute the set of K -points $X(K)$ that expands on the proofs of finiteness of $X(K)$ given by Voloch and Jeong. This, together with Szpiro's height bound for K -points on smooth nonisotrivial curves of genus at least two, proves the effective Mordell conjecture for regular curves in positive characteristic. (Received August 31, 2019)

1154-14-338 **Andrew Kobin*** (ak5ah@virginia.edu). *Root stacks in characteristic p .*

As stacks continue to become an essential part of a modern algebraic geometer's toolbox, researchers look to their local structure as a guide to their nature. Over the complex numbers, this local structure is called a complex orbifold, or 'orbit space of a manifold' under a cyclic group action. In this talk, I will describe an important construction, due to Cadman and independently to Abramovich-Graber-Vistoli, called a root stack which captures the notion of orbifold over an arbitrary field. Root stacks appear in lots of places, including in the classification of stacky curves in characteristic 0. After describing what is known, I will introduce a new construction, called an Artin-Schreier root stack, which allows for similar classification results in characteristic p . (Received September 01, 2019)

1154-14-475 **Huy Dang*** (hq4bz@virginia.edu). *Equal characteristic deformations of Artin-Schreier-Witt covers.*

We study equal characteristics deformations of cyclic covers of curves over an algebraically closed field of positive characteristic. We first examine the notion of Hurwitz tree equipped with the refined Swan conductors of a cyclic action on a formal disc. The existence of such a tree is known to give a necessary condition for the existence of an action whose ramification data fit into it. We prove that the conditions imposed by the Hurwitz tree's structure are also sufficient. (Received September 05, 2019)

1154-14-526 **Huy Dang** (hq4bz@virginia.edu), 141 Cabell Drive, 401 Kerchof Hall, Charlottesville, VA 22904, **Soumyadip Das** (soumyadipp@gmail.com), 8th Mile Mysore Road, Bangalore, 560059, India, **Kostas Karagiannis*** (kkaragia@math.auth.gr), 54124 Thessaloniki, Greece, **Andrew Obus** (andrew.obus@baruch.cuny.edu), 1 Bernard Baruch Way, New York, NY 10010, and **Vaidehee Thatte** (thatte@math.binghamton.edu), Binghamton, NY 13902-6000. *Lifting of wild cyclic-by-tame covers.* Preliminary report.

The generalized Oort Conjecture states that over an algebraically closed field k of positive characteristic $p > 0$ any cyclic-by-tame cover of smooth projective k -curves lifts to characteristic 0. Obus has shown that this conjecture holds given the existence of certain meromorphic differential forms on \mathbb{P}^1 with specified behavior under the Cartier operator. We will present a computational approach on the study of such forms and discuss in detail the

cases of D_{25} -covers and D_{27} -covers. If time permits, we will discuss our progress on the general case. (Received September 06, 2019)

1154-14-560 **Junliang Shen*** (j1shen@mit.edu), 77 Massachusetts Ave, Building 2, Room 2-232, Cambridge, MA. *Gopakumar-Vafa invariants and the $P=W$ conjecture.*

Perverse sheaves and their associated invariants appear naturally in the study of certain abelian fibrations. A striking example is the $P=W$ conjecture by de Cataldo, Hausel, and Migliorini connecting the topology of Hitchin systems and Hodge theory of character varieties via non-abelian Hodge theory. I will discuss recent progress on the $P=W$ conjecture as well as connections to Gopakumar-Vafa invariants for local Calabi-Yau varieties. (Received September 06, 2019)

1154-14-628 **Daniel Erman***, Van Vleck Hall, Madison, WI 53706, and **Steven V Sam** and **Andrew Snowden**. *Boundedness results in homological commutative algebra.*

I will survey some boundedness results related to Stillman's Conjecture, about bounds in homological commutative algebra based only on the degrees of the polynomials defining an ideal, or based on other data. This is joint work Steven V Sam and Andrew Snowden. (Received September 08, 2019)

1154-14-639 **Adam Afandi*** (afandi@math.colostate.edu), 3309 Warren Farm Dr., Fort Collins, CO 80526. *Linear Hyperelliptic Hodge Integrals.*

We find a closed form expression for linear Hodge integrals on the hyperelliptic locus. Specifically, we find a combinatorial expression for all intersection numbers on the hyperelliptic locus that have one λ -insertion, and powers of a ψ -class pulled back along the branch map. In order to do this, we use Atiyah-Bott localization on a stack of stable maps into the orbifold $[\mathbb{P}^1/\mathbb{Z}_2]$. (Received September 09, 2019)

1154-14-661 **Renzo Cavaliere*** (renzo@math.colostate.edu), 1874 Campus Delivery Mail, Fort Collins, CO 80524-1874. *A tour of hyperelliptic loci.*

Hyperelliptic loci are subvarieties of the moduli spaces of curves which parameterize curves that admit a degree two map to the projective line. They are one of the basic examples of tautological classes described by the geometric properties that the curves parameterize satisfy. This talk will survey what we know and what we would like to know about these classes, and highlight the similarities and differences between the collection of all hyperelliptic loci and the datum of a CohFT. Collaborative work with Tarasca and Schmitt will be presented. (Received September 09, 2019)

1154-14-667 **Sean W Perry*** (sperry9@fau.edu), 8612 SW 17th Ave, Stuart, FL 34997. *An upper bound on the number of solutions in a particular gravitational lensing ensemble.*

Due to the phenomenon of gravitational lensing, a single point-source of light may be observed as several images when a distribution of mass exists between the observer and the source. We prove an upper bound on the number of lensed images in the case where the mass doing the lensing consists of finitely many point masses. In this setting, the problem reduces to studying the zeros of a complex rational function of a single complex variable and its conjugate. (Received September 09, 2019)

1154-14-691 **Botong Wang*** (wang@math.wisc.edu), Van Vleck Hall, 480 Lincoln Dr., Madison, WI 53706. *Lyubeznik numbers of irreducible projective varieties.*

Lyubeznik numbers are invariants of singularities that are defined algebraically, but closely related to the topology of the singularity. In positive characteristics, it is a theorem of Wenliang Zhang that the Lyubeznik numbers of the cone of a projective variety do not depend on the choice of the projective embedding. Recently, Thomas Reichelt, Morihiko Saito and Uli Walther related the problem with the failure of Hard Lefschetz theorem for singular varieties. And they constructed examples of reducible complex projective varieties whose Lyubeznik numbers depend on the choice of projective embeddings. I will discuss their works and a generalization to irreducible projective varieties. (Received September 09, 2019)

1154-14-748 **Anatoly Libgober*** (libgober@uic.edu), Department of Mathematics, University of Illinois at Chicago, 851 S.Morgan Street., Chicago, IL 60607. *Singular and Virtual elliptic genera*. Preliminary report.

We compare invariants of singular varieties with values in the ring of Jacobi forms introduced in the works of Borisov-Libgober and Fantechi-Gottsche. We will discuss their role in mirror symmetry as well as the study of conifold and extremal transitions. (Received September 12, 2019)

1154-14-764 **Brendan Hassett*** (brendan_hassett@brown.edu) and **Yuri Tschinkel**. *Cycle class maps and birational invariants*.

We introduce new obstructions to rationality for geometrically rational threefolds arising from the geometry of curves and their cycle maps. (Received September 10, 2019)

1154-14-793 **Colby Kelln*** (ckelln@umich.edu), **Talia Blum** and **Henry Talbott**. *Unlikely Intersections and Portraits of Dynamical Semigroups*.

Classical algebraic dynamics studies the behaviors of single rational functions under iteration. Of particular interest are the possible phase portraits of preperiodic points; Baker proved that with finitely many exceptions every preperiodic portrait can be realized by a complex rational function. Following recent work by Hindes and Hyde-Zieve, we investigate portraits for several rational functions simultaneously acting on the same point set. In this case, we no longer expect these portraits to be realizable for arbitrary rational functions. This leads to several questions, such as which portraits with several rational functions have realizations, and what properties do the spaces of all such realizations possess?

We used a computer cluster to determine realization spaces for all portraits with several points and two polynomials of low degree. Surprisingly, many portraits had realization spaces with higher than expected dimension. We will present three main results: a sequence of portraits with positive-dimensional realization spaces for multiple rational functions acting on arbitrarily many points; a classification theorem for the realizable dimension of two-image portraits; and a realizable portrait with 28 quadratic polynomials acting on four points. (Received September 10, 2019)

1154-14-804 **Leslie C Wilson*** (les@math.hawaii.edu), **Massimo Ferrarotti** and **Elisabetta Fortuna**. *Algebraic approximation of analytic and semi-analytic sets*.

Two sets A and B in \mathbf{R}^n are said to be s -equivalent at x if $H(A \cap S_r; B \cap S_r) = o(r^s)$, where S_r is the sphere of radius r centered at x , and H is the Hausdorff distance. They are tangential s -equivalent if the embedded tangent bundles are s -equivalent. We will describe various results about approximating analytic or semi-analytic sets by algebraic sets up to s - or tangential s -equivalence. (Received September 10, 2019)

1154-14-831 **Nguyen-Bac Dang**, **Dragos Ghioca**, **Fei Hu** and **John Lesieutre***, jd@psu.edu, and **Matthew Satriano**. *Higher arithmetic degrees of rational maps*. Preliminary report.

Suppose that $f : X \dashrightarrow X$ is a dominant rational self-map of a variety defined over a number field. For a point P on X , Kawaguchi and Silverman have defined the arithmetic degree of f at P , a measure of the asymptotic growth rate of the heights of points $f^n(P)$. In this talk, I will introduce a definition of higher arithmetic degrees, measuring the growth rates of heights of higher-dimensional cycles. I will then describe efforts to develop a theory of arithmetic degrees in parallel to the much better established theory of dynamical degrees. This project is joint work with Nguyen-Bac Dang, Dragos Ghioca, Fei Hu, and Matthew Satriano. (Received September 11, 2019)

1154-14-1016 **Cristian Lenart*** (c1enart@albany.edu), Department of Mathematics and Statistics, State University of New York at Albany, 1400 Washington Avenue, Albany, NY 12222, **Kirill Zainoulline**, University of Ottawa, Canada, and **Changlong Zhong**, State University of New York at Albany. *Schubert calculus beyond K -theory*.

Modern Schubert calculus has been mostly studying the cohomology and K -theory (including their equivariant and quantum generalizations) of flag manifolds. The basic results for an arbitrary oriented cohomology theory have only been obtained recently; additional complexity is due to the dependence of the classes associated to Schubert varieties on their Bott-Samelson desingularizations. Our work in this area focuses on torus equivariant hyperbolic cohomology (a stalk version of elliptic cohomology). First, we generalize certain formulas for the equivariant Schubert classes in cohomology and K -theory. We then construct and study a canonical replacement of the Schubert basis (for partial flag varieties), using the Kazhdan-Lusztig basis of a certain Hecke algebra; geometric motivation for our construction is provided. As a byproduct, we give a new interpretation of several results in Kazhdan-Lusztig theory. (Received September 12, 2019)

1154-14-1019 **Felix Janda*** (janda@umich.edu), 1 Einstein Drive, Princeton, NJ 08540. *All genus Landau-Ginzburg/Calabi-Yau correspondence*. Preliminary report.

The Landau-Ginzburg/Calabi-Yau (LG/CY) correspondence is a physical correspondence between topological strings on a Calabi-Yau manifold and the corresponding Landau-Ginzburg singularity model. Witten established it as an instance of GLSM wall-crossing. In mathematics, in the example of quintic threefolds, Chiodo-Ruan have formulated and established the correspondence in genus zero. It is an explicit relationship between Gromov-Witten and FJRW invariants of quintics.

In my talk, I will discuss joint work in progress with S. Guo and Y. Ruan on the higher genus LG/CY correspondence for quintic threefolds. (Received September 12, 2019)

1154-14-1050 **Katrina Honigs*** (honigs@uoregon.edu), 4710 Center Way, Eugene, OR 97405. *Rational points and derived equivalence.*

The derived category of coherent sheaves on a variety connects to many different areas of study in algebraic geometry. However, there are many open questions about which properties of a variety are detected by this invariant. In this talk, I will discuss recent joint work with Addington, Antieau and Frei where we gave the first examples of derived equivalences between smooth, projective varieties where one variety has a \mathbb{Q} -rational point and the other does not. (Received September 12, 2019)

1154-14-1129 **Richard Rimanyi*** ([rimanyi@email.unc.edu](mailto:rimaryi@email.unc.edu)) and **Andrzej Weber**. *Schubert calculus in equivariant elliptic cohomology.*

Assigning characteristic classes to singular varieties is an effective way of studying the enumerative properties of the singularities. Initially one wants to consider the so-called fundamental class in H , K , or Ell , but it turns out that in Ell such a class is not well defined. However, a deformation of the notion of fundamental class (under the name of Chern-Schwartz-MacPherson class in H , motivic Chern class in K) extends to Ell , due to Borisov-Libgober. To make sense of the Borisov-Libgober class for a wider class of singularities we introduce a version of it, which now necessarily depends on new ('dynamical' or 'Kahler') variables. We obtain that this elliptic class of Schubert varieties satisfies two different recursions (Bott-Samelson, and R-matrix recursions). The second one relates elliptic Schubert calculus with Felder-Tarasov-Varchenko weight functions, and Aganagic-Okounkov stable envelopes. The duality between the two recursions is an incarnation of 3d mirror symmetry (and symplectic duality). (Received September 13, 2019)

1154-14-1163 **Kalila Joelle Sawyer*** (kalila.sawyer@uky.edu) and **David Jensen**. *Scrollar Invariants of Tropical Curves.*

The *scrollar invariants* of a special divisor D on a k -gonal algebraic curve X are a family of invariants that provide insight into the behavior of D and the geometry of X . In the classical case, the ranks of multiples cD of D form a convex sequence completely determined by the scrollar invariants, which are extremely useful but hard to compute in this setting. We use degeneration techniques to investigate this question in the tropical setting, where combinatorial tools provide helpful insight. We begin by defining scrollar invariants of tropical curves with a fixed rank 1 divisor. We examine the behavior of scrollar invariants under specialization, and compute these invariants for a much-studied family of tropical curves. Our examples highlight many parallels between the classical and tropical theories, but also point to some substantive distinctions. (Received September 13, 2019)

1154-14-1209 **Grayson Jorgenson*** (gjorgens@math.fsu.edu). *Linear recurrence sequences and the duality defect conjecture.*

It is conjectured that the dual variety of every smooth nonlinear subvariety of dimension $> 2N/3$ in projective N -space is a hypersurface, an expectation known as the duality defect conjecture. This would follow from the truth of Hartshorne's complete intersection conjecture but nevertheless remains open for the case of subvarieties of codimension > 2 . A combinatorial approach to proving the conjecture in the codimension 2 case was developed by Holme. This approach employs Segre classes to give a potential method of proving the duality defect conjecture by studying the positivity of certain homogeneous integer linear recurrence sequences. We will discuss the relationship with recurrence sequences and use it to prove that the conjecture holds in the codimension 3 case when N is odd. (Received September 13, 2019)

1154-14-1250 **Evangelia Gazaki*** (eg4va@virginia.edu), 141 Cabell Drive, Kerchof Hall, Charlottesville, VA 22904. *Zero-cycles over arithmetic fields.* Preliminary report.

In this talk I will discuss some big open conjectures about zero-cycles on smooth projective varieties over fields of arithmetic interest, namely algebraic number fields and p -adic fields. I will later present some progress for some products of elliptic curves. Part of this work is joint with Isabel Leal. (Received September 14, 2019)

1154-14-1326 **Hannah Larson** and **Isabel Vogt***, Stanford University, Department of Mathematics, 450 Serra Mall, Building 380, Stanford, CA 94305. *An enriched count of the bitangents to a smooth plane quartic curve.*

Recent work of Kass-Wickelgren gives an enriched count of the 27 lines on a smooth cubic surface over arbitrary fields, generalizing Segre's signed count of elliptic and hyperbolic lines. Their approach using \mathbb{A}^1 -enumerative geometry suggests that other classical enumerative problems should have similar enrichments when the answer is computed as the degree of the Euler class of a relatively orientable vector bundle. In this

talk, we consider the closely related problem of the 28 bitangents to a smooth plane quartic. Subtleties arise because the relevant vector bundle is not relatively orientable. (Received September 14, 2019)

1154-14-1359 **Jeff Achter*** (achter@math.colostate.edu), Department of Mathematics, Colorado State University, Fort Collins, CO 80523-1874, and **Sebastian Casalaina-Martin** and **Charles Vial**. *The arithmetic of intermediate Jacobians*.

Let X be a variety over a subfield K of \mathbb{C} . Although the construction of the (algebraic) intermediate Jacobian of $X_{\mathbb{C}}$ is resolutely transcendental, this abelian variety has a distinguished model over K . I'll discuss what this abelian variety can tell us about the arithmetic and geometry of X . (Received September 15, 2019)

1154-14-1373 **Leonardo C. Mihalcea** and **Camron M. Withrow*** (cwithrow@vt.edu). *Motivic Chern classes of Bott-Samelson strata and Kazhdan-Lusztig polynomials*.

Let X be a flag manifold G/B , and consider a Schubert subvariety Ω in X , and a Bott-Samelson resolution B of Ω . We calculate the push forward of the motivic Chern class of each Bott-Samelson stratum in B , then we expand this push forward in terms of the basis of motivic Chern classes of Schubert cells in Ω . The resulting coefficients are univariate polynomials, and we identify them with polynomials previously investigated by Deodhar in terms of the Hecke algebra. Following Deodhar's work, we reinterpret our results in terms of Deodhar's combinatorial formalism and investigate the relationship between motivic Chern classes of Bott-Samelson varieties and Kazhdan-Lusztig polynomials. This is joint work with Leonardo Mihalcea. (Received September 15, 2019)

1154-14-1381 **Jack Petok*** (jp58@rice.edu). *Kodaira dimension of moduli spaces of low degree special $K3^{[2]}$ -fourfolds*.

In this talk, we will consider the moduli space \mathcal{M} of $K3^{[2]}$ -fourfolds with a polarization of degree 2. This space has a dense open set parametrizing smooth EPW double sextics, and is closely related to rationality questions about Gushel-Mukai fourfolds. Following Hassett's work on cubic fourfolds, Debarre, Iliev, and Manivel have shown that the Noether-Lefschetz locus in \mathcal{M} is a countable union of special divisors \mathcal{M}_d , where the discriminant d is a positive integer congruent to 0, 2, or 4 modulo 8. We compute the Kodaira dimensions of these special divisors for all but finitely many discriminants. In particular, for each congruence class of discriminants modulo 8, we give explicit upper and lower bounds on the unique discriminant d_0 such that the following is true: \mathcal{M}_d is of general type if and only if $d \geq d_0$. We also discuss why one may want to look at \mathcal{M}_d when studying the transcendental Brauer groups of very general $K3$ surfaces. (Received September 16, 2019)

1154-14-1387 **Laure Flapan*** (lflapan@mit.edu) and **Jaclyn Lang** (lang@math.univ-paris13.fr). *Algebraicity of Hodge classes via algebraic Hecke characters*. Preliminary report.

We examine the relationship between having an algebraic Hecke character attached to the cohomology of a smooth projective variety X equipped with a finite-order automorphism and the algebraicity of some Hodge classes on the product X^n . (Received September 15, 2019)

1154-14-1392 **C Damiolini** (chiarad@princeton.edu), Department of Mathematics, Princeton University, Princeton, NJ 08540, **A. Gibney*** (angela.gibney@gmail.com), Department of Mathematics, Rutgers University, Piscataway, NJ 08854, and **N. Tarasca**, Department of Mathematics, Virginia Commonwealth University, Richmond, 23284. *On factorization and vector bundles of conformal blocks from vertex algebras*. Preliminary report.

Modules over conformal vertex algebras give rise to sheaves of coinvariants and conformal blocks on moduli of stable pointed curves. We show that under certain natural hypotheses, these sheaves satisfy the factorization property, a reflection of their inherent combinatorial nature. As an application, we prove they are vector bundles. These provide a generalization of vector bundles defined by integrable modules over affine Lie algebras at a fixed level. Satisfying factorization is essential to a recursive formulation of invariants, like ranks and Chern classes, and to produce new constructions of rational conformal field theories. (Received September 15, 2019)

1154-14-1396 **C. Damiolini** (chiarad@princeton.edu), Department of Mathematics, Princeton University, Princeton, NJ 08540, **A. Gibney*** (angela.gibney@rutgers.edu), Department of Mathematics, Rutgers University, Piscataway, NJ 08854, and **N. Tarasca** (tarasca@vcu.edu), Department of Mathematics & Applied Mathematics, Virginia Commonwealth University, Richmond, VA 23284. *Chern classes of vector bundles on the moduli space of curves from vertex algebras*. Preliminary report.

One can form vector bundles of coinvariants defined by modules over conformal vertex algebras, and these generalize vector bundles given by integrable modules at a fixed level over affine Lie algebras. I will talk about how one can determine the total Chern character of the vertex algebra bundles, given that appropriate hypotheses

hold, following the approach by A. Marian, D. Oprea, R. Pandharipande, A. Pixton, and D. Zvonkine for the classical case. We learn in particular that the classes are tautological. (Received September 15, 2019)

1154-14-1429 **Franco Rota*** (rota@math.rutgers.edu), Department of Mathematics - Hill Center, 110 Frelinghuysen Road, Piscataway, NJ 08854-8019. *The stability manifold of local orbifold elliptic quotients.*

In this paper, we investigate the stability manifold of local surface models of orbifold quotients of elliptic curves. In particular, we describe a component of the stability manifold which maps as a covering space onto the universal unfolding space of the mirror singularity. The construction requires a detailed description of the derived McKay correspondence for A_N surface singularities and a study of wall-crossing phenomena. The results presented are an analog of the work of Bridgeland and Thomas on Kleinian singularities in the context of simple elliptic singularities. At the same time, they extend Ikeda's result on arbitrary root systems of symmetric Kac-Moody Lie algebras to the case of elliptic root systems. Moreover, they represent an instance of a mirror symmetric principle. (Received September 15, 2019)

1154-14-1486 **Sarah Frei*** (sf31@rice.edu). *Derived equivalence and rational points.*

It is natural to ask which properties of a smooth projective variety are recovered by its derived category. In this talk, I will consider the question: is the existence of a rational point preserved under derived equivalence? In recent joint work with Nicolas Addington, Ben Antieau, and Katrina Honigs, we show that over \mathbb{Q} , the answer is no. We give two counterexamples: an abelian variety and a torsor over it, and a pair of hyperkaehler fourfolds. (Received September 15, 2019)

1154-14-1487 **Sarah Frei*** (sf31@rice.edu). *Moduli spaces of sheaves on surfaces.*

Moduli spaces of sheaves on surfaces provide rich and interesting examples of higher dimensional varieties, and yet many of their arithmetic properties when defined over an arbitrary field are still unknown. In this talk, I will discuss recent work, joint with Katrina Honigs, pertaining to those moduli spaces which arise in the study of hyperkaehler varieties: we show that over a finite field, two smooth projective moduli spaces of sheaves on a given K3 or abelian surface have the same number of points as soon as they have the same dimension. (Received September 15, 2019)

1154-14-1488 **Farkhod Eshmatov, Rustam Hamdam, Jacob Kesten*** (jgk3@rice.edu), **Samuel Mathers** and **Zafar Normatov**. *Transitivity on the Calogero-Moser Variety C_2 .*

We consider the Calogero-Moser Varieties C_n , defined as orbit spaces of the set $\{(X, Y) \in \mathbb{C}^{n \times n} \times \mathbb{C}^{n \times n} : \text{Rank}[XY - YX + I_n] \leq 1\}$ under the group action of $GL_n(\mathbb{C})$. These emerge through the study of n -particle systems in \mathbb{R} evolving in time. Since Calogero conjectured such systems are solvable and proved so using matrices of the above form, these matrix sets have been studied in detail, where it was shown that the elements of the space are expressible as polynomials in matrix traces. In our project, we consider the variety of dimension four, which we proved can be expressed as the quotient $C_2 := \mathbb{C}[x_1, \dots, x_5]/(x_4^2 - x_3x_5 - 1)$. We then define an action by the group of automorphisms of the free algebra $\mathbb{C}\langle x, y \rangle$ on C_2 . It was proven by Berest, et al. that this group action is 2-transitive, and conjecture that it is infinitely transitive, which is what we attempt to prove. We develop a method that allows us to prove up to 7-transitivity, and which should be generalizable to infinite-transitivity. Specifically, barring the proof of a technical lemma, we indeed show that the action is infinitely transitive.

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1154-14-1513 **Radu Laza, Gregory Pearlstein** and **Zheng Zhang*** (zheng.zhang-2@colorado.edu). *On the moduli space of pairs consisting of a cubic threefold and a hyperplane.*

The period map is a powerful tool for studying moduli spaces, and has been applied successfully to abelian varieties, K3 surfaces, cubic threefolds/fourfolds, and hyper-Kahler manifolds. However, for some interesting moduli problems (e.g. moduli spaces for pairs of varieties) there might be no obvious way to construct periods. Joint with R. Laza and G. Pearlstein, we construct a period map for cubic pairs consisting of a cubic threefold and a transverse hyperplane using a variation of the construction by Allcock, Carlson and Toledo (which allows us to encode a cubic pair as a "lattice polarized" cubic fourfold). The main result is that the period map induces an isomorphism between a GIT model of the moduli of cubic pairs and the Baily-Borel compactification of some locally symmetric domain. (Received September 16, 2019)

1154-14-1531 **Rachel Webb*** (webbra@umich.edu), **Felix Janda** and **Qile Chen**. *Virtual Cycle on the Moduli Space of Maps to a Complete Intersection.*

A driving question in Gromov-Witten theory is to relate the invariants of a complete intersection to the invariants of the ambient variety. In genus-zero this can often be done with a “twisted theory,” but this fails in higher genus. Several years ago, Chang-Li presented the moduli space of p -fields as a piece of the solution to the higher-genus problem, constructing the virtual cycle on the space of maps to the quintic 3-fold as a cosection localized virtual cycle on a larger moduli space (the space of p -fields). Their result is analogous to the classical statement that the Euler class of a vector bundle is the class of the zero locus of a generic section. I will discuss work joint with Qile Chen and Felix Janda where we extend Chang-Li’s result to a more general setting, a setting that includes standard Gromov-Witten theory of smooth orbifold targets and quasimap theory of GIT targets. (Received September 16, 2019)

1154-14-1605 **Levi Heath*** (levi.heath@colostate.edu). *Cycle-valued quantum Serre duality for quasimaps.* Preliminary report.

The moduli space of stable quasimaps is another compactification of the space of Kontsevich’s space of stable maps. In this talk we will walk through the sketch of a proof of quantum Serre duality for stable quasimaps. Originally proved by Givental, quantum Serre duality refers to a correspondence between, for instance, the Gromov-Witten invariants of a degree d hypersurface in \mathbb{P}^n and the Gromov-Witten invariants of the line bundle $\mathcal{O}(-d)$ over \mathbb{P}^n . We will show how a cycle-valued statement comparing virtual classes recovers a relationship between generating functions of quasimap invariants. (Received September 16, 2019)

1154-14-1622 **Yohsuke Matsuzawa***, 151 Thayer street, Providence, RI 02912. *On Kawaguchi-Silverman conjecture for endomorphisms of rationally connected varieties.*

Kawaguchi-Silverman conjecture asserts that arithmetic degrees of Zariski dense orbits of a rational self-map are equal to the dynamical degree of the map. I will talk about the proof of the conjecture for endomorphisms of smooth projective rationally connected varieties admitting an int-amplified endomorphism. This is a joint work with Shou Yoshikawa. (Received September 16, 2019)

1154-14-1652 **David E Anderson***, 231 W 18th Ave, Columbus, OH 43210. *The direct sum map for classical-type flag varieties.*

The direct sum map sends two flags in a vector space V to their direct sum in the vector space $V \oplus V$. I will describe some variations on this map, along with applications to singularities of Schubert varieties, formulas for Schubert polynomials, and the homology product on an infinite-dimensional Grassmannian. Parts of this come from joint work with W. Fulton, and with T. Ikeda, M. Jeon, and R. Kawago. (Received September 16, 2019)

1154-14-1660 **Y Cooper*** (yaim@math.ias.edu). *Computing descendent insertions for surfaces.*

The degeneration formula has proved to be a powerful tool for computing Gromov-Witten invariants of varieties. In this talk, we will discuss stationary Gromov-Witten invariants of surfaces. We will associate to each type of descendent condition an operator, and will discuss the algebra formed by these operators. (Received September 16, 2019)

1154-14-1703 **Matt Kerr*** (matkerr@wustl.edu), Dept. of Mathematics and Statistics, 1 Brookings Drive, Campus Box 1146, St. Louis, MO 63130-4899. *Specialization of cycles at conifold points.*

Families of toric hypersurfaces admit natural families of motivic cohomology classes arising from the torus. The resulting *higher normal functions* are closely related to solutions of GKZ systems, Feynman integrals, and even the irrationality of $\zeta(3)$. In this talk we consider the role played by their values at conifold points and how these give evidence for a conjecture in topological string theory. (Received September 16, 2019)

1154-14-1711 **Vance Blankers*** (blankersv@northeastern.edu). *Descendent Potentials on Hassett Spaces.*

The Deligne-Mumford compactification of $\overline{\mathcal{M}}_{g,n}$ is often considered the “standard” way to compactify the moduli space of curves. Hassett spaces form a family of alternative compactifications developed in 2002 by Brendan Hassett. In this talk we relate the intersection theory of ψ -classes on Hassett spaces to the same on $\overline{\mathcal{M}}_{g,n}$ through an explicit change of variables of their respective generating functions. As a consequence, we are able to use an asymptotic limit of Hassett spaces to establish the Witten conjecture for κ -classes on $\overline{\mathcal{M}}_g$, which leads to a recursive method for calculating all κ -class intersection numbers. This is joint work with Renzo Cavalieri. (Received September 16, 2019)

1154-14-1769 **Rongxiao Mi*** (rongxiao@cmsa.fas.harvard.edu), 20 Garden St, Cambridge, MA 02138, and **Mark Shoemaker**. *Genus-zero Gromov-Witten theory under extremal transitions*.

Extremal transitions are the basic building blocks of topological changes of Calabi-Yau 3-folds. In this talk, we will briefly survey the change of genus-zero Gromov-Witten theory under an extremal transition, and explain our result in the setting of toric hypersurfaces. This is based on joint work with Mark Shoemaker. (Received September 16, 2019)

1154-14-1772 **Ming Zhang*** (zhangming@math.ubc.ca) and **Yang Zhou**. *K-theoretic quasimap wall-crossing for GIT quotients*. Preliminary report.

For a large class of GIT quotients $X=W//G$, Ciocan-Fontanine—Kim—Maulik and many others have developed the theory of epsilon-stable quasimaps. The conjectured wall-crossing formula of cohomological epsilon-stable quasimap invariants for all targets in all genera has been recently proved by Yang Zhou.

In this talk, we will introduce permutation-equivariant K-theoretic epsilon-stable quasimap invariants with level structure and prove their wall-crossing formulae for all targets in all genera. In particular, it will recover the genus-0 K-theoretic toric mirror theorem by Givental-Tonita and Givental, and the genus-0 mirror theorem for quantum K-theory with level structure by Ruan-Zhang. It is based on joint work with Yang Zhou. (Received September 16, 2019)

1154-14-1803 **Matilde Marcolli***, Caltech, Mathematics Department, 1200 E California Blvd, Pasadena, CA 91125, and **Goncalo Tabuada**. *Motives of intersections of quadrics and the Feynman integral of the massive sunset graph*.

We prove that the Feynman quadrics-motive of the massive sunset graph is generically (in the mass parameters) not mixed-Tate. Moreover, we explicitly describe its extra complexity in terms of a Prym variety. (Received September 16, 2019)

1154-14-1953 **Nathan Priddis*** (priddis@math.byu.edu), 275 TMCB, Brigham Young University, Provo, UT 84604. *Mirror Symmetry for nonabelian Landau-Ginzburg models*.

Mirror symmetry is a phenomenon that has inspired many deep results in the area of algebraic geometry in the past several decades. Mirror symmetry essentially says that the A-model is equivalent in some particular way to the B-model. In the realm of mirror symmetry, one can consider what are known as Landau-Ginzburg (LG) models—a counterpart to the much-studied Calabi-Yau manifolds. LG models are built from the data of a pair (W,G) , where W is a quasihomogeneous potential function and G is a group of symmetries of W . In the past, much has been written about LG mirror symmetry, when G is a group comprised of what are called diagonal symmetries—meaning they are represented as diagonal matrices. More recently, there has been interest in understanding mirror symmetry for the case when G is no longer comprised of diagonal symmetries, but may in fact be nonabelian. We refer to these as nonabelian LG models. In this talk we will discuss mirror symmetry for LG models, and describe a conjectural construction of mirror symmetry for nonabelian LG models, together with some early results supporting this conjectured relationship between the LG A-model and the LG B-model. (Received September 16, 2019)

1154-14-2073 **Richard Rimanyi*** (rimaryi@email.unc.edu) and **Andrzej Weber** (aweber@mimuw.edu.pl). *Elliptic classes of Schubert varieties*.

We will review characteristic classes of singular varieties, with particular attention to the classes of Schubert varieties in equivariant elliptic cohomology. Formulas, recursions, and the relation with stable envelope classes will be presented. (Received September 17, 2019)

1154-14-2194 **Hannah Larson** and **Ravi Vakil*** (rvakil@stanford.edu). *Characteristic classes of vector bundles on spheres and Bott periodicity: algebra, geometry, topology*.

Hannah Larson has recently completely described (integrally) the "characteristic classes" of vector bundles on \mathbf{P}^1 -bundles, in Chow. Bott periodicity relates vector bundles on a topological space X to vector bundles on $X \times S^2$: the "moduli space" BU of complex vector bundles is basically the same as the "moduli space" maps of a sphere to BU . I'm not a topologist, so I will try to explain an algebraic or geometric incarnation, in terms of vector bundles on the Riemann sphere. The algebro-geometric incarnation of Bott periodicity is actually motivated by important current questions in geometry. This is joint work in progress with H. Larson, who did the heavy lifting. (Received September 17, 2019)

1154-14-2238 **Zachary Hamaker** and **Oliver Pechenik***, Department of Mathematics, University of Michigan, 530 Church St, Ann Arbor, MI 48103, and **Anna Weigandt**. *Gröbner geometry of Schubert polynomials through ice*. Preliminary report.

Knutson and Miller (2005) showed that the multidegree of a matrix Schubert variety X_w is the corresponding Schubert polynomial \mathfrak{S}_w . Moreover, after Gröbner degeneration with respect to any antidiagonal term order, the resulting irreducible components are naturally labeled by the pipe dreams for w . In later work with Yong (2009), they used diagonal term orders to obtain alternative combinatorics for certain X_w . We present further results in this direction, with connections to a neglected Schubert polynomial formula of Lascoux (2002) in terms of the square-ice model (recently rediscovered by Lam, Lee, and Shimozono in the guise of “bumpless pipe dreams”). (Received September 17, 2019)

1154-14-2484 **Mboyo Esole*** (mboyoesole@gmail.com), Boston, MA. *Elliptic fibrations and string theory: Crepant resolutions of Weierstrass models, characteristic numbers, and flops*.

I will review progress in the study of elliptic fibrations and their applications in string theory such as (1) Crepant resolutions of Weierstrass models of elliptic fibrations. (2) Topological invariants and characteristic numbers of elliptic threefolds and fourfolds. (3) The geometry of relative minimal models of elliptic fibrations. (Received September 17, 2019)

1154-14-2540 **Justin Hilburn*** (jhilburn@perimeterinstitute.ca), **Benjamin Gammage**, **Mikayel Mkrtchyan**, **Matej Penciak** and **Alex Sherman**. *Elliptic Coulomb Branches*. Preliminary report.

When studying irreducible components of the moduli space of vacua for supersymmetric gauge theories one is often led to study new and interesting classes of algebraic varieties. For example Nakajima quiver varieties and Cherkis bow varieties arise as the Higgs and Coulomb branches respectively of 3d $\mathcal{N} = 4$ quiver gauge theories.

More generally, Braverman, Finkelberg, and Nakajima have given a construction of the ring of functions on the Coulomb branch of an arbitrary 3d $\mathcal{N} = 4$ gauge theory as a certain cohomological convolution algebra. It is a physical heuristic that this construction can be extended to 4d $\mathcal{N} = 2$ and 5d $\mathcal{N} = 1$ theories by replacing ordinary cohomology with K-theory or elliptic cohomology.

In this talk I will describe what is known about Coulomb branches of 5d $\mathcal{N} = 1$ theories. (Received September 17, 2019)

1154-14-2648 **Bill F. Trok***, University of Kentucky, 715 Patterson Office Tower, Lexington, KY 40506. *Projective Duality, Unexpected Hypersurfaces and Hyperplane Arrangements*.

We say a finite set of points Z in P^2 admits unexpected curves in degree d , if the intersection of the ideal $I(Z)$ with the ideal $I(Q)^{d-1}$ is larger than “expected”. Surprisingly, it was shown that the degrees in which Z admits unexpected curve can be determined from combinatorial data of Z , and the Derivation bundle of the line arrangement dual to Z . We generalize this result to P^n , replacing Q with a general codimension 2 subspace. Connections to Terao’s Freeness Conjecture are discussed as well. (Received September 17, 2019)

1154-14-2665 **Gretchen L. Matthews*** (gmatthews@vt.edu), Department of Mathematics, Virginia Tech, Blacksburg, VA 24061. *Algebraic geometry code-based crypto*.

Code-based cryptography, introduced by McEliece in 1978, has received recent attention due to its potential for post quantum use. In this talk, we share recent results on the use of algebraic geometry codes in code-based cryptosystems. (Received September 17, 2019)

1154-14-2694 **Andrew Alaniz***, 377 Lockett Hall, Baton Rouge, LA 70803. *On some local invariants of geometric Langlands parameters*. Preliminary report.

For a simple complex algebraic group G , M. Kamgarpour and D. Sage have shown that the adjoint irregularity of an irregular singular flat G -bundle on the formal punctured disc is bounded from below by the rank of G ; moreover, the rank is realized by the formal Frenkel-Gross connection. This is a geometric analog of a conjecture of Gross and Reeder on the Swan conductor of arithmetic local Langlands parameters. In this talk I consider the irregularity of formal connections with respect to representations other than the adjoint representation. In particular, I discuss the conjecture that the minimum irregularity is always attained at the formal Frenkel-Gross connection, and I will present my results on this conjecture for type A Lie algebras. (Received September 17, 2019)

1154-14-2726 **Yi Hu** and **Jingchen Niu*** (jniu@math.arizona.edu). *Moduli of curves of genus one with twisted fields.*

We construct a smooth moduli stack of genus one weighted curves with twisted fields. This leads to a modular interpretation of Vakil-Zinger's desingularization of the moduli of genus one stable maps to projective spaces. The methods and ideas of this paper have been applied to obtain a modular resolution of the moduli of genus two stable maps by the same authors and are expected to extend to higher genera cases. (Received September 17, 2019)

15 ► *Linear and multilinear algebra; matrix theory*

1154-15-105 **Keller L Blackwell*** (kellerb@mail.usf.edu), **Neelima Borade** (nborad2@uic.edu), **Charles P Devlin VI** (chatrick@umich.edu), **Renyuan Ma** (renyuanma01@gmail.com), **Steven J Miller** (sjm1@williams.edu) and **Wanqiao Xu** (wanqiaox@umich.edu). *Eigenvalue Distributions of Random Iterated Block Matrices.*

RMT successfully models many properties of L -functions; however, there are situations where it is silent. One instance is the number theory process of Rankin-Selberg convolution, which creates a new L -function from an input pair. Our work investigates a possible RMT analogue of this process through the parallel study of random matrix ensembles constructed from existing families.

Let A be a random symmetric Toeplitz matrix with a palindromic first row and B denote a random real symmetric matrix; it is well-known these ensembles converge to the Gaussian and semi-circle distribution, respectively. We consider the “disco” $\mathcal{D}(A, B) = \begin{bmatrix} A & B \\ B & A \end{bmatrix}$. We adapt Wigner's method of moments to derive the limiting spectral measure of $\mathcal{D}(A, B)$; the primary obstacle is analysis of non-commutative matrix polynomials, which the method of moments reformulates as a combinatorial problem of independent interest. We prove convergence of the hybrid distribution, show it is sharply bounded away from the constituent distributions, explore generalizations of our construction, and discuss potential applications to open inquiries in number theory. (Received August 07, 2019)

1154-15-381 **Steve Kirkland*** (stephen.kirkland@umanitoba.ca), Department of Mathematics, University of Manitoba, Winnipeg, MB R3T 2N2, Canada. *Directed Forests and the Constancy of Kemeny's Constant.* Preliminary report.

Consider a discrete-time, time-homogeneous Markov chain on states $1, \dots, n$ whose transition matrix is irreducible. Denote the mean first passage times by m_{jk} , $j, k = 1, \dots, n$, and stationary distribution vector entries by w_k , $k = 1, \dots, n$. A surprising result of Kemeny reveals that the quantity $\sum_{k=1}^n m_{jk} w_k$, which is the expected number of steps needed to arrive at a randomly chosen destination state starting from state j , is independent of the initial state j . In this talk, we consider $\sum_{k=1}^n m_{jk} w_k$ from the perspective of algebraic combinatorics, and provide an intuitive explanation for its independence on the initial state j . The all minors matrix tree theorem is the key tool employed. (Received September 02, 2019)

1154-15-409 **Shaun Fallat*** (shaun.fallat@uregina.ca), Department of Mathematics and Statistics, University of Regina, Regina, Sask. S4S0A2, Canada, and **Mohammad Adm** (mjamathe@yahoo.com), Department of Applied Mathematics and Physics, Palestine Polytechnic University, Hebron. *On the maximum multiplicity of the k th largest eigenvalue of a tree.*

Given a graph G , we are interested in studying the maximum nullity over all real symmetric matrices $S(G)$ with a fixed number of negative eigenvalues. For the case of trees we re-derive a formula for this maximum nullity and completely describe its behaviour as a function of the number of negative eigenvalues. Using this analysis, we revisit some work on describing all partial inertias associated with trees and review an instance of the inverse eigenvalue problem for some special trees. (Received September 03, 2019)

1154-15-434 **Stephen B Robinson*** (sbr@wfu.edu) and **Klaus Schmitt.** *Discrete Resonance Problems Subject to Periodic Forcing.*

We consider the following discrete nonlinear problem which is subject to a periodic nonlinear forcing term:

$$Au = \lambda u + p(u) + h$$

where A is an $n \times n$ matrix with real components, $p : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a periodic forcing term, and $\langle h, \bar{\phi} \rangle = 0$, where $\bar{\phi}$ is an eigenvector of A^T , the transpose of A , corresponding to a simple real eigenvalue λ . Conditions

on these terms will be provided such that this problem will have infinitely many distinct solutions. The results here are motivated by some recent results for discrete systems and by results obtained for analogous boundary value problems for semilinear elliptic problems at resonance. (Received September 04, 2019)

1154-15-595 **Shuang Li*** (shuangli@mines.edu), **Gongguo Tang** and **Michael B. Wakin**. *Simultaneous Blind Deconvolution and Phase Retrieval with Tensor Iterative Hard Thresholding.*

Blind deconvolution and phase retrieval are both fundamental problems with a growing interest in signal processing and communications. In this work, we consider the task of simultaneous blind deconvolution and phase retrieval. We show that this non-linear problem can be reformulated as a low-rank tensor recovery problem and propose an algorithm named TIHT-BDPR to recover the unknown parameters. We include a series of numerical simulations to illustrate the effectiveness of our proposed algorithm. (Received September 08, 2019)

1154-15-602 **Jane Breen*** (jane.breen@ontariotechu.ca). *Kemeny's constant and random walks on graphs.* Preliminary report.

Kemeny's constant is an interesting and useful quantifier of how well-connected the states of a Markov chain are, and is calculated using the eigenvalues of the transition matrix. By considering the random walk on a simple, undirected graph, and the eigenvalues of the normalized Laplacian matrix of the graph, we can compute Kemeny's constant and regard this value as a graph parameter with a concrete interpretation in terms of the expected length of a random trip in the graph. In this talk we give a survey of known results, consider extremal graphs where Kemeny's constant is largest possible, and present new techniques in spectral graph theory which facilitate the computation of Kemeny's constant for these graphs. (Received September 08, 2019)

1154-15-686 **W. Gilbert Strang*** (gilstrang@gmail.com), Department of Mathematics, Massachusetts Institute of Technology, Room 2 - 245, Cambridge, MA 02139. *The big picture of linear algebra*

Linear algebra has surged in importance and we need to explain the key ideas in a memorable way. I have found two approaches that help students to understand the action of a matrix:

- (1) The Four Fundamental Subspaces (Row space, Nullspace, Column space, Left nullspace) Their dimensions / their orthogonality / their bases
- (2) Matrix factorizations that give particularly useful bases for those four subspaces

$$A = LU \quad A = QR \quad A = X\lambda X^{-1} \quad S = Q\lambda Q^T \quad A = U\sigma V^T \quad A = CMR$$

I hope to add something new about both of these approaches. (Received September 09, 2019)

1154-15-698 **Megan G Wendler*** (megan.wendler@wsu.edu), 1134 Markley Drive Apt 5, Pullman, WA 99163. *The almost semimonotone matrices.*

A matrix $A \in \mathbb{R}^{n \times n}$ is (strictly) semimonotone if for every nonzero vector $x \in \mathbb{R}^n$ with nonnegative entries, there is an index k such that $x_k > 0$ and $(Ax)_k$ is nonnegative (positive). A (strictly) semimonotone matrix has the property that every principal submatrix is also (strictly) semimonotone. Thus, it becomes natural to examine the almost (strictly) semimonotone matrices which are those matrices which are not (strictly) semimonotone but whose proper principal submatrices are (strictly) semimonotone. We characterize the 2×2 and 3×3 almost (strictly) semimonotone matrices and describe many of their properties. Then we explore general almost (strictly) semimonotone matrices, including the problem of detection and construction. (Received September 09, 2019)

1154-15-727 **Derek D Young*** (dyoung@ntholyoke.edu). *Some graphs whose maximum nullity and zero forcing number are the same.*

It is known that the zero forcing number of a graph is an upper bound for the maximum nullity of the graph. In this talk, we introduce a new graph parameter which acts as a lower bound for the maximum nullity of a graph. As a result, we show that the Aztec Diamond graph's maximum nullity and zero forcing number are the same. Another graph parameter that is considered is the vertex connectivity which is used to establish results on some circulant graphs. (Received September 10, 2019)

1154-15-806 **Bryan A Curtis*** (bcurtis6@uwyo.edu) and **Bryan L Shader**. *The Strong Inner Product Property.*

A sign pattern is a matrix whose entries belong to $\{0, 1, -1\}$. Each entry of a sign pattern represents the sign of a real number. We say that a sign pattern allows orthogonality if we can replace its entries with real numbers of the corresponding sign and obtain an orthogonal matrix. In this talk we address the question of which sign

patterns allow orthogonality. The talk focuses on the strong inner product property, a new tool for constructing sign patterns of orthogonal matrices, along with some applications. (Received September 10, 2019)

1154-15-820 **Hein van der Holst*** (hvanderholst@gsu.edu), **Marina Arav** and **Scott Dahlgren**. *Colin de Verdiere invariants and antipodal mappings.*

The Colin de Verdière graph invariant μ characterizes disjoint unions of paths, outerplanar graphs, and planar graphs as those graphs G for which $\mu(G) \leq 1$, $\mu(G) \leq 2$, and $\mu(G) \leq 3$, respectively. Arav, Hall, van der Holst, and Li introduced the signed graph invariant ν . For $k \in \{1, 2, 3\}$, the graphs G such that $\mu(G) \leq k$ can be described as those graphs G such that there exists an antipodal mapping of a certain cell complex associated with G into S^{k-1} . Surprisingly, a similar situation holds for the class of signed graphs (G, Σ) with $\nu(G, \Sigma) \leq 1$ and for the class of signed graphs (G, Σ) with $\nu(G, \Sigma) \leq 2$. In this talk, we will discuss this and give some results on signed graphs with $\nu(G, \Sigma) \leq 3$. (Received September 11, 2019)

1154-15-872 **Tyler J Gonzales***, Department of Mathematics, Hibbard Humanities Hall 508, 124 Garfield Avenue, Eau Claire, WI 54701. *Extensions of M -matrix Theory to Rectangular M -matrices.*

We study (square) M -matrices and generalizations of such matrices. M -matrices are important in many mathematical applications including biology, economics, and probability theory. M -matrices have been very extensively studied and there are more than fifty characterizations for a matrix to be an M -matrix. There are a number of generalizations of this notion including a study on what are called rectangular M -matrices. The objective of this project is to revisit the notion of rectangular M -matrices, by giving a better definition to it, as well as to study the extent to which some of the properties of square M -matrices could be generalized to this modified concept of rectangular M -matrices. (Received September 11, 2019)

1154-15-884 **Sarah Bockting-Conrad*** (sarah.bockting@depaul.edu). *Tridiagonal pairs of Racah type and the universal enveloping algebra $U(\mathfrak{sl}_2)$.*

Let \mathbb{F} denote a field and let V denote a vector space over \mathbb{F} with finite positive dimension. Let A, A^* denote a tridiagonal pair of Racah type with diameter $d \geq 1$. Let $\{U_i\}_{i=0}^d$ (resp. $\{U_i^\downarrow\}_{i=0}^d$) denote the first (resp. second) split decomposition of A, A^* . In an earlier paper, we associated with A, A^* a linear transformation $\psi : V \rightarrow V$ such that $\psi U_i \subseteq U_{i-1}$ and $\psi U_i^\downarrow \subseteq U_{i-1}^\downarrow$ for $0 \leq i \leq d$. One of the relations involving ψ was reminiscent of a defining relation for the universal enveloping algebra $U(\mathfrak{sl}_2)$. We explore this connection further. In doing so, we will give two natural $U(\mathfrak{sl}_2)$ -module structures for V and discuss how they are related. This leads to a number of interesting relations involving the operator ψ and other operators associated with A, A^* . (Received September 11, 2019)

1154-15-913 **Luyining Gan*** (lzg0027@auburn.edu), **Tin-Yau Tam** and **Ming Liao**. *Differential geometry on matrix groups.*

The matrix data is often taken from a special matrix group, the mean value may lie outside the group if they are averaged using the Euclidean distance. Many authors proposed to compute matrix means using the geodesic distance defined by an invariant Riemannian metric on either the space of positive definite matrices or some special matrix groups. This talk is about some differential geometric aspects of matrix groups, related to averaging matrix data, like geodesics. Additionally, we will also present some results about curvature of matrix groups. (Received September 11, 2019)

1154-15-1025 **Rachel Grotheer*** (rachel.grotheer@goucher.edu), **Shuang Li** (shuangli@mymail.mines.edu), **Anna Ma** (anna.ma@uci.edu), **Deanna Needell** (deanna@math.ucla.edu) and **Jing Qin** (jing.qin@uky.edu). *Stochastic Methods for Low-Rank Tensor Recovery.*

The low-rank matrix recovery problem is ubiquitous in data analysis applications, specifically in signal processing. Recently, work has been done to investigate the analogous problem for higher-dimensional data sets, namely, tensors. Since rank has several different definitions in the tensor setting, depending on the decomposition of the tensor in question, many different approaches to the problem have been generated. In this talk, we examine the recovery problem for tensors with low Tucker rank. In this case, the tensor decomposition is the Tucker decomposition, one example of which is the higher-order singular value decomposition. We apply a stochastic iterative hard-thresholding algorithm to the problem and demonstrate its effectiveness. The stochastic nature of the algorithm gives it many favorable properties that allow it to outperform a standard iterative hard-thresholding algorithm, especially in the large-scale setting. (Received September 12, 2019)

1154-15-1083 **Chassidy Bozeman***, cbozeman@mtholyoke.edu. *Tree covers and the positive semidefinite maximum nullity of a graph.*

For a simple graph $G = (V, E)$, let $\mathcal{S}_+(G)$ denote the set of real positive semidefinite matrices $A = (a_{ij})$ such that $a_{ij} \neq 0$ if $\{i, j\} \in E$ and $a_{ij} = 0$ if $\{i, j\} \notin E$. The maximum positive semidefinite nullity of G , denoted $M_+(G)$, is $\max\{\text{null}(A) \mid A \in \mathcal{S}_+(G)\}$. A tree cover of G is a collection of vertex-disjoint simple trees occurring as induced subgraphs of G that cover all the vertices of G . The tree cover number of G , denoted $T(G)$, is the cardinality of a minimum tree cover. It is known that the tree cover number of a graph and the maximum positive semidefinite nullity of a graph are equal for outerplanar graphs, and it was conjectured in 2011 that $T(G) \leq M_+(G)$ for all graphs. In this talk we will present results on the tree cover number of outerplanar graphs. In particular, we characterize connected outerplanar graphs whose tree cover number and maximum nullity is equal to the upper bound $\lceil \frac{n}{2} \rceil$, where n is the number of vertices in the graph. (Received September 13, 2019)

1154-15-1114 **Emily J King***, 1874 Campus Delivery, 111 Weber Bldg, Fort Collins, CO 80523-1874, and **Sören Dittmer** and **Peter Maass**. *ReLU-Singular Values and Gaussian Mean Width in Neural Networks.*

Feedforward neural networks are simply the composition of a sequence of functions alternating between various affine linear maps and a non-linear function called an activation function. One of the most common activation functions is the rectified linear unit ReLU, which maps negative components of a vector to zero and positive components to themselves. A layer of a network consists of a single affine linear map composed with the activation function. A new concept which generalizes singular values to the necessarily non-linear maps corresponding to layers with the ReLU activation function called ReLU-singular values will be introduced. One may leverage ReLU-singular values to prune the neural network. Gaussian mean width, which was originally introduced in high dimensional geometry may be used as another mathematical tool to analyze neural networks which seems to yield information about the success of the neural network in performing the task it was trained to do. (Received September 13, 2019)

1154-15-1224 **Rachel Grotheer**, **Shuang Li**, **Anna Ma*** (anna.ma@uci.edu), **Deanna Needell** and **Jing Qin**. *Iterative Hard Thresholding for Low CP-rank Tensors.*

Recovery of low-rank matrices from a small number of linear measurements is now well-known to be possible under various model assumptions on the measurements. Such results demonstrate robustness and are backed with provable theoretical guarantees. However, extensions to tensor recovery have only recently began to be studied and developed, despite an abundance of practical tensor applications. Recently, a tensor variant of the Iterative Hard Thresholding (IHT) method was proposed and theoretical results were obtained that guarantee exact recovery of tensors with low Tucker rank. In this talk, we present an IHT approach to approximating low-CP rank tensors and discuss connections between the low rank tensor approximation problem and problems that arise in data science. (Received September 14, 2019)

1154-15-1225 **Anna Ma*** (anna.ma@uci.edu) and **Denali Molitor**. *Randomized Kaczmarz for Tensors.* When data is large-scale, techniques such as the Randomized Kaczmarz algorithm and Randomized Gauss-Seidel algorithm are advantageous for solving linear systems of the form $Ax = y$. In this talk, we discuss an extension of the Randomized Kaczmarz algorithm to the setting where large-scale data takes on the form of a multidimensional array. Traditionally, multi-dimensional data, i.e., data in the form of high dimensional tensors, are often unfolded to be treated as a matrix-vector problem. In this work, we seek to preserve the tensor structure of the input data and provide theoretical guarantees for recovering underlying, unknown tensors from linear measurements using a tensorized version of the Randomized Kaczmarz algorithm. (Received September 14, 2019)

1154-15-1284 **Thomas R Cameron*** (thcameron@davidson.edu). *On the Graph Laplacian and the Rankability of Data.*

The concept of dominance relations and its connection to ranking dates back to Dodgson's voting method and Landau's study of animal societies. Recently, Anderson et al. proposed the concept of rankability, which refers to a dataset's inherent ability to be ranked in a meaningful way. In the process, they defined a measure of rankability that is based on the number of edge additions or deletions required to change a given digraph to a complete dominance graph, i.e., an acyclic tournament graph.

In this talk, we present a spectral-degree characterization of complete dominance graphs. Then, we use this characterization to define a new measure of rankability. The effectiveness of this measure is supported by several results for complete dominance graphs regarding the conditioning of their Laplacian spectrum, the effect of a single edge change on their Laplacian spectrum, and a sharp upper bound on the Hausdorff distance between

their Laplacian spectrum and that of an arbitrary directed graph with weights between zero and one. Finally, we analyze the rankability of several datasets from the world of chess and college football. (Received September 14, 2019)

1154-15-1364 **Christina Pospisil*** (pospisil.christina@gmx.de). *Generalization Theory for Linear Algebra II: Appropriate Inverses for Non-Injective and Non-Surjective Mappings.*

Continues the talk at JMM 2019 Generalization Theory for Linear Algebra I. In this second part of the Generalization Theory for Linear Algebra appropriate inverses for non-injective mappings as well as non-surjective mappings are presented. Another step for the definition of a general determinant theory. Moreover, applications in the natural sciences are investigated and presented. (Received September 15, 2019)

1154-15-1394 **Hubertus F von Bremen*** (hfvonbremen@cpp.edu), California State Polytechnic University, 3801 West Temple Ave, Department of Mathematics and Statistics, Pomona, CA 91754, and **Alan Krinik, Saif A Aljashamy, Aaron Kim, Jeremy Lin, Thuy Vu Dieu Vu, David Perez, Mac Elroyd Fernandez and Jeffrey Yeh.** *Closed form expressions for powers and exponentials of tridiagonal Toeplitz matrices.* Preliminary report.

In this work, we present closed form expressions for powers and exponentials of tridiagonal Toeplitz matrices. We present examples illustrating the use of these closed form expressions when solving the following problems: 1. Find the probability of going from state i to state j in n -steps where i and j are in $[0, H]$ and n is any natural number and H is some fixed natural number. We assume transitions follow a birth-death chains having constant probabilities p, r, q restricted to a strip of integer states in $[0, H]$. 2. Find the probability of going from state i to state j for any time t where i and j are in $[0, E]$. where E is a constant, even natural number. We assume that transitions are governed by a birth-death process having constant birth and death rates on the integer states within $[0, E]$. 3. Find the probability that in a two-candidate election that the winner was never behind the opponent throughout the counting of the votes. Voting is modeled here according to the birth-death chain described in problem 1. (Received September 17, 2019)

1154-15-1601 **Riley Borgard*** (rborgard@purdue.edu), **Haley Duba** (haley.duba@my.wheaton.edu), **Steven N Harding** (sharding@iastate.edu), **Chloe Makdad** (cmakdad@butler.edu), **Jay Mayfield** (jmayfiel@iastate.edu), **Randal Tuggle** (randal.tuggle@vikings.berry.edu) and **Eric Weber** (esweber@iastate.edu). *Solving Linear Systems on a Distributed Network with the Kaczmarz Method.*

The Kaczmarz method is an iterative algorithm to solve a linear system of equations. The advantage of Kaczmarz over other methods lies in its robustness while other methods often require the system to be well-conditioned. In this talk, we will discuss how to use Kaczmarz on a distributed computer network and strategies to accelerate the rate of convergence. (Received September 16, 2019)

1154-15-1709 **Emily P. Devine*** (epdevi21@g.holycross.edu), **David B. Damiano** and **Richard C. Schmidt.** *Understanding Behavioral Synchrony: Differences in Behavioral Synchrony in Adolescents with Autism Spectrum Disorder using Functional EEG Networks.*

People engaged in social interactions exhibit natural, unintentional coordination of their body movements. Although intense efforts have been made to localize behaviors in human brain activity, little is known about the functional networks that underlie human social interaction. We are interested in how behavioral synchronization corresponds to functional networks in the brain. To this end, we analyzed electroencephalograph activity recorded at 64 electrode locations on the scalp from an experiment in which participant pairs swing pendulums in different interpersonal coordination conditions (in-phase, anti-phase, unintentional, and intentional). Data is collected for both adolescent participants with Autism Spectrum Disorder (ASD) and Typically Developing (TD) adolescents. Using the weighted phase lag index as a pairwise electrode coordination measure, we compare functional networks in selected frequency bands. We apply network analysis techniques to evaluate and compare the dynamic expression of subgraphs in adolescents with ASD and TD adolescents, as social disconnection is one of the suggested tendencies of individuals with ASD. Finally, our analysis lends support to the hypothesized dysfunction of the mirror neuron system with regard to ASD. (Received September 16, 2019)

1154-15-1949 **Kapila Kottegoda*** (kottegi@math.ksu.edu). *Spanning tree modulus for secure broadcast games.*

The theory of p -modulus provides a general framework for quantifying the richness of a family of objects on a graph. When applied to the family of spanning trees, p -modulus has an interesting probabilistic interpretation. In particular, the 2-modulus problem in this case has been shown to be equivalent to the problem of finding a

probability distribution on spanning trees that utilizes the edges of the graph as evenly as possible. We use this fact to produce a game-theoretic interpretation of modulus by employing modulus to solve a secure broadcast game. (Received September 16, 2019)

1154-15-2300 **Dustin G. Mixon*** (mixon.23@osu.edu). *Matching component analysis for transfer learning.*

We introduce a new Procrustes-type method called matching component analysis to isolate components in data for transfer learning. Our theoretical results describe the sample complexity of this method, and we demonstrate through numerical experiments that our approach is indeed well suited for transfer learning. (Joint work with Charles Clum and Theresa Scarnati.) (Received September 17, 2019)

1154-15-2370 **Sarah Friday***, 1001 E University Ave, Georgetown, TX 78626, and **Jordan Smith, Aaron Waclawczyk** and **Fumiko Futamura**. *Diagonalizing the Undiagonalizable.*

Calculating powers of a defective matrix can be considerably difficult for arbitrarily large matrices. It was recently shown that for any defective matrix there exists an l -lift which embeds the matrix inside a larger, diagonalizable matrix, whose powers are easily calculable. This particular lifting, however, uses Lidskii's theorem to create the diagonalization which consequently perturbs the eigenvalues. Furthermore, the lifting does not allow for powers of the original matrix to be extracted from powers of the lifted matrix. We show an alternate way of lifting a defective matrix using some of its generalized eigenvectors such that the lifted matrix retains the eigenvalues of the original and also allows for one to extract the powers of the original from the powers of the lifted matrix. (Received September 17, 2019)

1154-15-2428 **Enzo Wandler*** (enzo.wandler@wsu.edu). *α -adjacency- A generalization of Skew Adjacency.*

B. Shader and W. So introduced the idea of the skew adjacency matrix. Their idea was to give an orientation δ to a simple undirected graph G from which a skew adjacency matrix $S(G^\delta)$ is created. The α -adjacency matrix extends this idea to an arbitrary field \mathbb{F} . To study the underlying undirected graph, the average α -characteristic polynomial can be created by averaging the characteristic polynomials over all the possible orientations. In particular, a Harary-Sachs theorem for the average α -characteristic polynomial is derived and used to determine a few features of the graph from the average α -characteristic polynomial. (Received September 17, 2019)

1154-15-2464 **Michael G Luby***, International Computer Science Institute, Suite 600, Berkeley, CA 94704. *Fountain codes and their applications.*

A fountain code is an erasure code that has unprecedented scalability in two dimensions: (1) Data objects of any size can be efficiently encoded and decoded; (2) The amount of encoding data that can be generated from a data object is essentially unlimited. In a typical case, encoding data is generated from a data object and sent in packets from a sender to a receiver. Packets can be lost between the sender and receiver, and thus only a portion of the encoding data arrives at the receiver. A fountain code has optimal recovery if the data object can be recovered as long as the size of the received portion of encoding data is at least the size of the data object, independent of which portion of encoding data is received.

Based on elementary linear algebra and random graphs, we give a simple description of a fountain code.

We describe some of the conceptual and practical usages of fountain codes. Conceptual usages include reliable broadcast delivery of multimedia data, reliable real-time high speed delivery of data over large distances, reliable distributed storage, and a new internet design based on fountain codes. Practical usages include integration of fountain codes into a number of commercial standards, and a number of commercial and defense deployments. (Received September 17, 2019)

1154-15-2604 **Pawan K Gupta*** (gupta.pawan@knights.ucf.edu) and **Marianna Pensky**. *Solution of Linear Ill-Posed Problems Using Random Dictionaries.*

In the present paper, we consider an application of overcomplete dictionaries to the solution of general ill-posed linear inverse problems. In the context of regression problems, there has been an enormous amount of effort to recover an unknown function using such dictionaries. One of the most popular methods, lasso, and its versions, is based on minimizing the empirical likelihood and unfortunately, requires stringent assumptions on the dictionary, the so-called, compatibility conditions. Though compatibility conditions are hard to satisfy, it is well known that this can be accomplished by using random dictionaries. In the present paper, we show how one can apply random dictionaries to the solution of ill-posed linear inverse problems. We put a theoretical foundation under the suggested methodology and study its performance via simulations and real-data example. (Received September 17, 2019)

1154-15-2841 **Beheshteh Tolouei Rakhshan*** (btolouei@purdue.edu). *Tensor Train Random Projection.*

Random projection methods reduce the dimension of points in a high dimensional vector space while preserving the distances between the points. In this work, we use the topic of dimensionality reduction by proposing a map benefiting from random projection and a tensor train (TT) decomposition, where we call it tensor train random projection. The map is formed by the inner product of a TT-tensor and the input data, where each core tensor in TT decomposition is drawn from the independent Gaussian random variables. This work is a novel use of tensor decomposition method and random projection which requires less memory compared to existing random projection methods. We provide a theoretical analysis of the expected value and variance of the proposed map. We present that our linear map is a good dimension reduction map as it satisfies Johnson-Lindenstrauss property, i.e., it has expected value isometry and vanishing variance properties. Also, the proposed map is a data-base friendly map as it uses not-too-much storage. Our results rely on first bounding the variance of the random projection maps and using these bounds we can obtain the expected isometry property with high probability. (Received September 18, 2019)

16 ► Associative rings and algebras

1154-16-64 **Luigi Ferraro*** (ferrarl@wfu.edu) and **W. Frank Moore** (moorewf@wfu.edu). *Differential graded algebra over quotients of skew polynomial rings by normal elements.*

Differential graded algebra techniques have played a crucial role in the development of homological algebra, especially in the study of homological properties of commutative rings carried out by Serre, Tate, Gulliksen, Avramov, and others. In our work, we extend the construction of the Koszul complex and acyclic closure to a more general setting. As an application of our constructions, we show that the Ext algebra of quotients of skew polynomial rings by ideals generated by normal elements is the universal enveloping algebra of a color Lie algebra, and therefore a color Hopf algebra. As a consequence, we give a presentation of the Ext algebra when the elements generating the ideal form a regular sequence, this generalizes a theorem of Bergh and Oppermann. It follows that in this case the Ext algebra is noetherian, providing a partial answer to a question of Kirkman, Kuzmanovich and Zhang. (Received July 27, 2019)

1154-16-615 **Feroz Siddique*** (siddiquf@uwec.edu) and **Daniel Bossaller** (dbossaller@jcu.edu). *A Generalization of Exchange Rings.*

We define and explore a new class of rings called unit-exchange rings that strictly contain the class of exchange rings. An element $a \in R$ is *left unit-exchange* if there exists a unit u and an idempotent e in R such that $e - ua \in R(a - aua)$. An element $a \in R$ is defined to be *right unit-exchange* if for some unit v and an idempotent f we have $f - av \in (a - ava)R$. We will show that the definition of unit-exchange is left-right symmetric for each element. We will show several classes of rings that are unit-exchange but not exchange. It is known, due to Kaplansky, that a von Neumann regular ring R is unit regular if and only if it has stable range one. Later Camillo and Yu extended this result to exchange rings. We first show that the property of stable range one is equivalent to the property that every left (right) unit lifts modulo every left (right) principal ideal. Using this property, we generalize Camillo and Yu's result and show that for a unit-exchange ring the notion of stable range one is equivalent to the property that the ring is partially unit-regular which is equivalent to the property that the ring is left (right) uniquely generated. (Received September 08, 2019)

1154-16-675 **Sultan Eylem Toksoy*** (eylem.toksoy@gmail.com), Ankara, Turkey. *On Copure-injectively Poor Modules.*

There is a wide research on the injectivity of modules involving notions derived from relative injectivity. The recent trend is to measure how close a module being injective; one of the notion is a module which is called as a poor module. Poor modules are those modules whose domain of injectivity is as small as possible. In a similar vein, another interesting family of modules has defined in our research; the copure-injectively poor modules are those modules whose copure-injectivity domain is as small as possible. In this work we study properties of copure-injectively poor modules. Although we do not know whether copure-injectively poor modules exist over arbitrary rings, rings over which every right R -module is copure-injectively poor is shown to be right CDS rings. Rings on which poor modules and copi-poor modules are the same are characterized. Also examples showing that neither condition being a copure-injectively poor module nor being a poor module imply the other in general are given. A copure-injectively poor module need not be pure-injectively poor in general and conversely. We prove

that over commutative (co-)noetherian rings a module is pi-poor if and only if it is copi-poor. Therefore it is obtained that copi-poor abelian groups coincide with pi-poor Abelian groups. (Received September 09, 2019)

1154-16-680 **K. Radler*** (katie.radler@slu.edu), **S. Esin**, **M. Kanuni**, **A. Koç** and **K. M. Rangaswamy**. *An ideal analogue to the property $\text{lcm}(a, b) * \text{gcf}(a, b) = ab$ for Leavitt path algebras.*

One of the elementary facts about integers is that given any two integers, the product of the greatest common divisor and the least common multiple is equal to the product of those two integers. We obtain an analogue of this for ideals in Leavitt path algebras by proving that given two ideals A, B of a Leavitt path algebra, we have $(A + B)(A \cap B) = AB$. (Received September 09, 2019)

1154-16-1174 **Richard M. Green*** (rmg@euclid.colorado.edu), Department of Mathematics, University of Colorado Boulder, Campus Box 395, Boulder, CO 80309-0395. *Generalized nil Temperley–Lieb algebras and particle configurations.*

A full heap is a certain type of infinite (but locally finite) partially ordered set whose elements are labeled by the vertices of a Dynkin diagram. Any full heap can be used to define a generalized nil Temperley–Lieb algebra, which in the special case of type affine A gives rise to the usual affine nil Temperley–Lieb algebra. I will describe the general construction of these algebras and show how, in certain cases, they arise as algebras of operators on particle configurations that come from statistical physics. (Received September 13, 2019)

1154-16-1226 **Jean Auger**, **Thomas Creutzig**, **Shashank Kanade** and **Matthew Rupert***, mrupert@ualberta.ca. *Braided Tensor Categories related to B_p Vertex Algebras.*

The B_p ($p \in \mathbb{Z}_{\geq 2}$) vertex operator algebras are important examples of logarithmic CFTs and appear as chiral algebras of type (A_1, A_{2p-3}) Argyres–Douglas theories. The first member of this series, the B_2 -algebra, are the well-known symplectic bosons also often called the $\beta\gamma$ VOA. I will discuss a construction of braided, rigid, non semi-simple tensor categories related to the B_p VOAs using their conjectural relation to the unrolled restricted quantum groups of \mathfrak{sl}_2 . Simple and projective objects, and their tensor products are determined along with their Hopf links. The latter are successfully compared to modular data of characters. (Received September 14, 2019)

1154-16-1365 **Briana Foster-Greenwood*** (brianaf@cpp.edu) and **Cathy Kriloff** (krilcath@isu.edu). *Lie Orbifold Algebras for Doubled Reflection Representations of the Symmetric Group.*

Lie orbifold algebras were introduced by Shepler and Witherspoon to complete the analogy “Weyl algebras are to symplectic reflection algebras as universal enveloping algebras are to what?” In particular, Lie orbifold algebras are deformations whose parameters specialize to yield universal enveloping algebras extended by groups. By analyzing PBW conditions, we classify Lie orbifold algebras arising as deformations of the skew group algebra $S(V)\#S_n$ where the symmetric group S_n acts on the vector space V by a doubled reflection representation. The algebras in this classification provide a generalization of rational Cherednik algebras in type A which have enjoyed much attention in connection with representation theory, physics, and algebraic geometry. (Received September 15, 2019)

1154-16-1808 **Lauren Grimley** and **Christine Uhl*** (cuhl@sbu.edu). *Truncated Quantum Drinfeld Hecke Algebras and Hochschild Cohomology.*

We consider deformations of quantum exterior algebras extended by finite groups. Among these deformations are a class of algebras which we call truncated quantum Drinfeld Hecke algebras in view of their relation to classical Drinfeld Hecke algebras. We give the necessary and sufficient conditions for which these algebras occur and briefly mention the explicit connection between them and Hochschild cohomology. (Received September 16, 2019)

1154-16-1932 **Rebin A. Muhammad*** (rm775311@ohio.edu), **Pinar Aydogdu** (paydogdu@hacettepe.edu.tr) and **Sergio Lopez-Permouth** (lopez@ohio.edu). *On Two-Value Graph Algebras.*

In this research we introduce a family of algebras, which we call it two-value graph algebras. We use this new algebras to answer questions related to study of a new topic which is called modules over infinite dimensional algebras. In particular, we show that there is a family of algebras which have simple bases up to congeniality but do not have any projective basis. We will explore amenability of bases in two-value graph algebras and characterize all basic graphs that give a commutative two-value algebras. (Received September 16, 2019)

1154-16-1962 **Joshua Jeishing Wen*** (jeiwenno@gmail.com). *Elliptic shuffle algebras revisited*. Preliminary report.

I will present ongoing work began at the MRC on investigating elliptic quantum group actions on equivariant elliptic cohomology of Hilbert schemes of points on the plane. By mirroring previous work of Feigin and Tsymbaliuk in K-theory, it seems natural to consider a shuffle algebra containing theta functions of higher degrees. This structure differs from previous notions of elliptic shuffle algebras defined by various authors. (Received September 16, 2019)

1154-16-2531 **Adam Wood*** (adam-wood@uiowa.edu). *Representation Theory of the Space of Holomorphic Polydifferentials*.

When a finite group G acts on a smooth projective curve over a field, one can define the space of holomorphic polydifferentials of the curve, which provides a representation of G . It is a classical problem to determine the decomposition of this representation into indecomposable subrepresentations. We survey previous work on this problem. We then use methods from algebraic geometry to describe the structure of the space of holomorphic polydifferentials when the base field has prime characteristic p and G is a group having cyclic Sylow p -subgroups. (Received September 17, 2019)

17 ► Nonassociative rings and algebras

1154-17-355 **Matt Szczesny*** (szczesny@math.bu.edu), **Jackson Walters** and **Brian Williams**. *"Holomorphic fibrations, factorization algebras, and toroidal vertex algebras"*.

Let X be a complex manifold, $\pi : E \rightarrow X$ a locally trivial holomorphic fibration with fiber F , and \mathfrak{g} a Lie algebra with an invariant symmetric form. We associate to this data a holomorphic prefactorization algebra $\mathcal{F}_{\mathfrak{g},\pi}$ on X in the formalism of Costello-Gwilliam. When $X = \mathbb{C}$, \mathfrak{g} is simple, and F is a smooth affine variety, we extract from $\mathcal{F}_{\mathfrak{g},\pi}$ a vertex algebra which is a vacuum module for the universal central extension of the Lie algebra $\mathfrak{g} \otimes H^0(F, \mathcal{O})[z, z^{-1}]$. As a special case, when F is an algebraic torus $(\mathbb{C}^*)^n$, we obtain a vertex algebra naturally associated to an $(n+1)$ -toroidal algebra, generalizing the affine vacuum module. (Received September 02, 2019)

1154-17-391 **M Elhamdadi**, **M Saito** and **Emanuele Zappala*** (zae@mail.usf.edu). *Heap cohomology and ternary self-distributive cohomology*.

Heaps are para-associative ternary operations bijectively exemplified by groups endowed with the operation $(x, y, z) \mapsto xy^{-1}z$. They are also ternary self-distributive, and therefore have a diagrammatic interpretation by framed links. Motivated by these properties, I will introduce heap cohomology and ternary self-distributive cohomology with abelian heap coefficients, with the purpose of defining framed link cocycle invariants. I will also explain the relation intercurring between heap cohomology, and ternary self-distributive and group cohomologies. I will briefly discuss, lastly, heap objects in symmetric monoidal categories. (Received September 03, 2019)

1154-17-405 **Florencia Orosz Hunziker*** (orosz@math.harvard.edu), **Jinwei Yang**, **Thomas Creutzig**, **Cuibo Jiang** and **David Ridout**. *Tensor categories associated to the Virasoro algebra*.

We discuss the application of the logarithmic tensor product theory developed by Huang, Lepowsky and Zhang to the categories of finite length modules for the Virasoro algebra. (Received September 03, 2019)

1154-17-522 **Ismail Demir*** (idemir@ncsu.edu). *Classification of Low Dimensional Complex Nilpotent Leibniz Algebras*.

Leibniz algebras are non-antisymmetric generalization of Lie algebras. Classification of all nilpotent Lie algebras is still unsolved and is very difficult problem. Due to lack of antisymmetry in Leibniz algebras, the problem of classifying all nilpotent Leibniz algebras is more complicated. We give classification of low dimensional complex nilpotent Leibniz algebras. We use the canonical forms for the congruence classes of matrices of bilinear forms and some algebraic invariants to obtain our result. (Received September 06, 2019)

1154-17-810 **Nicholas W. Mayers*** (nwm215@lehigh.edu). *The index of Lie poset algebras*.

Incidence, or poset, algebras can be given a Lie structure by taking the commutator product. These "Lie poset algebras" have only recently been introduced into the literature by Coll and Gerstenhaber, who showed that such algebras may be regarded as subalgebras of a simple Lie algebra \mathfrak{g} which lie between a Cartan and a Borel subalgebra of \mathfrak{g} . Here, we initiate the study of the index (an algebraic invariant) of Lie poset algebras in $\mathfrak{sl}(n)$. In particular, we provide general closed-form formulas for the index of such type-A Lie poset algebras

corresponding to posets of restricted height. Furthermore, we provide a combinatorial recipe for constructing all posets corresponding to type-A Frobenius (index zero) Lie poset algebras of heights zero, one, and two. We conclude by showing how this theory can be extended to the other classical types. (Received September 10, 2019)

1154-17-816 **Latham Boyle*** (lboyle@pitp.ca), 31 Caroline Street North, Waterloo, Ontario N2L 2Y5, Canada. *The standard model of particle physics: from noncommutative geometry or Jordan geometry?*

Ever since the early work of Kaluza and Klein a century ago, physicists have been intrigued by the idea that our 4-dimensional laws of physics may be unified geometrically as purely "gravitational" laws on some appropriate extension of ordinary four-dimensional spacetime. In the traditional Kaluza-Klein picture (which is incorporated in string theory), spacetime is augmented by an "internal space" which is imagined to be a continuous (usually 6-dimensional) manifold. An awkward feature of this picture is that it predicts an infinite number of particles/fields, in addition to the finite number we actually observe. In order to capture more directly the finite spectrum of particles we observe, it is natural to instead imagine that the internal space is some sort of finite/discrete space, and the problem is to determine what sort of space this should be. Following early work by Dubois-Violette, Kerner, Madore, and Connes 30 years ago, many subsequent authors have argued that the internal space is a kind of noncommutative geometry. I will suggest that a certain type of "Jordan geometry" (based on a certain Jordan algebra) may be more appropriate. (Received September 10, 2019)

1154-17-986 **Bakhrom Omirov*** (omirovb@mail.ru), National University of Uzbekistan, 4, University street, Tashkent, 100174, Uzbekistan. *Finite-dimensional nilpotent Leibniz algebras and their derivations.*

Leibniz algebras are generalizations of Lie algebras and they have been introduced by J.-L. Loday as a non-antisymmetric version of Lie algebras. These algebras preserve a unique property of Lie algebras - the right multiplication operators are derivations. Many classical results of the theory of Lie algebras were extended to the case of Leibniz algebras. For instance, the analogue of Levi's theorem for Leibniz algebras is also true. Namely, it is proved that any finite-dimensional Leibniz algebra is decomposed into the semidirect sum of solvable radical and semisimple Lie subalgebra. Therefore, the biggest challenge in the classification problem of finite-dimensional Leibniz algebras is the study of solvable part. The method of the description of solvable Lie algebras with a given nilradical which involve outer derivations of a nilradical was extended to the case of Leibniz algebras. Therefore, the problem of classification of finite-dimensional Leibniz algebras reduces to study of nilpotent one and their derivations. In this talk we are going to present results on nilpotent Leibniz algebras and their derivations. (Received September 12, 2019)

1154-17-1001 **Abror Khakimovich Khudoyberdiyev*** (khabror@mail.ru), National University of Uzbekistan, Olmazor region Talabalar street, 4, 100174, Tashkent, Uzbekistan. *On nilpotent and solvable Leibniz superalgebras.*

Lie superalgebras have been studied as the fundamental algebraic structures behind several areas of mathematical physics in 1970s. The problem of the description of nilpotent Lie superalgebras with maximal index of nilpotency have been studied in 2004 by Gómez J.R., Khakimdjano Yu., Navarro R.M. Leibniz superalgebras are generalizations of the Leibniz algebras, they naturally also generalize Lie superalgebras. Many works have been devoted to the description of nilpotent Leibniz superalgebras. In particular, the classification of nilpotent $n + m$ -dimensional Leibniz superalgebras with maximal nilindex and with nilindex $n + m$ is obtained. It should be noted that there is a method of description of solvable Leibniz algebras using their nilradicals. We apply this method for the description of solvable Leibniz superalgebras. It this talk we are going to present results about the description of solvable Leibniz superalgebras such that nilradical is the Lie superalgebra with maximal index of nilpotency. (Received September 12, 2019)

1154-17-1216 **Dwight Anderson Williams II*** (dwightawilliams@mavs.uta.edu) and **Dimitar Grantcharov**. *Basis of an infinite-dimensional tensor product representation of $\mathfrak{osp}(1|2n)$.* Preliminary report.

We consider the complex orthosymplectic Lie superalgebra $\mathfrak{osp}(1|2n)$ acting on the super vector space $\mathbb{C}[x_1, x_2, \dots, x_n] \otimes_{\mathbb{C}} \mathbb{C}^{1|2n}$, where $\mathfrak{osp}(1|2n)$ acts via differential operators on polynomials $\mathbb{C}[x_1, x_2, \dots, x_n]$ (Weyl representation). The resulting tensor product representation decomposes into the direct sum of two simple infinite-dimensional submodules. We provide an explicit basis for each of these modules by introducing certain differential operators. (Received September 14, 2019)

1154-17-1243 **Dan D. Graybill*** (dan.graybill@du.edu). *Strong Finite Generation of the Z_n Orbifolds of the Rank 2 Heisenberg System.*

The Rank 2 Heisenberg vertex operator algebra admits an action of Z_n for all n . This talk will discuss techniques used in our proof that the resulting orbifold for $n > 2$ is strongly finitely generated by only 8 elements. The proof relies on utilizing the decomposition of the Z_n orbifold as a module over the $SO(2)$ orbifold and requires insights from a careful analysis of the associated classical situation. (Received September 14, 2019)

1154-17-1399 **Vladimir V Kovalchuk*** (vladimir.kovalchuk@du.edu). *Structure of VOAs of type $W(2, N)$.* Preliminary report.

The problem of classifying vertex algebras by strong generating type has recently received some attention. As a basic but nontrivial case, we consider the classification of vertex algebras of type $W(2, N)$, that is, vertex algebras with a strong generating set consisting of a Virasoro field, and a primary field of weight N . Well-known examples include the principal W -algebras of sl_3 , sp_4 , and G_2 , which are of types $W(2, 3)$, $W(2, 4)$, and $W(2, 6)$ respectively, the singlet algebras, and certain Virasoro minimal model extensions. We shall give the complete classification for $N \leq 7$, and also discuss some strong restrictions about what can occur in the general case. This is a joint work in progress with Andrew Linshaw and Dan Graybill. (Received September 15, 2019)

1154-17-1449 **Michael Penn*** (mpenn@randolphcollege.edu). *Permutation Orbifolds of Vertex Operator Algebras.*

Orbifolds are an important example of vertex operator algebras that have garnered significant attention recently. In this talk, we examine the permutation orbifolds of several vertex operator algebras, including the Heisenberg vertex algebra, rank one lattice vertex algebra, and the three fold tensor product of the universal Virasoro vertex algebra, including some coincidental isomorphisms as appropriate. We also present some results regarding the characters of the associated orbifolds. This is based on joint work with A. Milas and H. Shao. (Received September 15, 2019)

1154-17-1563 **O'Neill Kingston*** (oneillk@iastate.edu) and **Jonas Hartwig.** *Crystal structure on Gelfand-Tsetlin-Zhelobenko patterns.*

In this talk, we begin by presenting the crystal structure of finite-dimensional irreducible representations of the Lie algebras \mathfrak{sl}_n in terms of Gelfand-Tsetlin patterns. We then define a crystal structure using the set of symplectic Zhelobenko patterns, parametrizing bases for finite-dimensional irreducible representations of \mathfrak{sp}_4 . This is obtained by a bijection with Kashiwara-Nakashima tableaux and the symplectic jeu de taquin of Sheats and Lecouvey. We offer some conjectures on the generalization of this structure to rank n . (Received September 16, 2019)

1154-17-1635 **Nicholas V. Russoniello*** (nvr217@lehigh.edu), **Nick W. Mayers** and **Vincent E. Coll.** *Unimodality for certain families of Frobenius seaweed algebras.*

If \mathfrak{g} is a Frobenius Lie algebra, then for certain $F \in \mathfrak{g}^*$ the natural map $\mathfrak{g} \rightarrow \mathfrak{g}^*$ given by $x \rightarrow F[x, -]$ is an isomorphism. The inverse image of F under this isomorphism is called a principal element. It has recently been established that if \mathfrak{g} is a Frobenius seaweed subalgebra of a classical Lie algebra, then the spectrum of the adjoint of a principal element consists of an unbroken set of integers whose multiplicities have a symmetric distribution. Extensive simulation suggest that the spectrum is unimodal – but the proof has been elusive in all classical types. Here, we establish unimodality in type A for certain families of maximal parabolic Frobenius seaweeds. We further provide explicit formulas to compute dimensions, yielding logarithmically concave sequences. (Received September 16, 2019)

1154-17-1688 **Katrina Barron*** (kbarron@nd.edu) and **Darlayne Addabbo.** *On generators and relations for higher level Zhu algebras for certain vertex operator algebras.*

We prove some recursion relations that occur in higher level Zhu algebras for general $n \in \mathbb{N}$ for certain vertex operator algebras, and give applications for determining a set of generators as well as further relations within these Zhu algebras and related Zhu algebras. (Received September 16, 2019)

1154-17-1693 **Darlayne Addabbo*** (daddabbo@nd.edu) and **Katrina Barron** (kbarron@nd.edu). *Higher level Zhu algebras for the Heisenberg vertex operator algebra and applications.*

We present results on the structure of the level $n \geq 2$ Zhu algebras for the Heisenberg vertex operator algebra and discuss techniques used in determining the structure of these algebras. We give an example clarifying the necessity of certain conditions in defining higher level Zhu algebras, and discuss applications of these results. (Received September 16, 2019)

1154-17-2239 **Andrew R. Linshaw***, 2390 S. York St., Denver, CO 80208. *A 3-parameter vertex algebra interpolating between the diagonal cosets of \mathfrak{sl}_n* . Preliminary report.

We sketch the construction of a 3-parameter vertex algebra of type $\mathcal{W}(2, 3, 4^2, 5^2, 6^6, 7^6, 8^{16}, 9^{24}, 10^{48}, 11^{80}, 12^{164}, \dots)$, which interpolates between the diagonal cosets $\text{Com}(V^{k_1+k_2}(\mathfrak{sl}_n), V^{k_1}(\mathfrak{sl}_n) \otimes V^{k_2}(\mathfrak{sl}_n))$ for all $n \geq 3$. The construction is based on Procesi's first and second fundamental theorems of invariant theory for the adjoint representation of \mathfrak{sl}_n , together with the notion of a nonlinear Lie conformal algebra which was introduced by De Sole and Kac. (Received September 17, 2019)

1154-17-2328 **Sachin Gautam*** (gautam.42@osu.edu), Department of Mathematics, The Ohio State U., 231 W 18th Ave, Columbus, OH 43210, and **Valerio Toledano Laredo**. *Irreducible representations of elliptic quantum groups*.

In 1995 G. Felder introduced an elliptic R-matrix, which satisfies a dynamical version of the Yang-Baxter equation. The elliptic quantum group of $\mathfrak{sl}(n)$ is then defined in the same vein as the usual R-matrices give rise to quantum groups via the RTT formalism of Faddeev, Reshetikhin and Takhtajan.

In this talk I will present a generalization of Felder's construction, which works uniformly for any symmetrisable Kac-Moody algebra, analogous to Drinfeld's new presentation of Yangians and quantum affine algebras. I will also explain a method of constructing representations of the elliptic quantum group using q-difference equations. In particular, we obtain a classification of irreducible representations, in terms of elliptic analogues of Drinfeld polynomials.

This talk is based on a joint work with V. Toledano Laredo. (Received September 17, 2019)

1154-17-2556 **Fei Qi*** (fei.qi@umanitoba.ca), Department of Mathematics, University of Manitoba, 420 Machray Hall, 186 Dysart Road, Winnipeg, Manitoba R3T2N2, Canada. *Meromorphic open-string vertex algebra on the two-dimensional sphere and its modules*. Preliminary report.

Meromorphic open-string vertex algebra (MOSVA hereafter) is a noncommutative generalization introduced by Yi-Zhi Huang in 2012. He also proved that the parallel sections of the tensor algebra bundles over a Riemannian manifold naturally generate a MOSVA, where vertex operators do not satisfy commutativity if the curvature is nonzero. We hope to give a mathematical construction of 2-dimensional nonlinear sigma-model using MOSVAs and modules. In this talk, I will present the example of MOSVA and modules built from the simplest nonflat Riemannian manifold, the 2-dimensional sphere, and discuss some interesting properties. (Received September 17, 2019)

1154-17-2669 **Jonas T Hartwig*** (jth@iastate.edu). *Relative representation theory*. Preliminary report.

A commonly seen paradigm in mathematics is to replace objects by morphism from (or to) a given base object. In representation theory, this naturally leads to restriction and induction functors. In this talk we argue that carefully selecting base algebras and their corresponding category of representations can be a systematic way to approach the construction of a well-behaved theory. We will draw upon a variety of examples from associative algebras and vertex algebras. (Received September 17, 2019)

1154-17-2741 **Nicholas Davidson*** (njd@reed.edu), **Jonathan Kujawa** and **Robert Muth**. *Webs of type P*. Preliminary report.

Webs are combinatorially defined diagrams which encode homomorphisms between certain representations of Lie (super)algebras. I will discuss joint work with Jon Kujawa and Rob Muth which uses webs to give a combinatorial description of the homomorphisms between tensor products of representations of the type P Lie superalgebra. (Received September 17, 2019)

1154-17-2751 **Thomas Creutzig, Shashank Kanade** and **Robert McRae*** (robertmcræ77@gmail.com). *Gluing vertex algebras*.

We show that under suitable conditions, module categories for two vertex operator algebras can be glued along a braid-reversed tensor equivalence to obtain a vertex operator algebra extension of the tensor product of the two original algebras. Conversely, a vertex operator algebra extension of two suitable commuting subalgebras yields a braid-reversed equivalence of module categories for the subalgebras. These results hold for example when the module categories under consideration are semisimple braided ribbon categories with possibly infinitely many objects; examples arise from affine Lie algebras at admissible and generic levels. (Received September 17, 2019)

18 ► *Category theory; homological algebra*

1154-18-732 **Markus Pflaum*** (markus.pflaum@colorado.edu), Department of Mathematics, University of Colorado Box 395, Boulder, CO 80309. *Localization in Hochschild Homology and Applications.*

We revisit localization in Hochschild homology. Following an original idea by N. Teleman we show that in the case of certain algebras of functions on a singular space or the convolution algebra over a Lie groupoid localization leads to a quasi-isomorphism between the Hochschild chain complex and the complex of global sections of a complex of fine sheaves over the underlying space respectively orbit space. This sheaf complex is called the diagonal complex. In several situations the computation of its homology sheaf appears to be manageable thus leading to a solution of the original problem to determine the Hochschild homology of the function or convolution algebra. This is partially based on joint work with H.-Ch. Herbig and with H. Posthuma, and X. Tang. (Received September 10, 2019)

1154-18-906 **Janina C. Letz*** (letz@math.utah.edu). *Local to global principles for generation time over commutative rings.*

In the derived category of modules over a commutative noetherian ring a complex G is said to generate a complex X if the latter can be obtained from the former by taking finitely many summands and cones. The number of cones needed in this process is the generation time of X . In this talk I will present some local to global type results for computing this invariant, and also discuss some applications of these results. (Received September 11, 2019)

1154-18-1262 **Colleen Delaney*** (crdelane@iu.edu). *Fusion rules for permutation defects in multilayer topological order.*

Although permutation orbifolding in conformal field theory is well established, a thorough understanding of its algebraic counterpart under the bulk-boundary correspondence of (2+1)D TQFTs and (1+1)D CFTs - namely permutation gauging of modular tensor categories - remains under development. It was only recently by the work of Corey Jones and Terry Gannon that the existence of these permutation gaugings was confirmed mathematically (see Corey Jones' talk also in the Special Session on Mathematical Aspects of CFT).

Motivated by the physics of anyons and symmetry defects in symmetry-enriched topological phases of matter, we show how to derive the fusion rings for permutation extensions of modular tensor categories from elementary considerations. This gives a foundation for a concrete construction of permutation gaugings at the combinatorial level and beyond. (Received September 16, 2019)

1154-18-2063 **John C. Baez, John D. Foley*** (foley@metsci.com) and **Joe Moeller**. *Network models from Petri nets with catalysts.*

Petri nets are a widely studied formalism for describing how collections of entities of different types can be processed to turn into other entities. More recently, network models were developed as a formal foundation for designing and tasking networks of agents. This work combines these two formalisms to serve complementary roles. We show how Petri nets with catalysts—whose counts at fixed places are preserved, like catalysts in chemistry—can be taken apart and put back together again with network models. Catalysts can be viewed as agent types that provide services for other types in the network. The “taking apart” step filters the Petri net by what can be done when some specific number of catalysts are available. The “putting back together” step is accomplished with the monoidal Grothendieck construction and has the upshot that catalysts—i.e. agents providing services—can be distinguished after this step. (Received September 17, 2019)

1154-18-2182 **David Weisbart** and **Adam Yassine***, University of California, Riverside, 900 University Ave., Riverside, CA 92521. *Generalized Span Categories in Classical Mechanics.*

Category theory is a language that is capable of reducing complicated problems to a very basic form. While the last several centuries have seen an intensive study of classical mechanics, many philosophical questions about classical mechanics remain that category theory may potentially answer. The current work initiates a study of the categorical structures that underlie classical mechanics. While span categories are useful in the categorification of systems, they do not appear capable of describing the composition of classical mechanical systems. We introduce the notion of a generalized span category and an augmentation of a generalized span category. We construct the augmented generalized span categories LagSy and HamSy that respectively provide a categorical framework for the Lagrangian and Hamiltonian descriptions of certain simple classical systems. The morphisms in the appropriate category contain all kinematical and dynamical information about these physical systems and composition of morphisms corresponds to the construction of systems from subsystems. A functor

from LagSy to HamSy translates between the Lagrangian and Hamiltonian perspectives. (Received September 17, 2019)

1154-18-2377 **Eric Berry, Daniel Berwick-Evans, Emily Cliff, Laura Wells Murray*** (lwells@nd.edu), **Apurva Nakade, Emma Phillips and Robert Joseph Rennie.** *2-groups and bundles on the moduli space of elliptic curves.* Preliminary report.

In this talk we will introduce the notion of 2-groups, a categorification of groups, and define 2-group bundles. Specifically looking at 2-group \mathcal{G} coming from an ordinary group G , we consider \mathcal{G} -bundles on elliptic curves, and construct a $U(1)$ -bundle on the moduli stack of principal G -bundles on elliptic curves. We compare our construction to the classical construction via transgression methods. (Received September 17, 2019)

1154-18-2383 **Laura Wells Murray*** (lwells@nd.edu). *Higher categories and factorization algebras.* Factorization algebras are a mathematical tool for modeling the observables of classical and quantum field theories. In this talk I will define equivariant factorization algebras, and compare them to a geometric version of factorization algebras which allows one to incorporate structures helpful for modeling physically relevant field theories. I will touch on the higher categorical structure that is involved in this comparison and how this can be used to describe a parameterized, or ‘smooth’, version of factorization algebras. (Received September 17, 2019)

1154-18-2453 **Radmila Sazdanovic*** (rsazdan@ncsu.edu), Department of Mathematics NC State University, PO Box 8205, Raleigh, NC 27695, and **Mikhail Khovanov.** *Diagrammatic categorification of the polynomial ring $\mathbb{Z}[x]$.*

We develop a diagrammatic categorification of the polynomial ring $\mathbb{Z}[x]$ that leads to categorification of some families of orthogonal polynomials including Chebyshev and Hermite polynomials. Our categorifications satisfy a version of the Bernstein-Gelfand-Gelfand reciprocity property with the indecomposable modules corresponding to the monomials and standard modules to the orthogonal polynomials in the Grothendieck ring. (Received September 17, 2019)

1154-18-2470 **Michael Horst*** (horst.59@osu.edu). *A Categorification of Group Cohomology.* Preliminary report.

We present a categorification of group cohomology by way of Picard categories, which are to be thought of as categories that behave like Abelian groups. After reviewing background information on Picard categories, we will present a construction for free Picard categories generated by groupoids and will do so by using a tensoring over groupoids to demonstrate the appropriate free-forgetful adjunction. We continue by discussing the framework for this cohomology of Picard categories, including categorical modules and derived functors. We will close by using free Picard categories to form projective resolutions and compute various specific examples. (Received September 17, 2019)

19 ► *K*-theory

1154-19-302 **Sayan Chakraborty, Xiang Tang*** (xtang@math.wustl.edu) and **Yijun Yao.** *Smooth Connes–Thom isomorphism, cyclic homology, and equivariant quantisation.* Preliminary report.

Using a smooth version of the Connes–Thom isomorphism in Grensing’s bivariant K -theory for locally convex algebras, we prove an equivariant version of the Connes–Thom isomorphism in periodic cyclic homology. As an application, we prove that periodic cyclic homology is invariant with respect to equivariant strict deformation quantization. (Received August 29, 2019)

1154-19-598 **Terry A Loring***, 1 University of New Mexico, Department of Mathematics and Statistics, MSC01 1115, Albuquerque, NM 87131. *A numerical approach to Chern numbers and unbounded Fredholm modules.* Preliminary report.

A quasiperiodic Hamiltonian H , where Hilbert space is built on the vertices of a quasiperiodic tiling of the plane, can exhibit the properties of a Chern insulator. Calculating the gaps in $\sigma(H)$ and labeling these by K -theory can be a challenge. Forming a Dirac-type operator from H and the position operators X and Y gives a way to relate a Fredholm index with the signature a finite-dimensional, but large, matrices. Hence the necessity of numerical K -theory.

Some of the work to be presented was joint with Hermann Schulz-Baldes (Received September 08, 2019)

1154-19-676 **Virgil Chan***, Department of Mathematical Sciences, IUPUI, 402 North Blackford, LD 270, Indianapolis, IN 46202. *An Explicit Formula for the Loday Assembly*. Preliminary report.

We give an explicit description of the Loday assembly map on homotopy groups when restricted to a certain subgroup coming from the Atiyah-Hirzebruch Spectral Sequence. This proves and generalises a formula about the Loday assembly map on the first homotopy group written down in the survey article by Lück and Reich. Furthermore, we use this new result to prove the Loday assembly for the integral group ring of cyclic group of order 2 is a bijection on the second homotopy group, and is an injection when one considers n -fold direct sum of cyclic group of order 2. (Received September 09, 2019)

1154-19-860 **Anna Duwenig*** (aduwenig@uovic.ca) and **Heath Emerson**. *Non-commutative Poincaré duality of the irrational rotation algebra*.

The irrational rotation algebra is known to be Poincaré self-dual in a KK-theoretic sense. The required K-homology fundamental class was constructed by Connes out of the Dolbeault operator on the 2-torus, but so far, there has not been an explicit description of the dual element. In this talk, I will geometrically construct a finitely generated projective module representing said K-theory class, by using a pair of transverse Kronecker flows on the 2-torus. (Received September 11, 2019)

1154-19-1279 **Sarah L Browne*** (slbrowne@ku.edu). *E-theory spectra*.

E-theory is an invariant of C^* -algebras and in particular is a collection of abelian groups defined in terms of homotopy classes of certain morphisms of C^* -algebras. This makes it a natural object to define in terms of stable homotopy groups. In my talk I will detail the framework we require, namely a spectrum of quasi-topological spaces, to represent the E-theory groups as a stable homotopy theory. I will highlight how we encode the Bott map and the product structure of E-theory in this construction. (Received September 14, 2019)

1154-19-2530 **Zachary J. Garvey*** (zachary.j.garvey.gr@dartmouth.edu). *Index Theory on Contact Manifolds with a Groupoid Action*. Preliminary report.

Heisenberg-elliptic operators are not elliptic, and consequently, their Fredholm index cannot be computed along the lines of the Atiyah-Singer index theorem. Erik van Erp and Paul Baum proved an index theorem for such operators using tools from non-commutative geometry. We outline a generalization of this theorem to the groupoid-equivariant setting. This verifies, as special cases, families and group-equivariant versions of the theorem. (Received September 17, 2019)

20 ► Group theory and generalizations

1154-20-92 **Zachary Mesyan***, Department of Mathematics, University of Colorado, Colorado Springs, CO 80918, and **James D. Mitchell** and **Yann H. Péresse**. *Topological Transformation Monoids*.

We investigate semigroup topologies on the full transformation monoid X^X of an infinite set X . We show that the standard pointwise topology is the weakest Hausdorff semigroup topology on X^X , show that this topology is the unique Hausdorff semigroup topology on X^X that induces the pointwise topology on the group $\text{Sym}(X)$ of all permutations of X , and construct $|X|$ distinct Hausdorff semigroup topologies on X^X . In the case where X is countable, we prove that the pointwise topology is the only Polish semigroup topology on X^X . (Received August 01, 2019)

1154-20-113 **Casey Donovan** (cdonoven@binghamton.edu) and **Luise-Charlotte Kappe*** (menger@math.binghamton.edu). *Finite coverings of algebraic structures I*.

We say a group has a finite covering if it is the union of finitely many proper subgroups. The minimal number of subgroups needed is called the covering number of the group. Finite covering have been investigated in other algebraic structures, such as loops, rings, and semigroups. No loop and hence no group or ring has covering number two. However, for every integer $n > 2$, there exists a loop with covering number n . The situation for groups is different. It is known which integers $n \leq 129$ are covering numbers. For rings, the situation is similar to the one for groups. The smallest integer > 2 for which it is not known whether it is the covering number of a ring or not is 13. However, the situation for semigroups is different. There exist semigroups with covering number two. This led us into further investigations of finite coverings of semigroups. (Received August 09, 2019)

1154-20-114 **Casey Donovan*** (cdonoven@binghamton.edu) and **Luise-Charlotte Kappe** (menger@math.binghamton.edu). *Finite coverings of algebraic structures II.*

The covering number of a semigroup S with respect to semigroups, $\sigma_s(S)$, is the minimum number of proper subsemigroups of S whose union is S . We have a complete characterization of covering numbers of finite semigroups. When S is a finite semigroup that is neither a group nor generated by a single element, $\sigma_s(S) = 2$; when S is a finite group, the covering number of S with respect to semigroups is equal to the covering number with respect to groups. The case of infinite semigroups is largely unsolved outside of specific cases. One worth highlighting is the following: for an infinite group G , $\sigma_s(G) = 2$ if and only if G has a non-trivial left-orderable quotient. (Received August 09, 2019)

1154-20-438 **John C Meakin*** (jmeakin@unl.edu), Department of Mathematics, University of Nebraska-Lincoln, Lincoln, NE 68588, and **Zhengpan Wang** (zpwang@swu.edu.cn), School of Mathematics and Statistics, Southwest University, Chongqing, Peoples Rep of China. *Graph inverse semigroups and Leavitt path algebras.* Preliminary report.

For each directed graph we construct an associated inverse semigroup that we call the *Leavitt inverse semigroup* of the graph. Leavitt inverse semigroups are closely related to Leavitt path algebras and graph inverse semigroups. They provide a certain amount of structural information about Leavitt path algebras. For example two graphs that have isomorphic Leavitt inverse semigroups have isomorphic Leavitt path algebras, but the converse is false. We study some topological aspects of the structure of graph inverse semigroups and Leavitt inverse semigroups and we provide necessary and sufficient conditions for two graphs to have isomorphic Leavitt inverse semigroups. (Received September 04, 2019)

1154-20-518 **Trevor Elliott Jack*** (trevor.jack@colorado.edu) and **Lukas Fleischer.** *Computational Complexity of Semigroup Properties.*

We will describe a nondeterministic logspace algorithm for the following class of problems. Given generators for a finite transformation semigroup, determine whether the semigroup satisfies a fixed equation. (Received September 06, 2019)

1154-20-551 **James Belk** and **Matthew C. B. Zaremsky*** (mzaremsky@albany.edu). *Twisted Brin-Thompson groups.* Preliminary report.

In joint work with Jim Belk we introduce versions of Brin-Thompson groups that are “twisted” by a group G . These are related to Röver-Nekrashevych groups $V_d(G)$ for G a self-similar group, except that for twisted Brin-Thompson groups, G can be any group. Twisted Brin-Thompson groups are always simple, and allow us to prove various results, for example, 1) every finitely generated group embeds as a quasi-isometrically embedded subgroup of a finitely generated simple group, 2) for each n there exists a simple group of type F_{n-1} but not F_n that contains every right-angled Artin group (RAAG), and 3) there exists a simple group of type F_∞ containing every RAAG. Since the class of groups embeddable into RAAGs is quite robust, the same is true for groups embeddable into our groups. (Received September 06, 2019)

1154-20-572 **Janusz Konieczny*** (jkoniecz@umw.edu), University of Mary Washington, Department of Mathematics, Fredericksburg, VA 22401. *Maximal Commutative Subsemigroups of a Finite Semigroup.*

Let S be a finite non-commutative semigroup. This talk will be concerned with the problem of finding maximal commutative subsemigroups of S , that is, commutative subsemigroups of S that are not properly included in any commutative subsemigroup of S .

For an element $a \in S$, denote by $C^2(a)$ the *second centralizer* of a in S , which is the set of all elements $b \in S$ such that $bx = xb$ for every $x \in S$ that commutes with a . It is clear that $C^2(a)$ is a commutative subsemigroup of S . Let M be any maximal commutative subsemigroup of S . Then $C^2(a) \subseteq M$ for every $a \in M$. We define the *commutative rank* (*c-rank*) of M as the minimum cardinality of a set $A \subseteq M$ such that $\bigcup_{a \in A} C^2(a)$ generates M .

I will present a general approach to the problem of finding maximal commutative subsemigroups of S of *c-rank* 1 and 2. Note that if S is a finite group, then the commutative subsemigroups of S are the abelian subgroups of S . Let S_n be the symmetric group on n elements. I will use the general approach to determine the maximal abelian subgroups of S_n of *c-rank* 1 and describe a class of maximal abelian subgroups of S_n of *c-rank* at most 2. (Received September 07, 2019)

1154-20-617 **J.P. Cossey*** (cossey@uakron.edu). *On the diameter of blocks of solvable groups.*

Let B be a p -block of a solvable group G . There is a natural graph on the irreducible Brauer characters of B where two characters θ and φ are adjacent if there exists an ordinary irreducible character χ such that $d_{\chi\varphi} \neq 0 \neq d_{\chi\theta}$.

We discuss some bounds on the diameter of this graph that extend earlier work. Time permitting, we compare our results on solvable groups to recent results on symmetric groups. (Received September 08, 2019)

1154-20-755 **Heiko Dietrich** and **Alexander Hulpke*** (hulpke@colostate.edu), Department of Mathematics, Colorado State University, Fort Collins, CO 80526. *Finding finite representations of finitely presented groups*. Preliminary report.

A principal tool for the study of finitely presented groups is to find representations through so-called quotient algorithms. Such algorithms take an existing quotient $\varphi: G \rightarrow Q$, and find homomorphisms $\psi: G \rightarrow N.Q$ for extensions $N.Q$ with N elementary abelian, such that φ factors through ψ . The p -quotient and the solvable quotient algorithms are examples of such algorithms, and have been used for a long time. Using a generalization of the p -covering group that arises from the radical factor of the representation module, and is parameterized by irreducible module type and number of generators, we can generalize the idea of the existing algorithms to the case of an arbitrary, nonsolvable (finite) Q . I will describe the algorithm and give examples of its performance. (Received September 10, 2019)

1154-20-801 **Aparna Upadhyay*** (aparnaup@buffalo.edu), 244 Mathematics Building, University at Buffalo, SUNY, Buffalo, NY 14260. *The Benson - Symonds invariant for Permutation modules*.

In a recent paper Dave Benson and Peter Symonds defined a new invariant $\gamma_G(M)$ for a finite dimensional module M of a finite group G . This invariant measures the non-projective proportion of $M^{\otimes n}$ in the limit and hence quantifies how close the module is to being projective. This invariant is not known for any infinite class of modules till date. In this talk, we will see some interesting properties of this invariant. We will go on to determine this invariant for permutation modules of the symmetric group corresponding to two-part partitions and present a combinatorial formula for the same using tools from representation theory and combinatorics. (Received September 10, 2019)

1154-20-841 **Michael Kinyon*** (mkinyon@du.edu), Department of Mathematics, 2390 S York St, University of Denver, Denver, CO 80208. *Conjugacy and the center of inverse monoids*.

In an inverse monoid S , there is a naive notion of what it means for two elements a, b to be conjugate: there exists $g \in S$ such that $b = g^{-1}ag$ and $a = gbg^{-1}$. This notion reveals more structure than at first meets the eye. For each $g \in G$, the (well-defined) map $\phi_g: gSg^{-1} \rightarrow g^{-1}Sg; a \mapsto g^{-1}ag$ turns out to be an *inner partial automorphism* of S . The mapping $\Phi: S \rightarrow \text{Inn}(S); g \mapsto \phi_g$ from S to the *inner partial automorphism monoid* of S is a surjective homomorphism. The kernel of Φ is the *central congruence* of S and the corresponding normal inverse subsemigroup is $Z(S) = \{a \in S \mid axa^{-1}a = aa^{-1}xa, \forall x \in S\}$, the (true) *center* of S .

In this talk, I'll elaborate on this and also discuss the connection with marginal inverse subsemigroups and how this is all generalized beyond inverse semigroups. This is joint work with David Stanovský (Charles Univ. Prague) and the generalizations (if I get to them) are joint work with João Araújo (Nova Univ. Lisbon), Janusz Konieczny (Mary Washington College) and António Malheiro (Nova Univ. Lisbon). (Received September 11, 2019)

1154-20-874 **David Milan*** (dmilan@uttyler.edu), **Aria Beaupré**, **Anthony Dickson** and **Christin Sum**. *On inverse hulls of Markov subshifts*. Preliminary report.

We consider the problem of determining when the inverse hulls of two Markov subshifts are isomorphic. For three important invariants—the entropy, the Bowen-Franks group, and the period—we give examples of Markov subshifts that differ in the invariant, yet give isomorphic inverse hulls.

The key tool in building the examples is an abstract characterization of the inverse semigroups that are isomorphic to inverse hulls of Markov subshifts. (Received September 11, 2019)

1154-20-956 **Thibault Godin*** (thib.godin@gmail.com). *Some new tools and examples for automaton groups of intermediate growth*.

Mealy automata are powerful tools to generate groups with unusual properties. Since the eighties, they have been used to solve several important group theoretical questions, and are especially ubiquitous in problems related to growth. The growth of a group (namely the function counting the number of elements in balls of growing radii in the Cayley graph) is a geometric way to understand and classify groups. The first example of a group of intermediate growth (whose growth function is bigger than any polynomial but smaller than any exponential function), was given by Grigorchuk in 1983 as an automaton group, and since these automaton groups have been the inspiration for all known examples of groups with intermediate growth. I will briefly explain how the underlying automaton structure gives a powerful leverage to understand it, and how this is amenable to computer experimentation. Then I will give several new examples of groups with intermediate growth and compare them

with previously known examples. based on an on-going work joint with Jérémie Brioussel (Received September 12, 2019)

1154-20-1031 **Farid AliniaEIFARD*** (aliniaEIFARD.farid@gmail.com), 2106-1651 Hardwood Street, Vancouver, BC V6G 1Y2, Canada. *Representation theory and combinatorial Hopf structures*. Preliminary report.

The concept of Hopf algebras originated from the theory of algebraic groups and algebraic topology in the mid 20th century. Hopf structures have numerous applications in many other mathematical branches, and now it is a familiar concept in representation theory as the class functions and superclass functions of some tower of groups have Hopf structures. In these Hopf structures, representation theoretic functors give the product and coproduct. In this talk, we give a brief introduction to normal lattice supercharacter theories, and then we construct a Hopf structure by using these supercharacter theories. (Received September 12, 2019)

1154-20-1111 **Rose Morris-Wright*** (rmorriswright@brandeis.edu). *An analogy of the curve complex for Artin groups of FC type*.

The curve complex is an important geometric construction for studying the mapping class group of a topological surface. This talk will construct an analogous simplicial complex for Artin groups of FC type. Artin groups are a generalization of braid groups that provide a rich field of examples and counter-examples for many algebraic, geometric, and topological properties. The complex of parabolic subgroups generalizes the curve complex for braid groups, when braid groups are considered as the mapping class groups of the punctured disc. I will discuss properties that the curve complex and the complex of parabolic subgroups share as well as some open questions about Artin groups that the complex of parabolic subgroups might address. (Received September 13, 2019)

1154-20-1167 **Rachel Skipper and Benjamin Steinberg*** (bsteinberg@ccny.cuny.edu). *Bireversible automata generating lamplighter groups*.

Grigorchuk and Zuk (2000) realized the lamplighter group $\mathbb{Z}_2 \wr \mathbb{Z}$ as an automaton group and performed spectral computations that led to a counterexample to the strong Atiyah conjecture on ℓ_2 -Betti numbers.

Shortly after Silva and Steinberg gave a construction of $A \wr \mathbb{Z}$ as an automaton group for any finite abelian group A using reversible automata and a representation as rational power series over finite rings.

In 2015 Bondarenko, D'Angeli and Rodaro realized the lamplighter group $\mathbb{Z}_3 \wr \mathbb{Z}$ as a bireversible automaton group. Previously known bireversible automaton groups tended to be non-amenable or virtually nilpotent, so this was quite a surprise.

Bondarenko and Savchuk announced a construction of $A \wr \mathbb{Z}$ as a bireversible automaton group for A an elementary abelian p -group with p an odd prime using rational power series over finite fields.

In this talk we discuss realizations of lamplighter groups $A \wr \mathbb{Z}$ with A finite abelian as rational series over finite rings. We show that bireversibility seems to impose a constraint on the 2-Sylow subgroups. (Received September 13, 2019)

1154-20-1169 **Robert D Gray and Benjamin Steinberg*** (bsteinberg@ccny.cuny.edu). *Report on homological finiteness conditions for one-relator monoids*. Preliminary report.

The word problem for one-relator monoids is a longstanding open question. Kobayashi asked in 2000 whether every one-relator monoid admits a finite complete rewriting system. A necessary condition to have a finite complete rewriting system is to satisfy the homological finiteness condition FP_∞ . Kobayashi also asked in 2000 whether every one-relator monoid is of type FP_∞ and proved that each one-relator monoid is of type FP_3 .

In this talk, we report on our progress toward proving that every one-relator monoid is indeed of type FP_∞ . Our techniques combine methods from algebraic topology (monoids acting on CW complexes), homological algebra and the theory of monoid van Kampen diagrams. (Received September 13, 2019)

1154-20-1193 **Bhama Srinivasan and C. Ryan Vinroot*** (vinroot@math.wm.edu). *Galois group action on Lusztig parameters*.

Consider a connected reductive group \mathbf{G} with connected center, defined over the algebraic closure of a finite field, and let F be a Frobenius map on \mathbf{G} with $G = \mathbf{G}^F$ the finite group of rational points. If χ is an irreducible complex character of G , we study the action on χ by an element of the absolute Galois group $\text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ on the character values of χ . In particular, if χ is described by Lusztig parameters (or Jordan decomposition), then we give the Lusztig parameters of the image of χ under the Galois action. As a result, we obtain a criterion for χ to be rational-valued. We will also discuss recent developments on similar results in some cases when the center of \mathbf{G} is disconnected, which is joint work with Mandi Schaeffer Fry and Stephen Trefethen. (Received September 13, 2019)

1154-20-1234 **Carolyn Abbott*** (cra2112@columbia.edu). *Ordering actions on hyperbolic spaces.*

Every group admits at least one action by isometries on a hyperbolic metric space, that is, a metric space in which all triangles are uniformly thin. Moreover, certain classes of groups admit many different actions on different hyperbolic metric spaces (in fact, often uncountably many). One such class of groups is the class of so-called acylindrically hyperbolic groups, which contains many interesting groups, such as mapping class groups, $\text{Out}(F_n)$, and right-angled Artin and Coxeter groups, among many others. In this talk, I will describe how to put a partial order on the set of actions of a given group on hyperbolic spaces which, in some sense, measures how much information about the group the action provides. This partial order defines a “poset of actions” of the given group. I will then describe the class of acylindrically hyperbolic groups and give some structural properties of the resulting poset of actions for such groups. (Received September 14, 2019)

1154-20-1248 **Justin Cantu*** (justincc@math.tamu.edu). *Periodic groups via orbital graphs.*

We introduce a method of proving when an infinite group of homeomorphisms of a Cantor set is periodic using the geometry of its orbital graphs. In doing so, we expand a recent class of infinite finitely generated periodic groups introduced by Volodymyr Nekrashevych. In particular, we generalize his concept of fragmentation to arbitrary groups of homeomorphisms of a Cantor set, and give examples of finitely generated groups that can be fragmented to produce groups of Burnside type. (Received September 14, 2019)

1154-20-1368 **James Belk, Collin Bleak, Martyn Quick and Rachel Skipper*** (skipper.26@osu.edu), 100 Math Tower, 231 W 18th Ave, Columbus, OH 43210. *Maximal subgroups of Higman-Thompson groups.* Preliminary report.

We define certain labellings of the infinite regular rooted tree and use them to find a large class of maximal subgroups of Higman-Thompson groups. (Received September 15, 2019)

1154-20-1393 **Robert Boltje*** (boltje@ucsc.edu) and **Philipp Perepelitsky** (philipp.perepelitsky@gmail.com). *p-permutation equivalences and auto-equivalences.*

Let $A \subseteq FG$ and $B \subseteq FH$ be two block algebras of finite groups G and H over an algebraically closed field F of characteristic $p > 0$. A p -permutation equivalence between A and B is an element in the representation group of (A, B) -bimodules that are direct summands of permutation bimodules and have twisted diagonal vertices. These equivalences, although only elements in a representation group, preserve all relevant invariants of blocks. We recall definitions and properties of p -permutation equivalences and report on new work on the finite group of auto-equivalences of blocks with a complete result for cyclic defect groups. This is joint work with Philipp Perepelitsky. (Received September 15, 2019)

1154-20-1530 **Nicolás Matte Bon***, EPFL SB MATH EGG, MA B3 514 (Bâtiment MA), Station 8, CH-1015 Lausanne, Switzerland, and **Michele Triestino**. *Groups of homeomorphisms of mapping tori over the Cantor set.*

To every Cantor dynamical system (i.e. a homeomorphism of the Cantor set), we associate a group of homeomorphisms on the real line. It arises as a group of homeomorphisms of the suspension flow of the dynamical system which preserves every orbit of the flow and acts on it by piecewise linear dyadic homeomorphisms. This gives a simple construction of groups that satisfy interesting properties among groups of homeomorphisms of real line, including simplicity and finite generation, and a fixed point property for actions on the circle. (Received September 16, 2019)

1154-20-1569 **Shawn T. Burkett*** (sburket1@kent.edu) and **Mark L. Lewis**. *GVZ-groups.*

A finite group G is called a GVZ-group if every character $\chi \in \text{Irr}(G)$ vanishes on $G \setminus Z(\chi)$, and is called flat if every conjugacy class is a coset of some subgroup. We will show that these two notions coincide, thereby obtaining a character-free definition of GVZ-groups. We obtain several other characterizations of GVZ-groups, and then use a Taketa-type argument to prove that the nilpotence class of a GVZ-group (such groups are necessarily nilpotent) is bounded above by the number of distinct degrees of its irreducible characters. (Received September 16, 2019)

1154-20-1571 **David Vogan***, Room 2-355, MIT, 77 Massachusetts Avenue, Cambridge, MA 02139. *Local Langlands conjecture for finite groups of Lie type.* Preliminary report.

In the late 1960s, Langlands’ study of automorphic forms led him to a remarkable conjecture about the representation theory of reductive groups over local fields. The simplest and most fundamental case of this conjecture says that if F is any local field, then the (infinite-dimensional) irreducible representations of $GL(n, F)$ are indexed (approximately) by the n -dimensional representations of the Galois group of F .

In the 1970s, Macdonald formulated and proved an analogue of Langlands’ conjecture for the finite group $GL(n, \mathbb{F}_q)$. I will explain how one can extend Macdonald’s formulation to any finite group of Lie type; what

results of Lusztig and Shoji offer toward proof of this extension; and how these questions are related to Langlands' (still unproven!) conjecture about local fields. (Received September 16, 2019)

1154-20-1609 **Petr Vojtěchovský*** (petr@math.du.edu), Department of Mathematics, University of Denver, 2390 S York St, Denver, CO 80208. *A large scale automated deduction project in nonassociative algebra*. Preliminary report.

An easy theorem in group theory states that a group modulo its center is isomorphic to the group of inner automorphisms. The ultimate goal of this project is to establish a generalization of the above result in the theory of loops. I will briefly survey results obtained by traditional means and then focus on recent developments in which automated deduction plays a key role. This is a large scale project from computational perspective and joint work with Michael Kinyon, J.D. Phillips and Robert Veroff. (Received September 16, 2019)

1154-20-1675 **Francis J Pastijn*** (francis.pastijn@marquette.edu), Mathematical and Statistical Sciences, Marquette University, Milwaukee, WI 53201-1881. *Bands and symmetry*.

The class of semigroups that can be embedded into a semigroup which has a transitive automorphism group forms a quasivariety of semigroups. Every semigroup in this quasivariety is either a band or is idempotent free. It can be shown that every band B can be embedded into a band B' which has a transitive automorphism group and such that B' is in turn embedded into a power of B : thus B and B' generate the same prevariety (and therefore also the same quasivariety, the same variety). If a subgroup of the automorphism group of a band B has the D -classes of B as its orbits, then B is a regular band. One shows that conversely, every regular band can be embedded into a regular band B such that $\text{Aut } B$ has a subgroup G where (i) the D -classes of B are the orbits of G , (ii) every element of B belongs to a semilattice transversal of B , and (iii) G acts as a transitive permutation group on the set of semilattice transversals. (Received September 16, 2019)

1154-20-1719 **Volodymyr Nekrashevych*** (nekrash@math.tamu.edu), TX. *Constructing simple groups using dynamical systems*. Preliminary report.

We will show how minimal dynamical systems and étale groupoids can be used to construct finitely generated simple groups with prescribed properties. For example, one can show that there are uncountably many different growth types (in particular quasi-isometry classes) among finitely generated simple groups, or embed the Grigorchuk group into a simple torsion group of intermediate growth. Other properties like torsion and amenability will be also discussed. (Received September 16, 2019)

1154-20-1876 **Meghan M. De Witt*** (mdewitt@stac.edu), Sparkill, NY 10976. *Using group-like interactions to motivate undergraduate research projects*. Preliminary report.

We explore several recent group theory based and group-like research projects accessible to undergraduate students. This includes studying the interactions of the symmetry groups of hypercubes, the group-like structure of swarms, and the group-like interactions that create the magnetosphere of the earth. These projects are designed for under-prepared students to have easy access and are inspired by scientific phenomenon. We explain how we were able to find and create such interdisciplinary problems for students to be able to tackle with little to no background knowledge. (Received September 16, 2019)

1154-20-1938 **Elizabeth Field*** (ecfield2@illinois.edu). *Trees, dendrites, and the Cannon-Thurston map*.

When $1 \rightarrow H \rightarrow G \rightarrow Q \rightarrow 1$ is a short exact sequence of three word-hyperbolic groups, Mahan Mitra (Mj) has shown that the inclusion map from H to G extends continuously to a map between the Gromov boundaries of H and G . This boundary map is known as the Cannon-Thurston map. In this context, Mitra associates to every point z in the Gromov boundary of Q an "ending lamination" on H which consists of pairs of distinct points in the boundary of H . We prove that for each such z , the quotient of the Gromov boundary of H by the equivalence relation generated by this ending lamination is a dendrite, that is, a tree-like topological space. This result generalizes the work of Kapovich-Lustig and Dowdall-Kapovich-Taylor, who prove that in the case where H is a free group and Q is a convex cocompact purely atoroidal subgroup of $\text{Out}(F_n)$, one can identify the resultant quotient space with a certain \mathbb{R} -tree in the boundary of Culler-Vogtmann's Outer space. (Received September 16, 2019)

1154-20-1942 **Kate Juschenko*** (kate.juschenko@gmail.com), 3606 Highland View, Austin, TX 78731, and **Peter Burton**. *Representations of products of free groups*. Preliminary report.

I will discuss representations of products of free groups, Connes' embedding problem and extensions of transport operators on free group. (Received September 17, 2019)

1154-20-1950 **Rostyslav Kravchenko*** (rkchenko@utexas.edu). *On the rigidity of rank gradient in a group of intermediate growth.*

We introduce and investigate the rigidity property of rank gradient in the case of the Grigorchuk group. We show that it is normally ($\log, \log \log$)-RG rigid. This is a joint work with R. Grigorchuk. (Received September 16, 2019)

1154-20-2049 **Stephen Trefethen*** (sjtrefethen@wm.edu). *Finite groups with the CUT property.*

A finite group G is said to have the CUT property if the integral group ring $\mathbb{Z}G$ has only trivial central units, where a central unit is trivial if it is of the form $\pm g$ for some $g \in Z(G)$. The question of classifying all finite groups with the CUT property was posed by Goodaire and Parmenter in 1986. Since then, abelian groups, metacyclic groups, nilpotent groups, and solvable groups with the CUT property have been classified. In this talk we present a complete list of the possible non-abelian composition factors of non-solvable finite groups with the CUT property. (Received September 17, 2019)

1154-20-2101 **Persi Diaconis***, Department of Statistics, Sequoia Hall, 390 Serra Mall, Stanford, CA 94305-4020. *Tensor Product Markov Chains.*

Let G be a finite group. Decomposing tensor products leads to interesting, challenging, useful math problems. In joint work with Georgia Benkhart, Martin Liebeck, and Pham Tiep, applications to simple probability problems are found. Building on work of Fulman and the hypergroup community, fix a prime p and let \hat{G} be the p -modular irreducibles. Fix one say r and build a random walk on \hat{G} by 'from t tensor with r and pick a component of the tensor product with probability proportional to its degree x multiplicity. Specialized to groups like $Sl(2, q)$ (with $q = p^a$), This gives 'easy to interpret' Markov chains which relate to Pitman's '2M-X' theorem and work of Bien, Bougeorl, O'Connell. Replacing q by Z/nZ leads to quantum groups. In all cases, the analysis required to give sharp rates of convergence to stationarity is challenging. (Received September 17, 2019)

1154-20-2121 **Trevor Arrigoni, Alexander Black, Jessica Harter, David Milan and Yansy Perez*** (yperez@uttyler.edu). *On Inverse Semigroups of Self-Similar Graph Actions.*

We discuss a recently discovered class of inverse semigroups called the inverse semigroups of self-similar graph actions. To describe this class, we characterized the idempotents, ideals, Green's Relations, and other properties common to inverse semigroups in general. Furthermore, we provide conditions that ensure a self-similar graph action is fundamental or 0-disjunctive and, more generally, when it satisfies properties of inverse semigroups relevant to the study of C^* -algebras. (Received September 17, 2019)

1154-20-2153 **Supun T Samarakoon*** (stsamster@math.tamu.edu). *Generalized Grigorchuk's Overgroups.*

Grigorchuk's Overgroup \tilde{G} , is a branch group of intermediate growth. It contains the first Grigorchuk's torsion group G of intermediate growth constructed in [1980], but also has elements of infinite order. Its growth is substantially greater than the growth of G . The group G , corresponding to the sequence 012012... , is a member of the family $\{G_\omega, \omega \in \Omega = \{0, 1, 2\}^{\mathbb{N}}\}$ consisting of groups of intermediate growth when sequence ω is not virtually constant. Following this construction we define generalized overgroups $\{\tilde{G}_\omega, \omega \in \Omega\}$ such that G_ω is a subgroup of \tilde{G}_ω for each $\omega \in \Omega$. We prove, if ω is eventually constant, then \tilde{G}_ω is of polynomial growth and if ω is not eventually constant, then \tilde{G}_ω is of intermediate growth. As a subset of the space \mathcal{M}_8 of marked groups with eight generators, the set $\{\tilde{G}_\omega, \omega \in \Omega\}$ of generalized overgroups is not complete. We describe the completion of it and explain a similarity and a difference with the completion of the classical Grigorchuk's family $\{G_\omega, \omega \in \Omega\}$. (Received September 17, 2019)

1154-20-2416 **Zoran Sunic*** (zoran.sunic@hofstra.edu). *Branch groups with (non-)trivial rigid kernel.* Preliminary report.

We provide a simple criterion for a self-similar, regular branch group to have a trivial rigid kernel. Namely, a self-similar, regular branch group has a trivial rigid kernel if and only if its closure is finitely constrained and a certain index related to the group coincides with the corresponding index for the closure. The criterion easily applies to the Hanoi Towers group (on 3 pegs) to confirm the known fact (Bartholdi-Siegenthaler-Zaleskii) that this group has nontrivial rigid kernel. An infinite family of groups is provided to which the criterion also applies and which provides new examples of regular branch groups with nontrivial rigid kernel (there is already such a family, constructed by Skipper). The groups in the family share many properties with the Hanoi Towers group. (Received September 17, 2019)

1154-20-2508 **Jonathan DH Smith** and **Stefanie G Wang*** (stwang@smith.edu), Burton Hall, Smith College, Northampton, MA 01063. *Numerical analysis on the growth of free quasigroup words.*

A quasigroup (Q, \cdot) is a set equipped with a (non)associative binary multiplication that satisfies the Latin square property. This gives rise to four equational identities that define a quasigroup. In this talk, we'll discuss the algorithm used to generate reduced free quasigroup words in s generators, called the s -peri-Catalan numbers and denoted P_n^s . For large values of n and s , the quasigroup identities play an insignificant role in quasigroup word reductions. This talk will discuss the numerical analysis of the growth of quasigroup words and its relationship to other free algebras. (Received September 17, 2019)

1154-20-2649 **Ariane Alexandrescu, Christine Cho, Dalyn Gillentine** and **Allison Quintana*** (aquintana@smith.edu), Smith College, Northampton, MA 01063, and **Marium Tapal**. *Invariants of \mathbb{Z} -linear quasigroups*. Preliminary report.

A quasigroup (Q, \cdot) is a set equipped with a (non)associative binary multiplication that satisfies the Latin square property. A quasigroup (A, \cdot) is said to be \mathbb{Z} -linear if $(A, +, 0)$ forms an abelian group and $x \cdot y = \rho(x) + \lambda(y)$, where ρ, λ are automorphisms of the underlying abelian group. This talk will discuss isomorphism invariants for permutation representations of a class of \mathbb{Z} -linear quasigroups. (Received September 17, 2019)

1154-20-2701 **Robin D Tucker-Drob*** (rtuckerd@math.tamu.edu), Department of Mathematics, Mailstop 3368, Texas A&M University, College Station, TX 77843-3368. *Superrigidity, Measure Equivalence, and Weak Pinsker Entropy*.

We show that the class B, of discrete groups which satisfy the conclusion of Popa's Cocycle Superrigidity Theorem for Bernoulli actions, is invariant under measure equivalence. We generalize this to the setting of discrete p.m.p. groupoids, and as a consequence we deduce that any nonamenable lattice in a product of two noncompact, locally compact second countable groups, must belong to B.

We also introduce a measure-conjugacy invariant called Weak Pinsker entropy and show that, if G is a group in the class B, then Weak Pinsker entropy is an orbit-equivalence invariant of every essentially free p.m.p. action of G . This is joint work with Lewis Bowen (Received September 17, 2019)

1154-20-2710 **Robin D Tucker-Drob*** (rtuckerd@math.tamu.edu), Department of Mathematics, Mailstop 3368, Texas A&M University, College Station, TX 77843-3368. *Groups with infinite FC-center have the Schmidt property*.

We show that any countable group with infinite FC-center has the Schmidt property, i.e., admits a free, ergodic, measure-preserving action on a standard probability space such that the full group of the associated orbit equivalence relation contains a non-trivial central sequence. As a consequence, we show that any countable, inner amenable group with property (T) has the Schmidt property. This is joint work with Yoshikata Kida. (Received September 17, 2019)

22 ► Topological groups, Lie groups

1154-22-284 **Lucas Mason-Brown*** (lbrown@mit.edu). *Unipotent Representations Attached to Principal Nilpotent Orbits*.

The Orbit Method is a conjectural correspondence between co-adjoint orbits of a Lie group G and its irreducible unitary representations. When G is nilpotent or simply connected and solvable, this correspondence is perfect and complete. But when G is reductive, serious problems arise. The worst of these problems have to do with the nilpotent orbits of G . As of yet, there is no general method for attaching unitary representations to nilpotent orbits. In this talk, I will construct and classify all of the representations attached to the *principal* nilpotent orbits. The solution is a special case of the Langlands correspondence for real reductive groups and will give rise to a Blattner-type formula for the K -multiplicities of the representations in question. (Received August 28, 2019)

1154-22-285 **Bill Casselman*** (cass@math.ubc.ca). *Computing unramified orbital integrals*. Preliminary report.

Recently, because of Langlands' proposals regarding the stable trace formula ('Beyond endoscopy'), it has become important to find explicit formulas for orbital integrals $\langle O_\gamma, f \rangle$ where g is a regular semi-simple element in the p -adic group G and f a function in the Hecke algebra. Work of Waldspurger, extended by Joyner, gives us such a formula, but it has not so far been clear how practical it is. I'll explain recent work on this question. (Received September 11, 2019)

1154-22-347 **Birgit Speh*** (bes12@cornell.edu). *Branching laws for representations of a non compact orthogonal group.*

Finite dimensional representations of classical groups were first treated systematically by Hermann Weyl in his famous book “The classical groups, their Invariants and Representations” 80 years ago. He classified in this book the irreducible representations $\Pi : SO(n) \rightarrow \text{Aut}(V)$ for finite dimensional vector space V . H. Weyl also considered the restriction of an irreducible representation to a subgroup and proved that the restriction of a finite dimensional representation is direct sum of finite dimensional representations. In 1938 famous branching rules describing for an irreducible representation Π of $SO(n)$ and an irreducible representation π of $SO(n-1)$ the multiplicity $\dim \text{Hom}_{SO(n-1)}(\Pi, \pi)$ were proved. A noncompact orthogonal group $SO(p, q)$ has also infinite dimensional irreducible representations. Unfortunately the restriction of an infinite dimensional representation of $SO(p, q)$ to a subgroup $SO(r, s)$ is often not a direct sum of irreducible representations. In this talk I will show that for infinite dimensional representations Π of $SO(n, 1)$ and infinite dimensional representations π of $SO(n-1, 1)$ we obtain very similar branching laws for $\dim \text{Hom}_{SO(n-1)}(\pi, \Pi)$ although the restriction of the representations is not a direct sum of irreducible representations. (Received September 18, 2019)

1154-22-351 **Kayue Daniel Wong*** (kayue.wong@gmail.com), **Dan Barbasch** and **Chao-ping Dong**. *On Dirac Cohomology for Complex Simple Lie Groups.*

Dirac cohomology is closely related to many aspects in representation theory, including (\mathfrak{g}, K) -cohomology and automorphic forms. In this talk, we will give a complete description of all irreducible, unitary representations with nonzero Dirac cohomology for complex classical Lie groups. We will also give some ideas on how these results can be extended to exceptional Lie groups.

This is a joint work with Dan Barbasch and Chao-ping Dong. (Received September 01, 2019)

1154-22-609 **Marisa Gaetz*** (mgaetz@mit.edu). *Dual Pairs in Complex Reductive Groups.* Preliminary report.

In Roger Howe’s 1989 paper, “Remarks on classical invariant theory,” Howe introduces the notion of a *dual pair of Lie subalgebras* – a pair $(\mathfrak{g}_1, \mathfrak{g}_2)$ of reductive Lie subalgebras of a Lie algebra \mathfrak{g} such that \mathfrak{g}_1 and \mathfrak{g}_2 are each other’s centralizers in \mathfrak{g} . This notion has a natural analog for algebraic groups; namely, a *dual pair of subgroups* is a pair (G_1, G_2) of reductive subgroups of an algebraic group G such that G_1 and G_2 are each other’s centralizers in G . We present substantial progress towards classifying the dual pairs of the complex classical groups $(GL(n, \mathbb{C}), SL(n, \mathbb{C}), Sp(2n, \mathbb{C}), O(n, \mathbb{C}), \text{ and } SO(n, \mathbb{C}))$ and their projective counterparts $(PGL(n, \mathbb{C}), PSp(2n, \mathbb{C}), PO(n, \mathbb{C}), \text{ and } PSO(n, \mathbb{C}))$. The classifications of dual pairs in $Sp(2n, \mathbb{C}), GL(n, \mathbb{C}), \text{ and } O(n, \mathbb{C})$ are known, but lack a unified explicit treatment; we provide such a treatment. Additionally, we classify the dual pairs in $SL(n, \mathbb{C})$ and $SO(n, \mathbb{C})$, and present partial progress towards classifying the dual pairs in $PGL(n, \mathbb{C})$ and $PSp(2n, \mathbb{C})$. (Received September 08, 2019)

1154-22-1165 **Judith A. Packer*** (packer@colorado.edu), Department of Mathematics, CB 395, University of Colorado, Boulder, Boulder, CO 80309-0395. *Direct integral decompositions of wavelet representations for crystallographic groups.* Preliminary report.

We study certain types of wavelets associated to crystallographic groups that generalize conventional wavelets in the two-dimensional plane. In this set-up, the group of integer translations is replaced by action by a fixed wallpaper symmetry group, and we choose our dilation matrix to be an odd integer times the identity, for some odd integer greater than or equal to 3. We use the wavelet sets for crystallographic groups constructed by K. Merrill. These sets allow us to decompose the unitary representation of the associated discrete semidirect product generated by the crystallographic group and the dilation matrix into irreducible components, thus generalizing results of L.-H. Lim, J. Packer, and K. Taylor from 2001. The wavelet representations of the discrete semidirect product groups are related to their regular representations by studying the associated group C^* -algebras. This work is joint with L. Baggett, K. Merrill, and K. Taylor. (Received September 13, 2019)

1154-22-1348 **Toshiyuki Kobayashi***, Graduate School of Mathematical Sciences, The University of Tokyo, Komaba 3-8-1, Meguro, Tokyo, Japan. *A program for branching problems in the representation theory of real reductive groups — Classification problem of symmetry breaking operators.* Preliminary report.

A symmetry breaking operator is an intertwining operator that arises from branching problems of representations, that is, an H -intertwining operator from an (irreducible) representation of G to that of the subgroup H . It may be an integral operator, and may be a more singular one such as a differential operator, when representations are realized geometrically.

In general, it is a hard problem to classify symmetry breaking operators. We plan to discuss a criterion for the space of symmetry breaking operators to be finite-dimensional, and a classification scheme of symmetry breaking operators with some examples for orthogonal groups.

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—. Symmetry Breaking for Representations of Rank One Orthogonal Groups II, *Lecture Notes in Math.* 2234, Springer, 2018. xv+342 pages. (Received September 15, 2019)

1154-22-1524 **Brian Hwang*** (bhwang@math.cornell.edu). *When Algebraicity Meets Orthogonality: Emergent Phenomena.*

For an automorphic representation to be associated with a Galois representation, it must satisfy an appropriate algebraicity condition. However, these conditions tend to interact poorly with natural operations when *even* special orthogonal groups are involved. For example, the Langlands functorial transfer from SO_{2n} to GL_{2n} fails to preserve algebraicity, unlike those from odd special orthogonal or symplectic groups. In certain contexts, evidence suggests that O_{2n} is the more natural group to work with, but this means leaving the classical framework of the Langlands program, as this group is not connected. Explicit geometric examples seem to say that in order for algebraicity to be defined here in a compatible way, it requires imposing certain "spin"-like conditions that are absent in the connected case. This raises some curious questions about the representations one expects to associate with the corresponding Galois representations and the symmetries of the corresponding geometric families. (Received September 16, 2019)

1154-22-1625 **Jeffrey D Adams*** (jda@math.umd.edu), College Park, MD 20742. *Computing Unipotent Representations.* Preliminary report.

Jim Arthur has conjectured the existence of certain representations of a reductive group over a local field, which he calls unipotent. In general there is no single well defined definition of this term. Even in the case of real groups there is more than one definition of the term unipotent. Furthermore, even in cases when there is a precise definition there is typically no known algorithm to explicitly compute these representations, say in terms of their Langlands parameters.

In this talk I will discuss some progress on computing unipotent representations in the real case. This is part of the Atlas of Lie Groups and Representations project. (Received September 16, 2019)

1154-22-1686 **William Graham*** (wag@uga.edu) and **Victor Kreiman** (kreiman@uwp.edu). *Equivariant K-theory and tangent spaces to Schubert varieties.*

We study the weights of the action of a maximal torus T on the tangent space to a Schubert variety at a T -fixed point, using the formulas for the restriction of K -theoretic classes of Schubert varieties to fixed points. Our results are uniform across types. When applied to Schubert varieties in cominuscle flag varieties, our result gives a uniform description of the weights of the tangent spaces. (Received September 16, 2019)

1154-22-1974 **Manoj B Karki*** (mbkarki@pvamu.edu), Prairie View A&M University, Department of Mathematics MS 2250, W.R. Banks Building, Suite 333 P.O. Box 519, Prairie View, TX 77446, and **Gerard Thompson.** *Four-dimensional Einstein Lie groups.*

Abstract. Index formulas for the curvature tensors of an invariant metric on a Lie group are obtained. The results are applied to the case of solvable Lie algebras that have a codimension one Abelian nilradical. It is argued that such algebras are associated with Einstein metrics only when the metric is of constant negative or zero curvature. Thereafter, all of the four-dimensional Lie algebras are examined with regard to the question of whether they correspond to Einstein metrics. For algebras that are not of the type solvable with codimension one Abelian nilradical, the space of derivations is found and used to reduce the metric and the Ricci tensor is calculated. It is then possible to say precisely which metrics are Einstein spaces. In the indecomposable case precisely four such Einstein spaces are obtained. Two of them are spaces of constant curvature. No examples are found in the decomposable case. (Received September 16, 2019)

1154-22-2353 **Mark Blumstein*** (mark.blumstein@gmail.com), **Louis Scharf**, **Edwin Chong** and **Chris Peterson**. *Irreducible Representations of Groups in Image Processing*.

Fundamental problems in image processing include: *Detection* of an object from a sample image, *classification* of images into different object classes, and *estimation* of transformation parameters between sightings of an object. Here we consider cases where the variability in an image set can be parameterized by a group G of deformations, and so the fundamental mathematical object is an orbit of images. We consider examples where orbits are approximated in (linear) G -representation spaces. A given covariant rule allows for *estimation* of transformation parameters, while invariants, in the sense of Classical Invariant Theory, provide statistics which may be used for *detection* and *classification*. (Received September 17, 2019)

26 ► *Real functions*

1154-26-20 **Ali Pirhadi*** (pirhadi@okstate.edu). *Real zeros of random cosine polynomials with palindromic blocks of coefficients*.

Out of many ways to construct a dependent random function, one is to force the coefficients to be palindromic. In general, it makes sense to ask how many real zeros a random cosine polynomial (of degree n) possesses if the coefficients are identically and normally distributed, and sorted in palindromic blocks of length ℓ . In this paper, we show that the expected real zeros of such a polynomial lying in one period is asymptotically $K_\ell \cdot 2n/\sqrt{3}$ where $K_\ell > 1$. That is to say, such polynomials have slightly more expected real zeros compared to the classical case with i.i.d. coefficients. (Received June 07, 2019)

1154-26-144 **George A Anastassiou*** (ganastss@memphis.edu), Department of Mathematical Sciences, University of Memphis, Memphis, TN 38152. *On the left fractional local general M-derivative*.

Here is introduced and studied the left fractional local general M- derivative of various orders. All basic properties of an ordinary derivative are established here. We also define the corresponding left fractional M-integrals. Important theorems are established such as: the inversion theorem, the fundamental theorem of fractional calculus, the mean value theorem, the extended mean value theorem, the Taylor's formula with integral remainder, the integration by parts. Our left fractional derivative generalizes the alternative fractional derivative and the local M-fractional derivative. (Received August 15, 2019)

1154-26-1241 **Amalia Culiuc*** (aculiuc@amherst.edu), **Wei Chen**, **Francesco Di Plinio**, **Michael Lacey** and **Yumeng Ou**. *Sparse bounds for Walsh multipliers*.

Motivated by a conjecture of Andrei Lerner regarding quantitative estimates for Marcinkiewicz multipliers and Littlewood-Paley square functions, we study the endpoint sparse bounds for these operators in the Walsh setting. Our result establishes sharp weighted corollaries in terms of the A_p characteristic in the case of the Walsh-Littlewood-Paley square function, and for a restricted range of exponents for Marcinkiewicz multipliers. (Received September 14, 2019)

1154-26-1301 **Fernando Lopez-Garcia*** (fal@cpp.edu). *A local-to-global technique for inequalities in spaces of functions*.

In this talk, we will discuss a certain local-to-global technique which has been applied to prove the validity of several inequalities in spaces of functions, such as the Fractional Poincaré inequalities and Korn and Conformal Korn inequalities among others. The validity of these inequalities depends on the geometry of the domain where the functions are defined. This technique is based on a certain decomposition of functions that extends the validity of the inequalities from cubes or regular domains to more general domains, including domains with fractal boundaries such as the Koch Snowflake and T-square. (Received September 14, 2019)

28 ► *Measure and integration*

1154-28-739 **Alexander M Henderson*** (henderson@math.ucr.edu), UCR Math Department / Skye 202, 900 University Ave, Riverside, CA 92521. *Local Fractal Zeta Functions*. Preliminary report.

In *Fractal Zeta Functions and Fractal Drums*, Lapidus *et al.* study the geometry of bounded subsets of Euclidean space by associating “fractal zeta functions” to these sets. These fractal zeta functions make it possible to rephrase many problems in geometry as problems in complex analysis. The theory can be generalized to a larger class of metric spaces, but the generalization relies on embeddings of a set into an ambient space of known

dimension. In this talk, we introduce a notion of “local fractal zeta functions,” which have many properties analogous to those of the fractal zeta functions described by Lapidus *et al.*, but which are defined without embeddings into an ambient space. (Received September 10, 2019)

1154-28-1098 **Vasileios Chousionis, Sean Li and Scott Zimmerman***
(scott.zimmerman@uconn.edu). *Singular integral operators in Carnot groups.*

The theory of singular integral operators has led to important discoveries in complex analysis and PDEs since the beginning of their modern treatment in the 1950’s, and questions about the boundedness of these operators lie at the core of their study. In recent years, several mathematicians have revealed the deep interaction between the geometric structure of a set and the boundedness of singular integral operators on it. In this talk, we will explore several recent results generalizing these interactions to the setting of 1-dimensional sets in Carnot groups. (Received September 13, 2019)

1154-28-1219 **Dmitriy Bilyk*** (dbilyk@math.umn.edu). *Discreteness of minimizing measures.*

The phenomenon of clustering of minimizing measures is often observed theoretically, numerically, or experimentally: in many situations, in particular for some attractive-repulsive potentials, the minimizers of the energy integral happen to be discrete or supported on very thin sets. We will discuss several manifestations of this phenomenon on the sphere and on projective spaces, in relation to the p-frame energies and other energies arising from problems of discrete geometry. The talk is based on joint work with A. Glazyrin, R. Matzke, J. Park, A. Vlasiuk. (Received September 14, 2019)

1154-28-1551 **Matthew Badger*** (matthew.badger@uconn.edu). *Rectifiability of measures: the identification problem.*

One goal of geometric measure theory is to understand how measures in the plane or higher dimensional Euclidean space interact with families of lower dimensional sets. An important dichotomy arises between the class of rectifiable measures, which give full measure to a countable union of the lower dimensional sets, and the class of purely unrectifiable measures, which assign measure zero to each distinguished set. There are several commonly used definitions of rectifiable and purely unrectifiable measures in the literature (using different families of lower dimensional sets such as Lipschitz images of subspaces or Lipschitz graphs), but all of them can be encoded using the same framework. In this talk, I will describe the framework for generalized rectifiability, review a selection of classical results on rectifiable measures in this context, and survey recent advances on the identification problem for Radon measures that are carried by Lipschitz or Hölder or $C^{1,\alpha}$ images of Euclidean subspaces, including theorems of Azzam-Tolsa, Badger-Schul, Badger-Vellis, Edelen-Naber-Valtorta, Ghinassi, Goering, Naples, Santilli, and Tolsa-Toro. (Received September 16, 2019)

1154-28-1927 **Troy Daniel Butler** (troy.butler@ucdenver.edu) and **Wenjuan Zhang***
(wenjuan.zhang@ucdenver.edu). *An Interactive Guide to Data-Consistent Solutions for Stochastic Inverse Problems, Part II: From (Potential) Chaos to Order.*

(Brief Note: We utilize Jupyter notebooks to re-create some of our published results in real-time, build a computational intuition, and aid in transparency and reproducibility.)

In part 2, we are motivated by practical issues involving the use of computationally expensive models to create a data-consistent solution to a stochastic inverse problem (SIP). Such models limit the number of times we may interrogate an accurate quantity of interest (QoI) map from input parameters to observable data.

We discuss the impact of using approximate QoI maps on the various measure structures associated on input and output spaces required to construct the data-consistent solution. A previous study focused on the special case where the approximate QoI maps converge essentially uniformly (i.e., in L^∞). We briefly review that work along with some of the numerical results before turning our attention to a unifying theory for cases involving QoI convergence in L^p for any $1 \leq p < \infty$. In particular, we consider polynomial chaos approximations to QoI maps and discuss the order of convergence of various densities used in constructing the data-consistent solution. Numerical examples illustrate various aspects of the theory. (Received September 16, 2019)

1154-28-1955 **Troy Butler*** (troy.butler@ucdenver.edu) and **Wenjuan Zhang**. *An Interactive Guide to Data-Consistent Solutions for Stochastic Inverse Problems, Part I: The Role of Set-Valued Inverses.*

(Brief Note: We utilize Jupyter notebooks to re-create some of our published results in real-time, build a computational intuition, and aid in transparency and reproducibility.)

Models are useful for simulating key processes and generating significant amounts of (simulated) data on quantities of interest (QoI) from a model solution. This data can be compared directly to observable data to

address many important questions in scientific modeling. However, many key characteristics governing system behavior described as input parameters in the model remain hidden to direct observation. Thus, scientific inference fundamentally depends on the formulation and solution of a stochastic inverse problem (SIP) to describe sets of probable model parameters.

We have recently developed a "Data-Consistent" approach to solve the SIP based on measure-theoretic principles and set-valued inverses. An attractive aspect of this approach is that requires only a single solution to an associated stochastic forward problem in order to construct the data-consistent solution to the SIP.

In part 1, we briefly summarize this approach including existence, uniqueness, and stability of solutions. A brief comparison to statistical Bayesian inference is also provided. (Received September 16, 2019)

1154-28-2449 **Vira Babenko*** (vira.babenko@drake.edu), Department of Mathematics and Computer Scienc, Drake University, Des Moines, IA 50311, and **Vladyslav Babenko**, Department of Mathematical Analysis, Oles Honchar Dnipro National University, Dnipro, 49000, Ukraine. *Best Approximation, Optimal Recovery, and Landau Inequalities for Derivatives of Hukuhara-type in function L-spaces.*

In this talk, we consider the problem of approximation of unbounded positively homogeneous operators in L-spaces using Lipschitz operators. We discuss its connection to the problem of computing modulus of continuity of the unbounded operator on the class of elements, as well as, to the problem of optimal recovery of an unbounded operator by a Lipschitz one on the class of elements given with an error. Moreover, in L - spaces and for positively homogeneous operators, the connection of the above-mentioned problems with inequalities of Landau - Kolmogorov type is studied. As applications, we consider the problem of approximation of unbounded operator, that for functions with values in some L-space puts in a correspondence Hukuhara-type derivatives, by Lipschitz operators. In addition, we compute the modulus of continuity of this operator and obtain exact Landau-Kolmogorov type inequalities. Further, we discuss the solution of the problem of the optimal recovery of this operator on the class of functions that have Hukuhara-type derivative with the given majorant of the modulus of continuity (in the case of optimal recovery, elements of this class are given with an error). (Received September 17, 2019)

1154-28-2466 **Bobby Wilson*** (blwilson@uw.edu), WA. *A Marstrand Theorem for Polynomial Norms.* We will discuss a generalization of Marstrand's Theorem for the class of norms determined by polynomials. (Received September 17, 2019)

30 ► *Functions of a complex variable*

1154-30-206 **Gerardo R. Chacon***, gerardo.chacon@gallaudet.edu, and **Gerardo A. Chacon.** *An invitation to analytic variable exponent function spaces.*

We will review recent progress made on the theory of analytic variable exponent spaces, which are analytic subspaces of some kind of L^p in which the exponent p is a measurable function. We will discuss basic properties of variable exponent Bergman and Hardy spaces, and we will see where differences lie with respect to the classical case. (Received August 23, 2019)

1154-30-217 **See Keong Lee***, School of Mathematical Sciences, Universiti Sains Malaysia, 11800 USM, Penang, Malaysia. *The starlikeness of a generalized Bessel function.*

The generalized Bessel function

$${}_a\mathcal{B}_{b,p,c}(z) := \sum_{k=0}^{\infty} \frac{(-c)^k}{k! \Gamma\left(ak + p + \frac{b+1}{2}\right)} \left(\frac{z}{2}\right)^{2k+p},$$

where $z \in \mathbb{D} := \{z : |z| < 1\}$, $a \in \mathbb{N} = \{1, 2, 3, \dots\}$ and $b, p, c \in \mathbb{R}$, satisfied an $a + 1$ -order differential equation. The function ${}_a\mathcal{B}_{b,p,c}$ does not belong to the class \mathcal{A} consisting of analytic functions f in \mathbb{D} with normalization $f(0) = 0 = f'(0) - 1$. In this work, the starlikeness of normalized function related to ${}_a\mathcal{B}_{b,p,c}$ are investigated. (Received August 24, 2019)

1154-30-723 **Roosbeh Gharakhloo*** (roosbeh.gharakhloo@colostate.edu) and **Alexander Its** (aits@iupui.edu). *A Riemann-Hilbert approach to asymptotic analysis of Toeplitz+Hankel determinants.*

In this talk I will describe the joint work with Alexander Its on the formulation and analysis of 4×4 Riemann-Hilbert problems for Toeplitz+Hankel determinants and the associated system of orthogonal polynomials, when the Hankel symbol is supported on the unit circle and also when it is supported on an interval $[a, b]$, $0 < a < b < 1$.

The distinguishing feature of this work is that in the formulation of the Riemann–Hilbert problem no specific relationship is assumed between the Toeplitz and Hankel symbols. I will also talk about how one arrives at a model Riemann–Hilbert problem for Toeplitz+Hankel determinants which lies at the core of both cases of Hankel weight support (real line or the unit circle). I will also discuss the remarkable fact that how certain assumptions in a work by Estelle Basor and Torsten Ehrhardt who studied these asymptotics from an operator-theoretic approach, can resurface in the analysis of the proposed model Riemann–Hilbert problem and allow for an explicit factorization. (Received September 10, 2019)

1154-30-738 **Caleb G Parks*** (cparks1000000@gmail.com). *Sampling in Analytic Tent Spaces*. Preliminary report.

Introduced by Coifman, Meyer, and Stein, the tent spaces have seen wide applications in Harmonic Analysis. Their analytic cousins have seen some applications involving the derivatives of Hardy space functions. Moreover, the tent spaces have been a recent focus of research. We introduce the concept of sampling sequences for analytic tent spaces analogously to the same concepts for Bergman spaces. We then characterize such sequences in terms of Seip’s lower uniform density. We accomplish this by exploiting a kind of Möbius invariance for the tent spaces. (Received September 10, 2019)

1154-30-895 **Javad Mashreghi*** (javad.mashreghi@mat.ulaval.ca), 1960 Boul. Laurier, Quebec, QC G1S 1M8, Canada. *Outer functions and uniform integrability*.

We show that, if f is an outer function and $a \in [0, 1)$, then the set of functions

$$\{\log |(f \circ \psi)^*| : \psi : \mathbb{D} \rightarrow \mathbb{D} \text{ holomorphic, } |\psi(0)| \leq a\}$$

is uniformly integrable on the unit circle. As an application, we derive a simple proof of the fact that, if f is outer and $\phi : \mathbb{D} \rightarrow \mathbb{D}$ is holomorphic, then $f \circ \phi$ is outer. (Received September 11, 2019)

1154-30-922 **Qi Han*** (qhan@tamusa.edu), Department of Science and Mathematics, Texas A&M University, San Antonio, TX 78224, and **Jingbo Liu** (jliu@tamusa.edu), Department of Science and Mathematics, Texas A&M University, San Antonio, TX 78224. *On differential independence of ζ and Γ* .

It is a profound result of Hölder in 1887 that the Euler gamma-function cannot satisfy any nontrivial algebraic differential equation in \mathbf{C} . Hilbert, in his lecture addressed before the International Congress of Mathematicians at Paris in 1900 for his famous 23 problems, stated in Problem 18 that the Riemann zeta-function cannot satisfy any such a differential equation either.

In 2007, Markus showed that $\zeta(\sin(2\pi z))$ cannot satisfy such a differential equation with polynomial coefficients in Γ and its derivatives; he conjectured ζ itself cannot satisfy such a differential equation with polynomial coefficients in Γ and its derivatives either.

Thus, one wonders if there is a nontrivial polynomial $\mathcal{P}(u_0, u_1, \dots, u_m; v_0, v_1, \dots, v_n)$ such that

$$\mathcal{P}(\zeta, \zeta', \dots, \zeta^{(m)}; \Gamma, \Gamma', \dots, \Gamma^{(n)}) \equiv 0. \quad (1)$$

In this joint work with Dr. Jingbo Liu, we show that ζ and Γ cannot satisfy some differential equations generated through a family \mathfrak{F} of functions $\mathcal{F}(u_0, u_1, \dots, u_m; v_0, v_1, \dots, v_n)$ which are continuous in (u_0, u_1, \dots, u_m) with polynomial coefficients of (v_0, v_1, \dots, v_n) . (Received September 11, 2019)

1154-30-1000 **Khang D Tran*** (khangt@mail.fresnostate.edu), Department of Mathematics, 5245 North Backer Avenue M/S PB108, Fresno, 93740, and **Maverick Zhang**. *Linear combinations of polynomials with three-term recurrence*.

We study the zero distribution of the sum of the first n polynomials satisfying a three-term recurrence whose coefficients are linear polynomials. We also extend this sum to a linear combination, whose coefficients are powers of $az + b$ for $a, b \in \mathbb{R}$, of Chebyshev polynomials. In particular, we find necessary and sufficient conditions on a, b such that this linear combination is hyperbolic. (Received September 12, 2019)

1154-30-1195 **Michael Epstein*** (michael.epstein@colostate.edu), **Boris Hanin** (bhanin@math.tamu.edu) and **Erik Lundberg** (elundber@fau.edu). *The Lemniscate Tree of a Random Polynomial*.

Given a generic complex polynomial $p(z)$, we can encode the topology of the graph of $|p(z)|$ with a labeled tree. More specifically, the branching structure of the tree, which we call a *lemniscate tree*, encodes the nesting structure of the singular components of the level sets of $|p(z)|$ that pass through critical points. One may therefore study the topological configuration of these singular components of level sets by studying the structure of the associated lemniscate tree. We study the amount of branching in a random lemniscate tree, and we compare the outcome for two different notions of “random”: we consider sampling uniformly at random from

the combinatorial class of all such trees associated to the generic polynomials of a given degree, and we also consider the random lemniscate tree associated to a random polynomial with i.i.d. zeros. (Received September 13, 2019)

1154-30-1329 **Erik Lundberg*** (elundber@fau.edu) and **Koushik Ramachandran**. *The geometry of random polynomials: a probabilistic counterpart to the Erdos lemniscate problem.*

Erdos, Herzog, and Piranian posed the extremal problem of determining the maximum length of a polynomial lemniscate $—p(z)—=1$ when p is a monic polynomial of degree n . In this talk, we study the length and topology of a random lemniscate whose defining polynomial has i.i.d. Gaussian coefficients. We show that the length approaches a constant. We also show that the average number of connected components is asymptotically n , and we observe a positive probability (independent of n) of a giant component occurring. (Received September 14, 2019)

1154-30-1885 **Malik Younsi***, malik.younsi@gmail.com. *Fekete polynomials and shapes of Julia sets.*

We prove that a nonempty, proper subset of the complex plane can be approximated in a strong sense by polynomial filled Julia sets if and only if the set is bounded and its interior has connected complement. The proof that such a set is approximable by filled Julia sets is constructive and relies on Fekete polynomials; illustrative examples will be presented. (Received September 16, 2019)

1154-30-1959 **Andrew P. Thomack*** (andrew.thomack@greenville.edu). *The Asymptotic Growth of the Expected Number of Zeros of Random Harmonic Polynomials.*

A harmonic polynomial is a complex function of the form $h(z) = p(z) + \overline{q(z)}$ where p and q are complex analytic polynomials. We build off of work by Li and Wei, using the Kac-Rice formula to study the expected number of zeros for different models of random harmonic polynomials. Our main focus is the asymptotic growth of the number of zeros of these polynomials as the degree increases, but in this talk we will also look at the shape of the first intensity functions of several models of random harmonic polynomials and note how they compare to their analytic analogs. (Received September 16, 2019)

1154-30-2001 **William Gryc, Loredana Lanzani, Jue Xiong*** (jue.xiong@colorado.edu) and **Yuan Zhang**. *Neumann boundary condition for $\bar{\partial}$.* Preliminary report.

Suppose Ω is a smooth bounded domain in \mathbb{C} and let $H^2(\Omega)$ denote the classical Hardy space on Ω . It is well known that $H^2(\Omega)$ is the data space of the Dirichlet problem for the $\bar{\partial}$ operator on Ω . The solution has an integral representation using the Cauchy kernel function. A natural question to ask is to identify the data space of the Neumann boundary condition for $\bar{\partial}$ on Ω . For the unit disc in \mathbb{C} , it turns out that we can solve the Neumann problem within the classical Dirichlet space. Moreover, the solution can be represented via the Dirichlet kernel function. For Ω smooth simply connected, a promising candidate is the generalization of the Dirichlet space on such domain. We'll also investigate the Neumann problem for domains in higher dimensions and its connections to the Dirichlet problem. This is a joint work inspired by the SCV workshop the speaker attended at AIM in April 2019. (Received September 17, 2019)

1154-30-2192 **Jared E Hoppis*** (jehoppis@gmail.com), Kansas State University, Department of Mathematics, Manhattan, KS 66506. *Modulus metrics in the discrete and the continuum.* Preliminary report.

The study of Modulus originated in the plane, related to conformal mappings, but there has been much recent work in the setting of networks. We will show how Modulus can be used to define a new family of metrics depending on a parameter p , both in the discrete case, as well as in Euclidean spaces and more general metric spaces.

After recalling two main aspects of these modulus metrics on graphs, namely, an important property of duality and a notion of anti-snowflaking, we will explore, generalizations of these concepts to the continuum setting. (Received September 17, 2019)

1154-30-2285 **Trevor J. Richards*** (trichards@monmouthcollege.edu), Monmouth, IL 61462. *Rouché's Theorem and the Geometry of Rational Functions.*

The logarithmic derivative of a product of meromorphic functions $f \cdot g$ may, by the magic of the product rule, be broken up into the sum of the logarithmic derivatives:

$$\frac{(fg)'}{fg} = \frac{f'}{f} + \frac{g'}{g}.$$

This presents a ready-made setting for an application of Rouché’s theorem. We exploit this fact to establish several results regarding the geometry of rational functions. Included is a “root-dragging” result for rational functions. (Received September 17, 2019)

1154-30-2524 **Michael Dougherty*** (michael.dougherty@colby.edu) and **Jon McCammond**. *The Intrinsic Noncrossing Combinatorics of the Space of Generic Complex Polynomials*. Preliminary report.

The space of degree- n complex polynomials with distinct roots appears frequently and naturally throughout mathematics, but its shape and structure could be better understood. In recent and ongoing joint work with Jon McCammond, we present a deformation retraction of this space onto a simplicial complex with rich structure given by the combinatorics of noncrossing partitions. In this talk, I will describe the deformation retraction and the resulting combinatorial data associated to each generic complex polynomial. We will also discuss a helpful comment from Daan Krammer and connections from our work to the ideas of Bill Thurston and his collaborators. (Received September 17, 2019)

1154-30-2561 **Kourosh Tavakoli*** (ktavakoli@okcu.edu). *On the Compositions of Infinitely Many Holomorphic Functions*.

In this research, we studied the limit behavior of the compositions of infinitely many holomorphic functions. We found certain classes of functions for which the corresponding compositions have constant limits. Also, we investigated several examples to observe the limit behavior of the infinite compositions. The relevant geometric properties were studied too. (Received September 17, 2019)

31 ► Potential theory

1154-31-124 **Stefan Steinerberger*** (stefan.steinerberger@gmail.com), 10 Hillhouse Avenue, New Haven, CT 06511. *A Nonlocal Transport Equation Describing Roots of Polynomials Under Differentiation*.

Let p_n be a polynomial of degree n having all its roots on the real line distributed according to a smooth function $u(0, x)$. One could wonder how the distribution of roots behaves under iterated differentiation of the function, i.e. how the density of roots of $p_n^{(k)}$ evolves. We derive a nonlinear transport equation with nonlocal flux

$$u_t + \frac{1}{\pi} \left(\arctan \left(\frac{Hu}{u} \right) \right)_x = 0 \quad \text{on } \text{supp } \{u > 0\},$$

where H is the Hilbert transform. This equation has three very different compactly supported solutions: (1) the arcsine distribution $u(t, x) = (1 - x^2)^{-1/2} \chi_{(-1,1)}$, (2) the family of semicircle distributions

$$u(t, x) = \frac{2}{\pi} \sqrt{(T - t) - x^2}$$

and (3) a family of solutions contained in the Marchenko-Pastur law. (Received August 12, 2019)

1154-31-2015 **Dmitriy Bilyk, Alexey Glazyrin, Ryan W Matzke*** (matzk053@umn.edu), **Josiah Park** and **Oleksandr Vlasiuk**. *Minimization of the p -frame energy*.

The p -frame energy on the sphere \mathbb{S}^d of a Borel probability measure μ is defined by the integral

$$I_p(\mu) \int_{\mathbb{S}^d} \int_{\mathbb{S}^d} |\langle x, y \rangle|^p d\mu(x) d\mu(y).$$

Under the p -frame potential, points repel each other when they are close but attract each other when they are far apart. In particular, the energy between two points is minimized when the points are orthogonal. This gives the problem of finding minimizers a distinct flavor, as most energy optimization problems involve energies, such as the Riesz energy, that are minimized by points being antipodal. We give a survey of recent results in minimization of the p -frame potential. (Received September 17, 2019)

1154-31-2263 **Alexander Reznikov*** (reznikov@math.fsu.edu), Tallahassee, FL 32309, and **Brian Simanek** and **Oleksandr Vlasiuk**. *Supports of solutions to the minimal log-energy problem with constraints*. Preliminary report.

If we look at a minimal log-energy problem on two non-intersecting curves, the support of the solution is well known. We discuss the problem when the solution is required to have prescribed masses on both curves. In particular, in this case, the support of the minimizing measure can have unexpected holes. (Received September 17, 2019)

32 ► *Several complex variables and analytic spaces*

1154-32-363 **Nataliia Goncharuk*** (natasha@urkud.name) and **Yuri Kudryashov**. *Polynomial foliations of C^2 : cheap complex limit cycles and leaves of high genus.*

Based on the joint work with Yu. Kudryashov. We present a simple criterion for an analytic foliation of a two-dimensional complex manifold to have infinitely many homologically independent complex limit cycles (i.e. closed loops on the leaves with non-trivial holonomy). I will also address our earlier results: for most of the foliations in CP^2 with a certain symmetry, most leaves have an infinite genus; each degree- n polynomial foliation in C^2 can be perturbed in the class of degree- n polynomial foliations in C^2 so that a perturbed foliation has a (non-algebraic) leaf with at least $(n+1)(n+2)/2 - 4$ handles. (Received September 02, 2019)

1154-32-428 **Sara Lapan*** (slapan@ucr.edu) and **Feng Rong**. *Existence of attracting domains in C^3 for some holomorphic maps tangent to the identity.*

In this talk, I will discuss holomorphic self-maps of C^3 that fix the origin and are tangent to the identity (i.e., $f(0) = 0$ and $df(0) = \text{Id}$). A well-known result of Hakim states that for such a map, if all directors of a characteristic direction have strictly positive real part, then there is a domain of attraction along that direction while such a domain does not exist if any of the directors have negative real part. I will introduce a family of maps that lie on the border line case where the directors have trivial real part. For this family, I will show that a domain of attraction does exist. Time permitting, I will show that small changes to the family can affect whether or not a domain of attraction exists. (Received September 03, 2019)

1154-32-597 **David B Massey*** (d.massey@northeastern.edu), Dept. of Mathematics, Northeastern University, Boston, MA 02115. *Characteristic Polar Cycles of Perverse Sheaves.*

Given a perverse sheaf, P , we will discuss the characteristic polar cycles of P and their relationship to the Chern-Schwartz-MacPherson class of P . We will discuss the calculation of the characteristic polar cycles and their relationship with the characteristic cycle of P . Our goal is to show that the characteristic polar cycles should be thought of as a calculable refinement of the data supplied by the Chern-Schwartz-MacPherson class of P , which uses only the stalk cohomology Euler characteristic data from P . When applied to the vanishing cycles along a function f , the characteristic polar cycles yield a refinement of the Chern-Schwartz-MacPherson class associated to the Euler characteristics of the Milnor fibers of f . (Received September 08, 2019)

1154-32-701 **Lukasz Kosinski** and **John E. McCarthy*** (mccarthy@wustl.edu). *Norm-preserving extensions of bounded holomorphic functions.*

Let V be an analytic subvariety of a domain Ω in C^n . When does V have the property that every bounded holomorphic function f on V has an extension to a bounded holomorphic function on Ω with the same norm?

An obvious sufficient condition is if V is a holomorphic retract of Ω . We shall discuss for what domains Ω this is also necessary. (Received September 09, 2019)

1154-32-808 **Yifei Pan** and **Yuan Zhang*** (zhangyu@pfw.edu). *Hölder solutions to $\bar{\partial}$ problem on product domains in C^n .*

In this talk, we study $\bar{\partial}$ problem on product domains in C^n . Due to an example of Stein which was later verified by Kerzman, there is in general no gain of derivatives phenomenon on product domains. Making use of an integral representation, we show that given a Hölder data, there is a Hölder solution at the same regularity level. (Received September 10, 2019)

1154-32-900 **Anne-Katrin Gallagher*** (gallagherannek@gmail.com), **Jiri Lebl** and **Koushik Ramachandran**. *The Poincare-Dirichlet inequality for planar domains.*

In this talk, we will describe joint work with Jiri Lebl and Koushik Ramachandran on necessary and sufficient potential-theoretic conditions for the Poincare-Dirichlet inequality to hold. (Received September 11, 2019)

1154-32-943 **Mei-Chi Shaw*** (shaw.1@nd.edu) and **Christine Laurent-Thiebaut**. *Holomorphic approximation via Dolbeault cohomology.*

In this talk we study holomorphic approximation and approximation of $\bar{\partial}$ -closed forms in complex manifolds. The classical Runge theorem and the Mergelyan property are extended to domains in complex manifolds. We characterize the Runge or Mergelyan property in terms of certain Dolbeault cohomology groups and geometric conditions. Holomorphic approximation is also related naturally to the mixed boundary problems for $\bar{\partial}$ on annuli. (joint work with Christine Laurent-Thiébaud). (Received September 12, 2019)

1154-32-999 **Jason Bell, Jeffrey Diller*** (diller.1@nd.edu) and **Mattias Jonsson**. *A transcendental dynamical degree.*

I will discuss an example of a rational self-map of the projective plane whose first dynamical degree turns out to be transcendental. (Received September 12, 2019)

1154-32-1528 **Alexander Nagel** and **Malabika Pramanik*** (malabika@math.ubc.ca). *On geometric and computational aspects of Bergman kernel estimates.*

The talk consists of two parts, with the common theme of Bergman kernel estimation.

(a) If a proper monomial map carries a complex domain $\Omega_1 \subseteq \mathbb{C}^n$ onto $\Omega_2 \subseteq \mathbb{C}^n$, how are the Bergman spaces of Ω_1 and Ω_2 related? We will discuss a structure theorem, with applications.

(b) Given a convex set $\Sigma \subseteq \mathbb{R}^n$, let $T_\Sigma \subseteq \mathbb{C}^n$ denote the tube domain over Σ . We will explore the connection of diagonal estimates for the Bergman kernel of T_Σ with certain caps of Σ of minimal volume. (Received September 16, 2019)

1154-32-2209 **Claire Marilynn Canner*** (cmc2511@g.rit.edu), 127 Loden Lane, Rochester, NY 14623. *Non-PCF Fractals: The Octacarpet and other $4N$ -Carpets.*

The study of analytic structures on self-similar fractal sets was initiated by physicists who discovered that heat flow on such sets had sub-Gaussian rather than Gaussian scaling, indicating that the fundamental physics of these sets was very different than on manifolds. These results were first made rigorous for sets with a finite ramification property, but in the late 1980s Barlow and Bass developed a corresponding theory on a class of generalized Sierpinski carpets. Their approach depends on taking a (weak) limit of Brownian motions on a suitable sequence of closed sets that intersect to the carpet. A key step in proving that the limiting object has sub-Gaussian scaling is showing that the resistance of the approximating domain of scale n is bounded above and below by ρ^n for a factor ρ that depends on the carpet. Computing the exact value of ρ remains an open problem.

We consider the resistance scaling problem for the octacarpet, and more generally for $4N$ -carpets, with the goal of showing analogous bounds for the resistance and obtaining numerical estimates for the resistance scaling factors. (Received September 17, 2019)

1154-32-2319 **Javier Falco, Paul M. Gauthier, Myrto Manolaki*** (myrto.manolaki@ucd.ie) and **Vassili Nestoridis**. *Polynomial and rational approximation in several complex variables.*

The celebrated theorem of Mergelyan states that if K is a compact subset of the complex plane with connected complement, then every continuous function on K which is holomorphic on its interior can be uniformly approximated on K by polynomials. In several complex variables the situation is far from being understood. In this talk, I will present some results on polynomial and rational approximation for products of planar compact sets and graphs of functions. In particular, I will introduce a natural function algebra which allows us to obtain new Mergelyan-type theorems. (Received September 17, 2019)

1154-32-2448 **Alexander J. Izzo*** (aizzo@bgsu.edu). *Extensions of the notions of polynomial and rational hull.*

Extensions of the notions of polynomial and rational hull will be presented. Among the applications of these new hulls is a very flexible method of constructing polynomial and rational hulls without analytic discs. (Received September 17, 2019)

1154-32-2478 **Purvi Gupta*** (purvi.gupta@rutgers.edu) and **Chloe U. Wawrzyniak**. *On the stability of the hull(s) of an n -sphere in \mathbb{C}^n via a nonlinear Riemann-Hilbert problem.*

The phenomenon of analytic continuation in several complex variables gives rise to various notions of hulls for compact sets in complex spaces. In certain cases, it is of interest to identify when these hulls can be geometrically described in terms of attached holomorphic discs (or varieties). This problem has been studied extensively for 2-spheres in \mathbb{C}^2 , where the hull also gives the solution to the so-called Levi-flat plateau problem. Far less is known about the hulls of generic n -spheres in \mathbb{C}^n when $n \geq 3$. In this talk, we will discuss a stability result in this direction. We will note the main distinctions from the $n = 2$ case and elaborate on the role of Riemann-Hilbert boundary problems in this study. (Received September 17, 2019)

1154-32-2493 **Katherine Brubaker*** (kbrubaker@ehc.edu). *Uniform estimates for the Monge-Ampère foliation on a compact Kähler manifold.*

The tradition of studying equations of the form $(\partial\bar{\partial}u)^n = 0$ via an associated foliation goes back to Bedford and Kalka. Dirichlet problems of an analogous ‘‘Monge-Ampère type’’ have played an important role in Kähler geometry. However, on a compact Kähler manifold M , such equations may not have even C^2 solutions.

A smooth solution to the homogeneous complex Monge-Ampère equation on $\bar{U} \times M$, where $U \subset \mathbb{C}$ is the unit disk, corresponds to a “Monge-Ampère foliation” of $U \times M$ by holomorphic disks. In a 2002 paper, Donaldson leveraged this foliation to show that the set of boundary functions for which a smooth solution exists is open.

To continue this line of inquiry, we prove uniform estimates on the leaves of Monge-Ampère foliations, showing that sequences of leaves converge to holomorphic disks. The goal is to understand, for a sequence of boundary functions, when the associated foliations converge to a Monge-Ampère foliation. (Received September 17, 2019)

1154-32-2601 **Tanya Firsova*** (tanyaf@math.ksu.edu). *Critical locus for complex Henon maps.* Preliminary report.

Henon maps are automorphisms of \mathbb{C}^2 and as such do not have critical points. Critical loci defined by E. Bedford, J. Smillie and J. Hubbard are a natural generalization of the critical points of polynomials. We'll discuss the properties of the critical locus. (Received September 17, 2019)

33 ► *Special functions*

1154-33-283 **Jonathan Novak*** (jinovak@ucsd.edu), UC San Diego, La Jolla, CA 92104. *A tale of two matrix integrals.*

I will explain recent work on the complex asymptotics of the Harish-Chandra/Itzykson-Zuber (HCIZ) integral and its additive counterpart, the Brezin-Gross-Witten (BGW) integral. (Received August 28, 2019)

1154-33-1352 **Bruce R Miller*** (bruce.miller@nist.gov), NIST, 100 Bureau Drive, Gaithersburg, MD 20899. *Writing Mathematics in the Digital Age.* Preliminary report.

Display of beautiful mathematics has long been possible thanks to Knuth's TeX. Display on the web has been a tricky proposition; doing so from the same document sources trickier still. But these are achievable and the situation is improving. Yet, the full potential of machine readable mathematics, for accessibility, computation, discovery, reuse and verification, is still elusive.

I will talk about the strategies we are developing in NIST's Digital Library of Mathematical Functions (<http://dlmf.nist.gov/>) project to go beyond mere presentation on the web towards human writable, machine readable mathematics. These include a collection of semantically oriented macros for special functions and mathematical concepts. Additionally, meta macros for defining semantic macros, as well as properties and relationships between objects are employed. Tools like LaTeXXML go beyond LaTeX to convert the documents into machine readable formats for the web, such as XML and MathML. Finally, catalogs of mathematical concepts and special functions along with their characteristics and properties are essential for interoperability. (Received September 15, 2019)

1154-33-1519 **Anton Dzhamay***, anton.dzhamay@unco.edu, and **Galina Filipuk** and **Alexander Stokes**. *Recurrence coefficients for discrete orthogonal polynomials with hypergeometric weight and discrete Painlevé equations.*

Over the last decade it became clear that the role of discrete Painlevé equations in applications is steadily growing. Thus, the question of recognizing some non-autonomous recurrence as a discrete Painlevé equation and understanding its place in Sakai's classification scheme, determining whether it is equivalent to some known (model) example, and especially finding an explicit change of coordinates transforming it to such example, is one of the central ones. Fortunately, Sakai's geometric theory provides an almost algorithmic procedure of answering this question. We illustrate this procedure by studying an example coming from the theory of discrete orthogonal polynomials. There are many connections between orthogonal polynomials and Painlevé equations, both differential and discrete. In particular, often the coefficients of three-term recurrence relations for orthogonal polynomials can be expressed in terms of solutions of some discrete Painlevé equation. In this work we study orthogonal polynomials with general hypergeometric weight and show that their recurrence coefficients satisfy, after some change of variables, the standard discrete Painlevé-V equation. We also provide an explicit change of variables transforming this equation to the standard form. (Received September 16, 2019)

34 ► *Ordinary differential equations*

1154-34-26 **Leonard Mushunje*** (leonsmushunje@gmail.com), Midlands state university, 9055 Senga Road, Gweru, Gweru, Midlands 00263, Zimbabwe. *Theoretical game modelling in the insurance sector.*

This paper discussed theoretical games that are played by insurers and the insured within the insurance sector. We aimed to model their trading relations in terms of their risks and returns, where the risks part was for the insured and the latter for the insurers. We used a two way competition model, that is, the predator-prey and the prey-predator models. We aimed to see the trading discrepancies that occur within insurance sector if we apply a destocking control factor that is, if we vary the number and power of insurers and the policyholders at different times ceteris paribus. We used the eigenvalue analysis through the Jacobean matrix method. Results suggested that, in the absence of interaction both risks and returns for both players grows and decays exponentially respectively under the predator pray model and true for the counter case. The eigenvalue analysis suggested that, there is always a stable co-existence gaming point and as well unstable points which are not conducive for profitable games in the insurance sector. However, these results proved to greatly depend on the values of parameters used in the models. (Received June 28, 2019)

1154-34-232 **Michael A. Karls*** (mkarls@bsu.edu), Department of Mathematical Sciences, Muncie, IN 47306. *Modeling Bubbles of Beer.*

The goal of this project is to set up and numerically solve a first-order nonlinear ODE system of three equations in three unknowns that models beer bubbles that form at the bottom of a glass and rise to the top. The system solution is then used to verify the model via data collected from a bubble rising in a glass of beer. (Received August 26, 2019)

1154-34-247 **Lale Asik*** (lale.asik@ttu.edu), 1108 Memorial Circle, Lubbock, TX 79409, and **Angela Peace.** *Effects of Excess Food-Nutrient Content on the Coexistence of Competing Consumers.*

Recent discoveries in ecological stoichiometry have indicated that food quality in terms of the phosphorus:carbon (P:C) ratio affects consumers whether the imbalance involves insufficient or excess nutrients. This phenomenon is called the “stoichiometric knife-edge.” In this study, we develop and analyze two consumers feeding on one producer model, which captures this phenomenon. Criteria for local stability and instability of the non-negative equilibria are obtained. The co-existence of the three species is also discussed. Finally, computer simulations are performed to investigate the dynamics of the system. (Received August 26, 2019)

1154-34-264 **Ariel N Leslie*** (ariel.bowman@mavs.uta.edu) and **Jianzhong Su** (su@uta.edu). *Mathematical Modeling of a Network of Neurons Regarding G1D Transport Deficiency Epilepsy Seizures.*

Epilepsy is known to be traced back to spatial and temporal patterns working in sequence. Previous models such as Wilson-Cowan (1972), introduced a model showing the dynamics of a network of neurons consisting of excitatory and inhibitory neurons. Taylor et. al (2014) then adapted the Wilson-Cowan model to epileptic seizures using thalamo-cortical based theory. Fan et. al (2018) projects that thalamic reticulus nuclei control spike wave discharges specifically in absence seizures. Our current work includes studying the EEG patterns to identify the single mechanism that causes G1D epileptic behavior. The goal is to find out how an entirely connected brain network shows the neuronal functionality as a unit regarding G1D. Our coupled thalamo-cortical model goes beyond a connection in a logical unidirectional pattern shown by Fan but in a bidirectional small world pattern more analogous to realistic seizure activity. Using our model, we are able to study stability analysis, parameter values which cause synchronized or more stable activity, identify a synchronization index, and uncertainty analysis regarding parameters that directly cause specific spiking behavior. We will show explicitly how our 32-unit network model is a more accurate picture of G1D and its limitations. (Received August 27, 2019)

1154-34-420 **Alfonso Castro*** (castro@g.hmc.edu), Department of Mathematics, Harvey Mudd College, Claremont, CA 91711. *Beyond the first semester on differential equations.*

Concepts and pedagogical strategies leading to the understanding of research questions on semilinear ordinary and partial differential equations will be discussed. The role of singular differential equations in finding the the spectrum of the Laplacian operator restricted to radial functions and the existence of radial solutions to semilinear differential equations will be considered. (Received September 04, 2019)

1154-34-690 **Lale Asik*** (lale.asik@ttu.edu), Department of Mathematics and Statistics, Texas Tech University, 1108 Memorial Circle, Lubbock, TX 79409-1042, and **Angela Peace**. *The Effects of Excess Food Nutrient Content on the Coexistence of Competing Consumer Species*.

Recent discoveries in ecological stoichiometry have indicated that food quality in terms of the phosphorus:carbon (P:C) ratio affects consumers whether the imbalance involves insufficient or excess nutrients. This phenomenon is called the “stoichiometric knife-edge.” In this study, we develop and analyze two consumers feeding on one producer model, which captures this phenomenon. Criteria for local stability and instability of the non-negative equilibria are obtained. The co-existence of the three species is also discussed. Finally, computer simulations are performed to investigate the dynamics of the system. (Received September 09, 2019)

1154-34-798 **Yun Kang*** (yun.kang@asu.edu), Sciences and Mathematics, Mesa, AZ 85212, and **Jun Chen** (yun.kang@asu.edu), **Komi Messan** (yun.kang@asu.edu), **Marisabel Rodriguez-Messan** (yun.kang@asu.edu), **Brian Sweeney** (yun.kang@asu.edu) and **Gloria Hoffman** (yun.kang@asu.edu). *Population Dynamics of Honeybees: Interplay Effects of Parasites and Nutrition*. Preliminary report.

Honeybees are important pollinators worldwide and pollinate about one-third of the food we consume. The incidence of honeybee colony collapsing has been increased under increasing stress due to global warming, pesticides, mites, viruses and nutrition status. In this talk we would start with experimental data and the related analysis from Dr. Gloria Hoffman. The data suggests that low initial bee populations lead to collapse in September while mites and viruses can lead to collapse in December. Feeding bee colonies also has a mixed effect, where it increases both bee and mite populations. Based on the data, we provide our modeling approach by using nonlinear distributed delay differential equations. Our proposed model includes both age structure of honeybees and mites. Some of our interesting founding from our proposed model is including but not limit to: (1) Initial populations are important for the survival of colony; (2) Mite can destabilize honeybee populations and potentially lead to the colony collapsing; and (3) delay term can also destabilize the honeybee-mite interaction dynamics. Our ongoing work is validating our proposed model with data. If time permits, we will also discuss how nutrient and seasonality affect honeybee population dynamics. (Received September 10, 2019)

1154-34-800 **Yun Kang*** (yun.kang@asu.edu), **Tao Feng**, **Jun Chen**, **Zhipeng Qiu** and **Daniel Charbonneau**. *Dynamics of task allocation in social insect colonies: Scaling effects of colony size versus work activities*. Preliminary report.

One of the key questions in study of social animals relates to the mechanisms underlying their organization of work. In this work, we propose an adaptive modeling framework on task allocations by incorporating variations both in task performance and the related metabolic rates. We study the scaling effects of colony size on the resting probability as well as the task allocation. We also numerically explore the effects of stochastic noise on the task allocation of social insect colonies. Our theoretical and numerical results show that: (a) changes in colony size can regulate the probability of the colony resting and the allocation of colony task, and the direction of regulation is related to the nonlinear metabolic scaling effects of tasks; (b) an enhanced response threshold can result in the appearance of a periodic solution. In this case, we observed interesting bubble phenomena in the task allocation of social insect colonies for the first time; and (c) stochastic noise causes the probability of colony resting and the allocation of colony task to fluctuate within a range, and the amplitude of the fluctuation is positively correlated with the intensity of the noise. (Received September 10, 2019)

1154-34-866 **Elpiniki Nikolopoulou***, 900 S Palm Walk, Tempe, AZ 85281. *Mathematical Modeling of an Immune Checkpoint Inhibitor and Its Synergy with an Immunostimulant*.

Immune checkpoint inhibitors (ICIs) are a novel cancer therapy that may induce tumor regression across multiple types of cancer. There has recently been interest in combining the ICIs with other forms of treatments, as not all patients benefit from monotherapy. We propose a mathematical model consisting of ordinary differential equations to investigate the combination treatments of the ICI avelumab and the immunostimulant NHS-muIL12. We validated our model using the average tumor volume curves provided in Xu et al. (2017). We initially analyzed a simple generic model without the use of any drug, which provided us with mathematical conditions for local stability for both the tumorous and tumor-free equilibrium. This enabled us to adapt these conditions for special cases of our model. Additionally, we conducted systematic mathematical analysis for the case that both drugs are applied continuously. Numerical simulations suggest that the two drugs act synergistically, such that, compared to monotherapy, only about one-third the dose of both drugs is required in combination for tumor control. (Received September 11, 2019)

1154-34-882 **James D Meiss*** (jdm@colorado.edu), Applied Mathematics, UCB 526, Boulder, CO 80309-0526, and **Rebecca A Mitchell**. *Optimizing Mixing in Aref's Blinking Vortex Flow*. Mixing of a passive scalar in a fluid flow results from a two part process in which large gradients are first created by advection and then smoothed by diffusion. We will discuss methods of designing efficient stirrers to optimize the mixing of a passive scalar in a two-dimensional nonautonomous, incompressible flow over a finite time interval. The flow is modeled by a sequence of area-preserving maps whose parameters change in time, defining a mixing protocol. As an example, we study a version of Aref's blinking vortex flow; here the stirrers are modeled as point vortices. The positions and strengths of the vortices represent parameters to be selected to optimize the stirring efficiency. Stirring efficiency is measured in two ways: a version of a "mix-norm"—a negative Sobolev seminorm, and the "mix-variance" a smoothed variance. A Perron-Frobenius operator is used to numerically advect the scalar. Various strategies for obtaining near-optimal protocols are compared with those obtained by random optimization methods. (Received September 11, 2019)

1154-34-891 **Yunxiang Bai*** (c00257292@louisiana.edu), 217 Maxim Doucet Hall, P.O. Box 43568, Lafayette, LA 70504, and **Aghalaya Vatsala**. *Generalized Monotone Method for Nonlinear Caputo Fractional Impulsive Differential Equations*. Preliminary report. Generalized monotone method is a useful technique to prove the existence of coupled minimal and maximal solutions when the nonlinear function is the sum of an increasing and decreasing functions. In this work, we develop generalized monotone method for Caputo fractional impulsive differential equations with initial conditions, using coupled lower and upper solutions of Type 1. For that purpose we develop comparison results for Caputo fractional impulsive differential equation. Further, under uniqueness assumption, we prove that the existence of the unique solution of the nonlinear Caputo fractional impulsive differential equation with initial conditions. (Received September 11, 2019)

1154-34-915 **Enahoro A Iboi***, **Abba B Gumel**, **Steffen Eikenberry**, **Silvie Huijben** and **Krijn Paaijmans**. *Long-lasting Insecticidal Nets and the Quest for Malaria Eradication: A Mathematical Modeling Approach*. Recent dramatic declines in global malaria burden and mortality can be largely attributed to the large-scale deployment of insecticidal-based measures, namely long-lasting insecticidal nets (LLINs) and indoor residual spraying (IRS). However, the sustainability of these gains, and the feasibility of global malaria eradication by 2040, may be affected by increasing insecticide resistance among the *Anopheles* malaria vector. We employ a new differential-equations based mathematical model, which incorporates the full, weather-dependent mosquito lifecycle, to assess the population-level impact of the large-scale use of LLINs, under different levels of *Anopheles* pyrethroid insecticide resistance, on malaria transmission dynamics and control in a community. (Received September 11, 2019)

1154-34-1002 **Oksana A Chkrebtii*** (oksana@stat.osu.edu) and **David A Campbell** (dac5@sfu.ca). *Adaptive step-size selection for state-space probabilistic differential equation solvers*. When models are defined implicitly by systems of differential equations with no closed-form solution, small local errors in finite-dimensional solution approximations can propagate into deviations from the true underlying model trajectory. Some recent perspectives in quantifying this uncertainty are based on Bayesian probability modeling: a prior is defined over the unknown solution and updated by conditioning on interrogations of the forward model. Improvement in accuracy via grid refinement must be considered in order for such Bayesian numerical methods to compete with state of the art numerical techniques. We apply principles of Bayesian statistical design to develop an adaptive probabilistic method to sequentially select time-steps for state-space probabilistic ODE solvers. We investigate the behaviour of local error under the adaptive scheme which underlies numerical variable step-size methods. Numerical experiments are used to illustrate the performance of such adaptive schemes, showing improved accuracy in terms of global error over uniform designs when small step lengths are considered. (Received September 12, 2019)

1154-34-1007 **Weston Roda** and **Michael Li*** (myli@ualberta.ca), Mathematical and Statistical Sciences, University of Alberta, Edmonton, Alberta T6G 2G1, Canada, and **Christopher Power**. *Mathematical Modeling of HIV/SIV Infection in Brain*. Preliminary report. Understanding HIV-1 replication and latency in different reservoirs is an ongoing challenge in the care of patients with HIV/AIDS. We use mathematical models to quantify the progression and predict the viral dynamics of HIV-1 and SIV infection within the brain during effective combination antiretroviral therapy (cART), and discuss the effects of the "shock-and-kill" strategy for eliminating latent reservoirs. (Received September 12, 2019)

- 1154-34-1062 **Basiru Usman*** (bzu0005@auburn.edu), 221 Parker Hall Auburn University, Auburn, AL 36849, and **Xiaoying Han** and **Peter E Kloeden**. *Upper semi-continuous convergence of attractors for a Hop field-type lattice model*.

To investigate dynamical behavior of the Hop field neural network model when its dimension becomes increasingly large, a Hop field-type lattice system is developed as the finite dimensional extension of the classical Hopf model. The existence of global attractors is established for both the lattice system and its finite dimensional approximations. Moreover, the global attractors for the finite dimensional approximations are shown to converge to the attractor for the infinite dimensional lattice system upper semi-continuously. (Received September 16, 2019)

- 1154-34-1080 **Peng Feng*** (pfeng@fgcu.edu), 11501 FGCU Blvd. S., Fort Myers, FL 33965, and **Menaka Navaratna**, 11501 FGCU Blvd. S., Fort Myers, FL 33965. *Role of Regulatory T Cells on a Tumor-Immune System with Immunotherapy*. Preliminary report.

Recent advance in the field of regulatory T cell reveals that it plays a vital role during immunotherapy. For example, a higher ratio between regulatory T cells and effector T cells within tumor tissue is associated with worse prognoses in many cancers, including ovarian cancer (Leffers et al., 2009), lung cancer (Tao et al., 2012), glioblastoma (Sayour et al., 2015). On the other hand, the tug war between regulatory T cells and effector T cells for interleukin-2 may chisel immune responses against cancer. In this work, we propose a mathematical model that studies the role of regulatory T cells during immunotherapy. We demonstrate mathematically, for the first time, that the initial ratio between regulatory T cells and effector T cells does impact the tumor recurrence time. We also demonstrate the effectiveness of utilization of IL-2 may flip the outcome of immunotherapy, providing further evidence that it may be clinically viable to modulate the consumption of IL-2 by Tregs. (Received September 13, 2019)

- 1154-34-1081 **Anthony M Bloch*** (abloch@umich.edu), Department of Mathematics, University of Michigan, Ann Arbor, MI 48109, and **Francois Gay-Balmaz** and **Tudor S Ratiu**. *Classical integrable Hamiltonian systems, Poisson structures and some generalizations*.

In this talk we discuss a geometric approach to some classical integrable Hamiltonian systems and their generalizations. The theory is motivated by some ideas in optimal control theory. The integrable systems discussed include the rigid body equations, geodesic flows on the ellipsoid, flows on Stiefel manifolds, and the Toda lattice flows. We discuss the Hamiltonian structure of these systems, relate our work to some work of Moser and discuss some generalizations. This is mainly joint work with Francois Gay Balmaz and Tudor Ratiu. (Received September 13, 2019)

- 1154-34-1149 **Xiaoying Han***, 221 Parker Hall, Department of Mathematics and Statistics, Auburn University, Auburn, AL 36849, **Yusuke Asai**, Sapporo, Japan, and **Peter E Kloeden**, Tuebingen, Germany. *Dynamics of Zika Virus Epidemic in Random Environment*.

A mathematical model for Zika virus dynamics under randomly varying environmental conditions will be introduced. In the model, the birth and loss rates for mosquitoes, and environmental influence are represented by random processes. Long term dynamics in terms of existence and geometric structures of random attractors and forward omega limit sets are investigated. In particular, sufficient conditions under which the prevalence of Zika virus among human beings decreases monotonically to zero, as well as conditions under which an epidemic occurs are established. (Received September 13, 2019)

- 1154-34-1317 **Marion Weeder mann*** (mweeder m@dom.edu), 7900 W Division Street, River Forest, IL 60305. *Modeling of Anaerobic Digestion*.

Anaerobic digestion, i.e., the decomposition of organic matter in the absence of oxygen, is utilized on an industrial scale to decompose waste material and generate green energy. Anaerobic digestion is a complex multi-stage process involving many different types of micro-organisms. A number of mathematical models has been developed to describe, better understand and optimize this process. In this talk I'll give an overview over these models and discuss the insights gained through these modeling efforts. (Received September 14, 2019)

- 1154-34-1427 **Preeti Dubey*** (pdubey1@luc.edu), Division of Hepatology, 2160 S First Ave, Maywood, IL 60153, and **Harel Dahari**. *Modeling the interplay between hepatitis D virus (HDV) and hepatitis B virus (HBV) during anti-HDV treatment*. Preliminary report.

Hepatitis D virus (HDV) is an infectious subviral agent that can only propagate in people infected with hepatitis B virus (HBV). The dynamics of treatment response in patients co-infected with HBV and HDV can be complex because change in one virus can have effect on the other virus. The prenylation inhibitor lonafarnib is the first antiviral treatment against HDV, which provided novel information about the interplay between the two viruses.

Interestingly, some lonafarnib-treated patients had a decline in HDV RNA while HBV DNA had an increase. To explain the observed kinetics, we developed a mathematical model and performed model calibrations. I will present the observed kinetics and modeling efforts along with estimated parameters and drug efficacy. (Received September 15, 2019)

1154-34-1428 **Mary M Ballyk***, Department of Mathematical Sciences, New Mexico State University, 1290 Frenger Mall, MSC 3MB / Science Hall 236, Las Cruces, NM 88003-8001, and **Ibrahim Jawarneh** and **Ross Staffeldt**. *A Nutrient-Prey-Predator Model: Stability and Bifurcations*.

We consider a model of a nutrient-prey-predator system in a chemostat with general functional responses, using the input concentration of nutrient as the bifurcation parameter. We study changes in the existence and the stability of isolated equilibria, as well as changes in the global dynamics, as the nutrient concentration varies. The bifurcations of the system are analytically verified and we identify conditions under which an equilibrium undergoes a Hopf bifurcation and a limit cycle appears. (Received September 15, 2019)

1154-34-1603 **Benjamin Steinhurst*** (bsteinhurst@mcdaniel.edu). *Fibonacci Word Fractals: Three Constructions and Introductory Analysis*. Preliminary report.

The first construction is as the scaling limit of curves produced by an L-system. The second is as the attractor of a derived iterated function system satisfying something similar to an open set condition. The third is as a projective limit. Using the third construction we will consider how lifting functions to higher approximations works and construct a compatible sequence of approximating Laplacians. (Received September 16, 2019)

1154-34-1629 **Jennifer M. Miller*** (jmiller@bellarmine.edu). *The effect of an environmental toxin on a competitive species model*.

We extend a differential equations model for competitive species to include an environmental toxin that affects the species. As such, we begin with a system of four differential equations corresponding to the two species and the toxin in each. We examine the long-term system behavior when the toxin affects the birth rate, death rate, or both. Without the toxin, species X would thrive and species Y would die out. The addition of the toxin allows for a stable steady state where the two species coexist. (Received September 16, 2019)

1154-34-1655 **Brian J Winkel*** (brianwinkel@simiode.org). *Eye Surgery Problems, Dry Ice Does a Job, and How Water Cooling while Room Heats Up – Modeling with Differential Equations*.

We will engage the audience in modeling scenarios we have used with students in modeling with differential equations using data. In retinal surgery inert gas is injected into the eye and doctors need to model the gas's diffusion out of the eye before permitting patients to fly. Otherwise bad things could happen! Dry ice sublimates, but how can we model it, and how should we not model it? Hot water cools in a room, but the room is heating up. What happens? All of these are readily available at www.simiode.org. (Received September 16, 2019)

1154-34-1661 **Ismail Abdulrashid*** (iza0009@auburn.edu), 221 Parker Hall, Department of Mathematics and Statistics, Auburn, AL 36849, and **Hakim Ghazzai**, **Xiaoying Han** and **Yehia Massoud**. *Optimal Control Treatment Analysis for the Predator-Prey Chemotherapy Model*.

In this talk, we investigate a non-autonomous model of chemotherapy cancer treatment with time-dependent infusion concentration of the chemotherapy agent. A predator prey type model is adopted to describe the interactions between the chemotherapy agent and cells, in which the chemotherapy agent is modeled as the prey being consumed by both cancer and normal cells, thereby affecting the population of both. We derive an optimal control for this model and provide necessary conditions for continuous application of chemotherapy treatment. Finally, we provide selected numerical results and find out that with the same amount of chemotherapy drug infused at the beginning of the treatment, normal cell population continually increase over time, while the tumor cell population is quickly driven to zero. An interesting aspect of this work indicates that for large volume of tumor, our numerical result shows that continuous treatment needs to be carried out in order to combat the tumor. (Received September 16, 2019)

1154-34-1748 **Corban Harwood*** (rharwood@georgefox.edu). *Lost at Sea: Introduction to Numerical Methods through Navigation*.

In this talk, we will discuss a two part modeling activity which guides two simultaneous discovery-based approaches to learning the basics of numerical methods for first order differential equations, by following the graphical and analytical perspectives of the forward Euler method and second order Taylor method. These methods are motivated by dead reckoning applied graphically to the velocity field over two dimensions to locate

a ship lost at sea. We also provide a guide for planning, facilitating, and assessing the two simultaneous activities which require groups to compare their results from different perspectives to conclude whether or not the ship is found. (Received September 16, 2019)

1154-34-1832 **Susmita Sadhu*** (susmita.sadhu@gcsu.edu). *Mixed-mode oscillations as long transient dynamics in a predator-prey model with timescale separation.*

Several ecosystems exhibit long transient behavior and may experience sudden transition to another state under seemingly constant environment. In this talk, I will present a singularly perturbed three-species predator-prey model featuring fast prey dynamics and slow dynamics of the predators, where chaotic transients are observed in a neighborhood of a “singular Hopf” bifurcation as the system approaches a periodic attractor. The transient dynamics consist of complex oscillatory patterns known as mixed mode oscillations (MMOs), featuring concatenation of long epochs of small amplitude oscillations and large amplitude oscillations. The transients could persist for thousands of generations, reflecting that dynamics on an ecological timescale can be completely different than asymptotic dynamics. The goal of this talk is to obtain conditions that will be used to determine whether a trajectory exhibits another cycle of MMO dynamics (leading to a population outbreak) before reaching its asymptotic state. (Received September 16, 2019)

1154-34-2018 **Eli E Goldwyn*** (goldwyn@up.edu), University of Portland, 5000 N Willamette Blvd, Portland, OR 97203. *The Effect of External Perturbations on Ecological Oscillators.*

Population interactions cause oscillations across a variety of ecological systems. These ecological population oscillators are subject to external forces including climatic effects and migration. One way of understanding the impact of these perturbations is to study their effect on the phase of the oscillations. In this talk, we introduce tools commonly used to describe the phase of oscillating phenomena in cardiac dynamics, circadian rhythms, firing neurons, and more recently, coupled predator-prey dynamics. We discuss applications of these methods to several common ecological models and present undergraduate research opportunities to extend these techniques to other models to address specific ecological issues and phenomena. (Received September 17, 2019)

1154-34-2032 **Dale Frymark*** (dale@math.su.se), Department of Mathematics, Stockholm University, Kräftriket 6, 106 91 Stockholm, Sweden, and **Annemarie Luger**. *Boundary Triples and Weyl m -functions for Powers of the Jacobi Differential Operator.* Preliminary report.

We present a scheme for using the method of boundary triples to approach the spectral theory of powers of the classical Jacobi differential operator. Operations in the boundary triple are regularizations of quasi-derivatives that are not generated by deficiency elements, making calculations much more accessible. The associated Weyl m -function for the n -th power of the operator, a $2n \times 2n$ matrix-valued Nevanlinna-Herglotz function, will be also be presented. (Received September 17, 2019)

1154-34-2081 **Mahbubur Rahman*** (mrahman@unf.edu), Department of Mathematics & Statistics, 1 UNF Drive, Jacksonville, FL 32224. *Stochastic approximations of Fredholm Volterra Integro-differential equation arising in mathematical neuroscience.* Preliminary report.

This paper extends the results of synaptically generated wave propagation through a network of connected excitatory neurons to a continuous model, defined by a Fredholm Volterra integro-differential equation, which includes memory effects of the past as well as parametric noise in the system. Stochastic approximation and numerical simulations are discussed. (Received September 17, 2019)

35 ► Partial differential equations

1154-35-19 **Peter Y Pang*** (matpyh@nus.edu.sg), Department of Mathematics, National University of Singapore, 10 Lower Kent Ridge Road, Singapore, 119076, Singapore. *Global existence of a two-dimensional chemotaxis-haptotaxis model with remodeling of non-diffusible attractant.*

In this talk, we deal with the cancer invasion model

$$\begin{aligned}u_t &= \Delta u - \chi \nabla \cdot (u \nabla v) - \xi \nabla \cdot (u \nabla w) + \mu u(1 - u - w) \\v_t &= \Delta v - v + u \\w_t &= -vw + \eta w(1 - w - u)\end{aligned}$$

in a bounded smooth domain in the plane with zero-flux boundary conditions, where χ , ξ , μ and η are positive parameters. Compared to previous mathematical studies, the novelty here lies in: first, our treatment of the full parabolic-parabolic chemotaxis-haptotaxis system; and second, allowing for positive values of η , reflecting processes with self-remodeling of the extracellular matrix. Under appropriate regularity assumptions on the initial data (u_0, v_0, w_0) , by using adapted L_p -estimate techniques, we prove the global existence and uniqueness

of classical solutions when μ is sufficiently large, i.e., in the high cell proliferation rate regime. (Received June 06, 2019)

1154-35-25 **Tatiana Toro*** (toro@uw.edu), University of Washington, Department of Mathematics, Box 354350, Seattle, WA 98195-4350. *Differential operators and the geometry of domains in Euclidean space.*

In this talk we will introduce an area of analysis that is concerned with the extent to which a differential operator, and the properties of its solutions, determine the geometry of the domain on which they are considered. We will initially describe the case where the differential operator sees the domain as a homogeneous medium. We will contrast this with several inhomogeneous cases and mention some recent results in that direction. The tools used come from analysis of partial differential equations, harmonic analysis and geometric measure theory. (Received June 26, 2019)

1154-35-50 **Suzanne Weekes*** (sweekes@wpi.edu). *A numerical and analytical study of dynamic materials.*

In this presentation, I will give an overview of some of the work that we have done on wave propagation through dynamic materials (DM). DM are spatio-temporal composites - materials whose properties vary in space and in time. Mathematically, we formulate the problem as linear, hyperbolic equations with spatio-temporally varying coefficients. Both analytic and computational means have been applied to the analysis of the effective properties of dynamic materials generated by certain dynamic microstructures. Spatio-temporal variability in the material constituents allows us to create effects that are unachievable through purely static (spatial) design. There are a host of geometries and effects that can be explored. For example, we have found that with dynamic laminates we are able to screen portions of the material from the effects of a wave disturbance. With checkerboard geometry in space-time, we see pulse compression and energy accumulation and recent work shows that these effects are structurally stable. (Received July 30, 2019)

1154-35-63 **Kamel M. Al-Khaled*** (kamel@just.edu.jo), Department of Math and Stat, Faculty of Science and Arts, Jordan University of Science and Technology, Irbid, 22110, Jordan. *Sinc and Solitary Wave Solutions to the Generalized Benjamin-Bona-Mahony-Burgers Equations.*

In this paper, we consider the generalized Benjamin-Bona-Mahony-Burgers equations (abbreviated BBMB). A variety of exact solutions for the MMBM equations are developed by means of the tanh method. A Sinc-Galerkin procedure is also developed for solving the MMBM equations. Sinc approximations to both derivatives and the indefinite integrals reduce the system to an explicit system of algebraic equations. It is shown that Sinc-Galerkin approximations produce an error of exponential order. A comparison between the two methods for the solution of BBMB equation is analyzed for their solutions. The study outlines the significant features of the Sinc method.

Keywords: Sinc-Galerkin method, Tanh method, Nonlinear PDEs, Numerical methods. (Received July 27, 2019)

1154-35-153 **Vlad C Vicol*** (vicol@cims.nyu.edu), Warren Weaver Hall, New York, NY 10012. *Convex integration and fluid turbulence.*

The Navier-Stokes and the Euler equations are the fundamental models describing incompressible homogenous fluids, with and without viscosity. Based on theories of fluid turbulence, one may expect that solutions of these equations have on average a finite degree of smoothness, in the infinite Reynolds number limit. These ideas go back to Kolmogorov '41 and to Onsager '49. As such, it is natural to consider weak or distributional solutions of the fluid equations. The behavior of weak solutions to the Navier-Stokes and Euler equations is however mysterious: the equations appear to be too soft at this low smoothness level. One may exhibit weak solutions which have finite kinetic energy, and have compact support in time: the fluid is fully at rest, then it starts to move, and then it goes back to sleep. In this talk, we survey a number of results concerning such wild weak solutions of the fluid equations. These works build on the groundbreaking works of De Lellis and Szekelyhidi Jr., who extended Nash's fundamental ideas on C^1 flexible isometric embeddings, into the realm of fluid dynamics. These techniques, which go under the umbrella name "convex integration", have fundamental analogies the phenomenological theories of turbulence. (Received September 01, 2019)

1154-35-182 **Nathan Pennington*** (nathanpennington@creighton.edu), 2500 California Plaza, Omaha, NE 68178. *PDE's and Big Data.*

In this talk we consider a problem at the intersection of PDE's and big data analysis called Continuous Data Assimilation. In these problems, instead of being given an entire initial condition, we are instead given the evaluation of the initial condition on a large (but finite) set. The goal is to show that any solution to the PDE that matches those observations is asymptotic to a unique, global solution of a modified version of the PDE with initial condition equal to zero. The equation is modified from its original form by the presence of feedback terms which depend on the observations. This talk will outline this method by applying it to a model of the Navier-Stokes equation. (Received August 20, 2019)

1154-35-226 **Michail E Filippakis*** (mfilip@unipi.gr), Department of Digital Systems, University of Piraeus, 126 Grigoriou Labraki Str, 18532, Piraeus-Gr, 18534 Piraeus, Greece. *Multiple and nodal solutions for nonlinear equations with a nonhomogeneous differential operator and concave-convex term-Nodal solutions for nonlinear problems.*

In this paper we consider a nonlinear parametric Dirichlet problem driven by a nonhomogeneous differential operator (special cases are the p -Laplacian and the (p, q) -differential operator) and with a reaction which has the combined effects of concave $((p - 1)$ -sublinear) and convex $((p - 1)$ -superlinear) terms. We do not employ the usual in such cases AR-condition. Using variational methods based on critical point theory, together with truncation and comparison techniques and Morse theory (critical groups), we show that for all small $\lambda > 0$ (λ is a parameter), the problem has at least five nontrivial smooth solutions. We also prove two auxiliary results of independent interest. The first is a strong comparison principle and the second relates Sobolev and H^1 -local minimizers for C^1 functionals. Then we consider a nonlinear nonhomogeneous Robin problem and with Morse Theory and variational methods we prove the existence of nontrivial smooth solutions.

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1154-35-235 **Pablo Raúl Stinga*** (stinga@iastate.edu), Iowa State University. *Regularity theory for nonlocal space-time master equations.*

We report on newly developed regularity theory for fractional powers of parabolic operators in divergence form. Our results include local degenerate parabolic extension problems, interior and boundary Harnack inequalities and sharp interior and global parabolic Schauder estimates. For the latter, we also prove a characterization of the correct intermediate parabolic Hölder spaces in the spirit of Sergio Campanato. This is joint work with Animesh Biswas (Iowa State University) (Received August 26, 2019)

1154-35-265 **J. Darby Smith*** (jsmit16@sandia.gov) and **Rich Lehoucq.** *An SDE Representation of Particle Transport for Low Power Implementation.*

A quantity of interest concerning particle transport is the angular flux Φ , defined as particle speed times the particle angular density. Traditionally, the PDE for angular flux is numerically approximated by first expressing Φ as a Neumann series with a physical interpretation, and then employing Monte Carlo techniques. With the advent of low power neuromorphic hardware, we have the ability to cheaply simulate large numbers of random walks. However, there is a fundamental tradeoff as these walks live in a coarser, discretized space. We have returned to the derivation of the transport equation and the random walk method looking for alternate forms and methods optimized for implementation on neuromorphic hardware and present the derivation of a jump-SDE for particle angular flux. (Received August 27, 2019)

1154-35-279 **Michael Benfield, Helge Kristian Jenssen and Irina A Kogan*** (iakogan@ncsu.edu). *A Generalization of an Integrability Theorem of Darboux.*

In his book "Systèmes Orthogonaux" (1910), Darboux stated a theorem (Théorème III) on local existence and uniqueness of solutions to PDE systems of the type

$$\partial_{x_i} u_\alpha(x) = f_i^\alpha(x, u(x)), \quad i \in I_\alpha \subseteq \{1, \dots, n\}, \quad \alpha = 1, \dots, m,$$

where, for a given point $\bar{x} \in \mathbb{R}^n$, the values for the unknown u_α are prescribed near \bar{x} along $\{x \mid x_i = \bar{x}_i \text{ for each } i \in I_\alpha\}$. The theorem was proven by Darboux only for $n = 2$ and 3 . We prove a generalization of Darboux's result, applicable to

$$\mathbf{r}_i(u_\alpha)|_x = f_i^\alpha(x, u(x)), \quad i \in I_\alpha \subseteq \{1, \dots, n\}, \quad \alpha = 1, \dots, m,$$

where $\{\mathbf{r}_i\}_{i=1}^n$ is a local frame of vector fields. The values for u_α are prescribed along a manifold Ξ_α transverse to the vector fields $\{\mathbf{r}_i \mid i \in I_\alpha\}$. We identify a geometric condition, the Stable Configuration Condition (SCC), that depends on both the frame and the data manifolds. Assuming the SCC and the integrability conditions are

satisfied, we establish local existence and uniqueness of a C^1 -solution for arbitrary n . If the SCC is not satisfied, we show that the uniqueness may fail. (Received August 28, 2019)

1154-35-291 **Avner Peleg*** (avpeleg@gmail.com) and **Debananda Chakraborty**. *Radiation dynamics in fast soliton collisions in the presence of nonlinear dissipation.*

We study the dynamics of emission of radiation (small-amplitude waves) in fast collisions between solitons of the nonlinear Schrödinger (NLS) equation in the presence of nonlinear dissipation, considering cubic loss as an example for the dissipation. We calculate the radiation dynamics by a perturbation technique with two small parameters: the cubic loss coefficient ϵ_3 and the reciprocal of the group velocity difference between the solitons $1/\beta$. We obtain very good agreement between the perturbation theory predictions and the results of numerical simulations with the full coupled-NLS propagation model for large β values, and good agreement for intermediate β values. Additional numerical simulations with four simplified NLS models show that the differences between perturbation theory and numerical simulations for intermediate β values are due to the effects of Kerr nonlinearity on inter-soliton interaction in the collision. Thus, our study demonstrates that the perturbation technique that was originally developed for studying radiation dynamics in fast soliton collisions in the presence of conservative perturbations can also be employed for studying soliton collisions in the presence of dissipative perturbations. (Received August 29, 2019)

1154-35-311 **T Bourni*** (tbourni@utk.edu), 227 Ayres Hall, 1403 Circle Drive, Knoxville, TN 37996, and **M Langford** (mlangfo5@utk.edu) and **G Tinaglia** (giuseppe.tinaglia@kcl.ac.uk). *On the existence of translating solutions of mean curvature flow in slab regions.*

We prove, in all dimensions $n \geq 2$, that there exists a convex translator lying in a slab of width $\pi \sec \theta$ in \mathbb{R}^{n+1} (and in no smaller slab) if and only if $\theta \in [0, \frac{\pi}{2}]$. In constructing such solutions we develop a compactness theory for the graphs of solutions of the Dirichlet problem of the translator equation which is similar to that of almost area minimizing currents. The usual dimension restriction is circumvented here due to the rotational symmetry of the solutions. We also obtain convexity and regularity results for translators which admit appropriate symmetries and study the asymptotics and reflection symmetry of translators lying in slab regions. This work is joint with Mat Langford and Giuseppe Tinaglia. (Received August 30, 2019)

1154-35-314 **Yannick Sire***, Krieger Hall, room 404, 3400 N. Charles Street, Baltimore, MD 21218. *Minimizers for the thin one-phase problem.* Preliminary report.

We consider the thin one-phase free boundary problem, associated to minimizing a weighted Dirichlet energy of the function in the half-space plus the area of the positivity set of that function restricted to the boundary. I will provide a rather complete picture of the (partial) regularity of the free boundary, providing content and structure estimates on the singular set of the free boundary when it exists. All of these results hold for the full range of the relevant weight related to an anomalous diffusion on the boundary. The approach does not follow the standard one introduced in the seminal work of Alt and Caffarelli. Instead, the nonlocal nature of the distributional measure associated to a minimizer necessitates arguments which are less reliant on the underlying PDE. This opens several directions of research that I will try to describe. (Received August 30, 2019)

1154-35-375 **Minh T Kha*** (minhkha@math.arizona.edu), Tucson, AZ, and **Peter Kuchment** and **Vladimir Lin**. *Some results on polynomial growth solutions to periodic elliptic equations on co-compact coverings.*

The classical Liouville theorem says that a harmonic function that grows polynomially is a harmonic polynomial and so, the space of all harmonic functions with a fixed polynomial growth in \mathbb{R}^n is of finite dimension. This leads to a natural problem concerning the finite dimensionality of the spaces of solutions of an assigned polynomial growth, estimates of their dimensions, and descriptions of the structures of these solutions for more general elliptic operators on certain non-compact manifolds. For elliptic operators with periodic coefficients on co-compact abelian Riemannian coverings, it is established by many authors previously that the dimensions of the spaces of all solutions with fixed polynomial growths are all finite and also, each of these solutions also admits a representation as a linear combination of polynomials with periodic coefficients. In this talk, we will present some results including a combination of the Riemann-Roch and Liouville theorems for periodic elliptic operators on abelian coverings and a result concerning the finite dimensionality of the spaces containing polynomial-like solutions to periodic equations on co-compact nilpotent coverings. This talk is based on two joint works with Peter Kuchment and Vladimir Lin. (Received September 02, 2019)

1154-35-389 **Kazuo Yamazaki*** (kyamazak@ttu.edu), Texas Tech University, 1108 Memorial Circle, Lubbock, TX 79409-1042. *Strong Feller property of the magnetohydrodynamics system forced by space-time white noise via regularity structures.* Preliminary report.

When a differential equation is forced by a random noise that is white in not only time, but also in space, the solution becomes too rough so that the non-linear term does not make any sense via a classical method. Such equations are called singular PDEs; a prominent example includes the KPZ equations. The theories of paracontrolled calculus and regularity structures are two recent novel methods with which non-trivial limit of approximation scheme may be obtained for such singular PDEs. We consider three-dimensional magnetohydrodynamics system forced by space-time white noise, apply the theory of regularity structures and prove the local well-posedness, as well as the strong Feller property. (Received September 03, 2019)

1154-35-419 **Jennifer Beichman*** (jbeichman@smith.edu). *Computing Change with Partial Differential Equations: Running a MAA NREUP at a Liberal Arts College.*

The MAA National Research Experience for Undergraduates Program is a funding source for REUs specifically for underrepresented minority students at the host institution. I, along with another colleague, applied for, were awarded, and ran a program in summer 2019. In this talk, I'll share our application and planning process, the problems we worked on, and share my thoughts on the research impact for myself and the students. (Received September 03, 2019)

1154-35-430 **Julienne Kabre*** (julienne_kabre@baylor.edu) and **Qin Sheng**. *A splitting approximation for the numerical solution of a self-adjoint quenching problem.*

Ideal combustion processes are often modelled via nonlinear reaction-diffusion equations with singular forcing terms. Our preliminary work considers the numerical solution of such a partial differential equation problem, where a self-adjoint operator with variable diffusion coefficient is considered. Traditional Peaceman-Rachford-Strang splitting is used for time stepping of the semi-discrete system of equations obtained. Conditions are derived to ensure the monotonicity, positivity, and linear stability of the finite difference method. Simulation experiments are provided to validate our splitting approximation. (Received September 03, 2019)

1154-35-493 **Timothy E. Faver*** (t.e.faver@math.leidenuniv.nl) and **Hermen Jan Hupkes**. *Micropteron traveling waves in diatomic Fermi-Pasta-Ulam-Tsingou lattices under the equal mass limit.*

The diatomic Fermi-Pasta-Ulam-Tsingou (FPUT) lattice is an infinite chain of alternating particles connected by identical nonlinear springs. We prove the existence of micropteron traveling waves in the diatomic FPUT lattice in the limit as the ratio of the two alternating masses approaches 1, at which point the diatomic lattice reduces to the well-understood monatomic FPUT lattice. These are traveling waves whose profiles asymptote to a small periodic oscillation at infinity, instead of vanishing like the classical solitary wave. We produce these micropteron waves using a functional analytic method, originally due to Beale, that was successfully deployed in the related long wave and small mass diatomic problems. Unlike the long wave and small mass problems, this equal mass problem is not singularly perturbed, and so the amplitude of the micropteron's oscillation is not necessarily small beyond all orders (i.e., the traveling wave that we find is not necessarily a nanopteron). The central challenge of this equal mass problem hinges on a hidden solvability condition in the traveling wave equations, which manifests itself in the existence and fine properties of asymptotically sinusoidal solutions (Jost solutions) to an auxiliary advance-delay differential equation. (Received September 05, 2019)

1154-35-504 **Nalin Fonseca*** (g_fonsek@uncg.edu), Dept of Mathematics & Statistics, UNCG, 116 Petty, 317 College Ave, Greensboro, NC 27412, and **Ratnasingham Shivaji, Byungjae Son** and **Keri Spetzer**. *Classes of reaction diffusion equations where a parameter influences the equation as well as the boundary condition.*

We study positive solutions to steady state reaction diffusion equations of the form:

$$\begin{cases} -\Delta u = \lambda f(u); \Omega, \\ \frac{\partial u}{\partial \eta} + \mu(\lambda)u = 0; \partial\Omega, \end{cases}$$

where $\lambda > 0$, Ω is a bounded domain in \mathbb{R}^N ; $N \geq 1$ with smooth boundary $\partial\Omega$, $\frac{\partial u}{\partial \eta}$ is the outward normal derivative of u , $\mu \in C([0, \infty))$ is strictly increasing such that $\mu(0) \geq 0$ and $f \in C^2([0, r_0])$ with $0 < r_0 \leq \infty$. If $r_0 < \infty$ we assume $f \in C^2([0, r_0])$ with $f(r_0) = 0$ and $f(s) \leq 0$ for $s \in (r_0, \infty)$, and if $r_0 = \infty$ we assume $\lim_{s \rightarrow \infty} f(s) > 0$ and $\lim_{s \rightarrow \infty} \frac{f(s)}{s} = 0$ (sublinear at ∞). Note here that the parameter λ influences both the equation and the boundary condition. We discuss existence, nonexistence, multiplicity and uniqueness results for the cases when (A) $f(0) = 0$, (B) $f(0) < 0$, and (C) $f(0) > 0$. We obtain existence and multiplicity results by the

method of sub-super solutions and uniqueness results by comparison principles and a priori estimates (Received September 05, 2019)

1154-35-505 **Jerome Goddard, Quinn Morris, Stephen Robinson and Ratnasingham Shivaji*** (shivaji@uncg.edu), Dept of Mathematics & Statistics, UNCG, 116 Petty, 317 College Ave, Greensboro, NC 27412. *An exact bifurcation diagram for a reaction diffusion equation arising in population dynamics.*

We analyze the positive solutions to

$$\begin{cases} -\Delta v = \lambda v(1-v); & x \in \Omega_0, \\ \frac{\partial v}{\partial \eta} + \gamma \sqrt{\lambda} v = 0; & x \in \partial\Omega_0, \end{cases}$$

where $\Omega_0 = (0, 1)$ or is a bounded domain in \mathbb{R}^n ; $n = 2, 3$ with smooth boundary and $|\Omega_0| = 1$, and λ, γ are positive parameters. Such steady state equations arise in population dynamics encapsulating assumptions regarding the patch/matrix interfaces such as patch preference and movement behavior. In this paper, we will discuss the exact bifurcation diagram and stability properties for such a steady state model (Received September 05, 2019)

1154-35-508 **Huijing Du***, 203 Avery Hall, Lincoln, NE 68588. *Multiscale modeling of epidermal-dermal interactions during skin wound healing.*

Following injury, skin activates a complex wound healing programme. While cellular and signalling mechanisms of wound repair have been extensively studied, the principles of epidermal-dermal interactions and their effects on wound healing outcomes are only partially understood. To gain new insight into the effects of epidermal-dermal interactions, we developed a multiscale, hybrid mathematical model of skin wound healing. The model takes into consideration interactions between epidermis and dermis across the basement membrane via diffusible signals, defined as activator and inhibitor. Simulations revealed that epidermal-dermal interactions are critical for proper extracellular matrix deposition in the dermis, suggesting these signals may influence how wound scars form. Our model also makes several theoretical predictions. Taken together, our model predicts the important role of signalling across the dermal-epidermal interface and the effect of fibrin clot density and wound geometry on scar formation. This hybrid modelling approach may be also applicable to other complex tissue systems, enabling the simulation of dynamic processes, otherwise computationally prohibitive with fully discrete models due to a large number of variables. (Received September 05, 2019)

1154-35-511 **Yan Li*** (yan_li1@baylor.edu), Sid Richardson Science Building, One Bear Place 97328, Waco, TX 76798, and **Ran Zhuo** (zhuoran1986@126.com), Huanghuai University, Zhumadian, Henan 463000, Peoples Rep of China. *Existence of positive solutions to nonlinear equations involving distinct fractional Laplacians.* Preliminary report.

Consider

$$\begin{cases} (-\Delta)^{\alpha/2} u(x) = f(x, u(x), v(x)), & x \in \Omega, \\ (-\Delta)^{\beta/2} v(x) = g(x, u(x), v(x)), & x \in \Omega, \\ u(x) = v(x) = 0, & x \in \mathbb{R}^n \setminus \Omega, \end{cases} \quad (1)$$

where $f, g \in C(\Omega, R, R)$, Ω is bounded in R^n and $\partial\Omega$ is C^2 . When the solution (u, v) is a priori bounded, under some assumptions on $f(x, t, s)$ and $g(x, t, s)$ about their super-linearity with respect to t and s near zero and infinity, we prove that there exists at least one positive solution (u, v) using the topological degree theory. (Received September 05, 2019)

1154-35-521 **Guy David, Max Engelstein, Mariana Smit Vega Garcia*** (mariana.smitvegagarcia@wwu.edu) and **Tatiana Toro.** *Regularity of almost minimizers with free boundary.*

We study almost minimizer for functionals which yield a free boundary, as in the work of Alt-Caffarelli and Alt-Caffarelli-Friedman. The almost minimizing property can be understood as the defining characteristic of a minimizer in a problem which explicitly takes noise into account. In this talk, we discuss the regularity of almost minimizers to energy functionals with variable coefficients. (Received September 06, 2019)

1154-35-539 **Brandon P. Ashley*** (brandonpashley@live.com). *Darboux Integrable Systems of Class $s = 2$.* Preliminary report.

Hyperbolic Darboux integrable exterior differential systems of class $s = 0, 1$, and 3 have been extensively studied (Goursat, Vessiot, Sokolov, etc.). In this talk, we present our recent work on the classification of hyperbolic Darboux integrable systems of class $s = 2$. For the equations in our classification, we calculate several geometric features including the Vessiot algebra, generalized symmetries, and zero curvature representations. (Received September 06, 2019)

1154-35-546 **Alex Kasman*** (kasmana@cofc.edu). *Producing and Understanding Quaternion-valued Solutions to the KdV Equation.*

Although both the KdV Equation and quaternions are already well-studied, I aim to convince you that this student research project generated some interesting and new results about their intersection. We utilize the unique properties of quaternions to develop new methods for producing and understanding the behavior of these non-commutative KdV solutions. The main theorem characterizing the non-linear super-position of breather solitons with other solutions gives what I believe are new results even in the real/commutative case. (Received September 06, 2019)

1154-35-563 **Adrianna Gillman***, Department of Applied Mathematics, Engineering Center, ECOT 225, 526 UCB, Boulder, CO 80309-0526. *An efficient and high order accurate direct solution technique for variable coefficient elliptic partial differential equations.*

For many applications in science and engineering, the ability to efficiently and accurately approximate solutions to elliptic PDEs dictates what physical phenomena can be simulated numerically. In this talk, we present a high-order accurate discretization technique for variable coefficient PDEs with smooth coefficients. The technique comes with a nested dissection inspired direct solver that scales linearly or nearly linearly with respect to the number of unknowns. Unlike the application of nested dissection methods to classic discretization techniques, the constant prefactors do not grow with the order of the discretization. The discretization is robust even for problems with highly oscillatory solutions. For example, a problem 100 wavelengths in size can be solved to 9 digits of accuracy with 3.7 million unknowns on a desktop computer. The precomputation of the direct solver takes 6 minutes on a desktop computer. Then applying the computed solver takes 3 seconds. The recent application of the algorithm to inverse media scattering also will be presented. (Received September 06, 2019)

1154-35-576 **Sayonita Ghosh Hajra*** (sayonita.ghoshhajra@csus.edu). *Generalized Two-phase Mass Flow Model.* Preliminary report.

Two-phase mass flow is characterized primarily by the relative motion between the solid and fluid phases that appear in natural phenomena such as avalanches and landslides. These flows are modeled by a system of non-linear partial differential equations. The most general model has been developed by Pudasaini (2012). We will discuss a recent progress in the study of this model, in particular, we will discuss its symmetries and consequences. This work is joint with Dr. S. Kandel, Dr. S. P. Pudasaini. (Received September 07, 2019)

1154-35-614 **N. Fonseka, J. Goddard, Q. Morris, R. Shivaji and B. Son*** (byungjae.son@maine.edu). *On the effects of the exterior matrix hostility and a U-shaped density dependent dispersal on a diffusive logistic growth model.*

In this talk, we will study positive solutions to a steady state model arising in population dynamics, namely,

$$\begin{cases} -\Delta u = \lambda u(1-u); & \Omega \\ \frac{\partial u}{\partial \eta} + \gamma \sqrt{\lambda}[(A-u)^2 + \epsilon]u = 0; & \partial\Omega \end{cases}$$

where λ is a domain scaling parameter, γ is a measure of the exterior matrix (Ω^c) hostility, and $A \in (0, 1)$ and $\epsilon > 0$ are constants. The boundary condition represents a case when the dispersal at the boundary is U-shaped, that is decreasing for lower densities and increasing for higher densities. We discuss nonexistence, existence, multiplicity and uniqueness results. In particular, we discuss the occurrence of an Allee effect for certain range of λ . (Received September 08, 2019)

1154-35-683 **Azmy S. Ackleh, Rainey Lyons*** (rainey@louisiana.edu) and **Nicolas Saintier.** *Finite Difference Schemes for a Structured Population Equation in the Space of Measures.*

We present two finite-difference methods for approximating solutions to a structured population model in the space of non-negative Radon Measures. The first method is a formally first-order upwind-type scheme and the second is high-resolution method of formally second-order. We prove that the two schemes converge to the solution in the Bounded-Lipschitz (or flat) norm. We will provide several numerical examples demonstrating the order of convergence and behavior of the schemes around singularities. In particular, these numerical examples show for smooth initial conditions, the upwind method is of first order and the high-resolution method is of second order. Furthermore, for singular solutions the second-order high-resolution method is shown to be superior to the first-order method. (Received September 09, 2019)

1154-35-685 **Janak R Joshi*** (jjoshi@cameron.edu), 214 NW 44th Street, Apt#D, Lawton, OK 73505. *Infinitely many solutions of semilinear equations on the exterior domains.*

We study the steady state reaction diffusion equations

$$\Delta u + K(r)f(u) = 0$$

with various boundary conditions on the exterior of the ball in \mathbb{R}^N such that $\lim_{r \rightarrow \infty} u(r) = 0$.

Here, $u(x, t)$ is an unknown spatiotemporal vector function, $\Delta u = \operatorname{div}(\nabla u)$ is a laplacian of u with respect to the space parameter x which represents the diffusion term, and $f(u)$ accounts for all reactions and interactions. We assume $f: \mathbb{R} \rightarrow \mathbb{R}$ is an odd locally lipschitz non-linear function such that there exists a $\beta > 0$ with $f < 0$ on $(0, \beta)$, $f > 0$ on (β, ∞) , and $K(|x|) \sim |x|^{-\alpha}$ for some $\alpha > 0$.

Reaction diffusion system can be interpreted as a mathematical model describing how the concentration of one or more substances vary over time and space under the influence of reaction factor and diffusion factor. (Received September 09, 2019)

1154-35-714 **Mostafa Rezapour*** (mostafa.rezapour@wsu.edu), Department of Mathematics and Statistics, Washington State University, Pullman, WA 99163, **Adebowale Sijuwade** (adebowale.sijuwade@wsu.edu), Department of Mathematics and Statistics, Washington State University, Pullman, WA 99164, and **Thomas J. Asaki** (tasaki@wsu.edu), Department of Mathematics and Statistics, Washington State University, Pullman, WA 99164. *Neural partial differential equation solver with fractional gradient descent.*

We present techniques that use both supervised and unsupervised learning methods to solve partial differential equations problems. The universal approximation theorem guarantees the construction of a feedforward neural network (FNN) that effectively serves as a universal approximator for a continuous function. First, we apply back-propagation neural network using a fractional gradient descent technique with an adjustable learning rate to solve diffusion equations and compare this approach against traditional gradient descent methods. This idea can be used to handle multivariable coupled integrodifferential equations such as those which arise in the field theory for predicting polymer microphase-separated structures. Next, for a given class of initial boundary value problems (IBVPs) and their solutions, we present a supervised machine-learning based solver. We define a set of pairs of well-posed diffusion IBVPs and their general solutions, and then train a model on it. The behavior of the IBVPs can be predicted using the obtained model, which can be trained by using cross-validation and minimizing the corresponding loss function of the equation to reduce error. (Received September 18, 2019)

1154-35-728 **Alejandro Velez-Santiago*** (alejandro.velez2@upr.edu), Call Box 9000, Mayaguez, PR 00681. *Approximation of quasi-linear Koch-type fractal energy functionals on varying Hilbert spaces.*

We study a quasi-linear evolution equation with nonlinear dynamical boundary conditions in a two dimensional domain with Koch-type fractal boundary. We consider suitable approximating pre-fractal problems in the corresponding pre-fractal varying domains. After proving existence and uniqueness results via standard semigroup approach, we prove that the pre-fractal solutions converge in a suitable sense to the limit fractal solution in varying Hilbert spaces. (Received September 10, 2019)

1154-35-737 **Jinping Zhuge*** (jpzhuge@math.uchicago.edu). *Weak maximum principle for biharmonic equations in quasiconvex Lipschitz domains.*

In dimension two or three, the weak maximum principle for biharmonic equation is valid in any bounded Lipschitz domains. In higher dimensions (greater than three), it was only known that the weak maximum principle holds in convex domains or C^1 domains, and may fail in general Lipschitz domains. In this talk, we will show the weak maximum principle in higher dimensions in quasiconvex Lipschitz domains, which is a sharp condition in some sense and recovers both convex and C^1 domains. The main ingredient is a reverse Hölder inequality in quasiconvex domains proved by a real variable argument. (Received September 10, 2019)

1154-35-752 **Luda Korobenko***, korobenko@reed.edu. *Log-Sobolev inequalities, doubling condition, and degenerate elliptic PDE's.*

I will discuss the connection between log-Sobolev type inequalities in general metric measure spaces and the doubling condition on the metric balls. It has been recently shown that the classical (q, p) -Sobolev inequality with $q > p$ implies the doubling condition. In particular, since Carnot-Carathéodory metric spaces associated to infinitely degenerate operators are non doubling with respect to the Euclidean measure, they do not support a classical Sobolev inequality. Weaker types of log-Sobolev inequalities have been shown to hold for such spaces, and then successfully applied in the DeGiorgi iteration scheme to establish regularity of weak solutions. I will show that even certain types of weak log-Sobolev inequalities imply the doubling condition, and establish the best possible (within a particular class) Sobolev inequality one can obtain in the non doubling case. (Received September 10, 2019)

1154-35-756

Olivier Pinaud* (pinaud@math.colostate.edu), Colorado State University, Department of Mathematics, Fort Collins, CO 80523. *Fractional White-Noise Limit and Paraxial Approximation for Waves in Random Media.*

This work is devoted to the asymptotic analysis of high frequency wave propagation in random media with long-range dependence. We are interested in two asymptotic regimes, that we investigate simultaneously: the paraxial approximation, where the wave is collimated and propagates along a privileged direction of propagation, and the white-noise limit, where random fluctuations in the back-ground are well approximated in a statistical sense by a fractional white noise. The fractional nature of the fluctuations is reminiscent of the long-range correlations in the underlying random medium. A typical physical setting is laser beam propagation in turbulent atmosphere. Starting from the high frequency wave equation with fast non-Gaussian random oscillations in the velocity field, we derive the fractional Itô-Schrödinger equation, that is, a Schrödinger equation with potential equal to a fractional white noise. The proof involves a fine analysis of the backscattering and of the coupling between the propagating and evanescent modes. Because of the long-range dependence, classical diffusion-approximation theorems for equations with random coefficients do not apply, and we therefore use moment techniques to study the convergence. (Received September 10, 2019)

1154-35-782

Connor Mooney* (mooneycr@math.uci.edu), 410C Rowland Hall, UC Irvine, Irvine, CA 92697-3875. *Minimizers of strictly convex functionals.*

A central problem in the calculus of variations is to determine the regularity of Lipschitz minimizers of $\int F(\nabla u)$, where F is convex. When F is smooth and uniformly convex, De Giorgi and Nash showed that minimizers are smooth. If the graph of F contains a line segment, minimizers are no better than Lipschitz. In the intermediate case that F is strictly convex but its second derivatives tend to zero or infinity on some set (which arises in many applications), it is reasonable to ask whether Lipschitz minimizers are C^1 . We will discuss recent results that answer this question positively in some cases and negatively in general, and highlight a connection between this problem and classical differential geometry. (Received September 10, 2019)

1154-35-785

Azmy S. Ackleh* (azmy.ackleh@louisiana.edu), Department of Mathematics, University of Louisiana at Lafayette, Lafayette, LA 70504-1010, and **Nicolas Saintier** and **Jakub Skrzeczkowski**. *Sensitivity equations for measure-valued solutions to transport equations.*

Transport type equations arise ubiquitously in the physical, biological and social sciences. They were used, for example, to model the dynamics of opinion formation and structured populations. Because of the natural setting of the space of measures for these equations, which allows for unifying discrete and continuous dynamics under the same framework, we consider the following transport equation in the space of bounded, nonnegative Radon measures $\mathcal{M}^+(\mathbb{R}^d)$:

$$\partial_t \mu_t + \partial_x(v(x)\mu_t) = 0.$$

We study the sensitivity of the solution μ_t with respect to a perturbation in the vector field, $v(x)$. In particular, we replace the vector field v with a perturbation of the form $v^h = v_0(x) + hv_1(x)$ and let μ_t^h be the solution of

$$\partial_t \mu_t^h + \partial_x(v^h(x)\mu_t^h) = 0.$$

We derive a partial differential equation that is satisfied by the derivative of μ_t^h with respect to h , $\partial_h(\mu_t^h)$. We show that this equation has a unique very weak solution on the space Z , being the closure of $\mathcal{M}(\mathbb{R}^d)$ endowed with the dual norm $(C^{1,\alpha}(\mathbb{R}^d))^*$. We also extend the result to the nonlinear case where the vector field depends on μ_t , i.e., $v = v[\mu_t](x)$. (Received September 10, 2019)

1154-35-826

Kamran Sadiq*, Altenbergerstrasse 69, 4040 Linz, Austria. *A Fourier approach to the inverse source problem in an absorbing and scattering medium with applications to Optical Molecular Imaging.*

We revisit the inverse source problem in a two dimensional absorbing and scattering medium and present a non-iterative reconstruction method using measurements of the radiating flux at the boundary. The approach is based on the Cauchy problem for a Beltrami-like equation for the sequence valued maps, and extends the original ideas of A. Bukhgeim from the non-scattering to the scattering media. Of novelty here, the medium has an anisotropic scattering property that is neither negligible nor large enough for the diffusion approximation to hold. The numerical realization of the proposed reconstruction method is also presented, which is amenable for such scattering media. The feasibility of the proposed algorithm is demonstrated in several numerical experiments, including simulated scenarios for parameters meaningful in Optical Molecular Imaging. This is joint work with Alexandru Tamasan and Hiroshi Fujiwara. (Received September 11, 2019)

1154-35-862 **P. Jameson Graber*** (jameson_graber@baylor.edu), One Bear Place #97328, Waco, TX 76798, and **Alan Mullenix, Frederic Bonnans** and **Laurent Pfeiffer**. *Weak solutions for variational mean field games of controls.*

Recently, Bonnans et al. proved existence and uniqueness of classical solutions to a system of PDE that models a mean field game in which players can choose a strategy based on the distribution of both states and controls. They show that the system has a variational structure, i.e. it is the optimality condition for an infinite dimensional convex minimization problem. This inspires us to extend the theory of weak solutions from Cardaliaguet et al. to this class of mean field games of controls. We need only growth rates on the data, rather than smoothness, and the diffusion term can be degenerate (even zero) rather than uniformly elliptic. (Received September 11, 2019)

1154-35-972 **Ryan Alvarado*** (rjalvarado@amherst.edu). *On the Dirichlet problem with boundary datum belonging to Hardy spaces in the upper-half space.*

The primary focus of this talk concerns the solvability of the Dirichlet problem for elliptic, second-order, homogeneous $M \times M$ systems with constant, complex coefficients in the upper-half space \mathbb{R}_+^n , with boundary datum belonging to the Hardy Space $HP(\mathbb{R}^{n-1})$, $p \in (\frac{n-1}{n}, 1]$. It turns out that the solution can be expressed in terms of a convolution between the S. Agmon, A. Douglis, and L. Nirenberg Poisson kernel for the given system and the boundary data from the corresponding Hardy space. A key tool in establishing nontangential maximal estimates for the solution constructed in this manner is a certain new, general criterion guaranteeing boundedness in HP of linear operators. The presented work is in collaboration with Marius Mitrea. (Received September 12, 2019)

1154-35-979 **Francisco Delgado-Vences*** (delgadovences@gmail.com), Alameda de Leon 2, Centro, 68000 Oaxaca de Juarez, Oaxaca, Mexico, and **Franco Flandoli**. *A numerical method for Kolmogorov equations in Hilbert spaces.*

We introduce a numerical method for Kolmogorov (FPK) equations in a Hilbert space. The method is based on the spectral decomposition of the Ornstein-Uhlenbeck semigroup associated with the Kolmogorov equation. This allows us to write the solution of the Kolmogorov equation as a deterministic version of the Wiener-Chaos Expansion. With this expansion, we reformulate the Kolmogorov equation as an infinite system of ordinary differential equations, and by truncation it, we set a linear finite system of differential equations. The solution of such system allows us to build an approximation to the solution of the Kolmogorov equations in a separable Hilbert space. As an example, we present the solution of a stochastic Fisher-KPP equation. Moreover, we will discuss the Continuous Dependence on the Initial Condition of the solution. (Received September 12, 2019)

1154-35-1040 **Zachary J. Hilliard*** (zachary.hilliard@wsu.edu), Department of Mathematics and Statistics, Washington State University, PO Box 643113, Pullman, WA 99164-3113, and **Lynn Schreyer** and **Nikolaos Voulgarakis**. *An Anisotropic Cahn-Hilliard Equation with Variable Mobility and Gravity Potential.*

When considering the variable mobility case, the Cahn-Hilliard equation can be viewed as either a 4th-order semi-linear partial differential equation or as a system of second order partial differential equations of the form

$$\begin{aligned} u_t &= \nabla \cdot b(u) \nabla w \\ w &= f(u) - \alpha \Delta u, \end{aligned}$$

where f is generally taken to be the derivative of a double well potential function, $\alpha > 0$, and the function b is bounded away from zero in the non-degenerate case. Here we analyze a generalization of this equation of the form

$$\begin{aligned} u_t &= \nabla \cdot b(u) \mathbf{K} \nabla w \\ w &= f(u) - \alpha \Delta u + \phi. \end{aligned}$$

When viewing this equation from the porous media perspective, u represents density, $b(u) \mathbf{K}$ conductivity, w the gravi-chemical potential, and ϕ the gravity potential. We present various properties of this equation which are demonstrated with numerical examples as well as proving the existence of a weak solution when taking no-flux boundary conditions on both u and w . (Received September 12, 2019)

1154-35-1052 **Plamen Stefanov*** (stefanov@math.purdue.edu). *Semiclassical Sampling and Discretization of Linear Inverse Problems.*

We study sampling of functions f and their images Af under Fourier Integral Operators A at rates sh with s fixed and h a small parameter. We show that the Nyquist sampling limit of Af and f are related by the canonical relation of A using semiclassical analysis. We apply this analysis to the Radon transform in the parallel and the fan-beam coordinates. We explain and illustrate the optimal sampling rates for Af , the aliasing artifacts,

and the effect of averaging (blurring) the data Af . We prove a Weyl type of estimate on the minimal number of sampling points to recover f stably in terms of the volume of its semiclassical wave front set. (Received September 12, 2019)

1154-35-1075 **Eric Stachura*** (eric.stachura@kennesaw.edu). *Analysis of the Bi-anisotropic Maxwell system in Lipschitz domains.*

We prove well-posedness of an initial boundary value problem for the time dependent, bi-anisotropic Maxwell system in a Lipschitz domain. In such a setting there are 8 material parameters, which are allowed to depend on space and time. We in particular taken into account memory effects and impose nonzero Dirichlet boundary data. Similar results in higher order Sobolev spaces are obtained as well, assuming the material parameters satisfy a certain multiplier property. (Received September 13, 2019)

1154-35-1175 **Alan Mullenix*** (alan_mullenix@baylor.edu), **P. Jameson Graber**, **Rob Kirby** and **Colin Cotter**. *Mixed finite element methods for a linearized multilayer shallow water model.*

Building on previous work by Cotter, Graber, and Kirby for global tides, we develop a multilayer shallow water model and a finite element method to compute solutions. We establish new energy estimates that rigorously prove there is a unique attracting solution in the long-time limit. (Received September 13, 2019)

1154-35-1186 **Alyssa D Genschaw***, alyssa.genschaw@uconn.edu, and **Steve Hofmann**. *Parabolic Measure.*

We will discuss parabolic measure associated to a uniformly parabolic divergence form operator. We will give a brief overview of some recent results, including a Bourgain-type estimate, a criterion for non-doubling parabolic measure to satisfy a weak reverse Hölder inequality, and that BMO-solvability implies scale invariant quantitative absolute continuity of parabolic measure with respect to surface measure. This is joint work with Steve Hofmann. (Received September 13, 2019)

1154-35-1213 **Oana Marin**, **Marieme Ngom*** (mngom@anl.gov) and **Barry Smith**. *Optimal distribution of dielectrics via boundary integral constrained optimization.*

We show a method for determining the optimal distribution of a collection of dielectrics with the objective that the power of the scattered waves achieves its maximum in a target region. The physical problem is modeled using the Helmholtz equation recast in the boundary integral formulation. The change of simulation framework is justified by an increase in accuracy due to a more rigorous treatment of the dielectric interfaces as well as the free-space boundary conditions via the Sommerfeld condition. Well-conditioning of the discretized linear system is guaranteed by using the Combined Field Integral Equation (CFIE) which incorporates both single and double layer formulations in the half-space plane. This new approach requires the derivation of continuous adjoint operators associated with the half-space Helmholtz kernels for single and double layer formulations. The derivatives are verified against finite differences evaluations of the gradient of the objective function. The optimization itself is performed using the Limited Memory Variable Metric (LMVM) algorithm as well as steepest descent methods, and two types of objective functions are considered. (Received September 13, 2019)

1154-35-1266 **Zilong Song***, zilong.song@ucr.edu, and **Xiulei Cao**, **Tzyy-Leng Horng** and **Huaxiong Huang**. *Selectivity of the KcsA potassium channel: Analysis and computation.*

Ion channels regulate the flux of ions through cell membranes and play significant roles in many physiological functions. In this talk, we present an analytical and computational study of a mathematical model of the KcsA potassium channel, including the effects of ion size (Bikerman model) and solvation energy (Born model). Under equilibrium conditions, we obtain an analytical solution of our modified PNP system, which is used to explain selectivity of KcsA of various ions (K^+ , Na^+ , Cl^- , Ca^{2+} and Ba^{2+}) due to negative permanent charges inside the filter region and the effect of ion sizes. Our results show that K^+ is always selected over Na^+ , as smaller Na^+ ions have larger solvation energy. As the amount of negative charges in the filter exceeds a critical value, divalent ions (Ca^{2+} and Ba^{2+}) can enter the filter region and block the KcsA channel. For the non-equilibrium cases, due to difficulties associated with a pure analytical or numerical approach, we use a hybrid analytical-numerical method to solve the modified PNP system. Our predictions of selectivity of KcsA channels and saturation phenomenon of the current-voltage (I-V) curve agree with experimental observations. (Received September 14, 2019)

- 1154-35-1289 **Rachidi B Salako*** (salako.7@osu.edu), **Dawit Denu** (ddenu@georgiasouthern.edu) and **Sedar Ngoma** (ngoma@geneseo.edu). *Existence of traveling wave solutions of a deterministic vector-host epidemic model with direct transmission.*

We consider an epidemic model with direct transmission given by a system of nonlinear partial differential equations and study the existence of traveling wave solutions. When the basic reproductive number of the considered model is less than one, we show that there is no nontrivial traveling wave solution. On the other hand, when the basic reproductive number is greater than one, we prove that there is a minimum wave speed c^* such that the system has a traveling wave solution with speed c connecting both equilibrium points for any $c \geq c^*$. Moreover, under suitable assumption on the diffusion rates, we show that there is no traveling wave solution with speed less than c^* . We conclude with numerical simulations to illustrate our findings. The numerical experiments supports the validity of our theoretical results. (Received September 14, 2019)

- 1154-35-1292 **Johann Rudi*** (jrudi@anl.gov), **Georg Stadler**, **Jiashun Hu**, **Micheal Gurnis** and **Omar Ghattas**. *Inference of Uncertain Parameters in Physical Models Governed by PDEs with Application to Earth's Mantle Convection.*

We consider maps from a set of parameters to a set of quantities of interest that we seek to calibrate to observational data. In particular, we focus on maps that incorporate physical models in form of PDEs. Computing the solution of the parameter-to-observable map constitutes a forward problem, while finding parameters such that the model output is consistent with observational data amounts to an inverse problem.

We present computational methods for large-scale inverse problems posed in a Bayesian statistical framework by introducing a Gaussian prior distribution for the uncertain parameters. The maximum a posteriori (MAP) estimate and an approximation of parameter uncertainties at this MAP point is obtained from the solution of an optimization problem governed by the model PDE and second-order derivatives (Hessians). Newton's method is used for solving the optimization problem, which requires first- and second-order derivatives of the parameter-to-observable map. These operations are performed in an efficient and scalable fashion using adjoint methods. This algorithmic approach is independent of the problem under consideration, however we showcase its strengths on a model of Earth's mantle convection with velocities at the surface as observations. (Received September 14, 2019)

- 1154-35-1346 **Benjamin Blake McMillan*** (mcmillanbb@gmail.com). *Conservation laws and parabolic Monge-Ampere equations.*

In this talk, I will describe how the geometry of an arbitrary parabolic second order equation governs the existence of its conservation laws, and conversely, how the existence of even a single conservation law puts strong geometric restrictions on a parabolic equation. In particular, I will describe the strong connection between conservation laws and parabolic Monge-Ampere equations. (Received September 15, 2019)

- 1154-35-1366 **L. M. Chasman*** (chasmanm@morris.umn.edu). *Vibrating Plates and Eigenvalues of the Bi-Laplacian.*

Many interesting problems involving low eigenvalues of the Laplacian can be generalized to their fourth-order counterpart the Bi-Laplacian, often with the addition of a parameter representing some physical quantity such as tension or compression. In this talk, we consider recent work on a Bi-Laplacian (plate) eigenvalue problem in comparison with the methods of the corresponding Laplacian (drum) problem. (Received September 15, 2019)

- 1154-35-1415 **Sara Calandrini*** (scalandrini@fsu.edu), **Max Gunzburger** and **Konstantin Pieper**. *Exponential Time Differencing for the Tracer Equations Appearing in Ocean Models.*

The tracer equations are part of the primitive equations used in ocean modeling and describe the transport of tracers, such as temperature, in the ocean. Depending on the number of tracers considered, several equations may be added to and coupled to the dynamics system. In many relevant situations, the time-step requirements of explicit methods imposed by the transport and mixing in the vertical direction are more restrictive than those for the horizontal, and this may cause the need to use very small time steps if a fully explicit method is employed. To overcome this issue, we propose an exponential time differencing (ETD) solver where the vertical terms (transport and diffusion) are treated with a matrix exponential, whereas the horizontal terms are dealt with in an explicit way. In my talk, I will describe the method and discuss the implementation challenges. I will present the computational speed-ups that can be obtained over other semi-implicit methods and analyze the advantages of the scheme in the case of multiple tracers. Finally, I will present comparisons with existing ocean models, to make sure that our ETD solver is able to reproduce similar results under the same physical conditions. To do so, the whole primitive equations system will be solved. (Received September 15, 2019)

1154-35-1447 **Carley R. Walker***, 118 College Drive, Hattiesburg, MS 39406. *Rapid Approximate Diagonalization of Variable-Coefficient Differential Operators using the Uncertainty Principle*. Preliminary report.

We propose to create a new numerical method for a class of time-dependent PDEs (second-order, one space dimension, periodic boundary conditions) that can be used to obtain more accurate and reliable solutions than traditional methods. Previously, it was shown that conventional time-stepping methods could be avoided for time-dependent mathematical models featuring a finite number of homogeneous materials, thus assuming general piecewise constant coefficients. This proposed method will avoid the modeling shortcuts that are traditionally taken, and it will generalize the piecewise constant case of energy diffusion and wave propagation to work for an infinite number of smaller pieces, or a smoothly varying coefficient. We hypothesize that by treating a smoothly varying function as a piecewise constant function with infinitely many pieces, this potential method can be realized. Through the Uncertainty Principle, we will expectedly formulate highly accurate eigenfunctions which will in turn help us produce a more efficient solution method that avoids traditional time-stepping. (Received September 15, 2019)

1154-35-1496 **Robert Buckingham*** (buckinrt@uc.edu), Department of Mathematical Sciences, University of Cincinnati, Cincinnati, OH 45221. *Multiple-pole solutions of the nonlinear Schrodinger equation*.

The focusing nonlinear Schrodinger equation admits pure soliton solutions whose spectral data consists of poles of arbitrary order. If N is the pole order, these solutions qualitatively resemble N solitons approaching a single point, interacting, and separating again. Using the nonlinear steepest-descent method, we carry out the first analysis of the large-order asymptotic behavior of such solutions. These multiple-pole solitons exhibit a remarkable degree of structure, including a near-field limit with rogue-wave-type behavior described by Painleve functions, and a far-field limit with decaying, non-oscillatory, and oscillatory regions. We give explicit formulas for the leading-order behavior in each of these regimes. This is joint work with Deniz Bilman and Dengshan Wang. (Received September 15, 2019)

1154-35-1525 **David G Costa*** (david.costa@unlv.edu), Department of Mathematical Sciences, University of Nevada Las Vegas, 4505 S. Maryland Pkwy, Las Vegas, NV 89154-4020. *Multiple Solutions of Elliptic PDEs in Exterior Domains of \mathbb{R}^2* .

We prove existence of extremal constant-sign solutions and sign-changing solutions in $D_0^{1,2}(\Omega)$ of the boundary value problem

$$-\Delta u = a(x)f(u) \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega = \partial B(0, 1),$$

where $\Omega = \mathbb{R}^2 \setminus \overline{B(0, 1)}$ and $a(x)$ is a nonnegative coefficient satisfying a suitable integrability condition. Our main tool is the Kelvin transform, which we show to be an order-preserving and isometric isomorphism between $D_0^{1,2}(\Omega)$ and $H_0^1(B(0, 1))$. Our approach allows to handle sub, super or asymptotically linear nonlinearities $f(u)$. This is joint work with S. Carl, M. Fotouhi and H. Tehrani. (Received September 16, 2019)

1154-35-1556 **Annegret Glitzky**, WIAS Berlin, Mohrenstrasse 39, 10117 Berlin, Germany, **Matthias Liero**, WIAS Berlin, Mohrenstrasse 39, 10117 Berlin, Germany, and **Grigor Nika***, WIAS Berlin, Mohrenstrasse 39, 10117 Berlin, Germany. *An existence result for a class of electrothermal drift-diffusion models with Gauss-Fermi statistics for organic semiconductors*.

We prove the existence of solutions for the stationary Van Roosbroeck system coupled to the heat equation by an iteration scheme and Schauder's fixed point theorem. The former consists of continuity equations for electrons and holes and a Poisson equation for the electrostatic potential, and the latter features source terms containing Joule heat contributions and recombination heat. Special features of organic semiconductors like Gauss-Fermi statistics and mobilities functions depending on the electric field strength are taken into account. The underlying solution concept is related to weak solutions of the Van Roosbroeck system and entropy solutions of the heat equation. Additionally, for data compatible with thermodynamic equilibrium, the uniqueness of the solution is verified. (Received September 16, 2019)

1154-35-1578 **Maeve L. McCarthy*** (mmccarthy@murraystate.edu), Mathematics & Statistics, Murray State University, Murray, KY 42071, and **K. Renee Fister** and **Seth F. Oppenheimer**. *Optimal control of sterile insect techniques*.

We develop an optimal control framework to investigate the introduction of sterile type mosquitoes to reduce the overall mosquito population. Mosquitoes are vectors of disease. The goal is to establish the existence of a solution given an optimal sterilization protocol as well as to develop the corresponding optimal control representation

to minimize the infiltrating mosquito population while minimizing fecundity and the number of sterile type mosquitoes introduced into the environment per unit time. (Received September 16, 2019)

1154-35-1591 **Hong Wang*** (hwang@math.sc.edu), Department of Mathematics, University of South Carolina, Columbia, SC 29208. *Variable-order fractional PDEs: modeling, computation and analysis*. Preliminary report.

Fractional partial differential equations (FPDEs) provide accurate descriptions of complex phenomena including anomalously diffusive transport, memory effect and long-range spatial interaction. However, existing FPDE models tend to introduce certain nonphysical singularity near the initial time and boundary of the domain. Consequently, numerical approximations to FPDEs, which were proved to converge under the artificially assumed full regularity of the true solutions that are in appropriate, often have compromised accuracy. The reason lies in the incompatibility between the nonlocality of the power law decaying tail of the solutions and the locality of the initial or boundary conditions.

We study a physically correct variable-order FPDE model with a variable order that varies smoothly near the initial time or boundary of the domain to account for the impact of the locality of the initial or boundary condition. In addition, variable-order FPDE models have already been applied in a variety of applications. We will discuss their numerical simulations, based on theoretically proven mathematical results as well as the identification of the parameters of the models. (Received September 16, 2019)

1154-35-1646 **Ikemefuna C Agbanusi*** (iagbanusi@coloradocollege.edu). *Singular Integrals and Singular Perturbations*.

This talk will highlight some unexpected—at least to the speaker—connections between convergence estimates for some singular perturbation problems and estimates for singular oscillatory integrals and multipliers that lie just beyond the Calderon-Zygmund theory. One example is the multiplier $m_\mu(\xi) = \frac{|\xi|}{(|\xi|^2 + \mu^2)^{\frac{1}{2}}}$ and its variable coefficient generalizations. The goal is to find estimates for the corresponding operator whose Fourier transform is $\widehat{T_\mu f}(\xi) := m_\mu(\xi)\hat{f}(\xi)$ as the parameter $\mu \rightarrow \infty$ (Received September 16, 2019)

1154-35-1697 **Alexandru Tamasan*** (tamasan@math.ucf.edu), Oviedo, FL 32816, and **Kamran Sadiq** (kamran.sadiq@oeaw.ac.at), Linz, Austria. *On the range characterization of the Radon transform in two dimensions*. Preliminary report.

Range characterization of the Radon transform has been known since the early 1960s in the works of Gelfand-Graev, Ludwig, and Helgason (the so called Cavalieri conditions). For Riemannian geometries, the range characterization has been provided in terms of the scattering relations by Pestov-Uhlmann in 2004. The equivalence between the two characterizations was recently shown by Monard 2018. A separate characterization has been given by the authors in 2014 in terms of a Hilbert transform corresponding to the A-analytic maps in the sense of Bukhgeim (introduced in 1995). In this talk I will present some recent results establishing the equivalence between the Hilbert transform characterization and the Cavalieri conditions. (Received September 16, 2019)

1154-35-1801 **Valmir Bucaj, David Damanik, Jake Fillman, Vitalii Gerbuz, Tom VandenBoom*** (thomas.vandenboom@yale.edu), **Fengpeng Wang and Zhenghe Zhang**. *Positive Lyapunov Exponents and a Large Deviation Theorem for Continuum Anderson Models, Briefly*.

We prove positivity of the Lyapunov exponent for 1D continuum Anderson models by leveraging some classical tools from inverse spectral theory. The argument is much simpler than the existing proof due to Damanik–Sims–Stolz, and it covers a wider variety of random models. Along the way we note that a Large Deviation Theorem holds uniformly on compacts. (Received September 16, 2019)

1154-35-1937 **Erin Compaan, Renato Luca' and Gigliola Staffilani*** (gigliola@mit.edu), MIT Room 2-251, 77 Massachusetts Avenue, Cambridge, MA 02138. *Pointwise convergence of the Schrödinger flow*.

In this paper we address the question of the pointwise almost everywhere limit of nonlinear Schrödinger flows to the initial data, in both the continuous and the periodic settings. Then we show how, in some cases, certain smoothing effects for the non-homogeneous part of the solution can be used to upgrade to a uniform convergence to zero of this part, and we discuss the sharpness of the results obtained. We also use randomization techniques to prove that with much less regularity of the initial data, both in continuous and the periodic settings, almost surely one obtains uniform convergence of the nonlinear solution to the initial data, hence showing how more generic results can be obtained. (Received September 16, 2019)

1154-35-1994 **Annalisa Calini*** (calinia@cofc.edu) and **Thomas Ivey** (iveyt@cofc.edu). *Integrable Geometric Flows for Curves in Pseudoconformal S^3* .

We consider evolution equations for curves in the 3-dimensional sphere S^3 that are invariant under the group $SU(2, 1)$ of pseudoconformal transformations, which preserves the standard contact structure on the sphere. In particular, we investigate how invariant evolutions of Legendrian and transverse curves induce well-known integrable systems and hierarchies at the level of their geometric invariants. (Received September 17, 2019)

1154-35-2191 **Elliott Z Hollifield*** (ezhollif@uncg.edu), 116 Petty Building, PO Box 26170, Greensboro, NC 27402, and **Maya Chhetri** and **Petr Girg**. *Fractional Laplacian problems involving logistic reaction terms*.

We discuss existence of positive solutions to fractional Laplacian problems involving logistic reaction terms. We use the method of sub- and super solutions to establish existence results. We also discuss numerical bifurcation diagrams and profiles of positive solutions, corresponding to theoretical results, using a Finite Element Method. (Received September 17, 2019)

1154-35-2268 **Suncica Canic***, canics@berkeley.edu. *Recent Progress on Moving Boundary Problems*.

Even though the interaction between fluids and solids is one of the most classical problems in fluid mechanics, mathematicians have only recently begun developing a systematic theory to study this class of problems. The strong nonlinearities make the coupled, nonlinear moving boundary problem(s) exceedingly difficult to study. In this talk I will survey the most recent developments in this area, and show how the mathematical theory we are developing influences real-life problems, such as optimal design of cardiovascular prostheses, and optimal design of bioartificial pancreas. (Received September 17, 2019)

1154-35-2306 **Maya Chhetri*** (maya@uncg.edu), **Petr Girg** and **Elliott Hollifield**. *Existence of positive solutions for a class of superlinear fractional Laplacian problems*.

We consider a class of nonlinear fractional Laplacian problems with Dirichlet external condition. Using degree theory, combined with a re-scaling technique, we discuss the existence of positive weak solutions for a class of superlinear problem when the bifurcation parameter is small. (Received September 17, 2019)

1154-35-2365 **Sohaib Nasir**, **Devraj Duggal** and **Isabel Allen*** (iallen2@wisc.edu). *On a boundary value problem for a time-fractional wave equation with the Riemann-Liouville and Atangana-Baleanu derivatives*. Preliminary report.

This paper examines a time-fractional wave differential equation that combines Riemann-Liouville and Atangana-Baleanu definitions of fractional derivatives. A solution is found using separation of variables and the Laplace Transform. (Received September 17, 2019)

1154-35-2432 **Stefania Patrizi*** (spatrizi@math.utexas.edu) and **Luis Caffarelli**. *A local vs nonlocal segregation model*.

Segregation phenomena occurs in many areas of mathematics and science: from equipartition problems in geometry, to social and biological processes (cells, bacteria, ants, mammals) to finance (sellers and buyers). Segregation problems model a situation of high competition for resources and involve a combination of diffusion and annihilation between populations. We present a new model in which two competing species have the same population dynamics but different dispersal strategies: one species moves following a local diffusion while the other species adopts a nonlocal diffusion. This is a joint work with Luis Caffarelli. (Received September 17, 2019)

1154-35-2456 **Rachel A. Neville*** (raneville@math.arizona.edu), Dept. of Mathematics, PO Box 210089, Tucson, AZ 85721. *Topological Methods for Characterizing Snow Surface Roughness*. Preliminary report.

Snowpack is at the interface of between the Earth and the atmosphere, influencing the movement of air. The geometry of the snow surface can undergo dramatic changes at various length scales due to snow accumulation, terrain features, and wind. The result is a surface pattern that exhibits multi-scale roughness that is spatio-temporal complexity. As the snowpack surface evolves, albedo, wind resistance, energy exchange, and meltwater production are affected. Characteristics of the snowpack surface are important input variables in snow-hydrologic and climate models, therefore accurate estimates of these parameters are needed.

Leveraging tools from topological data analysis, we develop an estimate of the surface roughness of snowpack in mountainous terrain. This method captures multi-scale roughness as well as directionality of roughness characteristics. This is applied to LIDAR data collected from several snowfields outside of Boulder, CO. We draw comparisons to other estimates. (Received September 17, 2019)

1154-35-2458 **Zhanerke Temirgaliyeva*** (temirgal@usc.edu). *On the existence of strong solution to the 3D incompressible Navier-Stokes equations with damping.*

We present a simple proof of the existence of a global unique strong solution to the Cauchy problem of the incompressible Navier-Stokes equations with damping $\alpha|u|^{\beta-1}u$ for any $\alpha > 0$, $\beta > 3$. (Received September 17, 2019)

1154-35-2461 **Joshua Lee Flynn*** (joshua.flynn@uconn.edu). *Sharp Caffarelli-Kohn-Nirenberg Type Inequalities on Iwasawa Groups.*

For an Iwasawa group G (i.e., the "boundary" of a real noncompact rank one symmetric space), it is shown that the L^2 -CKN inequalities are sharp for all parameter values except one exceptional case. To show this, generalized Kelvin transforms K_σ are introduced and shown to be isometries for certain weighted Sobolev spaces. An interesting transformation formula for the sub-Laplacian with respect to K_σ is also derived. (Received September 17, 2019)

1154-35-2474 **Subhash Subedi*** (ssubedi@coastal.edu), Coastal Carolina University, Conway, SC 29528, and **Aghalaya S. Vatsala** (vatsala@louisiana.edu), University of Louisiana at Lafayette, Lafayette, LA 70504. *Quenching in Two Dimensional Time Fractional Reaction-Diffusion Equation.*

We study the quenching problem for time Caputo-fractional reaction-diffusion equation with a nonlinear reaction term in a two-dimensional rectangular domain. In this work, we prove local existence and the quenching of the solution of Caputo fractional ordinary differential equation and Caputo fractional reaction-diffusion equation with a nonlinear reaction term in finite time. We establish the condition for quenching for the solution of the fractional ordinary differential equation and fractional reaction-diffusion equation. We also provide the upper bound for the quenching time of the solution of fractional ordinary and reaction-diffusion equation. The study of quenching behavior of the solution of fractional differential equation relies on the quenching behavior of the solution of integer order reaction-diffusion equation and method of upper and lower solution. (Received September 17, 2019)

1154-35-2507 **Svetlana Roudenko*** (sroudenko@fiu.edu), Department of Math & Stats, Florida International University, Miami, FL 33199. *Soliton stability in higher-dimensional generalization of KdV equation.*

We consider Zakharov-Kuznetsov (ZK) equation, a higher-dimensional generalization of the well-known KdV equation, which was introduced by Zakharov and Kuznetsov back in 1972, where they also asked the question about the stability of solitons in higher dimensions (the KdV is restricted as the one-dimensional model). We discuss the stability of solitary waves in the 3d ZK equation, proving that solutions in the energy space that are orbitally stable are also asymptotically stable, that is, as time goes to infinity, they converge to a rescaling and shift of the solitary wave $Q(x-t, y, z)$ in a certain rightward moving window. This is a joint work with Luiz Gustavo Farah, Justin Holmer, and Kai Yang. (Received September 17, 2019)

1154-35-2509 **Dohyun Kwon*** (dhkwon@g.ucla.edu), 520 Portola Plaza, Math Sciences Building 6363, Los Angeles, CA 90095, and **Alpár Richárd Mészáros**. *Nonlinear diffusion equations with discontinuous nonlinearities.*

In this talk, I will consider degenerate dependent diffusion equations with discontinuous nonlinearities. In this model, the intensity of the diffusion is discontinuous, which describes so-called self-organized criticality phenomena in sandpile models. In order to show the well-posedness of this problem, we consider its gradient flow formulation in the Wasserstein space. Relying on this approach, we discover a link to recent models on congested crowd dynamics. In particular, we introduce a pressure field which characterizes the emerging critical regions. This is a joint work with Alpár Richárd Mészáros (Durham University). (Received September 17, 2019)

1154-35-2522 **Svetlana Roudenko***, Department of Math & Stats, Florida International University, Miami, FL 33199. *Behavior of solutions in stochastic critical and supercritical NLS equation with additive or multiplicative noise.*

We study nonlinear Schrödinger (NLS) equation with focusing nonlinearity, subject to additive or multiplicative stochastic perturbations driven by an infinite dimensional Brownian motion. Under the appropriate assumptions on the space covariance of the driving noise, previously de Bouard and Debussche established the H^1 local well-posedness in a general case and global well-posedness in the mass-subcritical case. In our work we study the mass-critical, intercritical and energy-critical cases of NLS and obtain quantitative estimates on solutions under the so-called mass-energy threshold. This is joint work with Annie Millet. (Received September 17, 2019)

1154-35-2583 **Subas Acharya*** (sa.subas@gmail.com), **Alain Bensoussan**, **Dmitry Rachinskiy** and **Alejandro Rivera**. *Real Options with the General Investment Cost*.

We revisit the classical real options problem, with additional features. The classical situation concerns choosing the right timing and right amount for an investment. We consider a stochastic optimization problem, which involves an option of one-time investment and an option of closing the activities. The timing and amount of an investment and the timing of the closure are parameters to be optimized in order to maximize the expected value of the profit. We reduce the stochastic optimization problem to a deterministic variational inequality using the dynamic programming technique and discuss the properties of solutions to the variational inequality. A general type of investment cost and particular examples are considered. (Received September 17, 2019)

1154-35-2584 **Taige Wang*** (taige.wang@uc.edu), 2925 CGD, Department of Mathematical Sciences, Cincinnati, OH 45221, and **Guojie Zheng** and **Dihong Xu**. *Backward uniqueness property of a chemotaxis model*. Preliminary report.

This presentation is concerned with backward uniqueness property of a parabolic - elliptic coupled chemotaxis model proposed on a convex domain Ω with homogeneous Dirichlet boundary condition and L -infinity initial data. We derive the estimate that norm of the initial data in a certain function space can be majorized by that of solution on a bounded open subset ω at terminal moment $t = T$. (Received September 17, 2019)

1154-35-2638 **Svitlana Mayboroda*** (svitlana@umn.edu), **Guy David** and **Marcel Filoche**. *The landscape law for the integrated density of states*. Preliminary report.

The present paper establishes sharp non-asymptotic estimates from above and below on the integrated density of states of the Schrödinger operator $L = -\Delta + V$, using a counting function for the minima of the landscape, a solution to the equation $Lu = 1$. The results are deterministic, and rely on the new uncertainty principle. Narrowing down to the context of disordered potentials, we derive the best currently available bounds on the integrated density of states for the Anderson model in R^d . In particular, this settles the long-debated question of applicability of the landscape theory to the potentials associated to the Anderson localization. (Received September 17, 2019)

1154-35-2640 **Svitlana Mayboroda*** (svitlana@umn.edu). *Elliptic measure and Green function on domains with lower dimensional boundaries*. Preliminary report.

We will discuss some recent progress regarding the connections between the geometric and PDE properties of the lower dimensional sets. (Received September 17, 2019)

1154-35-2663 **Brian Jongwon Choi*** (choigh@bu.edu), 111 Sciarappa Street, Cambridge, MA 02141. *Global Well-posedness and Modified Strichartz Estimates for the Fourth Order Schrödinger Equation*.

We discuss the adiabatic limit of the Quantum Zakharov System on the real line and the 1-D torus for square-integrable initial data. On the real line, we show that solutions exhibit smoothing effects measured in the space-time Fourier restriction norm. The use of smoothing estimates proves to be more useful than Strichartz's estimates in this case. We show that as the quantum parameter goes to zero, we obtain the NLS limit when the time element is restricted to a compact interval. When the periodic boundary condition is imposed, we prove L^4 and L^6 versions of modified Strichartz estimates. These estimates have applications in showing that certain sub-critical nonlinear equations are well-posed at low regularities. (Received September 17, 2019)

1154-35-2829 **Chi Phan*** (chi@mail.usf.edu), Department of Mathematics and Statistics, University of South Florida, 4202 E Fowler Ave, CMC342, Tampa, FL 33620. *Random Dynamics for Stochastic Hindmarsh-Rose Equations with Multiplicative Noise*.

The longtime pullback dynamics of stochastic Hindmarsh-Rose equations with multiplicative noise in neurodynamics is investigated. The existence of a random attractor for this random dynamical system is proved through the exponential transformation and uniform estimates showing the pullback absorbing property and the pullback asymptotically compactness of this stochastic cocycle. (Received September 18, 2019)

1154-35-2834 **Jaffar Ali Shahul Hameed*** (jashahulhameed@fgcu.edu), 10501 FGCU Blvd., Fort Myers, FL 33965, and **Seshadev Padhi**. *Positive Solutions for a Derivative Dependent p -Laplacian Equation with Riemann-Stieltjes Integral Boundary Conditions*. Preliminary report.

In this talk, we will discuss the existence of two non-trivial positive solutions to a class of boundary value problems (BVP), involving a p -Laplacian, of the form

$$\begin{aligned}(\Phi_p(x'))' + g(t)f(t, x, x') &= 0, \quad t \in (0, 1), \\ x(0) - ax'(0) &= \alpha[x], \\ x(1) + bx'(1) &= \beta[x],\end{aligned}$$

where $\Phi_p(x) = |x|^{p-2}x$ is a one dimensional p -Laplacian operator with $p > 1$, a, b are real constants. Here α, β are given by Riemann-Stieltjes integrals

$$\alpha[x] = \int_0^1 x(t)dA(t), \quad \beta[x] = \int_0^1 x(t)dB(t),$$

where A and B are functions of bounded variations. We will use the fixed point index theory to establish our results. (Received September 18, 2019)

37 ► *Dynamical systems and ergodic theory*

1154-37-58 **Rhiannan Ruef*** (rhiannan.ruef677@myci.csuci.edu), Mathematics Program, California State University, Channel Islands, 1 University Dr., Camarillo, CA 93012, and **Yasa Syed** (yas33@scarletmail.rutgers.edu), Department of Mathematics, Rutgers University, Hill Center for the Mathematical Sciences, Piscataway, NJ 08854. *Modeling New Low-Energy Defibrillation Methods and Heartbeat Patterns in Canines*.

Standard defibrillation methods rely on large electric fields that are often painful and damaging to the heart. Many spatially 2-D studies of low-energy defibrillation methods require knowledge of the number, phase, and/or location of the rotating waves that cause fibrillation. Here, we introduce a 3-D method that terminates these waves independently of such properties by applying a monophasic or biphasic low-energy electric field to detach the waves' axes of rotation from the surfaces. In the biphasic model, we found 9.6 and 12.8 ms to be optimal timings for the pulse separation and duration, respectively, terminating all rotating waves 87.5 - 89.7 Unlike humans, the heartbeat pattern of normal dogs contains abrupt changes between long and short interbeat intervals. We present a new probabilistic model that defines these intervals as functions of the vagal and sympathetic stimuli from the nervous system and their probability of emerging from the sinus node, the heart's natural pacemaker. A computer model based on this algorithm replicated behavior seen in Poincaré plots ($(n+1)$ st vs. n th interbeat intervals) obtained experimentally from both normal and sinus node dysfunction dogs. (Received July 26, 2019)

1154-37-107 **Travis A Dillon*** (travis.a.dillon@lawrence.edu). *Entropy of \mathcal{S} -graph shifts*. Preliminary report.

Symbolic dynamics arose as a way to study dynamical systems by discretizing space. The primary tools in symbolic dynamics are collections of infinite symbolic sequences called shift spaces. Of particular interest is a shift space's entropy, which is an invariant of the system and a measure of its complexity.

One specific class of shift spaces, called \mathcal{S} -gap shifts, has been well-studied and is notable for its rich dynamical and combinatorial structure. In this talk, we generalize \mathcal{S} -gap shifts to a much larger class of shift spaces called \mathcal{S} -graph shifts. We then prove a formula for the entropy of \mathcal{S} -graph shifts, which extends the entropy formula for \mathcal{S} -gap shifts. This talk assumes no prior knowledge of dynamical systems or symbolic dynamics. (Received August 07, 2019)

1154-37-197 **Xin Ma*** (xma29@buffalo.edu). *Pure infinite dynamical systems and their C^* -algebras*. I will introduce two notions characterizing pure infiniteness of dynamical systems and explain how these notions are equivalent and imply pure infiniteness of the reduced crossed product C^* -algebras. When the dynamical system is minimal, I will explain how this result provide us a dichotomy between stably finiteness and pure infiniteness of reduced crossed product C^* -algebras. (Received August 22, 2019)

1154-37-215 **Olga Lukina*** (olga.lukina@univie.ac.at), Oskar-Morgenstern-Platz 1, 1090 Vienna, Austria. *Arboreal Cantor actions.*

Given an arboreal representation of the absolute Galois group of a field, we associate to it an action of a countably generated discrete group on a Cantor set. We then classify certain classes of arboreal representations by their topological dynamical properties. (Received August 24, 2019)

1154-37-216 **Olga Lukina*** (olga.lukina@univie.ac.at), Oskar-Morgenstern-Platz 1, 1090 Vienna, Austria. *Leaves with non-trivial holonomy in laminations.*

Hurder and Katok proved that in a Riemannian foliation of codimension n , the subset of leaves with non-trivial linear holonomy has measure zero. A natural topological generalization of a Riemannian foliation is a lamination with a Cantor set transversal with an equicontinuous action of the holonomy pseudogroup. For this class of foliated spaces, we specify the sufficient conditions under which the set of leaves with non-trivial holonomy has measure zero. We also give examples of laminations where the set of leaves with non-trivial holonomy has positive measure. (Received August 24, 2019)

1154-37-337 **Elizabeth Bradley*** (lizb@cs.colorado.edu), University of Colorado, 430 USB, Boulder, CO 80309, **Varad Deshmukh**, University of Colorado 430 USB, Boulder, CO 80309, and **James D Meiss** (jdm@colorado.edu), University of Colorado, Department of Applied Mathematics, Boulder, CO 80309. *Computational topology techniques for characterizing time-series data.*

Topological data analysis (TDA), while abstract, allows characterization of time-series data obtained from non-linear dynamical systems. Though it is surprising that abstract measures of structure like the Betti numbers could be useful in the analysis of real-world data, TDA lets us compare different systems and even do membership testing or change-point detection. However, TDA of time-series data is computationally expensive and involves a number of free parameters. This complexity can be obviated by coarse-graining, using a construct called the witness complex. The parametric dependence of this process gives rise to the concept of persistent homology: how shape changes with the scale of the analysis. Its results allow us to distinguish time-series data from different systems—e.g., the same note played on different musical instruments. (Received September 01, 2019)

1154-37-365 **Feng Fu*** (feng.fu@dartmouth.edu), 27 N. Main Street, 6188 Kemeny Hall, Hanover, NH 03755. *Mathematical models of adoptive cell transfer approach for cancer immunotherapy.* Preliminary report.

Recent continued breakthroughs in antibody research and advances in T-cell engineering techniques have begun to unleash the curative potential of cancer immunotherapy. Here our model takes into account two means of immunotherapy. First, administration of monoclonal antibodies with high specificity, which will result in an increase in the ability of immune cells to detect and eliminate cancer cells – its killing rate or efficacy. Second, adoptive immune cell transfer, characterized by the infusion of in-vitro engineered and personalized immune cells into patients. These two novel treatment methods can be combined to take advantage of the bistability phenomenon in cancer, and may be applied in concert as combination immunotherapy. We provide a quantitative mathematical framework to provide practical guidance for clinical assessment of immunotherapy. (Received September 02, 2019)

1154-37-380 **Vincent R Martinez*** (vmartinez@hunter.cuny.edu), 695 Park Ave, Department of Mathematics & Statistics, Hunter East 922, New York, NY 10065. *Unique ergodicity for the damped-driven stochastically forced Korteweg-de Vries equation.* Preliminary report.

In 1967, Foias and Prodi captured precisely a notion of finitely many degrees of freedom for the 2D incompressible Navier-Stokes equations. This notion has since led to several developments in the understanding of the long-time behavior of solutions to the NSE, particularly, in the context of turbulence. In this talk, we will discuss this property as it regards the issue of uniqueness of ergodic invariant measures for the stochastically forced, damped-driven Korteweg-de Vries equation. (Received September 02, 2019)

1154-37-417 **Maria-Veronica Ciocanel*** (ciocanel.1@osu.edu), **Bjorn Sandstede** and **Scott McKinley**. *Stochastic and continuum dynamics in cellular transport.*

The cellular cytoskeleton ensures the dynamic transport, localization, and anchoring of various proteins and vesicles. For example, in the development of egg cells into embryos, messenger RNA (mRNA) molecules bind and unbind to and from cellular roads called microtubules, switching between bidirectional transport, diffusion, and stationary states. Since models of intracellular transport can be analytically intractable, asymptotic methods are useful in understanding effective cargo transport properties as well as their dependence on model parameters.

We consider these models in the framework of partial differential equations as well as stochastic processes and derive the effective velocity and diffusivity of cargo at large time for a general class of problems. We illustrate applications of the proposed method to macroscopic models of protein localization and microscopic models of processive cargo movement by teams of molecular motor proteins. In the context of mRNA localization, our simulations incorporating the microtubule geometry suggest that anchoring of certain protein complexes may be necessary in promoting healthy development. (Received September 03, 2019)

1154-37-448 **Tomoki Ohsawa*** (tomoki@utdallas.edu), 800 W Campbell Rd, Richardson, TX 75080-3021. *Collective Heavy Top Dynamics.*

We construct a Poisson map \mathbf{M} that relates the heavy top dynamics with a canonical Hamiltonian system. It is well known that the heavy top dynamics can be described as a Hamiltonian system using the $(-)$ -Lie–Poisson bracket on the dual $\mathfrak{se}(3)^*$ of the Lie algebra of the special Euclidean group $\mathbf{SE}(3)$. Our map $\mathbf{M}: T^*\mathbb{C}^2 \rightarrow \mathfrak{se}(3)^*$ is Poisson with respect to the canonical Poisson bracket on $T^*\mathbb{C}^2 \cong T^*\mathbb{R}^4$ and the $(-)$ -Lie–Poisson bracket on $\mathfrak{se}(3)^*$. The essential part of this map is the momentum map associated with the cotangent lift of the natural right action of the semidirect product Lie group $\mathbf{SU}(2) \ltimes \mathbb{C}^2$ on \mathbb{C}^2 . This Poisson map gives rise to a canonical Hamiltonian system on $T^*\mathbb{C}^2$ whose solutions are mapped by \mathbf{M} to solutions of the heavy top equations. We show that the Casimirs of the heavy top dynamics and the additional conserved quantity of the Lagrange top correspond to the Noether conserved quantities associated with certain symmetries of the canonical Hamiltonian system. (Received September 04, 2019)

1154-37-470 **Vladimir Dragovic***, Department of Mathematical Sciences, 800 W. Campbell Road, FO 35, Richardson, TX 75080. *Ellipsoidal Billiards and Chebyshev-type polynomials.*

A comprehensive study of periodic trajectories of the billiards within ellipsoids in the d -dimensional Euclidean space is presented. The novelty of the approach is based on a relationship established between the periodic billiard trajectories and the extremal polynomials of the Chebyshev type on the systems of d intervals on the real line. The case study of trajectories of small periods is given. In particular, it is proven that all d -periodic trajectories are contained in a coordinate-hyperplane and that for a given ellipsoid, there is a unique set of caustics which generates $(d + 1)$ -periodic trajectories. A complete catalog of billiard trajectories with small periods is provided for $d = 3$. This is a joint work with Milena Radnovic. (Received September 04, 2019)

1154-37-506 **Jeffrey Diller and Kyounghee Kim*** (kim@math.fsu.edu), Department of Mathematics, Florida State University, Tallahassee, FL 32308. *Entropy of real rational surface automorphisms.*

We compare real and complex dynamics for automorphisms of rational surfaces that are obtained by lifting some quadratic birational maps of the plane. We will discuss examples where the entropy of the full (complex) automorphism is the same as its real restriction. Also, we exhibit different examples where the entropy is strictly decreased by restricting to the real part of the surface. (Received September 05, 2019)

1154-37-517 **Mauricio Andrés Díaz Raby*** (mauricio.diazraby@gmail.com), Collao Paula Pineda 153, 4030 Concepción, Concepcion, Chile. *Special Sensitive maps in Measure Dynamical Systems of type $F_{ip} \cap F_{ps} \cap F_{pubd}$ in uniform spaces.*

In this article we going to study the Sensitive system that can be described using a family of subsets of \mathbb{Z}^+ . We prove that a MDS-type 1 without equicontinuous points has $(F_{ip} \cap F_{ps} \cap F_{pd1})^*$ as the set of a F-recurrent and F-scattering Dynamical System. Later, We going to prove that for a MDS-type1 with F-sensitive has the same results if and only if the Dynamical system has Li-York weakly n -sensitive by $n > 2$. Later, we prove that a sensitive map has a non bijective map between two families with disjoint and uncountable scrambled sets in disjoint product system that are F-scattering. At the end, we going to discuss about the dependence of those theorems in a metric space with unbounded complexity function (Received September 06, 2019)

1154-37-523 **Yuri Latushkin and Xinyao Yang***, xinyao.yang@xjtlu.edu.cn. *Stability of a steady state in reaction diffusion systems arising in combustion theory in the one dimensional case.*

We prove that a steady state solution of a class of the reaction diffusion system is Lyapunov stable in the intersection of the Sobolev space and an exponentially weighted space. Special attention is given to a particular case, that is, the system of equations arising in combustion theory. The steady state solution considered herein is the end state of the traveling front associated with the system, and thus the current results complement recent papers by A. Ghazaryan, Y. Latushkin and S. Schechter where stability of the traveling fronts was investigated. (Received September 06, 2019)

- 1154-37-581 **Lu Lu*** (lu_lu_1@brown.edu), Providence, RI 02906, and **P Jin, Y Su and George Karniadakis**. *OpNet: Learning nonlinear operators for identifying differential equations*. Preliminary report.

It is well-known that neural networks can be used to approximate any continuous function to arbitrary accuracy. Another approximation result, which is yet more surprising but has not been appreciated enough, states that a neural network with one hidden layer can approximate accurately any nonlinear continuous operator. This approximation theorem indicates the potential application of neural networks to learn nonlinear operators from data. However, it only guarantees the small approximation error for the sufficient large network, and does not consider the optimization error and generalization error. To realize this theorem in practice, we propose operator networks (OpNets) to learn operators accurately and efficiently. OpNet significantly improves generalization by constructing two sub-networks. We test the performance of OpNet on identifying two types of operators, including dynamic systems (in the form of ordinary differential equations) and partial differential equations. (Received September 07, 2019)

- 1154-37-642 **Mark Piraino*** (mark.piraino@northwestern.edu). *The weak Bernoulli property for matrix Gibbs states*.

We study the ergodic properties of a class of measures on $\Sigma^{\mathbb{Z}}$ for which $\mu_{\mathcal{A},t}[x_0 \cdots x_{n-1}] \approx e^{-nP} \|A_{x_0} \cdots A_{x_{n-1}}\|^t$, where $\mathcal{A} = (A_0, \dots, A_{M-1})$ is a collection of matrices. The measure $\mu_{\mathcal{A},t}$ is called a matrix Gibbs state. In particular we give a sufficient condition for a matrix Gibbs state to have the weak Bernoulli property. We employ a number of techniques to understand these measures including a novel approach based on Perron-Frobenius theory. We find that when t is an even integer the ergodic properties of $\mu_{\mathcal{A},t}$ are readily deduced from finite dimensional Perron-Frobenius theory. We then consider an extension of this method to $t > 0$ using operators on an infinite dimensional space. Finally we use a general result of Bradley to prove the main theorem. (Received September 09, 2019)

- 1154-37-651 **Mohammad Farazmand*** (farazmand@ncsu.edu). *Physics-compatible metrics for clustering algorithms*.

Clustering of high-dimensional data is routinely used for understanding dynamics and developing efficient control strategies. Clustering algorithms often use a measure of distance between data points to assess their proximity. The familiar Euclidean distance is the prevalent choice of metric for clustering algorithms. While the Euclidean metric is useful in many applications, we show that it can lead to unsatisfactory results in the context of nonlinear dynamics. By leveraging the physics of the problem (such as its conserved quantities or its variational structure), we show how to devise more meaningful metrics that lead to physically relevant clusters. (Received September 09, 2019)

- 1154-37-668 **John R. Doyle*** (jdoyle@latech.edu), Louisiana Tech University, Mathematics and Statistics, Ruston, LA 71272, and **Joseph H. Silverman**. *Moduli spaces for dynamical systems with level structure*.

We construct moduli spaces for endomorphisms of projective space together with a dynamical notion of level structure—namely, marked points satisfying specified orbit relations. We will discuss various properties of these spaces, and we will mention a result that underscores a connection with the dynamical uniform boundedness conjecture of Morton and Silverman. (Received September 09, 2019)

- 1154-37-725 **Lori Alvin*** (lori.alvin@furman.edu) and **James Kelly**. *Topological Entropy of Markov Set-valued Functions*.

We investigate the entropy for a class of upper semi-continuous set-valued functions called Markov set-valued functions, which are a generalization of single-valued Markov interval functions. It is known that the entropy of a Markov interval function can be found by calculating the entropy of an associated shift of finite type. In this talk we construct a similar shift of finite type for Markov set-valued functions and use this shift space to find upper and lower bounds on the entropy of the set-valued functions. (Received September 10, 2019)

- 1154-37-749 **Abhishek Shivkumar*** (ashivkum@berkeley.edu), **Rebecca Alexandra Lopez**, **Mykhaylo Mykolaiovich Malakhov** and **Brianna Fitzpatrick**. *Attractor Reconstruction and Empirical Parameter Inference for Hydrogen-Oxygen Chemistry*.

Robust tools for characterizing nonlinear dynamical systems are indispensable in the development of in-space thrusters and other technologies of interest to the Air Force Research Laboratory. Although combustion can be easily simulated, the difficulty of experimentally observing a large number of chemical species complicates traditional methods for identifying system dynamics and ascertaining reaction rate coefficients. We utilize the attractor reconstruction procedure from convergent cross mapping to reconstruct the complete behavior of a

continuously stirred hydrogen-oxygen tank reactor model from time-lagged observations (shadow manifolds) of individual species. Having demonstrated that a shadow manifold can effectively capture the information present in the entire attractor, we describe a novel optimization metric for data-driven parameter inference that only requires knowledge of a single observable. The proposed method infers parameters by minimizing the Wasserstein distance between binned shadow manifolds of a given reference data set and trial solutions. We demonstrate the superiority of our metric over standard approaches and present proof-of-concept results for reaction coefficient inference. (Received September 10, 2019)

1154-37-796 **Andrew Dykstra*** (adykstra@hamilton.edu), **Nic Ormes** and **Ronnie Pavlov**. *The number of minimal subsystems of a transitive subshift of linear complexity.*

We bound the number of distinct minimal subsystems of a given transitive subshift of linear complexity. We also bound the number of generic measures such a subshift can support based on its complexity function. (Received September 10, 2019)

1154-37-818 **David Krumm*** (dkrumm@reed.edu), Portland, OR, **Diego Marques**, Brasília, DF, Brazil, and **Carlos Gustavo Moreira**, Rio de Janeiro, RJ, Brazil. *Algebraic preperiodic points of entire transcendental functions.* Preliminary report.

Motivated by questions in transcendental number theory, K. Mahler asked in 1976 whether there exists an entire transcendental function $f : \mathbb{C} \rightarrow \mathbb{C}$ with the property that $f(\overline{\mathbb{Q}}) \subseteq \overline{\mathbb{Q}}$ and $f^{-1}(\overline{\mathbb{Q}}) \subseteq \overline{\mathbb{Q}}$. Mahler's question was answered in the affirmative by Marques and Moreira in 2016. In this talk we will discuss some dynamical properties of this type of function f , in particular the structure of the directed graph of algebraic preperiodic points of f . (Received September 17, 2019)

1154-37-889 **Yunping Jiang*** (yunping.jiang@qc.cuny.edu), Department of Mathematics, 65-30 Kissena Blvd, Flushing, NY 11367. *Orders of Oscillation Motivated By Sarnak's Conjecture.*

In view of Sarnak's conjecture in number theory, I will define orders of oscillating sequences and mention the important role of them playing in number theory in this talk. For oscillating sequences (of order 1), we have proved that they are linearly disjoint from all MMA and MMLA flows. For oscillating sequences of order $d > 2$, I have proved that they are linearly disjoint from all affine distal flows as well as all nonlinear affine distal flows with Diophantine translations on the d -torus. We will present a detail explanation about how to use this result to study the linear disjointness of affine flows with zero topological entropy as well as associated nonlinear flows with Diophantine translations on the d -torus and oscillating sequences of order d in the arithmetic sense. One of the consequences is that Sarnak's conjecture holds for all the flows discussed in this talk. (Received September 11, 2019)

1154-37-1015 **Trevor Hyde***, 5734 S University Ave, Chicago, IL 60637, Chicago, IL 60637, and **John Doyle**, **Max Weinreich**, **Colby Kelln**, **Talia Blum** and **Henry Talbott**. *Portrait spaces for dynamical semigroups and unlikely intersections.* Preliminary report.

Given a dynamical portrait for several rational functions acting on a finite set, we initiate the study of the corresponding moduli space of realizations. In this talk we discuss the various ways in which these moduli spaces can have higher than expected dimension and present results explaining this phenomenon. Time permitting we will also survey the findings of large exhaustive computations we conducted for portrait spaces of cubic polynomials. (Received September 12, 2019)

1154-37-1053 **Kevin McGoff*** (kmcgoff1@unc.edu), Fretwell 376, 9201 University City Blvd., Charlotte, NC 28223, and **James P. Kelly**. *Entropy conjugacy for Markov multi-maps of the interval.*

We consider a class \mathcal{F} of Markov multi-maps on the unit interval. Any multi-map gives rise to a space of trajectories, which is a closed, shift-invariant subset of $[0, 1]^{\mathbb{Z}_+}$. For a multi-map in \mathcal{F} , we show that the space of trajectories is (Borel) entropy conjugate to an associated shift of finite type. Additionally, we characterize the set of numbers that can be obtained as the topological entropy of a multi-map in \mathcal{F} . (Received September 12, 2019)

1154-37-1079 **Felipe García-Ramos*** (felipegra@yahoo.com), Instituto de Física, UASLP, Av. Manuel Nava #6, Zona Universitaria, 78290 San Luis Potosi, Mexico, and **Sebastian Barbieri** and **Hanfeng Li**. *Markovian properties of continuous group actions: algebraic actions, entropy and the homoclinic group.*

We study topological Markov properties for continuous group actions on compact metric spaces (for symbolic systems these were introduced as topological Markov fields). Examples appear in the context of supports of equilibrium measures and expansive algebraic actions. With these properties we can give simple conditions to ensure the existence of off-diagonal asymptotic pairs and positive sofic entropy. We generalize previous results by Chung and Li. (Received September 13, 2019)

1154-37-1252 **Piotr Oprocha*** (oprocha@agh.edu.pl), AGH University of Science and Technology, Faculty of Applied Mathematics, al. Mickiewicza 30, 30-059 Krakow, Poland. *On completely scrambled systems.*

Let (X, T) be a dynamical system (continuous map T acting on a compact metric space (X, d)). A pair of points (x, y) is Li-Yorke chaotic if it is proximal ($\liminf_{n \rightarrow \infty} d(T^n x, T^n y) = 0$) but not asymptotic ($\liminf_{n \rightarrow \infty} d(T^n x, T^n y) > 0$). A set $A \subset X$ is scrambled, if any pair of distinct points of A is Li-Yorke chaotic.

Very quickly after publication of paper of Li and Yorke in 1975 it was realized that uncountable scrambled set can be constructed in many case. In fact, it exists in every dynamical system with positive topological entropy (Blanchard, Glasner, Kolyada and Maass, 2002). On the other hand, it seems intuitively that compactness prevents whole space to be scrambled (so-called completely scrambled system), however also such examples were developed.

The aim of this talk is present history of research on completely scrambled systems, including some recent results. (Received September 14, 2019)

1154-37-1328 **Sarah Koch*** (kochsc@umich.edu). *Multiplier spectra of PCF rational maps.*

We discuss the question of which algebraic numbers arise as multipliers of PCF rational maps, and to what extent this data determines the rational map in moduli space. This is very much work in progress, with X. Buff and A. Epstein. (Received September 14, 2019)

1154-37-1333 **Matthew P Clay**, **Michael Hofbauer** and **Nandor J Simanyi*** (simanyi@uab.edu), 4014 University Hall, 1402 10th Avenue South, Birmingham, AL 35294-1241. *On the ergodicity of Wojtkowski's falling ball system.* Preliminary report.

We consider the system of n (≥ 2) point masses m_1, \dots, m_n falling freely in the vertical half line $\{q | q \geq 0\}$ (so that $0 \leq q_1 \leq q_2 \leq \dots \leq q_n$) under constant gravitation and colliding with each other and the solid floor $q = 0$ elastically. In order to have a natural, invariant symplectic cone system, we assume that $m_1 \geq \dots \geq m_n$, but not all masses are equal. One is interested the ergodic properties, like hyperbolicity, ergodicity, mixing, etc of such systems. We survey the existing results, pose some challenging open questions, and sketch a roadmap for proving ergodicity of such systems with $m_1 > m_n$. An important feature of this research is a thorough understanding of why such systems are, in fact, not isomorphic to any semi-dispersive billiard flow. Let A be the subset of the phase space containing all phase points for which all velocities are strictly positive. We proved that the flow-invariant hull of A belongs to a single ergodic component of the flow. Therefore, in order to prove ergodicity, it is enough to show that almost every trajectory enters A , sooner or later. (Received September 15, 2019)

1154-37-1355 **Mrinal K Roychowdhury*** (mrinal.roychowdhury@utrgv.edu), 1201 West University Drive, Edinburg, TX 78539. *Optimal Quantization.*

The basic goal of quantization for probability distribution is to reduce the number of values, which is typically uncountable, describing a probability distribution to some finite set and thus approximation of a continuous probability distribution by a discrete distribution. It has broad application in signal processing, and data compression. Two main goals have been: finding the exact configuration of a so-called n-optimal set' which corresponds to the support of the quantized version of the distribution, and estimating the rate, called quantization dimension, at which some specified measure of the error goes to zero as n goes to infinity. Quantization dimension is also connected with other dimensions of dynamical systems, such as Hausdorff, packing, and box counting dimensions. I will talk about it. (Received September 15, 2019)

1154-37-1384 **Jan P Boronski***, boronski@agh.edu.pl, and **Jernej Cinc** and **Xiao-Chuan Liu**. *Parametrised Families of Rotational Attractors and the Accessible Rotation Numbers.*

We investigate the prime ends rotation numbers arising from parametrized Brown-Barge-Martin (BBM for short) embeddings of inverse limits of topological graphs as in [BCH13]. Among our results, we show the existence

of homeomorphisms of \mathbb{S}^2 with Lakes of Wada rotational attractors, that are arbitrarily close to the identity. With the help of reduced Arnold's family we also construct a parametrised family of Birkhoff-like cofrontier attractors, where for uncountably many parameter values the two accessible rotation numbers are irrational. This complements the negative resolution of Walker's Conjecture in [KLN15].

References

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[KLN15] Koropecski, A; Le Calvez, P.; Nassiri, M.; Prime ends rotation numbers and periodic points. Duke Math. J. 164:403-472, 2015 (Received September 15, 2019)

1154-37-1420 **Drew D Ash*** (dash@albion.edu), Albion College, 611 E Porter St, Dept. of Mathematics and Computer Science, Albion, MI 49224. *Strong Speedups of Minimal Cantor Systems and Strong Orbit Equivalence.*

Given a dynamical system (X, T) one can define a *speedup* of (X, T) as another dynamical system $S : X \rightarrow X$ where $S(x) = T^{p(x)}(x)$ for some $p : X \rightarrow \mathbb{Z}^+$. We call p the jump function. This talk will focus on *strong speedups*, a speedup where the jump function has one point of discontinuity. We will present a theorem characterizing when, given two minimal Cantor systems one system is a strong speedup of the other. The theorem is closely related to Giordano, Putnam, and Skau's characterization theorem for Strong Orbit Equivalence. This is joint work with Nic Ormes. Finally, time permitting, we will discuss current work, with Andrew Dykstra and Michelle LeMasurier, on speedups of Toeplitz flows with bounded jump functions. (Received September 15, 2019)

1154-37-1422 **David Kerr***, Department of Mathematics, Texas A&M University, College Station, TX 77843-3368, and **Hanfeng Li**, Department of Mathematics, SUNY at Buffalo, Buffalo, NY 14260-2900. *Orbit equivalence and Bernoulli rigidity.*

It was shown by Tim Austin that if an orbit equivalence between probability-measure-preserving actions of finitely generated amenable groups is integrable then it preserves entropy. We will show that the same conclusion holds for the maximal sofic entropy when the acting groups are countable and sofic and contain an amenable w-normal subgroup which is not locally virtually cyclic, and that it is moreover enough to assume that the Shannon entropy of the cocycle partitions is finite (what we call *Shannon orbit equivalence*). It follows that two Bernoulli actions of a group in the above class are Shannon orbit equivalent if and only if they are conjugate. (Received September 15, 2019)

1154-37-1425 **Michail E Filippakis*** (mfilip@unipi.gr), Department of Digital Systems, University of Piraeus, 122 Grigoriou Labraki Str, 18532, Piraeus-Gr, 18534 Piraeus, Greece, and **Marilena Poulou**. *A fractional nonlinear Schrödinger-Poisson system.*

We are concerned with the following damped fractional nonlinear Schrödinger Poisson system,

$$\begin{cases} u_t + \gamma u + i(-\Delta)^s u + iu\varphi = f \\ \pm(-\Delta)^t \varphi = |u|^2. \end{cases} \quad (1)$$

where $\gamma > 0$, $(-\Delta)^\alpha$ is the fractional Laplacian operator for $\alpha = s, t \in (0, 1)$,

The fractional Schrödinger equation provides us with a general point of view on the relationship between statistical properties of quantum-mechanical path and structure of the fundamental equations of quantum mechanics. First we analyse how the different orders of the Laplacian operator affect the existence and uniqueness of solutions as well as the existence of a global attractor. Next step to address is the discrete counterpart of the continuous dynamical systems. The aim is to prove that such a semi-discrete equation provides a discrete infinite dimensional dynamical system that possesses a global attractor.

The publication of this paper has been partly supported by the University of Piraeus Research Center. (Received September 16, 2019)

1154-37-1446 **Vadim Kaimanovich*** (vadim.kaimanovich@gmail.com). *Free and totally non-free boundary actions.* Preliminary report.

I will outline recent results (joint with Anna Erschler) on the stabilizers of the group action on its Poisson boundary: existence of a free boundary action for any group with infinite conjugacy classes, a complete description of the possible kernels of such actions, and an example of a totally non-free boundary action. (Received September 15, 2019)

1154-37-1471 **Artem Dudko** and **Kostya Medynets*** (medynets@usna.edu), 572C Holloway Road, Annapolis, MD 21402. *Invariant Random Subgroups of Full Groups.*

We classify the ergodic invariant random subgroups of block-diagonal limits of symmetric groups in the cases when the groups are simple and the associated dimension groups have finite dimensional state spaces. These block-diagonal limits arise as the transformation groups (full groups) of Bratteli diagrams that preserve the cofinality of infinite paths in the diagram. Given a simple full group G admitting only a finite number of ergodic measures on the path-space X of the associated Bratteli digram, we prove that every non-Dirac ergodic invariant random subgroup of G arises as the stabilizer distribution of the diagonal action on X^n for some $n \geq 1$. As a corollary, we establish that every group character χ of G has the form $\chi(g) = \text{Prob}(g \in K)$, where K is a conjugation-invariant random subgroup of G . (Received September 15, 2019)

1154-37-1539 **Thomas Silverman*** (tjsilver@umich.edu). *A non-archimedean λ -lemma and J -stability.*

In a celebrated paper published in 1983, R. Mañé, P. Sad, and D. Sullivan prove a result about holomorphic families of injections called the λ -Lemma with impressive applications to the complex dynamics of families of one-variable rational functions. In this talk, I will discuss the dynamics of families of one-variable rational functions parametrized by Berkovich spaces over a complete non-archimedean field, including a suitable non-archimedean analogue of the λ -Lemma. I will also explain how this can be used to prove the equivalence of two stability conditions in non-archimedean dynamics. (Received September 16, 2019)

1154-37-1568 **Ivan Chio*** (ichio@iu.edu), 402 N Blackford St, LD270, Indianapolis, IN 46202, and **Roland K.W. Roeder** (rroeder@math.iupui.edu), LD224Q, 402 N Blackford St, Indianapolis, IN 46202. *Chromatic Zeros On Hierarchical Lattices and Equidistribution on Parameter Space.*

Associated to any finite simple graph $\Gamma = (V, E)$ is the *chromatic polynomial* $P_\Gamma(q)$, which has the property that for any integer $k \geq 0$, $P_\Gamma(k)$ is the number of ways to properly colour the vertices of Γ using k colours. The degree of $P_\Gamma(q)$ is $|V|$. A hierarchical lattice is a sequence of graphs $\{\Gamma_n\}_{n=0}^\infty$ built recursively under a generating graph. For each $n \geq 0$, let μ_n be the probability measure

$$\mu_n := \frac{1}{|V_n|} \sum_{\substack{q \in \mathbb{C} \\ P_{\Gamma_n}(q)=0}} \delta_q.$$

We prove that if the generating graph is 2-connected, then the sequence of measures μ_n converges to some measure μ , called the *limiting measure of chromatic zeros* for $\{\Gamma_n\}_{n=0}^\infty$. For the Diamond Hierarchical Lattice (DHL), we show that its limiting measure has Hausdorff dimension 2.

The main techniques come from holomorphic dynamics, in particular we prove a new equidistribution result that relates the chromatic zeros of a hierarchical lattice to the bifurcation/activity current associated to a particular marked point. This is joint work with Roland Roeder. (Received September 16, 2019)

1154-37-1597 **David Kerr***, Department of Mathematics, Texas A&M University, College Station, TX 77843-3368, and **Hanfeng Li**, Department of Mathematics, SUNY at Buffalo, Buffalo, NY 14260-2900. *Continuous orbit equivalence, entropy, and sparse connectivity.*

We introduce a property sofic SC (sparse connectivity) for continuous actions of a countable sofic group G on a compact metrizable space and show that any two such actions which are continuously orbit equivalent have the same maximum sofic entropy. We show moreover that if G has a w-normal amenable subgroup which is neither locally finite nor virtually cyclic then every action of G has property SC. (Received September 16, 2019)

1154-37-1604 **Lien-Yung Kao*** (lkao@uchicago.edu), The University of Chicago, Department of Mathematics, Chicago, IL 60637. *Pressure metrics for Teichmüller spaces of punctured surfaces.*

Thurston pointed out that one can use variations of lengths of closed geodesics on hyperbolic surfaces to construct a Riemannian metric on the Teichmüller space. For closed surfaces cases, Wolpert proved this Riemannian metric is indeed the Weil-Petersson metric. McMullen proposed a thermodynamic formalism approach to this Riemannian metric and called it the pressure metric. In this talk, I will discuss how to extend this dynamics construction to non-compact finite area hyperbolic surfaces. If time permitted, I will also discuss relations between the pressure metric and Manhattan curves. (Received September 16, 2019)

1154-37-1612 **Jane M. Hawkins*** (jmh@math.unc.edu), Mathematics Dept, CB #3250, UNC at Chapel Hill, Chapel Hill, NC 27599. *An application of entropy to genomics*. Preliminary report.

The entropy of a dynamical system is a measure of its complexity, which in turn reflects how its evolution over time seems to exhibit randomness. A recent application is to use entropy to help classify and understand the evolution of some common viruses. The papilloma virus (PV) infects animals from birds, rabbits and cats to cattle, horses and humans, and while all types have some negative health effects, only a few are deadly. We discuss how entropy helps to solve the huge classification problem for PVs to better understand their evolution, with a view to eradicating the worst types. (Received September 16, 2019)

1154-37-1621 **Benjamin Hutz*** (benjamin.hutz@slu.edu), Saint Louis, MO. *Automorphism loci for endomorphisms of \mathbb{P}^1* . Preliminary report.

I will present some recent classification results for endomorphisms of \mathbb{P}^1 with nontrivial automorphisms stemming from the groups I worked with at the summer REU program at the Institute for Computational and Experimental Research in Mathematics (ICERM) in the summer of 2019. In characteristic 0, these results concern classifying families of maps of degree $2 - 4$ with non-trivial automorphisms. In characteristic $p > 0$, these results concern the realizability of finite subgroups of $PGL_2(\mathbb{F}_p)$ as automorphism groups. The focus will be open questions stemming from these projects. (Received September 16, 2019)

1154-37-1639 **Tamara Kucherenko*** (tkucherenko@ccny.cuny.edu) and **Daniel Thompson**. *Measures of maximal entropy on subsystems of topological suspension semi-flows*.

Given a compact topological dynamical system (X, f) with positive entropy and upper semi-continuous entropy map, and any closed invariant subset $Y \subset X$ with positive entropy, we show that there exists a continuous roof function such that the set of measures of maximal entropy for the suspension semi-flow over (X, f) consists precisely of the lifts of measures which maximize entropy on Y . This result has a number of implications for the possible size of the set of measures of maximal entropy for topological suspension flows. In particular, for a suspension flow on the full shift on a finite alphabet, the set of ergodic measures of maximal entropy may be countable, uncountable, or have any finite cardinality. (Received September 16, 2019)

1154-37-1649 **Akshat Das, Joanna Furno*** (jfurno@depaul.edu) and **Alan Haynes**. *Bounded Remainder Sets for Rotations on Products of Adelic Tori*. Preliminary report.

The cut-and-project construction uses dynamical systems to give a concrete and geometric way to create non-periodic sets with an abundance of repetitive structure. Alan Haynes, Michael Kelly, and Henna Koivusalo used this construction to find bounded remainder sets for irrational rotations on the torus. In joint work with Akshat Das and Alan Haynes, we generalize this result to the case of rotations on products of adelic tori. (Received September 16, 2019)

1154-37-1671 **Stefano Silvestri*** (ssilvestri@butler.edu), 4600 Sunset Avenue, Indianapolis, IN 46208, and **Rodrigo A Perez**. *Accessibility of the Boundary of the Thurston Set*.

Consider two objects associated to the Iterated Function System (IFS) $\{\lambda z + 1, \lambda z - 1\}$: the locus \mathcal{M} of parameters $\lambda \in \mathbb{D} \setminus \{0\}$ for which the corresponding attractor is connected; and the locus \mathcal{M}_0 of parameters for which the related attractor contains 0. The set \mathcal{M} can also be characterized as the locus of parameters for which the attractor of the IFS $\{\lambda z + 1, \lambda z, \lambda z - 1\}$ contains λ^{-1} . Exploiting the asymptotic similarity of \mathcal{M} and \mathcal{M}_0 with the respective associated attractors, we give sufficient conditions on $\lambda \in \partial\mathcal{M}$ or $\partial\mathcal{M}_0$ to guarantee it is path accessible from the complement $\mathbb{D} \setminus \mathcal{M}$.

This is joint work with Dr. Rodrigo Pérez. (Received September 16, 2019)

1154-37-1679 **Ana Anusic*** (ana.anusic@gmail.com), Departamento de Matematica Aplicada, IME-US, Rua de Matao 1010, Cidade Universitaria, 05508-090 Sao Paulo SP, Brazil, Sao Paulo, 05508-090, Brazil. *Folding points in inverse limits*.

Folding point is a point which is locally not homeomorphic to the zero-dimensional set of arcs. Williams' work from the 60s showed that hyperbolic one-dimensional attractors do not have folding points. Moreover, such attractors can be modeled as inverse limits on branched one-manifolds. We study the conditions which guarantee existence of folding points in general inverse limits on graphs with a single bonding map, implying the lack of hyperbolicity on the attractor of the extended (\mathbb{R}^2 or \mathbb{R}^3) system. Moreover, we distinguish certain types of folding points and show how their presence is affected by the dynamics of the underlying bonding map. (Received September 16, 2019)

1154-37-1704 **Thomas French*** (thomas.french@du.edu) and **Ronnie Pavlov** (rpavlov@du.edu). *Extender Entropy*.

For a word w in the language of a shift space X , the extender set $E_X(w)$ of w is the set of all pairs (s, u) of words so that suw is in the language of X . The set of all extender sets occurring for words of length n in X is denoted by $E_X(n)$. We define extender entropy to be the logarithmic growth rate of $|E_X(n)|$, and show that it is a useful conjugacy invariant. (Received September 16, 2019)

1154-37-1738 **Richard Montgomery*** (rmont@ucsc.edu). *Scattering in the N-Body Problem; Preliminary Report*.

The distances between bodies increase at an asymptotically linear rate for a large open set of solutions to the Newtonian N-body problem, solutions termed “hyperbolic” by Chazy in 1927. A McGehee-inspired coordinate change adds a manifold at infinity which gives these solutions a place to go: they tend to equilibria at infinity. By analyzing the flow at and near infinity we reprove the analytic asymptotic expansions of Chazy for these solutions, and we set up a scattering map associated to solutions hyperbolic in both time directions.

This is joint work with Guowei Yu, Natan Duignan and Rick Moeckel. (Received September 16, 2019)

1154-37-1744 **Jan Boronski** and **Jernej Činč*** (jernej.cinc@agh.edu.pl), AGH University of Science and Technology, al. Mickiewicza 30, 30-059 Krakow, Poland, and **Piotr Oprocha**. *On the Barge entropy conjecture*.

In 1920’s Knaster described the first example of a hereditarily indecomposable continuum which later received the name pseudo-arc due to the property that every of its subcontinua is homeomorphic to itself, yet the continuum is not homeomorphic to the arc. One of the basic questions in topological dynamics about such complicated spaces is, what are the possible topological entropies of their homeomorphisms. For the pseudo-arc the conjecture that possible topological entropies of its homeomorphisms can take any positive real value was posed by Marcy Barge in 1989. Until now, all known pseudo-arc homeomorphisms have had entropy 0 or ∞ . In this talk, I will overview some known results relating to the Barge entropy conjecture and discuss our recently obtained positive solution to the conjecture. The positive entropy homeomorphisms that we construct are periodic point free, except for a unique fixed point. The talk is based on a joint work with Jan P. Boroński and Piotr Oprocha. (Received September 16, 2019)

1154-37-1768 **James Keesling*** (kees@uf1.edu), Department of Mathematics, University of Florida, Gainesville, FL 32611-8105, and **Jerzy Dydak** and **Joanna Furno**, FL. *Some recent advances in the theory of generalized covering spaces*. Preliminary report.

There have been considerable advances in generalized covering spaces recently. We present some of these and give examples and applications of the advances. (Received September 16, 2019)

1154-37-1799 **Jakub Byszewski*** (jakub.byszewski@gmail.com). *A transitive map on a dendrite with zero topological entropy*.

Hoehn and Mouron constructed a map on the universal dendrite that is topologically weakly mixing but not mixing. We modify their example to show that there exists a weakly mixing dendrite map with zero topological entropy. This answers a question of Baldwin.

The talk is based on joint work with Fryderyk Falniowski and Dominik Kwietniak. (Received September 16, 2019)

1154-37-1805 **Eugen Andrei Ghenciu*** (ghenciue@uwstout.edu). *Iterated constructions generated by countable non specified shifts*. Preliminary report.

In this presentation we look at iterated constructions in n dimensional Euclidean spaces that are generated by countable (finite or infinite) non specified shifts. Particular attention will be paid to balanced and super multiplicative shifts. As a main result we prove the Bowen formula, which connects the Hausdorff dimension of the limit set of the iterated construction and the zero of the associated topological pressure. (Received September 16, 2019)

1154-37-1825 **Philip J Morrison*** (morrison@physics.utexas.edu). *Finite-dimensional reductions of infinite-dimensional Lie-Poisson brackets*.

The Lie-Poisson bracket associated with the group of canonical transformations (symplectomorphisms) has been known since 1980 to describe important field theories such as the Vlasov equation, the two-dimensional Euler equation for an ideal fluid, and many other physical systems. For purposes of computation it is desirable to produce semi-discrete projections in a structure preserving way. New ways to project onto finite-dimensional Poisson

manifolds will be described, along with concomitant low degree-of-freedom dynamics. (Received September 16, 2019)

1154-37-1834 **Genrich Belitskii** and **Victoria Rayskin*** (victoria.rayskin@tufts.edu). *Smooth localization of maps on non-smooth spaces. Applications and examples.*

The question of extension of locally defined maps to the entire space arises in many problems of analysis (e.g., local linearization of functional equations). A known classical method of extension of smooth local maps on Banach spaces uses smooth bump functions. However, such functions are absent in the majority of infinite-dimensional spaces. We suggest a new approach to localization of Banach spaces with the help of locally identical maps, which we call blid maps. In addition to smooth spaces, blid maps also allow to extend local maps on non-smooth spaces (e.g., $C^q[0, 1]$, $q = 0, 1, 2, \dots$).

For the spaces possessing blid maps, we show how to reconstruct a map from its derivatives at a point (see the Borel Lemma). We also demonstrate how blid maps assist in finding global solutions of cohomological equations having linear transformation of the argument. We present application of blid maps to local differentiable linearization of maps on Banach spaces.

We discuss differentiable localization for metric spaces (e.g., $C^\infty(\mathbb{R})$), prove an extension result for locally defined maps and present examples of such extensions for the specific metric spaces.

In conclusion, we formulate open problems. (Received September 16, 2019)

1154-37-1845 **Victoria Rayskin*** (victoria.rayskin@tufts.edu). *Nonchaotic Models and Predictability of the Users' Traffic Dynamics on Internet Platforms.*

Internet platforms' traffic defines important characteristics of platforms, such as price of services, advertisements, speed of operations. The traffic is usually estimated with the help of the traditional time series models (ARIMA, Holt-Winters, etc.), which are successful in short term extrapolations of sufficiently denoised signals.

We propose a dynamical system approach for the modeling of the underlying process. The method allows to discuss the global qualitative properties of the dynamics' phase portrait and long term tendencies. We prove that the proposed models are not chaotic, the long term prediction is reliable, and it explains the fundamental properties and trend of various types of digital platforms. Because of these properties, we call the flow of these models the *trending flow*. Utilizing the new approach, we construct the two-sided platform models for the volume of users, that can be applied to Amazon.com, Homes.mil or Wikipedia.org.

We consider a generalization of the two-sided platforms' models to multi-sided platforms. If the equations' are cooperative, the flow is trending, and it helps to understand the properties of the platforms and reliably predicts the long term behavior. (Received September 16, 2019)

1154-37-1868 **David Sumner Lipham*** (dliipham@aum.edu). *Exploding endpoints and a positive entropy conjecture.*

For each $a \in (-\infty, -1)$ define $f_a : \mathbb{C} \rightarrow \mathbb{C}$ by $f_a(z) = \exp(z) + a$. Let $E(f_a)$ be the set of finite endpoints of maximal rays in the Julia set $J(f_a)$. Let $\dot{E}(f_a)$ and $\ddot{E}(f_a)$ be the sets of *escaping* and *fast escaping* points in $E(f_a)$.

In 1988, Mayer proved $E(f_a) \cup \{\infty\}$ is connected. In 1996, Kawamura Oversteegen and Tymchatyn proved $E(f_a)$ is homeomorphic to the *almost zero-dimensional* irrational Hilbert space \mathfrak{E}_c . In 2017, Alhabib and Rempe-Gillen proved $\dot{E}(f_a) \cup \{\infty\}$ and $\ddot{E}(f_a) \cup \{\infty\}$ are connected. Thus $\dot{E}(f_a)$ and $\ddot{E}(f_a)$ are recently discovered almost zero-dimensional spaces with one-point connectifications. In this talk we present joint work with Jan J. Dijkstra on new topological characteristics of these spaces. We conjecture $\ddot{E}(f_a) \simeq \mathbb{Q} \times \mathfrak{E}_c$ and $\dot{E}(f_a) \simeq \mathfrak{E}$, where \mathfrak{E} is the rational Hilbert space.

Time permitting, we will also discuss a conjecture of Seidler and Kato linking the existence of positive entropy homeomorphisms to uncountable topological structure. We show an example by Tymchatyn re-opens the conjecture, and discuss other versions of the problem. (Received September 16, 2019)

1154-37-1960 **Lihong Zhao*** (lihong@uidaho.edu). *Population Dynamic Model of Microbial Interactions and Traits Associated with Microbiome.* Preliminary report.

Microbes are everywhere; they form complex communities which are essential in maintaining the balance of ecosystems and hosts' health. The generalized Lotka-Volterra (gLTV) model has been applied to microbiome data to infer interactions between microbes within the community and gain insight into the stability of microbial communities. However, this model has a few limitations: it ignores the interaction mechanism and assumes that microbial community dynamics are driven by pair-wise interactions only. We propose a resource-mediated model, as we know that in many microbial communities, interactions between microbes are typically indirect and mediated by molecules in the environment (e.g., resources, toxins), rather than direct as assumed in the

gLV model. In the model, we allow species to transition between various metabolic states. To gain insight into dynamics, we have rewritten the model in terms of the proportion of microbial abundance that is a given species. Rewriting the equations in this way uncovers relationships governing the evolutionary dynamics of the system explicitly in terms of the fitness of each species relative to the mean fitness of the entire population, and offers a way to model trait dynamics that emerge from the microbial interactions. (Received September 16, 2019)

1154-37-2003 **Robert G. Niemeyer*** (niemey1@msudenver.edu), Metropolitan State University of Denver, Department of Mathematical and Computer Scienc, PO Box 173362, Campus Box 38, Denver, CO 80217, and **Charles C. Johnson** (ccjohnson@gmail.com), 701 Moore Ave, Lewisburg, PA 17837. *The wild, elusive singularities of the T -fractal surface, Part I.*

This is Part I of a two-part talk on the T -fractal translation surface, a fractal analog of an interval exchange transformation and the geometry and topology of the elusive singularities of the T -fractal translation surface. In this talk, we will remind the audience of the basic definitions, construct the T -fractal translation surface from the so-called Quad T -surface. Such a construction takes advantage of the similarity inherent in the T -fractal. We will show that while the elusive points of the T -fractal billiard is a connected set, the corresponding elusive singularities of the T -fractal surface is a totally disconnected perfect set. (Received September 17, 2019)

1154-37-2007 **Charles C. Johnson*** (ccjohnson@gmail.com), Bucknell University, 701 Moore Ave, Lewisburg, PA 17837, and **Robert G. Niemeyer** (niemey1@msudenver.edu), Metropolitan State University of Denver, Department of Mathematical and Computer Scienc, PO Box 173362, Campus Box 38, Denver, CO 80217. *The wild, elusive singularities of the T -fractal surface, Part II.*

This is part II of a two-part talk on the T -fractal translation surface, a fractal analog of an interval exchange transformation and the geometry and topology of elusive singularities of the T -fractal translation surface. In this talk we will begin by concluding from our previous talk that the T -fractal translation surface is not a true surface, in the strictest sense of the definition. We will then construct what we believe to be a suitable analog of an interval exchange transformation on the T -fractal translation surface and prove that every elusive singularity has a linear approach. As such, we will show that such points constitute a Cantor set of wild singularities, indicating that the current definition of *wild singularities* may be extended to include the fractal case. (Received September 17, 2019)

1154-37-2014 **Joshua P. Bowman*** (joshua.bowman@pepperdine.edu). *Cantor sets arising from laminations on homothety surfaces.*

In the study of translation surfaces, a fundamental result is that any geodesic is either closed (homeomorphic to S^1), a saddle connection (homeomorphic to an interval), or dense in some subsurface (possibly with boundary). Homothety surfaces (also called dilation surfaces) are a mild generalization of translation surfaces, whose dynamical properties are just beginning to be explored. I will describe a family of genus-2 homothety surfaces on which every geodesic either accumulates on a closed loop or is dense in a lamination with Cantor cross-section. The Cantor sets that arise all have Hausdorff dimension zero, but their geometric structures are determined by continued fractions. This is based on joint work with Slade Sanderson. (Received September 17, 2019)

1154-37-2022 **Sean Gasioerek*** (sean.gasioerek@sydney.edu.au). *Inverse Magnetic Billiards: A Survey.* Consider a strictly convex set Ω in the plane, and a homogeneous, stationary magnetic field orthogonal to the plane whose strength is B on the complement of Ω and 0 inside Ω . The trajectories of a charged particle in this setting are straight lines concatenated with circular arcs of Larmor radius μ . We examine the dynamics of such a particle and call this *inverse magnetic billiards*. Comparisons are made to standard Birkhoff billiards and magnetic billiards, as some theorems regarding inverse magnetic billiards are consistent with each of these billiard variants while others are not. (Received September 17, 2019)

1154-37-2028 **Dominik Kwietniak***, Jagiellonian University in Krakow, Institute of Mathematics, ul. Łojasiewicza 6, 30-348 Kraków, Poland, and **Gabriel Fuhrmann**. *On the abundance of (non)tame group actions.*

Tame dynamical systems were introduced by Köhler in 1995. Tameness is a topological notion corresponding to compactness appearing in the compactness vs weak mixing dichotomy in the structure theory for measure preserving actions.

In recent years several authors developed the theory of tame systems revealing connections to other areas of mathematics like Banach spaces, circularly ordered systems, substitutions and tilings, quasicrystals, cut and project schemes and even model theory and logic.

During my talk, I will discuss tameness and nullness of regular almost automorphic G -actions utilising a generalised notion of semi-cocycle extensions (see “On tameness of almost automorphic dynamical systems for general groups” to appear in Bull. of the LMS for details). In particular, we show that every ergodic equicontinuous G -action on a compact metric space admits a regular almost automorphic extension which is non-tame as well as tame but non-null extension. In some sense, this complements a recent result of Glasner [Invent. Math. **211** (2018), no. 1, pp. 213–244]. We prove that such examples appear in well-studied families of group actions including Delone dynamical systems and symbolic systems (including Toeplitz flows over arbitrary G -odometers). (Received September 17, 2019)

1154-37-2029 **Dominik Kwietniak***, Jagiellonian University in Krakow, Institute of Mathematics, ul. Lojasiewicza 6, Kraków, 30-348, and **Marta Straszak**. *Minimal symbolic actions of amenable groups with prescribed entropy*. Preliminary report.

Let A be a finite alphabet. We show that for every congruent monotileable amenable group G and every $0 \leq \alpha < \log |A|$ there exists a minimal action of G on the subshift X of A^G with topological entropy α . Congruent monotileable amenable groups are a generalization of amenable residually finite groups. The class was introduced by Paulina Cecchi and María Isabel Cortez. In particular, this class contains all the infinite countable abelian and all the infinite countable virtually nilpotent groups. For abelian groups the minimal actions we construct are free. (Received September 17, 2019)

1154-37-2042 **Dominik Kwietniak***, Jagiellonian University in Krakow, Institute of Mathematics, ul. Lojasiewicza 6, 30-348 Kraków, Poland, and **Martha Lacka**. *Feldman-Katok convergence, f -bar, and entropy of nonuniformly hyperbolic measures*.

I will discuss a new tool: the Feldman-Katok pseudometric on orbits. It leads to a notion of convergence for invariant measures. We used it to show that the entropy of nonuniformly hyperbolic measures constructed using the method of Gorodetski, Ilyashenko, Kleptsyn, and Nalsky is zero. Furthermore, these measures are always Kakutani equivalent to an ergodic group rotation. Time permits, I will discuss a related result obtained with Bonatti and D

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az: Assuming robust transitivity, we prove that in the partially hyperbolic setting, there robustly exists an ergodic nonhyperbolic measure with full support and positive entropy. The novelty of this result is that we address all four conditions (robustness, ergodicity, positive entropy, and full support) together, while previous works (of Bochi, Bonatti and D

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az) dealt only with a subset of these conditions. (Received September 17, 2019)

1154-37-2132 **Iztok Banic** and **Judy Anita Kennedy*** (judy.kennedy@lamar.edu), PO Box 10047, Department of Mathematics, Lamar University, Beaumont, TX 77710, and **Uros Milutinovic** and **Piotr Minc**. *Characterizations of \mathcal{P} -like continua that do not have the fixed point property*.

We give two new characterizations of \mathcal{P} -like continua X that do not have the fixed point property. Both characterizations are stated in terms of open covers of X and fixed-point-free mapping patterns. An earlier characterization of \mathcal{P} -like continua with the fixed point property was given by Feuerbacher in 1994 based on a 1963 result of Mioduszewski. The Mioduszewski-Feuerbacher characterization is expressed in terms of almost commutative inverse diagrams. In contrast, our approach is more geometric, and it may potentially lead to new methods in the elusive search for a planar tree-like continuum without the fixed point property. (Received September 17, 2019)

1154-37-2165 **Kitty Yang*** (kyang@math.northwestern.edu), 2033 Sheridan Road, Evanston, IL 60208. *Mapping class groups of zero-entropy subshifts*.

Let (X, σ) be a subshift. A flow equivalence of two spaces is an orientation-preserving homeomorphism of the suspension spaces. The mapping class group of a subshift is the group of self-flow equivalences up to isotopy. We compute the mapping class group for various classes of zero-entropy subshifts. (Received September 17, 2019)

1154-37-2173 **Md Nazmul Hassan*** (hassan.du.math@gmail.com), 401 University oaks Blvd, College station, TX 77840, and **Angie Peace**. *Mechanistically derived toxicant-mediated predator-prey model under Stoichiometric constraints*.

Studies in ecological stoichiometry highlight that grazer dynamics are affected by insufficient food nutrient content (low phosphorus (P)/carbon (C) ratio) as well as excess food nutrient content (high P:C). Contaminant stressors affect all levels of the biological hierarchy, from cells to organs to organisms to populations to entire

ecosystems. Eco-toxicological modeling under the framework of ecological stoichiometry predicts the risk of bio-accumulation of a toxicant under stoichiometric constraints. In this paper, we developed and analyzed a Lotka–Volterra type predator-prey model which explicitly tracks the environmental toxicant as well as the toxicant in the populations under stoichiometric constraints. Analytic, numerical, slow-fast steady-state and bifurcation theory is employed to predict the risk of toxicant bio-accumulation under varying food conditions. In some cases, our model predicts different population dynamics, including wide amplitude limit cycles where producer densities exhibit very low values and may be in danger of stochastic extinction. (Received September 17, 2019)

1154-37-2198 **Sarah Bailey Frick*** (sarah.frick@furman.edu), 3300 Poinsett Hwy, Mathematics Department, Greenville, SC 29613. *Determining when a Bratteli-Vershik system is expansive.*

Downarowicz and Maass proved that Bratteli-Vershik diagrams with a uniformly bounded number of vertices are either expansive or topologically conjugate to an odometer. In joint work with Karl Petersen and Sandra Shields, we give criteria for when a simple properly ordered (unique maximal/minimal path) is topologically conjugate to an odometer. Furthermore, we give a class of simple, bounded width diagrams for which every possible ordering is expansive. (Received September 17, 2019)

1154-37-2249 **Dylan J Murphy*** (djmurphy@email.arizona.edu). *Additions and the Toda Hierarchy.* In a series of papers starting in 1986, McKean considered a special class of Darboux transformations of Schrödinger operators called “additions.” These transformations have the property of preserving the spectrum of the original operator, and turn out to have a close connection to the Korteweg-de Vries (KdV) hierarchy of integrable PDEs, including the fact that an “infinitesimal” addition generates the flows of all equations in the hierarchy.

We develop an analogous class of transformations on the space of Jacobi operators, extending the commutation methods developed by Gesztesy, Holden, Simon, and Zhao, and explore their connection to the Toda hierarchy of difference/differential equations (which may be regarded as a spatially discrete version of the KdV hierarchy). (Received September 17, 2019)

1154-37-2271 **Jonathan Meddaugh*** (jonathan_meddaugh@baylor.edu) and **Brian Raines** (brian_raines@baylor.edu). *Internal Chain Transitivity and ω -limit sets in Baire space.*

For a dynamical system $f : X \rightarrow X$ on a compact metric space, it is well-known that the ω -limit sets of f are internally chain transitive. Furthermore, under appropriate conditions on the map f and space X , internal chain transitivity completely characterizes the ω -limit sets.

In this talk, we allow X to be non-compact and explore the relation between internal chain transitivity and the ω -limit sets of a dynamical system. In particular, we will consider the shift map acting on Baire space, the space of sequences of natural numbers, as well as some invariant subspaces thereof. (Received September 17, 2019)

1154-37-2341 **Joshua Frisch*** (jfrisch@caltech.edu). *Characteristic Measures of Symbolic Dynamical Systems.*

One of the most important techniques in topological dynamics is the reduction of questions about topological dynamical systems to those about measurable dynamical systems. One of the principle techniques used to transfer knowledge from the measurable category to the topological category is finding invariant measures, those measures which are preserved under the transformation. In this talk I will discuss measures which are invariant not only to the transformation, but to the entire automorphism group of the dynamical system, called characteristic measures. In particular I will give a myriad of open questions about the existence of characteristic measures and one theorem. For 0 entropy symbolic dynamical systems, characteristic measures always exist. This is joint work with Omer Tamuz. (Received September 17, 2019)

1154-37-2375 **Darren Creutz***, creutz@usna.edu. *(Relatively) Contractive Actions of Lattices and Lie Groups.*

A measurable action of G on a probability space is contractive when it is the “opposite of measure-preserving”: $\sup_{g \in G} \nu(gB) = 1$ for every nonnull set B . A topological action of G on a compact metric space with a probability measure is contractible when every point mass is in the (weak*) closure of the orbit of the measure.

Every topological model of a contractive space is contractible, meaning the topological and measurable dynamics completely determine one another in this setting. These notions (and the relativized version of these notions), and particularly the interplay of topological dynamics and measurable dynamics, play a key role in various results (some due to the speaker) on the structure of actions of lattices and commensurators in semisimple

Lie groups. I will present these notions and some fundamental results then outline the role they play in these structural results. (Received September 17, 2019)

1154-37-2406 **Van Cyr, Aimee Johnson, Bryna Kra and Ayşe Şahin*** (ayse.sahin@wright.edu). *Low Complexity and the Loosely Bernoulli Property*. Preliminary report.

We present some results on the connection between the Loosely Bernoulli property and low complexity for ergodic, probability measure preserving \mathbb{Z} actions. (Received September 17, 2019)

1154-37-2418 **Van Cyr*** (van.cyr@bucknell.edu) and **Bryna Kra**. *Properties of low complexity symbolic systems and applications*.

The topological entropy of a subshift is the exponential growth rate of the number of words of different lengths in its language. For subshifts of entropy zero, finer growth invariants constrain their dynamical properties. In this talk we will survey how the complexity of a subshift affects the ergodic properties of the invariant measures it carries and constrains the properties of its orbits. Combinatorial and algebraic applications will be discussed. (Received September 17, 2019)

1154-37-2430 **Steven Collazos*** (colla054@umn.edu), Science 1535, 600 East Fourth Street, Morris, MN 56267, and **Duane Nykamp**. *Coding Properties of Firing Rate Models with Low-Rank Synaptic Weight Matrices*.

A theory in neuroscience proposes that groups of co-active neurons form a basis for neural processing. Following other researchers' work on threshold-linear networks, which are neural networks where the activation function is a rectifier, we model the collection of all possible ensembles of neurons—known as *permitted sets*, $\mathcal{P}_\Phi(W)$ —as a collection of binary strings that indicate which neurons are considered active. Here Φ is a function describing how neurons respond to inputs (i.e., an activation function) and W is a matrix whose entries represent the effective influence neurons in the network have among each other (i.e., a synaptic weight matrix). Unlike the threshold-linear regime, however, Φ of the neural networks we study is C^1 with finitely many discontinuities. We construct $\mathcal{P}_\Phi(W)$ by imposing a threshold on the responsiveness of the neuron to input at the steady state. Furthermore, when W is almost rank one, we prove that $\mathcal{P}_\Phi(W)$ is a convex code, i.e., a combinatorial neural code arising from a pattern of intersections of convex sets. (Received September 17, 2019)

1154-37-2463 **William Ott*** (ott@math.uh.edu) and **Edward Stout**. *Can the dimension of a fractal set or measure be inferred from projections?*

How do we analyze invariant sets and measures produced by infinite-dimensional dynamical systems? One can project these objects into (finite-dimensional) Euclidean spaces and then perform the analysis in \mathbb{R}^n . This approach raises an immediate question: How much information, if any, associated with the invariant object is lost when we project it? It turns out that the answer to this question depends on the structure of the infinite-dimensional space in which the invariant object lives. Here, we discuss recent projection results for fractal sets and measures in Banach spaces. (Received September 17, 2019)

1154-37-2538 **Carlos M. Ortiz Marrero*** (carlos.ortizmarrero@pnnl.gov). *Data-Driven State Space Discovery of Physical and Chemical Systems*. Preliminary report.

What is "scientific machine learning"? We will address this question by exploring various data-driven problems that arise in chemistry and engineering. In particular, we will present results related to state space reconstruction using delay embeddings, forecasting using domain-aware models, and state estimation using physics-informed neural networks. Moreover, we will highlight some key mathematical aspects that are driving further progress in the field. (Received September 17, 2019)

1154-37-2548 **Terrence Adams and Cesar E. Silva*** (csilva@williams.edu). *On ergodic properties for powers of finite and infinite measure-preserving transformations*. Preliminary report.

It is well-known that if an infinite measure-preserving transformation is conservative, then so are all its powers. Of course, this is not the case for ergodic transformations, even in the finite measure-preserving case. Also, in infinite measure there is a variety of different counterexamples. In this talk, we will examine conditions for dynamical properties of a transformation to extend to some of its powers. (Received September 17, 2019)

1154-37-2563 **Marco Antonio López*** (lopezma@wfu.edu). *Dimensions and measures of shrinking target sets*.

Generally speaking, a shrinking target set for a dynamical system $T: X \rightarrow X$ and a countable family of subsets $A_n \subseteq X$ is the set of points $x \in X$ whose orbits satisfy $T^n(x) \in A_n$ for infinitely many n .

Two main questions arise in this situation. If the shrinking target set has zero Lebesgue measure, what is its Hausdorff dimension? We will demonstrate how the theory of thermodynamic formalism helps us address this question. The other question is whether we can characterize the shrinking target sets according to whether they have zero or full measure (in the sense of Lebesgue or Hausdorff). In this direction, a dichotomy law is often obtained.

We will discuss such problems in the context of non-autonomous conformal iterated function systems. (Received September 17, 2019)

1154-37-2676 **Duong Nguyen*** (duong.nguyen@kzoo.edu) and **William Ott**. *Nonstationary Coupled Map Lattices*.

Coupled Map Lattices (CML) have been extensively studied due to their applications in physics and biology. From the dynamics point of view, several methods have been used to study CML analytically, most notably by Keller and Liverani. This talk will provide some background on CML and our study in nonstationary CML. (Received September 17, 2019)

1154-37-2706 **Brooks Emerick*** (bemerick@kutztown.edu), Lytle Hall 266, Kutztown University, 15200 Kutztown Rd, Kutztown, PA 19530. *Infected Host-Feeding in Semi-Discrete Host-Parasitoid Population Dynamics*. Preliminary report.

Discrete-time models are the traditional approach for capturing population dynamics of a host-parasitoid system. More recent work has introduced a semi-discrete framework for obtaining model update functions that connect host-parasitoid population levels from year-to-year. In particular, this framework uses differential equations to describe the host-parasitoid interaction during the time of year when they come in contact, allowing specific behaviors to be mechanistically incorporated. We use the semi-discrete approach to study the effects of infected-host-feeding, the tendency of female parasitoids to occasionally feed on an already infected host larvae. We find that infected-host-feeding does stabilize the system even with a time-dependent feeding rate, yielding a period-doubling bifurcation as the number of viable hosts increases. Overall, infected-host-feeding creates an inefficiency in the parasitoid reproduction habits from year to year effectively establishing coexistence between the two species. (Received September 17, 2019)

1154-37-2748 **A Perico*** (aperico@ucsc.edu), 1156 High St, Santa Cruz, CA 95064. *Magnetic Billiard on Polygons: The Square case*. Preliminary report.

We consider a polygon in a two-dimensional plane with a homogeneous constant magnetic field orthogonal to such plane, but inside the polygon, the magnetic field is zero. We study the dynamics of an electron with an initial velocity in this setting. Problems arise because we have a boundary with corners. In the square case, we generalize velocities with a rational slope and show numerical evidence of a chaotic-like behaviour with other initial conditions. (Received September 17, 2019)

1154-37-2754 **Lauren Lazarus*** (lauren.lazarus@trincoll.edu). *Comparison and machine classification of limit cycles from ODE and delayed oscillator models*. Preliminary report.

Many nonlinear systems exhibit oscillatory behavior in the form of limit cycles, including the common van der Pol equation and its variants. In this talk, I will discuss differences between the limit cycles governed by various oscillator models, and how the models' outputs may be classified through those differences. After standardizing the frequencies, phases, and amplitudes of the cycles by adjusting relevant model parameters, we find that distinctions remain between the shapes of the limit cycles. Using these features, can we train a machine learning method to classify output data from an unknown oscillator?

While many oscillators (including the van der Pol model above) are governed by ordinary differential equations of at least second order, limit cycles can also be caused by system delay. The comparison of limit cycles in this talk will therefore include multiple variants of a first-order delay limit cycle oscillator as well. (Received September 17, 2019)

1154-37-2767 **Christopher L Cox***, Clcox@tarleton.edu. *Mass distribution and persistent periodicity in no-slip billiards*.

No-slip billiards use a physically motivated collision model in which linear and angular momentum may be exchanged at boundary collisions. Particles of uniform mass distribution in a triangular billiard have been shown to exhibit persistent periodicity, with any set of initial conditions yielding periodic orbits of period six or eight. We present many new examples of persistently periodic billiards, and examples that suggest persistent periodicity becomes more prevalent for billiards with particles having certain decentralized mass distributions. (Received September 17, 2019)

1154-37-2788 **Nicholas Mandler*** (nickmandler101@gmail.com), 2737 Clement St, Apt. 3, San Francisco, CA 94121. *Parameterizing the canopy and the boundary of the connectedness locus of the IFS of two distinct affine maps.*

The connectedness locus of an IFS of two affine maps was investigated by Barnsley in the case of a pair of affine maps with identical corresponding linear map components. This talk will convey an escape time algorithm for determining the connectedness locus in the generalized case of distinct affine maps. It will then be demonstrated that this algorithm produces as a byproduct key information for determining the generalized analog to Mandelbrot and Frame's 'Canopy' or 'Shortest Path' in the attractor. The algorithm can also be used to parameterize the boundary of the connectedness locus as well as quickly compute the unstable locus of a topological-set / post-critically-finite set with respect to a parameterized space of systems. (Received September 18, 2019)

39 ► *Difference and functional equations*

1154-39-68 **Nalini Joshi*** (nalini.joshi@sydney.edu.au), School of Mathematics and Statistics F07, University of Sydney, NSW 2006, Australia, and **Nobutaka Nakazono**. *Consistency on a cuboctahedron.*

Integrable partial difference equations, known as lattice equations, are given by zeroes of polynomials of 4 variables, each variable being assigned to a vertex of a quadrilateral. When the quadrilateral is identified as a face of a cube, it is known that integrable lattice equations are consistent around cubes in N -dimensional lattices. Adler, Bobenko and Suris (2003, 2009) classified all polynomials with this consistency property. We report on an extension of the classification to quadrilaterals that arise in a lattice constructed from overlapping cuboctahedra. (Received August 29, 2019)

1154-39-1082 **Vlajko L Kocic*** (vkocic@xula.edu). *On the dynamics of certain classes of nonlinear discrete discontinuous population models.*

We present a survey of results about the dynamics of some discrete discontinuous population models. We study oscillations, the structure of semicycles, periodicity, attractivity, and bifurcations. We focus on the classical Williamson's population model and an equivalent model

$$x_{n+1} = (a - bh(x_n - c))x_n,$$

(h - Heaviside function) which was used in modeling the spread of West Nile epidemic. In addition, we consider discontinuous Beverton-Holt type difference equation and we address the dynamics of a general nonlinear population population model with two jump discontinuities exhibiting Allee-type effect. Some generalizations and several open problems are also discussed. (Received September 13, 2019)

1154-39-1451 **Kevin Ahrendt*** (kahrendt@mines.edu). *Solutions to a three point boundary value problem in nabla fractional calculus.* Preliminary report.

We consider the nabla fractional difference equation $\nabla \nabla_a^\nu x(t) = h(t)$ with three point boundary conditions $x(a) = 0$ and $\alpha x(a+k) = x(b)$, where $a+k$ is an interior point in the domain \mathbb{N}_a^b . We give the Green's function for the corresponding homogeneous boundary value problem, along with several bounds on the Green's function. With these bounds, and the contraction mapping theorem, we prove the existence and uniqueness of solutions to certain nonlinear, three point boundary value problems. (Received September 15, 2019)

1154-39-1637 **Danielle L Burton*** (dburton3@utk.edu). *Optimal Control of Harvest Timing in a Discrete Model.* Preliminary report.

Management decisions regarding harvest are complicated and important. In the difference equation setting, many order of events cases have been studied. Hiromi Seno proposed a model to study the timing of harvest between breeding seasons by taking a convex combination of the order of events cases reproduce-harvest and harvest-reproduce. We derive a new model that mechanistically incorporates harvest timing and share some preliminary numerical results. (Received September 16, 2019)

1154-39-1700 **Kevin Ahrendt** (kahrendt@mines.edu), **Areeba Ikram** (aikram@mines.edu) and **Larsen Kronstad*** (lkronsta@mines.edu). *An Extension of Regressive Functions from Discrete Nabla Calculus into the Complex Domain.* Preliminary report.

The nabla discrete exponential function is defined to be the solution of an initial value problem involving a difference equation. From this, we build hyperbolic and circular sinusoids with similar properties to the analogous continuous functions. In constructing such functions, we need to incorporate complex valued discrete

exponentials. By extending the definition of nabla regressive functions to include complex valued functions, we obtain an analogue to DeMoivre's Theorem for discrete sinusoids. (Received September 16, 2019)

1154-39-1872 **Robert Krueger*** (rkrueger@csp.edu) and **Eric Stachura** (eric.stachura@kennesaw.edu). *Tiling an n by 3 Hallway with 1 by 2 Tiles: An Interactive Presentation.*

Students will investigate difference equations through the context of tiling hallways. Students will observe patterns in the tiling which will lead to a difference equation model. Solutions will be calculated by iteration. Then students will be introduced to the concept of the shift operator which will enable them to solve for a closed form solution.

The session will be an interactive presentation of how this modeling scenario could be used in a differential equations classroom.

This work was recently published here: <https://www.simiode.org/resources/6417>. This was joint work completed along with Eric Stachura as part of the DEMARC Fellowship sponsored by SIMIODE and the National Science Foundation. (Received September 16, 2019)

1154-39-1954 **Brandon M. Tippings*** (tippings@math.arizona.edu). *A Non-Autonomous Discrete Painlevé Equation and Counting Maps.* Preliminary report.

Over the last few decades, the autonomous discrete Painlevé 1 equation has been studied from several different perspectives. One can view the d - P_1 equation as defining dynamics on the plane, or as a Quispel-Roberts-Thompson mapping. Another perspective is that of Okamoto and Sakai, where the dynamics are understood through the geometry of a special surface called 'the space of initial values'. Using these perspectives, one can gain insight into several related combinatorial problems, such as counting 4-valent planar maps, and blossom trees.

In this talk we will discuss an integrable deautonomization of the discrete Painlevé 1 equation. This specific equation arises from a recurrence relation for orthogonal polynomials with respect to an exponential weight, and has connections to counting 4-valent maps of any genus. From the various different perspectives, we can understand the non-autonomous dynamics and address questions from the 4-valent map counting problem. (Received September 16, 2019)

1154-39-1978 **Kevin Ahrendt** (kahrendt@mines.edu) and **Michael O'Leary*** (moleary@alumni.mines.edu). *Differentiability of a fractional difference operator and modeling applications.* Preliminary report.

We will present results about continuous derivatives with respect to the order of a nabla fractional difference operator. In particular, we examine the differentiability of the Riemann-Liouville fractional difference's order at integer values. As an application, we use least squares regression to find optimal orders for discrete fractional models from real world datasets. (Received September 16, 2019)

1154-39-1989 **William S Dula*** (william.dula@students.cau.edu) and **Ronald E Mickens.** *Exact and Nonstandard Finite Difference Schemes for a Modified Law of Cooling.*

We construct a first-order, nonlinear differential equation to model cooling and heating of a system embedded in a constant temperature environment. The equation generalizes the standard Newton "Law of Cooling" by including an additional nonlinear term which allows for the system to achieve the equilibrium temperature in a finite time. The major goal of this work is to demonstrate that finite difference schemes exist such that they are dynamically consistent with the major features of the physical system. Both exact and NSFD schemes are formulated and their numerics are investigated, including a detailed comparison of their corresponding numerical solutions. (Received September 17, 2019)

1154-39-2332 **Kevin Ahrendt** (kahrendt@mines.edu), **Areeba Ikram** (aikram@mines.edu) and **Rocco Marchitto*** (rmarchitto@mymail.mines.edu). *Sturm-Liouville Problems in Discrete Nabla Fractional Calculus.* Preliminary report.

We consider a discrete nabla fractional Sturm-Liouville boundary-value problem in the Caputo sense. We prove basic properties of the system including existence of solutions and develop analytical approximations to the system solutions. Furthermore, eigenvalue results analogous to continuous classical Sturm-Liouville systems are introduced and developed. Finally, we expound computational aspects and develop illustrations of sample solutions to the system. (Received September 17, 2019)

1154-39-2541 **Gulsah Yeni*** (gyq3f@mst.edu) and **Elvan Akin**. *Some Epidemic Models on Time Scales*.

The theory of time scales has been introduced in order to unify and extend continuous and discrete analysis. In this talk, we give a formulation of SIS (susceptible-infected-susceptible) and SIR (susceptible-infected-recovered) epidemic models with time dependent coefficients on time scales. After presenting the derivation of these models, we find exact solutions by the approach of the Bernoulli equation on time scales. We also discuss the asymptotic behavior of susceptibles and infectives and illustrate our results on different time scales in which we compare them to existing models in literature. (Received September 17, 2019)

40 ► Sequences, series, summability

1154-40-987 **Sukhdev Singh*** (singh.sukhdev01@gmail.com), Jalandhar-Delhi G.T Road, (NH-1), Phagwara, Punjab, India. *On the Orlicz function and bicomplex paranormed space $\mathbb{C}_2(s)$ of double sequences*.

In this paper, by using the Orlicz function, we introduce the paranormed sequence space $\mathbb{C}_2(s)$ which is the generalization of the space $\mathbb{C}_2(s_p)$ of all absolutely p -summable \mathbb{C}_2 -double sequences. Some topological properties of the space $\mathbb{C}_2(s)$ are examined and determine its α -, β - and γ -duals. Also, some classes of bicomplex matrix transformations are characterized from the space $\mathbb{C}_2(s)$ into other spaces of double sequences. (Received September 12, 2019)

41 ► Approximations and expansions

1154-41-66 **Aaron Michael Yeager*** (ayeager@ccga.edu), 348 Terrapin Trail, Brunswick, GA 31525. *Real Zeros of Random Sums with I.I.D. Coefficients*.

Let $\{f_k\}$ be a sequence of entire functions that are real valued on the real-line. We study the expected number of real zeros of random sums of the form $P_n(z) = \sum_{k=0}^n \eta_k f_k(z)$, where $\{\eta_k\}$ are real valued i.i.d. random variables. We establish a formula for the density function ρ_n for the expected number of real zeros of P_n . As a corollary, taking the random variables $\{\eta_k\}$ to be i.i.d. standard Gaussian, appealing to Fourier inversion we recover the representation for the density function previously given by Vanderbei through means of a different proof. Placing the restrictions on the common characteristic function ϕ of $\{\eta_k\}$ that $|\phi(s)| \leq (1 + as^2)^{-q}$, with $a > 0$ and $q \geq 1$, as well as that ϕ is three times differentiable with each the second and third derivatives being uniformly bounded, we achieve an upper bound on the density function ρ_n with explicit constants that depend only on the restrictions on ϕ . As an application we take the spanning functions of P_n to be $f_k(z) = p_k(z)$, $k = 0, 1, \dots, n$, where $\{p_k\}$ are Bergman polynomials on the unit disk. (Received July 29, 2019)

1154-41-167 **Simon Foucart***, foucart@tamu.edu. *Nonlinear Approximation and (Deep) ReLU Networks*.

This talk concerns the approximation power (sometimes called the expressive power) of deep neural networks. The most common results found in the literature prove that neural networks approximate functions with classical smoothness to the same accuracy as classical methods, e.g. approximation by polynomials or piecewise polynomials on prescribed partitions. However, approximation by neural networks depending on n parameters is a form of nonlinear approximation and as such should exhibit the increased efficiency of nonlinear approximation methods. We show that this is indeed the case. Furthermore, the performance of neural networks in targeted applications such as machine learning indicate that they actually possess even greater approximation power than traditional methods of nonlinear approximation, such as free knot splines or n -term approximation from a dictionary. We again show that this is indeed the case. To do so, we exhibit large classes of functions which can be efficiently captured by neural networks where classical nonlinear methods fall short of the task. Our work purposefully limits itself to studying the approximation of univariate functions by ReLU networks.

This is joint work with I. Daubechies, R. DeVore, B. Hanin, and G. Petrova. (Received August 17, 2019)

1154-41-601 **Longxiu Huang*** (huangl3@math.ucla.edu) and **Keaton Hamm**. *CUR Decompositions and Perturbations*.

This talk will discuss a useful tool in dimensionality reduction and low-rank matrix approximation called the CUR decomposition which decomposes a matrix by selecting representative columns and rows from it. We would present several equivalent formulations for CUR decomposition, and discuss a randomized row/column selection method to guarantee the exact CUR decomposition of a low-rank matrix with high probability. Additionally, a

novel perturbation analysis is performed on CUR approximations of noisy versions of low-rank matrices, which compares them with the putative CUR decomposition of the underlying low-rank part. (Received September 08, 2019)

1154-41-603 **Akram Aldroubi** (akram.aldroubi@vanderbilt.edu), Nashville, TN, **Longxiu Huang** (huangl3@math.ucla.edu), Los Angeles, CA, **Keri A. Kornelson** (kkornelson@ou.edu), Norman, OK, and **Ilya A. Krishtal*** (ikrishtal@niu.edu), DeKalb, IL. *The Prony-Laplace method for identifying burst-like forcing terms.*

We present a method for the recovery of a burst-like forcing term of an abstract differential equation from samples of the spectrogram of the solution. The method combines the ideas of the classical Prony and Laplace transform methods and dynamical sampling. (Received September 08, 2019)

1154-41-1069 **Willi Freeden*** (freeden@rhrk.uni-kl.de), Department of Mathematics, University of Kaiserslautern, 67653 Kaiserslautern, RP, Germany. *Lattice Point Identities, Shannon Sampling, and Antenna Problem.*

This talk demonstrates that Shannon-type sampling can be associated to newly-created types of Hardy-Landau lattice point identities of analytic number theory which, in the language of sampling, give a deepened insight, e.g., into over- and undersampling and the explicit representation of the aliasing error. Moreover, the reproducing kernel Paley-Wiener Hilbert space context resulting from the multivariate Shannon-type sampling theorem is discussed to handle the multivariate inverse antenna problem. The roots of this talk are twofold: (i) the work on multi-dimensional generalizations of the Euler summation formula to elliptic partial differential operators and adapted multi-dimensional Poisson summation formulas over potato-like regions and their application in number theory (W. Freeden, *Metaharmonic Lattice Point Theory*. CRC Press, Boca Raton, 2011) (ii) the work on multi-dimensional lattice point Shannon-type sampling and the application to the antenna problem (W. Freeden, M.Z. Nashed, *Lattice Point Identities and Shannon-Type Sampling*. CRC Press, Boca Raton, 2019). (Received September 13, 2019)

1154-41-1087 **Wenjing Liao***, School of Mathematics, Georgia Institute of Technology, Atlanta, GA, **Minshuo Chen**, School of Industrial & Systems Engineering, Georgia Institute of Technology, Atlanta, GA, **Haoming Jiang**, School of Industrial & Systems Engineering, Georgia Institute of Technology, Atlanta, GA, and **Tuo Zhao**, School of Industrial & Systems Engineering, Georgia Institute of Technology, Atlanta, GA. *Regression of functions on low-dimensional manifolds by deep ReLU networks.*

Many data in real-world applications lie in a high-dimensional space but are concentrated on or near a low-dimensional manifold. Our goal is to estimate functions on the manifold from finite samples of data. This talk focuses on an efficient approximation theory of deep ReLU networks for functions supported on low-dimensional manifolds. We constructed a ReLU network for such function approximation where the size of the network grows exponentially with respect to the intrinsic dimension of the manifold. When the function is estimated from finite samples, we proved that the mean squared error for the function approximation converges as the training samples increases with a rate depending on the intrinsic dimension of the manifold instead of the ambient dimension of the space. (Received September 13, 2019)

1154-41-1416 **Fazal Abbas*** (fabbas1@stetson.edu), 421 N Woodland Blvd, DeLand, FL 32723, and **Hermann J Eberl**. *A new approach to substrate flux approximation for Monod boundary value problem arises in the study of biofilms.*

We present an analytical approximation for the diffusive flux of a substrate into a reactive layer, in which the substrate is degraded according to Monod kinetics. This problem is described by a nonlinear two-point boundary value problem. The approximation is derived from Modified Adomian Decomposition Approach for boundary value problems and verified computationally, by comparison against a numerical solution of the problem. The analytical approximation is easy to evaluate and depends only on model parameters. It is shown that the approximation simplifies the study of biofilm reactor model arises in the waste water engineering. (Received September 15, 2019)

1154-41-1533 **Willi Freeden** and **Helga Nutz*** (nutz@cbm-ac.de), CBM - Gesellschaft für Consulting, Business und Management mbH, 66450 Bexbach, Germany, and **Rainer Rummel** and **Michael Schreiner**. *Multiscale regularization strategies for the tensorial satellite gravitational gradiometry (SGG) problem.*

Due to the measuring procedure, SGG provides information about the second order partial derivatives of the gravitational potential in the orbit of a satellite (Hesse tensor). In the context of inverse problems, the calculation

of the gravitational potential at the Earth's surface from its second order derivatives at satellite's height turns out to be exponentially ill-posed and, thus, it requires specific tensorial regularization procedures for its solution. The relation between the known tensorial measurements and the gravitational potential on the Earth's surface is known to be expressible by a tensorial linear integral equation of the first kind. This operator equation may be discussed in the framework of pseudodifferential operators as an invertible mapping between certain Sobolev spaces. In the talk a wavelet based regularization technique for the decorrelation of the scalar gravitational potential at the Earth's surface from tensorial SGG data is presented. The talk is based on the article "Satellite gravitational gradiometry: Methodological foundation and geomathematical advances" from W. Freeden, H. Nutz, R. Rummel, and M. Schreiner (In: W. Freeden, R. Rummel (Eds.): *Handbuch der Geodäsie*, Vol. 6, Springer Spektrum, Berlin, Heidelberg, 2019). (Received September 16, 2019)

1154-41-1560 **Douglas P Hardin*** (doug.hardin@vanderbilt.edu), Department of Mathematics, Vanderbilt University, Nashville, TN 37240, and **Peter Boyvalenkov, Peter Dragnev, Edward Saff** and **Maya Stoyanova**. *Linear programming bounds for packing and energy on the sphere*.

For a potential h defined on $[-1, 1)$ and a finite point configuration (or *code*) C on the unit sphere \mathbb{S}^{n-1} , the h -energy of C is given by

$$E(C, n, h) := \sum_{x \neq y \in C} h(\langle x, y \rangle).$$

In this talk, I will review classical linear programming bounds for spherical codes and present new 'universal' lower bounds for energy of the form

$$E(C, n, h) \geq N^2 \sum_{i=1}^m \rho_i h(\alpha_i),$$

where the nodes $\{\alpha_i\}$ and weights $\{\rho_i\}$ depend only on the cardinality N and dimension n and are obtained from a quadrature rule framework developed by Levenshtein in relation to maximal codes. These bounds coincide with those of Cohn and Kumar for the case of sharp codes. This is joint work with P. Boyvalenkov, P. Dragnev, E. Saff and M. Stoyanova. (Received September 16, 2019)

1154-41-1842 **Kathryn E Leonard***, kleonard.ci@gmail.com. *Non-negative tensor decompositions for dynamic topic modeling*. Preliminary report.

Success in non-negative matrix factorization for tasks such as gene expression, face modeling, source separation, and dynamic topic modeling, and the ease of using tensors as representations for multivariate data with a time component, prompt a search for non-negative tensor decompositions that capture salient qualities of data and their evolution over time. Non-negative tensor decomposition, though still not as well understood as its matrix cousin, has produced some interesting theoretical results. From an applications perspective, these results are only useful if they produce decompositions that capture important features in the data. We present experimental results for synthetic data that demonstrate promise for these decompositions, and also discuss some remaining challenges for more realistic datasets such as dynamic topic modeling. This is collaborative work from the 2019 Women in the Science of Data and Mathematics workshop group led by Deanna Needell and Jamie Haddock. (Received September 16, 2019)

1154-41-1980 **Anna C Aboud*** (aaboud@westmont.edu), aaboud@westmont.edu. *Kaczmarz and Least Absolute Deviations*.

The Kaczmarz algorithm iteratively reconstructs vectors in a Hilbert space using inner products against a sequence $\{e_n\}$. Because of its low complexity and computational efficiency, the algorithm is particularly well suited to large data sets, leading to many applications in the realm of data science. In the case of an inconsistent system, variations of the algorithm will converge to a least squares solution. There are cases, however, where a least absolute deviations (LAD) solution is preferred. As currently defined, the algorithm only converges in the context of a Hilbert space, rendering the search for an LAD solution fruitless. To address this limitation, we develop a dualized version of the Kaczmarz algorithm which can be applied in a Banach space setting. We will discuss the construction, ramifications, and application of this dualized algorithm. (Received September 17, 2019)

1154-41-2457 **Vladyslav Babenko***, Department of Mathematical Analysis, Oles Honchar Dnipro National University, Dnipro, 49000, Ukraine, **Vira Babenko**, Department of Mathematics and Computer Scienc, Drake University, Des Moines, IA 50311, and **Oleg Kovalenko**, Department of Mathematical Analysis, Oles Honchar Dnipro National University, 49000, Ukraine. *Korneichuk-Stechkin lemma and Ostrovskii type inequalities for functions with values in L -spaces.*

Korneichuk-Stechkin lemma plays an important role in solving many extremal problems of Approximation Theory. We will present a generalization of this lemma to the case of functions with values in L -spaces (semilinear isotropic metric spaces) and discuss various applications of the generalization. In particular, we will present a new sharp Ostrovskii type inequalities for functions with values in L -spaces. (Received September 17, 2019)

1154-41-2539 **Min Shu*** (min.shu@uconn.edu), 1 University Pl, Stamford, CT 06901, and **Wei Zhu** (wei.zhu@stonybrook.edu). *Extension of Rough Set Based on Positive Transitive Relation.*

Rough set-based data analysis methods have found great successes in data science and artificial intelligence. The application of rough set theory in incomplete information systems is a key problem in practice since missing values almost always occur in knowledge acquisition due to the error of data measuring, the limitation of data collection, or the limitation of data comprehension, etc. The existing rough set extension models based on tolerance or symmetric similarity relations typically discard one relation among the reflexive, symmetric and transitive relations, especially the transitive relation. In order to overcome the limitations of the current rough set extension models, we define a new relation called the positive transitive relation and then propose a novel rough set extension model built upon which. In comparison to the existing extension models, the proposed model has a better performance in processing the incomplete information systems while substantially reducing the computational complexity, taking into account the relation of tolerance and similarity of positive transitivity, and supplementing the related theories in accordance to the intuitive classification of incomplete information. (Received September 17, 2019)

42 ► *Fourier analysis*

1154-42-131 **Kyle Hambrook*** (kyle.hambrook@sjsu.edu) and **Robert Fraser**. *Explicit Salem Sets in Euclidean Space.*

A Salem set is a set which supports a probability measure whose Fourier transform decays at infinity as fast as is allowed by the Hausdorff dimension of the set. We exhibit explicit (i.e. non-random) Salem sets in d -dimensional Euclidean space of every possible Hausdorff dimension. This completely resolves an old problem of Kahane. The construction of the required measure combines Fourier analysis and algebraic number theory in an interesting way. (Received August 13, 2019)

1154-42-154 **Palle Jorgensen***, palle-jorgensene@uiowa.edu, Harvard Univ, Boston, and **Steen Pedersen**. *Orthogonal Fourier expansions for certain fractals.*

Starting with classical Fourier analysis, the talk focuses on classes of fractals, and a wider view of their harmonic analysis. We review a construction by the author and Pedersen of explicit orthogonal Fourier expansions for certain affine fractals. It has since branched off several new directions, each one dealing with aspect of the wider subject. The results presented will cover (among others) joint work with Steen Pedersen, then later, with Dorin Dutkay. Intuitively, it is surprising that any selfsimilar fractals at all, in fact, do admit orthogonal Fourier series. The general theme of Fourier series, and harmonic analysis, on Fractals has by now taken off in a number of diverse directions; e.g., (i) wavelets on fractals, or frames; (ii) non-commutative analysis on graph limits, (iii) discrete approximations; to mention only three. Two popular question are: “What kind of fractals admit Fourier series?” “If they don’t, then what alternative harmonic analysis might be feasible?” (Received August 16, 2019)

1154-42-218 **Jameson Cahill***, Department of Mathematical Sciences, 1290 Frenger Mall, Science Hall 236, Las Cruces, NM 88003. *Complete translation invariant measurements.*

In image and audio signal classification, a major problem is to build stable representations that are invariant under rigid motions and, more generally, to small diffeomorphisms. Translation invariant representations of signals are of particular importance. The existence of such representations is intimately related to classical invariant theory, inverse problems in compressed sensing and deep learning. We construct low dimensional representations of signals that are invariant under finite unitary group actions, as a special case we establish the existence of low-dimensional set of measurements which separates the orbits of any cyclic group action, of which

translation is one example. Furthermore our map is Lipschitz with respect to the natural metric on the space of orbits. Our construction is closely related to methods use in phase retrieval, so we will give an overview of these methods. (Received August 24, 2019)

1154-42-513 **Kasso A Okoudjou*** (kasso@math.umd.edu), 77 Massachusetts Ave, Cambridge, MA 02139. *Spectral properties of pseudodifferential operators on fractals.*

The spectral analysis of the Laplacian on a class of self-similar fractals which includes the Sierpinski gasket (SG) has resulted in a rich body of work that is part of the theory of harmonic analysis on fractals. Some of the important features of this fractal Laplacian include the existence of high multiplicity eigenvalues along with large gaps in its spectrum. An interesting consequence of some of these features is the fact Fourier series on fractals have "nicer" convergence properties. In this talk, I will show how these features can be used to give descriptions of the spectra of pseudodifferential operators on fractals. In particular, I will consider some Schrödinger operators on SG. (The talk is based on joint work with M. Ionescu, L. Rogers, and R. Strichartz.) (Received September 05, 2019)

1154-42-694 **Terry Harris*** (terence2@illinois.edu). *Improved bounds for restricted projection families via Fourier restriction.*

Let $\gamma : [0, 1] \rightarrow S^2$ be a curve in the sphere in \mathbb{R}^3 satisfying the non-degeneracy condition $\det(\gamma, \gamma', \gamma'') \neq 0$. Given a set $A \subseteq \mathbb{R}^3$ of Hausdorff dimension at most 2, it is conjectured that the orthogonal projection of A onto the 2-dimensional plane orthogonal to $\gamma(t)$ has the same Hausdorff dimension as A , for almost every $t \in [0, 1]$. In this talk, I will present some improved lower bounds for this problem obtained via Fourier restriction. I will also present two generalisations of this problem in higher dimensions. (Received September 09, 2019)

1154-42-879 **Chun-Kit Lai*** (cklai@sfsu.edu), Th 928, 1600 Holloway Ave, San Francisco State University, San Francisco, CA 94132, and **Yang Wang**. *Fuglede's spectrum is rational in dimension one.*

A bounded measurable set $\Omega \subset \mathbb{R}^d$ is called a spectral set if it admits some exponential orthonormal basis $\{e^{2\pi i\lambda \cdot x} : \lambda \in \Lambda\}$. Fuglede's conjecture states that a spectral set is equivalent a translational tile. It remains open when $d = 1, 2$ and was disproved in higher dimensions.

In this talk, we show that in dimension one $d = 1$, any spectrum Λ with $0 \in \Lambda$ of a spectral set with Lebesgue measure normalized to 1 must be rational. Combining this with all previous results due to Lagarias, Wang, Iosevich and Kolountzakis, the Fuglede's conjecture on \mathbb{R}^1 is now equivalent to the validity of the corresponding conjecture on all cyclic groups \mathbb{Z}_n . (Received September 11, 2019)

1154-42-1036 **Ahmed I Zayed*** (azayed@depaul.edu), Department of Mathematical Sciences, DePaul University, Chicago, IL 60614. *On the Reconstruction of a Class of Signals Bandlimited to a Disc in the Linear Canonical Transform Domain.*

The linear canonical transform is a generalization of the fractional Fourier transform which in turn is a generalization of the Fourier transform. It arose in optics and some signal processing applications because it can model many general optical systems. It has been used to solve problems in physics and quantum mechanics and it is associated with the homogeneous special group $SL(2, \mathbb{R})$, which is represented by the unimodular matrix

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}; \quad ad - bc = 1.$$

The reconstruction of a bandlimited signal in the linear canonical transform domain from its samples at a discrete set of points has been developed in one and several variables, but in the latter case it has primarily been obtained for signals that are bandlimited to a parallelepiped.

In this talk we present a reconstruction formula for signals that are bandlimited to a disc in the linear canonical transform domain. This formula includes, as special cases, sampling theorems for signals that are bandlimited to a disc in the fractional Fourier transform and the Fourier transform domains. (Received September 12, 2019)

1154-42-1125 **Tom Needham** and **Clayton Shonkwiler*** (clay@shonkwiler.org). *Hamiltonian Group Actions on Frame Spaces.*

A frame for a finite-dimensional complex vector space is simply a spanning set. Frames include bases, but also spanning sets that are larger than a basis, and hence more suitable for applications where robustness to noise and erasures are important.

Since complex vector spaces are naturally symplectic, the space of frames is a symplectic manifold. Moreover, it admits natural Hamiltonian actions of a number of compact Lie groups, and several submanifolds of frames of

particular interest, including Parseval frames and unit-norm frames, arise naturally as level sets of the momentum maps associated with these group actions.

Level sets of momentum maps of Hamiltonian group actions are important objects in symplectic geometry, and many powerful tools can be used to study them. After introducing this perspective, I will describe how it gives new insight into some important problems in frame theory, including the frame homotopy conjecture and the Paulsen problem. (Received September 13, 2019)

1154-42-1185 **John E Herr*** (jeherr@butler.edu). *Computation of Fourier Series with Respect to the Cantor Measure via the Kaczmarz Algorithm*. Preliminary report.

We show how the Kaczmarz algorithm can be used to construct series of the form $\sum_{n=0}^{\infty} c_n e^{2\pi i n x}$ for functions in $L^2(\mu)$ where μ is any singular measure on the unit circle, even when there does not exist an orthogonal basis or frame of functions of the form $e^{2\pi i \lambda x}$. We explain how the coefficients of these series can be numerically approximated, and we demonstrate explicit Fourier-type series for select functions with respect to the ternary Cantor measure. (Received September 13, 2019)

1154-42-1203 **Adel Faridani*** (faridani@math.oregonstate.edu), Department of Mathematics, Oregon State University, Corvallis, OR 97331-4605, and **Hussain Al-Hammali**. *On the peak value problem for non-uniform sampling*.

The problem of finding an upper bound for the infinity norm of signals from their sampled values is called the peak value problem. We consider the peak value problem in the context of non-uniform sampling.

A sampling set Λ is called a set of stable sampling if there exists a constant K such that

$$\|f\|_{\infty} \leq K \sup_{\lambda \in \Lambda} |f(\lambda)| \quad \text{for all } f \in B_{\sigma}^{\infty},$$

where B_{σ}^{∞} denotes the space of bounded signals f whose (distributional) Fourier transform \hat{f} has compact support in the interval $[-\sigma, \sigma]$.

We derive an upper bound for K that is valid for any set of stable sampling Λ for which the maximum distance between two neighboring points is less than π/σ . We also provide numerically computed upper bounds for K for some specific sampling sets Λ that do not satisfy the above condition. (Received September 13, 2019)

1154-42-1220 **David Beltran*** (dbeltran@math.wisc.edu) and **José Madrid** (jmadrid@math.ucla.edu). *Endpoint Sobolev continuity for the fractional maximal function*.

Let M_{β} denote the non-centered fractional maximal function on \mathbb{R}^d , where $0 < \beta < d$. A well known result of Kinnunen says that the $L^p - L^q$ bounds satisfied by M_{β} are preserved at the derivative level, that is,

$$\|\nabla M_{\beta} f\|_q \leq \|\nabla f\|_p, \quad \text{for } \frac{1}{q} = \frac{1}{p} - \frac{\beta}{d}$$

and $1 < p \leq \infty$. Moreover, there are some instances in which the map $f \mapsto |\nabla M_{\beta} f|$ satisfies strong bounds for the endpoint input space $W^{1,1}$. This is in contrast to the weak-type bounds satisfied by M_{β} for L^1 functions.

As the map $f \mapsto |\nabla M_{\beta} f|$ ceases to be sublinear, its boundedness from $W^{1,p}$ to L^q does not immediately imply that is a continuous map between such function spaces. The first affirmative result in this direction was obtained by Luiro in the non-endpoint case $1 < p \leq \infty$.

For the endpoint space $W^{1,1}$, Madrid proved the continuity of such map in dimension $d = 1$. In this talk, we will present higher dimensional results. In particular, continuity from $W^{1,1}(\mathbb{R}^d)$ to $L^{d/(d-\beta)}(\mathbb{R}^d)$ holds if $1 \leq \beta < d$ and if $0 < \beta < 1$ for radial functions. (Received September 14, 2019)

1154-42-1408 **Roza Aceska*** (racaska@bsu.edu), Muncie, IN, and **Yeon Hiang Kim** (kim4y@cmich.edu). *Time-Variant System Approximation via Later-Time Samples*. Preliminary report.

We propose a reconstruction algorithm for a time-variant system $u = u(x, t)$ from its later-time samples at a single spatial location x_0 . We construct a sequence of approximations of the unknown initial state $u(x, 0)$ and, as a consequence, of all later-time states. We show that a careful selection of the sequence of later-time samples $\{u(x_0, t_j)\}_{j \geq 1}$ provides for good convergence rates. Applications include approximations of solutions to special classes of linear evolutionary PDEs. (Received September 15, 2019)

1154-42-1424 **Shujie Kang*** (kangsj@math.umd.edu), 4176 Campus Drive - William E. Kirwan Hall, College Park, MD 20742. *Universal Minimizers of the p -Frame Potentials*.

The p -frame potential of a frame $\{v_i\}_{i=1}^N \subset \mathbb{R}^d$ is defined as $FP_{p,N,d} = (\sum_{i < j} |\langle v_i, v_j \rangle|^p)^{1/p}$. Here we investigate the minimizers of $FP_{p,N,d}$. In the special case $p = 2$, the minimizers were characterized as tight frames; and the $p = \infty$ case is related to the line packing problem. We focus on the special case $d = 2$. We prove that the

minimizers of the p -frame potential in \mathbb{R}^2 are universal for large enough p including infinity, and that they can be obtained from spherical t -designs. In fact, we extend techniques developed by Cohn and Kumar involving absolutely continuous functions and ultraspherical polynomials. In addition, we present numerical results for the minimal $FP_{p,5,2}$ when $p < 2$. (Received September 15, 2019)

1154-42-1677 **Michael Perlmutter*** (perlmutter@msu.edu), **Feng Gao, Guy Wolf and Matthew Hirn.**
The Scattering Transform for Geometric Deep Learning.

The scattering transform is a mathematical model of convolutional neural networks (CNNs) introduced for functions defined on Euclidean space by Stephané Mallat. It differs from traditional CNNs by using predesigned, wavelet filters rather than filters which are learned from training data. This leads to a network which provably has desirable mathematical properties such as translation invariance and diffeomorphism stability. Moreover, in situations where the wavelets can be designed in correspondence to underlying physics, it can produce numerical results which rival state of the art CNNs. However, many data sets of interest have an intrinsically non-Euclidean structure and are better modeled as graphs or manifolds. This motivates us to construct geometric versions of the scattering transform using the spectral decompositions of Laplace-Beltrami operator and Graph Laplacian. We will discuss applications of these networks to a variety of geometric deep learning tasks and show that analogously to its Euclidean predecessor, the manifold scattering transform possesses desirable invariance and stability properties with respect to the actions of the isometry and diffeomorphism groups. (Received September 16, 2019)

1154-42-1692 **Joris Roos*** (jroos@math.wisc.edu). *L^p improving estimates for maximal spherical and circular averages.*

For a given compact set of radii E we will discuss L^p improving properties of maximal spherical averages with a supremum over E in dimensions two and higher. Our results are sharp for a large class of E . A surprising feature is that results depend on the Assouad dimension of the set E and also its Assouad spectrum, a notion that was recently introduced by J. M. Fraser and H. Yu. This talk is based on joint works with Tess Anderson, Kevin Hughes and Andreas Seeger. (Received September 16, 2019)

1154-42-1835 **Benjamin Baker Bruce*** (bbuce@math.wisc.edu), Department of Mathematics, Van Vleck Hall, 480 Lincoln Drive, Madison, WI 53706. *Fourier Restriction to a Hyperbolic Cone.*

We resolve the Fourier restriction problem for a conical surface in \mathbb{R}^4 whose cross sections are hyperbolic paraboloids. Earlier work of S. Lee established an optimal L^2 -based bilinear restriction theorem, as well as the full range of off-scaling linear restriction estimates, for this “hyperbolic cone.” We obtain the remaining scale-invariant estimates by adapting a bilinear-to-linear argument introduced by B. Stovall in the context of restriction to the hyperbolic paraboloid in \mathbb{R}^3 . (Received September 16, 2019)

1154-42-2107 **Calvin F Hotchkiss*** (hotchkiss@udel.edu). *Fourier Bases for Certain Fractal L^2 Spaces.*

We discuss the generation of Fourier bases for fractal L^2 spaces generated by iterated function systems. In particular, we consider a certain iterated function system, whose invariant set is a skewed Sierpinski gasket, $S = \{(x, y) \in \mathbb{R}^2 : x \in C_3, y \in C_3, x + y \in C_3\}$, where C_3 is the standard middle-thirds Cantor set. We show the existence of sequences of exponentials which form orthonormal bases on $L^2(S)$, including an infinite set of such bases. (Received September 17, 2019)

1154-42-2134 **Jeremy Schwend*** (jschwend@math.wisc.edu). *Optimal $L^p \rightarrow L^q$ Estimates for Euclidean Averages Over Prototypical Hypersurfaces in \mathbb{R}^3 .*

We find the precise range of $(\frac{1}{p}, \frac{1}{q})$ for which local averages along graphs of a class of two-variable polynomials in \mathbb{R}^3 are bounded (at least in the restricted-weak sense) from L^p to L^q , given the hypersurfaces have Euclidean measure. We derive these results using non-oscillatory geometric methods, for a model class of polynomials bearing a strong connection to the general case of graphs of real-analytic functions. (Received September 17, 2019)

1154-42-2135 **Laura Cladek, Polona Durcik*** (durcik@caltech.edu), **Ben Krause and Jose Madrid.** *Discrete directional maximal function along the primes.*

We study a discrete directional maximal operator along the set of the prime numbers. We show existence of a set of lattice points near a large discrete sphere, for which the ℓ^2 norm of the maximal operator with supremum taken over all large scales grows with an epsilon power in the number of the directions. (Received September 17, 2019)

1154-42-2180 **Theresa C Anderson*** (tcanderson@purdue.edu). *Discrete variants of maximal functions: spheres, primes and triangles*. Preliminary report.

We explore, through examples, how different variants of the spherical maximal function have drastically different boundedness properties. (Hang on, this discrete ride has a few curves). (Received September 17, 2019)

1154-42-2670 **Brendan Marshall Williams*** (brendwil@umich.edu), Brendan Williams, 10835 Bunton Road, Willis, MI 48191, and **Cyril Cordor** (ccordor@umich.edu), Cyril Cordor, 10971 Oak Lane Apt. 4111, Belleville, MI 48111. *Fast Phase Retrieval from Masked Phaseless Fourier Measurements*.

The phase retrieval problem is a fascinating inverse problem that arises in molecular imaging methods like X-ray crystallography as well as in other areas such as speech processing and quantum mechanics. To acquire measurements for a test sample, a coherent beam of radiation is focused on it, causing the beam to scatter. The physics of the diffraction process dictates that the measurements are the squared magnitude of the Fourier transform of the signal admitted by the sample. Because distinct signals can have the same magnitude, different phase factors can generate multiple solutions for the same problem. To encode phase information, we utilize the common approach of acquiring masked (or windowed) measurements. Discrete Fourier analysis is used in conjunction with spectral analysis of strategically constructed, circulant-like matrices to recover the phase accurately and efficiently. However, even today there remain obstacles to flawless phase retrieval such as robustness to noise and computational efficiency. Many phase recovery algorithms used in practice are heuristic in nature and so have no mathematical assurances of obtaining a correct solution. The result of our research was an accurate and efficient 2D phase retrieval algorithm with a strong mathematical backing. (Received September 17, 2019)

43 ► *Abstract harmonic analysis*

1154-43-38 **Tefera Worku*** (tworku3@yahoo.com), Addis Ababa, Ethiopia. *Utilizing advanced probabilistic object in cost cutting when building a smart grid and identifying problem areas + recommend viable approach to solutions*.

Background Math Material: Deriving that the space of Homogeneous polynomials in n variable and of degree $m = (m+n-1)\dots(m+1)/(n-1)!$. Then a discussion of how that relates and gets applied to pressing current and pressing as well as timely Economic and Infrastructural drives. (Received July 09, 2019)

1154-43-241 **Bradley Currey** and **Vignon Oussa*** (voussa@bridgew.edu). *HRT conjecture and linear independence of translates on the Heisenberg group*. Preliminary report.

In this talk, we will establish the relationship between the HRT Conjecture and linear independence of translation systems on the Heisenberg group. We will show that the HRT Conjecture is equivalent to the conjecture that co-central translates of square-integrable functions on the Heisenberg group are linearly independent. This result affirmatively answers a question asked at the HRT workshop in Saint Louis University in 2016. (Received August 26, 2019)

1154-43-1493 **Radu Balan*** (rvbalan@math.umd.edu), Department of Mathematics, University of Maryland, College Park, MD 20742, and **Naveed Haghani**. *Permutation Invariant Representations*. Preliminary report.

Two matrices of same size are called permutation equivalent if they are equal to one another up to a row permutation. The problem asks for an Euclidian embedding of the quotient space induced by the row permutation equivalence relation. This problem admits several equivalent formulations. We shall discuss representations inspired by results from commutative algebra theory, measure theory, and reproducing kernel Hilbert space theory. This problem has direct application to graph classification problems where the underlying network has a natural equivariance property. (Received September 15, 2019)

1154-43-1813 **Emily J King*** (emily.king@colostate.edu). *Introduction to Group Actions in Harmonic Analysis*.

Group actions have implicitly played a role in harmonic analysis since its inception in the work of Fourier on solutions of the heat equation. Namely, one can generate the classical Fourier basis as an action of the group \mathbb{T} on $L^2([0, 1])$. Hundreds of years later in the 20th century, the two most important transforms in applied harmonic analysis arose, the wavelet transform and the short-time Fourier transform, which were created using projective unitary representations of the affine group and the Weyl-Heisenberg group, respectively. Today, some of the most active research areas in harmonic analysis involve the generation of a set as the orbit of a (semi-)group action, in particular in dynamic sampling and in open problems in finite frame theory, like Zauner's conjecture.

There are also cutting edge methods in data analysis which generalize the algebraically generated transforms in harmonic analysis to domains like graphs and neural networks, e.g., diffusion wavelets and the scattering transform. This talk will serve as the introduction to the AMS Special Session on Group Actions in Harmonic Analysis. (Received September 16, 2019)

1154-43-1988 **Anna C Aboud***, aaboud@westmont.edu. *A Dualized Kaczmarz Algorithm.*

Because of its low complexity and computational efficiency, the Kaczmarz algorithm is particularly well suited to solve large linear systems, leading to many applications in the realm of data science. In the case of an inconsistent system, current variations of the algorithm will converge to a least squares solution. There are situations, however, where it is more desirable to minimize the ℓ^1 norm, pursuing a least absolute deviations solution. To address this need, we develop a dualized version of the algorithm within a Banach space setting. We will explore some of the difficulties encountered when transferring the algorithm from a Hilbert space to a Banach space, and will present necessary and sufficient conditions for certain forms of convergence. (Received September 17, 2019)

1154-43-2555 **John Jasper*** (john.jasper@sdstate.edu), Brookings, SD 57006. *Harmonic equiangular tight frames and their combinatorial generalizations.*

It is a well established phenomenon that optimal packings of points in metric spaces often exhibit large amounts of symmetry. One of the richest classes of optimal packings, the so-called equiangular tight frames (ETFs), are no exception. Indeed, a large family of ETFs known as harmonic ETFs arise as the orbit of a single vector under the action of some abelian group. However, when we look closely at these harmonic ETFs we often observe that the group that seems to be calling the shots can actually be replaced by a more common combinatorial object. In this talk we will see a couple of instances of this generalization from algebra to combinatorics and how these generalizations greatly enrich the theory of ETFs. (Received September 17, 2019)

1154-43-2632 **Wojciech Czaja*** (wojtek@math.umd.edu), Department of Mathematics, University of Maryland, College Park, MD 20742. *Fourier Scattering Transforms as Efficient Feature Extractors.*

We present a construction of a family of feature extractors which combine Mallat's scattering transform framework with the benefits of time-frequency representations. We do this by introducing a class of frames, called uniform covering frames, which includes a variety of semi-discrete Gabor systems and other Fourier-based representations. We then incorporate these frames into an iterative neural network-like structure, to generate our candidate features., which we aggregate into a new scattering transformation. This approach proves advantageous in several data-related applications, ranging from spectral imaging to signal analysis and communications. This is joint work with Weilin Li, Ilya Kavalero, and Rama Chellappa. (Received September 17, 2019)

1154-43-2654 **Ehssan Khanmohammadi*** (khanmohe@union.edu). *Some harmonic analysis problems arising from Kirillov's character formula.*

In 1960s Kirillov proposed an elegant formula for the character of an irreducible representation of a Lie group that fitted nicely into his geometric quantization program. Kirillov's formula is striking for its simplicity and generality, but as is well known, it can fail to correctly express the character in a variety of ways. In this talk, I take an operator-algebraic approach to understanding this issue better, and present some results and problems that originate from my study of Kirillov's character formula. (Received September 17, 2019)

44 ► *Integral transforms, operational calculus*

1154-44-1708 **Steven Patrick Flynn***, spflynn@ucsc.edu. *Integral Geometry on Contact Manifolds.*

The classical X Ray Transform (the Radon Transform in two dimensions), maps a function on Euclidean space to a function on the space of lines on this Euclidean space by integrating the function over the given line. Inverting the X ray transform has wide-ranging applications, including to medical imaging and seismology. Much work has been done to understand the problem of inverting the X-ray transform for Euclidean space, Euclidean domains, and more generally for compact Riemannian manifolds with boundary where the lines become geodesics. I formulate a subRiemannian version of the X-ray transform, on the simplest subRiemannian manifold, the Heisenberg group. I report initial progress in showing that this subRiemannian X-ray transform is injective, and so potentially invertible, and discuss progress towards its generalization to 2 step Nilpotent groups and general contact manifolds. (Received September 16, 2019)

45 ► *Integral equations*

1154-45-424 **Marta D’Elia*** (mdelia@sandia.gov), **Christian Vollmann** (vollmann@uni-trier.de) and **Max Gunzburger** (mgunzburger@fsu.edu). *Nonlocal models with approximate nonlocal neighborhoods: towards fast nonlocal FEM.*

Numerical solution of nonlocal models via FEM in 2 or 3D can be prohibitively expensive. This is due to the fact that points in a domain interact with a neighborhood of points. Standard neighborhoods are Euclidean balls; this creates computational challenges in terms of assembling and accuracy of FE matrices. We propose approximate neighborhoods that make the assembling process easier and faster. We analyze the new discretized nonlocal equation and present numerical results. (Received September 03, 2019)

1154-45-837 **Yanzhi Zhang***, Department of Mathematics and Statistics, Missouri University of Science and Technology, 400 W 12th St, Rolla, MO 65409, and **Siwei Duo**. *Nonlocal Problems with the Tempered Fractional Laplacian.*

In this talk, I will present our recent work on nonlocal problems with the tempered integral fractional Laplacian. An accurate numerical method is presented to discretize the d -dimensional (for $d = 1, 2$, or 3) tempered integral fractional Laplacian, together with the rigorous numerical analysis. Since our method yields a (multilevel) Toeplitz stiffness matrix, one can design fast algorithms via the fast Fourier transform for efficient simulations. Finally, we study the tempered nonlocal effects on the solutions of various fractional PDEs, including the Allen-Cahn equation and Gray-Scott equations. (Received September 11, 2019)

1154-45-1543 **James M Scott*** (jscott66@vols.utk.edu) and **Tadele Mengesha**. *Asymptotic analysis of a coupled system of nonlocal equations with oscillatory coefficients.* Preliminary report.

In this paper we study the asymptotic behavior of solutions to strongly coupled systems of integral equations with oscillatory coefficients. The system of equations is motivated by the peridynamic model of the deformation of heterogeneous media that additionally accounts for short-range forces. We consider the vanishing nonlocality limit on the same length scale as the heterogeneity and show that the system’s effective behavior is characterized by a coupled system of local equations that are elliptic in the sense of Legendre-Hadamard. This effective system is characterized by a fourth-order tensor that shares properties with Cauchy elasticity tensors that appear in the classical equilibrium equations for linearized elasticity. (Received September 16, 2019)

1154-45-1656 **Marta D’Elia**, **Cynthia Flores**, **Xingjie Li**, **Petronela Radu*** (pradu@unl.edu) and **Yue Yu**. *Nonlocal operators: properties and decompositions.*

The emergence of nonlocal theories as promising models in different areas of science (continuum mechanics, biology, image processing) has led the mathematical community to conduct varied investigations of systems of integro-differential equations and the underlying operators. In this talk I will discuss properties of nonlocal operators, counterparts of local versions, and present some recent results on Helmholtz-Hodge type decompositions of nonlocal operators. Applications of these results will be discussed, as well as connections with other theories. (Received September 16, 2019)

1154-45-1995 **M. D’Elia** and **Cynthia Flores*** (cynthia.flores@csuci.edu), One University Drive, Bell Tower East, Camarillo, CA 93012, and **Xingjie Li**, **Petronela Radu** and **Yue Yu**. *On Theoretical Aspects of Nonlocal Helmholtz Decomposition of a Vector Field.*

Nonlocal theories have been introduced in the mechanics of solids where the propagation of cracks and other discontinuities hinder the use of classical differential operators. The study of integral operators has been central to the formulation of nonlocal systems and the analysis of their corresponding solutions. By replacing classical differential operators with integral operators, nonlocal frameworks allow the consideration of solutions with little to no regularity (L^2 -level). Moreover, a collection of nonlocal tools can be identified that is useful for analyzing the Helmholtz-Hodge Decompositions (HHD) of a vector field into its divergence-free, curl-free, and harmonic components. In this talk, we will discuss preliminary well-posedness results for nonlocal material science models where the interaction kernel of the integral operator is weakly singular and we will motivate the development of theory for HHD in the nonlocal setting. The work presented is part of the recent Women in Mathematics of Materials (WIMM) Workshop at University of Michigan. (Received September 17, 2019)

1154-45-2644 **Udita N. Katugampola*** (uditanalin@yahoo.com), Department of Mathematical Sciences, Florida Polytechnic University, Lakeland, FL 33805. *New Fractional-like Derivative with Dynamic Memory: Theory and Applications.* Preliminary report.

One of the advantages of the non-local formulation of the fractional derivative is that it carries a memory effects. The question is that “Do we need to use the memory starting at a fixed point as in the Riemann-Liouville

definition of the fractional integral or can it be dynamic?" The purpose of this note is to address that question in detail. It turns out that the fractional-like derivative defined in this way is not equivalent to any familiar fractional derivative even though there are several interesting relationships between them. As applications, we study several nonlocal models such as population dynamics. (Received September 17, 2019)

1154-45-2806 **Oana Marin***, 455 Talcott Ave, Lemont, IL 60439, and **Marieme Ngom**. *Uncertainty quantification of surface tension in a rising bubble using boundary integrals*. Preliminary report.

We study effects of surface tension on a multi-phase problem, specifically on the problem of a rising bubble. Surface tension is a constant measured via experiments, and thus prone to error. By employing uncertainty quantification we assess the effects on pressure difference, mean curvature and other quantities of interest. Addressing this issue has been difficult using volumetric methods, however a boundary integral based formulation assures that the only errors stem from numerical discretization and mathematical model. Additionally we showcase how proper algebraic treatment can lead to an efficient treatment on this given problem. (Received September 18, 2019)

46 ► *Functional analysis*

1154-46-88 **Carla Farsi** (leonardhuang@unr.edu), **Leonard Tristan Huang*** (leonardhuang@unr.edu), **Alex Kumjian** and **Judith Packer**. *Cocycles on Deaconu-Renault Groupoids, and KMS States for Generalized Gauge Dynamics*.

In this talk, we introduce a new technique of constructing KMS states on the C^* -algebra of the Deaconu-Renault groupoid associated to a commuting family of k surjective local homeomorphisms on a compact metrizable space. This technique is based on two things: (i) a new result that classifies all continuous \mathbb{R} -valued 1-cocycles on a Deaconu-Renault groupoid in terms of commuting families of Ruelle transfer operators, and (ii) a generalized Ruelle-Perron-Frobenius Theorem for such commuting families. We explain how our technique can be applied to discretize higher-rank graphs to recover existing results and to produce new ones. We also present the first known analysis of KMS states on the C^* -algebras of topological higher-rank graphs. We hope that our work will appeal to C^* -algebraists and ergodic theorists alike. (Received September 17, 2019)

1154-46-117 **Kelly Bickel*** (kelly.bickel@bucknell.edu), **James Pascoe** and **Alan Sola**. *Singular Rational Functions on the Polydisk*.

Rational inner functions (RIFs) on the bidisk have well-understood behavior near their boundary singularities; in particular, many things are known about their derivative integrability, zero and unimodular level set behavior, and non-tangential regularity. In this talk, I will discuss singular RIFs in dimensions three and higher, with a focus on degree $(1, m, n)$ RIFs on the tridisk. While some two-variable results generalize, non-trivial RIF examples will show that much of the nice behavior seen in the two-variable case is lost in higher dimensions. Time permitting, I will discuss recent explorations concerning rational (not necessarily inner) functions on the bidisk. (Received August 09, 2019)

1154-46-205 **Michel L Lapidus*** (lapidus@math.ucr.edu), University of California, Riverside, Department of Mathematics, Riverside, CA 92521-0135. *An Introduction to Noncommutative Fractal Geometry*.

We provide an introduction to noncommutative fractal geometry, based on work of the presenter (Topological Methods in Nonlinear Analysis, special issue dedicated to Jean Leray; Contemporary Math.), and, in the case of the Sierpinski gasket and other fractals (such as certain fractal trees), by E. Christensen, C. Antonescu and the presenter (Advances in Math.), as well as by J. Sarhad and the presenter. The latter work applies to a broader class of fractals, including the harmonic gasket, which is a primary example of a "fractal manifold". Special emphasis is placed on the recovery of the Hausdorff dimension, the Hausdorff measure and the geodesic metric of the underlying fractals. (Received August 23, 2019)

1154-46-237 **Masoud Khalkhali*** (masoud@uwo.ca) and **Shahab Azarfar**. *From random spectral triples to spectral curves and topological recursion*.

Recently suggested matrix models to probe quantum gravity based on Dirac operators on finite spectral triples, pose very challenging analytic problems. In particular their large N limits have only been studied by computer simulations. There are also conjectures about existence of phase transition in the limit laws. In this talk I will show how new techniques developed in modern random matrix theory, namely topological recursion and the theory of Riemann surfaces (more precisely spectral curves), can be effectively applied and yield rigorous results

(even for more general models). The Schwinger-Dyson equations satisfied by the connected correlators W_n of the corresponding multi-trace formal 1-Hermitian matrix model are derived. I will show that the coefficients $W_{g,n}$ of the large N expansion of W_n 's enumerate discrete surfaces, called stuffed maps, whose building blocks are of particular topologies. The spectral curve $(\Sigma, \omega_{0,1}, \omega_{0,2})$ of the model can be investigated in detail. In particular, I will give an explicit expression for the fundamental symmetric bidifferential $\omega_{0,2}$ in terms of the formal parameters of the model. This is Joint work with S. Azarfar. (arXiv:1906.09362). (Received August 26, 2019)

1154-46-243 **Ray Cheng*** (rcheng@odu.edu). *The zero sets of ℓ_A^p are nested.*

For $0 < p \leq \infty$, the space ℓ_A^p is defined to be the set of analytic functions on the unit disk \mathbb{D} with Taylor coefficients belonging to the sequence space ℓ^p . The main result is that when $1 < p < \infty$, the zero sets for ℓ_A^p depend on p .

Theorem Let $1 < p_1 < p_2 < \infty$. There exists a sequence W of distinct points in \mathbb{D} such that there is a nontrivial function belonging to $\ell_A^{p_2}$ that vanishes at all the points of W , and any function in $\ell_A^{p_1}$ vanishing on W must vanish identically.

Rather than go through the details of the proof, which is long and technical, we will highlight some of the underlying ideas, which may be applicable to other problems about ℓ_A^p and other function spaces. These concepts and tools include Birkhoff-James orthogonality and an associated Pythagorean theorem; a notion of "inner" function for ℓ_A^p , a zero set criterion based on an extremal problem and its dual; and some counting tricks. (Received August 26, 2019)

1154-46-282 **Terje Høim*** (thoim@fau.edu), Jupiter, FL 33458, and **D. A. Robbins** (david.robbins@trincoll.edu), Tucson, AZ 85750. *Module bundles and module amenability.*

Let A and B be Banach algebras, with A a B -bimodule. The notion of B -module amenability for A was introduced by M. Amini in [Semigroup Forum **69** (2004), 243-254]. In this paper, we take X to be a compact Hausdorff space, and let $\{A_x : x \in X\}$ and $\{B_x : x \in X\}$ be collections of Banach algebras such that each A_x is B_x -module amenable. We investigate the module amenability of certain spaces of A_x -valued functions over X , when these function spaces are viewed as modules over spaces of B_x -valued functions. (Received September 01, 2019)

1154-46-326 **Martin E. Walter*** (martin.walter@colorado.edu), Campus Box 395, Department of Mathematics, University of Colorado, Boulder, CO 80309. *Negative Definite Functions and Matrix-valued Euler-Phi functions.* Preliminary report.

The length, $N[\sigma]$, of a permutation σ , can be defined such that when N is applied to an abelian group A in its regular representation, N is a negative definite function, and the values of its Fourier transform can be described using the Euler ϕ -function, also known as Euler's totient function. In particular, if $A = C_n$, a cyclic group of order n , generated by $a \in A$, then $N[a^i] = n - \gcd[n, i]$, $i = 1, \dots, n$, where \gcd means greatest common divisor. The function N is associated with an $n \times n$ self-adjoint matrix, \mathbf{N} , whose eigenvalues are the values of the Fourier transform of N . We generalize this to any abelian group.

This leads to the definition of a matrix-valued Euler-phi function. The classical Euler-phi function and the classical Mobius function are just different representations of this matrix-valued Euler-phi function. (Received September 01, 2019)

1154-46-370 **Ben Hayes*** (brh5c@virginia.edu), **David Jekel**, **Brent Nelson** and **Thomas Sinclair**. *Pinsker algebras for 1-bounded entropy I.*

I will discuss joint work with Jekel-Nelson-Sinclair where we discuss Pinsker algebras in the context of 1-bounded entropy, implicitly defined in work of Jung and explicitly in previous work of mine. The 1-bounded entropy is a von Neumann algebra invariant defined as a modification of the free entropy dimension, and measures "how many" finitary approximations a tracial von Neumann algebra M has. Pinsker algebras, defined in analogy with ergodic theory, are maximal subalgebras of 1-bounded entropy zero. My talk will be mostly on the definition of 1-bounded entropy and Pinsker algebras, and their main properties. I will discuss how one can use random matrix models to show certain subalgebras are Pinsker, and how this relates to maximal amenable subalgebras of free group factors. David Jekel will present more of the details in a follow up talk. (Received September 02, 2019)

1154-46-408 **Ionut Chifan** and **Sayan Das*** (sayan-das@uiowa.edu), 14 Maclean Hall, Department of Mathematics, The University of Iowa, Iowa City, IA 52246. *On a Question of D. Shlyakhtenko*. Preliminary report.

Two groups Γ and Λ are said to be W^* -equivalent if their group von Neumann algebras $L(\Gamma)$ and $L(\Lambda)$ are isomorphic. The groups Γ and Λ are said to be measure equivalent (M.E.) if they admit (stably) orbit equivalent free p.m.p. actions. D. Shlyakhtenko proposed the question whether measure equivalence implies W^* equivalence.

In this talk, I shall give a large class of examples of icc groups Γ and Λ , which are measure equivalent, but not W^* equivalent. This talk is based on a joint work with Prof. Ionut Chifan. (Received September 03, 2019)

1154-46-472 **Rajan Puri***, Wake Forest University, and **Boris Vainberg**. *On critical value of the coupling constant in exterior elliptic problems*.

We consider exterior elliptic problems with coefficients stabilizing at infinity and study the critical value β_{cr} of the coupling constant (the coefficient at the potential) that separates operators with a discrete spectrum and those without it. The dependence of β_{cr} on the boundary condition and on the distance between the boundary and the support of the potential is described. The discrete spectrum of a non-symmetric operator with the FKW boundary condition (that appears in diffusion processes with traps) is also investigated. (Received September 04, 2019)

1154-46-488 **Meredith Sargent*** (sargent@uark.edu) and **Alan Sola**. *Old Results and New Observations for Optimal Approximants on the Bidisk*.

In recent years, optimal polynomial approximants have been used to study cyclicity of functions in Dirichlet-type spaces on the complex unit disk, with particular interest being paid to the connection between the location of the zero sets of the optimal approximants and cyclicity, as well as a correspondence between optimal approximants and orthogonal polynomials. Some progress has been made to extend these ideas to the bidisk, where this question was also studied by electrical engineers for polynomials in the Hardy space. In this talk, the engineering results are reviewed, and new observations regarding zero sets and cyclic functions are discussed. (Received September 05, 2019)

1154-46-543 **Palle E. T. Jorgensen**, **Sooran Kang** and **Myung-Sin Song*** (msong@siue.edu), Department of Mathematics & Statistics, Southern Illinois University Edwardsville, Box 1653, Edwardsville, IL 62026, and **Feng Tian**. *Dimension Reduction and Kernel Principal Component Analysis*.

First, the background of the classical linear framework of Principal Component Analysis will be introduced. Then non-linear data-dimension reduction will be discussed. In nonlinear case, kernel-Principal Component Analysis is used instead with manifold and feature space transforms. The results extend earlier work for probabilistic Karhunen-Loève transforms on compression of wavelet images which were algorithms for optimization, selection of efficient bases, or components, which serve to minimize entropy and error; and hence to improve digital representation of images, and hence of optimal storage, and transmission. Several new theorems for data-dimension reduction will be presented, and with the use of frames in Hilbert space, and a new Hilbert-Schmidt analysis, we identify when a choice of Gaussian kernel is optimal. (Received September 06, 2019)

1154-46-547 **David A Jekel*** (davidjekel@math.ucla.edu). *Pinsker Algebras for 1-bounded Entropy II*. Preliminary report.

I will discuss joint work with Hayes, Nelson, and Sinclair, in which we use free entropy techniques to prove maximal amenability for certain subalgebras of II_1 factors. Continuing from Ben Hayes' talk, I will present some tools and details of our proof. A key idea is that if a random matrix models have exponential concentration and are supported on a "subexponentially large" microstate space, then most of the mass is actually localized to a small ball. A convenient way to map from one microstate space to another (e.g. for different generators) is provided by a new functional calculus, introduced in my own work, that can express an arbitrary element of a (Connes-embeddable) tracial von Neumann algebra $M = W^*(X_1, \dots, X_n)$ as $f(X_1, \dots, X_n)$, where f is a function that is defined and L^2 -uniformly continuous on all self-adjoint tuples from \mathcal{R}^ω . (Received September 06, 2019)

1154-46-569 **Shen Lu*** (shen.lu@colorado.edu). *Constructing finitely generated projective modules for noncommutative solenoids*.

For a fixed prime p , a noncommutative solenoid as defined by Frédéric Latrémolière and Judith Packer is a twisted group C^* -algebra $C^*\left(\mathbb{Z}\left[\frac{1}{p}\right] \times \mathbb{Z}\left[\frac{1}{p}\right], \sigma\right)$, where $\mathbb{Z}\left[\frac{1}{p}\right] := \left\{\frac{j}{p^k} \in \mathbb{Q} : j \in \mathbb{Z}, k \in \mathbb{N}\right\}$ is an additive discrete group and σ is a \mathbb{T} -valued group 2-cocycle (multiplier) on $\mathbb{Z}\left[\frac{1}{p}\right] \times \mathbb{Z}\left[\frac{1}{p}\right]$. In this talk, we discuss two different

ways of constructing projective modules over the NC solenoids: by writing the NC solenoids as direct limits of noncommutative tori and by constructing the Heisenberg equivalence bimodule of Rieffel. We use these constructions to discuss some recent progress made in classifying the irrational NC solenoids up to Morita equivalence. (Received September 07, 2019)

1154-46-611 **Xin Li, Tron Omland and Jack Spielberg*** (jack.spielberg@asu.edu). *C*-algebras of Artin-Tits monoids*. Preliminary report.

Semigroup C-algebras* have been studied intensively by Xin Li in recent years. The first example was, arguably, Coburn's theorem characterizing the Toeplitz algebra, and there followed work by Douglas, Murphy, Cuntz, Nica, etc. Crisp and Laca studied right-angled Artin monoids, and showed that they were of the same paradigm as the Cuntz algebras, namely, the situation that Nica termed *amenable*. They also noted that Artin-Tits monoids of finite type fall outside this paradigm. We consider in particular the finite type case, and describe the structure of the *C**-algebras. We find a dichotomy of sorts depending on the number of relations in the presentation of the group. (Received September 08, 2019)

1154-46-613 **Marc A. Rieffel***, rieffel@math.berkeley.edu. *Dirac operators for matrix algebras converging to coadjoint orbits*. Preliminary report.

In the high-energy quantum-physics literature one finds statements such as "matrix algebras converge to the sphere". Earlier I provided a general setting for understanding such statements, in which the matrix algebras are viewed as compact quantum metric spaces, and convergence is with respect to a quantum Gromov-Hausdorff-type distance. The general setting is matrix algebras converging to coadjoint orbits. But physicists want, even more, to treat structures on coadjoint orbits such as such as vector bundles, Riemannian metrics, and Dirac operators, and they want to approximate these by corresponding structures on matrix algebras.

Recently I have worked out what the corresponding "cotangent bundles" should be for the matrix algebras. These can then be used to define corresponding Riemannian metrics. I will very briefly indicate how some of this works. I am now trying to work out what the Dirac operators for the matrix algebras should be for the corresponding Riemannian metrics, and how they relate to the Dirac operators for the coadjoint orbits. (In the physics literature there are at least 3 inequivalent suggestions for what the Dirac operators on the matrix algebras should be in the case of the 2-sphere.) In my talk I will report on my findings. (Received September 08, 2019)

1154-46-621 **Dejenie Alemayehu Lakew*** (dejenieal@gmail.com), 7933 Arbor Ponds Terr, New Kent, VA 23124. *On Orthogonal Components of a Solution to a Cauchy BVP*. Preliminary report.

Let D be the Dirac operator and $k \geq 3$. It will be shown that the Sobolev Space $W^{(k-1),2}(\Omega)$ ($fork \geq 1$) has an orthogonal decomposition given by

$$W^{(k-1),2}(\Omega) = A^{k,2}(\Omega) \oplus D^k(W_0^{(2k-1),2}(\Omega))$$

where D^k is the k^{th} order Dirac operator and $A^{k,2}(\Omega) = Ker D^k \cap W^{(k-1),2}(\Omega)$, so that any function ψ in $W^{(k-1),2}(\Omega)$ can be written as an orthogonal sum $\psi = \phi \uplus \{\phi\}$ with $\phi \in A^{k,2}(\Omega)$ and $\{\phi\} \in D^k(W_0^{(2k-1),2}(\Omega))$.

(Received September 08, 2019)

1154-46-665 **Alvaro Arias*** (aarias@du.edu), University of Denver, and **Vladimir Kovalchuk** (vladimir.kovalchuk@du.edu), University of Denver. *Geodesics in the Banach-Mazur distance*.

We show that there are uncountably many geodesics between any two non-isometric n -dimensional normed spaces. We construct two explicit geodesics that can be used to describe all the points of the other geodesics. (Received September 09, 2019)

1154-46-674 **Valentin Deaconu*** (vdeaconu@unr.edu), Department of Math and Stat, University of Nevada, Reno, NV 89557-0084. *Group actions on C*-algebras of a vector bundle*. Preliminary report.

We consider *C**-algebras constructed from compact group actions on complex vector bundles $E \rightarrow X$ endowed with a Hermitian metric. An action of G by isometries on $E \rightarrow X$ induces an action on the *C**-correspondence $\Gamma(E)$ over $C(X)$ consisting of continuous sections, and on the Cuntz-Pimsner algebra \mathcal{O}_E , so we can study the crossed product $\mathcal{O}_E \rtimes G$. If the action is free and $\text{rank } E = n$, then we prove that $\mathcal{O}_E \rtimes G$ is Morita-Rieffel equivalent to a field of Cuntz algebras \mathcal{O}_n over the orbit space X/G . If the action is fiberwise, then $\mathcal{O}_E \rtimes G$ becomes a continuous field of crossed products $\mathcal{O}_n \rtimes G$. For transitive actions, we show that $\mathcal{O}_E \rtimes G$ is Morita-Rieffel equivalent to a graph *C**-algebra. (Received September 09, 2019)

1154-46-757 **Ian Charlesworth** and **Brent Nelson*** (brent@math.msu.edu). *Free Stein Irregularity and Dimension.*

Given an n -tuple X of non-commutative random variables, its free Stein discrepancy relative to the semicircle law (the non-commutative analogue of classical Stein discrepancy relative to the Gaussian distribution) measures how "close" the distribution of X is to the semicircle law. By considering free Stein discrepancies relative to a broader class of laws, one can define a quantity called the free Stein irregularity. I will discuss this quantity and show how it can be related to other free probabilistic quantities such as the free Fisher information and the non-microstates free entropy dimension. I will also show how it can be easily computed for a number of interesting examples. (Received September 10, 2019)

1154-46-788 **Mutasim Mim*** (mutasim.mim16@stjohns.edu), 8000 Utopia Parkway, New York, NY 11439, **Mikhail Ostrovskii** (ostrovsm@stjohns.edu), 8000 Utopia Parkway, New York, NY 11439, and **Seychelle S Khan** (seychelle.khan16@my.stjohns.edu), 8000 Utopia Parkway, New York, NY 11439. *Isometric copies of l_∞^n and l_1^n in transportation cost spaces on finite metric spaces.* Preliminary report.

We prove the following results: (a) If a metric space contains $2n$ elements, the transportation cost space on it contains a 1-complemented isometric copy of l_1^n . (b) An example of a finite metric space whose transportation cost space contains an isometric copy of l_∞^4 . Transportation cost spaces are also known as Arens-Eells, Lipschitz-free, or Wasserstein 1 spaces. (Received September 10, 2019)

1154-46-1033 **Cheng Cheng** and **Ingrid Daubechies***, ingrid@math.duke.edu. *Phase Retrieval for Windowed Fourier Transform samples.*

When a square integrable function $f \in L^2(\mathbb{R})$ consists of several "components" that are widely separated in time-frequency space, it is hard to retrieve f from the knowledge of the magnitudes $(|\langle f, g_{m\alpha, n\beta} \rangle|)_{m, n \in \mathbb{Z}}$, where $g_{a,b}(x) = \exp(2\pi i ax)g(x-b)$. The instability of the phase retrieval problem (determining f from the $|\langle f, g_{a,b} \rangle|$) has been discussed in a number of recent papers. When g is a Gaussian, and one considers a and b as continuous variables, Grohs and Rathmair expressed this instability, in *Stable Gabor Phase Retrieval and Spectral Clustering*, in terms of a Cheeger constant that quantifies the "delocalization" of $|\langle f, g_{a,b} \rangle|$. We show a similar result for the discrete case, where $a = m\alpha$, $b = n\beta$, with $m, n \in \mathbb{Z}$; we extend this to other functions g than Gaussians.

The presentation will start by a brief review, focusing on the link between "separation" and "instability", before describing the authors' recent results and open problems. (Received September 12, 2019)

1154-46-1072 **J. Alejandro Chávez-Domínguez***, Department of Mathematics, University of Oklahoma, Norman, OK 73019-3103. *Constructing tight unit norm frames for finite-dimensional Banach spaces using group actions.* Preliminary report.

Frames for Hilbert spaces, as overcomplete versions of bases, are quite useful in applications because they provide decompositions that are more robust. Those frames that consist of vectors of norm one and are additionally tight (FUNTFs) have even more computational advantages, e.g. they minimize the reconstruction error due to the loss of a single coefficient.

FUNTFs for finite-dimensional Banach spaces have recently been introduced, and they have been shown to enjoy some of the same desirable properties as their Hilbertian counterparts (such as the aforementioned reconstruction error minimization). However, very few examples of such FUNTFs for Banach spaces are known.

In this talk we generalize, from the Hilbertian to the Banach case, various results regarding the construction of FUNTFs using group actions. (Received September 13, 2019)

1154-46-1106 **Therese Landry*** (tland004@ucr.edu), **Michel Lapidus** and **Frederic Latremoliere**. *Convergence of Spectral Triples on the Sierpinski Gasket.*

One of the fundamental tools of noncommutative geometry is Connes' spectral triple. Michel Lapidus, Erik Christensen, and Cristina Ivan have developed a spectral triple for the Sierpinski gasket fractal that recovers the Hausdorff dimension, the geodesic metric, and the \log_2 3-dimensional Hausdorff measure. The Gromov-Hausdorff distance is an important tool of Riemannian geometry, and building on the earlier work of Marc Rieffel, Frederic Latremoliere introduced a generalization of the Gromov-Hausdorff distance to the quantum compact metric space. An extension of this new technique in noncommutative geometry- the Gromov-Hausdorff propinquity- to the setting of spectral triples will be used to show how the Lapidus, Christensen, and Ivan spectral triple on the Sierpinski gasket can be written as a limit of spectral triples on its n th level approximations. This talk will develop ideas underlying joint work in progress with Michel Lapidus and Frederic Latremoliere. (Received September 14, 2019)

1154-46-1124 **Benjamin W. Passer*** (bpasser@uwaterloo.ca) and **Vern I. Paulsen**. *Approximation Properties of Matrix Convex Sets and Operator Systems*.

A matrix convex set X , by definition, is built up of many different matrix levels X_n . We show that the smallest and largest matrix convex sets over X_n do not necessarily approximate X to arbitrary precision as n approaches infinity. In particular, successful approximation corresponds to the lifting property and exactness of the corresponding operator system. (Received September 13, 2019)

1154-46-1179 **Krishnendu Khan*** (krishnendu.khan@vanderbilt.edu). *On Product Rigidity*.

It is well known that the passage of the weak closure of the group ring of a discrete group Γ , denoted by $L(\Gamma)$ tends to forget the algebraic properties of the group. However, over the last few years many examples where $L(\Gamma)$ "remembers" various algebraic properties of Γ were obtained via advancement of deformation/rigidity theory. In my talk, I shall give new classes of examples of groups for which the "product feature" is remembered by the group von Neumann algebra. This talk is based on joint work with Prof. Ionut Chifan and Sayan Das. (Received September 13, 2019)

1154-46-1464 **Alexander Gorokhovskiy*** (gorokhov@colorado.edu) and **Henri Moscovici**. *Higher indices and cyclic cohomology*. Preliminary report.

We give a uniform construction of the higher indices of elliptic operators associated to Alexander-Spanier cocycles of either parity in terms of a pairing between the K -theory and the cyclic cohomology of the algebra of complete symbols of pseudodifferential operators. This construction allows us to get a new insights into some classical results on traces of pseudodifferential operators and obtain a number of new results. (Received September 15, 2019)

1154-46-1497 **Becky Armstrong, Lisa Orloff Clark, Kristin Courtney, Ying-Fen Lin, Kathryn McCormick*** (kamccorm@umn.edu) and **Jacqui Ramagge**. *Twisted Steinberg Algebras*.

We introduce twisted Steinberg algebras, which generalize complex Steinberg algebras, and are a purely algebraic notion of Renault's twisted groupoid C^* -algebras. In particular, for each ample Hausdorff groupoid G and each locally constant 2-cocycle σ on G taking values in the complex unit circle, we study the complex $*$ -algebra $A(G, \sigma)$ consisting of locally constant compactly-supported functions on G , with convolution and involution twisted by σ . We introduce a "discretized" analogue of Kumjian's twist over an étale Hausdorff groupoid, and show that there is a one-to-one correspondence between these discretized twists and locally constant 2-cocycles on G . In this setting, we also define a twisted Steinberg algebra and show that the two definitions agree. Under the additional hypothesis that G is effective, we prove Cuntz-Krieger and graded uniqueness theorems for $A(G, \sigma)$, and we show that simplicity of $A(G, \sigma)$ is equivalent to minimality of G . (Received September 15, 2019)

1154-46-1547 **Karen R. Strung***, krstrung@gmail.com, and **Robin J. Deeley** and **Ian F. Putnam**. *Constructions in minimal amenable dynamics and applications to classification of C^* -algebras*.

We study the existence of minimal dynamical systems, their orbit and minimal orbit-breaking equivalence relations, and their applications to the classification of C^* -algebras. We show that given any finite CW-complex there exists a space with the same K -theory and cohomology that admits a minimal homeomorphism. The proof relies on the existence of homeomorphisms on point-like spaces, together with existence results for skew product systems due to Glasner and Weiss.

To any minimal dynamical system one can also associate minimal equivalence relations by breaking orbits at small subsets. Using the groupoid C^* -algebra construction we can associate K -theory groups to minimal dynamical systems and orbit-breaking equivalence relations. We show that given arbitrary countable abelian groups G_0 and G_1 we can find a minimal orbit-breaking relation such that the K -theory of the associated C^* -algebra is exactly this pair. These results have important applications to the Elliott classification program for C^* -algebras. In particular, we make a step towards determining the range of the Elliott invariant of the C^* -algebras associated to minimal dynamical systems with mean dimension zero and their minimal orbit-breaking relations. (Received September 16, 2019)

1154-46-1555 **Corey Jones*** (jones.6457@osu.edu). *Braided tensor categories from finite von Neumann algebras*.

Connes introduced an abelian subgroup $\chi(M) \leq \text{Out}(M)$ of a von Neumann algebra M . Jones defined a quadratic form κ on this subgroup. By results of Joyal and Street, abelian groups together with quadratic forms define a braided tensor category. In this talk we explain how to generalize this braided tensor category to include not

necessarily invertible bimodules in the case M is a finite von Neumann algebra. We will discuss some examples and some conjectures. Based on joint work with Vaughan Jones. (Received September 16, 2019)

1154-46-1572 **Robert Calderbank, Ingrid Daubechies, Daniel Freeman*** (daniel.freeman@slu.edu) and **Nikki Freeman**. *Stable phase retrieval from magnitude of point evaluation for infinite dimensional subspaces of $L_2(R)$.*

Let H be a real Hilbert space and let $(\phi_j)_{j \in J} \subseteq H$. We say that $(\phi_j)_{j \in J}$ does phase retrieval if whenever $x, y \in H$ and $(|\langle x, \phi_j \rangle|)_{j \in J} = (|\langle y, \phi_j \rangle|)_{j \in J}$ we have that $x = y$ or $x = -y$. In the case that $(\phi_j)_{j \in J}$ is a frame, the analysis operator $\Theta(x) = (\langle x, \phi_j \rangle)_{j \in J}$ is an isomorphic embedding of H into $l_2(J)$. Thus phase retrieval using a frame is equivalent to recovering $f \in \Theta(H) \subseteq l_2(J)$ from $|f|$ (up to a unimodular scalar).

Phase retrieval is always stable for finite dimensional spaces. On the other hand, it is known that phase retrieval using frames is always unstable for infinite dimensional Hilbert spaces. This is equivalent to phase retrieval from point evaluation is always unstable for infinite dimensional subspaces of l_2 . In contrast to this, we present a construction of infinite dimensional subspaces of $L_2(R)$ where phase retrieval from point evaluation is stable. By restricting to subspaces, we have new finite dimensional examples where phase retrieval is uniformly stable independent of the dimension. (Received September 16, 2019)

1154-46-1585 **David Kerr***, Department of Mathematics, Texas A&M University, College Station, TX 77843-3368, and **Robin Tucker-Drob**, Department of Mathematics, Texas A&M University, College Station, TX 77843-3368. *Dynamical alternating groups and property Gamma.*

We show that the alternating group of a topologically free action of a countably infinite amenable group on the Cantor set has property Gamma (and in particular is inner amenable) and that there are large classes of such groups which are simple, finitely generated, and nonamenable. (Received September 16, 2019)

1154-46-1641 **Marcel Bischoff*** (bischoff@ohio.edu), **Simone Del Vecchio** and **Luca Giorgetti**. *Conformal subnets and compact hypergroups.* Preliminary report.

Given a rational chiral conformal field theory A it is well understood how to obtain extensions $B \supseteq A$. Namely, they arise from commutative algebras in $\text{Rep}(A)$. The converse problem of characterizing subtheories $B \subseteq A$ with the same Virasoro symmetry seems harder. One gets examples by taking fixed points A^G under the action of a subgroup G of the automorphism group of A . But it is well-known that many examples are not of this form.

In the framework of local conformal nets, we propose a notion of a “quantum operation” on a conformal net A which generalize the one of an automorphism of A . We show that taking fixed points under a set of quantum operations gives a conformal subnet $B \subseteq A$. Conversely, we show that for any “discrete” subnet $B \subseteq A$ the quantum operations fixing the B form a compact hypergroup whose fixed point is exactly B . (Received September 16, 2019)

1154-46-1701 **J E Pascoe*** (pascoe@uf1.edu). *Advances in realization theory for special classes of noncommutative functions.*

We will describe new methods for obtaining realizations for certain classes of noncommutative functions which encode their desirable properties. For example, matrix monotone functions have Nevanlinna-type representations, matrix convex functions have butterfly type realizations, and plurisubharmonic can be (essentially) uniquely decomposed into alternating convex function and an analytic function.

This talk with survey recent advances on the topic, many of which have been obtained by the author and various subsets of collaborators. (Received September 16, 2019)

1154-46-1706 **Michael Penrod*** (mxp053@shsu.edu). *Variable Lebesgue Spaces: A Comparison.*

The classical Lebesgue space $L^p(\mathbb{R})$ is a well understood Banach space when $1 \leq p \leq \infty$. Many results come from this classical setting in which p is a fixed constant. However, when we replace this constant exponent p with a variable exponent function $p(\cdot)$, many well known definitions and results change, such as the norm, Holder’s Inequality, and the Dominated convergence Theorem. In this presentation, we define the variable Lebesgue space $L^{p(\cdot)}(\mathbb{R})$. We then compare $L^p(\mathbb{R})$ with $L^{p(\cdot)}(\mathbb{R})$ by seeing what happens to these well-know theorems in the variable setting. We finish by considering an equality of conjugate exponent norms in the constant exponent setting, and determine whether this equality holds in the variable exponent setting. (Received September 16, 2019)

1154-46-1712 **Shashank Sule*** (ssule20@amherst.edu), AC 1794, 16 Barrett Hill Dr., Keefe Campus Center, Amherst College, Amherst, MA 01002, and **Sreeram Venkat** (srvenkat@ncsu.edu) and **Tian Lan** (1155091994@link.cuhk.edu.hk). *Sobolev Orthogonal Polynomials on the Sierpinski Gasket*. Preliminary report.

Building on the theory of Legendre orthogonal polynomials on the Sierpinski Gasket (SG), we develop a theory of Sobolev orthogonal polynomials on SG. Initially, we define several notions of a Sobolev inner product on SG using powers of the canonical Laplacian. We use these inner products to find general recurrence relations connecting the Sobolev polynomials to the Legendre polynomials on SG. We then analyze the finer properties of the Sobolev inner product by presenting estimates for the L^2 , L^∞ and H^m norms of the polynomials and studying their convergence properties with respect to the parameters in the H^m inner product. We also highlight the major differences and similarities between the polynomials on SG and those on \mathbb{R} resulting from the properties of the self-similar measure and the Laplacian. Finally, we study the properties of zero sets of polynomials and develop fast computational tools to explore applications to quadrature and interpolation on SG. (Received September 16, 2019)

1154-46-1766 **Michael Anshelevich*** (manshel@math.tamu.edu), Department of Mathematics, Mailstop 3368, Texas A&M University, College Station, TX 77843-3368. *Algebraic aspects of Hermitian Brownian motion*. Preliminary report.

We will describe the combinatorics associated with the Hermitian matrix-valued Brownian motion, represent this process on an appropriate Fock space, and use this representation to define multiple stochastic integrals with respect to it. (Received September 16, 2019)

1154-46-1767 **Robert L Hingtgen*** (rhingtgen@mercyhurst.edu), 501 E 38th St., Erie, PA 16546. *Existence and Uniqueness of Solutions to the Weakly Nonlinear Schrodinger-Newton System*. Preliminary report.

The existence of a family of static spherically symmetric solutions to the Schrodinger-Newton system has been known since the late nineties. Numerical analysis strongly suggests that each solution is unique, corresponding to a number of zeroes on the half-line. The SN system can be viewed as a nonlinear perturbation of the one dimensional Hydrogen atom on the half-line. We show existence and uniqueness of static solutions for small values of the perturbation parameter via an implicit function theorem for Fredholm maps. (Received September 16, 2019)

1154-46-1786 **Lara Ismert*** (ismertl@erau.edu). *Analytic Vectors of a Weakly-Defined Derivation*.

In 2015, E. Christensen provided a much needed formalism to a derivation δ_D on $\mathcal{B}(\mathcal{H})$ implemented by commutation with an unbounded self-adjoint operator D . Specifically, the domain of such a derivation is ambiguous without further conditions on the domains of the resulting commutators $[iD, x]$ where x belongs to $\mathcal{B}(\mathcal{H})$. Restricting the domain of δ_D to the subalgebra of bounded operators x which make $[iD, x]$ bounded on the domain of D , Christensen shows that the domain of δ_D is weak operator topology (WOT) dense in $\mathcal{B}(\mathcal{H})$. In a subsequent paper, Christensen studies higher powers of δ_D and their respective domains, but does not provide a density statement for these (strictly) smaller subalgebras of $\mathcal{B}(\mathcal{H})$. We show that all domains of higher powers of δ_D are WOT-dense in $\mathcal{B}(\mathcal{H})$, and in fact, the analytic vectors for δ_D are WOT-dense in $\mathcal{B}(\mathcal{H})$. (Received September 16, 2019)

1154-46-1846 **Weihua Li*** (wli@colum.edu). *Unitary topological free entropy*. Preliminary report.

Motivated by the book "The semicircle law, free random variables and entropy" written by Fumio Hiai and Dénes Petz, we introduce the unitary topological free entropy, and obtain an exact formula of the unitary topological free entropy for a single unitary. (Received September 16, 2019)

1154-46-1857 **S. Kaliszewski*** (kaliszewski@asu.edu), **Magnus B. Landstad** and **John Quigg**. *Tensor-product coaction functors*.

Recent work by Baum, Guentner, and Willett, and further developed by Buss, Echterhoff, and Willett introduced a crossed-product functor that involves tensoring an action with a fixed action (C, γ) , then forming the image inside the crossed product of the maximal-tensor-product action.

Here we discuss an analogue for a fixed coaction (D, ζ) of a discrete group. Composing our tensor-product coaction functor with the full (action) crossed product of an action reproduces the crossed-product functor of the above-mentioned authors. We show that every such tensor-product coaction functor is exact, and that the tensor-product coaction functor associated to the dual coaction on $\ell^\infty(G) \rtimes G$ is minimal.

Our techniques involve a " G -balanced Fell bundle" $\mathcal{A} \otimes_G \mathcal{D}$, whose cross-sectional C^* -algebra embeds faithfully in the maximal tensor product $A \otimes_{\max} D$. (Received September 16, 2019)

- 1154-46-1893 **Elizabeth Gillaspay*** (elizabeth.gillaspay@mso.umt.edu), **Anna Duwenig**, **Rachael Norton**, **Sarah Reznikoff** and **Sarah Wright**. *Cartan subalgebras in the C^* -algebras of non-principal groupoids*.

When a C^* -algebra has a particularly nice abelian subalgebra known as a Cartan subalgebra, that C^* -algebra necessarily arises from a groupoid. (A groupoid is a generalization of a group where multiplication is only partially defined.) However, sometimes groupoid C^* -algebras admit unexpected Cartan subalgebras. This implies the existence of multiple different groupoid models for the same C^* -algebra. When does this happen, and how are these groupoids related? In this talk, we will consider an example of a 2-step nilpotent group for which these questions have intriguing answers, and (time permitting) discuss how this example fits into a broader context. (Received September 16, 2019)

- 1154-46-1901 **Danny W Crytser*** (dannycrytser@gmail.com), 118 Valentine Hall, Dept Math - SLU, Canton, NY 13617. *Pure extension property for étale groupoid crossed product C^* -algebras*. Preliminary report.

We analyze the pure extension property (PEP) for crossed products of unital C^* -algebra by locally compact Hausdorff étale groupoids. We show that if \mathcal{G} is such a groupoid and A is a \mathcal{G} - C^* -algebra, then the inclusion $A \subset A \rtimes_r \mathcal{G}$ has the PEP exactly when the action of \mathcal{G} on the spectrum \widehat{A} is free. This extends previous work of Zarikian on crossed products by discrete groups. (Received September 16, 2019)

- 1154-46-1917 **J. E. Pascoe** and **Ryan Tully-Doyle*** (rtullydoyle@newhaven.edu). *Automatic real analyticity and a regal proof of a commutative multivariate Löwner theorem*.

We adapt the “royal road” method used to simplify automatic analyticity theorems in noncommutative function theory to several complex variables. We show that certain families of functions must be real analytic if they have certain nice properties on one dimensional slices. Let $E \subset \mathbb{R}^d$ be open. A function $f : E \rightarrow \mathbb{R}$ is *matrix monotone lite* if $f(\varphi_1(t), \dots, \varphi_d(t))$ is a matrix monotone function of t whenever $t \in (0, 1)$, the φ_i are automorphisms of the upper half plane, and the tuple $(\varphi_1(t), \dots, \varphi_d(t))$ maps $(0, 1)$ into E . We use the “royal road” to show that a function is matrix monotone lite if and only if it analytically continues to the multi-variate upper half plane as a map into the upper half plane. Moreover, matrix monotone lite functions in two variables are locally matrix monotone in the sense of Agler-McCarthy-Young. (Received September 16, 2019)

- 1154-46-1957 **Sofya S Masharipova*** (ssm57@pitt.edu), 450 Schoolhouse Road, Johnstown, PA 15904, and **Shukhrat M Usmanov** (susmanov@concord.edu), P.O.Box 1000 F2, Concord University, Athens, WV 24712. *Theorem of isomorphism and theorem of reduction for real non-commutative L_p -spaces*. Preliminary report.

Non-commutative real valued L_p -spaces, $1 \leq p < \infty$, associated with real W^* -algebras of arbitrary types, are constructed. For real algebras of type III we use the scheme of construction of non-commutative L_p -spaces, given by U. Haagerup. Some non-trivial examples of real valued non-commutative L_p -spaces are considered. The theorem of isomorphism for real non-commutative L_p -spaces is proven. Also, the theorem of approximation of real valued L_p -spaces built on real W^* -algebras of type III by real L_p -spaces constructed on finite real W^* -algebras (the reduction theorem) is obtained. (Received September 16, 2019)

- 1154-46-2184 **Almut Burchard*** (almut@math.toronto.edu), Department of Mathematics, 40 St. George Street, Rm. 6290, Toronto, ON M5S 2E4, Canada. *A geometric stability result for Riesz-potentials*. Preliminary report.

Riesz’ rearrangement inequality implies that integral functionals (such as the Coulomb energy of a charge distribution) which are defined by a pair interaction potential (such as the Newton potential) that decreases with distance are maximized (under appropriate constraints) only by densities that are radially decreasing about some point. I will describe recent and ongoing work with Greg Chambers on the stability of this inequality for the special case of the Riesz-potentials in n dimensions (given by the kernels $|x - y|^{-(n-\lambda)}$), for densities that are uniform on a set of given volume. For $1 < \lambda < n$, we bound the square of the symmetric difference of a set from a ball by the energy difference of the corresponding uniform distributions. (Received September 17, 2019)

- 1154-46-2241 **C. Sinan Gunturk*** (gunturk@cims.nyu.edu) and **Nguyen T. Thao**. *Random products of projections in Hilbert space: Regularity, absolute convergence, statistics of displacements*.

Let $\mathbf{V} := (V_1, \dots, V_N)$ be a finite collection of closed linear subspaces of a real Hilbert space H with the associated orthogonal projection operators $P_i : H \rightarrow V_i$, $i = 1, \dots, N$. For any $x_0 \in H$ and any sequence (i_n) taking values in $\{1, \dots, N\}$, consider the forward trajectory $x_{n+1} := P_{i_n}(x_n)$. It is well-known that (x_n) always converges weakly. Norm convergence holds provided \mathbf{V} satisfies a mild angular condition known as “innate regularity,” but otherwise can fail for general \mathbf{V} when $N \geq 3$.

We show that innate regularity implies a stronger sense of norm convergence: For any $\gamma > 0$, there exists $C := C(\mathbf{V}, \gamma) < \infty$ such that

$$\sum_{n=0}^{\infty} \|x_{n+1} - x_n\|^\gamma \leq C \|x_0\|^\gamma$$

uniformly for all x_0 and (i_n) . This result is interesting for $\gamma < 2$. In particular, $\gamma = 1$ yields that the displacement series $\sum (x_{n+1} - x_n)$ converges absolutely in H . Quantifying the constant $C(\mathbf{V}, \gamma)$ as $\gamma \rightarrow 0$, we also derive an effective bound on the distribution function of the displacement norms.

The result extends naturally to relaxed projections and projections on affine subspaces with nonempty intersection. (Received September 17, 2019)

1154-46-2307 **Alexander A. Katz**, Dep. of Math&CS, SJC, St. John's University, 8000 Utopia Parkway, SJH-334-G, Queens, NY 11439, **Roman Kushnir** (kushnir_roman@yahoo.com), Department of Mathematical Sciences, University of South Africa, P.O.Box 392 UNISA, Pretoria, 0003, South Africa, and **Mark Ustayev*** (mark.uuuu@gmail.com), Department of Mathematical Sciences, University of South Africa, P.O.Box 392 UNISA, Pretoria, 0003, South Africa. *On Gelfand-Naimark theorem for real locally W^* -algebras.*

It is shown in the sequel that each projective limit of a projective family of real W^* -algebras is real topologically *-isomorphic to a concrete real locally von Neumann algebra of continuous unbounded operators acting on some real pre-Hilbert space. (Received September 17, 2019)

1154-46-2338 **Oleg Friedman*** (oleg.friedman@touro.edu), Department of Mathematics, Lander College for Men / Touro, 75-31 150th Street, Kew Gardens Hills, NY 11367, and **Alexander A Katz** (katza@stjohns.edu), Dep. of Math&CS, SJC, St. John's University, 8000 Utopia Parkway, SJH-334-G, Queens, NY 11439. *Equivalent definitions of real locally C^* -algebras.*

We establish the following main result in the sequel: For a complete real lmc *-algebra A the following conditions are equivalent:

- 1) A is a strongly regular real lmc *-algebra.
- 2) A is a projective limit of a projective family of real C^* -algebras equipped with projective topology.
- 3) A is topologically real *-isomorphic and homeomorphic to a closed *-subalgebra of real admissible operators $L(\mathbf{H}^R)$, where \mathbf{H}^R is a real locally Hilbert space.
- 4) A is topologically real *-isomorphic and homeomorphic to a closed real *-subalgebra of admissible operators $L(\mathbf{H}^C)$, where \mathbf{H}^C is a locally Hilbert space.
- 5) Let $B = A + iA$ be a complexification of A . There exists a topology τ_B on B , such that
 - a) $\tau_B|_A = \tau_A$ (A naturally embedded in B).
 - b) (B, τ_B) is complex locally C^* -algebra. (Received September 18, 2019)

1154-46-2385 **George A. Elliott*** (elliott@math.toronto.edu). *The classification problem for amenable C^* -algebras.*

Sixty years after Glimm's classification of UHF algebras by supernatural numbers, followed by Bratteli's generalization to arbitrary approximately finite-dimensional C^* -algebras (AF algebras) using Bratteli diagrams, which turned out to be equivalent to certain ordered group information, the classification of all reasonably well behaved unital simple (separable amenable) C^* -algebras, in terms of a simple (K-theoretical) invariant, has now been achieved. The non-unital case seems not far behind. Non-simple C^* -algebras, or less well behaved simple C^* -algebras, still present a considerable challenge. (Received September 17, 2019)

1154-46-2667 **Dawn E. Archey***, 4001 W McNichols Rd, Detroit, MI 48221. *Applications of Large Subalgebras of C^* -Algebras.*

We will introduce the concept of a large subalgebra of a C^* -algebra and briefly describe its historical predecessors. Next we will provide samples of theorems obtained using large subalgebras. We will finish with a few examples to indicate the strength of these theorems. This talk is based on joint work with Julian Buck and N. Christopher Phillips and on joint work with Maria Stella Adamo, Marzieh Forough, Magdalena Georgescu, Ja A Jeong, Karen Strung, and Maria Grazia Viola. (Received September 17, 2019)

1154-46-2678 **Aqeeb A Sabree*** (sabreea@xavier.edu), Department of Mathematics, 3800 Victory Pkwy, Cincinnati, OH 45207. *Positive Definite Kernels, Harmonic Analysis, and Boundary Spaces: Drury-Arveson Theory, and Related.*

A reproducing kernel Hilbert space (RKHS), $\mathcal{H} \subset \mathcal{F}(X, \mathbb{C})$, is a Hilbert function space where the values $f(x)$, for $f \in \mathcal{H}, x \in X$, are reproduced from the inner product $\langle \cdot, \cdot \rangle_{\mathcal{H}}$:

$$f(x) = \langle f(\cdot), k_x(\cdot) \rangle_{\mathcal{H}}, \quad \text{where } k_x \in \mathcal{H}.$$

In this setting, we have an explicit correspondence between reproducing kernel Hilbert spaces and reproducing kernel functions—also called positive definite functions. My research studies the duality between positive definite functions and their boundary spaces for specific RKHSs. We will define these notions and cover an overview of the subject area. Ultimately, this presentation will provide the audience with a look at applications of RKHSs to harmonic analysis, signal/image analysis, mathematical physics, and machine learning algorithms. (Received September 17, 2019)

47 ► Operator theory

1154-47-103 **Stephan Ramon Garcia*** (stephan.garcia@pomona.edu), 610 N College Ave, Claremont, CA 91711. *Nonvanishing Minors and Uncertainty Principles.*

Fourier uncertainty principles play a major role in harmonic analysis, mathematical physics, and number theory. In 2005, Tao used a nonvanishing minors result for the DFT matrix to establish the best possible Fourier uncertainty principle for fields of prime order, a result independently discovered by Biró and Meshulam. We prove an optimal generalization of the BMT principle for functions that enjoy certain symmetries. The discrete cosine matrix and its generalizations play a central role. This is joint work with D. Katz and G. Karaali. (Received August 07, 2019)

1154-47-109 **Ramesh Garimella*** (rameshg@uca.edu), Department of Mathematics, University of Central Arkansas, Conway, AR 72035. *Construction of the exact solution of an operator equation.* Preliminary report.

Let X be an infinite dimensional complex separable Banach space, and $B(X)$ be the Banach algebra of all bounded linear operators on X . Let A be an element of $B(X)$. In this talk, under the assumption that zero is not in the convex hull of the spectrum of A , using a step by step iterative process we naturally construct an integral form of the exact solution of the operator equation $AB + BA = C$ when C is of rank-one. We extend the solution in a natural way to the cases when C is of finite rank and eventually compact operator on X . (Received August 08, 2019)

1154-47-156 **Rene Ardila***, ardilar@gvsu.edu. *Automorphism groups of unit balls of intertwiners.* Preliminary report.

Let E be a W^* -correspondence and let $H^\infty(E)$ be the associated Hardy algebra. The elements of $H^\infty(E)$ can be realized in terms of $B(H)$ -valued functions on the open unit ball $\mathbb{D}((E^\sigma)^*)$. We show a number of results related to the automorphism groups of both $H^\infty(E)$ and $\mathbb{D}((E^\sigma)^*)$. We find a matrix representation for these groups and describe several features of their algebraic structure. (Received August 16, 2019)

1154-47-191 **Cheng Chu*** (chengchu813@gmail.com), Quebec City, QC, Canada. *Reducing Subspaces of de Branges-Rovnyak Spaces.*

The de Branges-Rovnyak spaces are Hilbert spaces contractively contained in the Hardy space of the unit disk that are invariant by the backward shift operator.

We consider the square of backward shift operator when restricted to a de Branges-Rovnyak space, and study its reducing subspaces. In a special case, this operator is a truncated Toeplitz operator acting on a model space, and its reducibility was characterized by Douglas and Foias using model theory. We use a function theoretical approach to extend their result to more general cases. (Received August 21, 2019)

1154-47-207 **B T Kidane*** (berhanu.kidane@ung.edu), 132 Wood Lake Drive, Apt 605, Athens, GA 30606. *A Boundary Norm for Weighted Dirichlet Spaces.*

The estimates for the bounds of the weighted Dirichlet Corona Theorem norm is computed using a norm (a boundary norm) on the corresponding weighted harmonic Dirichlet space, which is defined on the boundary of the unit circle. In this paper, we establish the equivalence between the norms of the weighted Dirichlet space and the corresponding harmonic weighted Dirichlet spaces. (Received August 23, 2019)

1154-47-220

J. William Helton, Igor Klep, Scott McCullough and Jurij Volčič*(volcic@math.tamu.edu). *Noncommutative polynomials describing convex sets.*

The free semialgebraic set \mathcal{D}_f determined by a hermitian noncommutative polynomial $f \in M_d(\mathbb{C}\langle x, x^* \rangle)$ is the closure of the connected component of $\{X : f(X, X^*) > 0\}$ containing the origin. When L is a hermitian monic linear pencil, the free semialgebraic set \mathcal{D}_L is called a free spectrahedron. Since it is the feasible set of a linear matrix inequality (LMI), it is evidently convex. Conversely, it is well-known that every convex free semialgebraic set is a free spectrahedron. This talk presents a solution to the basic problem of determining those f for which \mathcal{D}_f is convex. A consequence is an effective probabilistic algorithm that not only determines if \mathcal{D}_f is convex, but if so, produces its LMI representation. Of independent interest is a subalgorithm based on a Nichtsingulärstellensatz: given a linear pencil L and a free spectrahedron \mathcal{D} , it determines if L takes only invertible values on the interior of \mathcal{D} . Lastly, if \mathcal{D}_f is convex and $f \in \mathbb{C}\langle x, x^* \rangle$ is irreducible, then f is quadratic. (Received August 24, 2019)

1154-47-239

Mark E. Mancuso* (mark.mancuso@wustl.edu). *Γ -convex sets and polynomials.*

Biconvexity and partial convexity are ubiquitous in systems engineering and the area of semidefinite programming. We provide a general framework, known as Γ -convexity, for partial matrix convexity that includes many well-studied examples as special cases. Hahn-Banach separation theorems in this setting are discussed. Γ -convexity for free polynomials is also examined for certain cases of interest. This is joint work with Michael Jury, Igor Klep, Scott McCullough, and J. E. Pascoe. (Received August 26, 2019)

1154-47-276

Constanze Liaw* (liaw@udel.edu). *Singular parts of matrix-valued Aleksandrov–Clark measures.*

To obtain matrix-valued Aleksandrov–Clark (AC) measures, fix a matrix-valued pure contraction b on \mathbb{D} . Here b may be non-inner and/or non-extreme. For each unitary matrix α , the linear fractional transformation $(I + b(z)\alpha^*)(I - b(z)\alpha^*)^{-1}$ has non-negative real part. So, Herglotz’s representation theorem associates a matrix-valued measure μ^α . The collection $\{\mu^\alpha : \alpha \text{ unitary}\}$ forms the family of matrix-valued AC measures, which stand in bijection with matrix-valued pure contractions. A description of the measures’ absolutely continuous parts is easily obtained in terms of non-tangential boundary values of b .

The singular parts μ_s^α are harder. We present a matrix-valued version of Nevanlinna’s result relating non-tangential boundary limits with the measures’ point masses. The connection to Carathéodory angular derivatives is more subtle than in the scalar setting. Aleksandrov spectral averaging yields restrictions on the singular parts. We have a directional version of the mutual singularity of μ_s^α and μ_s^β , $\alpha \neq \beta$ both unitary. This presentation is based on joint work with R.T.W. Martin and S. Treil. (Received August 28, 2019)

1154-47-313

Michael R. Pilla*, mpilla@iu.edu. *Spectra of Composition Operators on the Drury-Arveson Space of Analytic Functions on the Unit Ball in \mathbb{C}^2 .*

A composition operator with symbol ϕ is a linear operator defined by $C_\phi f = f \circ \phi$. Given an analytic map ϕ of the unit ball into itself, we study the solutions to the equation $C_\phi f = \lambda f$ for all complex numbers λ and $f : \mathbb{B}_2 \rightarrow \mathbb{C}$ in the Drury-Arveson space which is defined as the Hilbert function space with kernel $k(z, w) = \frac{1}{1 - \langle z, w \rangle}$.

On the unit disk \mathbb{D} , models of iteration and intertwining maps have given way to great progress in solving the above eigenvalue equation on the Hardy space $H^2(\mathbb{D})$. We apply similar techniques to the Drury-Arveson space in \mathbb{B}_2 and discuss recent progress on the solution when ϕ has a boundary fixed point and has certain intertwining properties. (Received August 30, 2019)

1154-47-335

Bishnu P Sedai* (bishnu.sedai@minotstateu.edu), 525 22ND AVE NW APT E68, Minot, ND 58703. *Trace Formulas for Perturbations of Operators with Hilbert-Schmidt Resolvents.*

Trace formulas for self-adjoint perturbations V of self-adjoint operators H such that V is in Schatten class were obtained in the works of L.S. Koplienko, M.G. Krein, and the joint paper of D. Potapov, A. Skripka and F. Sukochev. In this talk, we discuss an analogous trace formula under the assumptions that the perturbation V is bounded and the resolvent of H belongs to Hilbert-Schmidt class. (Received August 31, 2019)

1154-47-359

Vladimir Bolotnikov* (vladi@math.wm.edu), Williamsburg, VA 23185-8795, and **Joseph A Ball**. *Observability operators and backward-shift invariant subspaces of certain reproducing kernel Hilbert spaces over the unit ball.* Preliminary report.

It is known that subspaces of the Hardy space H^2 of the unit disk occur as the range of an observability operator as the range of an observability operator associated with a discrete-time linear system with stable state-dynamics, as well as the functional-model space for a Hilbert space contraction operator. Parallel results are known in several multivariable settings, in particular, when the space H^2 is replaced by the Drury-Arveson space, the reproducing kernel Hilbert space over the unit ball \mathbb{B}^d in complex d -dimensional space \mathbb{C}^d based on the

d -variable Szegő reproducing kernel $(1 - z_1\bar{\zeta}_1 - \dots - z_d\bar{\zeta}_d)^{-1}$, whereas the backward-shift operator is replaced by the d -tuple $(M_{z_1}^*, \dots, M_{z_d}^*)$ of adjoints of shift operators $M_j : f(z) \rightarrow z_j f(z)$.

In the talk (based on the ongoing joint project with J.A. Ball), we will discuss backward-shift invariant subspaces of reproducing kernel Hilbert spaces with reproducing kernels $(1 - z_1\bar{\zeta}_1 - \dots - z_d\bar{\zeta}_d)^{-n}$ for $n > 1$. (Received September 02, 2019)

1154-47-514 **Dan-Virgil Voiculescu*** (dvv@math.berkeley.edu). *K-theory of the commutant mod trace-class of a hermitean operator.*

The talk will be about how the commutant in the title relates to classical results on trace-class perturbations. Hints at generalization and open problems may be included. (Received September 05, 2019)

1154-47-535 **Robert T.W. Martin*** (robert.martin@umanitoba.ca) and **Eli Shamovich** (shamovic@post.bgu.ac.il). *Blaschke-Singular-Outer factorization of non-commutative functions.* Preliminary report.

Any uniformly bounded analytic function in the open unit disk in the complex plane has a Blaschke-Singular-Outer factorization, where the Blaschke factor contains all information about where (and to what degree) the function vanishes, and the singular and outer factors are pointwise invertible. We investigate analogues of this factorization for uniformly bounded non-commutative holomorphic functions in a non-commutative disk or row-ball in several matrix-variables. (Received September 06, 2019)

1154-47-585 **Konrad Aguilar*** (konrad.aguilar@gmail.com). *Quantum metrics on the tensor product of a commutative C^* -algebra and an AF C^* -algebra.*

Given a compact metric space X and a unital AF algebra A equipped with a faithful tracial state, we place quantum metrics on the tensor product of $C(X)$ and A given established quantum metrics on $C(X)$ and A from work with Bice and Latremoliere. We prove the inductive limit of $C(X)$ tensor A given by A is a metric limit in the Gromov-Hausdorff propinquity. We show that our quantum metric is compatible with the tensor product by producing a Leibniz rule on elementary tensors and showing the diameter of our quantum metric on the tensor product is bounded above the diameter of the Cartesian product of the quantum metric spaces. We provide continuous families of $C(X)$ tensor A which extends our previous results with Latremoliere on UHF algebras. (Received September 07, 2019)

1154-47-588 **Tepper L Gill*** (tgill@howard.edu), 2300 6th st NW, Washington, DC 20059. *New Approach to Operator theory which contradicts the Enflo Conclusion.*

Kuelbs showed that every separable Banach space \mathcal{B} can be continuously and densely embedded into a separable Hilbert space \mathcal{H} . In this talk, we show that \mathcal{B}^* , the dual space of \mathcal{B} , has a new representation, which depends directly on \mathcal{H} . This allows us to provide an explicit representation for every compact operator on \mathcal{B} as a limit of operators of finite rank. This shows that the conclusions of Enflo as well all other negative conclusions related to it are implicitly based on the assumption that the dual space is uniquely defined. (Received September 07, 2019)

1154-47-828 **Gelu F Popescu*** (gelu.popescu@utsa.edu). *Multi-Toeplitz Operators Associated With Noncommutative Domains.*

We present recent results concerning multi-Toeplitz operators associated with noncommutative domains $\mathbf{D}_q^n(H) \subset B(H)^n$, $m, n \geq 1$, where $B(H)$ is the algebra of all bounded linear operators on a Hilbert space H . These operators are acting on the full Fock space with n generators and have as symbols free pluriharmonic functions on the interior of the domain. Several classical results from complex analysis concerning harmonic functions have analogues in our noncommutative setting. In particular, we show that the bounded free pluriharmonic functions are precisely those which are noncommutative Berezin transforms of multi-Toeplitz operators, and solve the Dirichlet extension problem in this setting.

We also present a Brown-Halmos characterization of the multi-Toeplitz operators associated with the non-commutative m -hyperball (the case when $q = Z_1 + \dots + Z_n, m \geq 2$), which is a noncommutative version of Eschmeier and Langendörfer recent commutative result. Our result shows that the multi-Toeplitz operators are characterized by an algebraic equation involving the universal model (W_1, \dots, W_n) of the noncommutative m -hyperball. Many of these results admit extensions to poly-domains. (Received September 11, 2019)

1154-47-885 **Kari Eifler*** (keifler@math.tamu.edu). *Variations of the Graph Isomorphism Game.*

Non-local games give us a way of observing quantum behaviour through the observation of only classical data, and there are several different mathematical models to consider. The graph isomorphism game is an important

example of a synchronous non-local game in quantum information theory, and it has been well studied. I will discuss variations of the Graph Isomorphism Game as well as applications. (Received September 11, 2019)

1154-47-920 **Svetlana Jitomirskaya** and **Wencai Liu*** (liuwencai1226@gmail.com), Blocker 221C, College Station, TX 77843. *Constructions of asymptotically hyperbolic manifolds with singular continuous spectrum embedded into the essential spectrum of the Laplacian.*

In this talk, we will construct asymptotically hyperbolic Riemannian manifolds with singular continuous spectrum embedded in the absolutely continuous spectrum (essential spectrum) of the Laplacian. Moreover, our constructions provide sharp curvature bounds. This is joint work with S. Jitomirskaya. (Received September 11, 2019)

1154-47-950 **Dorin Ervin Dutkay***, ddutkay@gmail.com. *Orthonormal bases generated by Cuntz algebras.*

The Cuntz algebra is generated by N isometries subject to the Cuntz relations

$$S_i^* S_j = \delta_{ij} I, \quad \sum_{i=1}^N S_i S_i^* = I.$$

We will show how Fourier bases, Walsh bases and their generalization, Fourier bases on fractals can be generated by iterating some Cuntz isometries on a finite family of vectors. (Received September 12, 2019)

1154-47-1095 **Paul S. Muhly*** (paul-muhly@uiowa.edu), Department of Mathematics, 2 West Washington St., Iowa City, IA 52242. *W^* -Categories: the Womb of Noncommutative Function Theory.*

Noncommutative function theory is, *inter alia*, a function theory based on intertwining relations among spaces of matrices. So is the more general *matricial function theory* that Baruch Solel and I have been developing. I will describe how W^* -categories help to organize both theories. (Received September 13, 2019)

1154-47-1215 **Mariusz Tobolski*** (mtobolski@impan.pl). *Noncommutative principal bundles and their classification.* Preliminary report.

A classical result from topology states that if X is a principal G -bundle over a paracompact space M , then there exists a map from M to the classifying space BG , and the homotopy class of this map classifies all principal G -bundles isomorphic with X . The aim of this talk is to find an analog of this result in the realm of noncommutative topology, where instead of topological spaces and groups, we consider C^* -algebras and quantum groups respectively. First, we introduce the notion of a locally trivial noncommutative principal bundle in the setting of compact quantum group actions on C^* -algebras. Then, for a compact quantum group \mathbb{G} , we define the noncommutative classifying space $C(B\mathbb{G})$ and prove that it classifies all locally trivial noncommutative principal \mathbb{G} -bundles. (Received September 13, 2019)

1154-47-1229 **Elodie Pozzi***, Saint Louis University, 220 N Grand Blvd, Saint Louis, MO 63122. *A two-dimensional inverse problem in magnetism.*

Inverse problems have known a recent development in many fields like signal processing, medical imaging and more recently paleomagnetism. Broadly speaking, an inverse problem consists in reconstructing from a set of measurements the original source. We consider a two dimensional inverse problem in magnetism to estimate the net moment represented by the mean value of a function supported on an interval K of the real line from the partial knowledge of the magnetism on another interval S located on the parallel line to K at height h . We will see how this question can be rephrased using complex analysis, harmonic analysis and operator theory. To estimate the mean value, we will construct and solve a constrained approximation problem. This talk is based on a joint work with Juliette Leblond, INRIA, France. (Received September 14, 2019)

1154-47-1341 **Samir Raouafi*** (szr0067@auburn.edu), Department of Mathematics and Statistics, Auburn University, 221 Parker Hall, Auburn, AL 36849. *Operators with minimal pseudospectra and connections to normality.*

Normal operators are well understood in operator theory because the spectral theorem holds for them. Several mathematicians have therefore paid attention to investigate some conditions under which certain operators or matrices are normal. It is well-known that normal operators have minimal pseudospectra and the converse is not valid in general. In this talk, we shall discuss some new characterizations of the normality of certain operators in terms of only one pseudospectrum. We will also examine the class of non-normal operators with minimal pseudospectra and derive some applications to the numerical range. (Received September 15, 2019)

1154-47-1367 **Rachael M. Norton*** (rnorton9@fitchburgstate.edu). *A Nevanlinna-Pick theorem in the context of the weighted Hardy algebra over a W^* -correspondence.*

Since the original proof of the Nevanlinna-Pick theorem in 1915, many noncommutative generalizations have arisen and two proof strategies have emerged: via a commutant lifting theorem or via a displacement equation. In this talk, we present a generalized Nevanlinna-Pick theorem in the setting of the weighted Hardy algebra over a W^* -correspondence, which generalizes a theorem proved by the author in 2017. The proof relies on a displacement equation and avoids commutant lifting. Time permitting, we compare our result with a similar theorem proved by Jennifer Good in 2017 using her weighted commutant lifting theorem. (Received September 15, 2019)

1154-47-1385 **Edward J Timko*** (edward.timko@umanitoba.ca), 15 Carlton Street, Apartment 705, Winnipeg, Manitoba R3C1N8, Canada. *Spectral Properties of Model d -Tuples.*

In this talk, we examine the spectral properties of compressions Z of the coordinate multiplication d -tuple to co-invariant subspaces of special reproducing kernel Hilbert spaces. The co-invariant subspaces arise as orthogonal complements of multiplier ideals, and the reproducing kernel Hilbert spaces are all complete NP and possess unitarily-invariant kernels over the unit ball. We show that several different notions of joint spectrum coincide for Z and we connect the spectra with function-theoretic properties of the given ideal. We then use these results to study constrained d -tuples of operators possessing an absolutely continuous functional calculus associated with the model. This is joint work with Raphael Clouatre. (Received September 15, 2019)

1154-47-1413 **Martin Gebert*** (mgebert@math.ucdavis.edu), Department of Mathematics, UC Davis, Davis, CA 95616, and **Constanza Rojas-Molina** (crojasmo@u-cergy.fr), Université de Cergy-Pontoise, France. *Lifshitz tails for the fractional Anderson model.* Preliminary report.

We consider the fractional Anderson model $(-\Delta)^a + V_\omega$ on $\ell^2(\mathbb{Z}^d)$ with $0 < a < 1$. Here $-\Delta$ is the discrete negative Laplacian and V_ω is the standard random potential consisting of iid random variables appearing in the Anderson model. We show that the integrated density of states of this model exhibits Lifshitz tails at the lower edge of the spectrum with Lifshitz exponent $d/(2a)$. We also comment on extensions of the latter to more general operators and open problems. (Received September 15, 2019)

1154-47-1462 **Kit C Chan** and **Serge P. Phanzu*** (sphanzu@bgsu.edu), Bowling Green State University, Department of Mathematics and Statistics, Bowling Green, OH 43403. *Pure Quasinormal Operators have Supercyclic Adjoints.* Preliminary report.

We prove that every pure quasinormal operator has a supercyclic adjoint. It follows that if an operator has a pure quasinormal extension then the operator has a supercyclic adjoint. Our result generalizes a theorem of Wogen who proved that every pure quasinormal operator has a cyclic adjoint. (Received September 15, 2019)

1154-47-1502 **Rodrigo Matos*** (matosrod@msu.edu), 619 Red Cedar Road, C531, East Lansing, MI 48824. *Dynamical Contrast on Highly Correlated Anderson Models.*

We present simple examples where Schrödinger operators on graphs obtained from infinitely many identical copies of the Anderson model have contrasting dynamical behavior, depending on how these models are “decorated”. Spectral contrast between the models will also be presented. Moreover, one of our examples consists of a Schrödinger operator with purely absolutely continuous spectrum where the transient and recurrent components coexist. Based on joint work with Rajinder Mavi and Jeffrey Schenker. (Received September 16, 2019)

1154-47-1506 **Menevse Eryuzlu*** (meryuzlu@asu.edu), **John Quigg** and **Steven Kaliszewski.** *Exact Sequences in the Enchilada Category.*

Imprimitivity theorems provide a fundamental tool for studying the representation theory. It was shown that all of imprimitivity theorems can be viewed as natural isomorphisms between various crossed-product functors among certain equivariant categories. In the proof, we see that there exists a category (the *enchilada category*) in which objects are C^* -algebras, and the morphisms from A to B are the isomorphism classes of $A - B$ correspondences. We study whether exact sequences exist in this category and try to see if the crossed-product functors preserve exact sequences. Our goal is to determine whether we can have a better understanding of the Baum-Connes conjecture by using enchilada categories. (Received September 16, 2019)

1154-47-1620 **Greg Knese*** (geknese@wustl.edu), Washington University in St. Louis, One Brookings Drive, Dept of Math, CB 1146, Saint Louis, MO 63130. *Kummert's approach to realization revisited.*

We give a simplified exposition of Kummert's approach to proving that every matrix-valued rational inner function in two variables has a minimal unitary transfer function realization. A slight modification of the approach extends to rational functions which are isometric on the two-torus and we use this to give a largely elementary new proof of the existence of Agler decompositions for every matrix-valued Schur function in two variables. We use a recent sums of squares result of Dritschel to prove two variable matrix-valued rational Schur functions always have finite-dimensional contractive transfer function realizations. Finally, we prove that two variable matrix-valued polynomial inner functions have transfer function realizations built out of special nilpotent linear combinations. We also will present some open problems about finite dimensional realizations. (Received September 16, 2019)

1154-47-1626 **Joseph A. Ball***, joball@math.vt.edu, **Gregory Marx**, marxg@vt.edu, and **Victor Vinnikov**, vinnikov@cs.bgu.ac.il. *Jordan decomposition for free noncommutative kernels.* Preliminary report.

A *completely positive (cp) kernel* on Ω to $\mathcal{L}(\mathcal{A}, \mathcal{L}(\mathcal{Y}))$ (bounded linear operators from the C^* -algebra \mathcal{A} to bounded linear operators on the Hilbert space \mathcal{Y}) in the sense of Barreto-Bhat-Liebscher-Skeide is a function $k: \Omega \times \Omega \rightarrow \mathcal{L}(\mathcal{A}, \mathcal{L}(\mathcal{Y}))$ which satisfies the cp condition: $\sum_{i,j=1}^N y_i^* k(\omega_i, \omega_j) (a_i^* a_j) y_j \geq 0$ for all ω_i 's in Ω , a_i 's in \mathcal{A} , y_i 's in \mathcal{Y} for $i = 1, \dots, N$, $N = 1, 2, \dots$. A *free noncommutative (nc) cp kernel* is a quantized version of the BBLS cp kernel, whereby one allows the point set to include matrices over the level-1 set of points Ω and demands that the kernel function respect direct sums and similarities via complex matrices for the matrix-point arguments in a natural way. When one drops the cp condition on such a kernel K , one is led to the notion of free nc kernel. We show that any free nc kernel over a finite point set Ω has a Jordan decomposition, i.e., one can write the kernel as a four-fold linear combination of completely positive kernels. (Received September 16, 2019)

1154-47-1696 **William Feldman*** (wfeldman@uark.edu), Dept. of Mathematics, University of Arkansas, Fayetteville, AR 72701. *A vector lattice of nonlinear operators on Banach lattices satisfying a disjointness condition.* Preliminary report.

It is established that a class \mathcal{S} of nonlinear operators between Banach lattices with quasi-interior points (or a dense ideals in these lattices) form a vector lattice. This vector lattice is characterized in terms of properties of the representation of the Banach lattice as continuous extended real-valued functions on a compact space. The characterizations are most insightful in the case of Dedekind complete Banach lattices where the representation is on an extremally disconnected space. The operator T is in the class \mathcal{S} if there exists a natural number n (dependent on T) so that if infimum of the image of any collection of n pairwise disjoint positive elements in E is zero and T satisfies conditions related to orthogonal additivity and monotonicity. Various limits of operators in \mathcal{S} are analyzed. (Received September 16, 2019)

1154-47-1731 **Leonard Huang** and **Lara Ismert*** (ismertl@erau.edu). *The Covariant Stone-von Neumann Theorem for Actions of Abelian Groups on C^* -algebras of Compact Operators.*

In this talk, we formulate and prove a version of the Stone-von Neumann Theorem for every C^* -dynamical system of the form $(G, \mathbb{K}(\mathcal{H}), \alpha)$, where G is a locally compact Hausdorff abelian group and \mathcal{H} is a Hilbert space. The novelty of our work stems from our representation of the Weyl Commutation Relation on Hilbert $\mathbb{K}(\mathcal{H})$ -modules instead of just Hilbert spaces, and our introduction of two additional commutation relations, which are necessary to obtain a uniqueness theorem. (Received September 16, 2019)

1154-47-1847 **Michael Jury**, **Igor Klep**, **Mark E. Mancuso** and **Scott McCullough*** (sam@uf1.edu), Department of Mathematics, University of Florida, Gainesville, FL 32611-8105, and **James E. Pascoe**. *Partial Matrix Convexity.* Preliminary report.

Linear Matrix Inequalities (LMIs), matrix convex sets and matrix convex free polynomials are related by the facts that a free set is matrix convex if and only if it is determined by LMIs; a free polynomial is matrix convex if and only if it has an LMI inspired algebraic certificate; and the positivity set of a matrix concave free polynomial is matrix convex. In this talk we will introduce a framework for generalizing these ideas to various notions of partial convexity, focusing most of our attention on the case of Bimatrix Inequalities (BMIs) and the corresponding notions of xy -matrix convex sets and xy -matrix convex polynomials. Given two sets of freely noncommuting variables $x = (x_1, \dots, x_g)$ and $y = (y_1, \dots, y_h)$, a monic xy -pencil is an expression of the form

$$L(x, y) = I + \sum A_j x_j + \sum B_k y_k + \sum C_{p,q} x_p y_q + \sum C_{p,q}^* y_q x_p.$$

Here the $A_j, B_k, C_{p,q}$ are matrices, A_j and B_k are selfadjoint, and of course I is the identity matrix. The pencil L is naturally evaluated at a tuple $(X, Y) = (X_1, \dots, X_g, Y_1, \dots, Y_h)$ of matrices of the same size using the Kronecker tensor product giving output $L(X, Y)$. The inequality (in the Loewner ordering) $L(X, Y) \succeq 0$ is a Bimatrix Inequality (BMI). (Received September 16, 2019)

1154-47-2000 **Ilya Kachkovskiy*** (ikachkov@msu.edu), Department of Mathematics, Wells Hall, 619 Red Cedar Rd, East Lansing, MI 48824. *Ballistic transport for one-frequency Schrödinger operators*. Preliminary report.

Let $H(x)$ be a family of quasiperiodic operators on $\ell^2(\mathbb{Z})$:

$$(H(x)\psi)(n) = \psi(n+1) + \psi(n-1) + \varepsilon v(x+n\omega)\psi(n).$$

Suppose $v \in C^\omega(\mathbb{T})$ is an analytic 1-periodic function. Then, for $0 < \varepsilon < \varepsilon_0(v)$ and for Diophantine irrational ω , the operators $H(x)$ satisfy strong ballistic transport. In other words, let

$$X\psi(n) = n\psi(n)$$

be the coordinate operator, and consider its quantum evolution:

$$X(T) = e^{iH(x)T} X e^{-iH(x)T}.$$

Then the following strong limit exists (on a dense set):

$$\text{s-}\lim_{T \rightarrow \infty} \frac{1}{T} X(T) = Q,$$

and the operator Q has trivial kernel. Previous results in this area either require choosing a subsequence of time scales, or establish ballistic lower norm bounds without strong convergence. (Received September 17, 2019)

1154-47-2258 **Amudhan Krishnaswamy-Usha*** (amudhan@math.tamu.edu) and **Ken Dykema**. *Spectral operators in finite von Neumann algebras*.

For operators in a finite von Neumann algebra, Haagerup and Schultz showed there exist certain invariant projections, which behave well with respect to the spectral measure. We show a relation between the spectrality of operators (as defined by Dunford), and the angles between the Haagerup-Schultz projections. We then use this to show that a broad class of operators are not spectral, including Voiculescu's circular operator. This is joint work with Ken Dykema. (Received September 17, 2019)

1154-47-2272 **J Schenker** and **F Z Tilocco***, tiloccof@msu.edu, and **S Zhang**. *Diffusion in the mean for a periodic Schrödinger equation perturbed by a fluctuating potential*.

We consider the evolution of a quantum particle hopping on a cubic lattice in any dimension and subject to a potential consisting of a periodic part and a random part that fluctuates stochastically in time. If the random potential evolves according to a stationary Markov process, we obtain diffusive scaling for moments of the position displacement, with a diffusion constant that grows as the inverse square of the disorder strength at weak coupling. More generally, we show that a central limit theorem holds such that the square amplitude of the wave packet converges, after diffusive rescaling, to a solution of a heat equation.

Additionally, we will consider how the addition of a random, stochastically evolving, potential affects the transport properties of the one dimensional Dimer model. (Joint work with J. Schenker and S. Zhang) (Received September 17, 2019)

1154-47-2308 **Méric Augat*** (maugat@wustl.edu). *Free Analysis and Free Rational Automorphisms*. Preliminary report.

Recent advances in *Free Analysis* have shown that an endomorphism φ of the free algebra $\mathbb{C}\langle x_1, \dots, x_g \rangle$ is an automorphism if and only if the induced polynomial mapping $p = (\varphi(x_1), \dots, \varphi(x_g))$ is injective on the set of all g -tuples of $n \times n$ matrices, taken over all n .

It is natural to try and find a similar condition for when an endomorphism of the free skew field (free division algebra) is an automorphism. In this talk we discuss the conjectured injectivity condition and the difficulties that arise in trying to prove it.

This talk is related to free analysis, realizations of nc rational functions, and invertibility in the tensor product of skew fields. (Received September 18, 2019)

1154-47-2421 **Chian Yeong Chuah*** (chian_chuah@baylor.edu). *A characterization for l_2 -radial completely positive multipliers on free groups*. Preliminary report.

In this talk, we will give a brief account about the relationship between radial positive-definite functions on free groups and the moments of probability measures on the interval $[-1, 1]$. The case for the commutative setting is proven by Bochner. Meanwhile, Haagerup and Knudby proved the case for l_1 radial positive definite function.

We explore the case for l_2 radial positive definite functions (completely positive Fourier multipliers) on free groups. (Received September 17, 2019)

1154-47-2589 **Benjamin E Lifthartke*** (benlis213@ksu.edu). *Special constructions of k -graphs and their properties.*

The graph C^* -algebras form an interesting class of example C^* -algebras to study. Each such algebra is constructed from a directed graph, and the properties of the algebra can be observed through paths formed on the graph. One can also construct a higher-rank analog of directed graphs and build C^* -algebras from these. In a similar fashion, the properties of the C^* -algebra constructed from a higher-rank graph are observed through path equivalences in the graph itself. In this talk, I will explore a few interesting examples of these higher-rank graphs and discuss some of the aforementioned properties. (Received September 17, 2019)

1154-47-2620 **Stephanie Prah1*** (stephanie.prah1@huskers.unl.edu). *Determining Hyperrigidity of Operator Systems.*

An operator system \mathcal{S} that generates a C^* -algebra \mathcal{A} is said to be *hyperrigid* in \mathcal{A} if for every non-degenerate representation π of \mathcal{A} on a Hilbert space, the only unital completely positive map extending $\pi|_{\mathcal{S}}$ to \mathcal{A} is π itself. This notion has strong connections to the non-commutative Choquet boundary of an operator system and its C^* -envelope. In fact, if an operator system \mathcal{S} is hyperrigid in \mathcal{A} , then \mathcal{A} must be the C^* -envelope of \mathcal{S} . This implies that hyperrigidity can be thought of as a property intrinsic to the operator system rather than relative to a C^* -algebra. We discuss some recent results in determining whether an operator system \mathcal{S} is hyperrigid in its C^* -envelope, even if \mathcal{S} is viewed in a C^* -algebra that is not the C^* -envelope. (Received September 17, 2019)

1154-47-2642 **Lijing Sun*** (sun2@uwm.edu), Department of Mathematical Sciences, University of Wisconsin-Milwaukee, Milwaukee, WI 53201, and **Xiaoying Lin**. *Some estimates on the Hausdorff operator.*

Recently, sufficient conditions for the H^p boundedness of the one-dimensional Hausdorff operator were given by Lifyand and Miyachi. In this paper, we obtain new sufficient conditions for the H^p boundedness of the one-dimensional Hausdorff operator. The results of Lifyand and Miyachi and the results of this paper are mutually independent. More importantly, our method in the proof allows us to study the high dimensional Hausdorff operator and fractional Hausdorff operator. We then obtain $H^p(R^n) \rightarrow L^q(R^n)$ and $L^p(|x|^\gamma dx) \rightarrow L^q(|x|^\gamma dx)$ boundedness for the high dimensional (fractional) Hausdorff operator. (Received September 17, 2019)

1154-47-2700 **Mitch Hamidi*** (hamidim@erau.edu). *Rigidity of Crossed Products Arising from \mathbb{Z} Acting on the Disc Algebra.*

We show that conjugacy of operator algebraic dynamical systems yields completely isometrically isomorphic crossed products, and we give a geometric characterization of conjugacy for two dynamical systems arising from the integers acting on the disc algebra. This is progress towards answering an open question of E. Katsoulis and C. Ramsey from 2015. (Received September 17, 2019)

1154-47-2772 **Victor Vinnikov*** (vinnikov@math.bgu.ac.il). *Rational nc power series around a matrix centre and the free skew field.*

Noncommutative (nc) formal power series can be identified with germs of nc functions at zero. They are obviously the completion of the ring of nc polynomials. Rational nc formal power series can be characterized by a finiteness condition (the associated infinite Hankel matrix has finite rank) which is closely related to minimal realizations.

A nc rational expression that is defined at a matrix point can be expanded in generalized power series around this point, namely the Taylor–Taylor series of nc function theory. In this talk I will survey the developments of the last few years that generalize what happens around the centre zero (which by translation is of course the same as any scalar point) to the case of a general matrix centre. The main feature is that the coefficients of the power series are no longer arbitrary: they have to satisfy certain nontrivial relations depending on the centre. There is again a finiteness condition for nc rational power series that is closely related to minimal realizations. This leads in particular to an immediate construction of the skew field of nc rational functions out of the “local” rings of rational nc powers series around matrix centres. (Received September 17, 2019)

1154-47-2825 **Pamela I Delgado*** (pid2@pitt.edu), 166 Thackeray Ave, Pittsburgh, PA 15213, **Christopher J Lennard** (lennard@pitt.edu), 166 Thackeray Ave, Pittsburgh, PA 15213, and **Jeromy Sivek** (sivek@temple.edu), 1805 N Broad St, Philadelphia, PA 19122. *Generalization of iterates of Cesàro Averaging for functions to include non negative real powers, and Banach limits invariant under these new operators.* Preliminary report.

For each $\alpha > 0$ we define a new operator J^α on L^∞ , such that for every $n \in \mathbb{N}$ we have that $(J^n)(f)$ coincides with taking the n -th iteration of the Cesàro Averaging operator to the function f . We also construct Banach limits that are invariant under these operators. (Received September 18, 2019)

1154-47-2831 **Safeer Hussain Khan*** (safeerhussain5@yahoo.com), Department of Math, Stat and Phy, Qatar University, Doha, 2713, Qatar. *Further generalized hybrid mappings and attractive points.*

Our purpose is to (i) introduce further generalized hybrid (FGH) mappings (ii) introduce common attractive points (CAP) (iii) write and use Picard-Mann hybrid iterative process for two mappings. We establish the existence and approximation of common attractive points of FGH mappings by using an iterative process due to the author generalized to the case of two mappings in Hilbert spaces without closedness condition. Our class of FGH mappings not only constitutes a simple generalization of normally generalized hybrid mappings but also contains the class of quasi-nonexpansive mappings when it has a fixed point contrary to "widely more generalized hybrid" mappings. It also contains the class of quasi-contractive mappings and in turn, contractive mappings, Kannan mappings, Chatterjea mappings, Zamfirescu mappings. We use a generalization of Picard-Mann hybrid iterative process (introduced earlier by author) for two mappings for approximating CAP.. This process reduces to Mann and at the same time deals with common attractive points. In view of the above discussion we can conclude that the common attractive points obtained by our new mapping using a faster iterative process will not only be more general but also simpler. (Received September 18, 2019)

49 ► *Calculus of variations and optimal control; optimization*

1154-49-240 **Jinlu Li*** (jli@shawnee.edu), Department of Mathematics, Shawnee State University, Portsmouth, OH 45662. *Set Optimization Problems on Ordered Sets and Applications.* Preliminary report.

The results in this talk mainly come from two joint papers by the speaker and Professor Christiane Tammer. In this talk, we define and construct some ordering relations on the power sets of ordered sets, which provides rules for ordering subsets. On the power sets of preordered sets, we introduce three preordering relations based on the given preorder relations, which are called the power preorder, upward power preorder and downward power preorder, respectively. (a) Several properties of these three preordering relations are provided; (b) We consider the order-clusters with respect to these preorders on the power sets, introduce the concepts of order-clustered fixed point and prove some order-clustered fixed point theorems; (c) We consider optimization problems of set-valued mappings on ordered sets; (d) Under these preordering relations, by applying the order-clustered fixed point theorems, we prove some existence theorems of generalized Nash equilibriums for set-valued mappings; (e) We prove the solvability of generalized variational inequality problems with set-valued mappings; (f) Efficiencies and Pareto efficiencies of set-valued mappings are defined and their solvability is investigated. (Received August 26, 2019)

1154-49-339 **Tan Cao, Giovanni Colombo, Boris Mordukhovich and Dao Nguyen*** (dao.nguyen2@wayne.edu). *Discrete Approximation of a Controlled Sweeping Process over polyhedral sets with perturbations.*

The result addresses a class of optimal control problems governed by a perturbed sweeping (Moreau) process with the moving convex polyhedral set, where the controls are used to determine the best shape of the moving set and select the best perturbation in order to optimize the given Bolza-type problem which depends on controls and state variables as well as their velocities. Using the method of discrete approximations, we approximate the optimal control problem under consideration by a sequence of well-posed finite-dimensional problems whose optimal solutions strongly converge to that of the original controlled sweeping problem under less restrictive assumptions on the controls and state variables. This approach together with the advanced tools of variational analysis and generalized differentiation allows us to efficiently derive the necessary optimality conditions for the discretized control problems. The obtained results can be considered as necessary sub-optimality conditions and

hence propose a numerical scheme for searching optimal solutions to the original controlled perturbed sweeping process. One numerical example is presented to illustrate such established necessary sub-optimality conditions. (Received September 01, 2019)

1154-49-476 **Yoshihiro Tonegawa*** (tonegawa@math.titec.ac.jp), 2-12-1 Ookayama, Meguro-Ku, Tokyo, 152-8551, Japan. *An existence theorem for Brakke flow with fixed boundary condition.*

Suppose that we are given an arbitrary closed countably n -rectifiable set in a strictly convex $n + 1$ dimensional domain, with the assumption that the set has finite n -dimensional Hausdorff measure and the complement is not connected. Then, we show that there exists a non-trivial Brakke flow starting from the given set and fixing the boundary data for all time. The result gives a non-trivial solution of the Plateau-type problem in a varifold setting, which should allow soap film like singularities. (Joint work with Salvatore Stuvard of U. Texas Austin.) (Received September 05, 2019)

1154-49-960 **Ebrahim Sarabi***, 301 S. Patterson Ave, Oxford, OH 45056. *Stability properties of Lagrange multipliers in constrained optimization problems.*

This talk focuses mainly on providing a second-order variational analysis of critical and noncritical Lagrange multipliers of optimization problems. Then we discuss its application to the sequential quadratic programming algorithm for constrained optimization problems.

The talk is based on a joint work with Boris Mordukhovich. (Received September 12, 2019)

1154-49-1206 **Salvatore Stuvard*** (stuvard@math.utexas.edu). *A capillarity theory approach to the analysis of soap films.*

Minimal surfaces, namely surfaces with everywhere vanishing mean curvature, are the classical model for soap films spanning a given boundary wire. A major drawback with this model is the complete lack of a length scale capturing the mechanical properties of soap films due to surface tension. In this talk, I will discuss how this issue can be overcome by modeling soap films as “thick” sets of finite perimeter enclosing a small given volume and satisfying a suitable spanning condition. I will investigate the existence theory for the corresponding variational problem, as well as the geometry of the minimizers. Then, I will show how this approach allows to recover the classical Plateau’s problem in the limit as the enclosed volume approaches zero. Open problems and future directions of research will conclude the presentation.

This is based on joint works obtained in collaboration with Darren King (UT Austin), Francesco Maggi (UT Austin), and Antonello Scardicchio (ICTP Trieste). (Received September 13, 2019)

1154-49-1331 **Ellina Grigorieva*** (krasavizha@yahoo.com), PO BOX 425464, Denton, TX 76204, and **Evgenii Khailov**. *Program- positional strategies for a Lotka -Volterra type cancer control model.*

In this work, we use the Lotka – Volterra competition model to describe the interaction of cancer and healthy cells. For the controllable system under consideration, the goal is to minimize the terminal functional, which is a weighted difference in the concentrations of cancer and healthy cells at the final time point of the treatment period. Three types of cancer treatment are considered. Using the Pontryagin maximum principle for each type of treatment, the properties of optimal controls are analytically established and confirmed by the corresponding numerical calculations. Also, this research presents the results of analytical and numerical studies of the terminal control problem for the model with two variants of control restrictions (Received September 14, 2019)

1154-49-1349 **Christian Gerhards*** (christian.gerhards@geophysik.tu-freiberg.de). *Inverse Magnetization Problems and Potential Field Separation.*

The Hardy-Hodge decomposition characterizes the contributions of a magnetization that can be determined uniquely from knowledge of the corresponding magnetic field. Based on this, it is possible to derive a geophysically reasonable setup that allows the unique separation of magnetic fields generated from sources on two separate spheres. If the source on the outer sphere is a compactly supported magnetization, then the two magnetic fields can be distinguished even if only the overall magnetic field is known on an external surface. We provide a rigorous derivation of that result as well as an iterative approximation scheme for the underlying minimization problem that allows the computation of Fourier coefficients of each of the two magnetic fields. (Received September 15, 2019)

1154-49-1467 **Sergiy Koshkin, Cameron Spiess and Zachary Zalles*** (z1z3@rice.edu), 3401 Southhill Circle, Austin, TX 78703. *Optimal allocation in annual plants with non-linear utility functions.*

Optimal allocation strategies for annual plants in unpredictable environments depend on a choice of utility function, which represents risk/reward preferences. We study two-compartment (vegetative and reproductive) allocation models with non-linear utility, and show that optimal strategies for linear utilities (the most commonly used) are dramatically different from those for concave ones, which are associated with risk aversion. In particular, while the former are typically bang-bang, the latter include a period of graded allocation (mixed growth). Concave utilities reflect bet-hedging behavior, and are more realistic than linear ones used for their technical convenience. Although the most complete results are obtained for the power utility functions, our results are robust over a range of modeling choices, and raise questions about applicability of linear utility models. (Received September 15, 2019)

1154-49-1861 **Mau Nam Nguyen*** (mnn3@pdx.edu), 724 SW Harrison Street, Portland, OR 97201. *Relative Interiors for Graphs of Convex Set-Valued Mappings and Applications to Generalized Differential Calculus.*

In this talk, we present a simple proof of a theorem by Rockafellar for representing relative interiors of graphs of convex set-valued mappings in terms of relative interiors of their domains and function values. Based on this theorem, we develop a geometric approach to convex generalized differential calculus in finite dimensions. This approach allows us to obtain natural and rather simple proofs of basic results of convex subdifferential calculus and also derive new results of convex analysis concerning optimal value functions, normals to inverse images of sets under set-valued mappings, and calculus rules for coderivatives of set-valued mappings. (Received September 16, 2019)

1154-49-1902 **Thomas J. Clark*** (tom.clark@dordt.edu) and **Anil B. Venkatesh** (avenkatesh@adelphi.edu). *Optimizing Flappy Bird Flight Paths.* Preliminary report.

We investigate several optimal path problems in various norms. Each problem corresponds to a formulation of the “Flappy Bird” video game where a player navigates a bird through a maze of pipes. Mathematically we aim to minimize the total acceleration required to navigate through a set of points for several interpretations of net acceleration. We present exact solutions to these various minimization problems as well as a numerical implementation that computes solutions via a convex optimization scheme. (Received September 16, 2019)

1154-49-1952 **Youssef Qranfal***, 550 Huntington Ave, Boston, MA 02115. *An Algorithm to Segment Noisy Medical Images.*

An implementation of a Markov random fields image segmentation algorithm works well for the purpose of denoising and segmenting medical images. One of the main contributions here is the ability for a user to manipulate online the image so as to achieve clear delineation of objects of interest in the image. Results are presented for images that are generated by SPECT and MRI. The method presented is effective at denoising as well as segmenting tissue types, organs, lesions, and other features of medical images. (Received September 16, 2019)

1154-49-1969 **Rene Vidal*** (rvidal@jhu.edu), 302B Clark Hall, 3400 N Charles St, Baltimore, MD 21218. *Mathematics of Deep Learning.*

The past few years have seen a dramatic increase in the performance of recognition systems thanks to the introduction of deep networks for representation learning. However, the mathematical reasons for this success remain elusive. For example, a key issue is that the neural network training problem is non-convex, hence optimization algorithms may not return a global minima. In addition, the regularization properties of algorithms such as dropout remain poorly understood. The first part of this talk will overview recent work on the theory of deep learning that aims to understand how to design the network architecture, how to regularize the network weights, and how to guarantee global optimality. The second part of this talk will present sufficient conditions to guarantee that local minima are globally optimal and that a local descent strategy can reach a global minima from any initialization. Such conditions apply to problems in matrix factorization, tensor factorization, and deep learning. The third part of this talk will present an analysis of the optimization and regularization properties of dropout for matrix factorization. Examples from neuroscience and computer vision will also be presented. (Received September 16, 2019)

1154-49-2006 **Haoyang Cindy Cao**, Berkeley, CA, **Xin Guo*** (xinguo@berkeley.edu), Berkeley, CA 94720, and **Mathieu Lauriere**, Princeton, NJ. *MFGs and GANs*. Preliminary report.

In this talk, we will discuss some interesting connections between GANs and MFGs, and propose several new computational approach for MFGs via GANs. (Received September 17, 2019)

1154-49-2230 **Michael R Kelly*** (mikelly@transy.edu), 300 North Broadway, Lexington, KY 40508, and **Suzanne Lenhart** and **Michael G Neubert**. *Marine reserves and optimal dynamic harvesting when fishing damages habitat*.

Marine fisheries are a significant source of protein for many human populations. In some locations, however, destructive fishing practices have negatively impacted the quality of fish habitat and reduced the habitat's ability to sustain fish stocks. Improving the management of stocks that can be potentially damaged by harvesting requires improved understanding of the spatiotemporal dynamics of the stocks, their habitats, and the behavior of the harvesters. We develop a mathematical model for both a fish stock as well as its habitat quality. Both are modeled using nonlinear, parabolic partial differential equations, and density-dependence in the growth rate of the fish stock depends upon habitat quality. The objective is to find the dynamic distribution of harvest effort that maximizes the discounted net present value of the coupled fishery-habitat system. The value derives both from extraction (and sale) of the stock and the provisioning of ecosystem services by the habitat. Optimal harvesting strategies are found numerically. (Received September 17, 2019)

1154-49-2437 **Cornelia Mihaila*** (cmihaila@math.uchicago.edu). *A problem in shape optimization*. Preliminary report.

I will discuss recent work on a problem in shape optimization. (Received September 17, 2019)

1154-49-2535 **Brian Krummel*** (bkrummel@purdue.edu), Department of Mathematics, 150 N. University Street, West Lafayette, IN 47906. *Relative isoperimetric inequality outside a convex set*.

We consider a higher codimension isoperimetric inequality for an $(m+1)$ -dimensional area minimizing submanifold R which lies outside a convex subset K of \mathbb{R}^{n+1} and is bounded by an m -dimensional submanifold T and a free boundary along the convex set K . We show that the isoperimetric ratio is greater than or equal to that of an $(m+1)$ -dimensional flat half-disk. A key aspect of our approach is obtaining estimates showing that when the isoperimetric ratio of R is close the least value of the isoperimetric ratio, then the mass of T and R cannot concentrate along K . This extends prior work of Choe, Ghomi, and Ritore in the case $m = n$ and proves a conjecture of Choe in the case of relative area minimizers. (Received September 17, 2019)

1154-49-2668 **Weldon A. Lodwick*** (weldon.lodwick@ucdenver.edu), Mathematical and Statistical Sciences, University of Colorado Denver, Denver, CO 80204. *Interval Valued Probability Measures as a Unifying Theory for Optimization Under Generalized Uncertainty*.

This talk uses a new formal definition of interval valued probability measure (IVPM) based on a measure theoretic foundation and shows that generalized uncertainty data occurring in optimization models, in particular, possibilistic optimization, can be formulated in the IVPM framework. We will first develop interval-valued probability measure theory and then look at how it is used in optimization. Interval-valued probability measure integration theory will be presented if there is time.

This is joint work with Dr. K. David. (Received September 17, 2019)

1154-49-2699 **M Tyler Moncur*** (moncur@math.byu.edu), **Michael Nelson** and **Tyler Jarvis**. *Hyperparameters for Convolutional Neural Networks*.

Neural networks are extremely versatile, in effect being universal function approximators, but it can be extremely challenging to find the right set of parameters and hyperparameters. Model training is both expensive and difficult due to the large number of parameters and sensitivity to hyperparameters such as learning rate and architecture. Hyperparameter searches are notorious for requiring tremendous amounts of processing power and human resources. Hettinger recently showed how to compute optimal initializations and learning rates for fully connected neural networks that use the Crelu activation function, which can reduce the cost of model training and hyperparameter searches significantly. We generalize Hettinger's work from fully connected neural networks to convolutional neural networks. Our work also suggests a new method of handling boundary values resulting from zero-padded convolutional layers. (Received September 17, 2019)

51 ► Geometry

1154-51-22 **Joonsoo Lee*** (joonyboy123@gmail.com), 521 Floyd St, Englewood Cliffs, NJ 07632, and **Jiho Justin Lee** (jhlee00502@gmail.com), Canterbury School, 101 Aspetuck Ave, New Milford, CT 06776. *Grid dissections of tangential quadrilaterals.*

For any integer $n \geq 2$, a square can be partitioned into n^2 smaller squares via a *checkerboard-type* dissection. Does there such a *class preserving grid dissection* exist for some other types of quadrilaterals? For instance, is it true that a tangential quadrilateral can be partitioned into n^2 smaller tangential quadrilaterals using an $n \times n$ grid dissection? We prove that the answer is affirmative for every integer $n \geq 2$. (Received June 08, 2019)

1154-51-149 **Alice Kwon*** (akwon@gradcenter.cuny.edu). *Fully Augmented Links in the Thickened Torus.*

A class of links called fully augmented links in the three-sphere have been studied by Adams, Purcell, and Chesebro-Deblois-Wilton. Alternating links in the thickened torus were studied by Champanerkar-Kofman-Purcell and have been of recent interest. I will be talking about fully augmented links in the thickened torus and discuss the geometric properties of their complements in the thickened torus. I will also be showing volume density convergence of fully augmented links. (Received August 15, 2019)

1154-51-160 **Kyle Andrlle, Anh Tran and J Mealy*** (jmealy@austincollege.edu). *Space-time staircase metric tori.*

After a brief introduction to the category, staircase metric (SCM) geometry, its accompanying methodology, and its extension to the space-time category, we utilize said framework to construct and investigate a variety of compact SCM space-time manifolds – including specifically such surfaces of higher genus. In particular, we consider various new signature (1,1) SCM tori, and then study properties of the time-like geodesics. Specific families of these tori are defined and analyzed; some feature more exotic closed time-like geodesics. Others utilize a more complex boundary quotient scheme than in previous work, and feature large collections of long, yet incomplete time-like geodesics. (Received August 16, 2019)

1154-51-166 **Xianzhe Dai and Junrong Yan*** (j_yan@math.ucsb.edu), Department of Mathematics, SH6607, UC, Santa Barbara, Santa Barbara, CA 93106. *Witten deformation on noncompact manifolds.*

In an extremely influential paper, Witten introduced a deformation of the de Rham complex by considering the new differential $d + df$, where f is a Morse function. It turns out that Witten deformation has great applications including Bismut-Zhang/Cheeger-Muller theorem as well as being instrumental in the developing of Floer homology theory. Motivated by Landau-Ginzberg model, we study the case of noncompact manifolds. We are able to prove that the cohomology of the Witten deformation acting on the complex of smooth L^2 forms is isomorphic to cohomology of Thom-Smale complex and relative cohomology of the pair (M, U) for some open set U in noncompact case. In the end, we will apply our main results to the case of compact manifolds with boundaries. This is joint work with Xianzhe Dai. (Received September 08, 2019)

1154-51-316 **Jerzy Dydak*** (jdydak@utk.edu), Department of Mathematics, University of Tennessee, Ayres Hall 227, Knoxville, TN 37966. *Linear algebra and unification of geometries in all scales: Evolution of concepts.* Preliminary report.

We present the evolution of concepts leading to an idea of unifying small scale (topology, proximity spaces, uniform spaces) and large scale (coarse spaces, large scale spaces). It relies on an analog of multilinear forms from Linear Algebra. As an application we get simple proofs of results generalizing well-known theorems from coarse topology. A new result (at least to the author) is the following

A coarse bornologous function $f : X \rightarrow Y$ of metrizable large scale spaces is a large scale equivalence if and only if it induces a homeomorphism of Higson coronas. (Received August 30, 2019)

1154-51-596 **Michel L Lapidus*** (lapidus@math.ucr.edu), University of California, Riverside, Department of Mathematics, Skye Hall, 900 University Ave., Riverside, CA 92521-0135. *Fractal Complex Dimensions and Zeta Functions, With Applications to Fractal Geometry.*

We give an overview of some of the results obtained by the author and his collaborators in developing the theory of fractal complex dimensions (see, e.g., [1]-[3]), which generalize to the complex domain the notion of Minkowski (or box) dimension. Complex dimensions are defined as the singularities (e.g., poles) of analytic continuations of fractal zeta functions associated with arbitrary bounded subsets of Euclidean spaces. Examples of applications include various characterizations of Minkowski measurability, along with fractal tube formulas (which enable us to detect and precisely express the oscillations that are intrinsic to fractal geometries). If time permits, we also

plan to discuss several open problems in this area. References: [1] M. L. Lapidus and M. van Frankenhuysen, *Fractal Geometry, Complex Dimensions and Zeta Functions*, 2nd edn., Springer, 2013. [2] M. L. Lapidus, G. Radunovic and D. Zubrinic, *Fractal Zeta Functions and Fractal Drums: Higher-Dimensional Theory of Complex Dimensions*, Springer, 2017. [3] M. L. Lapidus, *An Overview of Complex Dimensions: From Fractal Strings to Fractal Drums*, and *Back*, *Contemp. Math.* (AMS), 731 (2019), 143-265. (Received September 08, 2019)

1154-51-633 **Adam-Christiaan van Roosmalen, Réamonn Ó Buachalla and Zhaoting Wei*** (zwei3@kent.edu). *Idempotent elements and holomorphic structures on quantum complex projective spaces*. Preliminary report.

The quantum complex projective space $\mathbb{C}P_q^n$ is defined as the quotient of quantum groups $SU_q(n+1)/U_q(n)$. It is already known that $\mathbb{C}P_q^n$ has a noncommutative holomorphic structure. In this talk we will give explicit constructions of non-trivial idempotent elements on $\mathbb{C}P_q^n$. Inspired by a work of Polishchuk in 2006 we use these idempotent elements to construct non-standard holomorphic structures on $\mathbb{C}P_q^n$. (Received September 09, 2019)

1154-51-707 **Dahye Cho*** (dahye.cho@gmail.com), Mathematics Department, Stony Brook University, Stony Brook, NY 11794-3651. *Symplectic Cohomology of Affine Varieties*. Preliminary report.

Symplectic cohomology is a Hamiltonian Floer theory for some open symplectic manifolds including affine varieties. I will describe a symplectic criteria on stratified uniruledness of affine varieties and its application to birational geometry and the minimal model program. (Received September 11, 2019)

1154-51-991 **Jiawei Zhou*** (jiaweiz5@uci.edu) and **Li-Sheng Tseng** (lstseng@math.uci.edu). *A twist on TTY-algebra of symplectic manifolds*.

On symplectic manifolds, there is a novel algebra of special differential forms, constructed by Tsai, Tseng and Yau. For simplicity we just call it TTY-algebra. We will review this algebra and introduce a twist on it, motivated by consideration of fiber bundles over symplectic manifolds and sections on them. This twist leads to a natural flatness condition for such bundles which we will discuss, and we can generalize this flatness condition for a twist on A-infinity algebras. (Received September 12, 2019)

1154-51-1090 **Peter Koroteev*** (pkoroteev@math.berkeley.edu), University of California, Department of Mathematics, Berkeley, CA 94720. *Branes and DAHA Representations*.

It was shown by Oblomkov that spherical double affine Hecke algebra (DAHA) arises via geometric quantization of the Calogero-Moser space \mathcal{M} . I shall describe representation theory of $\mathfrak{sl}(2)$ DAHA in terms of geometry of \mathcal{M} . We conjecture an equivalence between the representation category of $\mathfrak{sl}(2)$ DAHA and a certain extension of the Fukaya category of \mathcal{M} . In particular, under this relation, finite dimensional representations of DAHA correspond to compact Lagrangian cycles (branes). (Received September 13, 2019)

1154-51-1267 **Bahar Acu*** (baharacu@northwestern.edu), Evanston, IL 60208. *Planarity in higher-dimensional contact manifolds*.

Planar contact manifolds, those that correspond to an open book decomposition with genus zero pages, have been intensively studied to understand several aspects of 3-dimensional contact topology. In this talk, we present a higher-dimensional notion of planarity, iterated planarity, and provide several generalizations of results for planar contact 3-manifolds, to higher dimensions. This is joint work with A. Moreno. (Received September 14, 2019)

1154-51-1296 **Abigail R. Ward*** (arward@stanford.edu). *Homological mirror symmetry for the Hopf surface*.

The Hopf surface is a complex surface which is non-Kähler, placing it outside of the realm of traditional homological mirror symmetry; however one can still define a mirror space to the Hopf surface by defining a “non-algebraic Landau Ginzburg model.” We will give a sketch of this construction of the mirror space, as well as show some evidence of the HMS correspondence. (Received September 15, 2019)

1154-51-1308 **Zahra Sinaei***, sinaei@math.umass.edu. *Convex functional and the stratification of the singular set of their stationary points*.

In this talk, I discuss partial regularity of stationary solutions and minimizers u from a set $\Omega \subset \mathbb{R}^n$ to a Riemannian manifold N , for the functional $\int_{\Omega} F(x, u, |\nabla u|^2) dx$. The integrand F is convex and satisfies some ellipticity, boundedness and integrability assumptions. Using the idea of quantitative stratification I show that the k -th strata of the singular set of such solutions are k -rectifiable. (Received September 14, 2019)

1154-51-1791 **Bahar Acu, Orsola Capovilla-Searle*** (oac5@duke.edu), **Agnes Gaddled, Aleksandra Marinkovic, Emmy Murphy, Laura Starkston and Angela Wu.** *Complements of toric divisors in toric 4 manifolds.*

We consider toric 4 manifolds with certain toric divisors that have normal crossing singularities. The normal crossing singularities can be smoothed, changing the topology of the complement. We develop a method to draw the Stein handlebody decomposition for the complement of such toric divisors. (Received September 16, 2019)

1154-51-1940 **Maxwell Auerbach, Adam Hodapp and Rebecca Whitman*** (rwhitman@wellesley.edu). *Shortest Paths in Generalizations of the Sierpinski Carpet.* Preliminary report.

In this talk we address several questions about shortest paths between two points in a 3-parameter family of fractals that naturally generalize the Sierpinski carpet. In particular, we find shortest geodesic paths between opposite corners of fractals in a 2-parameter subfamily and use this to prove results about the diameter of these fractals. We provide bounds on the shortest path length between opposite corners in some other cases by using computational methods and linear programming. We also describe special circumstances when shortest geodesic paths must be monotonic with respect to each coordinate. (Received September 16, 2019)

52 ► Convex and discrete geometry

1154-52-21 **Yi Won Stefan Kim***, The Taft School, 110 Woodbury Rd, Watertown, CT 06795, and **Andrew Woojong Lee**, Choate Rosemary Hall, 333 Christian St, Wallingford, CT 06492. *On the polygon determined by the short diagonals of a convex polygon.*

Let K be a convex pentagon in the plane and let K_1 be the pentagon bounded by the diagonals of K . It has been conjectured that the maximum of the ratio between the areas of K_1 and K is reached when K is an affine regular pentagon. In this paper we prove this conjecture. We also show that for polygons with at least six vertices the trivial answers are the best possible. (Received June 08, 2019)

1154-52-23 **Taeyang David Park*** (taeyangpark0801@gmail.com), Peddie School, 201 South Main St, Hightstown, NJ 08520, and **Seo Yeong Shauna Kwag** (kwags@blair.edu), Blair Academy, 2 Park St, Blairstown, NJ 07825. *On the largest axis-parallel empty rectangle among points in a square.*

Given S , a set of n points contained in the unit square $Q = [0, 1]^2$, let $f(S)$ denote the area of the largest axis-parallel rectangle that does not contain any of the points of S in its interior. Further, let $f(n)$ be the minimum value of $f(S)$ over all sets S of n points in Q . In 2009, Dumitrescu and Jiang proved that $f(2) = (3 - \sqrt{5})/2$, $f(4) = 1/4$, and the following general bounds for $f(n)$:

$$(1.25 - o(1)) \cdot \frac{1}{n} \leq f(n) \leq 4 \cdot \frac{1}{n}.$$

We show that $f(3) = 0.3079\dots$, $0.2192 < f(5) < 0.2215$, $0.1835 < f(6) < 0.1962$, and we improve the bounds in the general case:

$$(1.31 - o(1)) \cdot \frac{1}{n} \leq f(n) \leq 1.91 \cdot \frac{1}{n}.$$

(Received June 08, 2019)

1154-52-97 **Alvaro Carbonero** (carboa1@unlv.nevada.edu), **Beth Anne Castellano** (castelle@lafayette.edu), **Charles Kulick*** (charles.kulick@scranton.edu) and **Karie Schmitz** (karie.schmitz@gmail.com). *Exploring Preference Orderings Through Discrete Geometry.* Preliminary report.

Consider $n + 1$ points in the plane: a set S consisting of n points along with a distinguished vantage point v . By measuring the distance from v to each of the points in S , we generate a preference ordering of S . The maximum number of orderings possible is given by a fourth-degree polynomial (related to Stirling numbers of the first kind), found by Good and Tideman (1977), while the minimum is given by a linear function. We investigate intermediate numbers of orderings achievable by special configurations S . This work is motivated by a voting theory application, where an ordering corresponds to a preference list. We also consider this problem for points on the sphere, where our results are similar to what we found for the plane. Other variants of the original problem inspired by voting theory are developed. These include using a weighted distance function and also using two vantage points. (Received August 04, 2019)

1154-52-106 **Deane Yang*** (deane.yang@courant.nyu.edu). *Minkowski Problems*.

The discrete Minkowski problem involves prescribing the normal directions and areas of the faces of a convex polytope. The smooth version involves prescribing the Gauss curvature of a closed convex hypersurface as a function of the outer unit normal and is equivalent to a Monge-Ampère PDE. The most general version involves prescribing the surface area measure of a convex body.

The extensions of the Brunn-Minkowski theory for convex bodies introduced by Erwin Lutwak have led to a rich new set of Minkowski problems. One particularly interesting case is the logarithmic Minkowski problem, which involves prescribing the cone volume measure. Parallels and contrasts between the classical and logarithmic Minkowski problems will be presented. (Received August 07, 2019)

1154-52-684 **Joseph W. Iverson*** (jwi@iastate.edu) and **Dustin G. Mixon** (mixon.23@osu.edu).
Doubly transitive lines: Symmetry implies optimality.

Since the work of L.F. Tóth on regular figures, it has been widely observed that optimal solutions to packing problems frequently display extraordinary symmetries. For instance, spheres centered on points in the Leech lattice give an optimal packing in 24 dimensions, while lines through antipodal vertices of an icosahedron give an optimal packing in two-dimensional projective space. In this talk, we demonstrate an extreme case of this phenomenon for line packings: symmetry can be a sufficient condition for optimality. Specifically, consider n lines spanning a space of dimension $d < n$. If the lines have a doubly transitive automorphism group, then they are optimally packed in projective space. In fact, unit norm representatives for the lines reach equality in the Welch bound to create an equiangular tight frame. We will explain this phenomenon, and then discuss progress toward a classification of all doubly transitive lines. (Received September 09, 2019)

1154-52-792 **Ralph Morrison*** (10rem@williams.edu), 33 Stetson Ct, Williamstown, MA 01267.
Undergraduate research topics in tropical geometry.

Tropical geometry transforms questions in algebraic geometry into combinatorial ones, drawing from discrete geometry and graph theory. In this talk I'll talk about some specific project ideas coming from the tropical world, and also how to generate lots more ideas in parallel with algebro-geometric results. Many of these projects will draw on tropical plane curves and chip-firing games on graphs. (Received September 10, 2019)

1154-52-1285 **Opal J Graham*** (ograham@math.fsu.edu), 800 Ocala Rd, Ste 300-194, Tallahassee, FL 32304. *Rigidity of Points, Circles, and Spheres*. Preliminary report.

The rigidity of configurations of points and configurations of circles in the Riemann Sphere is attained by Beardon and Minda using a maximal amount of conformal invariant information. Crane and Short generalize to configurations of points and spheres in higher dimensions. For example, in order to uniquely place a collection of spheres, the inversive distance between every pair must be known. We look at how rigidity of collections of positive Lorentz vectors with independent subcollections translates to rigidity statements for ideal points, spheres, hyperbolic points, and combinations of the three. These statements cut down on the amount of conformal invariant data used. (Received September 14, 2019)

1154-52-1574 **Jeewoo Lee*** (jlee1397@townsendharris.org), 240-19 69th Ave, Little Neck, NY 11362, and **Kelvin Kim** (kelvin2kim@gmail.com), 19 Peach Hill Ct, Ramsey, NJ 07446. *The Largest Angle Bisection Procedure*.

The *largest angle bisection* procedure is the operation which partitions a given triangle, T , into two smaller triangles by constructing the angle bisector of the largest angle of T . Applying the procedure to each of these two triangles produces a partition of T into four smaller triangles. Continuing in this manner, after n iterations, the initial triangle is divided into 2^n small triangles. We prove that as n approaches infinity, the diameters of all these 2^n triangles tends to 0, the smallest angle of all these triangles is bounded away from 0, and that, with the exception of T being an isosceles right triangle, the number of dissimilar triangles is unbounded. (Received September 16, 2019)

1154-52-1644 **Dustin G. Mixon** and **Hans Parshall*** (parshall.6@osu.edu). *Optimal packings in real projective spaces*.

An optimal (real) projective packing is a set of n unit vectors in \mathbf{R}^d with minimal coherence. We will discuss recent proofs of optimality of several projective packings using ideas from combinatorics, convex optimization, and real algebraic geometry. (Received September 16, 2019)

1154-52-1806 **E. B. Saff*** (edward.b.saff@vanderbilt.edu), Department of Mathematics, Vanderbilt University, Stevenson Center Building 1, Nashville, TN 37215, and **D. P. Hardin** and **M. Petrache**. *Unconstrained Polarization (Chebyshev) Problems: Basic Properties and Asymptotics*.

We introduce and study the unconstrained polarization (or Chebyshev) problem that requires to find an N -point configuration that maximizes the minimum value of its potential over a set A in p -dimensional Euclidean space. This problem is compared to the constrained problem in which the points are required to belong to the set A . We find that for Riesz kernels $1/|x - y|^s$ the optimum unconstrained configurations concentrate close to the set A and based on this fundamental fact we recover the same asymptotic value of the polarization as for the more classical constrained problem on a class of d -rectifiable sets. We also investigate the new unconstrained problem in special cases such as for spheres and balls and discuss several open problems. (Received September 16, 2019)

1154-52-1854 **Michael P Hitchman*** (mhitchm@linfield.edu). *On student-centered tiling research*.

This talk provides a brief introduction to techniques developed to address tiling questions in the integer lattice, with an emphasis on finding tile invariants, linear combinations among the number of copies of each tile that must persist in any tiling of a region. We also discuss some possible tiling projects for those interested in getting started in this area of research. A tiler's toolbox for finding tile invariants draws on content from combinatorics, number theory, group theory, and topology. Given their hands-on nature and their relevance to the undergraduate curriculum, tiling questions continue to be a lively subject for student-centered research. (Received September 16, 2019)

1154-52-2297 **Dustin G. Mixon*** (mixon.23@osu.edu). *Doubly transitive lines*.

We are interested in finite subsets of complex projective space that maximize the minimum distance. Many of the known optimal codes exhibit extraordinary symmetry. This talk focuses on a special case of this phenomenon: doubly transitive lines. Such lines are necessarily optimal codes in projective space. We leverage the underlying symmetries to obtain a partial classification of doubly transitive lines, namely those with almost simple symmetries. (Joint work with Joseph Iverson.) (Received September 17, 2019)

1154-52-2403 **Nina Zubrilina*** (nizubrilina@gmail.com). *Zeros of Optimal Functions in the Cohn-Elkies Linear Program*.

In a recent breakthrough, Viazovska and Cohn, Kumar, Miller, Radchenko, Viazovska solved the sphere packing problem in \mathbb{R}^8 and \mathbb{R}^{24} , respectively, by exhibiting explicit optimal functions, arising from the theory of weakly modular forms, for the Cohn-Elkies linear program in those dimensions. These functions have roots exactly at the lengths of points of the corresponding optimal lattices: $\{\sqrt{2n}\}_{n \geq 1}$ for the E_8 lattice, and $\{\sqrt{2n}\}_{n \geq 2}$, for the Leech lattice. The constructions of these optimal functions are in part motivated by the locations of the zeros. But what are the roots of optimal functions in other dimensions? We prove that distances between root lengths are bounded from above for $n \geq 1$ and not bounded from below for $n \geq 2$. We further prove that the root lengths have to be arbitrarily close for arbitrarily long, that is, for any $C, \varepsilon > 0$, there is an interval of length C on which the root lengths are at most ε apart. Finally, we establish a technique that allows one to improve a non-optimal function in some cases. (Received September 17, 2019)

1154-52-2579 **Kevin Li, Christopher Xue*** (christopher.xue@yale.edu), **Jack Hirsch** and **Jackson Petty**. *Isoperimetric Tilings of Closed Hyperbolic Surfaces*.

The regular hexagon is the least-perimeter tile of the Euclidean plane for any given area. On hyperbolic surfaces, this "isoperimetric" problem differs for every given area, as solutions do not scale. However, there exist tilings of hyperbolic surfaces that preserve much of the symmetry of the Euclidean solution. In particular, a regular k -gon with area $(k - 6)\pi/3$ tiles certain hyperbolic surfaces with edges meeting in 3's at 120-degree angles. Cox conjectured that on closed hyperbolic surfaces, such tiles are isoperimetric. We prove his conjecture for tilings by a collection of curvilinear polygons. (Received September 17, 2019)

1154-52-2655 **Josiah Park*** (j.park@gatech.edu), Georgia Institute of Technology, 686 Cherry St NW, Atlanta, GA 30332. *Symmetry and dimensionality for generalized frame energies*.

How does one spread lines (through the origin), or points on a sphere so as to minimize energy? We recently observed peculiarities in limiting problems of the above type, proving through linear programming methods that tight designs appear as discrete minimizers for "frame-like" continuous energies. These observations are an extension of a type of "universality" behavior that such configurations are now well-known to exhibit. A large question that remains is what can be said about the dimensionality of minimizers of such energies, and more generally how such information can be extracted from characteristics of the associated potential function for an

interaction energy. I will talk about developments in this area as represented in work with D. Bilyk, A. Glazyrin, R. Matzke and O. Vlasiuk. (Received September 17, 2019)

1154-52-2681 **Henry Cohn*** (cohn@microsoft.com). *Universal optimality and its failure.*

When should we expect the solution of an optimization problem to solve other related problems at the same time? Generically, there is no reason for this to happen, but linear programming bounds have an odd tendency to single out unusually nice solutions, in particular universal optima. In this talk I'll survey what's known about this phenomenon for particle configurations in several different ambient spaces. (Received September 17, 2019)

53 ► Differential geometry

1154-53-30 **Ryad Ghanam*** (raghanam@vcu.edu), VCUQ, Al-Luqta St, Education City, Doha, Qatar, and **Gerard Thompson*** (gerard.thompson@utoledo.edu), Dept of Mathematics, University of Toledo, Toledo, OH 43606. *Symmetries of the Eikonal Equation on a Lorentzian space.* Preliminary report.

In this talk we present our preliminary results about the symmetries on the Eikonal equation on a Lorentzian space in dimensions 2 and 3. We consider flat Lorentzian and spaces of constant curvature. In either case, we identify the symmetry algebra for any dimension and use the symmetries to produce solutions. (Received July 01, 2019)

1154-53-91 **Nabil Kahouadji***, Mathematics Department, Northeastern Illinois University, Chicago, IL 60625. *Isometric Immersions of Pseudo-Spherical Surfaces via PDEs.*

Pseudo-spherical surfaces (PSS) are surfaces of constant negative Gaussian curvature. A way of realizing such a surface in 3d space is obtained by rotating the graph of a curve called tractrix around the z-axis. There is a remarkable connection between the solutions of the sine-Gordon eq and PSS, in the sense that every generic solution of this eq can be shown to give rise to a PSS. Furthermore, the sine-Gordon eq has the property that the way in which the PSS corresponding to its solutions are realized geometrically in 3d space is given in closed form through some remarkable explicit formulas. The sine-Gordon eq is but one member of a very large class of differential eqs whose solutions likewise define PSS. These were defined and classified by Chern, Tenenblat and others, and include almost all the known examples of integrable PDEs. This raises the question of whether the other eqs enjoy the same remarkable property as the sine-Gordon eq when it comes to the realization of the corresponding surfaces in 3d space. We will see that the answer is no, and will provide a full classification of hyperbolic and evolution eqs. The classification results will show, among other things, that the sine-Gordon eq is quite unique in this regard amongst all integrable eqs. (Received August 01, 2019)

1154-53-229 **Marco Radeschi*** (marco.radeschi@gmail.com), 272 Hurley, Notre Dame, Notre Dame, IN 46617. *Laplacian algebras and singular Riemannian foliations in spheres.*

We exhibit a one-to-one correspondence between spherical manifold submetries (a concept generalizing closed singular Riemannian foliations in spheres) and a special class of polynomial algebras, namely maximal Laplacian algebras.

We provide some applications, such as solving the Inverse Invariant Theory problem for orthogonal representations of finite groups, and characterizing transnormal systems with closed leaves. (Received August 25, 2019)

1154-53-234 **Robert L. Bryant*** (bryant@math.duke.edu), Duke Mathematics Department, PO Box 90320, Durham, NC 27708-0320. *On solitons for the closed G_2 -Laplacian flow.* Preliminary report.

For G_2 -structures on 7-manifolds, there is a natural analog of the Ricci-flow studied in Riemannian geometry, namely, one considers a 1-parameter family $\sigma = \sigma(t)$ of G_2 -structures on a given 7-manifold that satisfies the equation

$$\frac{d\sigma}{dt} = \Delta_\sigma \sigma$$

with a specified initial G_2 -structure $\sigma(0) = \sigma_0$.

When the 1-parameter family σ moves by diffeomorphism and scaling, we say that σ is a *soliton* for the G_2 -Laplacian flow. The most interesting case is when the initial G_2 -structure is *closed*.

In this talk, I will describe some of what is known about the existence and local generality of solitons for this flow, concluding with a discussion of the still-unsolved problem of the generality of the *gradient* solitons, which are of great interest in the theory of G_2 -structures. (Received August 26, 2019)

1154-53-328 **Gloria Mari-Beffa*** (maribeff@math.wisc.edu), Van Vleck Hall, 480 Linden drive, Madison, WI 53705, and **Annalisa Calini**. *Twisted polygonal evolutions in $\mathbb{R}P^m$ and discretizations of Adler-Gel'fand-Dikii flows.*

In this talk we will describe a recently found discretization of a well known completely integrable system of PDEs (the Adler-Gel'fand-Dikii flow, due to Adler). To define the differential-difference system and to show the discretization is itself completely integrable we will link it to polygonal evolutions in $\mathbb{R}P^m$ and to their background geometry. This relationship will allow us to define Hamiltonian structures for the system, and to prove that they are compatible and can be used to generate integrals of the motions (i.e., it is Liouville integrable). This is joint work with Annalisa Calini and Jin Ping Wang. (Received August 31, 2019)

1154-53-336 **S. D. Jung*** (sdjung@jejunu.ac.kr), Department of Mathematics, Jeju National University, Jeju 63243, South Korea, and **K. Richardson** (k.richardson@tcu.edu), Department of Mathematics, Texas Christian University, Fort Worth, TX 76109. *Basic Dolbeault cohomology on a transverse Kähler foliation.* Preliminary report.

Vaisman manifolds, Calabi-Eckmann manifolds and Sasaki manifolds are not Kähler, but they carry a transverse Kähler structure. In this talk, we will give some properties of basic Dolbeault cohomology on a transverse Kähler foliation. In particular, we study the Kodaira-Serra duality and hard Lefschetz theorem on a transverse Kähler foliation. (Received September 01, 2019)

1154-53-361 **Thomas A. Ivey***, 66 George St., Charleston, SC 29403. *Bäcklund Transformations and Darboux Integrability, Revisited.* Preliminary report.

About a decade ago Jeanne Clelland and I published a paper concerning Bäcklund transformations between the wave equation and other hyperbolic Monge-Ampere systems \mathcal{I} defined on 5-manifolds. We proved that if such a transformation exists, has one-dimensional fibers, and satisfies some mild genericity conditions, then the prolongation of \mathcal{I} is Darboux-integrable. We also argued that, conversely, Darboux-integrability at the 2-jet level implies the existence of such a transformation; however, Anderson and Fels later showed that the equation $z_{xy} = \sqrt{(1 - u_x^2)}\sqrt{(1 - u_y^2)}/\sin z$ is a counterexample.

In this talk I'll discuss a generalization of the first argument – establishing Darboux integrability – to a broader class of decomposable systems, and I'll briefly analyze the problem with the second argument. This is joint work with Jeanne Clelland and Naghmana Tehseen. (Received September 02, 2019)

1154-53-527 **Karsten Grove*** (kgrove2@nd.edu), **Adam Moreno** (adam.j.moreno.39@nd.edu) and **Peter Petersen** (petersen@math.ucla.edu). *The boundary conjecture for Alexandrov leaf spaces.*

A basic conjecture in Alexandrov geometry states that the boundary of an Alexandrov space X with its induced length metric is itself an Alexandrov space with the same lower curvature bound as that of X .

In joint work with Adam Moreno and Peter Petersen we confirm this conjecture in the case where X is the leaf space of a singular Riemannian foliation with closed leaves on a closed Riemannian manifold. (Received September 06, 2019)

1154-53-559 **Brian D Harvie*** (bharvie@math.ucdavis.edu), 1 Shields Avenue, Mathematical Sciences Building, Davis, CA 95616. *The Non-Star-Shaped Regime of Inverse Mean Curvature Flow.* Mean Curvature Flow (MCF) is a geometric flow which has garnered much attention from geometers since the 1980s. The study of MCF naturally motivates one to examine similar extrinsic flows, and among these is Inverse Mean Curvature Flow (IMCF).

Since the late 1990s, IMCF has been an increasingly important tool both in differential geometry and in general relativity. There appears to be an analogy between MCF and IMCF in the conditions one can impose on an initial surface to guarantee certain flow behavior: for MCF, requiring an initial surface to be convex assures the flow surfaces eventually resemble round spheres, while requiring an initial surface be star-shaped produces the same behavior for IMCF. While MCF over non-convex surfaces has been studied extensively (e.g. neckpinch singularities), the same cannot be said for IMCF over non-star-shaped surfaces.

This talk will focus on a recent paper of mine which characterizes some properties of non-star-shaped IMCF. Several of these results are analogues of very well-known properties of MCF (e.g. the avoidance principle, existence of finite-time singularities). Therefore, this talk will provide a link between the two flows, thus also providing a friendly introduction to the subject of geometric flows for newcomers. (Received September 06, 2019)

1154-53-574 **Sema Salur*** (salur@math.rochester.edu), Department of Mathematics, University of Rochester, Rochester, NY 14627. *Geometric Structures on Manifolds with Special Holonomy*. Preliminary report.

A 7-dimensional Riemannian manifold (M, g) is called a G_2 manifold if the holonomy group of its Levi-Civita connection of g lies inside of $G_2 \subset SO(7)$. Equivalently, a G_2 manifold is a 7-dimensional Riemannian manifold with a vector cross product \times on its tangent bundle, and a harmonic 3-form $\varphi \in \Omega^3(M)$ such that

$$\varphi(u, v, w) = g(u \times v, w)$$

G_2 manifolds have many applications in differential geometry and physics, and it is a very active area of research. One major problem in the field is a lack of an existence theorem that gives necessary and sufficient conditions for a 7-dimensional manifold to admit a G_2 metric. In this talk we give a report of recent research on this existence problem. (Received September 07, 2019)

1154-53-616 **Igor Zelenko*** (zelenko@math.tamu.edu), Department of Mathematics, Mailstop 3368, Texas A&M University, College Station, TX 77843-3368, and **Curtis Wade Porter**. *2-nondegenerate CR structures of hypersurface type: bigraded Tanaka prolongation for canonical absolute parallelism and maximally symmetric models*.

The talk is devoted to the local geometry of 2-nondegenerate CR manifolds M of hypersurface type. An absolute parallelism for such structures was recently constructed independently by Isaev-Zaitsev, Medori-Spiro, and Pocchiola in the minimal possible dimension ($\dim M = 5$), and for $\dim M = 7$ in certain cases by C. Porter. We develop a bigraded analog of the Tanaka prolongation procedure to construct a canonical absolute parallelism for these CR structures in arbitrary (odd) dimension with Levi kernel of arbitrary admissible dimension. In the talk we will describe the main notions and constructions of this bigraded version Tanaka theory. We also describe the hypersurface realization of maximally symmetric models for such structures with one dimensional Levi kernel in arbitrary odd dimensions using the relation between CR-structures and systems of PDEs via Segre varieties. (Received September 08, 2019)

1154-53-626 **Taylor J Klotz*** (taylor.klotz@colorado.edu). *Aspects of the Theory of Cascade Linearization*. Preliminary report.

Cascade Feedback linearizable control systems are ones that are “composed” of two smaller linear control systems. The second of the linear systems arises from reducing an equivalent form of the original control system to a lower rank system that is linearizable. This talk will present recent existence results for a system to have this second linearization property for a control system. (Received September 08, 2019)

1154-53-663 **Bang-Yen Chen*** (chenb@msu.edu), 2231 Tamarack Drive, Okemos, MI 48864. *Two-numbers and applications*.

The 2-number is a numerical invariant which assigns an integer to a Riemannian manifold M introduced in one of my joint articles with Professor T. Nagano. In this talk, I will present relationships between 2-number and several important areas of mathematics. Also, I will mention applications of 2-number to some important subjects in mathematics. At the end of my talk, I will present three conjectures on 2-number. (Received September 09, 2019)

1154-53-669 **Jo Nelson** and **Morgan Weiler*** (morgan.weiler@rice.edu), Rice University, Department of Mathematics – MS 136, PO Box 1892, Houston, TX 77005. *The embedded contact homology of Seifert fiber spaces*.

We investigate the embedded contact homology of Seifert fiber spaces via contact forms whose Reeb vector field generates the circle action. This extends the thesis of Farris. (Received September 09, 2019)

1154-53-705 **Christopher G Moseley*** (cgm3@calvin.edu), Calvin University, Department of Mathematics and Statistics, 1740 Knollcrest Circle SE, Grand Rapids, MI 49546. *Lie algebras of homogeneous sub-Riemannian Engel manifolds*. Preliminary report.

Previous work by J. Clelland, G. Wilkens and C. Moseley on sub-Finsler Engel 4-manifolds classified homogeneous sub-Finsler structures with nonzero Cartan scalar. In current work we consider sub-Finsler Engel 4-manifolds with Cartan scalar equal to zero, in which the sub-Finsler structure descends to a sub-Riemannian structure on the base manifold. We present a complete classification of all Lie algebras corresponding to homogeneous sub-Riemannian Engel manifolds. This is joint work with G. Wilkens. (Received September 09, 2019)

1154-53-721 **Lawrence Mouillé*** (lmoui001@ucr.edu). *Positive intermediate Ricci curvature with symmetries*. Preliminary report.

Positive intermediate Ricci curvature, denoted $\text{Ric}_k > 0$, is a condition which interpolates between positive sectional curvature and positive Ricci curvature. Though this curvature condition has existed in the literature for several decades, few structure results or non-trivial examples exist. In this talk, I will illustrate new examples of $\text{Ric}_k > 0$. I will also describe upper bounds on the symmetry rank for manifolds with $\text{Ric}_k > 0$ that generalize the Grove-Searle bound for positive curvature and the Wilking bound for quasi-positive curvature. Finally, I will present a classification result for manifolds with $\text{Ric}_k > 0$ and large symmetry rank. (Received September 10, 2019)

1154-53-767 **Dusa McDuff*** (dusa@math.columbia.edu). *Symplectic embedding problems*. Preliminary report.

This talk will discuss recent progress made in understanding the question of when one region in Euclidean space embeds symplectically into another. In particular we will discuss recent work by Kyler Siegel and Siegel–McDuff about the stabilized symplectic embedding problem for ellipsoids. (Received September 10, 2019)

1154-53-781 **Susan Tolman*** (tolman@illinois.edu). *Non-Hamiltonian symplectic circle actions*.

Frankel proved many years ago that symplectic circle actions on compact, connected Kähler manifolds with fixed points must be Hamiltonian. In contrast, as McDuff proves, there exist non-Hamiltonian symplectic circle actions on compact, connected symplectic manifolds with fixed tori. Several years ago, the author constructed a non-Hamiltonian symplectic circle action on a compact, connected six dimensional manifolds with isolated fixed points, thus answer a question posed by McDuff. However, that example had 32 fixed points. More recently, in joint work with D. Jang, we have been able to construct many more examples, with as few as 10 fixed points. (Received September 10, 2019)

1154-53-823 **Jayadev Athreya, Subhojoy Gupta and Heather Lee*** (heathml@uw.edu). *Polynomial quadratic differentials on \mathbb{P}^1* . Preliminary report.

I will talk about $GL(2, \mathbb{R})^+$ dynamics and symplectic structure on the moduli space of polynomial quadratic differentials on \mathbb{P}^1 . (Received September 11, 2019)

1154-53-858 **Jason Cantarella***, Boyd Graduate Studies Research Center, Athens, GA 30602, and **Tetsuo Deguchi, Clayton Shonkwiler and Erica Uehara**. *Random graph embeddings and entanglements in topological polymers*. Preliminary report.

The mechanical properties of elastic materials like rubber and collagen depend on the topology of the network of polymer strands which make up the material. This topology is that of a random embedding of an extremely complicated random graph.

In this talk, we discuss the theory of Gaussian random embeddings of graphs (much of which is classical), give a simplified and clarified picture of the existing theory, and derive new results on the expected geometry of a random embedding of a fixed graph. These results predict experimental results of Tezuka et al. for synthetic polymers of known graph type.

We then use our theorem to numerically study the expected geometry of a simple model of random graphs relevant to the “t-rex” polymer system of Honda et al. (Received September 11, 2019)

1154-53-886 **Ryad Ghanam*** (raghanam@vcu.edu), Doha, Qatar, and **Gerard Thompson** (gerard.thompson@utoledo.edu), Toledo, OH. *Symmetries of the Eikonal Equation*.

We consider the n -dimensional Eikonal equation on a flat metric. We prove that the infinitesimal algebra of Lie symmetries of the Eikonal equation is $\mathfrak{o}(n+1, 2)$ when there are n independent variables. We also give an explicit basis that is aligned with the standard basis coming from the standard matrix representation of $\mathfrak{o}(n+1, 2)$ thereby making it possible to read off inequivalent one dimensional symmetry vector fields. The symmetries are used to construct various solutions of the Eikonal equation. (Received September 11, 2019)

1154-53-1041 **David Sykes*** (dgsykes@math.tamu.edu) and **Igor Zelenko**. *Local geometry of 2-nondegenerate CR structures: classification of symbols and maximally symmetric homogeneous models*. Preliminary report.

We give a classification of a basic invariant of the local differential geometry of hypersurface-type CR structures having a uniformly degenerate Levi form with 1-dimensional kernel. The invariant is a pair consisting of a real line of nondegenerate Hermitian forms and a complex line of self-adjoint antilinear operators. It was introduced by Curtis Porter and Igor Zelenko, and they classified elements in the family of these pairs for which the line of antilinear operators is closed under the operation of taking each operator’s cube. We generalize this, giving

a complete classification of these invariants by first obtaining a more general classification of pairs consisting of a nondegenerate Hermitian form and a symmetric form on finite dimensional vector spaces. We use this classification to give upper bounds for the dimension of the symmetry group of a CR structure corresponding to a given invariant, and we give necessary conditions for such an invariant to be associated with a homogeneous CR manifold. The talk is based on joint work with Igor Zelenko. (Received September 17, 2019)

1154-53-1067 **Makiko Sumi Tanaka*** (tanaka_makiko@ma.noda.tus.ac.jp), **Hiroyuki Tasaki** (tasaki@math.tsukuba.ac.jp) and **Osami Yasukura** (yasukura@u-fukui.ac.jp).
Maximal antipodal sets related to G_2 .

An antipodal set of a Riemannian symmetric space M , introduced by B. -Y. Chen and T. Nagano in 1980's, is a subset A of M on which $s_x(y) = y$ holds for any elements x, y in A , where s_x denotes the geodesic symmetry at x on M . We explicitly describe maximal antipodal sets of compact Riemannian symmetric spaces related to the compact connected simple Lie group of type G_2 by realizing it as the automorphism group of the octonions. Moreover, we observe a close relation between the algebraic structure of the octonions and the Fano plane by using these explicit descriptions. (Received September 13, 2019)

1154-53-1233 **Hiroshi Tamaru*** (tamaru@sci.osaka-cu.ac.jp), Department of Mathematics, Osaka City University, Osaka, 585-8585, Japan. *Left-invariant metrics and submanifold geometry of noncompact symmetric spaces.*

For a left-invariant Riemannian metric on a given Lie group, we can construct a submanifold, where the ambient space is a noncompact Riemannian symmetric space consisting of all left-invariant metrics on that Lie group. We expect that nice left-invariant metrics (such as Einstein or Ricci soliton) are corresponding to nice submanifolds. In this talk, we introduce our framework, and mention some results related to such correspondence. (Received September 14, 2019)

1154-53-1260 **Noelle Sawyer*** (nsawyer@wesleyan.edu). *Partial marked length spectrum rigidity for negatively curved surfaces.* Preliminary report.

The marked length spectrum of a metric on a compact Riemannian manifold records the length of the shortest closed curve in each free homotopy class. It is known that a negatively curved metric on a compact Riemannian manifold is uniquely determined by its marked length spectrum up to isometry. My results show that under certain conditions on the excluded homotopy classes, a partial marked length spectrum also uniquely determines such a metric. (Received September 14, 2019)

1154-53-1383 **Mita Banik*** (mbanik@ucsc.edu). *Toric pseudo-rotations and quantum cohomology.*

In this talk we are going to discuss some deformations of the quantum product due to the existence of toric pseudo-rotations on a symplectic manifold. In the presence of such pseudo-rotations, we will discuss the restrictions on the minimal Chern number N and for the special case $N = n + 1$, where $2n$ is the dimension of the manifold, we will relate the quantum cohomology of the manifold with that of $\mathbb{C}P^n$. (Received September 15, 2019)

1154-53-1453 **Elahe Khalili Samani*** (ekhalili@syr.edu). *Positive curvature and fundamental group.*

A 1960s question of Chern asks if every abelian subgroup of the fundamental group of a Riemannian manifold with positive sectional curvature is cyclic. While this is not true in general, there are some positive results in the presence of symmetry. I will discuss some new structural results along these lines. I will also discuss an application to an infinite family of positively curved manifolds. (Received September 15, 2019)

1154-53-1729 **Nicholas D Brubaker*** (nbrubaker@fullerton.edu), Department of Mathematics, California State University, Fullerton, 154 McCarthy Hall, Fullerton, CA 92834.
Instabilities and bifurcations of minimal surfaces.

A classical result, extending Bernstein's theorem for graphs, states that stable complete minimal surfaces in \mathbf{R}^3 must be planar. Thus, for every nonplanar complete minimal surface, there exists a compact subdomain and a boundary preserving variation over that subdomain that reduces the value of the area functional. Embedded in this result is the requirement that the area of the subdomain must be sufficiently large, otherwise the surface patch will be approximately planar and, thus, locally area-minimizing. This small/large subdomain dichotomy exists for all smooth nonplanar minimal surfaces and induces a change in the index of the surface. Index changes produce, among other results, bifurcations where multiple minimal surfaces with the same boundary appear. In this talk, we review results on index changes in some classical minimal surfaces and then discuss a few specific examples, from work with T. Murphy and K.O. Negrón, that extend such results. (Received September 17, 2019)

1154-53-1787 **Yun Oh*** (ohy@andrews.edu). *V^T -Rectifying submanifolds.*

The idea of rectifying curves in \mathbb{E}^3 was introduced in 2003 by B.-Y. Chen and he extended the notion to the rectifying submanifold in \mathbb{E}^n in 2016. The following year he introduced the rectifying submanifold in an arbitrary Riemannian manifold associated with a torqued vector field V . In this talk, we investigate an another idea of rectifying submanifold in terms of the normal component of a torqued vector field V . Some recent results will be discussed. (Received September 16, 2019)

1154-53-1796 **Marcos M. Alexandrino*** (malex@ime.usp.br), Instituto de Matemática e Estatística, Sao Paulo, SP 05508-090, Brazil. *On Mean curvature flow of Singular Riemannian foliations: Non compact cases.*

In this talk we discuss the mean curvature flow (MCF) of a regular leaf of a closed generalized isoparametric foliation as initial datum, generalizing previous results of Radeschi and first author. We show that, under bounded curvature conditions, any finite time singularity is a singular leaf, and the singularity is of type I. We also discuss the existence of basins of attraction, how cylinder structures can affect convergence of basic MCF of immersed submanifolds and make a few remarks on MCF of non closed leaves of generalized isoparametric foliation. This talk is based on a joint work with Leonardo F. Cavenaghi, Icaro Gonçalves (see preprint arXiv:1909.04201) (Received September 16, 2019)

1154-53-1830 **Rachelle DeCoste, Lisa DeMeyer, Meera Mainkar and Allie Ray*** (adray@bsc.edu). *Singularity of Two-Step Nilpotent Lie Algebras Constructed from Graphs.* Preliminary report.

We start by introducing two methods for constructing a two-step nilpotent metric Lie algebra, one from a simple graph and one from a directed edge-labelled graph where the edge labels may repeat. Previous results about the singularity of Lie algebras for the simple graph construction will be briefly discussed. When introducing repeated edge labels in the construction, the Euclidean deRham (or Abelian) factor becomes a consideration. We will focus on certain classes of graphs and discuss how the Euclidean deRham factor and singularity of these Lie algebras may be determined by properties of the graphs. (Received September 16, 2019)

1154-53-1970 **Peter Quast and Takashi Sakai*** (sakai-t@tmu.ac.jp), Minami-Osawa, Hachioji-shi, Tokyo 192-0397, Japan. *Natural Γ -symmetric structures on R -spaces.* Preliminary report.

The notion of Γ -symmetric spaces was introduced by Lutz in 1981 generalizing k -symmetric spaces. In this talk, we consider Γ -symmetric structures on some R -spaces using Γ -symmetric triples introduced by Goze and Remm. We give a characterization in terms of root systems and classification of R -spaces that admit a certain natural Γ -symmetric structure, where $\Gamma = (\mathbb{Z}_2)^m$. For $\Gamma = \mathbb{Z}_2$ we recover the symmetric R -spaces.

In 1988, Chan and Nagano studied antipodal sets of compact symmetric spaces. The notion of antipodal sets can be extended to Γ -symmetric spaces. We study antipodal sets of R -spaces with respect to their natural Γ -symmetric structures, and show that a maximal antipodal set is given as an orbit of the Weyl group. That is a generalization of a result on maximal antipodal sets of symmetric R -spaces due to Tanaka and Tasaki. (Received September 16, 2019)

1154-53-2225 **Amir Babak Aazami*** (aaazami@clarku.edu), 950 Main St., Worcester, MA 01776. *The Ehlers-Kundt Conjecture in General Relativity.*

There is a beautiful class of spacetimes in Einstein's theory of General Relativity, known as pp-wave spacetimes. They model radiation in its various forms, and they are rich in beautiful properties. Among these is a particularly nice subclass, known as the plane wave spacetimes, which generalize electromagnetic plane waves in Maxwell's theory. In the 60s, Jurgen Ehlers and Wolfgang Kundt posed a (deceptively) simple conjecture:

“Prove the plane waves to be the only complete gravitational pp-waves.”

Even now, this question remains open – though a lot of progress has been made (including just last year). In this talk I will review this beautiful conjecture, describe where it stands today, and mention many of the principal actors. Surprisingly, this conjecture permits a very elegant viewpoint from dynamical systems theory, and therefore its resolution will be of interest to that community as well. (No background in General Relativity will be required for this talk.) (Received September 17, 2019)

1154-53-2287 **Ziva Myer*** (zmyer@math.duke.edu). *Legendrian Invariants.* Preliminary report.

In contact geometry there are a few different tools used to construct invariants for Legendrian submanifolds: pseudoholomorphic curves, generating families, and constructible sheaves. While the underlying theory of these techniques employ vastly distinct flavors of mathematics, there has been some evidence that Legendrian invariants defined through these methods contain the same information. I will elaborate more on this phenomenon and

highlight where some of my past, current, and future projects appear in this story. (Received September 17, 2019)

1154-53-2409 **Jeanne N. Clelland*** (jeanne.clelland@colorado.edu) and **Taylor J. Klotz**. *Beltrami fields with nonconstant proportionality factor*. Preliminary report.

A Beltrami field on an open set $U \subset \mathbb{R}^3$ is a vector field \mathbf{u} on U satisfying the PDE system

$$\operatorname{curl} \mathbf{u} = f\mathbf{u}, \quad \operatorname{div} \mathbf{u} = 0$$

for some smooth function $f : U \rightarrow \mathbb{R}$, called the *proportionality factor*. When f is constant, \mathbf{u} is called a *strong Beltrami field*. Strong Beltrami fields are well-studied, but in this talk we consider the question: What nonconstant functions f can occur as the proportionality factor for a Beltrami field on an open subset $U \subset \mathbb{R}^3$? Enciso and Peralta-Salas showed that such functions are *rare*; in particular, they must lie in the kernel of a 6th-order, nonlinear partial differential operator.

By applying Cartan's method of moving frames and the theory of exterior differential systems, we can say more about the space of functions f that may occur as proportionality factors. We also consider the related question: For any such f , how large is the space of associated Beltrami fields? It turns out that the answer to this question depends crucially upon the geometry of the level surfaces of f . (Received September 17, 2019)

1154-53-2506 **Shoo Seto*** (shoos@uci.edu), 419 Rowland Hall, Irvine, CA 92697. *First eigenvalue estimates of p -Laplace type operators on Kahler manifolds*. Preliminary report.

We give a lower bound estimate for the first eigenvalue of the p -Laplacian on Kahler manifolds. We then generalize the p -Laplace operator to act on differential forms and give a generalization of a lower bound estimate by Gallot-Meyers. (Received September 17, 2019)

1154-53-2722 **Zhiqin Lu*** (zlu@uci.edu), Department of Mathematics, University of California, Irvine, 92697. *The p -spectra of hyperbolic spaces*. Preliminary report.

In this talk, we compute the p -spectrum of the Laplacian on k -forms on hyperbolic spaces. The work generalizes the previous results of Donnelly and Lohoue-Rychener. This is joint with Charalambous. (Received September 17, 2019)

54 ► General topology

1154-54-323 **H. Zhang*** (zhanghon@grinnell.edu) and **A. Ledesma Alonso** (ledesmaa@grinnell.edu). *Artworks and Articles Meet Mapper and Persistent Homology*.

Since its recent birth, topological data analysis (TDA) has proven to be a very useful tool when studying large and high-dimensional data sets. In this talk, we present our application of two TDA tools, persistent homology and the Mapper algorithm, to the Metropolitan Museum of Art (MET) data set and two scholarly literature databases: arXiv and Google Scholar. For the MET data, we use the Mapper Algorithm to guide feature selection in building a logistic regression model for classifying public-domain and non-public-domain artworks. Then we use persistent homology to help differentiate between certain collections of artworks. For the arXiv data, we use persistent homology to derive a general sense of the shape of the data. With help of the Mapper Algorithm, we further explore the point cloud by analyzing trends and features in visualizations. For the Google Scholar data, we find that there are interesting correlations between academic category, number of pages, number of references, and published date time of a paper. (Received September 01, 2019)

1154-54-440 **Pawel Krupski** and **Murat Tuncali*** (muratt@nipissingu.ca). *Maps of rank $\leq m$ revisited*.

Let $f : X \rightarrow Y$ be a function and let \mathfrak{m} be an infinite cardinal. Then we say that the rank $r(f)$ of f is $\leq \mathfrak{m}$ if

$$|\{y \in Y : |f^{-1}(y)| > 1\}| \leq \mathfrak{m}.$$

If $\mathfrak{m} = \aleph_0$ then f is of countable rank. In this talk, we present some general properties and invariants of rank $\leq \mathfrak{m}$ maps. We also show there are close relationships between them and monotone maps. Monotone maps on surfaces are approximated by countable rank monotone maps if the set of local separating points of the range space has the countable closure. (Received September 04, 2019)

1154-54-525 **Lynne Yengulalp*** (yengull@wfu.edu), Winston-Salem, NC 27101. *Micro-homogeneity*.

A space X is homogeneous if for every two points x and y in X there is an auto-homeomorphism of X taking x to y . A local version of homogeneity is micro-homogeneity: given any two points x and y in X there are open neighborhoods U and V and a homeomorphism from U to V taking x to y .

We present some examples and show that some classical results about homogeneous spaces can be generalized to micro-homogeneous spaces. For example, van Douwen's result that the cardinality of a homogeneous space cannot exceed $2^{\pi w(X)}$ is also true for micro-homogeneous spaces. (Received September 06, 2019)

1154-54-778 **Greg Bell*** (gcbell@uncg.edu) and **C. Neil Pritchard**. *On coarse properties of persistence diagrams.*

A persistence diagram is one of two common ways to visualize the evolution of the homology of a Rips complex as the parameter increases. In this talk we examine the coarse geometry of the space of persistence diagrams in a Wasserstein metric. In particular, we give a simple proof that it has infinite asymptotic dimension, among other results. (Received September 10, 2019)

1154-54-838 **Craig R Guilbault** and **Molly A Moran*** (mmoran@coloradocollege.edu). *Quasi \mathcal{Z} -Structures*. Preliminary report.

Bestvina defined a \mathcal{Z} -structure on a group G to generalize the theory of boundaries of CAT(0) and hyperbolic groups. An interesting question to ask in this setting is: given two groups that are quasi-isometric where one group is known to admit a \mathcal{Z} -structure, must the other group also admit a \mathcal{Z} -structure? From previous work, we have a partial answer to this question, but there is one roadblock in giving a complete answer. In this talk, we will discuss this roadblock and provide a modified definition of \mathcal{Z} -structures where we hope to give a more complete answer to this question. (Received September 11, 2019)

1154-54-1133 **Jacob Pichelmeyer*** (jacobpichelmeyer@gmail.com), 3129 Lundin Dr, Apt 12, Manhattan, KS 66503. *Genera of knots in the complex projective plane.*

Let $K : S^1 \rightarrow S^3$ be a knot and M be a smooth closed four-dimensional manifold. The M -genus of K is the least genus among all smooth, orientable surfaces Σ smoothly and properly embedded in $M \setminus B^4$ such that $\partial\Sigma = K$. The M -genera has been computed for all 2,977 prime knots up to twelve crossings in the cases where M is the four-sphere S^4 or $S^2 \times S^2$. In the case of CP^2 , there are 4,000+ prime knots up to twelve crossings along with their mirrors for which computation of the CP^2 -genus is non-trivial. Of these, the CP^2 -genus was known for only 8 such knots. We have obtained both obstruction and construction results that have allowed the computation of 146 more prime knots of twelve crossings or less for which computation of the CP^2 -genus is nontrivial, along with several infinite families. We present background on this topic, explanation of how the constructions and obstructions were obtained, and how the computations were made using these results. (Received September 13, 2019)

1154-54-1377 **Veronica Martinez de la Vega*** (vmvm@matem.unam.mx), Mexico, and **D Darji**, **Alejandro Illanes** (illanes@matem.unam.mx) and **Jorge Martinez Montenaje**. *Topological Mixing and UPE*. Preliminary report.

In this talk we give relations and conditions for the maps mixing, weakly mixing and UPE to be equivalent on dendrites. This results allow us to generalize theorem of Darji & Kato that states the that if X is a G -like continuum for some graph G and G admits a 2-upe homeomorphism, then X is indecomposable. With this new results we can state weaker conditions and still obtain and indecomposable continuum. (Received September 15, 2019)

1154-54-1546 **Chaim Even-Zohar*** (chaim@ucdavis.edu), **Joel Hass**, **Nati Linial** and **Tahl Nowik**. *Universal Knot Diagrams*.

We study collections of planar curves that yield diagrams for all knots. We say that a sequence of closed immersed planar curves U is universal if every knot can be obtained from all but finitely many curves in U , by some choice of the overcrossing and undercrossing arcs at each crossing point. This definition includes some well-studied cases.

In particular, we show that a very special class called potholder curves carries all knots. This has implications for realizing all knots and links as special types of meanders and braids.

Our work raises quantitative questions about the efficiency of various classes of curves that represent all knots. Another major challenge is to characterize universal families of planar curves by necessary and sufficient conditions. We will discuss these two lines of research. (Received September 16, 2019)

1154-54-1727 **Margaret I Doig***, margaretdoig@creighton.edu. *Computational grid Floer homology: explorations of crossing change and knot genus.*

We present a recent program designed to calculate knot genus and other related invariants using grid Floer homology, and we use it to explore the typical effect of a random crossing change on a knot's genus. We also outline possible applications in studying the stability of naturally occurring knots such as those found in proteins

and DNA. We provide instructions for accessing the program via web browser in the hopes that others may use it for investigating questions in low-dimensional topology. (Received September 16, 2019)

1154-54-2113 **Jared Holshouser***, JaredHolshouser@southalabama.edu, and **Chris Caruvana**, chcaru@iu.edu. *Selection Games: Equivalence and Duality.*

In studying the principle of closed discrete selection (first discovered by Tkachuk) we unearthed connections between a variety of selection games on a topological space and on its space of continuous functions. Moreover, these connections have more to do with the topological objects than with strategies for the games. In this talk we will discuss classes of games which are equivalent/dual and showcase a proof technique which shows these relationships without directly working with strategies. (Received September 17, 2019)

1154-54-2220 **Brian E Raines*** (brian_raines@baylor.edu). *Folding points for inverse limits generated by a sequence of unimodal maps.*

We use the notion of a postcritical ω -limit set to classify the folding points in inverse limit spaces generated by a sequence of unimodal maps. (Received September 17, 2019)

1154-54-2228 **Joshua R Mirth*** (mirth@math.colostate.edu), Weber Building, 841 Oval Drive, Fort Collins, CO 80524. *Morse theory for Wasserstein spaces.*

Topological data analysis studies the topology of spaces obtained by thickening a set of data. For example, if the data is a metric space M then one can build the Vietoris–Rips simplicial complex on M . This can be viewed as a subset of the space $\mathcal{P}(M)$ of probability measures on M . We study the topology of $\mathcal{P}(M)$. In particular, if M is a manifold and $\mathcal{P}(M)$ is equipped with the 2-Wasserstein distance, it inherits a type of differential structure from M . Using ideas inspired by classical Morse theory, we determine the homotopy type of certain Wasserstein spaces. (Received September 17, 2019)

1154-54-2465 **Clara Buck*** (clara.buck28@gmail.com), **Sean Gallagher** and **Aniruddha Nadiga**. *On the Knotting Probability of Random Equilateral Hexagons.*

The rigidity of equilateral geometric knots makes them useful in certain applications of knot theory. An n -sided geometric knot is a polygon in \mathbb{R}^3 consisting of n straight edges and no self-intersections. The space of n -sided geometric knots is a $3n$ -dimensional manifold where path components determine geometric knot type. We will consider the submanifold of equilateral knots, consisting of embedded n -sided polygons with unit length edges. The space of equilateral hexagons up to translation modulo the rotation group, $\text{Pol}(6)/\text{SO}(3)$, is 6 dimensional and can be parameterized by 3 action coordinates from the interior of a moment polytope and 3 angle coordinates from the three-torus. Results from symplectic geometry show that the map between these action-angle coordinates and $\text{Pol}(6)/\text{SO}(3)$ is measure preserving. We use this parameterization to randomly sample 50 million equilateral hexagons and find that 0.01383% are non-trivial knots. Furthermore, we prove that a portion of the moment polytope consists of unknots, lowering the previous theoretical upper bound on the knotting probability of random equilateral hexagons. (Received September 17, 2019)

1154-54-2488 **Mustafa Hajij**, **Jesse S F Levitt*** (jslevitt@usc.edu) and **Radmila Sazdanovic**. *Learning from Filtrations on the Space of Knots.*

The authors discuss how to ‘best’ analyze knot invariant statistics on the space of knots. Consideration is given to filtrations based on any invariant that produces a well-ordered set. A way to measure which is the logical way to arrange knot tabulations is proposed based on the results. A variety of results and conjectures from statistics and data science that remain consistent under the ‘best’ filtration will be presented. (Received September 17, 2019)

1154-54-2717 **Nicholas Cazet** (nccazet@ucdavis.edu) and **Jane Park*** (jane.park@sjsu.edu). *Minimal Lattice Tiled Torus Surfaces Embedding Lattice Tiled Torus Knots.* Preliminary report.

Lattice tiled surfaces (or just “tiled” surfaces) exist as connected subsets of $(\mathbb{Z} \times \mathbb{R}^2) \cup (\mathbb{R} \times \mathbb{Z} \times \mathbb{R}) \cup (\mathbb{R} \times \mathbb{Z}^2) \subset \mathbb{R}^3$ which edges lie in $(\mathbb{Z}^2 \times \mathbb{R}) \cup (\mathbb{Z} \times \mathbb{R} \times \mathbb{Z}) \cup (\mathbb{R} \times \mathbb{Z}^2)$. Unlike conventional surfaces, and thus conventional knots, sitting in \mathbb{R}^3 , tiled surfaces have combinatorial consequences, and concepts of minimal surface area. In our talk, we will discuss the minimal tile tori for a few small tile (p, q) -torus knots. (Received September 17, 2019)

55 ► Algebraic topology

1154-55-151 **Chad M. Topaz*** (cmt6@williams.edu). *Topological Data Analysis of Collective Behavior*. From nanoparticle assembly to synchronized neurons to locust swarms, collective behaviors abound anywhere in nature that objects or agents interact. The study of collective behavior typically involves large data sets generated by experiment and/or simulation. This talk presents topological data analysis (TDA) as an approach for carrying out data science tasks in the context of collective behavior. The key approach is to characterize a system's dynamics via the time-evolution of topological invariants called Betti numbers, accounting for persistence of topological features across multiple scales. First, we use TDA to perform exploratory data analysis on the seminal aggregation model of Vicsek et al. (1995), identifying dynamical events that traditional methods do not. Next, we use TDA to choose between unbiased correlated random walk models of Nilsen et al. (2013) that describe motion tracking experiments on pea aphids. Finally, we investigate parameter recovery in the collective motion model of D'Orsogna et al. (2006). Machine learning methods with inputs derived from topology yield classification accuracy superior to ones with inputs derived from order parameters commonly used in biology and physics. This talk assumes no prior knowledge of topology or machine learning. (Received August 16, 2019)

1154-55-303 **Jonathan A Campbell*** (jonalfcam@gmail.com). *The Cyclotomic Trace and the Characteristic Polynomial*.

The cyclotomic trace is a powerful tool for computations in algebraic K-theory, and associated invariants have recently been used in number theory. However, the trace map is notoriously difficult to understand concretely. In this talk I'll describe work that relates the trace map to much more familiar invariants: zeta functions and the characteristic polynomial. The key input is a bicategorical view of the trace, and allows for tractable computations. This viewpoint simultaneously makes the trace more hands-on and more powerful.

This is joint work with John Lind, Cary Malkiewch, Kate Ponto and Inna Zakharevich (Received August 29, 2019)

1154-55-304 **Jonathan A Campbell*** (jonalfcam@gmail.com). *Some Homotopical Computations in Physics*.

In this talk I'll describe some homotopical computations that lead to classifications of symmetry protected phases. The computations are in agreement with physical literature, despite wildly different methods. In some sense, this can be regarded as an "experimental" verification of the cobordism hypothesis. (Received August 29, 2019)

1154-55-352 **Michał Adamaszek, Henry Adams*** (adams@math.colostate.edu) and **Florian Frick**. *Vietoris–Rips complexes of spheres*.

If a dataset is sampled from a manifold, then as more and more samples are drawn, the persistent homology of the Vietoris–Rips complex of the dataset converges to the persistent homology of the Vietoris–Rips complex of the manifold. But very little is known about the homotopy types of Vietoris–Rips complexes of manifolds. An exception is the case of the circle: as the scale parameter increases, the Vietoris–Rips complex of the circle obtains the homotopy types of the circle, the 3-sphere, the 5-sphere, the 7-sphere, . . . , until finally it is contractible. The Vietoris–Rips complex of the n -sphere first obtains the homotopy type of the n -sphere, and then next the $(n + 1)$ -fold suspension of a (topological) quotient of the special orthogonal group $SO(n + 1)$ by an alternating group A_{n+2} . Nothing is known at later scales, although one could conjecture that the critical scale parameters of Vietoris–Rips complexes of spheres are related to Lovász' strongly self-dual polytopes. (Received September 02, 2019)

1154-55-454 **Kevin P Knudson***, PO Box 118105, Gainesville, FL 32611. *Approximate triangulations of Grassmannians*. Preliminary report.

Schubert cell decompositions of Grassmannians have been known for decades, but ask for a specific triangulation of one of these manifolds and you'll be greeted with "Umm, triangulate the Schubert cells?" In this talk I will use persistent homology to construct simplicial complexes that approximate Grassmannians in the sense that their vertices lie on embedded Grassmannians and their homology is correct. This problem was suggested to me by Vidit Nanda and is still very much in progress. (Received September 04, 2019)

1154-55-483 **Hitesh Gakhar*** (gakharhi@msu.edu), 619 Red Cedar Road, Wells Hall, East Lansing, MI 48824, and **Jose A Perea** (joperea@msu.edu). *Sliding window embeddings of quasiperiodic functions*.

Classically, Sliding Window Embeddings were used in the study of dynamical systems to reconstruct the topology of underlying attractors from generic observation functions. In 2015, Perea and Harer used sliding window

embeddings of L^2 periodic functions and persistent homology to develop a technique for recurrence detection in time series data. We define a quasiperiodic function as a superposition of periodic functions with non-commensurate harmonics. As it turns out, sliding window embeddings of quasiperiodic functions are dense in high dimensional tori. In this talk, I will present some results for this case and present strategies to study their persistent homology using a persistent Künneth formula. (Received September 05, 2019)

1154-55-515 **Woojin Kim*** (kim.5235@osu.edu) and **Facundo Mémoli**. *Spatio-temporal persistent homology for dynamic metric spaces.*

We study the problem of characterizing the time evolution of dynamic metric spaces within the framework of topological data analysis. Popular instances of dynamic metric spaces include flocking/swarming behaviors in animals and social networks in the human sphere. We will discuss (1) how to induce multiparameter persistent homology as a topological summary of dynamic metric spaces, and (2) how to express the stability of this summarization process. In order to address stability, we define a new distance between dynamic metric spaces which extends the standard Gromov-Hausdorff distance on metric spaces. Also, we propose poly-time algorithms for the classification of dynamic metric spaces. A preprint with these results is available on <https://arxiv.org/abs/1812.00949>. Some demos are available in <https://research.math.osu.edu/networks/formigrams/>. (Received September 05, 2019)

1154-55-587 **Jun Hou Fung*** (jhfung@math.harvard.edu), Harvard University, Department of Mathematics, 1 Oxford Street, Cambridge, MA 02138. *Strict units of commutative ring spectra.*

Just as an ordinary commutative ring has a multiplicative group of units, a E_∞ -ring spectrum R also has a spectrum of units $gl_1 R$, which plays an important role for example in twisted cohomology theories. However, these spectra are typically very large, and to understand twists by Eilenberg-Mac Lane spaces or to isolate those units that come from geometry, it sometimes suffices to study the space of *strict units* of R . Previously, Hopkins and Lurie have computed the strict units of Morava E-theories, but much remains unknown about them in general.

In this talk, I will introduce these strict units and illustrate various methods for computing them for other commutative ring spectra R , and how these calculations relate to other interesting questions in homotopy theory. (Received September 07, 2019)

1154-55-619 **Agnes Beaudry*** (agnes.beaudry@colorado.edu), 107 Pine Glade Road, Nederland, CO 80466. *Homotopy theoretic methods in phases of matter.*

In this talk, I will discuss some interactions between homotopy theory and the classification of phases of matter, such as the connection between group cohomology, cobordism and the Freed-Hopkins classification. (Received September 08, 2019)

1154-55-754 **Edward Tymchatyn*** (tymchat@math.usask.ca), **Wojciech Debski**, **Kazuhiro Kawamura** and **Murat Tuncali**. *Inverse systems with simplicial bonding maps and cell structures.*

For a topologically complete space X and a sufficiently large family of normal, closed covers of X , we give a construction of an inverse system of simplicial complexes with simplicial bonding maps such that the inverse limit is homotopy equivalent to X . A connection with cell structures (i.e. certain inverse systems of graphs) is discussed. (Received September 10, 2019)

1154-55-822 **Yasuaki Hiraoka*** (hiraoka.yasuaki.6z@kyoto-u.ac.jp), Yoshida Ushinomiya-cho, Sakyo-ku, Kyoto, Kyoto 606-8501, Japan. *Limit theorems of persistent homology.*

The persistent homology of a stationary point process in a Euclidean space is studied in this talk. As a generalization of continuum percolation theory, we study higher dimensional topological features of the point process such as loops, cavities, etc. in a multiscale way. We prove the strong law of large numbers for persistence diagrams as the window size tends to infinity and give a sufficient condition for the support of the limiting persistence diagram to coincide with the geometrically realizable region. We also discuss the central limit theorem and the large deviation principle for persistent Betti numbers. Furthermore, a generalization to multi-parameter persistent homology is also studied in this talk. (Received September 11, 2019)

1154-55-893 **Luis G. Polanco*** (polanco2@msu.edu), 428 South Shaw Ln, Engineering Bldg, Rm. 1515, East Lansing, MI 48824, and **Jose Perea** (joperea@msu.edu), 428 South Shaw Lane, Engineering Bldg, Rm 1512, East Lansing, MI 48824. *Coordinatizing Data With Lens Spaces and Persistent Cohomology.*

We introduce here a framework to construct coordinates in *finite* Lens spaces for data with nontrivial 1-dimensional \mathbb{Z}_q persistent cohomology, for $q > 2$ prime. Said coordinates are defined on an open neighborhood of the data, yet constructed with only a small subset of landmarks. We also introduce a dimensionality reduction scheme in S^{2n-1}/\mathbb{Z}_q (Lens-PCA: LPCA), and demonstrate the efficacy of the pipeline \mathbb{Z}_q -persistent cohomology $\Rightarrow S^{2n-1}/\mathbb{Z}_q$ coordinates \Rightarrow LPCA, for nonlinear (topological) dimensionality reduction. (Received September 11, 2019)

1154-55-935 **Zhen Huan*** (huanzhen84@yahoo.com), Center for Mathematical Sciences, Huazhong University of Science and Technology, 1037 Luoyu Road, Wuhan, Hubei 430074, Peoples Rep of China, and **Nathaniel Stapleton.** *Level Structures and Morava E-theory.*

It is a historical problem how elliptic cohomology can classify the geometric structures on the corresponding elliptic curve. Strickland proved that the Morava E-theory of the symmetric group modulo a certain transfer ideal classifies the power subgroups of its formal group. Stapleton proved this result for generalized Morava E-theory via transchromatic character theory. And Huan proved that the subgroups of the Tate curve can be classified in the same way using quasi-elliptic cohomology. In this talk we show Strickland's theorem is also true for the classification of the level structures of generalized Morava E-theory via Hopkins-Kuhn-Ravenel character theory. This result gives further indications that Strickland's result holds for elliptic cohomology theories. (Received September 12, 2019)

1154-55-949 **Elizabeth Munch*** (muncheli@msu.edu), **Jose Perea** and **Firas A Khasawneh.** *Featurization of Persistence Diagrams using Template Functions for Machine Learning Tasks.*

The persistence diagram is an increasingly useful tool from Topological Data Analysis, but its use alongside typical machine learning techniques requires mathematical finesse. The most success to date has come from methods that map persistence diagrams into Euclidean space in a way which maximizes the structure preserved; this process is commonly referred to as featurization. In this talk, we describe a mathematical framework for featurization using "template functions". These functions are general as they are only required to be continuous and compactly supported. We will show applications for two exemplar template function families applied to synthetic and real data sets. (Received September 12, 2019)

1154-55-1271 **Clover May*** (clovermay@math.ucla.edu). *Decomposing C_2 -equivariant spectra.* Preliminary report.

Computations in $RO(G)$ -graded Bredon cohomology can be challenging and are not well understood, even for $G = C_2$, the cyclic group of order two. A recent structure theorem for $RO(C_2)$ -graded cohomology with coefficients in the constant Mackey functor \mathbb{F}_2 substantially simplifies computations. The structure theorem says the cohomology of any finite C_2 -CW complex decomposes as a direct sum of two basic pieces: cohomologies of representation spheres and cohomologies of spheres with the antipodal action. This decomposition lifts to a splitting at the spectrum level. In joint work with Dan Dugger and Christy Hazel we extend this result to a classification of compact modules over the Eilenberg-MacLane spectrum $H\mathbb{F}_2$. (Received September 14, 2019)

1154-55-1272 **Ningchuan Zhang*** (nzhang28@illinois.edu). *Analogs of Dirichlet L-functions in $K(1)$ -local homotopy theory.*

The relation between Eisenstein series and the J -homomorphism is an important topic in $K(1)$ -local homotopy theory. Both sides are related to the special values of the Riemann ζ -function. This relation is most clearly understood in the context of elliptic cohomology and topological modular forms.

Number theorists have studied the twistings of the Riemann ζ -functions and Eisenstein series by Dirichlet characters. In this talk, we investigate the analogs of Dirichlet character twistings in homotopy theory. We will introduce the Dirichlet twists of the J -spectrum. The homotopy groups of these Dirichlet J -spectra are related to the special values of the Dirichlet L -functions, and thus to congruences of the twisted Eisenstein series. We will explain the connection between Dirichlet J -spectra and the twisted Eisenstein series by generalizing Katz's algebro-geometric explanation of congruences of the normalized Eisenstein series. (Received September 14, 2019)

1154-55-1277 **Gunnar Erik Carlsson*** (gunnar@gunnarcarlsson.com), Department of Mathematics, Serra Street, Stanford, CA 94305. *Parametrized homotopy theory and motion planning problems.*

There are many problems in engineering that involve situations where one is given the topology of a region and is required to navigate the complement of this region. This comes up in robotics, sensor net technology, and other situations. It is interesting to consider problems where the set varies with time. Alexander duality is a natural tool to use for problems of this type, but it is not sufficient in many situations. We will survey topological results on these problems, and discuss some new directions involving techniques beyond homology. (Received September 14, 2019)

1154-55-1450 **Haibin Hang** (hhang@math.fsu.edu), Department of Mathematics, Florida State University, Tallahassee, FL 32306-4510, and **Washington Mio*** (mio@math.fsu.edu), Department of Mathematics, Florida State University, Tallahassee, FL 32306-4510. *Data analysis with continuously parametrized zigzag modules and beyond.* Preliminary report.

Persistence modules and zigzag modules are basic objects of study in topological data analysis. In this work, we generalize and unify these concepts in the framework of correspondence modules, which use partial linear relations between vector spaces as a replacement for linear maps. We prove a decomposition theorem for correspondence modules that leads to barcode representations of such modules. This allows us to formulate persistent homology in more general settings and analyze data using barcodes or persistence diagrams containing richer information. (Received September 15, 2019)

1154-55-1527 **Hessel Posthuma*** (h.b.posthuma@uva.nl), Korteweg-de Vries Institute for Mathematics, P.O. Box 94248, 1090 GE, Amsterdam, Netherlands. *Twisted K-theory and the classification of topological insulators.*

In this talk I will sketch, following Freed and Moore, how an exotic version of twisted equivariant K-theory classifies topological phases of free fermions in condensed matter physics. This version of K-theory turns out to be mathematically interesting in its own right. After that I will illustrate the theory with computations in some examples with crystal symmetries, based on joint work with de Boer, Kruthoff and Stehouwer. (Received September 16, 2019)

1154-55-1564 **Audun Myers, Elizabeth Munch and Firas A Khasawneh*** (khasawn3@egr.msu.edu), Department of Mechanical Engineering, 428 S. Shaw Lane, Rm 2503, East Lansing, MI 48824. *Topological Data Analysis for Detecting Dynamic State Changes via Nodal Networks.*

In this talk, we explore topological measures for identifying the state of a dynamic system by examining its time series. Specifically, we investigate embedding time series into a graph via two different methods: 1) Using Takens embedding and then mapping the embedded points into a network by connecting each node to its k -nearest neighbors, and 2) constructing ordinal networks by running a window of size n over the time series. The latter converts the time-indexed time series into an integer-indexed sequence of symbols or motifs. We study the resulting network using scores derived from their persistence diagrams, which are constructions from Topological Data Analysis (TDA). We apply our approach to several examples, show the results for both types of embeddings using novel TDA-based scores, and compare the resulting scores to existing methods which utilize graph-based scores. (Received September 16, 2019)

1154-55-1575 **Prasit Bhattacharya*** (pb9wh@virginia.edu). *Revisiting homotopy associative structures of Moore spectra.*

The Moore spectrum $M_p(i)$ is nothing but the cofiber of p^i -map on the sphere spectrum, yet the nice multiplicative properties of the sphere spectrum are not carried through to $M_p(i)$. In fact, it is conjectural that $M_p(i)$ is not an A_∞ ring spectrum. What can be the potential obstructions? In this talk, I will outline a method that will indicate that the obstructions to higher associativity of $M_p(i)$ could well be among the familiar Greek letter elements in the ‘image of J ’ part of the stable homotopy groups of spheres. This work is joint with N.Kitchloo. (Received September 16, 2019)

1154-55-1666 **Dominic Culver, Vitaly Lorman, Carl McTague, Brian Shin and Dylan Wilson*** (dwilson@math.harvard.edu). *Complex oriented cohomology of Nakajima quiver varieties.* Preliminary report.

Inspired by work of Yang and Zhao on cohomological Hall algebras, we study the convolution algebras and representations that arise when computing the Morava K-theory of Nakajima quiver varieties. This is joint work with Dominic Culver, Vitaly Lorman, Carl McTague, and Brian Shin. (Received September 16, 2019)

1154-55-1670 **Michael A. Hill*** (mahill@ksu.edu). *Abe Homotopy Groups.*

In condensed matter physics and elementary particle physics there are certain singularities called topological excitations that can be classified using homotopy groups. Sometimes these singularities can interact, requiring stronger invariants known as Abe homotopy groups.

After looking at the connection between homotopy groups and Abe homotopy groups, we will discuss how looking at equivalence classes of Abe homotopy up to an action of conjugation by the elements of the fundamental group gives useful information about models for physical systems. (Received September 16, 2019)

1154-55-1725 **Katharine L Adamyk*** (katharine.adamyk@colorado.edu). *Classification of $\mathcal{A}(1)$ -modules.* Preliminary report.

The Steenrod algebra, \mathcal{A} , arises from operations on cohomology (with coefficients in $\mathbb{Z}/p\mathbb{Z}$) that interact nicely with the stabilization of topological spaces. For $p = 2$, \mathcal{A} can be generated by a set of elements, the Steenrod squares, indexed by the nonnegative integers. The subalgebra of \mathcal{A} generated by the first 2^n Steenrod squares is denoted by $\mathcal{A}(n)$. Any \mathcal{A} -module inherits an $\mathcal{A}(n)$ -module structure, but not all $\mathcal{A}(n)$ -modules can be lifted to an \mathcal{A} -module. In this talk, we will focus on a classification of certain $\mathcal{A}(1)$ -modules that is useful for determining which $\mathcal{A}(1)$ modules can be lifted. (Received September 16, 2019)

1154-55-1754 **Christy Hazel*** (chazel@uoregon.edu). *Equivariant fundamental classes in $RO(C_2)$ -graded cohomology.*

Let C_2 denote the cyclic group of order two. Given a manifold with a C_2 -action, we can consider its equivariant Bredon $RO(C_2)$ -graded cohomology. In this talk, we show how a version of the Thom isomorphism theorem in $RO(C_2)$ -graded cohomology in constant $\mathbb{Z}/2$ coefficients can be used to develop a theory of fundamental classes for equivariant submanifolds. We then show these classes can be used to understand the cohomology of any C_2 -surface in constant $\mathbb{Z}/2$ coefficients, including the ring structure. (Received September 16, 2019)

1154-55-1794 **Robert H Stolz*** (rstolz@ucdavis.edu). *Modeling pathways of DNA unlinking by site-specific recombination.*

In Escherichia Coli, replication of circular chromosomes yields topologically linked DNA molecules. Topo IV, one of the type-II topoisomerases in E. coli, plays a major role in the decatenation of the newly replicated chromosomes. It has been shown that in the absence of Topo IV, site-specific recombinases XerC/D, in cooperation with the translocase FtsK, can also unlink the replication links in a stepwise manner. However, the topological pathways preferred by this enzyme complex are unknown. We use computational methods to model the recombination pathways as topological reconnection events in the simple cubic lattice and measure transition probabilities between relevant knotted and linked topologies of up to ten crossings. Our results give strong support to the stepwise unlinking pathway explored in previous studies and demonstrate the application of computational knot theory to address complex biological problems. (Received September 16, 2019)

1154-55-1814 **Indu Rasika Churchill***, State University of New York at Oswego, Oswego, NY 13126. *2-Cocycle Invariant and Oriented Singular Knots.*

The use of algebraic structures to study unoriented singular knots was introduced in 2017 by Elhamdadi, Hajji, Nelson and the speaker. In 2018, they were extended to the oriented case. We will review these algebraic structures and present some examples. Moreover, we extend the notion of quandle cocycle invariants to oriented singular knots. We will provide explicit examples of pairs of singular knots with the same number of colorings but distinguish between them using the quandle cocycle invariant, thus showing that the quandle cocycle invariant is a good enhancement. The talk is based on a joint preliminary work. (Received September 16, 2019)

1154-55-1817 **Gabe Angelini-Knoll and Dominic Culver*** (dculver@illinois.edu). *Towards the Topological Hochschild Homology of the second truncated Brown-Peterson spectrum.* Preliminary report.

In this talk, I will discuss work in progress towards computing the topological Hochschild homology of $BP\langle 2 \rangle$. In particular, I will describe work of Angelini-Knoll-Salch on a computational device, known as the THH-May spectral sequence, designed to compute THH. I will then indicate how this tool can be used to compute the THH of $BP\langle 2 \rangle$ with coefficients in the Adams summand. (Received September 16, 2019)

1154-55-1818 **Sarah Klanderma*** (kländer2@msu.edu). *Expanding Tools for Topological CoHochschild Homology.*

Hochschild homology (HH) is a classical algebraic invariant of rings that can be extended topologically to be an invariant of ring spectra, called topological Hochschild homology (THH). There exists a dual theory for coalgebras called coHochschild homology (coHH), and in recent work Hess and Shipley defined an invariant of coalgebra

spectra called topological coHochschild homology (coTHH). In this talk we will discuss coTHH calculations and the tools needed to do them. (Received September 16, 2019)

1154-55-1828 **Dmitriy Morozov*** (dmorozov@lbl.gov). *From Nanoporous Materials to Cosmology: Applied Topology at the National Labs.*

The talk will discuss the role of applied topology in the National Laboratory research, focusing on both the opportunities and the challenges. Specifically, we will show how combining persistent homology and machine learning helps identify nanoporous materials suitable for methane storage. And how the massive data sets generated by cosmologists require new algorithmic research in computational topology. (Received September 16, 2019)

1154-55-1888 **Meng Guo*** (mguo1@perimeterinstitute.ca), 31 Caroline St. N, Waterloo, ON N2L2Y5, and **Michael J Hopkins**. *Twisted tangential Pin^+ -bordism.*

We compute the bordism group of a Pin^+ manifold with a w_1 -twisted integer lift of tangential w_4 for dimension ≤ 14 . It is homotopy orbit of $\text{MSpin} \wedge K(\mathbb{Z}, 4)_+$ by a $\mathbb{Z}/2$ -action. We find its manifold generators up to dimension 14 and compute the group action. We apply homotopy orbit special sequence for computations. (Received September 16, 2019)

1154-55-1920 **Elizabeth Vidaurre***, evidaurre@molloy.edu, and **Robin Belton, Robyn Brooks, Stefania Ebli, Brittany Terese Fasy, Nicole Sanderson, Catherine Ray and Lisbeth Fajstrup**. *Towards Directed Collapsibility.*

In the directed setting, the spaces of directed paths between fixed initial and terminal points are the defining feature for distinguishing different directed spaces. The simplest case is when the space of directed paths is homotopy equivalent to that of a single path; we call this the trivial space of directed paths. Directed spaces that are topologically trivial may have non-trivial spaces of directed paths, which means that information is lost when the direction of these topological spaces is ignored. We define a notion of directed collapsibility in the setting of a directed Euclidean cubical complex using the spaces of directed paths of the underlying directed topological space relative to an initial or a final vertex. In addition, we give sufficient conditions for a directed Euclidean cubical complex to have a contractible or a connected space of directed paths from a fixed initial vertex. We also give sufficient conditions for the path space between two vertices in a Euclidean cubical complex to be disconnected. Our results have applications to speeding up the verification process of concurrent programming and to understanding partial executions in concurrent programs. (Received September 16, 2019)

1154-55-1972 **Matthew Spong*** (matt.spong@gmail.com), 16/671 Park St, Brunswick, VIC 3056, Australia, and **Zhen Huan**. *Twisted quasi-elliptic cohomology and a twisted Chern character map.* Preliminary report.

Let G be a finite group and X be a G -space. In a recent paper, Huan constructed the quasi-elliptic cohomology theory. This is a variant of Tate K -theory assigning a $\mathbb{Z}[q^\pm]$ -module $QEll(X//G)$ to the orbifold $X//G$. In this talk, given a 3-cocycle α , we present a construction of an α -twisted version of $QEll$. Furthermore, we also discuss the construction of a twisted Chern character map from the latter object to an α -twisted version of Devoto's G -equivariant elliptic cohomology, which appeared in a recent paper of Berwick-Evans. In the future, we expect to use these constructions to define twisted power operations in quasi-elliptic cohomology, and to compare them to operations in Devoto's theory. (Received September 16, 2019)

1154-55-1981 **Safia Chettih***, chettihs@southwestern.edu, and **L Bonatto, A Linton, S Raynor, M Roberston and N Wahl**. *A combinatorial model for an honest ∞ -operad.* Preliminary report.

The natural definition of gluing on metric Sullivan diagrams is not associative on the nose, but we expect it to be associative up to all higher homotopies. This suggests that metric Sullivan diagrams, together with the structure induced by gluing, should form an ∞ -prop of some sort. We approach the problem through the cacti operad; as a space, cacti include into metric admissible fat graphs, of which metric Sullivan diagrams are a quotient. We describe an ∞ -operad inspired by the cacti operad, as well as work towards promoting this construction to an ∞ -prop. (Received September 17, 2019)

1154-55-1984 **Kiran Luecke*** (kiranluecke@berkeley.edu). *Elliptic Cohomology at the Tate Curve.*

Kitchloo and Morava gave a strikingly simple picture of elliptic cohomology at the Tate curve in terms of K -theory. In this talk I will present an equivariant version of their construction which relates equivariant elliptic cohomology to the loop groups and twisted K -theory saga of Freed–Hopkins–Teleman. Some applications and further directions will be discussed. (Received September 17, 2019)

1154-55-2092 **Shane Clark*** (shane.clark@uky.edu), 715 Patterson Office Tower, Lexington, KY 40506. *Periodic Points: Vanishing and Realizability.*

The Reidemeister trace of an endomorphism of a CW complex is a lower bound for the number of fixed points (up to homotopy) of that endomorphism. For an endomorphism f , the Reidemeister trace of f^n is a lower bound for the number of fixed points of f^n , however it can be a far from an optimal lower bound.

There are many related invariants that refine the Reidemeister trace and have different strengths and weaknesses in regards to their computation and realizability. In this talk we will describe a classes of spaces where these invariants can be computed and realized using the classical Lefschetz fixed point theorem. (Received September 17, 2019)

1154-55-2120 **Antonio Rieser*** (antonio.rieser@cimat.mx). *Vietoris-Rips homology through the looking glass.*

Although the Vietoris-Rips complex is over 90 years old, very little is known about the properties of the Vietoris-Rips homology. We describe several new ways in which the Vietoris-Rips complex may be defined, and which help to clarify several of its basic properties. We show that these definitions reduce to classical one on graphs and metric spaces, and we then explore variations of continuity for functions between spaces for which the Vietoris-Rips homology is functorial. We conclude with a definition of homotopy which respects the Vietoris-Rips homology functor (i.e. such that two homotopic functions induce the same map) and, finally, with a discussion of the Kunneth theorem for Vietoris-Rips homology. (Received September 17, 2019)

1154-55-2154 **Brett A. Jefferson*** (brett.jefferson@pnnl.gov). *An Investigation Using Homology and Inverted Distances.* Preliminary report.

Persistent homology has been gaining traction as a tool for studying patterns in real-world data-sets. We have sought to understand a slight variant on the typical computation of homology of a metric space. This exposition will survey that work. We include early findings from computations on samples from common spaces (the circle, the disc, and the square), one conjecture and one lemma regarding 1-dimensional cycles, and an application to simulated data. (Received September 17, 2019)

1154-55-2313 **Peter J Bonventre*** (peterbonventre@uky.edu), **Bertrand Guillou** and **Nathaniel Stapleton.** *Power operations and transfers in equivariant cohomology theory.*

A commutative ring spectrum E gives rise to a cohomology theory with power operations. These are naturally multiplicative, and become additive after modding out by a transfer ideal. When E is a genuine G -spectrum, the associated cohomology theory is valued in the more general setting of (structured) Mackey functors. In this talk, I'll present joint work showing these structures are suitably compatible for a commutative G -ring spectrum E , modulo a (potentially larger) transfer ideal. This is the first stage in a program to describe C_2 -equivariant global Real K-theory. (Received September 17, 2019)

1154-55-2321 **Amy Qing Hai Li*** (qli@wellesley.edu). *Constructing Partial Configuration Spaces Using Limits.*

Given a topological space X , the configuration space $\text{Conf}(k, X)$ consists of k -tuples of distinct points on X . For example, $\text{Conf}(2, X)$ is homeomorphic to $X \times X$ with the diagonal removed. We can generalise this idea to *partial* configuration spaces $\text{Conf}(n_1, n_2, \dots, n_k, X)$ consisting of $(n_1 + n_2 + \dots + n_k)$ -tuples of points in X where elements in each set of n_i points are free to coincide with each other, but points from different sets must be distinct. To study these spaces, we use methods from category theory. In this talk, we will cover some background on functors between categories and limits of diagrams. By considering a particular functor between an indexing category and the category of topological spaces, we will show that a partial configuration space $\text{Conf}(n_1, n_2, \dots, n_k, X)$ is the limit of a diagram of ordinary configuration spaces. (Received September 17, 2019)

1154-55-2380 **Zhouli Xu*** (xuzhouli@mit.edu), Massachusetts Institute of Technology, Department of Mathematics 2-173, Cambridge, MA 02139. *Motivic t-structures and stable homotopy groups of spheres.*

I will discuss recent work that there exists a t -structure on the category of motivic cellular \widehat{S} -modules over fields of characteristic 0, with the heart equivalent to MU_*MU -comodules. This gives us computational tools for \mathbb{R} -motivic and C_2 -equivariant stable stems. This is joint work with Dan Isaksen, Hana Jia Kong and Guozhen Wang. (Received September 17, 2019)

1154-55-2469 **Daniel Scofield***, daniel.scofield@fmarion.edu, and **Radmila Sazdanovic**, rsazdanovic@math.ncsu.edu. *On extreme Khovanov homology, the Jones polynomial, and the girth of a link.*

In this talk we utilize relations between Khovanov and chromatic graph homology to determine extreme Khovanov groups and corresponding coefficients of the Jones polynomial. We define the girth of a link and describe its properties. (Received September 17, 2019)

1154-55-2536 **Michelle H Feng***, mhfeng@math.ucla.edu, and **Mason A Porter**. *Cities, voting, and spider DUIs: Case studies in spatial applications of topological tools.*

Persistent Homology (PH) has been used to study the topological characteristics of data across a variety of scales. In this talk, we will focus on a variety of spatial applications, as the geometric and topological features of PH are well suited to exploring data sets which are embedded in space. We will introduce two novel constructions for transforming network-based data into simplicial complexes suitable for PH computations and compare these constructions to state of the art. Additionally, we will apply these constructions to a variety of geographic and spatial applications, including voting data, cities and urban networks, and biological networks (i.e. spiders under the influence). We will highlight the computational performance of our constructions and discuss the implications of the PH computations for identifying and classifying certain features in our various data sets. In particular, we discuss spatial patterns which emerge in each case, and how those patterns relate to existing scholarship in the relevant area. (Received September 17, 2019)

1154-55-2582 **Daniel Berwick-Evans** and **Arnav Tripathy*** (tripathy@math.harvard.edu). *On the geometric model for de Rham elliptic cohomology.*

I will discuss aspects of the geometric model for elliptic cohomology, or topological modular forms. (Received September 17, 2019)

1154-55-2671 **Kerry M. Luse*** (lusek@trinitydc.edu) and **Mark E. Kidwell**. *The Alexander polynomial of a rational link.*

Our recent work showed certain terms of the Alexander polynomial $\Delta(x, y)$ of a rational link are related to the number and length of monochromatic twist sites in a particular diagram that we call the standard form. If the rational link has a reduced alternating diagram with no self-crossings, then $\Delta(-1, 0) = 1$. If the standard form of the rational link has M monochromatic twist sites, and the j th monochromatic twist site has m_j crossings, then $\Delta(-1, 0) = \prod_{j=1}^M (m_j + 1)$. Our proof employs Kauffman's clock moves and a lattice for the terms of $\Delta(x, y)$ in which the y -power cannot decrease. We conjecture that $\Delta(-1, 0) = 1$ holds for all 2-component alternating links with no self-crossings. (Received September 17, 2019)

1154-55-2679 **Danuta Kolodziejczyk*** (dkolodz@mini.pw.edu.pl), Warsaw University of Technology. *Polyhedra with Finite Depth II.*

A domination in a category \mathcal{C} is a morphism $f : X \rightarrow Y$, $X, Y \in \text{Ob}\mathcal{C}$, such that there exists a morphism $g : Y \rightarrow X$ with $fg = \text{id}_X$. Then Y is dominated by X (we write $X \geq Y$). In the following, \mathcal{C} is the homotopy category of CW -complexes, or the shape category of compacta.

Let P be a polyhedron. We may ask, if each sequence $P \geq X_1 \geq X_2 \geq \dots$ contains only finitely many different homotopy types of X_i or, does there exist an integer l_P (depending only on P) such that each sequence as above contains $\leq l_P$ different homotopy types? The answers are known only in dimension 1. It should be noted that there exist polyhedra (even 2-dimensional) dominating infinitely many different homotopy types or shapes (DK, 1996).

These questions are closely related to the problems of K. Borsuk (1967, 1975): "Is it true that two ANR 's homotopy dominating each other have the same homotopy type?" and "Are the homotopy types of two quasi-homeomorphic ANR 's equal?"

We will show that for some polyhedra (including all 2-dimensional polyhedra) the answers depend only on the properties of the fundamental group $\pi_1(P)$, and will present new positive results. (Received September 17, 2019)

57 ► *Manifolds and cell complexes*

1154-57-40 **Yu Pan***, 77 Massachusetts ave, Cambridge, MA 02139, and **Dan Rutherford**. *All the augmentations come from immersed Lagrangian fillings.*

Augmentations are tightly connected to embedded exact Lagrangian fillings. However, not all the augmentations of a Legendrian knot come from embedded exact Lagrangian fillings. In this talk, we show that all the augmentations come from possibly immersed exact Lagrangian fillings. In particular, for a 1-dimensional Legendrian knot in a 1-jet space, take an immersed exact Lagrangian filling that can be lifted to an embedded Legendrian L . For any augmentation of L , we associate an induced augmentation of the Legendrian knot, whose homotopy class only depends on the compactly supported Legendrian isotopy type of L and the homotopy class of its augmentation. This is a joint work with Dan Rutherford in progress. (Received July 13, 2019)

1154-57-41 **Yu Pan***, yupan@mit.edu. *Augmentations and exact Lagrangian surfaces.*

Augmentations are some algebraic invariants of Legendrian that are tightly related to both embedded and immersed exact Lagrangian fillings. We will talk about various relations between embedded and immersed exact Lagrangian surfaces using tools related to augmentations. (Received July 13, 2019)

1154-57-175 **Marion Campisi*** (marion.campisi@sjsu.edu), **Ryan Blair**, **Maggy Tomova** and **Scott Taylor**. *Distortion and the bridge distance of knots.*

The distortion of an embedding of a knot is the supremum of the ratio of the distance between a pair of points along the knot and the distance between the points in \mathbb{R}^3 . The distortion of a knot type is the infimum of the distortion over all embeddings. We extend techniques due to Pardon to show that there is a lower bound on the distortion of a knot in \mathbb{R}^3 proportional to the minimum of the bridge distance and the bridge number of the knot. We also exhibit an infinite family of knots for which the minimum of the bridge distance and the bridge number is unbounded and Pardon's lower bound is constant. (Received August 19, 2019)

1154-57-186 **Ryo Ohashi*** (ryoohashi@kings.edu), 133 North River Street, Wilkes-Barre, PA 18711. *The finite group actions on I -bundle over the projective plane \mathbb{P}^2 .* Preliminary report.

A finite G -action on a manifold M is a monomorphism $\varphi : G \rightarrow \text{Homeo}(M)$, where G is a finite group. In this talk, M is an I -bundle over the projective plane \mathbb{P}^2 , where $I = [0, 1]$. We will discuss all finite G -actions on $\mathbb{P}^2 \times I$.

A method is to study actions on \mathbb{P}^2 . Notice that its universal covering space is the 2-sphere \mathbb{S}^2 . Thus, we can lift any acting groups on \mathbb{P}^2 to \tilde{G} on \mathbb{S}^2 . It has been known the finite group actions on \mathbb{S}^2 , which is a subgroup of some permutation group S_n . This process enables us to analyze the finite G -actions on \mathbb{P}^2 by observing a fundamental region on \mathbb{S}^2 with the aid of an appropriate triangulation on \mathbb{S}^2 .

After establishing the actions on \mathbb{P}^2 , we may have optimistic feeling to describe the actions on $\mathbb{P}^2 \times I$, the I -bundle over the projective plane since I admits only \mathbb{Z}_2 -action. However, we may not always obtain $G \times \mathbb{Z}_2$. In fact, there are pitfalls in order to reach the conclusion which will be addressed in the talk. (Received August 21, 2019)

1154-57-188 **Charles Frohman*** (charles-frohman@uiowa.edu), IA, and **Joanna Kania-Bartoszyńska** (jkaniaba@nsf.gov). *A geometric field theory for three-manifolds.*

Given a compact three-manifold M equipped with a trace equivalence class of representations of its fundamental group into $SL_2\mathbb{C}$ whose image and its image of its restriction to the fundamental groups of all boundary components is Zariski dense in $SL_2\mathbb{C}$ we show how to reduce the Kauffman bracket skein module of the three-manifold to give a state space that is part of a field theory. (Received August 21, 2019)

1154-57-199 **Heather A Dye*** (heatheranddye@gmail.com) and **Aaron Kaestner**. *Virtual Parity Alexander Polynomial.*

In this talk, we define the parity virtual Alexander polynomial following the work of BDGGHN and Kaestner and Kauffman. The properties of this invariant are explored and some examples. In particular, the invariant demonstrates that many virtual knots can not be unknotted by crossing change on only odd crossings. (Received August 22, 2019)

- 1154-57-324 **Jesús A. Álvarez López*** (jesus.alvarez@usc.es), Department/Institute of Mathematics, 15782 Santiago Compostela, Spain, **Yuri A. Kordyukov** (yurikor@matem.anrb.ru), Institute of Mathematics, 112 Chernyshevsky str., Ufa, 450008, Russia, and **Eric Leichtnam** (ericleichtnam@math.jussieu.fr), Institut Mathématiques de Jussieu-PRG, CNRS, Batiment Sophie Germain (bureau 740), Case 7, 75205 Paris, France. *A trace formula for foliated flows.*
- Let \mathcal{F} be a smooth codimension one foliation on a compact manifold M . A flow ϕ^t on M is said to be foliated if it maps leaves to leaves. If moreover the closed orbits and preserved leaves are simple, then there are finitely many preserved leaves, which are compact, forming a compact subset M^0 , and a precise description of the transverse structure of \mathcal{F} can be given. A version of the reduced leafwise cohomology, $\overline{HI}(\mathcal{F})$, is defined by using distributional leafwise differential forms conormal to M^0 . The talk will be about our progress to define distributional traces of the induced action of ϕ^t on $\overline{H}^r I(\mathcal{F})$, for every degree r , and to prove a corresponding Lefschetz trace formula involving the closed orbits and leaves preserved by ϕ^t . The formula also involves a distributional version of the η -invariant of M^0 . This kind of distributional trace formula was conjectured by Christopher Deninger, and it was proved by the first two authors when $M^0 = \emptyset$. (Received August 31, 2019)
- 1154-57-386 **Naoko Kamada*** (kamada@nsc.nagoya-cu.ac.jp), 1 Yamanohata, Mizuho-cho, Mizuho-ku, Nagoya, Nagoya 467-8501, Japan. *Constructing almost classical virtual links from virtual links.*
- Any classical link diagram admits an Alexander numbering. If a virtual link diagram admits an Alexander numbering, it is said to be almost classical. Almost classical virtual link diagrams have a similar property to classical link diagrams. In this talk, we give a map from the set of virtual link diagrams to that of almost classical virtual link diagrams. It induces a map from the set of virtual links to that of almost classical virtual links. (Received September 03, 2019)
- 1154-57-435 **Seiichi Kamada*** (kamada@math.sci.osaka-u.ac.jp). *Motions and their actions on the fundamental groups and quandles of H-trivial links.*
- An H-trivial link of type (m, n) is a link in 3-space which is ambient isotopic to the split union of m Hopf links and a trivial link with n components. We discuss motions and their actions on the fundamental groups and quandles for H-trivial links. We first recall a recent result, with C. Damiani, on the ring group for small (m, n) , and then introduce a result with C. Damiani and R. Piergallini on a generating set of the motion group for any (m, n) . (Received September 04, 2019)
- 1154-57-495 **Nicolas Petit*** (nicolas.petit@emory.edu), Oxford College of Emory University, Oxford, GA 30054. *The multi-variable affine index polynomial.*
- We will introduce a multi-variable version of the Affine Index Polynomial for virtual links. This invariant generalizes the original Affine Index Polynomial and is computable on both compatible and incompatible diagram. We will discuss its definition, some properties and a couple of examples. (Received September 05, 2019)
- 1154-57-519 **Blake Mellor*** (blake.mellor@lmu.edu) and **Sean Nevin**. *Virtual Rational Tangles.*
- We use Kauffman's bracket polynomial to define a complex-valued invariant of virtual rational tangles that generalizes the well-known fraction invariant for classical rational tangles. We provide a recursive formula for computing the invariant, and use it to compute several examples. (Received September 06, 2019)
- 1154-57-610 **Felipe Castellano-Macias** and **Nicholas Owad*** (nick@owad.org). *Computation of tunnel numbers for low crossing knots.*
- Tunnel number is a notoriously hard invariant to compute in general. But for knots with low crossing number, we can get useful bounds from many important results, such as Lackenby's classification of alternating tunnel number one knots and others. These bounds have completely computed the tunnel number of 11 and 12 crossing alternating knots, allowing us to greatly increase the known values of tunnel numbers. We discuss some of the other ways we can compute tunnel number and how we can push these bounds further to find more tunnel numbers. (Received September 08, 2019)
- 1154-57-637 **Paul Beirne*** (paul.beirne@ucdconnect.ie), 72 Oaktree Road, Merville, Stillorgan, Dublin, A94 TA44, Ireland. *Knot invariants and coefficient stability.*
- In 2006, Dasbach and Lin observed stability in the coefficients of the N^{th} colored Jones polynomial for alternating knots. This observation and its consequences have sparked a flurry of activity in both number theory and quantum topology. For example, Garoufalidis, Le and Zagier conjectured identities which have a striking resemblance to those occurring in the classical setting of Rogers and Ramanujan. In this talk, we discuss these

developments and a higher order stability formula for an infinite family of pretzel links. (Received September 09, 2019)

1154-57-724 **Agnese Barbensi***, barbensi@maths.ox.ac.uk. *Double branched cover of knotoids and applications to proteins.*

Knotoids are a generalisation of knots that deals with open curves. In the past few years, they've been extensively used to classify entanglement in proteins. Through a double branched cover construction, we prove a 1-1 correspondence between knotoids and strongly invertible knots. We characterise forbidden moves between knotoids in terms of equivariant band attachments between strongly invertible knots, and in terms of crossing changes between theta-curves. Finally, we present some applications to the study of the topology of proteins. This is based on joint works with D.Buck, H.A.Harrington, M.Lackenby and with D. Goundaroulis. (Received September 10, 2019)

1154-57-730 **Keiko Kawamuro***, 14 MacLean Hall, Iowa City, IA 52242. *Right-veering and twist-left-veering open books.*

Given a contact 3-manifold, detecting tight or overtwisted is a fundamental problem. Via the Giroux-correspondence, studying contact 3-manifolds is equivalent to studying open books. Honda, Kazez, and Matić proved that non-right-veering open books support overtwisted contact structures. In this talk, I will define twist-left-veering open books that I introduced with Tetsuya Ito, and show that if a right-veering open book is twist-left veering then it supports an overtwisted contact structure. I also give comparison of twist-left-veering and Wand's inconsistency and give applications. (Received September 10, 2019)

1154-57-892 **Maggie Miller*** (maggie@princeton.edu). *Dehn surgery on links vs. the Thurston norm.* Let L be an $n > 1$ -component link in a rational homology sphere Y with pairwise nonzero linking numbers. Let S be a Thurston norm-minimizing surface in the complement X of L . By work of Gabai, S is the leaf of a taut foliation on X . Note that $H_2(X, \partial X; \mathbb{R})$ is rank n .

I show that if $[S]$ is primitive and outside a dimension- $(n-2)$ subset of $H_2(X, \partial X; \mathbb{R})$, then $Y_{\partial S}(L)$ admits a taut foliation containing \widehat{S} as a leaf, so \widehat{S} is norm-minimizing (and we conclude e.g. \widehat{S} is essential, and if S is not genus-zero then the surgered manifold is not $S^1 \times S^2$). In particular, when $n = 2$, there are only finitely many primitive classes where no taut foliation extends after surgery.

In this short talk, I will motivate the theorem and sketch the overall proof method. (Received September 11, 2019)

1154-57-971 **Micah Chrisman*** (chrisman.76@osu.edu). *Slice obstructions in the lower central series of the extended virtual knot group.* Preliminary report.

Let Σ be a closed oriented surface and K a knot in $\Sigma \times [0, 1]$. Then K is said to be (virtually) slice if there is a compact connected oriented 3-manifold W and a disc D smoothly embedded in $W \times [0, 1]$ such that $\partial W = \Sigma$ and $\partial D = K$. Here we prove that the lower central series of the extended virtual knot group \bar{G} is a concordance invariant for knots in thickened surfaces. Numerical concordance invariants for knots can be extracted from the nilpotent quotients which are analogous to Milnor's $\bar{\mu}$ -invariants of classical multi-component links in S^3 . There are 92800 virtual knots having at most six classical crossings. We apply our invariants to determine their slice status. Using our invariants together with all previously known slice obstructions, we reduce to 4 the number of virtual knots having unknown slice status. In particular, the new invariants can obstruct sliceness for many virtual knots having trivial Rasmussen invariant, graded genus, generalized Alexander polynomial, affine index polynomial, and parity projection. (Received September 12, 2019)

1154-57-994 **Louis H Kauffman*** (kauffman@uic.edu), Math, UIC, 851 South Morgan Street, Chicago, IL 60607-7045. *Quandles and Biquandles in Virtual Knot Theory.* Preliminary report.

This talk is a discussion of structures and examples of the use of biquandles and quandles in virtual knot theory and their relationship with quantum invariants and polynomial invariants of virtual knots and links. (Received September 12, 2019)

1154-57-1104 **Eleni Panagiotou***, 415 EMCS Building Dept 6956, 615 McCallie Ave, Chattanooga, TN 37403, and **Louis H. Kauffman**. *The bracket polynomial of a polygonal curve in 3-space.*

We introduce a method to measure entanglement of chains in 3-space that extends the notion of knot and link polynomials to open chains in 3-space. We define the bracket polynomial of curves in 3-space and show that it has real coefficients and is a continuous function of the chain coordinates. This is used to define the Jones polynomial of curves in 3-space in a way that it is applicable to both open and closed chains in 3-space. For open chains, it has real coefficients and it is a continuous function of the chain coordinates and as the endpoints

of the chain tend to coincide, the Jones polynomial of the open chain tends to that of the resulting knot. For closed chains, it is a topological invariant, as the classical Jones polynomial. We show how these measures attain a simpler expression for polygonal chains. (Received September 13, 2019)

1154-57-1139 **Robert C Haraway, III*** (bobbyciii@fastmail.com), 506 MSCS Building, Oklahoma State University, Stillwater, OK 74074, and **Neil R Hoffman**. *On the complexity of cusped non-hyperbolicity.*

We show that the problem of showing that a cusped 3-manifold M is not hyperbolic is in NP, assuming S^3 -recognition is a coNP problem. Our key contributions are a certificate that a manifold is $T^2 \times I$ and a certificate that an irreducible 3-manifold is toroidal, both verifiable in polynomial time. (Received September 13, 2019)

1154-57-1332 **Alissa S Crans*** (acrans@lmu.edu), **Marco Bonatto** and **Glen Whitney**. *The Structure of Hom Quandles.*

If A is an abelian quandle and Q is a quandle, the hom set $\text{Hom}(Q, A)$ of quandle homomorphisms from Q to A has a natural quandle structure. We show that it suffices to consider only medial source quandles, and therefore the structure theorem of Jedlicka et. al. provides a characterization of the Hom quandle. In the particular case when the target is 2-reductive this characterization takes on a simple form that makes it easy to count and determine the structure of the Hom quandle. (Received September 14, 2019)

1154-57-1347 **Ryan Moag** and **Boris Goldfarb*** (bgoldfarb@albany.edu). *An application of nerve complexes in imputation.*

This work uses a well-known tool from topological data analysis, the Mapper algorithm, to leverage geometric shape of data sets when performing imputation of missing values. This is an important step in data preparation for statistical and machine learning tasks. I will describe the general strategy and show results of experiments that demonstrate effectiveness of the method in the presence of complicated homotopy theoretic features and non-linear correlations in data. (Received September 15, 2019)

1154-57-1353 **Scott Baldridge*** (sbaldrid@math.lsu.edu), Department of Mathematics, Baton Rouge, LA 70803. *A new cohomology for planar trivalent graphs with perfect matchings.* Preliminary report.

In this lecture, I will describe a simple-to-compute polynomial invariant of a planar trivalent graph with a perfect matching (think: Jones polynomial for graphs). This polynomial when evaluated at 1 counts the number of 3-edge colorings of the graph that has the same color for all the perfect matching edges. (If the count is non-zero, it implies the map is 4-colorable.) I will then discuss how to categorify this polynomial to get a Khovanov-like cohomology theory for planar trivalent graphs. This is the first "TQFT" for graphs that gives combinatorial information instead of topological invariants. (Received September 15, 2019)

1154-57-1409 **Puttipong Pongtanapaisan*** (puttipong-pongtanapaisan@uiowa.edu) and **Daniel Rodman**. *Knots with Keen Weakly Reducible Bridge Spheres.*

A bridge sphere for a knot is a sphere transverse to the knot separating the maxima of the knot from the minima. Some simple closed curves on the bridge sphere will bound essential disks in the knot complement on at least one side. In this talk, I will exhibit infinitely many examples of knots, whose canonical bridge spheres admit a unique disjoint pair of simple closed curves bounding essential disks on distinct sides. Such a bridge sphere is said to be keen weakly reducible. This is joint work with Daniel Rodman. (Received September 15, 2019)

1154-57-1430 **Nancy Scherich** and **Sherilyn Tamagawa*** (shtamagawa@davidson.edu). *Virtual Niebrzydowski Algebras and Trivalent Spatial Graphs.* Preliminary report.

In this talk, we will introduce virtual Niebrzydowski algebras, algebraic structures with two ternary operations and a partially defined multiplication, whose axioms are motivated by the Reidemeister moves for Y-oriented virtual trivalent spatial graphs and handlebody-links. We will also show examples of the counting invariant on some trivalent spatial graphs. (Received September 15, 2019)

1154-57-1431 **Mohamed Elhamdadi**, **Masahico Saito*** (saito@usf.edu) and **Emanuele Zappala**. *Skein for Yang-Baxter homology.* Preliminary report.

Homology theories for the Yang-Baxter equation (YBE) have been developed and studied, with applications to knot invariants and deformation theories. We introduce a skein computation for the YB homology for the R-matrix corresponding to the Jones polynomial. A homology for such a matrix was defined by Przytycki and Wang by normalizing the R-matrix appropriately. We modify the skein relation accordingly for this normalization.

Diagrammatic computations of low dimensional homology groups are given, and an annihilation result that measures torsions is presented. (Received September 15, 2019)

1154-57-1456 **Sujoy Mukherjee*** (mukherjee.166@osu.edu) and **Dirk Schuetz** (dirk.schuetz@durham.ac.uk). *Arbitrarily large torsion in Khovanov homology*. Preliminary report.

Khovanov homology, an invariant of knots and links, is a categorification of the Jones polynomial. My talk will focus on some new results regarding torsion in Khovanov homology while keeping a very important question in mind. What does torsion in Khovanov homology signify? We will see how torsion in Khovanov homology behaves under the connected sum operation for certain knots and links. Using this, I will establish that torsion in Khovanov homology can be arbitrarily large. (Received September 15, 2019)

1154-57-1518 **Mark Clifford Hughes***, 324 TMCB, Provo, UT 84602, and **Thomas Liddle, Jamison Moody** and **Spencer Reschke**. *Reinforcement learning for constructive proofs in knot theory*. Preliminary report.

Low-dimensional topology has numerous examples of problems whose solutions require constructing sequences of operations taken from a fixed set of moves. Examples include showing that two handle diagrams represent the same smooth 4-manifold by Kirby moves, trivializing a group presentation by Andrews-Curtis moves, or constructing slice surfaces of a knot by Morse modifications to a diagram. In this talk I will focus on this last problem, and discuss early results on how deep reinforcement learning may provide an avenue for constructing specific examples of genus minimizing slice surfaces, and computing the slice genus. In particular, I will describe the framework of deep Q-learning, as well as some recent enhancements such as dueling network architectures, prioritized experience replay, and asynchronous methods, and outline some results. (Received September 16, 2019)

1154-57-1526 **Seung Yeop Yang*** (seungyeop.yang@knu.ac.kr). *An introduction to the normalized set-theoretic Yang-Baxter homology theory*.

A homology theory for the set-theoretic Yang-Baxter equation is introduced by Carter, Elhamdadi, and Saito in 2002. We define the normalized homology theory of certain set-theoretic solutions to the Yang-Baxter equation, such as Rump quasigroups, biquandles, etc., which is a modified form of their set-theoretic Yang-Baxter homology. We, moreover, introduce homological and homotopical knot invariants obtained from the normalized homology theory. (Received September 16, 2019)

1154-57-1595 **Samantha Allen*** (samantha.g.allen@dartmouth.edu) and **Charles Livingston**. *Unknotting with a single twist*.

Ohyama showed that any knot can be unknotted by performing two full twists, each on a set of parallel strands. We consider the question of whether or not a given knot can be unknotted with a single full twist, and if so, what are the possible linking numbers associated to such a twist. It is observed that if a knot can be unknotted with a single twist, then some surgery on the knot bounds a rational homology ball. With this, a wider range of tools become available, including Casson-Gordon invariants and a set of obstructions that Aceto-Golla developed using Heegaard-Floer theory. Using these tools, if a knot K can be unknotted with a single twist of linking number l , we give restrictions on the genus, signature function, Upsilon function, and V and ν^+ invariants of K in terms of l and the sign of the twist. This talk is based on joint work with Charles Livingston. (Received September 16, 2019)

1154-57-1606 **Douglas D Knowles*** (douglas.d.knowles.gr@dartmouth.edu), 66 Gould Road, West Lebanon, NH 03784, and **Ina Petkova**. *Bordered Heegaard Floer Homology over \mathbb{Z}* . Preliminary report.

Using nice diagrams, we provide an integral lift of bordered Heegaard Floer Homology in the case of torus boundary. We will briefly discuss applications to knot complements. This is joint work with Ina Petkova. (Received September 16, 2019)

1154-57-1750 **Rhea Palak Bakshi*** (rhea_palak@gwu.edu) and **Józef H. Przytycki**. *Categorification of the Kauffman Bracket Skein Module of the Projective Space and Lens Spaces*.

We unify the methods used by Asaeda, Przytycki and Sikora to categorify the Kauffman bracket skein modules of twisted I bundles over surfaces and Gabrovšek's categorification of the KBSM of the projective space. We speculate on applying a similar method for categorifying the KBSM of lens spaces. (Received September 16, 2019)

1154-57-1905 **Harrison Chapman*** (hchaps@gmail.com) and **Andrew Rechnitzer**. *A Markov chain sampler for knot diagrams*.

Knot diagrams provide a model for entanglement in physical polymers whose complexity is precisely the number of crossing regions where the polymer comes close to itself. As a DNA model, these crossing regions may be viewed as sites for the strand-passage action of type II topoisomerases.

Diagrams of fixed type become exponentially rare, and so efficient random generation is a difficult problem. We describe a new Markov chain Monte Carlo sampler for knot diagrams representing any fixed knot type, whose transitions are natural diagram moves. We prove that the limiting distribution of this Markov chain is the uniform distribution. Using this sampler, we examine a number of classical polymer statistics in the knot diagram model. We then explore topoisomerase unknotting pathways for circular DNA in the diagram model. (Received September 16, 2019)

1154-57-2005 **Jozef H. Przytycki*** (przytyck@gwu.edu). *Two-(co)cycle invariants of links from column unital Yang-Baxter operators*.

We describe how to define 2-(co)cycle invariants of links from column unital (e.g. stochastic) Yang-Baxter matrix. We show how to decompose the third Reidemeister move into boundary operation

$$\partial_3 = d_3^r + d_2^l + d_1^r - (d_1^l + d_2^r + d_3^l)$$

Despite the fact that I made this observation in May of 2015, the 2-(co)cycle invariants from Yang-Baxter operators which are not set theoretic, are still waiting to be studied. (Received September 17, 2019)

1154-57-2025 **Radmila Sazdanovic*** (rsazdan@ncsu.edu), Department of Mathematics NC State University, PO Box 8205, Raleigh, NC 27695, and **Pawel Dlotko**. *Applied topology methods in knot theory*. Preliminary report.

A multitude of knot invariants, including quantum invariants and their categorifications, have been introduced to aid with characterizing and classifying knots and their topological properties. Relations between knot invariants and their relative strengths at distinguishing knots are still mostly elusive. We use Ball Mapper by P. Dlotko, exploratory data analysis technique that builds graph based landscapes of high dimensional data using a single parameter- radius, to examine the structure of data consisting of various knot invariants. Our focus is on the Jones and Alexander polynomial, as well as the signature with the goal of gaining deeper insights into the relations between these invariants and related conjectures. (Received September 17, 2019)

1154-57-2226 **Olivia Del Guercio*** (delgur@gmail.com), 1 Chapin Way, Northampton, MA 01063, and **Elise Catania, Jack Kendrick, Hana Sambora and Sarangoo Chimgee**. *An Invariant of p-colorable Knots*. Preliminary report.

In this work we compute a knot invariant known as the dihedral linking number for all 3-colorable knots up to 12 crossings. Generally, a linking number is used to describe how many times two distinct knots wind around each other. When a knot has a valid 3-coloring, it lifts to two knots in a separate three-dimensional space known as a branched cover. The dihedral linking number is the linking number of these two knots. This number could potentially allow for the differentiation of mutant knots. We are also looking into the dihedral linking numbers produced by coloring knots with any prime number of colors, which will allow for analysis of a greater number of knots. (Received September 17, 2019)

1154-57-2276 **Radmila Sazdanovic*** (rsazdan@ncsu.edu), Department of Mathematics NC State University, PO Box 8205, Raleigh, NC 27695, and **Alex Chandler, Adam Lowrance and Victor Summers**. *Computing torsion of Khovanov homology*.

In the integral Khovanov homology of links, the presence of odd torsion is rare. Khovanov homology of homologically thin links only contains \mathbb{Z}_2 torsion. We prove a local version of this result and apply it to an infinite family of 3-braids, strictly containing all 3-strand torus links. This provides a partial answer to Sazdanovic and Przytycki's conjecture that 3-braids have only \mathbb{Z}_2 torsion in Khovanov homology. We provide explicit computations of integral Khovanov homology for all links in this family. Additionally, we prove an upper bound on the order of the torsion part of Khovanov homology in terms of the crossing number of the link. (Received September 17, 2019)

1154-57-2302 **Stanislav Jabuka, Beibei Liu and Allison H Moore***, Virginia Commonwealth University, Department of Mathematics, 1015 Floyd Avenue Box 842014, Richmond, VA 23284-2014. *Knot Graphs and Gromov Hyperbolicity*. Preliminary report.

The Gordian graph is a construction for organizing the set of knots, in which each vertex represents the isotopy type of a knot, and two vertices are connected by an edge whenever the corresponding knots are related by a

crossing change. This a countably infinite graph and every vertex has infinite valence. Other variants of this graph (for example, the $H(2)$ -Gordian graph) can be defined in a similar manner, and capture information about other local moves on knots. The broad structure of such knot graphs is not well-understood. We'll discuss some results in this area, and in particular, prove that many knot graphs are not Gromov hyperbolic. (Received September 17, 2019)

1154-57-2346 **Taylor E Martin*** (taylor.martin@shsu.edu), 1900 Ave I, Huntsville, TX 77340. *Link concordance, Milnor invariants, and low-order solvability*. Preliminary report.

In the 1950's, Milnor introduced numerical invariants that generalize linking number; these have since been known as Milnor's $\bar{\mu}$ -invariants. Since their introduction, Milnor invariants have proven extremely useful in many aspects of the study of links, including link concordance, finite type invariants, and virtual links. Unfortunately, only the first non-vanishing Milnor invariants are well-defined; Habegger and Lin showed that the indeterminacy of Milnor invariants correspond to the indeterminacy of representing a link as the closure of a string link.

In this talk, we will discuss aspects of the role of Milnor invariants in studying link concordance - in particular, as they are used in studying Cochran, Orr, and Teichner's n -solvable filtration of the (string) link concordance group. We will also discuss some new ideas of how we may hope to embrace the indeterminacy of Milnor's invariants in the presence of non-trivial linking number. (Received September 17, 2019)

1154-57-2527 **Dale Koenig** and **Anastasiia Tsvietkova*** (a.tsviet@rutgers.edu). *NP-hard problems naturally arising in knot theory*.

Abstract. Many problems that lie at the heart of classical knot theory can be formulated as decision problems, with an algorithm being a solution. Despite the lack of polynomial algorithms, few problems in knot theory were previously known to be NP-hard or NP-complete. We consider decision problems related to Reidemeister moves, to unlinking and splitting by crossing changes, and to detecting alternating links and sublinks. We prove that many of these problems are NP-hard. (Received September 17, 2019)

1154-57-2529 **James Belk, Justin Lanier, Dan Margalit** and **Rebecca R Winarski*** (rebecca.winarski@gmail.com). *Applying topology to polynomial recognition*.

Topological branched self-covers of \mathbb{C} satisfying certain finiteness conditions arise naturally in the study of complex dynamics. They can be viewed as generalizations of mapping classes. Thurston proved that such a map is either equivalent to a polynomial or else has an obstructing invariant multicurve. For branched self-covers of \mathbb{C} , we give an algorithm to determine if it is equivalent to a polynomial or obstructed. In the case that the map is equivalent to a polynomial, we determine *which* polynomial it is equivalent to by adapting tools used to study mapping class groups. This is joint work with Jim Belk, Justin Lanier, and Dan Margalit. (Received September 17, 2019)

1154-57-2592 **Felipe Castellano-Macias*** (castellanomacias.f@husky.neu.edu) and **Nicholas Owad**. *The tunnel number of all 11 and 12 crossing alternating knots*.

Given a knot K , tunnel number is a knot invariant which can be defined as one less than the Heegaard genus of $S^3 \setminus N(K)$, or equivalently, as the minimum number of properly embedded disjoint arcs α_i required to make $S^3 \setminus N(K \cup \{\alpha_i\})$ a handlebody. Morimoto, Sakuma, and Yokota computed the tunnel numbers of all knots with 10 or less crossings. Using exhaustive techniques together with results from Lackenby and many others, we compute the tunnel number of all 1655 alternating 11 and 12 crossing knots. These techniques also allow us to compute the tunnel number of some non-alternating knots with 11 and 12 crossings. (Received September 17, 2019)

1154-57-2631 **Marion Campisi** and **Luis Torres*** (luis.torres@sjsu.edu). *The disk complex and topologically minimal surfaces in the 3-sphere*. Preliminary report.

David Bachman introduced topologically minimal surfaces as generalizations of incompressible and strongly irreducible surfaces. These surfaces have been useful in problems that deal with stabilization, amalgamation, and isotopy of Heegaard splittings and bridge spheres for knots. In this talk, we discuss joint work with Marion Campisi which shows that the disk complex of a compact, orientable surface in the 3-sphere is homotopy equivalent to a wedge of spheres, all of the same dimension. This leads to a proof that genus $g > 1$ Heegaard surfaces of the 3-sphere are topologically minimal with index $2g - 1$. (Received September 17, 2019)

1154-57-2757 **Eric Samperton*** (smprtn@illinois.edu). *Coloring invariants of knots and links are often intractable*.

Invariants are useful for studying knots, but there is a trade-off: one should expect a stronger invariant to be harder to compute. Coloring invariants are a particular class of invariants of knots coming from finite groups.

Given a finite group G , we can count the number of homomorphisms (“ G -colors”) $\pi_1(S^3 \setminus K) \rightarrow G$. Of course, the complexity of the G -coloring invariant depends on G . If G is abelian, the invariant is trivial. On the other end, if G is nonabelian simple, then I will show that computing is hard, in a precise complexity-theoretic sense. Time-permitting, I’ll discuss applications to symmetry-enriched topological phases. This is joint work with Greg Kuperberg. (Received September 17, 2019)

58 ► *Global analysis, analysis on manifolds*

1154-58-266 **Corey Shanbrom*** (corey.shanbrom@csus.edu). *A survey of the monster tower.*

Prolonging the plane with its tangent bundle yields a tower of manifolds equipped with rank 2 Goursat distributions. In 2001, Montgomery and Zhitomirskii showed that the germ of every Goursat flag appears within this tower, which they called the *monster tower*. Subsequent investigations analyzed orbits of symmetry group actions, the natural correspondence between points of the tower and prolongations of plane curves, and generalizations to higher base dimensions.

The monster tower has also been studied by algebraic and enumerative geometers who know it as the Semple tower, the compactification of a space of curvilinear data. Moreover, it appears in control theory as the configuration space for the kinematic model of a truck pulling many trailers. Here, we survey the literature on this ubiquitous object and describe an ongoing project which aims to unify the existing approaches. (Received August 27, 2019)

1154-58-568 **Yuhao Hu*** (yuhao.hu@colorado.edu). *Absolute Equivalence and Linearization via a Cartan Formalism.*

In 1914, Élie Cartan introduced the notion of ‘absolute equivalence’ between two differential systems. Seven decades later, through the works of Gardner, Shadwick, Sluis and others, this idea of Cartan found its applications in control theory. In particular, it allows us to describe and study the so-called ‘dynamic equivalence’ in geometric terms. An interesting open problem is to decide whether a given control system is dynamically linearizable. In this talk, I’ll present some recent progress towards solving this problem. (Joint work with J. N. Clelland.) (Received September 07, 2019)

1154-58-583 **Jonathan Adam Belcher***, jonathan.belcher@colorado.edu, and **Markus J. Pflaum**, Boulder, CO 80309. *Bridge Cohomology: Generalizing Hochschild and Cyclic Cohomologies.*

The connection between Hochschild and cyclic cohomologies with generalized De Rham homology and index theories for arbitrary algebras has long been established by the work of Connes, Karoubi, Loday, Feigin, Tsygan, et al. Here we generalize these cohomology theories even further, essentially creating a theory that establishes a step-wise bridge between the two, which we call “Bridge Cohomology”. Motivation for this construction comes from trying to generalize the Hochschild-Kostant-Rosenberg-Connes theorem to manifolds with boundary, and applications in tracial constructions in certain classes of pseudodifferential operators. We end with a geometric example and extend the theorems by Hochschild-Kostant-Rosenberg and Connes to manifolds with boundary. Further geometric and topological interests of this theory include extending Chern-Weil theory to manifolds with boundary via pairings between bridge cohomology and higher K-theories. (Received September 07, 2019)

1154-58-629 **Steven Hurder*** (hurder@uic.edu). *Classification of weak solenoids.*

In this talk, we address the classification problem for equicontinuous foliated spaces, from the viewpoint of classifying homogenous spaces over profinite groups, and the properties of the adjoint action of the isotropy group of the action. For homogeneous spaces over a Lie group, this classification is well-known. But for quotients of profinite groups, this is a subtle problem, which is tractable when the acting group G is nilpotent, but is far from understood otherwise. Various invariants of these actions have been discovered in a series of works with Olga Lukina. For example, the discriminant is a profinite group associated to a weak solenoid, which is a measure of the local non-homogeneity of the leaf space. The discriminant can be either a finite group, possibly trivial, or a Cantor group, in which case the algebraic properties of this group yield topological invariants of the weak solenoid. Another wild case is when the global monodromy admits non-Hausdorff elements for its action, such as happens for actions of weakly branch groups. The classification of wild actions presents many puzzles. (Received September 08, 2019)

1154-58-1121 **Georges Habib** and **Ken Richardson*** (k.richardson@tcu.edu), Dept. of Mathematics, TCU Box 298900, Fort Worth, TX 76129. *New cohomological invariants of foliations.*

Given a smooth foliation on a closed manifold, basic forms are differential forms that can be expressed locally in terms of the transverse variables. The space of basic forms yields a differential complex, because the exterior derivative fixes this set. The basic cohomology is the cohomology of this complex, and this has been studied extensively. Given a Riemannian metric, the adjoint of the exterior derivative maps the orthogonal complement of the basic forms to itself, and we call the resulting cohomology the “antibasic cohomology”. Although these groups are defined using the metric, the dimensions of the antibasic cohomology groups are invariant under diffeomorphism and metric changes. If the underlying foliation is Riemannian, the groups are foliated homotopy invariants that are independent of basic cohomology and ordinary cohomology of the manifold. For this class of foliations we use the codifferential on antibasic forms to obtain the corresponding Laplace operator, develop its analytic properties, and prove a Hodge theorem. We then find some topological and geometric properties that impose restrictions on the antibasic Betti numbers. (Received September 13, 2019)

1154-58-1618 **Gregory W Moore***, NHETC and Department of Physics and Astronomy, Rutgers University, 126 Frelinghuysen Road, Piscataway, NJ 08855. *Smooth invariants of four-dimensional manifolds and quantum field theory.*

Quantum Field Theory offers an interesting perspective on many topological and geometric invariants in mathematics. Insights from physics can lead to new and unexpected predictions about concrete mathematical quantities, predictions that are amenable to rigorous analysis. This talk focuses on the renowned example of Donaldson and Seiberg-Witten invariants in the theory of four-manifolds. In this case physics predicts an interesting equality between a path integral of an “ultraviolet” (UV) and an “infrared” (IR) quantum field theory. The physical relation between the UV and IR theories allows one to derive the relation between Donaldson and Seiberg-Witten invariants. The key to the analysis is a finite-dimensional integral called the “u-plane integral.” The talk will describe some recent insights into the u-plane integral. For all four-manifolds with $b_2^+ = 1$ it can be identified with the constant term in the Fourier expansion of a Mock modular (or Jacobi) form. The physical approach also leads to several generalizations of the relation of Donaldson and Seiberg-Witten invariants. Finally, time permitting, a further generalization to families of four-manifolds will be described. (Received September 16, 2019)

60 ► *Probability theory and stochastic processes*

1154-60-86 **Indranil SenGupta*** (indranil.sengupta@ndsu.edu), Department of Mathematics, North Dakota State University, NDSU Dept # 2750, Minard Hall 408E12, Fargo, ND 58108-6050. *Refinement of the Barndorff-Nielsen and Shephard model with machine learning: analysis of various price indices.*

A commonly used stochastic model for the derivative and commodity market analysis is the Barndorff-Nielsen and Shephard (BN-S) model. At first, an application of the BN-S model will be presented to find an optimal hedging strategy for the oil commodity from the Bakken, a new region of oil extraction that is benefiting from fracking technology. Though this model is very efficient and analytically tractable, it is known that this model suffers from the absence of long-range dependence and many other issues. In this presentation, with the implementation of various machine learning algorithms, a simple way of improving the BN-S model will be proposed. This resulting model is more efficient and has fewer parameters than the superposition models that are used in practice to improve the BN-S model. The refined BN-S model will be implemented in the analysis of various price indices. (Received July 31, 2019)

1154-60-89 **Arka Ghosh**, **Steven Noren*** (snoren@iwu.edu) and **Alex Roitershtein**. *On the range of the transient frog model on \mathbb{Z} .*

We observe the frog model, an infinite system of interacting random walks, on \mathbb{Z} with an asymmetric underlying random walk. For certain initial frog distributions we construct an explicit formula for the moments of the leftmost visited site, as well as their asymptotic scaling limits as the drift of the underlying random walk vanishes. We also provide conditions in which the lower bound can be scaled to converge in probability to the degenerate distribution at 1 as the drift vanishes. (Received August 01, 2019)

1154-60-90 **Michael Roberts*** (michael.roberts.1@ndus.edu). *Hypothesis Testing of Lévy Processes from Two Streams of Data.*

In this presentation, we present the testing of four hypotheses on two streams of observations that are driven by Lévy processes. This is applicable for sequential decision making on the state of two-sensor systems. In one case, each sensor receives or does not receive a signal obstructed by noise. In another, each sensor receives data driven by Lévy processes with large or small jumps. In either case, these give rise to four possibilities. Infinitesimal generators are presented and analyzed. Bounds for infinitesimal generators in terms of *super-solutions* and *sub-solutions* are computed. (Received August 01, 2019)

1154-60-128 **Yu-Sin Chang*** (changy@msoe.edu), 1025 North Broadway, Milwaukee, WI 53202. *Markovian Consistency of Multivariate Markov Processes.* Preliminary report.

The Markovian consistency theory studies the Markov property of the coordinate process of a multivariate Markov process with respect to different filtrations. In this paper, we investigate necessary and sufficient conditions for the component of a continuous-time multivariate Markov process, taking values in continuous state space, to have a prescribed marginal law; moreover, the component is Markov in its natural filtration, but not Markov in the filtration of the multivariate process. We address verifiable conditions in terms of transition characteristics of the multivariate Markov process. Besides, we extend the results in Rogers and Pitman (1981) to the class of time-inhomogeneous multivariate Markov processes with coordinate projections. In particular, if the multivariate Markov process is homogeneous, upon applying the conditional expectation operator, the component may be a homogeneous or an inhomogeneous Markov process in its natural filtration. Finally, we give examples in cases of discrete and continuous state spaces. In continuous state example, we show that a class of Archimedean survival processes whose components are Markovian in their natural filtrations, respectively, satisfies our sufficient condition for the component to be Markov in its natural filtration. (Received August 12, 2019)

1154-60-185 **William A Massey*** (wmassey@princeton.edu), ORFE Department, Sherrerd Hall, Princeton University, Princeton, NJ 08544. *A Feynman Approach to Dynamic Rate Markov Processes.*

Physics successfully uses mathematics to describe nature but it also broadens this language by inspiring the creation of new mathematics. A prominent example of this positive feedback loop is the development of calculus and differential equations by Isaac Newton to invent 17th century physics.

In this talk, we use more physics inspired mathematics to understand the random evolution of time-inhomogeneous or dynamic rate Markov processes. First, we use the 19th century physics concept of a Hamiltonian for time-homogeneous or constant rate Markov processes. The Kolmogorov forward and backward differential equations, that govern the dynamics of Markov transition probabilities, parallel the Heisenberg and Schrödinger pictures of early 20th century quantum mechanics.

Richard Feynman during the middle in the 20th century created a new framework for quantum mechanics. By reintroducing the 18th century physics concept of a Lagrangian, he incorporates both of these quantum pictures and includes relativistic effects. He then introduces an operator calculus to formulate the time-ordered exponential. Applying these techniques to dynamic rate Markov processes gives us a transition probability analysis that reveals their fundamental sample path structure. (Received August 21, 2019)

1154-60-203 **Cory Ball*** (cball12015@fau.edu), cball12015@fau.edu, and **Hongwei Long.** *The Volatility Change-Point Problem for Diffusion Processes.*

In a 2008 paper, De Gregrio and Iacus estimated the volatility change point of a stochastic process under the assumption of equality of the Euler approximation. In the paper, they falsely claim the equivalence of two estimators. In this talk, we approach the problem without the assumption of equality of the Euler approximation. Instead, we consider two separate cases. The first case has a volatility function that is bounded away from zero, and the second case assumes ergodicity. In both cases, we are able to show consistency and determine asymptotic behavior. Finally, simulations indicate that the proposed estimator is more accurate than that of De Gregrio and Iacus. (Received September 05, 2019)

1154-60-318 **J Darby Smith*** (jsmit16@sandia.gov), **Scott A McKinley** and **William O Hancock.** *Molecular Motor Proteins: Designing experiments through stochastic inference.*

Molecular motor proteins form the basis for directed intracellular transport of structures such as vesicles and organelles. Motor protein properties, such as the unburdened velocity, detachment rate, and stall force have a rich history of estimation through experiment. Typically, a tracking particle is observed as a proxy for motor position, and the position is traced over time. Through the use of ad-hoc methods, estimates for these properties are calculated. When considering detachment dynamics, the critical force is another motor protein property of

interest. The critical force is not well understood and no generally agreed upon estimates exist for various motor types. Here we examine two experimental methods in an effort to understand force-dependent stepping and detachment for motor proteins and provide guidance for designing future experiments. To accomplish this, we simulate experimental data and employ an SDE approximation model for statistical inference and uncertainty quantification. We further explore a reduced experimental model to display distinct experimental regimes and to guide future experimentation. (Received August 30, 2019)

1154-60-364 **Danny Nam*** (dhnam@princeton.edu), Department of Mathematics, Princeton University, Princeton, NJ 08544, and **Shankar Bhamidi, Oanh Nguyen and Allan Sly**. *The contact process on random graphs.*

The contact process describes an elementary epidemic model, where each infected vertex gets healed at rate 1 while it passes its disease to each of its neighbors independently at rate λ . On the infinite d -regular tree with the initial infection at its root, [Pemantle '92] proved that the process has three different phases depending on λ : extinction, weak survival, and strong survival. In this talk, we show that the phase diagram of the contact process on a Galton-Watson tree depends on the tail of the offspring distribution in the following sense: the extinction-survival threshold is strictly positive if and only if the tail has an exponential decay. In such cases, we further achieve the first-order asymptotics for the location of the threshold. We will also discuss analogous results for Erdős-Rényi and other random graphs. Joint work with Shankar Bhamidi, Oanh Nguyen and Allan Sly. (Received September 02, 2019)

1154-60-384 **Lam Ho*** (lam.ho@dal.ca). *Modelling phenotypic evolution using continuous-time Markov processes.*

Continuous-time Markov processes have been used extensively for modelling the evolution of a phenotype along an evolutionary tree. Estimating parameters of these evolutionary models provide insight into many important macroevolution questions. However, parameter estimation is challenging due to the fact that species are related to each other according to an evolutionary tree. In this talk, I will present recent developments which cut directly to the heart of this challenge. (Received September 02, 2019)

1154-60-401 **Shanshan Lv*** (slv@truman.edu), 100 E. Normal Avenue, Kirksville, MO 63501, and **K. Krishnamoorthy**. *Prediction Intervals for Hypergeometric Distribution*

The problem of constructing prediction intervals (PIs) for a future sample from a hypergeometric distribution is addressed. Simple closed-form approximate PIs based the Wald approach, the joint sampling approach, and a fiducial approach are proposed and compared in terms of coverage probability and precision. Construction of the proposed PIs are illustrated using an example. (Received September 03, 2019)

1154-60-402 **Cheng Ouyang*** (couyang@math.uic.edu), MSCS (M/C 249), 851 South Morgan Street, Chicago, IL 60607. *Moment estimates for some renormalized parabolic Anderson models.* Preliminary report.

We consider a parabolic Anderson model with Gaussian noise whose space time covariance function is singular. We shall give some information about the moments of the solution when the stochastic heat equation is interpreted in the Skorohod sense. Of special interest is the critical case, for which one observes a blowup of moments for large times. (Received September 03, 2019)

1154-60-439 **Michael Salins*** (msalins@bu.edu), 111 Cummington Mall, Department of Mathematics and Statistics, Boston, MA 02215. *Uniform large deviations principles.*

There are three definitions of uniform large deviations principles that are used extensively in the literature: Freidlin and Wentzell's uniform large deviations principle (FWULDP), Dembo and Zeitouni's uniform large deviations principle (DZULDP), and Dupuis and Ellis's uniform Laplace principle (ULP). These three definitions are equivalent when describing uniformity over compact sets of parameters (usually initial conditions for stochastically perturbed dynamical systems). In the context of exit time problems for stochastic partial differential equations, however, the assumption of compact sets of initial conditions is too restrictive because compact sets in infinite dimensional Banach spaces have no interior. I demonstrate that the FWULDP, DZULDP, and ULP are not equivalent in the absence of compactness and that of the three definitions, only Freidlin and Wentzell's definition is applicable for proving uniformity with respect to initial conditions in non-compact sets. I also introduce the notion of the equicontinuous uniform Laplace principle (EULP) and prove that it is equivalent to the FWULDP without any compactness assumptions. (Received September 04, 2019)

1154-60-496 **Tatyana Shcherbina***, Fine Hall, Washington Road, Princeton, NJ 08540. *Universality for random band matrices.*

Random band matrices (RBM) are natural intermediate models to study eigenvalue statistics and quantum propagation in disordered systems, since they interpolate between mean-field type Wigner matrices and random Schrodinger operators. In particular, RBM can be used to model the Anderson metal-insulator phase transition (crossover) even in 1d. In this talk we will discuss some recent progress in application of the supersymmetric method (SUSY) and transfer matrix approach to the analysis of local spectral characteristics of some specific types of 1d RBM. (Received September 05, 2019)

1154-60-529 **Leonid Petrov*** (lenia.petrov@gmail.com), 141 Cabell Drive, Kerchof Hall, P.O. Box 40013, Charlottesville, VA 22902. *From Yang-Baxter equations to stochastic systems.*

Yang-Baxter equation (a class of local equivalence transformations preserving the partition function, i.e., the probability normalizing constant) is an indicator of integrability in two-dimensional models of statistical mechanics. I will explain how it can be utilized in a stochastic context, which brings new Markov chains on some well-studied models such as lozenge tilings, Totally Asymmetric Simple Exclusion Process (TASEP), and random matrices. (Received September 06, 2019)

1154-60-557 **Alexander Moll*** (a.moll@northeastern.edu). *Soliton Quantization and Random Partitions.*

In this talk we present exact Bohr-Sommerfeld quantization conditions for the multi-phase and multi-soliton solutions of the classical Benjamin-Ono equation. As an application, we use the theory of coherent states to construct a distinguished regularization of the critical Benjamin-Ono Cauchy problem with random periodic initial data sampled from a log-correlated Gaussian field. We find that the conserved quantities of the random multi-phase solutions in our regularization define Jack measures on partitions, a special case of Borodin-Corwin's Macdonald measures. As a consequence, we realize old and new asymptotic results for random partitions as semi-classical and small dispersion asymptotics of our regularization. Our results suggest that random matrix universality captures quantum corrections to the well-known edge and bulk universality for classical dispersive shock waves. (Received September 06, 2019)

1154-60-575 **Robert G Smits*** (rsmits@nmsu.edu). *Modelling Initial Biofilm Formation with Stochastic Differential Equations.* Preliminary report.

The onset of biofilm formation often begins with a complex attachment process of motile bacteria adhering to a surface, where the attachment is governed by a weak potential. The attachment and detachment process can be modelled as a PDE with generalized Wentzell boundary conditions, sometimes referred to as slowly adhering or sticky. A stochastic differential equation corresponding to the PDE will be discussed along with its numerical simulation. (Received September 07, 2019)

1154-60-672 **Amol Aggarwal*** (amolaggarwal@g.harvard.edu), Harvard Science Center, 1 Oxford Street, Office 324F, Cambridge, MA 02138. *Universality for Lozenge Tiling Local Statistics.*

We consider uniformly random lozenge tilings of essentially arbitrary domains and show that the local statistics of this model around any point in the liquid region of its limit shape are given by the infinite-volume, translation-invariant, extremal Gibbs measure of the appropriate slope. In this talk, we outline a proof of this result, which proceeds by locally coupling a uniformly random lozenge tiling with a model of Bernoulli random walks conditioned to never intersect. Central to implementing this procedure is to establish a local law for the random tiling, which states that the associated height function is approximately linear on any mesoscopic scale. (Received September 09, 2019)

1154-60-689 **Mela Hardin*** (melahardin@asu.edu) and **Nicolas Lanchier**. *Probability of Consensus in Spatial Opinion Models with Confidence Threshold.*

Interacting particle systems is a field of probability theory devoted to the rigorous analysis of certain types of models that arise in other fields such as physics, biology, and economics. One popular example of such systems is the voter model for the dynamics of opinions. We study two spatially explicit stochastic opinion models that are variants of the voter model. Both processes are characterized by two finite connected graphs – the spatial graph and the opinion graph. The spatial graph represents the social network describing how individuals interact. The opinion graph represents the topological structure of the opinion space. Representing opinions on a graph induces a distance between opinions which we use to measure disagreements among individuals. Pairs of neighbors on the spatial graph interact at rate one. Each interaction results in a local change of opinion only if the two interacting individuals do not disagree too much, which we quantify using a confidence threshold. We

study and derive lower bounds for the probability that, after an almost surely finite time, the system reaches consensus for some finite connected spatial graphs in the two models. (Received September 09, 2019)

1154-60-762 **Christopher Coscia*** (christopher.s.coscia.gr@dartmouth.edu), Department of Mathematics, 27 N. Main Street, 6188 Kemeny Hall, Hanover, NH 03755. *Exact Mixing and the Thorp Shuffle*. Preliminary report.

The Thorp Shuffle is a model for a riffle shuffle of a deck with $n = 2m$ cards with the following simple description: place the top m cards of the deck in the right hand and the bottom m cards in the left hand, then drop the bottom card from each hand, in the order determined by a fair coin flip, atop a new pile, repeating until all n cards have been dropped into a new permutation of the deck. We view a sequence of shuffles as a Markov chain on S_n in the usual way, and study stopping rules for the Thorp shuffle that observe the sequence of states and determine when to stop, having reached the uniform distribution exactly. (Received September 10, 2019)

1154-60-775 **Sevak Mkrtychyan*** (sevak.mkrtychyan@rochester.edu). *The point processes at turning points of large lozenge tilings*.

In the thermodynamic limit of the lozenge tiling model the frozen boundary develops special points where the liquid region meets with two different frozen regions. These are called turning points. It was conjectured by Okounkov and Reshetikhin that in the scaling limit of the model the local point process near turning points should converge to the GUE corners process. We will discuss various results showing that the point process at a turning point is the GUE corner process and that the GUE corner process is there in some form even when at the turning point the liquid region meets two frozen regions of arbitrary (non-lattice) rational slope. The last regime arises when weights in the model are periodic in one direction with arbitrary fixed finite period. The weights we study introduce a first order phase transition in the system. We compute the limiting correlation functions near this phase transition and obtain a process which is translation invariant in the vertical direction but not the horizontal. (Received September 10, 2019)

1154-60-787 **Matt Roberts** and **Jason Schweinsberg*** (jschwein@math.ucsd.edu). *A Gaussian particle distribution for branching Brownian motion with an inhomogeneous branching rate*. Preliminary report.

Motivated by the goal of understanding the evolution of populations undergoing selection, we consider branching Brownian motion in which particles independently move according to one-dimensional Brownian motion with drift, each particle dies at a constant rate, and the rate at which a particle splits into two is a linear function of the position of the particle. We show that, under certain assumptions, after a sufficiently long time, the empirical distribution of the positions of the particles is approximately Gaussian. This provides mathematically rigorous justification for results in the Biology literature indicating that the distribution of the fitnesses of individuals in a population over time evolves like a Gaussian traveling wave. (Received September 10, 2019)

1154-60-795 **Mark D. Dela*** (mddela@cupp.edu). *Numerically Solving a Rank-Based Forward Backward Stochastic Differential Equation by Applying the Least-Squares Monte Carlo Method*.

We describe a technique on how to numerically solve a rank-based FBSDE, a particular type of forward-backward stochastic differential equation (FBSDE) where the evolution of the forward component is described by a ranked-based diffusion (ranked-based SDE). To solve for the backward component of the rank-based FBSDE, we apply the Least-Squares Monte Carlo (LSMC) Method as documented by Gobet, Lemor, and Warin. This technique necessarily entails repeated simulation of the forward component, so we simulate the ranked-based SDE using a scheme presented by Ichiba and Karatzas, which takes advantage of the result that the gap process associated with the rank-based SDE is a particular kind of reflected Brownian motion. We implement both the LSMC Method and simulation of the rank-based SDE scheme in R and provide numerical results demonstrating the validity of the implementation and then apply this implementation to a rank-based SDE example. (Received September 10, 2019)

1154-60-815 **Alexandru Hening***, alexandru.hening@tufts.edu, and **Dang H Nguyen, Sergiu C Ungureanu** and **Tak Kwong Wong**. *Asymptotic harvesting of populations in random environments*.

We consider the harvesting of a population in a stochastic environment whose dynamics in the absence of harvesting is described by a one dimensional diffusion. Using ergodic optimal control, we find the optimal harvesting strategy which maximizes the asymptotic yield of harvested individuals. To our knowledge, ergodic optimal control has not been used before to study harvesting strategies. However, it is a natural framework because the optimal harvesting strategy will never be such that the population is harvested to extinction—instead the harvested population converges to a unique invariant probability measure. When the yield function

is the identity, we show that the optimal strategy has a bang–bang property: there exists a threshold such that whenever the population is under the threshold the harvesting rate must be zero, whereas when the population is above the threshold the harvesting rate must be at the upper limit. (Received September 10, 2019)

1154-60-845 **Dang H Nguyen***, 345 Gordon Palmer Hall Box 870350, Tuscaloosa, AL 35401.
Stochastic Functional Kolmogorov Equations.

This work is concerned with stochastic functional Kolmogorov equations, which are nonlinear stochastic differential equations depending on the present as well as the past states. Our main motivations stem from a wide variety of applications in biological and ecological systems. A longstanding question of fundamental importance pertaining to biology and ecology is: What are the minimal necessary and sufficient conditions for long-term persistence and extinction or for long-term coexistence of interacting species of a population? Our aim here is to answer this question when environmental noise, time delays, and past dependence have to be taken into consideration. While there are many excellent treatises of stochastic Kolmogorov systems under stochastic differential equation setup, the work on stochastic Kolmogorov systems with past dependence is still scarce. The main techniques used in this paper include the newly developed functional Itô formula and asymptotic coupling and Harris-like theory for infinite dimensional systems specialized to functional equations. General theorems for stochastic functional Kolmogorov equations are developed first. Then these results are applied to a number of application areas. In contrast to the existing literature, sharp conditions are obtained. (Received September 11, 2019)

1154-60-875 **Jacopo Borga, Enrica Duchi and Erik Slivken*** (eslivken@dartmouth.edu),
Dartmouth College, Kemeny Hall, HB 6188, Hanover, NH 03755. *Almost square permutations are typically squares.*

A (almost-) square permutation is a permutation where (almost) all points are records, either maximums or minimums, from the left or the right. We give a probabilistic approach to computing the first-order enumeration of almost-square permutations of size $n+k$ with n records and k non-records. We use the language of permuton limits to describe the points of an almost-square permutation scaled to fit in the unit square. A uniformly random almost-square permutation has a permuton limit with a simple geometric description. If k is fixed, the permuton can be viewed as a rectangle embedded in $[0, 1]^2$ whose edges have slope ± 1 and whose bottom corner is given by a $\beta(k+1, k+1)$ random variable. The bottom corner is uniform if $k=0$ and concentrated at $1/2$ if k is large. If k is increasing but small relative to n , then the permuton limit is a square. (Received September 11, 2019)

1154-60-880 **Deena R Schmidt*** (drschi@unr.edu). *Network Structure and Dynamics of Biological Systems.*

Many biological systems in nature can be represented as a dynamic model on a network. Examples include gene regulatory systems, neuronal networks, food webs, epidemics spreading within populations, social networks, and many others. A fundamental question when studying biological processes represented as dynamic models on networks is to what extent the network structure is contributing to the observed dynamics. In other words, how does network connectivity affect a dynamic model on a network? I will introduce a variety of network topologies and discuss biologically-inspired dynamic models that evolve on such networks. I will focus on several open problems in this area geared for undergraduate research. (Received September 11, 2019)

1154-60-883 **Brian C Hall*** (bhall@nd.edu), **Bruce K Driver** and **Todd Kemp**. *Random matrices in the general linear group.*

In light of the central limit theorem, the standard Gaussian random matrix models, such as the GUE and Ginibre ensembles, can be constructed as sums of small, independent random matrices. In my talk, I will discuss a “multiplicative” random matrix model constructed as a *product* of independent random matrices close to the identity. This model can be described as *Brownian motion in the general linear group* $GL(N; \mathbb{C})$.

In the large- N limit, Brownian motion in $GL(N; \mathbb{C})$ converges to Biane’s free multiplicative Brownian motion b_t . I will describe joint work with Bruce Driver and Todd Kemp in which we compute the Brown measure of b_t . The Brown measure of b_t is the natural candidate for the limiting eigenvalue distribution of Brownian motion in $GL(N; \mathbb{C})$.

The support of the Brown measure is a certain domain Σ_t in the plane, which is simply connected for $t \leq 4$ and doubly connected for $t > 4$. The Brown measure itself displays a remarkably simple structure. The talk will be self-contained and have lots of pictures. (Received September 11, 2019)

1154-60-952 **Nhu N, Nguyen*** (nhu.math.2611@gmail.com) and **George Yin**. *Stochastic Partial Differential Equation Models for Spatially Dependent Predator-Prey Equations*.

Stemming from the stochastic Lotka-Volterra or predator-prey equations, this work aims to model the spatial inhomogeneity by using stochastic partial differential equations (SPDEs). Compared to the classical models, the SPDE models are more versatile. To incorporate more qualitative features of the ratio-dependent models, the Beddington-DeAngelis functional response is also used. To analyze the systems under consideration, first existence and uniqueness of solutions of the SPDEs are obtained using the notion of mild solutions. Then sufficient conditions for permanence and extinction are derived. (Received September 12, 2019)

1154-60-980 **Po-Han Hsu***, Department of Mathematics, Louisiana State University, Baton Rouge, LA 70803, and **Padmanabhan Sundar**, Department of Mathematics, Louisiana State University, Baton Rouge, LA 70803. *Exponential inequalities for exit times of stochastic Navier-Stokes equations*.

Exponential estimates for probability of exit from a ball of radius r by time T for solutions of the two-dimensional stochastic Navier-Stokes equations will first be derived, and then studied in the context of Wentzell-Freidlin type large deviations principle. The existence of similar such estimates will also be presented for a suitable class of stochastic evolutions with multiplicative noise. (Received September 12, 2019)

1154-60-998 **Barbara H Margolius*** (b.margolius@csuohio.edu), Dept Mathematics, Cleveland State University, 2121 Euclid Ave, Cleveland, OH 44120, and **Yiqiang Zhao** (yiqiangzhao@cunet.carleton.ca), School of Mathematics and Statistics, Carleton University. *Quasi-Stationary Distributions in the context of continuous time Markov Processes with time-varying periodic transition rates*. Preliminary report.

We consider Markov processes with time-varying periodic transition rates evolving in a domain where there is a set of absorbing states that form a trap. The process is said to be killed when it hits the absorbing set and it is assumed that this happens almost surely. We investigate the behavior of the process conditioning on its not having been absorbed. We consider processes with transition matrices having a block structure.

Time-varying periodic transition rates arise in processes with a diurnal, weekly or seasonal variability such as call center traffic, rush hour traffic, weather or emergency service systems. (Received September 12, 2019)

1154-60-1026 **Jinqiao Duan*** (duan@iit.edu), RM 115B, Rettaliata Engineering Center, Illinois Institute of Technology, Chicago, IL 60616. *Most Probable Transition Pathways in Stochastic Dynamical Systems*. Preliminary report.

Dynamical systems arising in engineering and science are often subject to random fluctuations. The noisy fluctuations may be Gaussian or non-Gaussian, which are modeled by Brownian motion or α -stable Levy motion, respectively. Non-Gaussianity of the noise manifests as nonlocality at a “macroscopic” level. Stochastic dynamical systems with non-Gaussian noise (modeled by α -stable Levy motion) have attracted a lot of attention recently. The non-Gaussianity index α is a significant indicator for various dynamical behaviors.

The speaker will present recent work on most probable transition pathways between metastable states, for stochastic dynamical systems with non-Gaussian Levy noise. This is joint work with Ying Chao. (Received September 12, 2019)

1154-60-1034 **Jiajun Hoo*** (jiajun.hoo@gmail.com). *Chip-Firing in Abelian Sandpiles*.

Chip-firing processes, and the ideas of self-organized criticality related to chip-firing, contain information of significance to several scientific fields. For example, self-organized criticality can be used to model and understand pink noise (also known as $\frac{1}{f}$ noise). Moreover, this same idea of self-organized criticality can be used to describe seemingly complex phenomena such as the stock market, or cranial neural activity. This paper discusses, in particular, the Abelian Sandpile model and related groups. In particular, we are interested in the identity of such groups under sandpile addition, in both typical sandpiles and sandpiles with chip-firing rules constructed using non-trivial arithmetic structures. We explore the Sandpile Groups related to the class of cyclic graphs C_n , and further characterize the number of elements of each Sandpile Group relating to the graph with various sinks with reference to the associated arithmetic structure. (Received September 12, 2019)

1154-60-1044 **Martin V. Hildebrand*** (mhildebrand@albany.edu), Department of Mathematics and Statistics, University at Albany, SUNY, 1400 Washington Ave., Albany, NY 12222. *A multiplicatively symmetrized version of the Chung-Diaconis-Graham random process*. Preliminary report.

This work looks at random processes of the form $X_{n+1} = a_n X_n + b_n \pmod{p}$ where (a_0, b_0) , (a_1, b_1) , (a_2, b_2) , ... are i.i.d. with $P(a_n = (p+1)/2) = P(a_n = 2) = 1/2$ and $P(b_n = 1) = P(b_n = 0) = P(b_n = -1) = 1/3$, p is odd,

and $X_0 = 0$. This work shows that order $(\log p)^2$ steps are sufficient for X_n to be close to uniformly distributed on the integers mod p . Also order $(\log p)^2$ steps are necessary for X_n to be close to uniformly distributed in the integers mod p . A consequence is that there are doubly stochastic matrices P_1 and P_2 such that at least one row of $(0.5P_1 + 0.5P_2)^m$ will be close to uniform only for m much larger than values of m , i.e. order $(\log p) \log(\log p)$, which suffice to make all rows of P_1^m and P_2^m close to uniform. (Received September 12, 2019)

1154-60-1091 **Vishesh Jain*** (visheshj@mit.edu). *A combinatorial approach to the quantitative invertibility of random matrices.*

Let $s_n(M_n)$ denote the smallest singular value of an $n \times n$ random matrix M_n . We will discuss a novel combinatorial approach (in particular, not using either inverse Littlewood–Offord theory or net arguments) for proving statements of the following form for quite general random matrix models: there exist constants $c, C > 0$ such that for all $\eta \geq 0$, $\Pr(s_n(M_n) \leq \eta) \lesssim n^C \eta + \exp(-\Omega(n^c))$. (Received September 13, 2019)

1154-60-1096 **Ian Charlesworth, Zhiheng Li, Kyle P Meyer, Drew T Nguyen and Jennifer Pi***, jspi@uci.edu, and **Annie K Raichev**. *Characterizations of Infinitesimal Non-Crossing Bi-Free Probability.* Preliminary report.

Free probability is a non-commutative analogue of probability introduced in the 1980s by Voiculescu, where the notion of independence is replaced by the condition of free independence. Biane, Goodman, and Nica adapted the combinatorics to give a Type B free probability, arising from the Type B non-crossing partitions; Belinschi and Shlyakhtenko showed this notion to be equivalent to an infinitesimal free independence for a time-indexed family of states satisfying the freeness conditions to first order. More recently, Voiculescu has introduced bi-free probability, an extension of free probability to handle simultaneously left and right actions on free product spaces. After covering the necessary background, this talk will demonstrate how these extensions can be combined to give an infinitesimal bi-free probability. In particular, it will be shown that the bi-free generalizations of the combinatorial and analytic characterizations of infinitesimal free independence are equivalent. (Received September 13, 2019)

1154-60-1130 **Mark Huber*** (mhuber@cmc.edu), Claremont McKenna College, 850 Columbia AV, Claremont, CA 91711. *Using particle-distribution duality to determine mixing time.*

Quantum mechanics has long had particle-wave duality, two ways of viewing the same physical process. Similarly, Markov operators have a particle-distribution duality: they can be viewed as a particle evolving randomly from state to state or as a distribution which is acted upon by the Markov operator. By combining the particle and distribution ways of viewing a Markov chain, we create a special index process that can be used to give a stopping time where the particle is exactly in the stationary distribution. The expected value of the stopping time is one measure of the mixing time of the chain. As an example, we show that the mixing time of a simple symmetric random walk with partially reflecting boundaries on a line graph with n states is exactly $3n^2$. (Received September 13, 2019)

1154-60-1143 **Jacob Richey*** (jfrichey@uw.edu), C-138 Padelford, Box 354350, Seattle, WA 98195-4350. *Recent results on the phase transition for activated random walk.*

In this talk we will discuss activated random walk, an interacting particle system that exhibits a phase transition on infinite domains and self-organized criticality on finite domains. In the infinite version, the system is initialized with density μ of particles which perform independent simple random walk, fall asleep at rate λ , and are woken up if another particle moves to the same site. There are two possible limiting behaviors: local fixation, where each site is visited finitely many times a.s., or non-fixation, where each site is visited infinitely many times a.s. Current research is focused on determining where the transition between these two phenomena occurs in terms of μ and λ , and many questions still remain – even on \mathbb{Z} . We will present a novel statistical idea to establish non-fixation, mention some recent results that follow from it, and point to open problems in this vein. (Received September 13, 2019)

1154-60-1154 **Jean-Pierre Fouque*** (fouque@pstat.ucsb.edu). *MFG vs. Directed Chain Game.* Preliminary report.

We compare Nash equilibria for MFGs and for Directed Chain Games. (Received September 13, 2019)

1154-60-1173 **Skip Garibaldi***, 4320 Westerra Ct, San Diego, CA 92121. *Uncovering lottery shenanigans.*

Mathematicians since Euler have been interested in studying lotteries. Analyzing the games is made easier because the probabilities involved are clearly defined, yet the very large number of participants and variances in outcomes lead to subtle questions. Focusing on specific games provides an additional source of interest, because

each game has individual quirks. When looking at the outcomes of real games, it sometimes appears that people are up to shenanigans. Digging deeper, it may turn out that the apparent anomalies are not nefarious at all, just an illusion resulting from the oceans of tickets that are sold. Sometimes the scheme is real, but, surprisingly, entirely legal. And sometimes it is mostly illegal or an indicator of a larger criminal enterprise. This talk focuses on the latter kind of shenanigans. (Received September 13, 2019)

1154-60-1200 **Danielle Middlebrooks*** (dmiddle1@math.umd.edu), MD. *Quantifying Flows in Time-Irreversible Markov Chains: Application to a Gene Regulatory Network.*

Transition path theory (TPT) is a framework used to study the statistical properties of reactive trajectories. Reactive trajectories are those trajectories by which a random walker transits from one subset in the state-space to another disjoint subset. We develop analytical and computational tools based on TPT in order to quantify flows in time irreversible Markov Chains. These tools are applied to a gene regulatory network modeling the dynamics of the Budding Yeast cell cycle. (Received September 13, 2019)

1154-60-1204 **Serges Love Teutu Talla*** (seteu1@morgan.edu), Morgan State University, Department of Mathematics, 1700 East Cold Spring Lane, Baltimore, MD 21251, and **Isabelle Kemajou-Brown** (elisabeth.brown@morgan.edu), Morgan State University, Department of Mathematics - Actuarial Science, 1700 East Cold Spring Lane, Baltimore, MD 21251. *An Optimal Portfolio Control Problem.* Preliminary report.

In this talk, we will present steps of solution for a risk-sensitive optimal control problem under regime-switching in which the diffusion coefficients depends on the control. Our technique is based on the stochastic maximum principle using the expectation of the Hyperbolic Absolute Risk Aversion (HARA) function reduced to an exponential integral form. (Received September 13, 2019)

1154-60-1218 **Isabelle Kemajou-Brown*** (elisabeth.brown@morgan.edu), Morgan State University, Dept of Mathematics, Actuarial Science, 1700 E cold Spring Lane, Baltimore, MD 21251. *Risk Sensitive Markov Regime-switching Portfolio Optimization with Partial Information.* Preliminary report.

In this talk, we consider a maximum principle for a partially observed risk sensitive optimal control portfolio problem under Markovian regime switching. We compare the result to a fully observed risk sensitive Markov regime switching financial market problem. We use constant absolute risk aversion (CARA) utility function. (Received September 14, 2019)

1154-60-1227 **Fan Yang, Horng-Tzer Yau and Jun Yin*** (jyin@math.ucla.edu), Department of Mathematics, 6172 Math Science Hall, UCLA, los angeles, CA 90095. *Universality and Delocalization of random band matrices.*

In this talk, we discuss some recent work related to a main conjecture on random matrix theory, i.e. phase-transition conjecture on random matrix theory. The prediction says that phase-transition occurs at the band width regime $W \sim N^{1/2}$. For high dimensional matrix, i.e. $x, y \in Z^d$, H_{xy} , there exists some similar stimulation results.

Based on the development of studying on resolvent, i.e., $G = (H - z)^{-1}$, we obtained some results on low and high dimension cases. In this talk, we will introduce these work and the main ideas and tools used in these work.

They are jointed work with H.T. Yau, Yang Fan, etc.

Best Jun (Received September 14, 2019)

1154-60-1230 **Kyle Luh*** (kluh@cmsa.fas.harvard.edu), 20 Garden St., Cambridge, MA 02138. *Eigenvalue Gaps in Sparse Random Matrices.*

We discuss some recent developments in the smallest gap size of sparse random matrices. As an example, we show that the eigenvalues of the adjacency matrix of sparse Erdős-Rényi graphs are distinct. We also obtain quantitative tail bounds on the gap sizes. One consequence is that for sparse random graphs weak and strong nodal domains are the same, answering a question of Dekel, Lee and Linial. This is joint work with Van Vu and Patrick Lopatto. (Received September 14, 2019)

1154-60-1320 **Erik Davis and Sunder Sethuraman*** (sethuram@math.arizona.edu). *Approximating geodesics via random points.*

Given a 'cost' functional F on paths γ in a domain $D \subset \mathbb{R}^d$, in the form $F(\gamma) = \int_0^1 f(\gamma(t), \dot{\gamma}(t)) dt$, it is of interest to approximate its minimum cost and geodesic paths. Let X_1, \dots, X_n be points drawn independently from D

according to a distribution with a density. Form a random geometric graph on the points where X_i and X_j are connected when $0 < |X_i - X_j| < \epsilon$, and the length scale $\epsilon = \epsilon_n$ vanishes at a suitable rate.

For a general class of functionals F , using a form of Gamma convergence, we show that the minimum costs and geodesic paths, with respect to approximating discrete 'cost' functionals, built from the random geometric graph, converge almost surely in various senses to those corresponding to the continuum cost F , as the number of sample points diverges. (Received September 14, 2019)

1154-60-1342 **Erin Beckman, Keisha Cook, Nicole Eikmeier, Sarai Hernandez-Torres*** (saraiht@math.ubc.ca) and **Matthew Junge**. *Chase-escape with death on trees*.

Chase-escape is a competitive growth process in which red particles spread to adjacent uncolored sites while blue particles overtake and kill adjacent red particles. We can think of this model as prey escaping from pursuing predators. If the red particles spread fast enough, both particle types occupy infinitely many sites with positive probability. Otherwise, both almost surely occupy only finitely many sites. We introduce the modification that red particles die at some rate. When the underlying graph is a d -ary tree, chase-escape with death exhibits a new phase in which blue almost surely occupies finitely many sites, while red reaches infinity with positive probability. Moreover, the critical behavior, which we precisely characterize, is different with the presence of death. Many of our arguments make use of novel connections to analytic combinatorics. (Received September 16, 2019)

1154-60-1350 **E. Bayraktar, A. Cecchin, A. Cohen and F. Delarue*** (delarue@unice.fr). *Restoration of uniqueness for mean field games on a finite state space with a common noise*.

We here discuss how a common noise may help for restoring uniqueness to mean field games on a finite state space. The key point is to address the unique solvability of the corresponding master equation on the finite dimensional simplex and to invoke earlier results on the smoothing properties of diffusions with values in manifolds with corners. Interestingly enough, our result opens the door to selection principles for standard mean field games on a finite state space without uniqueness. This question is addressed in a companion talk by A. Cecchin in the framework of potential games.

This is a joint work with E. Bayraktar, A. Cecchin and A. Cohen (Received September 15, 2019)

1154-60-1357 **Boris Hanin*** (bhanin@math.tamu.edu), **Mihai Nica** and **Grigoris Paouris**. *Random Matrix Products in Deep Learning*.

Deep learning is the study and application of neural networks. Each network is a non-linear family of functions, and such families form the backbone for many state of the art machine learning tasks ranging from computer vision to natural language processing.

Several numerical stability questions about neural networks reduce to studying products of N independent matrices of size $n \times n$ in the regime where both the size n and the number of terms N tends to infinity. I will discuss joint work with Mihai Nica (Toronto) as well as work with Grigoris Paouris (Texas A&M) about the linear statistics and spectral theory of these matrix models. (Received September 15, 2019)

1154-60-1397 **Gerardo Rubino*** (gerardo.rubino@inria.fr), INRIA, Campus de Beaulieu, 35042 Rennes, France. *Tight bounds of performance metrics computed on Markovian queuing models*. Preliminary report.

Except in a few fundamental models, the only way to evaluate the performance associated with a queue is to use numerical techniques or simulation. These methods have several drawbacks, and concerning the former, one of them is how to deal with open models accepting an unbounded number of customers. Several years ago, based on results by Semal and Courtois, Munz, De Souza e Silva and Goyal developed a technique allowing to derive bounds of dependability measures on a finite Markovian model, that are particularly tight in case of rare events. These techniques were improved in a few works, including ours, where in particular we showed how to apply them to infinite state models of queues, basically in light traffic conditions. In this talk we will present some new improvements of those procedures, having as an objective to obtain tight bounds of classic performance metrics, using more general models, always in a Markovian setting. (Received September 15, 2019)

1154-60-1472 **Kseniya Klyachko***, kklyachko@albany.edu. *Random Processes of the Form*
 $X_{n+1} = AX_n + B_n \pmod{p}$.

While examining the random process of the form $X_{n+1} = AX_n + B_n \pmod{p}$ where $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ is a fixed matrix, B_0, B_1, B_2, \dots are independent and identically distributed on $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and $X_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, we come upon the Fibonacci sequence. Keeping in mind the goal of bounding the rate of convergence of this process to the uniform distribution, we discuss the Fourier Transform and its role in this setting. We also introduce an expansion we call the Fibonary expansion useful in analyzing the Fourier Transform. Generalizing to the random process where $A = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$, we use the β -ary expansion in analyzing the Fourier transform. To achieve the sought after rate of convergence we restrict A to non-trivial diagonalizable 2×2 matrices with no eigenvalues of 1 over \mathbb{C} , nonnegative integer entries and determinant 1. (Received September 15, 2019)

1154-60-1474 **Hui-Hsiung Kuo** (kuo@math.lsu.edu), **Sudip Sinha*** (ssinha4@lsu.edu) and **Jiayu Zhai** (zhai@math.umass.edu). *Stochastic Differential Equations with Anticipating Initial Conditions*.

In this paper, we study the solutions of a stochastic differential equation with various anticipating initial conditions. We show that the conditional expectation of the solution of such a stochastic differential equation is not simply the solution of the corresponding stochastic differential equation with initial condition taken as the conditional expectation of the anticipating initial condition. We derive the conditional expectation of the solution in general and apply it to the special case of anticipating initial condition given by Hermite polynomials. We also extend the class of initial conditions to functions of Wiener integrals. (Received September 17, 2019)

1154-60-1511 **Shirshendu Chatterjee, David Sivakoff** and **Matthew Wascher*** (wascher.1@osu.edu), Cockins Hall, 1958 Neil Ave, Columbus, OH 43210. *Survival dynamics for the contact process with avoidance on Z, Z_n , and the star graph*.

We consider the contact process with avoidance, a modified contact process, on directed graphs in which each healthy vertex can avoid each of its infected neighbors at rate α by turning off the directed edge from that infected neighbor to itself until the infected neighbor recovers. This model presents a challenge because, unlike the classical contact process ($\alpha = 0$), it has not been shown to be an attractive particle system. We study the survival dynamics of this model on the lattice Z , the cycle Z_n , and the star graph. On Z , we show there is a phase transition in λ between almost sure extinction and positive probability of survival. On Z_n , we show that as the number of vertices $n \rightarrow \infty$, there is a phase transition between log and exponential survival time in the size of the graph. On the star graph, we show that as $n \rightarrow \infty$ the survival time is polynomial in n for all values of λ and α . This contrasts with the classical contact process where the survival time on the star graph is exponential in n for all values of λ . (Received September 16, 2019)

1154-60-1576 **Mauro Maggioni*** (mauro@math.jhu.edu), 3400 N Charles St, Department of Mathematics, Baltimore, MD 21218, and **Fei Lu, Jason Miller, Sui Tang** and **Ming Zhong**. *Learning Interaction laws in particle- and agent-based systems*.

We consider the following inference problem for a system of interacting particles or agents: given only observed trajectories of the system, we are interested in estimating the interaction laws between the particles/agents. We consider both the mean-field limit (i.e. the number of particles going to infinity) and the case of a finite number of agents, with an increasing number of observations; in this talk we will mostly focus on the latter. We show that at least in the particular setting where the interaction is governed by an (unknown) function of pairwise distances, under a suitable coercivity condition that guarantees the well-posedness of the problem of recovering the interaction kernel, statistically and computationally efficient, nonparametric, suitably-regularized least-squares estimators exist. Our estimators achieve the optimal learning rate for one-dimensional (the variable being pairwise distance) regression problems with noisy observations. We discuss several examples, including extensions to agent systems with different types of agents, second-order systems, and families of systems with parametric interaction kernels. We also conduct numerical experiments to test the large time behavior of these systems, especially in the cases where they exhibit emergent behavior. (Received September 16, 2019)

1154-60-1583 **Manuel E. Lladser*** (manuel.lladser@colorado.edu), Department of Applied Mathematics, Campus Box 526 UCB, University of Colorado, Boulder, CO 80309, and **Stephen R. Chestnut** and **Javiera Barrera**. *Approximating linear functionals of transient Markov chains*. Preliminary report.

Linear functionals of Markov chains occur in wide-ranging domains. Despite their ubiquitous use, approximating their distribution is a challenging problem in transient regimes because of the infeasibility of exact computations and lack of stationarity to justify Gaussian, Poisson, and compound Poisson approximations. In this talk, I will review recent work with Stephen Chestnut and ongoing work with Javiera Barrera to approximate the distribution of these functionals in the presence of a so-called “regeneration” set; in particular, our setting encompasses Harris-recurrent Markov chains. (Received September 16, 2019)

1154-60-1663 **Shan Shan*** (sshan@math.duke.edu), **Douglas M Boyer**, **Sayan Mukherjee** and **Ingrid Daubechies**. *Probabilistic models on fibre bundles*.

We propose probabilistic models on fibre bundles for learning the data generating process. The main tool is diffusion kernel and we used it in two ways. First, we build from diffusion kernel on fibre bundle a *projected kernel* that generates robust representations of data, and we test that it outperforms regular diffusion maps under noise. Second, the diffusion kernel gives rise to a natural covariance function when defining Gaussian process (GP) on fibre bundle. To demonstrate the uses of GP on fibre bundle, we apply it to simulated data on a Möbius strip for the problem of prediction and regression. For an example of real-world application, we use probabilistic models on fibre bundles to study the evolutionary process on anatomical surfaces, like teeth and bones. The proposed machinery, relating diffusion processes to probabilistic models on fibre bundles, provides a unified framework for ideas from a variety of different topics such as geometric operators, dimension reduction, regression and Bayesian statistics. (Received September 16, 2019)

1154-60-1680 **Jordan S. Ellenberg*** (ellenber@math.wisc.edu). *Geometry, Inference, and Democracy*.

Every ten years, after the census, each U.S. state is divided into legislative districts. This might sound like a dry technical exercise, but it’s now red-hot political issue. That’s because it’s now clear that the same group of voters can send a Republican or Democratic majority to make the laws, depending who gets to draw those districts. The problem of gerrymandering — drawing districts with an intent to lock in control by one political faction — intertwines mathematics, law, and politics into an uncuttable knot. How can we tell whether a district map was gerrymandered? How much unfairness is too much? What does a “fair map” even mean, anyway? I’ll talk about the rapidly moving current mathematical developments in this area, and the way these developments have translated (and sometimes failed to translate) into legal doctrines when mathematicians meet the judicial system. (Received September 16, 2019)

1154-60-1685 **Elliot Paquette*** (paquette.30@osu.edu) and **Gaultier Lambert** (glambert@kth.se). *Distributional approximation of the characteristic polynomial of a Gaussian beta-ensemble*.

Conjecturally, the characteristic polynomials of many canonical classes of random matrices converge to universal random measure, the Gaussian multiplicative chaos. We develop machinery towards showing this convergence for the Gaussian beta-ensemble. We start from a standard representation for this polynomial as an entry in a product of independent random two-by-two transfer matrices. For a point z in the complex plane, at which the transfer matrix is to be evaluated, this product of transfer matrices splits into three independent factors, each of which can be understood as a diffusively perturbed dynamical system. Conjecturally, these factors converge to an exponential of a Gaussian field, the stochastic Airy function, and a diffusion similar to the stochastic sine equation. We show partial progress in establishing this conjecture, by giving an effective approximation for the polynomial that holds in the complex plane. (Received September 16, 2019)

1154-60-1690 **Andrey Sarantsev***, 1664 N Virginia St, Reno, NV 89557, and **Tomoyuki Ichiba** and **Michael Ludkovski**. *Dynamic Contagion in a Banking System*.

We consider a dynamic model of interconnected banks. New banks can emerge, and existing banks can default, creating a birth-and-death setup. Microscopically, banks evolve as independent geometric Brownian motions. Systemic effects are captured through default contagion: as one bank defaults, reserves of other banks are reduced by a random proportion. After examining the long-term stability of this system, we investigate mean-field limits as the number of banks tends to infinity. Our main results concern the measure-valued scaling limit which is governed by a McKean-Vlasov jump-diffusion. The default impact creates a mean-field drift, while the births and defaults introduce jump terms tied to the current distribution of the process. Individual dynamics in the limit is described by the propagation of chaos phenomenon. In certain cases, we explicitly characterize the limiting average reserves. (Received September 16, 2019)

- 1154-60-1785 **Christopher Janjigian***, 155 S 1400 E, Salt Lake Cty, UT 84106, and **Elnur Emrah** and **Timo Seppalainen**. *Geodesics in inhomogeneous exponential last passage percolation*. Preliminary report.

We study geodesics in the exactly solvable model of directed last passage percolation with exponential weights, where the weights are independent but not identically distributed. The impact of inhomogeneity can be seen at many different levels in the model. In this talk, we investigate the structure of geodesics and how they differ from the classical, homogeneous model with i.i.d. weights. (Received September 16, 2019)

- 1154-60-1826 **Pooja Agarwal**, **Mackenzie Simper*** (msimper@stanford.edu) and **Rick Durrett**. *The q -voter model on the torus*.

In the usual voter model, each vertex on a graph has opinion 0 or 1. A vertex changes its opinion at rate u , where u is the fraction of neighbors with opposite opinion. In the q -voter model, a vertex changes its opinion at rate u^q . Mean-field calculations suggest that there should be coexistence between opinions if $q < 1$, and clustering if $q > 1$. We use the machinery of voter model perturbations to show that the conjectured behavior holds for q close to 1 on the three-dimensional torus. (Received September 16, 2019)

- 1154-60-1858 **Ankan Ganguly*** (ankan_ganguly@brown.edu), Ankan Ganguly, Brown University, Applied Mathematics, Box F, Providence, RI 02912, and **Kavita Ramanan** (kavita_ramanan@brown.edu). *Asymptotic Properties of Interacting Stochastic Processes on Large Sparse Graphs*. Preliminary report.

We study discrete state, possibly non-Markovian, jump processes on large sparse graphs. These models arise in a variety of fields including population biology, traffic modeling, and load balancing. Such models are typically not amenable to exact analysis or (for large graphs) numerically tractable. Under broad conditions on the dynamics and graph structure, which allows for some asymmetry and heterogeneity in the dynamics, we will present a novel autonomous characterization of the marginal dynamics at a neighborhood of a typical particle in the limit as the number of particles goes to infinity. This complements earlier work of Lacker et al (2019) for homogeneous and symmetrically interacting diffusions. We also show how our characterization can be applied to a variety of interesting problems. This talk is based on joint work with Kavita Ramanan. (Received September 16, 2019)

- 1154-60-1916 **Semere Gebresilasie*** (habtemicaels@wit.edu), 48 Woodward St, Everett, MA 02149. *Pricing Variance Swaps for the Discrete BN-S Model*. Preliminary report.

We introduce a discrete Barndorff-Nielsen and Shephard stochastic volatility model where the non Gaussian Ornstein Uhlenbeck process describes the instantaneous variance of an underlying asset. We developed a closed form solution for the variance swaps for financial markets driven by a Barndorff-Nielsen and Shephard (BN-S) type Levy process. We present a discrete time model to determine the arbitrage free pricing of variance swaps alongside simulations to provide numerical results in support of our findings. Model fitting and parameter estimation are included. (Received September 16, 2019)

- 1154-60-1918 **Sean O'Rourke** and **Noah N. Williams*** (williamsnn@appstate.edu), Department of Mathematical Sciences, 121 Bodenheimer Dr., Appalachian State University, Boone, NC 28608. *Partial linear eigenvalue statistics for i.i.d. random matrices*.

Let X_n be an $n \times n$ matrix with independent and identically distributed entries whose moments match those of the entries of the real or complex Ginibre ensemble to third order. We study the asymptotic fluctuations of the partial linear eigenvalue statistics $\sum_{j=1}^{n-k} f(\lambda_j)$, where f is a test function, $\lambda_1, \dots, \lambda_n$ are the eigenvalues of X_n/\sqrt{n} , and the contributions of k randomly selected eigenvalues are removed from the sum. We consider the cases where k is finite and where $\min\{k, n-k\}$ tends to infinity. (Received September 16, 2019)

- 1154-60-2020 **Christopher Janjigian** (janjigia@math.utah.edu), 155 S 1400 E, Salt Lake City, UT 84112, **Firas Rassoul-Agha*** (firas@math.utah.edu), 155 S 1400 E, Salt Lake City, UT 84112, and **Timo Seppalainen** (seppalai@math.wisc.edu), Van Vleck Hall, 480 Lincoln Dr., Madison, WI 53706. *Geometry of geodesics through Busemann measures in directed last-passage percolation*.

We consider planar directed last-passage percolation on the square lattice with general i.i.d. weights and describe the geometry of the full set of semi-infinite geodesics in a typical realization of the random environment. The main tool is the Busemann functions viewed as a stochastic process indexed by the asymptotic direction. In the exactly solvable exponential model we give a complete characterization of the uniqueness and coalescence structure of the entire family of semi-infinite geodesics. Part of our results concerns the existence of exceptional (random) directions in which new interesting shock structures occur.

Joint work with Christopher Janjigian and Timo Seppalainen (Received September 17, 2019)

1154-60-2039 **Alan Krinik*** (ackrinik@cupp.edu), **Hubertus von Bremen**, **Saif A. Aljashamy**, **David Perez**, **Jeremy Lin**, **Thuy Vu Dieu Lu**, **Jeffrey Yeh**, **Aaron Kim** and **Mark Dela**. *Finite birth-death-like Markov chains with generalized catastrophe transitions*. Preliminary report.

Certain classes of finite birth-death-like chains whose transition probabilities are redefined to include generalized catastrophe probabilities are shown to have a one-step transition probability matrix, P , having distinct eigenvalues that are explicitly known and described by formulas that scale up as n , the number of states, increases. The eigenvalues of P for these Markov chains are shown to correspond to known eigenvalues of certain tridiagonal matrices. These conclusions follow from applying known linear algebraic properties of dual Markov chains. (Received September 17, 2019)

1154-60-2097 **Nguyet Nguyen*** (ntnguyen01@ysu.edu), Youngstown State University, and **Paa Kwesi Mensah**, Progressive Insurance, Cleveland Ohio. *Hidden Markov Model for Weather Forecast*.

A Hidden Markov Model is a stochastic tool for representing the probability distribution over sequences of observations. The machine learning model has been extensively used in areas such as speech recognition applications, predicting economic regime and quite recently in the field of weather forecasting. In this talk, we use the Hidden Markov Model for multiple observation sequences of weather data. First, we choose the number of states for the hidden Markov model by using the Akaike Information Criteria and the Bayesian Information Criteria. Then, we used the selected model for predicting the weather in some stations in the USA. Last, we compare our predictions to the weather forecast from the National Weather Channel and the actual weather data. (Received September 17, 2019)

1154-60-2102 **Galyna V. Livshyts**, **Konstantin Tikhomirov*** (ktikhomirov6@gatech.edu) and **Roman Vershynin**. *Invertibility of inhomogeneous random matrices*.

Let A be an $n \times n$ random matrix with independent entries such that $\mathcal{L}(a_{ij}, 1) \leq b$ for some $b \in (0, 1)$, and $\mathbb{E}\|A\|_{HS}^2 \leq Kn^2$ for $K \geq 1$. We show that the smallest singular value $\sigma_n(A)$ of A satisfies

$$\mathbb{P}\left\{\sigma_n(A) \leq \frac{\varepsilon}{\sqrt{n}}\right\} \leq C\varepsilon + 2e^{-c\varepsilon}, \quad \varepsilon \geq 0,$$

where $c, C > 0$ may only depend on b and K . (Received September 17, 2019)

1154-60-2206 **Abishek Sankararaman***, abishek@utexas.edu, and **Emmanuel Abbe** and **Francois Baccelli**. *Community Detection on Euclidean Random Graphs*.

In this talk, we consider the problem of community detection on Euclidean random geometric graphs where each vertex has two latent variables: a binary community label and a \mathbb{R}^d valued location label which forms the support of a Poisson point process of intensity λ . A random graph is then drawn with edge probabilities dependent on both the community and location labels. In contrast to the stochastic block model (SBM) that has no location labels, the resulting random graph contains many more short loops due to the geometric embedding. We consider the recovery of the community labels, partial and exact, using the random graph and the location labels. We establish phase transitions for both sparse and logarithmic degree regimes, and provide bounds on the location of the thresholds, conjectured to be tight in the case of exact recovery. We also show that the threshold of the distinguishability problem, i.e., the testing between our model and the null model without community labels exhibits no phase-transition and in particular, does not match the weak recovery threshold (in contrast to the SBM). (Received September 17, 2019)

1154-60-2240 **Jianjun Paul Tian*** (jtian@nmsu.edu), 259 SH, Department of Mathematical Sciences, New Mexico State University, Las Cruces, NM 88001. *Stochastic growth pattern of untreated human glioblastomas predicts the survival time for patients*. Preliminary report.

Glioblastomas are highly malignant brain tumors. Knowledge of growth rates and growth patterns is useful for understanding tumor biology and planning treatment logistics. Based on untreated human glioblastoma data collected in Trondheim, Norway, we first fit the average growth to a Gompertz curve, then find a best fitted white noise term for the growth rate variance. Combining these two fits, we obtain a new type of Gompertz diffusion dynamics, which is a stochastic differential equation (SDE). Newly collected untreated human glioblastoma data in Seattle, US, verifies our model. Instead of growth curves predicted by deterministic models, our SDE model predicts a band with a center curve as the tumor size average and its width as the tumor size variance over time. Given the glioblastoma size in a patient, our model can predict the patient survival time with a prescribed probability. Our model can be applied to studies of tumor treatments. As a demonstration, we investigate different protocols of surgical resection using our model and provide detailed strategies. This is a joint work with Ziwei Ma, Ben Niu, Tuan Phan, Philip Maini, Eric Holland, et al. (Received September 17, 2019)

1154-60-2264 **SangJoon Lee*** (sangjoon.lee@uconn.edu). *Quasi-Limiting Behavior of a Drifted Brownian Process.*

A Quasi-Stationary Distribution for a Markov process with an absorbing state is a distribution on the state space which, if taken as an initial distribution, is invariant under the law of the process conditioned not to be absorbed. This talk will first review results on the existence and domain of attraction of such distribution for a Brownian Motion with drift. Then I will discuss the limiting behavior of the process when the initial distribution is not in the domain of attraction of any QSD, by presenting the explicit corresponding scale factor and the scaled limiting distribution. (Received September 17, 2019)

1154-60-2320 **Sunday A. Asogwa** and **Jebessa B. Mijena*** (jebessa.mijena@gcsu.edu), 231 W. Hancock St., CBX 17, Milledgeville, GA 31061, and **Erkan Nane.** *Non-existence results for space-time fractional stochastic partial differential equations.*

Consider non-linear time-fractional stochastic reaction-diffusion equations of the following type,

$$\partial_t^\beta u_t(x) = -\nu(-\Delta)^{\alpha/2} u_t(x) + I^{1-\beta}[b(u) + \sigma(u) \dot{F}(t, x)]$$

in $(d+1)$ dimensions, where $\nu > 0$, $\beta \in (0, 1)$, $\alpha \in (0, 2]$. The operator ∂_t^β is the Caputo fractional derivative while $-(-\Delta)^{\alpha/2}$ is the generator of an isotropic α -stable Lévy process and $I^{1-\beta}$ is the Riesz fractional integral operator. The forcing noise denoted by $\dot{F}(t, x)$ is a Gaussian noise. These equations might be used as a model for materials with random thermal memory. We derive non-existence (blow-up) of global random field solutions under some additional conditions, most notably on b , σ and the initial condition. (Received September 17, 2019)

1154-60-2354 **Erika B Roldan-Roa***, e.rolدان.roa@gmail.com. *Topological and Geometric Analysis of Discrete Random Growth Models.*

Discrete Random Growth Models (DRGMs) are important models for the dynamical behavior of biological processes, and the interaction between the local and global properties of those systems. In previous work, we applied topological, combinatorial, and geometric techniques to the Eden Cell Growth Model, which is a DRGM process on the regular tessellation in the plane. Here, we extend this analysis to two-dimensional non-regular cell growth processes. Most significantly, we show that the asymptotic geometry and topology (measured in terms of the rank of the first homology group) of non-regular DGRMs are substantially different from that of the Eden Growth Model. This is joint work with Iancarlo Espinoza and Victor Perez-Abreu (CIMAT). (Received September 17, 2019)

1154-60-2373 **David Eric Weisbart*** (dweisbart@gmail.com), 7694 Rotunda Court, Riverside, CA 92507. *p -Adic Brownian Motion as a Limit of Discrete Time Random Walks.*

A large class of diffusion equations in the p -adic setting give rise to p -adic analogs of brownian motion. We show that these stochastic processes are limits of discrete time random walks on grids. The current work generalizes previous results by allowing for diffusion equations associated to Vladimirov operators with exponents greater than zero rather than greater than one, as was previously studied. We obtain stronger results in this more general setting as well. In particular, the proof of the weak-* convergence of the measures associated to discrete time processes to their continuum limits was previously valid only for path spaces with compact time intervals. We extend this proof to the setting of path spaces with unbounded time intervals.

This current study is a part of a larger program to extend physical models to non-Archimedean settings. Since the Archimedean axiom is fundamentally a statement about subdivision of physically measurable quantities and measurement fails at the Planck scale, the study of physics at ultra small distance and time scales may involve non-Archimedean structures. We will comment on the relationship of the current study to the broader research theme and pose some open problems that should merit further investigation. (Received September 17, 2019)

1154-60-2431 **Philip Speegle*** (pspeegle@crimson.ua.edu), 1901 5th Ave E, Tuscaloosa, AL 35401, and **R. Oliver VandenBerg.** *The Voter Model on Bipartite Graphs.*

Given a coloring on a graph, the Voter Model is a Markov Chain where the next time is given by a vertex chosen at random changing color to that of a randomly chosen neighbor. Given enough time, this will reach an absorbing state where all vertices are the same color. Conditioning on not reaching this absorbing state, it will approach a quasi-stationary distribution (QSD). Using techniques from probability and linear algebra, we analyzed both these distributions themselves as well as how the probabilities approach these distributions over time on star graphs and complete bipartite graphs. When we calculated the general transition matrix, we proved a formula to find any term in its eigenvector along with its normalization constant, allowing us to explicitly find its distribution. (Received September 17, 2019)

1154-60-2433 **Karl Liechty***, klichty@depaul.edu. *A deformation of the Tracy-Widom GOE distribution*. Preliminary report.

In random matrix theory the Tracy-Widom GOE distribution describes the location of the largest eigenvalue of large real symmetric random matrices. It also describes height fluctuations certain random growth models in the KPZ universality class, and can be characterized as the maximum value of the Airy process minus a parabola. We use this characterization to study a natural transition between the Tracy-Widom GOE distribution and the Gaussian distribution. Namely we study the maximum value of the Airy process with wanderers whose marginal distributions correspond to spiked complex Hermitian random matrix models like GUE. We present both Fredholm determinant and Painlevé formulas for the distribution of this maximum. This is joint work with Daniel Remenik. (Received September 17, 2019)

1154-60-2455 **Jonathan M. Wells*** (wellsj@reed.edu), 3203 Southeast Woodstock Boulevard, Portland, OR 97202-8199. *Hyperpfaffian descriptions of β -ensembles when β is a perfect square*. Preliminary report.

We use combinatorial identities in the shuffle and exterior algebra to present hyperpfaffian formulations for the partition functions of β -ensembles with arbitrary probability measure, when β is a square integer. This is an analogue of the de Bruijn integral identities for the $\beta = 1$ and $\beta = 4$ ensembles of Hermitian matrices in random matrix theory. (Received September 17, 2019)

1154-60-2653 **Nicolas Broutin, Luc Devroye and Nicolas Fraiman***, 337 Hanes Hall CB #3260, Chapel Hill, NC 27599. *Recursive functions on conditional Galton-Watson trees*.

A recursive function on a tree is a function in which each leaf has a given value, and each internal node has a value equal to a function of the number of children, the values of the children, and possibly an explicitly specified random element. The value of the root is the key quantity of interest in general. In this talk, we describe the limit behavior when the leaf values are drawn independently from a fixed distribution, and the tree is a random Galton-Watson tree conditional on its size. (Received September 17, 2019)

1154-60-2695 **George Yin, Le Yi Wang and Thu Thi Le Nguyen*** (fw6622@wayne.edu), 31133 Cedar Ridge Ln, Madison Heights, MI 48071. *Switching Stochastic Approximation and Applications to Networked Systems*.

This work investigates the interaction between control and communications in networked systems by studying a class of stochastic approximation algorithms that accommodate random network topology switching processes, time-varying functions, nonlinear dynamics, additive and non-additive noises, and other uncertainties. Interaction among control strategy and the multiple stochastic processes introduces critical challenges in such problems. By modeling the random switching as a discrete-time Markov chain and studying multiple stochastic uncertainties in a unified framework, it is shown that under broad conditions, the algorithms are convergent. The performance of the algorithms is further analyzed by establishing their rate of convergence and asymptotic characterizations. Simulation case studies are conducted to evaluate the performance of the procedures in various aspects (Received September 17, 2019)

1154-60-2732 **Taylor Meredith*** (taylor.meredith@nyu.edu), **Iddo Ben-Ari, Hugo Panzo, Xioran Tan and Jonah Green**. *A Finite-Memory Elephant Random Walk and the Central Limit Theorem for Additive Functionals of Markov Chains*.

The Central Limit Theorem (CLT) for additive functionals of Markov chains is a well known result with a long history. In this talk, we present applications of the CLT to two finite-memory versions of the Elephant Random Walk. Our results solve the problem of cumbersome computations for higher order moments and thus, we use these moments to prove limit theorems. We present a more accessible derivation of the CLT for additive functionals of finite state Markov chains, which is based on positive recurrence, the CLT for IID sequences and linear algebra, and which focuses on characterization of the variance. (Received September 17, 2019)

1154-60-2762 **Josiah Park*** (j.park@gatech.edu), Georgia Institute of Technology, 686 Cherry St NW, Atlanta, GA 30332. *On the minimum ℓ_p -norm of random matrices*. Preliminary report.

We prove small-ball estimates for the minimum value of $\|Ax\|_p$ for x on the sphere and a random matrix A , using only weak assumptions on the distribution of the entries. Using random-rounding, an efficient way to discretize the sphere, such results and other extensions of estimates for singular value may be derived. I will speak about these results, obtained in collaboration with G. Livshyts. (Received September 17, 2019)

1154-60-2802 **Scott A McKinley***, 6823 St Charles Ave, New Orleans, LA 70118, and **Christel Hohenegger**. *Anomalous Diffusion of Foreign Particles in Biological Fluids*.

The last twenty years have seen a revolution in tracking data of biological agents across unprecedented spatial and temporal scales. An important observation from these studies is that path trajectories of living organisms can appear random, but are often poorly described by classical Brownian motion. The analysis of this data can be controversial because practitioners tend to rely on summary statistics that can be produced by multiple, distinct stochastic process models. Furthermore, these summary statistics inappropriately compress the data, destroying details of non-Brownian characteristics that contain vital clues to mechanisms of transport and interaction. In this talk, I will describe the stochastic integro-differential equation framework we use to model this behavior and the associated statistical challenges that have arisen from recent work on the movement of foreign agents, particularly synthetic microparticle probes, in human mucus. (Received September 18, 2019)

62 ► *Statistics*

1154-62-10 **Nancy Reid*** (reid@outstat.toronto.edu), University of Toronto, Toronto, Ontario M5S 3G3, Canada. *In Praise of Small Data: Statistical and Data Science*.

The over-promotion of “Big Data” has perhaps settled down, but the data are still there, and the rapid development of the new field of data science is a response to this. As more data become available, the questions asked become more complex, and big data can quickly turn into small data. Statistical science has developed an arsenal of methods and models for learning under uncertainty over its 200-year history. Some thoughts on the interplay between statistical and data science, their interactions with science, and the ongoing relevance of statistical theory, will be presented, and illustrated through a number of examples. (Received September 17, 2019)

1154-62-158 **Tingran Gao*** (tingrangao@galton.uchicago.edu), 5747 S Ellis Avenue Jones 316, Chicago, IL 60637-1441, and **Zhizhen Zhao** (zhizhenz@illinois.edu), 324 CSL, 1308 W Main St, Urbana, IL 61801. *Learning the Geometry of Data Through Many Representations*.

Fibre bundles serve as a natural geometric setting for many learning problems involving non-scalar pairwise interactions. Modeled on a fixed principal bundle, different irreducible representations of the structural group induce many associated vector bundles, encoding rich geometric information for the fibre bundle as well as the underlying base manifold. An intuitive example for such a learning paradigm is phase synchronization—the problem of recovering angles from noisy pairwise relative phase measurements—which is prototypical for a large class of imaging problems in cryogenic electron microscopy (cryo-EM) image analysis. We propose a novel nonconvex optimization formulation for this problem, and develop a simple yet efficient two-stage algorithm that, for the first time, guarantees strong recovery for the phases with high probability. We demonstrate applications of this multi-representation methodology that improve denoising and clustering results for cryo-EM images. This algorithmic framework also extends naturally to general synchronization problems over other compact Lie groups, with a wide spectrum of potential applications. (Received August 16, 2019)

1154-62-726 **Amy Pitts***, amy.pitts1@marist.edu, and **Sujata Patil**, patils@mskcc.org. *Missing data in cancer studies*. Preliminary report.

Missing data is a common occurrence when working with medical-related datasets. When analysts are faced with missing values, decisions need to be made on how to deal with them before completing analyses. In this work, we provide an in depth look on how missing values can affect statistical inference, and also techniques on how to deal with missing data. Using simulation data, the authors evaluate how the three missing data mechanisms (MCAR, MAR, and MNAR) influence the parameter estimates in a model with a continuous outcome and two continuous covariates. In all cases the outcome was set to missing according to the different mechanisms. The simulations are evaluated using bias, mean squared error and coverage. The same datasets were used to evaluate the complete case, and two different imputation methods. We applied the complete case method as well as multiple imputation to a breast cancer data set. Through simulations and the case study, we observed that multiple imputation reduces the amount of bias in regression when the outcome is missing. With easy implementation in standard statistical software, multiple imputation should be considered when a dataset has an abundance of missing values. (Received September 10, 2019)

- 1154-62-1021 **Yujia Ding*** (yujia.ding@cgu.edu), **Qidi Peng**, **Hansen Chen** and **Zhengming Song**. *Arbitrary Rectangle-range Generalized Elastic Net Penalty Model and Variable Selection Consistency*.

We propose an arbitrary rectangle-range generalized elastic net penalty least squares method (ARGEN) for variable selection. It can be applied in high dimensional sparse linear regression models. Many commonly used methods such as lasso, ridge and elastic net are not only extended but also improved by ARGEN. We proved that ARGEN has variable selection consistency under certain condition which extends the Nonnegative Irrepresentable Condition in elastic net. To construct the estimator of ARGEN, we establish a new algorithm, which is an extension of multiplicative updates approach and is shown to has estimation consistency. We run multiple simulations to show that ARGEN is applicable to more complicated problems and at the same time has better performance, compared with that of commonly used methods. The rectangle-range constrained index tracking problem in stock market is studied in the latter part. The results indicate that ARGEN has small tracking error and is successful in assets selection. (Received September 12, 2019)

- 1154-62-1030 **Menggang Yu*** (meyu@biostat.wisc.edu) and **Rui Chen**. *Tailored Optimal Post-Treatment Surveillance for Cancer Recurrence*.

A substantial rise in the number of cancer survivors has led to urgent management questions regarding effective post-treatment surveillance strategies for cancer recurrence. Current surveillance guidelines provided by a number of professional societies all warn against overly aggressive surveillance, especially for low risk patients, but all fail to provide more specific directions to accommodate underlying heterogeneity of cancer recurrence. Therefore it is imperative to develop data-driven strategies that can tailor the surveillance schedules to recurrence risk in this era of stricter insurance regulations, provider shortages, and rising costs of health care. Due to a lack of statistical methods for optimizing surveillance scheduling in presence of competing risks, we propose a general approach that uses an intuitive loss function and dynamic programming for optimization of early detection of recurrence before death. The proposed strategies can tailor to patient risks of recurrence, in terms of both intensity and amount of surveillance. Using general three-state Markov models, our method is flexible and includes earlier works as special cases. We illustrate our method in both simulation studies and an application to breast cancer surveillance. (Received September 12, 2019)

- 1154-62-1078 **Emad M Abdurasul*** (abduraem@jmu.edu), 273 Emerald DR, Harrisonburg, VA 22801. *Small Sample Inference for The Product Limit survival function Estimator*. Preliminary report.

Our contribution is developing a saddlepoint-based method for generating small sample confidence interval for product limit estimator (PLE), under the proportional hazards model. In the process, we derive the exact distribution of these estimators and developed mid-population tolerance interval for it. Our saddlepoint method depends upon the Mellin transform of the zero-truncated survival estimator. This transform is inverted via saddlepoint approximations to yield highly accurate approximations to the cumulative distribution function of the respective cumulative hazard function estimator and this distribution function is then inverted to produce our saddlepoint confidence interval. Then we compare our saddlepoint confidence interval with confidence interval that is obtained from competing large sample method as well as that obtained from the exact distribution. In our simulation study, we found that the saddlepoint confidence interval is very close to the confidence interval derived from the exact distribution, while being much easier to compute, and outperforms the competing large sample method in terms of coverage probability. (Received September 13, 2019)

- 1154-62-1103 **Ramchandra Rimal***, 4393 Andromeda Loop N, Orlando, FL 32826, and **Marianna Pensky**. *Estimation in the Sparse Popularity Adjusted Stochastic Block Model*.

Consider a network with its adjacency matrix $A_{ij} \sim \text{Ber}(P_{ij})$. Since majority of real life networks are sparse, we consider the sparse PABM model. While majority of sparse network models are based on a rather unrealistic assumption that the maximum connection probability is bounded above by a small quantity, the flexibility of the PABM allows to model some of the probabilities of connections between the nodes as identical zeros. We estimate the probability matrix P . Our estimation technique involves the penalized optimization procedure. We estimate the matrix of the probability of connection between nodes by minimizing the squared differences between the blocks of the matrix A to its best rank one approximation over the set of all possible clustering matrix. We use the oracle inequality to find the upper bound of the estimation error and clustering error. (Received September 13, 2019)

1154-62-1142 **Thomas J. Faulkenberry*** (faulkenberry@tarleton.edu), Department of Psychological Sciences, Tarleton State University, Stephenville, TX 76401. *Closed form Bayes factor techniques for measuring evidential value from analysis of variance models.*

The analysis of variance is one of the most frequently used techniques for assessing group differences in experimental design. In a Bayesian framework, the goal of the analysis of variance is to assess the relative evidence between two competing models: \mathcal{H}_0 , where all group means are equal, and \mathcal{H}_1 , where at least one group mean is different from the others. In this talk, I will discuss recent work on developing methods for computing Bayes factors from analysis of variance summaries. The Bayes factor, defined as the ratio of marginal likelihoods for two competing models, represents the factor by which the prior odds for \mathcal{H}_1 over \mathcal{H}_0 is updated after observing data. Particularly, I will discuss a choice of prior distribution that yields Bayes factors with a simple closed form structure without integral representation. These results allow for a number of nice applications which I will discuss, including a web application that applied researchers can use to measure the evidential value of their own data. (Received September 13, 2019)

1154-62-1152 **Marie V Ozanne*** (mozanne@mtholyoke.edu). *Bayesian compartmental models and reproductive numbers for an infection with multiple infectious sources and transmission modes.*

Stochastic compartmental models comprise a class of techniques that can be used to study infection transmission dynamics. While various models have been developed to accommodate infections with an exposed (but not infectious) class or a less infectious carrier state, they do not accommodate an infection with two different infectious groups that are potentially equally important to maintaining infection in a population. We propose a Bayesian Susceptible, Asymptomatic, sYmptomatic, recoVered, Removed (SAYVR) model to address this scenario. We also present an Infection Source-specific Empirically Adjusted Reproductive Number (ISEARN) to quantify contributions from each of these two infectious classes to maintaining infection in a population of interest. We apply these methods to study the transmission dynamics of visceral leishmaniasis, a disease with two infectious class reservoirs, in the Americas. (Received September 13, 2019)

1154-62-1201 **Vasileios Maroulas*** (vmaroula@utk.edu), 1403 Circle Dr., Knoxville, TN 37996, and **Farzana Nasrin** and **Christopher Oballe**. *A Bayesian Framework for Persistent Homology.*

Persistence diagrams offer a way to summarize topological and geometric properties latent in datasets. While several methods have been developed that utilize persistence diagrams in statistical inference, a full Bayesian treatment remains absent. This talk, relying on the theory of point processes, presents a Bayesian framework for inference with persistence diagrams relying on a substitution likelihood argument. In essence, we model persistence diagrams as Poisson point processes with prior intensities and compute posterior intensities by adopting techniques from the theory of marked point processes. We then propose a family of conjugate prior intensities via Gaussian mixtures to obtain a closed form of the posterior intensity. Finally we demonstrate the utility of this Bayesian framework, packaged in R under BayesTDA, with a classification problem in materials science using Bayes factors. (Received September 13, 2019)

1154-62-1270 **Anna Little*** (litt1119@msu.edu), 2420 Alpha St., Lansing, MI 48910, and **Qiang Sun** and **Yuying Xie**. *Exact Cluster Recovery via Classical Multidimensional Scaling.*

Classical multidimensional scaling is an important tool for dimension reduction in many applications. Yet few theoretical results characterizing its statistical performance exist. This work provides a theoretical framework for analyzing the quality of embedded samples produced by classical multidimensional scaling. This lays the foundation for various downstream statistical analyses, and we study its performance in the setting of clustering noisy data. Our results provide scaling conditions on the sample size, ambient dimensionality, between-class distance and noise level under which classical multidimensional scaling followed by a clustering algorithm can recover the cluster labels of all samples with high probability. Numerical simulations confirm these scaling conditions are sharp in low, moderate, and high dimensional regimes. Applications to both human RNAseq data and natural language data lend strong support to the methodology and theory. (Received September 14, 2019)

1154-62-1615 **Anna Grim** and **Alona Kryshchenko*** (alona.kryshchenko@csuci.edu), 1 University Dr, Camarillo, CA 93012, and **Linda Ness**, **Patricia Medina**, **Boris Iskra**, **Randy Paffenroth**, **Sui Tang**, **Megan Owen** and **Melissa Ngamini**. *Analysis of Simulated Crowd Flow Exit Data: Visualization, Panic Detection, and Exit Time Convergence, Attribution and Estimation.*

This paper describes the results of exploratory analyses of black box simulation data modeling crowds exiting different configurations of a one-story building. The simulation data was created using the SteerSuite platform.

Exploratory analysis was performed on the simulation data without knowledge of simulation algorithm. The analysis effort provided a hands-on introduction to issues in crowd dynamics. Analyses focused on visualization, panic detection, exit convergence pattern discovery, identification of parameters influencing exit times, and estimation of exit times. A variety of mathematical and statistical methods were used: k-means clustering, principal component analysis, normalized cut grouping, product formula representation of dyadic measures, logistic regression, auto-encoders, and neural networks. The combined set of results provided insight into the algorithm and the behavior modeled by the algorithm and revealed the need for quantitative features modeling and distinguishing the shapes of the building configurations. (Received September 16, 2019)

1154-62-1616 **Xu (Sunny) Wang*** (xwang@wlu.ca), Department of Mathematics, 75 University Avenue West, Waterloo, Ontario N2L 3C5, Canada, and **Hugh Chipman** (hugh.chipman@acadiau.ca), Department of Mathematics and Statistics, Acadia University, 15 University Ave, Wolfville, NovaScotia B4P 2R6, Canada. *A New Constrained Mixture Models for Drug Discovery Data with Innovative Iterative Algorithms.*

It has been well-known that Statistics has played a very important role in drug discovery: facilitate and speed up the drug finding process. Statistical learning in drug discovery seeks a good classifier that separates chemical compounds into active and inactive classes. The active compounds are those that inhibit disease virus and are taken as drug candidates. However, the characteristics of drug data imply many challenges for structure modeling and identification of active compounds. Among these characteristics are the rarity of active compounds, the large volume of compounds tested by high-throughput screening, sub-set governed activity and the complexity of molecular structure and its relationship to activity. Due to the challenges of drug discovery data, we develop a new statistical learning model based on mixture models: Constrained Mixture Discriminant Analysis (CMDA) model. This method is designed to catch multiple mechanisms that lead to activity, explore the subsets of descriptors and be easily interpreted (e.g. identify important descriptors). The Expectation-Maximization, is used to estimate the parameters of the CMDA model. Comparing to the popular MclusteDA model, CMDA outperforms MclusteDA in terms of the parameter estimation and the active compound identification. (Received September 16, 2019)

1154-62-1640 **Charles Chen*** (charles.chen.training@gmail.com) and **Mason Chen** (mason.chen.training@gmail.com). *STEAMS Methodology.* Preliminary report.

Introduce STEAMS Methodology: Science, Technology, Engineering, AI, Mathematics, and Statistics. Use 2-3 Case Studies to demonstrate STEAMS methods. In addition to STEM, adding AI can discover the more insights on Science. Separating Statistics from Math can draw more practical risk management and reliable decision making. (Received September 16, 2019)

1154-62-1647 **Charles Chen*** (charles.chen.training@gmail.com) and **Mason Chen** (mason.chen.training@gmail.com). *STEAMS Methodology.* Preliminary report.

Introduce STEAMS Methodology: Science, Technology, Engineering, AI, Mathematics, and Statistics. Use 2-3 Case Studies to demonstrate STEAMS methods. In addition to STEM, adding AI can discover the more insights on Science. Separating Statistics from Math can draw more practical risk management and reliable decision making. (Received September 16, 2019)

1154-62-1673 **Aradhana Soni*** (sonia1@etsu.edu) and **Anant Godbole** (godbolea@etsu.edu). *Some properties of the First Return to the Origin (FRO) Distribution.* Preliminary report.

It is well known that a random walk beginning at $(m, 0)$ will return to origin in n steps with a probability of $m/n * (((n + m)/2)/2^n)$. In this talk, we will explore further some properties of this distribution. (Received September 16, 2019)

1154-62-1699 **Andrey Sarantsev*** (asarantsev@unr.edu), **Taran Grove** and **Akram Reshad.** *Long-Term Bayesian Analysis of Stock and Bond Markets.* Preliminary report.

We present long-term outlook on the USA stock and bond markets, measured by S&P 90/500, 1-year T-bills, and 10-year T-bills. We use various time series models with different time steps. (Received September 16, 2019)

1154-62-1821 **Sheng Gao***, 800 Lakeshore Drive, Birmingham, AL 35229, and **Mingwei Sun**, 800 Lakeshore Drive, Birmingham, AL 35209. *A Penalized Neural Network Process for Viable Selection and Application in Economics.*

Big data represents a new era in data exploration and utilization, which keeps generating large-volume and complex data sets all the time. Many statistical techniques and computer algorithms including deep learning skills have been created to provide new methods of analyzing big data. In this paper, we research the performances of

penalized regressions including lasso, elastic net, and ridge and compare with the Ordinary Least Squared(OLS) model in various simulation scenarios. Furthermore, a new method combines the penalized regression with non-linear neural network model is proposed for variable selection and data prediction. Moreover, we investigated the effects of different regression criteria including Mean Square Error(MSE), Akaike's information criterion (AIC), Bayesian information criterion(BIC) and AIC correction (AICc) to the different models. Last but not least, a real-data application is implemented with these models and the result shows our new method has a better performance than the others. (Received September 16, 2019)

1154-62-1865 **Mingwei Sun*** (msun1@samford.edu), 800 Lakeshore Drive, Birmingham, AL 35229, and **Patrick Wang.** *A Time-lagged Penalized Regression Model and Applications to Economic Modeling.*

Variable selection has been an important topic in high-dimensional data analysis. Penalized regressions which achieve variable selection and coefficient estimation simultaneously have enjoyed varying degrees of success in many different fields in recent years. In this paper, a time-lagged penalized regression model which considers the time-delayed effect in a data set and inherits the benefits of penalized regressions is proposed. The model identifies the lag times which make the correlations between the dependent and each independent variables highest and transforms the data based on the lags. An application of the new method to economical modeling which usually contains many lagged variables is illustrated. The result shows that the proposed model can discover some hidden variables that are not included in the linear penalized regression models and has a better prediction performance. Therefore, it could explain the data more comprehensively and objectively. (Received September 17, 2019)

1154-62-2046 **Angel R Pineda*** (angel.pineda@manhattan.edu), Department of Mathematics, Manhattan College, 4513 Manhattan College Parkway, Riverdale, NY 10471. *Optimizing constrained reconstruction in magnetic resonance imaging for signal detection.*

Constrained reconstruction in magnetic resonance imaging (MRI) allows the use of prior information through constraints to improve the reconstructed images. Constrained reconstruction leads to images which appear clearer than reconstructions without constraints but because the methods are typically non-linear, the reconstructed images have artifacts whose structure is hard to predict. In this work, we compared different methods of optimizing the regularization parameter using a total variation constraint in the spatial domain and sparsity in the wavelet domain for one-dimensional (2.56x) acceleration using variable density under-sampling. We compared the mean squared error (MSE), structural similarity (SSIM) and the area under the receiver operating characteristic (AUC) using a linear discriminant for detecting a small and a large signal with a signal-known-exactly (SKE) task with varying backgrounds. Our results show that the AUC dependence on regularization parameters depend on the imaging task (i.e. the signal being detected). We also found that a model-based reconstruction enforcing data agreement with no prior information did statistically as well as models which included total variation or wavelet sparsity. (Received September 17, 2019)

1154-62-2460 **Vindya Kumari Pathirana*** (vindya.pathirana@uconn.edu), Waterbury, CT. *Dynamically Adoptive Mahalanobis Based k-Nearest Neighbor Forecasting in Time Series Data.* Preliminary report.

Nearest Neighbor Algorithms which are among the most popular non-linear pattern recognition methods outperform the available linear forecasting methods when consider the high frequency foreign exchange data. In our previous work, we provided evidence that Mahalanobis distance- based k-nearest neighbor procedure outperforms the traditional Euclidean distance-based algorithm by comparing both the forecasting accuracy and trading performances. In this work, we identify couple of important facts, which can improve the k-NN algorithm even further. For highly volatile time instances, uniformly selected neighbors might not be that accurate. Instead of choosing a fixed number of neighbors at each time instance, we employed dynamically adoptive number of neighbors for the nearest neighbor forecasting algorithm. The performances were compared in two ways: (i) forecast accuracy and (ii) transforming their forecasts into a more effective technical trading rule. (Received September 17, 2019)

1154-62-2472 **Israel A Almodovar-Rivera*** (israel.almodovar@upr.edu), San Juan, PR 00936. *Kernel-estimated Nonparametric Overlap-Based Syncytial Clustering.*

Standard clustering algorithms usually find regular-structured clusters such as ellipsoidally- or spherically-dispersed groups, but are more challenged with groups lacking formal structure or definition. Syncytial clustering is the name that we introduce for methods that merge groups obtained from standard clustering algorithms in

order to reveal complex group structure in the data. Here, we develop a distribution-free fully-automated syn-
cycial clustering algorithm that can be used with k-means and other algorithms. Our approach computes the
cumulative distribution function of the normed residuals from an appropriately fit k- groups model and calculates
the nonparametric overlap between each pair of groups. (Received September 17, 2019)

1154-62-2504 **Jeremy L Thompson*** (jeremy.thompson@colorado.edu), **Valeria Barra**, **Jed Brown**,
Yohann Dudouit and **Oana Marin**. *Preconditioning with BDDC and FDM for High
Order Finite Elements with libCEED*. Preliminary report.

Matrix-free representations of high-order finite element operators are less expensive than sparse matrices, both
with respect to the FLOPs needed for evaluation and the memory transfer needed for a matrix-vector multiply
(Received September 18, 2019)

1154-62-2564 **William Rosenthal*** (william.rosenthal@pnnl.gov) and **Francesca Grogan**
(francesca.grogan@pnnl.gov). *"Sintering" models and measurements: data assimilation
for microstructure prediction of nylon component SLS additive manufacturing*.

Selective laser sintering (SLS) printers drive high-throughput polymer additive manufacturing. However, ther-
mal, feedstock, and exposure variations can introduce significant microstructure variability in the same batch of
components. Phase-field models have been developed to simulate material microstructure evolution and kinetic
during synthesis. We develop sensitivity analyses and introduce an adaptive sampling Bayesian algorithm
to estimate significant parameters and uncertainties in a 3D phase-field model for nylon-12 polymer synthesis,
including system free energy, interfacial energy, and sintering kinetics. In a high-throughput DIRAC-automated
computational design loop, we validate the model through comparison to high-resolution 3D CT images of com-
ponents built with varying orientations throughout the build chamber, as well as to partial sintering artifacts
identified by laser exposure metadata. We quantify uncertainties in phase-field initial and operating conditions
by developing a stochastic feedstock model from laser diffractometry and 3D CT imaging, and by analyzing
real-time infrared thermographic movies taken throughout the build process. (Received September 17, 2019)

1154-62-2580 **Michelle R. DeDeo*** (mdedeo@unf.edu), 1 UNF Dr., Dept. of Mathematics & Statistics,
Jacksonville, FL 32224. *Data Science: Challenges in the Quintessential Interdisciplinary
Field*. Preliminary report.

In 2008, Anderson in a Wired Magazine editorial entitled The End of Theory: The Data Deluge Makes the
Scientific Method Obsolete stated, ‘Out with every theory of human behavior, from linguistics to sociology. . .
Who knows why people do what they do? The point is they do it, and we can track and measure it with
unprecedented fidelity. With enough data, the numbers speak for themselves’. But do they?

In the 1980’s, Cox interestingly stated, “Most real life statistical problems have one or more non-standard
features. There are no routine statistical questions; only questionable statistical routines.” Box & Draper later
took this a step further by stating that, “All models are wrong, but some are useful.”

Here we discuss the roles of scientists and engineers versus statisticians, distinctions and similarities between
open data versus big data, challenges in evolving techniques for current and future data scientists, and updating
old concerns over statistical analyses with the current challenges that data science poses. (Received September
17, 2019)

1154-62-2790 **Marisa Blackman*** (mblackman@smith.edu), Department of Mathematics and Statistics,
Smith College, Northampton, MA 01060. *Statistical analysis of jackal skull length*.
Preliminary report.

This project analyzes jackal skull length data to ascertain differences between *Canis Aureus* and *Canis Lupaster*,
which have been mistakenly classified as the same species. (Received September 18, 2019)

1154-62-2795 **Annika King*** (jarvsi@math.byu.edu), 302 TMCB, Department of Mathematics,
Brigham Young University, Provo, UT 84602, and **Cristina Lange** and **Tyler J Jarvis**.
Using Markov-Chain Monte Carlo methods to study gerrymandering in Utah.

We present a mathematical analysis of Utah’s 2012 political redistricting, using Markov-chain Monte Carlo
methods to construct a large ensemble of alternative district plans that satisfy the legal requirements of contiguous
districts with equal population. We compare the legislature’s adopted plan in terms of many different measures
for redistricting fairness, including partisan bias, mean-median score, efficiency gap, and the percentage of
Republican vs Democartic voters in each district. We use precinct-level election results from the 2010 United
States Senate election to estimate the distribution of voters’ political parties, and we use the GerryChain library
written by the Metric Geometry and Gerrymandering Group to construct the ensemble. (Received September
18, 2019)

65 ► Numerical analysis

- 1154-65-126 **Xinyun Zhu***, Department of Mathematics, UTPB, Odessa, TX 79762. *A variant of relaxed triangular splitting preconditioners for generalized saddle point problems from Navier–Stokes Equations.* Preliminary report.

Based on the dimensional split preconditioner, in this paper, a variant of relaxed triangular splitting preconditioner (VRTS) is presented and discussed, in which a simple and feasible way is designed for the selection of the parameters. Spectral properties of the VRTS preconditioned matrix are analyzed in detail. Theoretical analysis shows that when applying the VRTS preconditioner within a Krylov subspace method, the less computational cost is required at each iteration than some state-of-the-art approaches. Finally, some numerical experiments arising from the discretization of Navier–Stokes equations are given to illustrate and validate the efficiency and robustness of the presented preconditioners with using GMRES(#) as an iterative solver. (Received August 12, 2019)

- 1154-65-163 **Mark A Iwen*** (markiwen@math.msu.edu), **Bosu Choi** and **Toni Volkmer**. *On Best s -Term Approximation Guarantees for Bounded Orthonormal Product Bases in Sublinear-Time.*

In this talk we will discuss fast and memory efficient numerical methods for learning the best s term approximation of functions of many variables in terms of a given bounded orthonormal product bases. Let \mathcal{B} be a finite Bounded Orthonormal Product Basis (BOPB) of cardinality $|\mathcal{B}| = N$. Herein we will develop methods that rapidly approximate any function f that is nearly sparse in the BOPB, that is, f of the form

$$f(\mathbf{x}) \approx \sum_{b \in \mathcal{S}} c_b \cdot b(\mathbf{x})$$

with $\mathcal{S} \subset \mathcal{B}$ where $|\mathcal{S}| = s$ is much less than N . Our method has a runtime of just $(s \log N)^{O(1)}$, uses only $(s \log N)^{O(1)}$ function evaluations on a fixed and nonadaptive grid, and not more than $(s \log N)^{O(1)}$ bits of memory. We emphasize that nothing about \mathcal{S} or any of the coefficients c_b is assumed in advance other than that $\mathcal{S} \subset \mathcal{B}$ has $|\mathcal{S}| \leq s$. Both \mathcal{S} and its related coefficients c_b will be learned from the given function evaluations by the developed method. Note that for $s \ll N$, the runtime $(s \log N)^{O(1)}$ will be less than what is required to simply enumerate the elements of the basis \mathcal{B} once. (Received August 16, 2019)

- 1154-65-228 **Qin Sheng*** (qin_sheng@baylor.edu), Department of Mathematics, Casper, Baylor University, Waco, TX 76798-7328. *A review and expectation of the numerical stabilities for nonlinear Kawarada equations.* Preliminary report.

This talk concerns the numerical stability of the nonlinear and highly singular quenching type partial differential equation problems. Utilizing one-dimensional sample problems, we show important physical backgrounds and characteristics of their solutions. Standard Crank-Nicolson schemes are used. While traditional linear stability analysis is accomplished by freezing the underlying source functions of the reaction-diffusion equations, the exploration of the nonlinear stability is proposed and carried out based on proper conservations. Interactive discussions are anticipated throughout this talk over aforementioned approaches. Simulation examples will be provided. (Received August 25, 2019)

- 1154-65-272 **James A Rossmannith*** (rossmani@iastate.edu), 411 Morrill Road, Ames, IA 50011, and **Christine Wiersma** (cwiersma@iastate.edu), 411 Morrill Road, Ames, IA 50011. *Lax-Wendroff Schemes for Quasi-Exponential Moment-Closure Approximations.*

In many applications, the dynamics of gas and plasma can be accurately modeled using kinetic Boltzmann equations. These equations are integro-differential systems posed in a high-dimensional phase space. If the system is sufficiently collisional the kinetic equations may be replaced by a fluid approximation that is posed in physical space (i.e., a lower dimensional space than the full phase space). The precise form of the fluid approximation depends on the choice of the moment-closure. In general, finding a suitable robust moment-closure is still an open scientific problem.

In this work we consider a specific moment-closure based on a nonextensive entropy formulation. In particular, the true distribution is replaced by a Maxwellian distribution multiplied by a quasi-exponential function. We develop a high-order, locally-implicit, discontinuous Galerkin scheme to numerically solve resulting fluid equations. The numerical update is broken into two parts: (1) an update for the background Maxwellian distribution, and (2) an update for the non-Maxwellian corrections. We also develop limiters that guarantee that the inversion problem between moments of the distribution function and the parameters in the quasi-exponential function is well-posed. (Received August 27, 2019)

- 1154-65-300 **Simon Tavener*** (simon.tavener@colostate.edu), Department of Mathematics, Colorado State University, Fort Collins, CO 80523, and **Jehanzeb Chaudhry** and **Don Estep**. A posteriori error analysis for domain decomposition.

Domain decomposition methods are widely used for the numerical solution of partial differential equations on parallel computers. We develop an adjoint-based *a posteriori* error analysis for overlapping multiplicative Schwarz domain decomposition and for overlapping additive Schwarz. In both cases the numerical error in a user-specified functional of the solution (quantity of interest), is decomposed into a component that arises as a result of the finite iteration between the subdomains, and a component that is due to the spatial discretization. The spatial discretization error can be further decomposed into the errors arising on each subdomain. This decomposition of the total error can then be used as part of a two-stage approach to construct a solution strategy that efficiently reduces the error in the quantity of interest. (Received August 29, 2019)

- 1154-65-301 **Jolene Britton***, jhout001@ucr.edu, and **Yulong Xing**, xing.205@osu.edu. High order well-balanced discontinuous Galerkin methods for blood flow simulation through arteries with living man equilibrium.

The simulation of blood flow in arteries can be modeled by a system of conservation laws and have a range of applications in medical contexts. This system of partial differential equations is in the same vein as the shallow water equations. We present well-balanced discontinuous Galerkin methods for the blood flow model which preserve the general living man equilibrium. Schemes for preserving the well-balanced property with zero-velocity, known as the man-at-eternal-rest steady state, have been recently been addressed, however we focus on the development of schemes that consider the more general living man equilibrium with non-zero velocity. Recovery of well-balanced states via a careful choice of projection, appropriate source term approximations, and approximations of the numerical fluxes are the key ideas. Numerical examples will be presented to verify the well-balanced property, high order accuracy, and good resolution for both smooth and discontinuous solutions. (Received August 29, 2019)

- 1154-65-325 **Ronald E. Mickens*** (rmickens@cau.edu), Clark Atlanta University, Atlanta, GA 30314, and **Talitha M. Washington** (talitha.washington@howard.edu), Howard University, Washington, DC 20059. NSFD Schemes: A Methodology for Constructing Structure Preserving Discretizations for Differential Equations.

The differential equations of most interest for numerical analysis have their genesis in mathematical models of important physical phenomena. However, a major difficulty is the occurrence of numerical instabilities (NIs), i.e., solutions of the numerical schemes not corresponding to any solutions of the differential equations. NIs arise when critical features of the differential equations are not incorporated into the discretizations. The nonstandard finite difference (NSFD) methodology directly deals with these issues. NSFD is based on the concept of “dynamical consistency” and leads to the appearance of denominator functions in the discretization of derivative terms, as well as the requirement that non-local representations be used for the discretization of functions of the dependent variables. A tool for the construction of valid NSFD schemes is the method of sub-equations. We will discuss various issues related to dynamical consistency, denominator functions, and non-local representations, and illustrate the NSFD methodology by using it to discretize elementary, but nontrivial differential equations. We will conclude with a summary of the successes of the NSFD methodology and present several unresolved issues available for future investigations. (Received August 31, 2019)

- 1154-65-412 **Pavel B Bochev*** (pbboche@sandia.gov), MS 1320, Albuquerque, NM 87185, and **Paul Kuberry** and **Kara Peterson**. Explicit Partitioned Methods based on Monolithic Formulations of the Coupled Problem.

Traditional explicit partitioned schemes exchange boundary conditions between subdomains and are related to iterative methods for the coupled problem. Thus, they may require multiple subdomain solves, acceleration, or optimized transmission conditions to be accurate and stable.

We present new synchronous partitioned methods derived from a monolithic formulation of the coupled problem in which the transmission condition is enforced by a Lagrange multiplier. We transform the resulting Differential Algebraic Equation (DAE) to a Hessenberg index-1 form in which the algebraic equation defines the Lagrange multiplier as an implicit function of the states.

We eliminate the multiplier and reduce the DAE to a system of ODEs for the states. Explicit time integration both discretizes this system in time and decouples it. Thus, temporal accuracy and stability of our formulation are governed solely by the accuracy and stability of the explicit scheme and are not subject to additional stability considerations.

We establish sufficient conditions for the formulation to be well-posed and prove that mortar finite elements are stable for the Lagrange multiplier. We show that in this case the condition number of the Schur complement is bounded by a constant. (Received September 03, 2019)

1154-65-433 **Andrea Aspri*** (andrea.aspri@ricam.oeaw.ac.at). *A data-driven iteratively regularized Landweber iteration.*

In this talk I will present a data-driven iteratively regularized Landweber iteration for solving linear and nonlinear ill-posed inverse problems. The method takes into account training data, which are used to estimate the interior of a black box, which is used to define the iteration process. I will show convergence and stability results for the scheme in the infinite dimensional Hilbert spaces and then I will discuss some numerical experiments. (Received September 04, 2019)

1154-65-465 **Hector D. Ceniceros*** (ceniceros@ucsb.edu), Department of Mathematics, University of California, Santa Barbara, CA 93106. *Machine Learning from Polymer Self Consistent Field Theoretic Simulations to Accelerate Phase Discovery.*

Self consistent field theory (SCFT) has been a valuable coarse-grained method for the study of many equilibrium polymer solutions. However, SCFT simulations are computationally expensive and the process of exploring the parameter space for new polymer phases is a formidable task. In this talk, we will discuss our first steps to use machine learning tools to accelerate this process. More specifically, our program consists of two problems: (1) the learning of the free energy map as a function of the model parameters and of the polymer segment density and (2) the determination of a polymer segment density that minimizes a fitness function. We will present different approaches for (1), including simple kernel ridge regression and deep learning and discuss, time permitting, local vs global optimization strategies for (2). (Received September 04, 2019)

1154-65-478 **Zhu Wang*** (wangzhu@math.sc.edu). *Efficient algorithm for simulating ensembles of parameterized flow problems.*

Many computational fluid dynamics applications require multiple simulations of a flow under different input conditions. In this talk, we consider such settings for which one needs to perform a sequence of simulations based on the Navier-Stokes equations, each having different initial condition data, boundary condition data, forcing functions, and/or coefficients such as the viscosity. For such settings, we propose ensemble methods to accelerate the solutions. The main idea is to manipulate the time-stepping scheme so that all the problems could share a common coefficient matrix, then, instead of solving a sequence of linear systems with one right-hand-side vector, the method needs to solve one linear system with multiple right-hand-sides. The computational efficiency is then improved by using block iterative algorithms. Rigorous analyses are given proving the conditional stability and establishing error estimates for the proposed algorithms. Numerical experiments are presented to illustrate the analyses. (Received September 08, 2019)

1154-65-494 **Yue Cao and Shuwang Li***, 10 W. 32nd St., Room 220, Chicago, IL 60616. *A kernel-free boundary integral method for boundary value and interface problems in doubly-connected domain.* Preliminary report.

In this talk, we propose a kernel-free boundary integral method (KFBIM) for variable coefficients partial differential equations (PDEs) defined in a doubly-connected domain. We are interested in boundary value problems (BVP) and interface problems. Without requiring the analytical form of the Green's function, the KFBIM computes boundary or volume integrals by equivalently solving an interface problem on Cartesian mesh in the finite difference framework. The coupled integral equations are then solved using a Krylov subspace iterative method. The method is second order accurate in space, and its complexity is linearly proportional to mesh node number. Numerical examples show the method is robust for variable coefficients PDEs with large diffusion coefficients ratio. (Received September 05, 2019)

1154-65-516 **Yekaterina Epshteyn*** (epshteyn@math.utah.edu). *Grain Structure, Grain Growth and Evolution of the Grain Boundary Network.*

Cellular networks are ubiquitous in nature. Most technologically useful materials arise as polycrystalline microstructures, composed of a myriad of small monocrystalline cells or grains, separated by interfaces, or grain boundaries. Grain boundaries play an essential role in determining the properties of materials across a wide range of scales. During grain growth (also termed coarsening), an initially random grain boundary arrangement reaches a steady state that is strongly correlated to the interfacial energy density. In this talk, we will discuss recent progress on modeling, simulation and analysis of the evolution of the grain boundary network in polycrystalline materials. (Received September 05, 2019)

1154-65-533 **Bashar Zogheib*** (bzogheib@auk.edu.kw), Abdalla Ibn Mosad Street, 1111 Salmiya, Hawali, Kuwait. *Surface Grid Generation Using R*.

Grid generation is a critical component in the process of finding a numerical solution to partial differential equations. Many methods have been created for this purpose. In this work an attempt is made to create an accurate surface mesh. This algorithm will be created in R and will be submitted to be included as a package in R. Although there are some packages in the literature that create meshes, not many of them turned into a user-friendly software. Therefore, there is a need to add new methods to the literature in a user-friendly software, R. (Received September 06, 2019)

1154-65-536 **Henry Adams, Lara Kassab*** (lara.kassab@colostate.edu) and **Deanna Needell**. *Matrix Completion for Structured Observations Using Iteratively Reweighted Algorithms*. Preliminary report.

The task of filling-in or predicting missing entries of a matrix from a subset of known entries is known as *matrix completion*. In today's data-driven world, data completion is essential whether it is the main goal or a pre-processing step. In recent work, a modification to the standard nuclear norm minimization for matrix completion have been made to take into account the *structural differences* between observed and unobserved entries. One example of such structural difference is when the unobserved entries have lower magnitudes than the observed entries or are sparse. Standard semidefinite programming tools do not work efficiently for solving large nuclear norm minimization problems. We propose adjusting an Iteratively Reweighted Least Squares (IRLS) algorithm for low-rank matrix completion to take into account structural differences between observed and unobserved entries. (Received September 06, 2019)

1154-65-537 **Pavel B. Bochev** and **Giacomo Capodaglio***, gcapodaglio@fsu.edu, and **Marta D'Elia** and **Max D. Gunzburger**. *Modeling and approximation of interface problems in nonlocal diffusion*.

In this talk, I will present recent results on the nonlocal modeling of interface problems in the context of heterogeneous materials. First, I will lay out the mathematical foundation of the problem, which is based on a minimization of the nonlocal energy of the system. Such an approach provides a useful mean to identify transmission conditions leading to a well-posed interface problem. A theoretical study of local limits shows that under certain sufficient conditions on the kernel function of the nonlocal operator the classical formulation is recovered, guaranteeing physical consistency.

Next, I will discuss the approximation of the nonlocal interface problem using the finite element method, and present several numerical results that support the theoretical predictions obtained analytically. More specifically, I will consider 1D and 2D examples to show that the proposed discretized nonlocal interface models achieve the appropriate order of convergence expected from the finite element method. Moreover, results on numerical convergence to the local limits will also be reported, with a specific focus on the behavior of the nonlocal solution at the interface.

Finally, I will discuss future work and currently open questions. (Received September 06, 2019)

1154-65-555 **Benjamin Peherstorfer***, Courant Institute, New York University, New York City, NY 10012. *Learning reduced dynamical-system models from data*.

This work introduces a method for learning low-dimensional models from data of high-dimensional black-box dynamical systems. The novelty is that the learned models are exactly the reduced models that are traditionally constructed with model reduction techniques that require full knowledge of governing equations and operators of the high-dimensional systems. Thus, the learned models are guaranteed to inherit the well-studied properties of reduced models from traditional model reduction. The key ingredient is a new data sampling scheme to obtain re-projected trajectories of high-dimensional systems that correspond to Markovian dynamics in low-dimensional subspaces. The exact recovery of reduced models from these re-projected trajectories is guaranteed pre-asymptotically under certain conditions for finite amounts of data and for a large class of systems with polynomial nonlinear terms. Numerical results demonstrate that the low-dimensional models learned with the proposed approach match reduced models from traditional model reduction up to numerical errors in practice. The numerical results further indicate that models fitted to re-projected trajectories are predictive even in situations where models fitted to trajectories without re-projection are inaccurate and unstable. (Received September 06, 2019)

1154-65-584 **Xiaofan Li*** (lix@iit.edu), 10 W 32nd St, RE220, Chicago, IL 60616. *Numerical simulations of macroscopic quantities for stochastic differential equations with α -stable processes.*

The mean first exit time, escape probability and transitional probability density are utilized to quantify dynamical behaviors of stochastic differential equations with non-Gaussian, α -stable type Lévy motions. Taking advantage of the Toeplitz matrix structure of the time-space discretization, a fast and accurate numerical algorithm is proposed to simulate the nonlocal Fokker-Planck equations on either a bounded or infinite domain. Under a specified condition, the scheme is shown to satisfy a discrete maximum principle and to be convergent. (Received September 07, 2019)

1154-65-592 **Robert Gower, Denali Molitor*** (dmolitor@math.ucla.edu), **Jacob Moorman** and **Deanna Needell.** *Adaptive sketch-and-project methods for solving linear systems.*

We present new adaptive sampling rules and convergence guarantees for iterative sketch-and-project methods for solving linear systems. To deduce the new sampling rules, we note that the progress of one step of the sketch-and-project method depends directly on a sketched residual. Based on this insight, we derive a 1) max-distance sampling rule, which samples the sketch with the largest sketched residual 2) a proportional sampling rule, which samples proportional to the sketched residual, and finally 3) a capped sampling rule, which is a generalization of recently introduced adaptive sampling rules for the Kaczmarz method. We provide a global linear convergence theorem for each sampling rule and show that the max-distance rule enjoys the fastest convergence guarantee. This finding is verified in numerical experiments and leads us to conclude that the max-distance sampling rule is superior both experimentally and theoretically to the capped sampling rule. We provide numerical insights into implementing the adaptive strategies so that the per iteration cost is of the same order as using a fixed sampling strategy in certain settings. (Received September 07, 2019)

1154-65-623 **Xiu Ye*** (xxye@ualr.edu), 2801 S. University Ave, Little Rock, AR 72223. *Finite element methods with discontinuous approximations.*

In this presentation, different finite element methods with discontinuous approximations will be discussed including IPDG, HDG and specially WG finite element methods as well as the relations between them. In addition, new stabilizer free discontinuous finite element methods will be introduced. (Received September 08, 2019)

1154-65-643 **Thi-Thao-Phuong Hoang*** (tzh0059@auburn.edu), **Wei Leng, Lili Ju, Zhu Wang** and **Konstantin Pieper.** *Conservative Explicit Local Time-Stepping Schemes for the Shallow Water Equations.*

In this talk, we present explicit local time-stepping schemes of second and third order accuracy for the shallow water equations. The system is discretized in space by a C-grid staggering method, namely the TRiSK scheme adopted in MPAS-Ocean, a global ocean model with the capability of resolving multiple resolutions within a single simulation. The time integration is designed based on the strong stability preserving Runge-Kutta methods, but different time step sizes can be used in different regions of the domain through the coupling of coarse-fine time discretizations on the interface, and are only restricted by respective local CFL conditions. The proposed local time-stepping schemes preserve all important properties in the discrete sense, such as exact conservation of the mass and potential vorticity and conservation of the total energy within time-truncation errors. Moreover, they inherit the natural parallelism of the original explicit global time-stepping schemes. Extensive numerical tests are presented to illustrate the performance of the proposed algorithms. (Received September 09, 2019)

1154-65-670 **Jamie Haddock*** (jhaddock@math.ucla.edu), 520 Portola Plaza, Los Angeles, CA 90095, and **Anna Ma.** *Analyzing Hybrid Randomized and Greedy Projection Methods.* Preliminary report.

Stochastic iterative algorithms have gained recent interest for solving large-scale systems of equations, $Ax=y$. One such example is the Randomized Kaczmarz (RK) algorithm, which acts only on single rows of the matrix A at a time. While RK randomly selects a row, Motzkin's algorithm employs a greedy row selection; the Sampling Kaczmarz-Motzkin (SKM) algorithm combines these two strategies. In this talk, we present a convergence analysis for SKM which interpolates between RK and Motzkin's algorithm. (Received September 09, 2019)

1154-65-688 **Malgorzata Peszynska*** (mpesz@math.oregonstate.edu), Oregon State University, Mathematics, Corvallis, OR 97331, and **Azhar Alhammali** (alhammaz@oregonstate.edu), Imam Abdulrahman, Bin Faisal University, Dammam, Saudi Arabia. *Numerical analysis of a biofilm-nutrient model involving a variational inequality.*

We consider a coupled PDE system for biofilm and nutrient dynamics in porous media. Of interest is modeling at the interface scale of microns at which a sharp interface between the biofilm phase and the surrounding fluid

is visible. The biofilm phase is where the biofilm concentration $B(x, t)$ reaches its maximum, beyond which the biomass spreads through the interface. Our model involves a variational inequality which models the constraint $B \leq B^*$ as well as nonlinear diffusivity and Monod coupling terms. The solutions are of low regularity thus we apply the low order finite element discretization, and we prove optimal convergence of our scheme, roughly of order $O(\Delta t + h)$. Numerical experiments in $d = 1, 2, 3$ confirm this rate. We also illustrate the importance of choosing a particular type of nonlinear diffusivity model which gives results qualitatively resembling experimental and imaging results. (Received September 09, 2019)

1154-65-746 **Jia Zhao*** (jia.zhao@usu.edu), **Yuezheng Gong** and **Qi Wang**. *Arbitrarily High-order Unconditionally Energy Stable Schemes for Thermodynamically Consistent Gradient Flow Models*.

We present a systematical approach to developing arbitrarily high order, unconditionally energy stable numerical schemes for thermodynamically consistent gradient flow models that satisfy energy dissipation laws. Utilizing the energy quadratization (EQ) method, We formulate the gradient flow model into an equivalent form with a corresponding quadratic free energy functional. Based on the equivalent form with a quadratic energy, we adopt the Gaussian collocation method to discretize the equivalent form with a quadratic energy, arriving at an arbitrarily high-order scheme for gradient flow models. Schemes derived using both approaches are proved rigorously to be unconditionally energy stable. Numerical results will be shown to illustrate their accuracy and effectiveness. (Received September 10, 2019)

1154-65-758 **Xucheng Meng**, **Thi-Thao-Phuong Hoang** and **Lili Ju*** (ju@math.sc.edu), Department of Mathematics, University of South Carolina, Columbia, SC 29208, and **Zhu Wang**. *Localized exponential time differencing methods for shallow water equations: algorithms and numerical investigations*. Preliminary report.

Exponential time differencing (ETD) has been proven to be very effective for solving stiff evolution problems in the past decades due to rapid development of matrix exponential algorithms and computing capacities. While direct parallelization of the ETD methods is rarely of good efficiency due to the required data communication, the localized exponential time differencing (LETD) approach was recently introduced for extreme-scale phase field simulations of coarsening dynamics, which displayed excellent scalability in modern supercomputers. The main idea is to use domain decomposition techniques to reduce the size of the problem, so that one instead only solves a group of smaller-sized subdomain problems simultaneously using the locally computed products of matrix exponentials and vectors. In this talk, we first propose and discuss overlapping LETD Runge-Kutta schemes for the rotating shallow water equations and their implementation algorithms. Numerical experiments are then presented to compare the performance of the LETD-RK schemes with the classic explicit RK time steppings to demonstrate the advantages of the proposed methods. (Received September 10, 2019)

1154-65-760 **Brian E. Moore*** (brian.moore@ucf.edu). *Structure-Preserving Exponential Integrators with Applications for Damped-Driven NLS*.

Many nonlinear PDEs have invariants or conservative properties (such as energy, momentum, mass, etc.) which can be preserved in numerical simulations by various schemes. In the presence of driving forces or damping terms those properties are altered, so that numerical preservation of the properties is more challenging. For cases in which the forcing and/or damping is linear with time-dependent coefficients, the properties often satisfy a linear differential equation and can be exactly preserved through discretization using exponential integrators. This talk presents a general framework for constructing methods that exactly preserve dynamic changes in a number of properties (energy, momentum, mass, etc.) which are effected by damping and/or driving forces. The resulting exponential methods are generalizations of other commonly used methods, such as Runge-Kutta, discrete gradient, finite difference, and collocation methods. To demonstrate their effectiveness, the methods are applied to several variations of damped-driven Nonlinear Schrödinger equations. In many cases, higher accuracy and efficiency are both observed in structure-preserving algorithms when they are compared to other standard schemes. (Received September 10, 2019)

1154-65-821 **Xianyi Zeng*** (xzeng@utep.edu), Bell Hall 202, 500 W University Ave, El Paso, TX 79968, and **Mashriq Saleh** and **Jianjun Paul Tian**. *A finite volume method for infiltration dynamics in tumor growth models*.

We address numerical challenges in free boundary problems described by spherically symmetric conservation laws that arise in the modeling of tumor growth due to immune cell infiltrations. In particular, we normalize the radial coordinate to transform the free boundary problem to a fixed boundary one, and utilize finite volume methods to discretize the resulting equations. The conventional finite volume methods fail to preserve constant

solutions and the incompressibility condition, and they typically lead to inaccurate solutions, if not wrong at all. These issues are addressed in a new finite volume framework with segregated flux computations that satisfy sufficient conditions for ensuring the so-called totality conservation law and the geometric conservation law. First- and second-order methods are constructed within this framework, and their numerical performances are assessed by various tests, including the prediction of a “rim” near the tumor boundary in a PDGF-driven glioma simulation. (Received September 11, 2019)

1154-65-836 **Xiaoming Wang*** (wangxm@sustech.edu.cn). *A second order in time energy stable variable step method for the Cahn-Hilliard equation.*

We propose a novel second-order in time variable step method for the Cahn-Hilliard equation. The scheme is a proper combination of variable BDF2, convex splitting, and viscous regularization at the discrete level. With the aid of a novel discrete Gronwall type inequality, we are able to show that the error is second-order in time and energy stable under a mild restriction on the ratio of the successive step-sizes. Such a result is new even for the linear case. (Received September 11, 2019)

1154-65-914 **Zhen Chao*** (chaozhen@uwm.edu), 2616 N Frederick Ave, APT 125, Milwaukee, WI 53211, and **Dexuan Xie** and **Ahmed H. Sameh**. *Preconditioners for nonsymmetric indefinite linear systems.*

In this talk, we present algorithms for solving nonsymmetric indefinite linear systems by considering the augmented linear systems resulting from a weighted linear least squares problem. Even though the augmented system is more ill-conditioned than the original linear system, one can construct preconditioned GMRES methods for solving these augmented systems capable of obtaining reasonable approximation of the solution in fewer iterations than the classical ILU preconditioned GMRES method for solving the original linear system. More specifically, we present two different preconditioners for these augmented systems, examine the spectral properties of these preconditioned augmented systems, and report numerical results to illustrate the effectiveness of these preconditioners. (Received September 11, 2019)

1154-65-983 **Andrew P Miller*** (andrew.miller@uconn.edu) and **Dmitriy Leykekhman**. *The Discrete Green's Function: Positivity in Two Dimensions Versus Three Dimensions using Piecewise Linear Finite Elements.* Preliminary report.

In this talk we will briefly introduce the finite element method using piecewise linear functions for the Laplace problem. We will then recall some basic properties of the continuous Green's function ($G^{x_0}(x)$), introduce the Discrete Green's function ($G_h^{x_0}(x)$), and discuss the differences in the basic properties of the Discrete Green's function. Finally, we briefly discuss the implication of a positive Discrete Green's function and proving a Discrete Harnack Inequality as well as compare the arguments used for proving positivity in two dimensions versus three dimensions and the challenges to the three dimensional case. (Received September 12, 2019)

1154-65-984 **JaEun Ku***, 401 Mathematical Sciences, Oklahoma State University, Stillwater, OK 74074. *Flux based finite element methods.*

In this talk, we present a new finite element method based on flux variables. In many applications, the flux variables are often the quantity of interest. To approximate the flux variable accurately and efficiently, one transforms the second-order equations into a system of first-order and approximates both the primary and flux variables simultaneously. While this indeed produces accurate approximations for the flux variables, the resulting algebraic system is large and expensive to solve. We present a new method approximating the flux variables only without approximation of the primary variable. If necessary, the primary variable can be recovered from the flux approximation with the same order of accuracy. We also consider the conservation of mass. This new approach can be considered as a reduced version of the standard mixed finite element methods. (Received September 12, 2019)

1154-65-1051 **Shawn W. Walker*** (walker@math.lsu.edu), **Juan-Pablo Borthagaray** (jpb@umd.edu) and **Ricardo H. Nochetto** (rhn@math.umd.edu). *The Uniaxially Constrained Q-tensor Model for Nematic Liquid Crystals.*

We consider the one-constant Landau-de Gennes model for nematic liquid crystals, with traceless tensor field \mathbf{Q} as the order parameter. We constrain \mathbf{Q} to be uniaxial: $\mathbf{Q} = s(\mathbf{n} \otimes \mathbf{n} - (1/3)\mathbf{I})$ where \mathbf{n} is a director field and s is the scalar degree of orientation. Building on similarities with the one-constant Ericksen energy, we propose a structure-preserving finite element method for the computation of equilibrium configurations. We prove stability and consistency of the method without regularization, and Γ -convergence of the discrete energies towards the continuous one as the mesh size goes to zero. We give an alternating direction gradient flow algorithm for the solution of the discrete problem, and we show that such a scheme decreases the energy monotonically. Finally,

we illustrate the method's capabilities by presenting some numerical simulations in two and three dimensions including non-orientable line fields. (Received September 12, 2019)

1154-65-1076 **Paul A Kuberry*** (pakuber@sandia.gov), Mailstop 1320, P.O. Box 5800, Albuquerque, NM 87125, and **Pavel B Bochev, Nathaniel A Trask** and **Mauro Perego**. *Generalized Moving Least-Squares approximation of variational problems for advection-diffusion problems.*

In most finite element methods, the mesh is used to both represent the domain and to define the finite element basis. As a result, the quality of such methods is tied to the quality of the mesh and may suffer when the latter deteriorates.

We present an approach by extending the Generalized Moving Least-Squares (GMLS) regression technique to approximation of bilinear forms, which separates the discretization of the PDE from the discretization of the domain. We make use of mesh quantities only for the integration of the GMLS polynomial basis. Our approach yields a non-conforming discretization of the weak equations that can be handled by standard discontinuous Galerkin or interior penalty terms.

Analysis of the approach will be presented along with numerical results demonstrating convergence and stability in the context of advection-diffusion. (Received September 13, 2019)

1154-65-1094 **Alessandro Alla, Max Gunzburger*** (mgunzburger@fsu.edu), **Martin Hess, Annalisa Quaini** and **Gianluigi Rozza**. *A localized reduced-order modeling approach for PDEs with bifurcating solutions.*

Reduced-order models (ROMs) are low-dimensional discretizations of PDEs that more efficiently treat settings that require multiple solutions such as optimization and UQ. Although ROMs are successful in many cases, ROMs built for the efficient treatment of bifurcating solutions as input parameter values change have not received much attention. In such cases, the parameter domain can be subdivided into subregions that corresponds to a different branch of solutions. ROM approaches such as proper orthogonal decomposition (POD) results in global low-dimensional bases that do not respect the large differences in solutions corresponding to different subregions. In this work, we develop and test a new ROM specifically aimed at bifurcation problems. In the new method, the k-means algorithm is used to cluster snapshots so that within cluster snapshots are similar to each other and are dissimilar to those in other clusters. This is followed by the construction of local POD bases, one for each cluster. The method can detect the cluster a new parameter point belongs to, after which the local basis for that cluster is used to determine a ROM solution. Numerical examples show the effectiveness of the method both for when bifurcations cause continuous and discontinuous changes in the solution. (Received September 13, 2019)

1154-65-1101 **Andrea Louise Bertozzi*** (bertozzi@math.ucla.edu), 520 Portola Plaza, Department of Mathematics, UCLA, Los Angeles, CA 90095. *Graphical Models in Machine Learning, Networks, and Uncertainty Quantification.*

I will speak on semi-supervised and unsupervised graph models for classification using similarity graphs and for community detection in networks. The equivalence between the graph min-cut problem and total variation minimization on the graph for an assignment function allows one to cast graph-cut variational problems in the language of total variation minimization, thus creating a parallel between low dimensional data science problems in Euclidean space (e.g. image segmentation) and high dimensional clustering. This talk reviews on a class of methods build around diffuse interface models, developed by the Author and collaborators. Semi-supervised learning with a small amount of training data can be carried out in this framework with diverse applications ranging from hyperspectral pixel classification to identifying activity in body worn video. It can also be extended to the context of uncertainty quantification with Gaussian noise models. The problem of community detection in networks also has a graph-cut structure and algorithms are presented for the use of threshold dynamics for modularity optimization. With efficient methods, this allows for the use of network modularity for unsupervised machine learning problems with unknown number of classes. (Received September 13, 2019)

1154-65-1126 **Susanne Brenner, Lina Ma, Minah Oh*** (ohmx@jmu.edu), **Li-yeng Sung** and **Kening Wang**. *P1 Finite Element Methods for Elliptic Optimal Control Problems.*

In this talk, I will talk about P1 finite element methods for two elliptic state-constrained optimal control problems. One is the elliptic state-constrained distributed optimal control problem with Neumann boundary conditions on general polygonal domains. The other is the weighted elliptic optimal control problem arising from the axisymmetric elliptic state-constrained distributed optimal control problem with Dirichlet boundary conditions after performing dimension reduction. (Received September 13, 2019)

1154-65-1189 **Axel G R Turnquist*** (agt6@njit.edu). *Towards Convergent Finite-Difference Schemes for the Monge-Ampère PDE on the Sphere*. Preliminary report.

The optimal transport problem has been found to apply to a wide variety of modern applications, among those include machine learning, geometric optics, seismology, meteorology, and image retrieval and processing. Some recent numerical methods solve the optimal transport problem using its PDE formulation, known in its most simple case as the Monge-Ampère equation, a fully nonlinear second-order elliptic PDE. One particular approach to solving this PDE formulation is by using wide-stencil finite difference schemes. We use these ideas to devise a numerical scheme for solving the PDE formulation of the optimal transport problem on the sphere. This requires taking the geometry into account and also reformulating the discrete comparison principle to derive a convergence result. (Received September 13, 2019)

1154-65-1212 **Jacob D Moorman***, 520 Portola Plaza, Los Angeles, CA 90095, and **Denali Molitor, Robert M Gower** and **Deanna Needell**. *On Comparing Adaptive Sampling Rules for Sketch-and-Project Methods*.

We derive a partial order on the guaranteed convergence rates of adaptive sampling rules for sketch-and-project methods for solving linear systems. The partial order is inherited from a partial order on the sampling rules themselves. As a direct result, we conclude that the max-distance rule has the fastest guaranteed convergence rate among all adaptive sampling rules. Applying this result to Kaczmarz and coordinate descent methods for solving linear systems shows that Motzkin's method and the Gauss-Southwell rule have the fastest guaranteed convergence rates among their respective families of adaptive sampling rules. (Received September 13, 2019)

1154-65-1274 **Rayan Saab*** (rsaab@ucsd.edu) and **Brian P Preskitt**. *Phase Retrieval from Locally Supported Measurements: Improved Algorithms and Robustness*.

We study an approach to solving the phase retrieval problem as it arises in ptychography. In ptychography, small overlapping sections of an unknown sample (or signal, say $x_0 \in \mathbb{C}^d$) are illuminated one at a time, often with a physical mask between the sample and light source. The corresponding measurements are the noisy magnitudes of certain corresponding Fourier transform coefficients. The goal is to recover the original signal from such measurements.

The algorithmic framework we study herein relies on first inverting a linear system of equations to recover a fraction of the entries in $x_0 x_0^*$ and then using non-linear techniques to recover the magnitudes and phases of the entries of x_0 . Our contributions are three-fold. First, we expand the theory studying which measurement schemes (i.e., masks, shifts of the sample) yield invertible linear systems, including an analysis of the conditioning of the resulting systems. Second, we analyze a class of improved magnitude recovery algorithms and, third, we propose and analyze algorithms for phase recovery in the ptychographic setting where large shifts — up to 50% the size of the mask — are permitted. (Received September 14, 2019)

1154-65-1303 **Ethan Dudley*** (etdudley@ncsu.edu), **Arvind Saibaba** and **Alen Alexanderian**. *Monte Carlo Estimators for the Schatten-p Norm*.

Computing the Schatten-p norm of a positive semidefinite matrix is important to scientific computing. Our motivation stems from optimal experimental design, where the Schatten-p norm defines a design criterion, known as P-optimal criterion. When the matrix is large, computing the Schatten-p norm is computationally expensive as it requires computing all the eigenvalues of the matrix. We propose a matrix-free method to estimate the Schatten-p norm using a Monte Carlo estimator and derive a bound on the number of samples to accurately estimate the Schatten-p norm with a given probability. To efficiently compute the Schatten-p norm for large values of p, we use a low order Chebyshev polynomial approximation and extend our error analysis to this case as well. Finally, since the Schatten-p norm converges to the spectral norm as $p \rightarrow \infty$, we discuss the accuracy of the Monte Carlo estimators for the spectral norm. We demonstrate the performance of our proposed estimators on several test matrices and through an application to optimal experimental design of inverse problems. (Received September 15, 2019)

1154-65-1321 **Isao Yamada*** (isao@sp.ce.titech.ac.jp), 2-12-1-S3-60, Ookayama Meguro-ku, Tokyo, 152-8552, Japan, and **Masao Yamagishi** (myamagi@sp.ce.titech.ac.jp), 2-12-1-S3-60, Ookayama Meguro-ku, Tokyo, 152-8552, Japan. *Hierarchical convex optimization by the hybrid steepest descent method with proximal splitting operators*.

In this talk, we demonstrate how the modern proximal splitting operators can be plugged into the hybrid steepest descent method (HSDM) for their applications to the hierarchical convex optimization problems which require further strategic selection of a most desirable vector from the set of all solutions of the convex optimization. The proposed technique can approximate iteratively a viscosity solution of the standard convex optimization problem,

where the 1st stage cost function is given as a superposition of multiple nonsmooth convex functions, involving linear operators, while its viscosity solution is a minimizer of the 2nd stage cost function which is differentiable convex function with Lipschitzian gradient. We also present an application of the proposed technique to a certain hierarchical enhancement of the support vector machine. (Received September 14, 2019)

1154-65-1382 **Joseph Munar*** (joey.munar@rice.edu), **Isaiah Meyers** (isaiah.meyers@utexas.edu), **Tom Overman** (tom.overman@mavs.uta.edu) and **Eric Neville** (eric.neville@ucdconnect.ie). *A Parallel-in-Time Multigrid Approach to Constrained Optimization.*

Optimization problems in areas such as weather prediction, traffic flow, or resource allocation are often time dependent. Unfortunately, traditional sequential-in-time algorithms have become bottlenecks in solution model generation. As such, parallelization is a crucial requirement to achieve high-performance computing. The OMGrit algorithm is a parallel-in-time multigrid method designed to solve time-dependent constrained optimization problems. Further, OMGrit is able to optimize systems simultaneously coupled forwards- and backwards-in-time such as molecular systems and economic models. For quadratic problems, the algorithm solves the linear Karush-Kuhn-Tucker system of the optimization problem by decomposing the domain and performing a multigrid relaxation scheme to approximate the solution of the reduced system. This project introduces an extension of OMGrit to general nonlinear optimization problems and investigates the performance on both linear and nonlinear problems. Results show a promising level of speedup and robustness compared to more popular optimization solvers without any sacrifice in accuracy. (Received September 15, 2019)

1154-65-1432 **Leo Rebholz*** (rebholz@clemson.edu). *New results for the EMAC formulation of the Navier-Stokes equations.* Preliminary report.

After a review and derivation of the EMAC (Energy-Momentum-Angular momentum Conserving) formulation for the Navier-Stokes equations, we discuss new results concerning numerical implementation, analysis, testing, and application to reduced order modeling. (Received September 15, 2019)

1154-65-1441 **Kyle Allaire*** (kyle.allaire@uconn.edu) and **Dmitriy Leykekhman** (dmitriy.leykekhman@uconn.edu). *Discrete Maximal Parabolic Regularity for Time Discontinuous Galerkin Finite Element Schemes.* Preliminary report.

For parabolic problems $u_t + Au = f$, where A is a second order elliptic operator, maximal parabolic regularity is an important established tool for studying nonlinear operators and general problems where sharp regularity results are required. Recently, there has been growing interest in establishing similar results for time discrete approximations. All known results are for self-adjoint operators, A . In this talk, we'll discuss such results for discontinuous Galerkin time schemes and describe how these results can be extended to non-self-adjoint operators, such as advection-diffusion-reaction problems. (Received September 15, 2019)

1154-65-1460 **Timo Heister*** (heister@clemson.edu), Clemson, SC 29634, and **Luca Heltai** and **Natasha Sharma**. *A comparison of discretizations for the Biharmonic equation.* Preliminary report.

We consider different finite element discretizations for the fourth order Biharmonic equation and discuss a-priori analysis, implementation, and practical limitations. This includes the C0 interior penalty scheme, C1 finite elements, and mixed formulations. (Received September 15, 2019)

1154-65-1465 **Ahmet Ozkan Ozer*** (ozkan.ozerk@wku.edu), 1906 College Heights Hill Blvd, Department of, Western Kentucky University, Bowling Green, KY 42101. *Convergent semi-discrete finite difference approximations for the boundary control of partial differential equations.*

Space-discretized Finite Difference approximations for the well-known one-dimensional partial differential equations (PDEs), i.e. wave equation, beam equation, heat equation, do not preserve the so-called observability or controllability features of the PDEs as the mesh parameter tends to zero. This is mainly due to the loss of the uniform gap among the eigenvalues of the approximated finite dimensional model. To obtain a uniform gap, and therefore, an exact observability result, we consider an indirect filtering technique which involves adding viscosity terms to the PDEs. After filtering, as the mesh parameter goes to zero, the approximated solution space covers the whole infinite-dimensional solution space, and a uniform gap is achieved. Both the discrete multipliers and the non-harmonic Fourier series are utilized for proving the main results.

To show the strength of this technique, the PDEs for the Rayleigh (Kirchhoff) beam is considered. (Received September 15, 2019)

1154-65-1537 **Rene Carmona** and **Mathieu Lauriere*** (lauriere@princeton.edu), 98 Charlon Street, Princeton, NJ 08540. *Machine Learning Methods for Mean Field Control and Mean Field Games.*

In this talk, we propose several numerical methods for mean field control problems and mean field games, both in the ergodic setting and the finite time horizon setting. These methods are based on machine learning tools such as function approximation via neural networks and optimization relying on stochastic gradient descent. We investigate the numerical analysis of these methods and prove bounds on the approximation error. We then consider numerical test cases, including examples which are difficult to tackle with deterministic methods such as numerical schemes based on finite differences. If time permits, we will also discuss model-free methods for mean-field problems in a reinforcement learning framework. This is based on joint work with Rene Carmona (Princeton University). (Received September 16, 2019)

1154-65-1584 **Alexander M. Powell***, Vanderbilt University, Department of Mathematics, Nashville, TN 37240, and **Jonathan Ashbrock**, Vanderbilt University, Department of Mathematics, Nashville, TN 37240. *Stochastic Markov gradient descent for training low-bit neural networks.*

We discuss stochastic Markov gradient descent (SMGD) as an algorithm for training neural networks with low-bit weights. SMGD utilizes only low-bit weight vectors at every stage of the training process. We prove theoretical error bounds for SMGD and also show that the approach performs well numerically. (Received September 16, 2019)

1154-65-1683 **Kiran K Mainali*** (kirankumar.mainali@mavs.uta.edu), 3732 Aspen Brook Lane, Fort Worth, TX 76244. *Optimizing ℓ_1 Loss Regularizer for Sylvester Type LASSO Problem and Its Application to EEG Inverse Problem.*

We revisit the current state-of-the-art ℓ_1 solvers to solve the EEG inverse problem under the sparsity assumption in solutions. We introduce the Sylvester type LASSO model and how it can be deployed into the EEG inverse problem. Conversion of the problem from Sylvester form to regular multi-task LASSO form will increase the size of the problem drastically for which the benchmark ℓ_1 solvers cannot be employed because of huge memory requirement and computational complexity. In this talk, we present our novel ideas to handle the large size data matrices by extracting their structures. The proposed algorithm to solve the weighted ℓ_1 problem with reweighted techniques to improve the solution will be discussed. (Received September 16, 2019)

1154-65-1705 **Shuang Liu*** (shuang@email.sc.edu), 1523 Greene Street, Department of Mathematics, University of South Carolina, Columbia, SC 29208, and **Xinfeng Liu** (xfliu@math.sc.edu), 1523 Greene Street, Department of Mathematics, University of South Carolina, Columbia, SC 29208. *A level-set method combining explicit exponential time differencing scheme (eETD) with adaptive Krylov subspace for a class of stiff reaction-diffusion system with free boundary.* Preliminary report.

The spreading of new or invasive species is a central topic in ecology. The systems of reaction-diffusion equations coupled with free boundary defined by Stefan condition have been widely used to better understand the nature of spreading behaviors of new species. From mathematical modeling point of view, it is a challenge to perform numerical simulations of the free boundary problems, due to the moving boundaries, the topological changes and the stiffness of the system.

In our work, we have incorporated explicit exponential time differencing method (eETD) with adaptive Krylov subspace and level set method to solve reaction-diffusion equations with free boundary in 2D. We introduce three different temporal schemes: Runge-Kutta, Crank-Nicolson and explicit ETD scheme with adaptive Krylov subspace for handling such stiff systems. Numerical examples are examined to illustrate the efficiency, accuracy and consistency for different approaches, and it can be shown that adaptive Krylov eETD is superior to other approaches in terms of stability and efficiency by direct comparison. (Received September 16, 2019)

1154-65-1720 **Aaron F Rapp*** (af rapp@uncg.edu), **Tom Lewis** and **Yi Zhang**. *Convergence analysis of symmetric dual-wind discontinuous Galerkin methods for the obstacle problem.*

A discontinuous Galerkin (DG) finite-element interior calculus is used as a common framework to describe various DG approximation methods for second-order elliptic problems. In this presentation, we will discuss the dual-wind discontinuous Galerkin method and its application to the obstacle problem with Dirichlet boundary conditions, $-\Delta u \geq f$ on Ω with $u = g$ on $\partial\Omega$ and $u \geq \psi$ on Ω . Our analytical results will be presented. Numerical examples that verify these results will also be presented. (Received September 16, 2019)

1154-65-1746 **Taufiqar R Khan*** (khan@clemsun.edu), O-201 Martin Hall, Box 340975, School of Mathematical and Statistical Science, Clemson University, Clemson, SC 29634-0975, and **Shyla Rae Kupis** and **Vincent Barrra**. *Machine learning approach for image reconstruction in electrical impedance tomography inverse problem*. Preliminary report.

In this presentation, we will introduce the inverse problem in electrical impedance tomography and describe the ill-posedness associated with the image reconstruction. We will then describe how machine learning approach can be used to solve the inverse problem. We will present the appropriate algorithm for machine learning approach and demonstrate the efficacy of the proposed technique using simulations. (Received September 16, 2019)

1154-65-1749 **Shyla Rae Kupis*** (skupis@g.clemson.edu), M-306 Martin Hall, School of Mathematical & Statistical Sciences, Clemson, SC 29634, and **Taufiqar R Khan**. *Development of a Wavelet-Based Parameter Representation to Improve Solutions to the Electrical Impedance Tomography Inverse Problem*. Preliminary report.

Electrical impedance tomography (EIT) is a method for non-invasively imaging the internal spatial distribution of an object's electrical resistivity with applications to a plethora of fields, such as medical imaging and geophysics. Since image reconstruction requires solving for the electrical conductivity distribution from an unstable or ill-posed EIT inverse problem, a wavelet-based parameter mesh is implemented to improve parameter resolvability. By reconstructing the electrical conductivity using a select few wavelet coefficients, we are able to enforce sparsity in the parameter space and, therefore, mitigate the ill-posedness of the EIT inverse problem. Lastly, the reconstructed electrical conductivity distributions with and without the use of wavelet coefficients from various synthetic examples at different noise levels are compared in terms of computational effort and accuracy. (Received September 16, 2019)

1154-65-1798 **Carol S. Woodward*** (woodward6@llnl.gov), **David J. Gardner**, **John Loffeld** and **Daniel R. Reynolds**. *Multirate Time Integration Methods and Their Deployment in the SUNDIALS Library*.

The advent of exascale systems will allow for simulating more complex models than ever before. As a result, scientific simulations, such as in climate, combustion, power grid, and hydrological sciences, will include more physics, resulting in a growing number of changing time scales. Starting with Gear (1984) and continuing with Savcenko et al. (2007), Guenther et al. (2016), and others, multirate methods were developed to provide robustness and speed up on multirate applications. These methods lower computational cost as they use small time steps only for fast evolving components and larger steps elsewhere. While multirate methods have been in use in some applications, little work has been done to develop methods that are efficient, have high stability, and have high order of accuracy. In addition, no multirate methods appeared in any general-purpose time integration software package until the last year. The SUNDIALS library, a combined development library by Lawrence Livermore National Laboratory and Southern Methodist University, has multiple types of multirate time integrators deployed. This presentation will overview multirate methods, discuss known strengths and weaknesses, discuss their addition to SUNDIALS, and show results to date of applying them in applications. (Received September 16, 2019)

1154-65-1851 **Konstantin Pieper*** (pieperk@ornl.gov), **K. Chad Sockwell** (kcsockw@sandia.gov) and **Max Gunzburger** (mgunzburger@fsu.edu). *Exponential time differencing for mimetic multilayer ocean models*.

Towards the goal of designing accurate time-stepping methods for global ocean models, we develop a time discretization framework based on exponential integrators for a stacked rotating shallow-water ocean model. The methods are based on a splitting of the forcing term into a linear rotating multi-layer wave-operator and a non-linear residual, capturing the advective forces. Solution strategies for the linear part are based on skew-adjoint Krylov methods. The resulting exponential integrators can take large time steps up to the advective time scale, independent of the speed of internal and external gravity waves. Additionally, the vertically coherent structure of the fastest waves can be used to compress the wave operator into a few vertical modes. In a special case, employing a reduction only to the barotropic component, we obtain a method with similar features to the well-known split-explicit method. Numerical experiments in the context of the SOMA testcase show that the methods are stable over decade-long simulation horizons and accurately reproduce solution statistics. (Received September 16, 2019)

1154-65-1911 **Olena Burkovska*** (burkovskao@ornl.gov) and **Max Gunzburger** (mgunzburger@fsu.edu). *A nonlocal diffuse interface model with a nonsmooth potential*.

We propose a nonlocal variant of the Cahn–Hilliard diffuse interface model with a nonsmooth potential of double-well obstacle type. We discuss different variants of the nonlocal contributions in the model such as nonlocal

operators and nonlocal boundary conditions. In contrast to the local setting, the proposed model allows for sharp interfaces in the solution for a certain critical (non-zero) value of the interface parameter. Here, the choice of the obstacle potential plays an important role in our analysis, since it guarantees the strict separation of the substance into pure phases for nontrivial interactions. Mathematically, this introduces additional inequality constraints that, in a weak form, lead to a coupled system of variational inequalities, which at each time instance can be restated as a constrained optimization problem. We analyze a discretization of the problem in space and time based on finite elements and implicit-explicit time stepping methods that can be realized efficiently. We provide numerical experiments to support our theoretical findings in one and two spatial dimensions. (Received September 16, 2019)

1154-65-1965 **Joshua Lee Padgett*** (joshua.padgett@ttu.edu), Broadway and Boston, Lubbock, TX 79416. *A nonlinear splitting algorithm for preserving asymptotic features of stochastic singular differential equations.*

In this talk we present a nonlinear splitting algorithm for approximating stochastic singular differential equations. In particular, we focus on problems whose singularities induce finite-time blow-up of either the solution, or its derivative, with respect to the expectation of the given norm. The proposed splitting algorithm allows for the careful handling of the singular and stochastic parts, separately. We also develop an adaptive time-stepping algorithm, based on the self-similarity of the true solution of the underlying system, which guarantees that the numerical approximation captures the asymptotic features of the problem—such as blow-up rates and blow-up time. Moreover, we provide convergence and stability results for the general abstract setting (which includes finite difference, finite element, and spectral discretizations of the spatial differential operators), demonstrating the robustness of the proposed algorithm. If time permits, we will briefly mention how the proposed method can be generalized to derive methods of arbitrarily high order. Numerical experiments will be provided to verify the theoretical results. (Received September 16, 2019)

1154-65-2051 **Stefan Wild***, wild@anl.gov. *Manifold Sampling for Robust Calibration.*

We adapt a manifold sampling algorithm for the composite nonsmooth, nonconvex optimization formulations of model calibration that arise when imposing robustness to outliers present in training data. We demonstrate the approach on objectives based on trimmed loss and highlight the challenges of solving these optimization problems. Initial results demonstrate that the method has favorable scaling properties. Savings in time on large-scale problems arise at the expense of not certifying global optimality in empirical studies, but our method also extends to cases where the loss is computed by a black-box oracle. (Received September 17, 2019)

1154-65-2054 **Peter Richtarik***, peter.richtarik@kaust.edu.sa. *A Stochastic Derivative Free Optimization Method with Momentum.*

We consider the problem of unconstrained minimization of a smooth objective function in \mathbb{R}^d in setting where only function evaluations are possible. We propose and analyze stochastic zeroth-order method with heavy ball momentum. In particular, we propose, SMTP, a momentum version of the stochastic three-point method (STP) (Bergou.2018). We show new complexity results for non-convex, convex and strongly convex functions. We test our method on a collection of learning to continuous control tasks on several MuJoCo (Todorov.2012) environments with varying difficulty and compare against STP, other state-of-the-art derivative-free optimization algorithms and against policy gradient methods. SMTP significantly outperforms STP and all other methods that we considered in our numerical experiments. Our second contribution is SMTP with importance sampling which we call SMTP-IS. We provide convergence analysis of this method for non-convex, convex and strongly convex objectives. (Received September 17, 2019)

1154-65-2057 **Stephen Becker***, Stephen.Becker@Colorado.EDU. *Stochastic Subspace Descent.*

We describe and analyze a family of algorithms that generalize block-coordinate descent, where we assume one can take directional derivatives (for low-precision optimization, this can be approximated with finite differences, hence this is similar to a 0th order method). The method generalizes randomized block coordinate descent. We prove almost-sure convergence of the algorithm at a linear rate (under strong convexity) and convergence (with convexity). Furthermore, we analyze a variant similar to SVRG but that does not require the finite-sum structure in the objective, and for isotropic random sampling, we use Johnson-Lindenstrauss style arguments to provide a non-asymptotic, probabilistic convergence results. Numerical examples are provided for selecting landmark points in Gaussian process regression, and in PDE-constrained optimization (shape optimization). This is joint work with Luis Tenorio, David Kozak, and Alireza Doostan. (Received September 17, 2019)

- 1154-65-2059 **Albert Berahas***, albertberahas@lehigh.edu. *A Theoretical and Empirical Comparison of Gradient Approximations in Derivative-Free Optimization: Algorithms, Convergence Analysis and Noise.*

This is the first of two talks on gradient approximations in derivative-free optimization (DFO). In this part, we present and analyze a general line search DFO algorithm. To this end, we derive a simple condition on the accuracy of gradient approximations which guarantees fast and reliable convergence rates for the method. We analyze the convergence properties even when this condition is only satisfied with some sufficiently large probability at each iteration. We also present results for the case where the function evaluations are contaminated with some level of noise. Joint work with Liyuan Cao, Krzysztof Choromanski and Katya Scheinberg. (Received September 17, 2019)

- 1154-65-2061 **Katya Scheinberg***, katyascheinberg@gmail.com. *A Theoretical and Empirical Comparison of Gradient Approximations in Derivative-Free Optimization: Gradient Approximations, Noise and Numerical Experiments.*

This is the second of two talks on gradient approximations in derivative-free optimization (DFO). In this part, we analyze several methods for approximating the gradient of a function using only function values. The methods differ in the number of functions sampled, the choice of the sample points and the way in which the gradient approximations are defined. For each method, we derive bounds on the number of samples and the sampling radius which guarantee the favorable convergence properties for a line search or fixed step size descent method presented in Part I. We verify our theoretical findings with numerical experiments on synthetic problems as well as on reinforcement learning problems. Joint work with Albert S. Berahas, Liyuan Cao and Krzysztof Choromanski. (Received September 17, 2019)

- 1154-65-2077 **Andrea Carracedo Rodriguez*** (crandrea@vt.edu) and **Serkan Gugercin**. *The AAA algorithm for parametric dynamical systems.*

The AAA algorithm is a popular method that builds a rational approximation of single variable functions from data. In many applications, the dynamics of the system may depend on various parameters. Hence we have extended the AAA algorithm to approximate multivariate functions. This extension is also data-driven, i.e., it only requires function evaluations. We present several numerical examples to illustrate the effectiveness of the algorithm. We include multi-input/multi-output examples to show how the algorithm may be extended to matrix-valued functions. (Received September 17, 2019)

- 1154-65-2104 **Alexander Zaitzeff** and **Selim Esedoglu*** (esedoglu@umich.edu), Department of Mathematics, 530 Church St., Ann Arbor, MI 48105, and **Krishna Garikipati**. *Variational extrapolation.*

Many problems in inverse problems and imaging have been addressed via variational models, requiring the minimization of a challenging cost function. Efficient optimization algorithms have been developed for many.

As is well known, once an efficient algorithm is found for the stationary problem of minimizing a cost function, the same algorithm can be used to generate an approximation to the dynamic problem of gradient flow, by solving a sequence of optimizations. This is a powerful approach to extending the know-how developed in imaging to applications where the dynamics, not just the finding of a stationary point, is important. The resulting scheme for the evolutionary problem is unconditionally stable, but is typically only first order accurate in time.

I will describe how a black-box optimization algorithm for a stationary problem can be used to generate high order accurate in time approximations to gradient flow, simply by calling the algorithm a few times per time step, while maintaining unconditional energy stability. The strategy is universal, making very few assumptions on the cost function. It can be seen as a variational analogue of Richardson extrapolation, the standard version of which lacks the stability guarantees of the new method. (Received September 17, 2019)

- 1154-65-2114 **Alexander Zaitzeff** and **Selim Esedoglu***, Department of Mathematics, 530 Church St., Ann Arbor, MI 48105, and **Krishna Garikipati**. *High order accurate versions of threshold dynamics.*

Threshold dynamics is an elegant algorithm for simulating the motion by mean curvature of interfaces in any dimension. It reduces the computation of this evolution to alternating the two simple steps of convolution and thresholding, and can therefore be implemented very efficiently.

In its standard form, threshold dynamics is at most first order accurate in time. Various extensions of it have been proposed that improve this order of accuracy. I will present the first such extension that comes with a stability guarantee. (Received September 17, 2019)

1154-65-2140 **Junping Wang*** (jwang@nsf.gov), 2415 Eisenhower Avenue, Alexandria, VA 22314.
Primal-Dual Weak Galerkin Finite Element Methods.

Weak Galerkin (WG) finite element method is a numerical technique for PDEs where the differential operators in the weak form are reconstructed by using a framework that mimics the theory of distributions for piecewise polynomials. The usual regularity of the approximating functions is compensated by carefully-designed stabilizers. In this talk, the speaker will discuss a primal-dual weak Galerkin (PD-WG) approach for some model PDE problems for which the usual numerical methods are difficult to apply. The idea of PD-WG will be first illustrated using linear transport equations, and will then be applied to general div-curl systems. (Received September 17, 2019)

1154-65-2149 **Chartese Darnel Jones*** (cdj192@msstate.edu), P.O. Box MA, Mississippi State, MS 39762. *Non-Local Blockwise Implementation Method with Selective Features for Speckle Image Denoising.*

Speckle noise occurs in a wide range of medical images due to sampling and digital degradation. Understanding how noise can be present in images have led to multiple denoising techniques. Most of these denoising techniques assume equal noise distribution. When the noise present in the image is not uniform, the resulting denoised image becomes of less than the highest standard or quality. The Non-local means (NLM) filter restores every pixel in the original image by computing a weighted average of non-local neighborhoods using a robust similarity measure. The NLM algorithm takes advantage of the high degree of redundancy of any natural image. Also, the NLM algorithm is very accurate since all pixels contribute for denoising at any given pixel. However, due to nonlocal averaging, one major drawback is computational cost. For this research, we will discuss new denoising techniques based on non-local means for images contaminated by speckle noise. We introduce blockwise and selective denoising methods based on NLM technique to enhance computational efficiency. Our numerical results show that the new methods are more efficient than the conventional NLM method. (Received September 17, 2019)

1154-65-2237 **Jeffrey Alan Furst Hittinger*** (hittinger1@llnl.gov), Center for Applied Scientific Computing, Box 808, L-561, Livermore, CA 94551. *Are PDEs still relevant? Scientific Computing in the Age of Big Data and Extreme Computing.*

Traditionally, scientific computing has been dominated by simulation based primarily on discrete approximations of Integral Equations, ODEs, and/or PDEs. As a community, we have seen decades of tremendous developments in both computer capabilities and numerical methods that have allowed us to model complex physical problems. Even with all these advances, there are still limitations in our modeling capabilities based on PDEs, particularly when issues of weak solutions, multiscale behavior, uncertainty, and optimization are involved, and we continue to see new advancements PDE-based simulation. However, the computing ecosystem is changing. Computer technologies are becoming more complex through hardware specialization, and the proliferation of inexpensive sensors and computing technologies are driving a new reliance on data-driven computing. Claims are being made that data-driven models will replace traditional PDE models and that traditional simulation will be less important moving forward. Is this true? What would be the trade-offs? Is this even a binary choice? Can we harness the unique strengths of both PDE and data-driven models to solve problems that today are intractable? (Received September 17, 2019)

1154-65-2250 **Xuemei Chen*** (xchen@nmsu.edu) and **Jing Qin**. *Regularized Random Kaczmarz Algorithm*. Preliminary report.

In this talk, we will provide theoretical guarantees for the convergence of a Kaczmarz-type algorithm that minimizes a strongly convex function with linear constraints. Such model can be utilized for recovery of signals (vectors), images (matrices), and tensors, and have applications including image deblurring and low-rank tensor recovery. This is joint work with Jing Qin. (Received September 17, 2019)

1154-65-2252 **Victor Ginting*** (vginting@uwyo.edu). *An A Posteriori Error Estimation for Numerical Solution of Richards Equation*. Preliminary report.

We first discuss several time integration techniques of system of ordinary differential equations characterized by strong nonlinear coupling of the unknown variables. This system is a result of spatial discretization (using such as finite element or finite difference) of the Richards Equation, which is a governing mathematical principle for modeling water infiltration through a subsurface. The nature of Richards Equation is further complicated by the fact that the rate of change of the quantity of interest represented by a time derivative is also nonlinear. We formulate a general framework of the numerical time integration as a discontinuous Galerkin method in temporal variable. The actual implementation of a particular scheme is realized by imposing certain finite element space in

time variable to the variational equation and appropriate "variational crime" in the form of numerical quadrature for calculating the integration in the formulation. Once this is in place, we derive an adjoint-based error estimator for the approximate solution from the scheme. The adjoint problem is obtained from appropriate linearization of the nonlinear system. Several numerical examples are presented to illustrate performance of the scheme and the error estimator. (Received September 17, 2019)

1154-65-2254 **Victor Churchill*** (victor.a.churchill.gr@dartmouth.edu). *Image reconstruction via edge-masked regularization.*

We present a reconstruction method for edge-sparse images that uses approximate edge locations to enforce a sparsity penalty more precisely than standard ℓ_1 regularization methods. Specifically, an edge detection informs a reconstruction where ℓ_2 regularization is applied away from edges. Since the difficulty of the problem is effectively shifted from reconstruction to edge detection, we also discuss several methods for detecting edges from data that are acquired as non-uniform Fourier samples as in synthetic aperture radar. (Received September 17, 2019)

1154-65-2312 **Halyun Jeong*** (hajeong@math.ubc.ca), , Canada, and **Kateryna Melnykova** and **Ozgur Yilmaz**. *Iterative re-weighted Kaczmarz algorithm for compressed sensing.* Preliminary report.

The main goal of compressed sensing is to recover a sparse signal from a few linear measurements. In this talk, a new iterative algorithm for such sparse recovery problems is proposed. The new method uses weighted Kaczmarz updates and thus it is applicable for big data. We provide extensive numerical comparison of the new algorithm with the iterative re-weighted least squares and the iterative hard thresholding. (Received September 17, 2019)

1154-65-2343 **Katarzyna Swirydowicz*** (katarzyna.swirydowicz@nrel.gov), National Renewable Energy Laboratory, 15013 Denver W Pkwy, Golden, CO 80401, and **Stephen Thomas**, **Daniel Bielich** and **Julien Langou**. *Reorthogonalization without iteration: stable pipelined Krylov iterations.* Preliminary report.

Recently alternative versions of Gram-Schmidt orthogonalization algorithms have been formulated. $O(\varepsilon)$ level orthogonality between the resulting basis vectors is achieved in CGS-2 (classical Gram-Schmidt algorithm with re-orthogonalization) with only one global reduction on a parallel computer for each basis vector formed. We have applied the one-reduce theorem to derive new Arnoldi-QR based eigenvalue algorithms and GMRES linear system solvers. In this talk we demonstrate how the recursive application of a projector can be applied to the pipelined Lanczos/conjugate gradient algorithm in order to maintain orthogonal Krylov vectors. (Received September 17, 2019)

1154-65-2350 **Tiffany N Jones*** (tnjones@math.arizona.edu), Department of Mathematics, The University of Arizona, 617 N. Santa Rita Ave., Tucson, AZ 85721-0089. *Solving highly oscillatory wave equations with an asymptotically stable dual-scale compact method.*

A dual-scale numerical algorithm for solving highly oscillatory Helmholtz equations in polar coordinates is presented and analyzed. Decomposing the axisymmetric radial domain and associated governing equations provides the potential for optical computations via interconnected micro and macro domains. These coupled equations are subsequently discretized utilizing a compact strategy for increased efficiency and high radial accuracy.

With a focus on highly oscillatory solution features, a rigorous analysis of the numerical method showed it to be asymptotically stable at high wavenumbers, supporting the algorithmic effectiveness and reliability. Furthermore, spectral norm analysis of the amplification matrices reveals necessary constraints for conventional stability. Numerical self-focusing beam propagation simulations, including those conducted with a range domain scaling factors, reinforce these findings. (Received September 17, 2019)

1154-65-2361 **Sanwar Ahmad*** (suahmad@colostate.edu), Department of Mathematics, Fort Collins, CO 80523, and **Taufiqar Khan** (khan@clemsun.edu), Clemson, SC 29634. *A hybrid approach combining analytical and iterative regularization methods for Electrical Impedance Tomography.* Preliminary report.

Electrical impedance tomography (EIT) is an imaging method that has been gaining more popularity due to its ease of use and non-invasiveness. EIT can potentially be used as an alternative to traditional imaging techniques, such as computed tomography (CT) scans, to reduce the damaging effects of radiation on the tissue. In EIT, the inner distribution of resistivity, which corresponds to different resistivity properties of different tissues, is estimated from the voltage potentials measured on the boundary of the object being imaged. In this paper, we discuss a direct method for solving the EIT inverse problem using mollifier regularization. A comprehensive numerical and computational comparison for EIT is presented. Based on the comparative results, a novel hybrid

method combining the mollifier and iterative method, iteratively regularized Gauss-Newton method, is proposed. (Received September 17, 2019)

1154-65-2372 **Thowhida Akther*** (thowhida@colostate.edu), Department of Mathematics, Fort Collins, CO 80523, **Shyla Kupis** (skupis@clemonson.edu), Clemson, SC 29634, and **Taufiqar Khan** (khan@clemonson.edu), Clemson, SC 29634. *Reconstructing the Diffusion Coefficient for Diffuse Optical Tomography Problems Using A Variationally Constrained Nonlinear Optimization Framework*. Preliminary report.

Diffuse Optical Tomography (DOT) is an emerging modality for soft tissue imaging since it is a benign form of non-ionizing radiation with various medical applications. However, the inverse problem is unstable during the reconstruction of the diffusion coefficient due to its ill-posedness. To address this issue, typical approaches involve using nonlinear techniques, e.g., the Gauss-Newton method, in which the forward model constraints are implicitly incorporated into the inverse problem. In this paper, we solved the one-dimensional DOT inverse problem by formulating it as a variationally constrained non-linear optimization problem with Newton's iteration and then extended this method to solve the two-dimensional inverse problem. We lastly present the results from our optimization framework and the accuracy of the reconstructed distribution of the diffusion coefficient parameter at different noise levels. (Received September 17, 2019)

1154-65-2376 **Jeonghun Lee*** (jeonghun_lee@baylor.edu), Sid Richardson Science Building, One Bear Place #97328, Waco, TX 76798. *A hybridized discontinuous Galerkin method for the Stokes equations with symmetric stress tensor approximation*.

In most hybridized discontinuous Galerkin methods for the Stokes equations, the formulation of first order differential equations which has the gradient of fluid velocity, the fluid velocity, and the fluid pressure as unknowns. However, this formulation uses a pseudo stress tensor instead of physical stress tensor, so it is not physically valid for problems with traction boundary conditions.

In this work we discuss construction and error analysis of a hybridized discontinuous Galerkin method for the Stokes equations which avoids the shortcomings. More specifically, we use a formulation using the symmetric gradient of fluid velocity as one of its unknowns, and the stress tensor is approximated by a symmetric tensor finite element space. As a consequence, we obtain a numerical method with optimal error estimates, such that traction boundary conditions are consistent with physical models and angular momentum is preserved exactly. (Received September 17, 2019)

1154-65-2408 **James V Lambers*** (james.lambers@usm.edu), School of Mathematics and Natural Sciences, The University of Southern Mississippi, 118 College Dr #5043, Hattiesburg, MS 39402. *Krylov Subspace Spectral Methods for Problems in Acoustics*. Preliminary report.

This talk provides an overview of the application of techniques for approximating bilinear forms involving matrix functions to PDEs of interest in acoustics. Featured work includes (1) the use of these to measure the sensitivity of solutions of PDEs, (2) the application of Krylov Subspace Spectral (KSS) Methods to the parabolic equation for acoustic pressure, and (3) KSS methods for wave propagation problems featuring periodic media and shock waves. (Received September 17, 2019)

1154-65-2444 **Bruce A. Wade*** (bruce.wade@louisiana.edu), Department of Mathematics, PO Box 43568, Maxim Doucet Hall 217, University of Louisiana, Lafayette, LA 70504-3568. *Smoothing Properties and Dimensional Splitting with Exponential Time Differencing Schemes for Advection-Diffusion-Reaction Systems*. Preliminary report.

Exponential Time Differencing (ETD) schemes for advection-diffusion-reaction systems are introduced and analyzed for their smoothing properties when applied to systems with nonsmooth or mismatched data. Several dimensional splitting strategies are presented, with an analysis of speedup. Robust performance under a variety of types of problems is empirically developed. (Received September 17, 2019)

1154-65-2454 **Andreas C Aristotelous*** (aaristotel@wcupa.edu). *Discontinuous Galerkin Finite Element Schemes for Diffuse Interface Models*.

Partial differential equation models based on Cahn-Hilliard type equations will be introduced and numerically solved. Discontinuous Galerkin Finite Element Methods for the numerical solution of the equations will be presented. For the underlined schemes: solvability, energy stability, convergence and error estimates will be established. Simulation results will be provided. (Received September 17, 2019)

1154-65-2617 **Lukas C. Erikson*** (lukas.erikson@gmail.com), Lukas.Erikson@gmail.com, and **Suzanna Stephenson, Daniel Christensen, Natalie Larsen and Tyler Jarvis.** *A Hybrid Multivariate Root-finding Method For Smooth Functions.*

We present a multivariate numerical rootfinding algorithm to find all the common zeros in a given compact region in \mathbb{R}^n of a system of functions. The method builds on ideas of Nakatsukasa, Noferini and Townsend to subdivide the original search interval and approximate the functions with Chebyshev polynomials. It then uses a variant of Telen and Van Barel's method to find the roots in each subdomain by computing eigenvectors or eigenvalues of the Chebyshev form of certain Möller-Stetter matrices constructed with a well-chosen basis. We compare the algorithm, in terms of accuracy and speed, to other popular numerical rootfinding algorithms, such as Bertini and Chebfun2. In many instances, this algorithm outperforms all known competitors. (Received September 17, 2019)

1154-65-2634 **Tommaso Buvoli*** (tbuvoli@ucmerced.edu), **Stéphane Gaudreault** (stephane.gaudreault2@canada.ca) and **Mayya Tokman** (mtokman@ucmerced.edu). *New Developments in Exponential Integration for Large Scale Stiff Systems.*

Over the past two decades, exponential integrators have emerged as an efficient class of time-stepping methods for solving stiff partial differential equations. For many application problems they can provide more accuracy and efficiency compared to fully implicit and semi-implicit integrators. In this talk we present a variety of exponential integrators including hybrid implicit-exponential (IMEXP) methods that combine exponential and rational functions. We will address theoretical and practical issues arising from the development of a fast integrators for different classes of problems. Construction and implementation of specific schemes for concrete applications such as shallow water equations and reactive flows will be discussed. (Received September 17, 2019)

1154-65-2646 **Hayden J Ringer*** (hringer@mathematics.byu.edu), 146W 1230N, Apt 335, Provo, UT 84604. *Multivariate Rootfinding Using Chebyshev Approximation and Möller-Stetter Matrices.*

Root-finding algorithms are key kernels in many areas of scientific computing. However, there are few robust methods for finding all, or even several, roots of multivariate systems. We present a method for finding all the common roots of a system of multivariate smooth functions lying within a compact set in \mathbb{R}^n . Our method utilizes multivariate Chebyshev polynomials to approximate smooth functions to high precision, and then uses a generalized form of the companion matrix, known as a Möller-Stetter matrix, to find the roots of the approximate polynomial system. We explore the numerical properties that the algorithm exploits in order to avoid a number of obstacles. We compare our method to other popular multivariate root-finding methods, including Chebfun2 and Bertini. (Received September 17, 2019)

1154-65-2730 **Vani Cheruvu*** (vani.cheruvu@utoledo.edu), 2801 W Bancroft Street, MS 942, Dept. of Mathematics and Statistics, Toledo, OH 43615, and **Sujeewa Hapuarachchi.** *Numerical solutions of the backward heat equation with time-dependent variable coefficient.* Preliminary report.

It is well known that solving the backward heat equation is challenging due to its severe ill-posedness and hence regularization is required. Hapuarachchi et al. considered the backward heat equation with a time-dependent variable coefficient and an approximation problem derived from the original equation by introducing a small perturbing parameter. They have used modified-quasi boundary value method to regularize the equation. We are currently working to obtain numerical results for various cases of this work. Classical numerical solutions (if exists) may not depend continuously on the boundary and final data since they cannot handle the severe ill-posedness and our current effort is to obtain numerical solutions to the approximation problem. (Received September 17, 2019)

68 ► Computer science

1154-68-201 **Ian Kaye** and **Narad Rampersad*** (n.rampersad@uwinnipeg.ca). *The abelian complexity of infinite words and the Frobenius problem.*

The classical Diophantine problem of Frobenius is the following: Given a pair of relatively prime positive integers a and b , what is the largest positive integer not representable as a non-negative linear combination of a and b ? We study a variation on this problem due to Dekking. Consider an infinite word \mathbf{x} over an alphabet $\{0, 1, \dots, k-1\}$ and a semigroup homomorphism $S : \{0, 1, \dots, k-1\}^* \rightarrow \mathbb{N}$. Let $\mathcal{L}_{\mathbf{x}}$ denote the set of factors of \mathbf{x} . What

conditions on S and the abelian complexity of \mathbf{x} guarantee that $S(\mathcal{L}_{\mathbf{x}})$ contains all but finitely many elements of \mathbb{N} ? We examine this question for some specific infinite words \mathbf{x} having different abelian complexity functions. We say that \mathbf{x} has the *Frobenius property* if $S(\mathcal{L}_{\mathbf{x}})$ contains all but finitely many elements of \mathbb{N} for every map S such that $\gcd(S(0), S(1), \dots, S(k-1)) = 1$. Dekking showed that no Sturmian word has the Frobenius property. We show that the ordinary paperfolding word does not have the Frobenius property and we give an example of an infinite binary word with non-maximal abelian complexity that does have the Frobenius property. (Received August 22, 2019)

1154-68-398 **Mireille Boutin*** (mboutin@purdue.edu). *Highly Likely Clusterable Data With No Cluster.*

Data generated as part of a real-life experiment is often quite organized. So much so that, in many cases, projecting the data onto a random line has a high probability of uncovering a clear division of the data into two well-separated groups. In other words, the data can be clustered with a high probability of success using a hyperplane whose normal vector direction is picked at random. We call such data “highly likely clusterable.” The clusters obtained in this fashion often do not seem compatible with a cluster structure in the original space. In fact, the data in the original space may not contain any cluster at all. This talk is about this surprising phenomenon. We will discuss empirical ways to detect it as well as how to exploit it to cluster datasets, especially datasets consisting of a small number of points in a high-dimensional space. We will also present a possible mathematical model that would explain this observed phenomenon. This is joint work with Sangchun Han, Tarun Yellamraju, and Alden Bradford. (Received September 05, 2019)

1154-68-404 **Anna Konstorum***, akonsto@super.org, and **Suzy V. Torti, Frank M. Torti** and **Reinhard C. Laubenbacher**. *Identifying critical pathways for ferroptosis using a systems approach.*

Ferroptosis is a newly recognized form of regulated cell death that has the potential to be employed for cancer treatment. Sensitivity to ferroptosis has a critical dependence on lipid peroxidation events, but there does not exist a framework to predict ferroptosis sensitivity that encompasses systems-level information in a quantitative manner. We utilize a stochastic multistate discrete framework to model ferroptosis sensitivity, and incorporate an asynchronous update scheme and continuity constraints into the model to improve consistency with biological data. We simulate all possible combinations of input variables to explore the space of ferroptosis sensitivity. The model results show that ferroptosis is an outcome of a balance between pro- and anti-oxidant mechanisms acting on lipid peroxidation substrates. The model captures known ferroptosis results, as well as provides novel predictions for combinatorial input parameter settings. We show experimental validation for a subset of these predictions using a primary ovarian cell culture model. (Received September 03, 2019)

1154-68-786 **Sarah Cannon*** (scannon@cmc.edu) and **Will Perkins** (math@willperkins.org). *Counting independent sets in unbalanced bipartite graphs.*

We study the hard-core model (independent sets) on bipartite graphs using the cluster expansion from statistical physics. When there is a sufficient imbalance in the degrees or fugacities between the sides (L, R) of the bipartition, we can rewrite the hard-core partition function in terms of deviations from independent sets that are empty on one side of the bipartition and show this expression has a convergent cluster expansion. This has interesting algorithmic and probabilistic consequences. On the algorithmic side, we address an open problem in approximate counting and give a polynomial-time algorithm for approximating the partition function for a large class of bounded degree bipartite graphs; this includes, among others, the unweighted biregular case where the degrees satisfy $d_R \geq 7d_L \log d_L$. Our approximation algorithm is based on truncating the cluster expansion. On the probabilistic side, we also prove that the hard-core model on such graphs exhibits exponential decay of correlations by utilizing connections between the cluster expansion and joint cumulants. (Received September 10, 2019)

1154-68-803 **Keaton Hamm*** (hamm@math.arizona.edu) and **Nick Henscheid**. *Wasserstein ISOMAP for Image Manifold Learning*. Preliminary report.

In many data-driven problems, a fundamental task is to find a dimensionality reduction map which faithfully represents the low-dimensional structure of the data. One method which addresses this problem is ISOMAP, which attempts to learn geodesics of a manifold which fits the data and uses these to embed the data into a smaller Euclidean space. A key step in this process is to associate a weighted graph to the data whose edge weights are the Euclidean distance between the points, but this procedure has some known drawbacks in learning manifolds of images. To get around this problem, we propose the use of the Wasserstein metric on the image

space to faithfully give geodesics between images. We will discuss the theory for certain image manifolds and illustrate the effectiveness of this method on several examples. (Received September 10, 2019)

1154-68-1086 **Stephen M Watt*** (smwatt@uwaterloo.ca), David R. Cheriton School of Computer Science, University of Waterloo, 200 University Ave W, Waterloo, ON N2L 3G1, Canada. *Progress in Mathematical Information and Knowledge Bases.*

This presentation will sum up what we have heard in the session on mathematical information in the digital age of science. Significant progress has been made recently in mathematical knowledge management, and this has spurred us on to more ambitious goals. As we conclude this session, it is helpful to review the current practical challenges and the most significant opportunities. (Received September 13, 2019)

1154-68-1109 **Y Cooper*** (yaim@math.ias.edu). *The loss function of overparameterized neural networks.*

Training modern neural networks relies on the use of gradient descent and related methods to minimize a non-convex loss function. Though this loss function is not convex, gradient descent comes close to finding global minima remarkably often in many real world settings. This is well established empirically, but in the early stages of being understood theoretically. In this talk, we will discuss recent progress in understanding some geometric properties of the loss function. This can help us understand the ways in which it is not convex, as well as ways in which it is better behaved than an arbitrary nonconvex function. (Received September 13, 2019)

1154-68-1176 **Cristina Garcia-Cardona*** (cgarciac@lanl.gov). *Learning Convolutional Sparse Representations.*

Convolutional sparse representation (CSR) is a successful technique that has been applied to a broad range of problems in signal and image processing, computer vision and machine learning. In a convolutional sparse representation, sums of a set of convolutions with dictionary filters (basis elements) are used to construct the model. The optimization is computed over the entire signal domain, yielding representations that are very sparse both spatially and across the filter indices. However, estimating the CSR for a specific data set usually requires the solution of a computationally expensive optimization problem. Recently, some of these optimization problems have been de-constructed and unrolled into a feed-forward network structure with the aim of reducing processing times. We present some applications of this unrolling concept, and discuss how this framework is more amenable to theoretical analysis and helps to bridge the gap of interpretability of more common convolutional neural networks. (Received September 13, 2019)

1154-68-1257 **Soledad Villar*** (soledad.villar@nyu.edu), 60 5th ave, New York, NY 10011. *Graph neural networks for combinatorial optimization problems on graphs.*

Graph neural networks are natural objects to express functions on graphs with relevant symmetries. In this talk we introduce graph neural networks and explain how they are being used to learn algorithms for combinatorial optimization problems on graphs from data (like clustering, max-cut and quadratic assignment), in supervised and unsupervised manners. We show a connection between universal approximation of invariant functions and the graph isomorphism problem. (Received September 14, 2019)

1154-68-1302 **Amber Hu*** (amber.hu@yale.edu), **Rodrigo Leonardo** (rodleonardoc@outlook.com) and **Mohammad Uzair** (muzair.bscecs@seecs.edu.pk). *Fusing Visual and Textual Information to Determine Content Safety.*

In advertising, identifying content safety is a significant concern since advertisers do not want brands to be associated with unsafe content. At the same time, publishers would like to open as much inventory as possible. Thus, a fine balance must be achieved to satisfy both advertisers and publishers. In this talk, we propose a multimodal machine learning framework that fuses visual and textual information from web pages to produce content safety predictions. The primary focus is on late fusion, a multimodal approach that involves combining final outputs of separate modalities, such as computer vision and natural language processing, to arrive at a decision. We developed a fully automated framework for multilabel classification using late fusion techniques. We also introduce work in early fusion, which involves extracting and fusing intermediate features from separate models to produce predictions. Our algorithms are applied to data provided by GumGum, Inc., a company that leverages machine learning in the advertising industry. Both of our late and early fusion methods obtain significant improvements over state-of-the-art algorithms. This work was completed at RIPS-Los Angeles. (Received September 14, 2019)

1154-68-1338 **Nathan Wiebe*** (nathan.wiebe@pnnl.gov). *Generative training of quantum Boltzmann machines with hidden units*. Preliminary report.

In this talk we provide a method for fully quantum generative training of quantum Boltzmann machines with both visible and hidden units while using quantum relative entropy as an objective. This is significant because prior methods were not able to do so due to mathematical challenges posed by the gradient evaluation. We present two novel methods for solving this problem. The first proposal addresses it, for a class of restricted quantum Boltzmann machines with mutually commuting Hamiltonians on the hidden units, by using a variational upper bound on the quantum relative entropy. The second one uses high-order divided difference methods and linear-combinations of unitaries to approximate the exact gradient of the relative entropy for a generic quantum Boltzmann machine. Both methods are efficient under the assumption that Gibbs state preparation is efficient and that the Hamiltonian are given by a sparse row-computable matrix. (Received September 16, 2019)

1154-68-1345 **Zafeirakis Zafeirakopoulos*** (zafeirakopoulos@gtu.edu.tr), Institute of Information Technologies, Computer Engineering Building, Gebze Technical University, 41400 Kocaeli, Turkey. *Using Machine Learning in Algebra and Geometry*.

The everyday research of a mathematician is typically a series of elaborated guesses that they try to prove or disprove. Often, making the guess, i.e., conjecture, is the most time consuming part. Experience is a factor that reduces the required effort, by aggregating the efforts put for previous conjectures. This is done by introducing a bias towards certain guesses.

In recent years, machine learning, a toolbox for making elaborated guesses, saw a lot of progress and attracted a lot of attention. Machine learning techniques, such as neural networks, have enormous impact in many scientific and engineering fields.

Applying the standard machine learning techniques to mathematical problems is not trivial. Most techniques are developed for applications in computer vision or in general for problems where the data have fixed dimensions. Advances in recurrent neural networks allow for variable size input. In particular, autoencoders can help in this direction.

In this talk we will see how to use machine learning for problems in algebra and polyhedral geometry. We focus on two problems, namely polytope volume prediction and real root counting. We will see how to solve the variable size representation problem and how to use the networks for mathematical exploration. (Received September 15, 2019)

1154-68-1370 **Ellen Gasparovic*** (gasparoe@union.edu). *The Relationship Between the Intrinsic Cech and Persistence Distortion Distances for Metric Graphs*.

We will look at two persistence-based distances that one may define for metric graphs and discuss progress toward establishing their discriminative capacities. This is joint work with Maria Gommel, Emilie Purvine, Radmila Sazdanovic, Bei Wang, Yusu Wang, and Lori Ziegelmeier. (Received September 15, 2019)

1154-68-1573 **Hassan Rafique*** (hassan-rafique@uiowa.edu), **Tong Wang** and **Qihang Lin**. *Model-Agnostic Linear Competitors - When Interpretable Models Compete and Collaborate with Black-box Models*. Preliminary report.

Driven by the increasing need for model interpretability, interpretable models have become strong competitors for black-box models in many real applications. In this paper, we propose a novel type of model where interpretable models compete and collaborate with black-box models. We present the Model-Agnostic Linear Competitors (MALC) for partially interpretable classification. MALC is a hybrid model that uses linear models to locally substitute an (any) black-box model, capturing subspaces that are most likely to be in a class while leaving the rest of the data to the black box. MALC brings together the interpretable power of linear models and good predictive performance of a black-box model. We formulate the training of a MALC model as a convex optimization, where predictive accuracy and transparency (defined as the percentage of data captured by the linear models) balance through a carefully designed objective function, and solve it with the accelerated proximal gradient method. Experiments show that MALC can effectively trade prediction accuracy for transparency and provide an efficient frontier that spans the entire spectrum of transparency. (Received September 16, 2019)

1154-68-1633 **Nicole Eikmeier*** (eikmeier@grinnell.edu). *Higher-order Preferential Attachment Models*.

Preferential attachment models are a common class of graph models which have been used to explain why power-law distributions appear in the degree sequences of real network data. Among other properties of real-world networks, they commonly have non-trivial clustering coefficients due to an abundance of triangles as well as power-laws in the eigenvalue spectra. In this talk we present a Higher-Order Generalized Preferential

Attachment Model that, by construction, has nontrivial clustering. In this model we allow for the addition of an arbitrary number of edges, producing more complex structures than in a traditional preferential attachment model. (Received September 16, 2019)

1154-68-1742 **Benjamin Rossman***, Bahen Centre for Information Technology, Room 6214, Toronto, ON M5S2E4, Canada. *Choiceless Polynomial Time.*

The choiceless computation model of Blass, Gurevich and Shelah (1999, 2022) is an algorithmic framework for computing isomorphism-invariant properties of unordered structures. Machines in this model have the power of parallel execution, but lack the ability to make arbitrary choices. For example, a choiceless algorithm cannot freely select an arbitrary neighbor of a vertex in an unordered graph, but may execute a subroutine in parallel over all neighbors. In this talk, I will give an overview of results in the choiceless model and discuss the intriguing open question whether every polynomial-time graph property admits a choiceless poly-time algorithm. (Received September 16, 2019)

1154-68-1800 **Camila Alexandra Ramirez***, cramirez@exptechinc.com. *On Complex-Valued Equivariant Neural Networks for Radio Frequency Fingerprinting.*

Identifying Internet of Things (IoT) devices by their Radio Frequency (RF) fingerprint has important security implications. As the number of devices grows, current authentication mechanisms become susceptible to spoofing attacks. To combat this problem, we exploit hardware imperfections in the RF transmit chain and extract device-specific features uniquely identifying an emitter. RF propagation channels degrade signals, when measured at a receiver, following a convolutional model. Assuming that test data reflects the diversity of channels in a train set, a device fingerprint should be repeatable and accuracy comparable. However, when this assumption is broken, the classifier performs poorly. That is, the learned classifier is not invariant to the RF channel degradation. In this talk, we use an algebraic representation of signal processing, where channels are defined as an algebra and signals as a module over the channel-algebra. Modeling channel degradation as convolution, and given that raw RF signals are complex-valued, we define an N-dimensional invariant distance metric with which we construct equivariant convolutional layers. We then build a complex-valued neural network invariant to channel degradation and test the efficacy of our method on synthetic and real WiFi datasets. (Received September 16, 2019)

1154-68-1850 **William Mark Severa*** (wmsevera@sandia.gov), PO Box 5800, MS 1327, Albuquerque, NM 87185. *Neural-Inspired Computation for Efficient Scientific Computing via Random Walks and Surrogate Models.*

The growth in the scale of scientific computing workloads has increased dramatically over the last decade. The Department of Energy's Exascale Computing Project is strong evidence of this, wherein the goal is to accelerate development of exa-op computers to tackle the world's largest computing problems. However, a key limitation in building these large systems is the amount of energy required to run them. In this talk we discuss some of our recent advances towards leveraging low-energy neural computing methods for HPC applications. We contend that these platforms can offer an extraordinary performance-per-Watt advantage, not just in machine learning, but also in aid of scientific workloads. First, we overview a neuromorphic-compatible algorithm for Markov random walks that uses a spiking neural network approach to implement a density-based random walk across a graph. We describe both algorithmic details and hardware results using IBM's TrueNorth. Then, we discuss a class of graph algorithms that can be approached through similar means for high levels of parallelization. Lastly, we briefly describe a growing effort for using deep learning, specifically convolutional neural networks and long short term memory, for fast surrogates of climate (ice sheet) modeling. (Received September 18, 2019)

1154-68-1853 **Hum Nath Bhandari*** (hbhandari@rwu.edu), One Old Ferry Road, Bristol, RI. *Developing Hybrid PSO Algorithm Models Using the Cyclic Coordinate Descent (CCD) Local Optimizer.* Preliminary report.

The particle swarm optimization (PSO) algorithm has been successfully applied in many scientific and engineering fields to solve global optimization problems. We present a hybrid implementation of this algorithm which harnesses the simplicity and effectiveness of the base PSO algorithm with a global restart component and a local optimizer (called the MPSO-CCD). This implementation is easy to parallelize, and we also relate our experience with this parallel version (PMP SO-CCD). We present a few of the standard benchmark problems to illustrate the effectiveness and resilience of these implementations. (Received September 16, 2019)

1154-68-1859 **Rodrigo Rojas Moraleda*** (rodrigo.moraleda@nct-heidelberg.de), AG Jaeger, Im Neuenheimer Feld 460, 69120 Heidelberg, Germany, and **Bénédicte Lenoir, Wei Xiong, Nektarios A. Valous and Niels Halama.** *Detection and segmentation of capillary structures in biomedical images based on a computational topology framework.*

The development of automated methods for supporting human experts provides tangible benefits in the context of clinical decision-making. Automated segmentation of capillary structures in histology is a difficult task; this is due to the wide variability of shapes and sizes adopted by the capillaries once the biological sample is prepared and fixed on a glass slide. Topology and homology, on the other hand, provide convenient tools for classifying spaces that are less sensitive to the geometry of the objects.

This work introduces a method for the segmentation of aggregated cells into capillary forms in histological images based on the principles of persistent homology. The approach deals with changing the representation of a histological image into a collection of simplicial complexes. Vertices represent cells and edges represent relationships between cells. Homology classes dimensions one and two, are identified in order to perform the segmentation.

Images acquired from histological sections of ovarian tissue are used to demonstrate the effectiveness of quantifying lymphatic vessels. The accuracy is verified against expert annotations. Results are provided on a per-object basis. (Received September 16, 2019)

1154-68-2004 **Adam Brown, Omer Bobrowski, Elizabeth Munch and Bei Wang*** (beiwang@sci.utah.edu), Scientific Computing and Imaging Institute, University of Utah, 72 South Central Campus Drive, Salt Lake City, UT 84112. *Probabilistic Convergence and Stability of Random Mapper Graphs.* Preliminary report.

We study the probabilistic convergence between the mapper graph and the Reeb graph of a topological space X equipped with a continuous function $f : X \rightarrow R$. Our techniques are based on the interleaving distance of constructible cosheaves and topological estimation via kernel density estimates. Following Munch and Wang (2018), we first show that the mapper graph of (X, f) approximates the Reeb graph of the same space. We then construct an isomorphism between the mapper of (X, f) to the mapper of a super-level set of a probability density function concentrated on (X, f) . Finally, building on the approach of Bobrowski et al. (2017), we show that, with high probability, we can recover the mapper of the super-level set given a sufficiently large sample. We introduce a variant of the classic mapper graph of Singh et al. (2007), referred to as the enhanced mapper graph. We show that the enhanced mapper graph reduces the information loss during summarization and may be of independent interest. Our work is the first to consider the mapper construction using the theory of cosheaves in a probabilistic setting. It is part of an ongoing effort to combine sheaf theory, probability, and statistics, to support topological data analysis with random data. (Received September 17, 2019)

1154-68-2036 **Anna C Gilbert*** (annacg@umich.edu), 2074 East Hall, 530 Church Street, Ann Arbor, MI 48109. *Sparse Metric Repair.*

Suppose we are given a distance or similarity matrix for a data set that is corrupted in some fashion, find a sparse correction or repair to the distance matrix so as to ensure the corrected distances come from a metric; i.e., repair as few entries as possible in the matrix so that we have a metric. I will discuss generalizations to graph metrics, applications to (and from) metric embeddings, and algorithms for variations of this problem. I will also touch upon applications in machine learning and bio-informatics. (Received September 17, 2019)

1154-68-2274 **Laura Balzano** (girasole@umich.edu), Ann Arbor, MI, **Hanbaek Lyu*** (colourgraph@gmail.com), Los Angeles, CA 90025, and **Deanna Needell** (deanna@math.ucla.edu), Los Angeles, CA 90025. *Online nonnegative matrix factorization for Markovian data.*

Nonnegative matrix factorization (NMF) is one of the fundamental tools in dictionary learning problems, which gives an approximate representation of complex data sets in terms of a reduced number of extracted features. While Markov chain Monte Carlo (MCMC) provides one of the most versatile sampling techniques across many disciplines, most of the online NMF algorithms in the literature assume independence between consecutive data matrices and the case of dependent data sequences remains largely unexplored. In this paper, we show that the well-known online NMF algorithm for i.i.d. stream of data proposed in [MBPS10], in fact converges almost surely to the set of critical points of the expected loss function, even when the data matrices form a Markov chain satisfying a mild mixing condition. Furthermore, we extend the convergence result to the case when we can only approximately solve the optimization problems in the online NMF algorithm. Lastly, we demonstrate that one can learn features from MCMC trajectories arising from spin systems, generative models, and prior distribution on data corpus, which is now theoretically guaranteed by our results. (Received September 17, 2019)

1154-68-2284 **Thomas Strohmer*** (strohmer@math.ucdavis.edu), Department of Mathematics, Davis, CA 95616. *A Rigorous Framework for Data Clustering.*

Organizing data into meaningful groups is one of the most fundamental tasks in data analysis and machine learning. While spectral clustering has become one of the most popular clustering techniques, a rigorous and meaningful theoretical justification has still been elusive so far. I will propose a convex relaxation approach, which gives rise to a rigorous theoretical analysis of spectral clustering. We do this by deriving deterministic bounds of finding optimal graph cuts via a natural and intuitive proximity condition related to the spectrum of the graph Laplacian. Moreover, the proposed approach provides theoretical guarantees for community detection. I will discuss extensions and applications of our framework. (Received September 17, 2019)

1154-68-2423 **Sifan Wang and Paris Perdikaris*** (pgp@seas.upenn.edu), 3401 Walnut Street, Rm. 527A, Philadelphia, PA 19104. *Overcoming gradient pathologies in physics-informed neural networks.*

The widespread use of neural networks across different scientific domains often involves constraining them to satisfy certain symmetries, conservation laws, or other domain knowledge. Such constraints are often imposed as soft penalties during model training and effectively act as domain-specific regularizers of the empirical risk loss. Physics-informed neural networks is an example of this philosophy in which the outputs of dense neural networks are constrained to approximately satisfy a given set of partial differential equations. In this work we identify and analyze a fundamental mode of failure of such approaches that is related to an imbalance in the magnitude of the back-propagated gradients during model training. To address this limitation we propose a re-weighting procedure that resembles the effect of an adaptive learning rate for balancing the interplay between different terms in composite loss functions. We also propose a novel neural network architecture that is more resilient to such gradient pathologies. Taken together, our developments provide new insight into the training of constrained neural networks and consistently improve the predictive accuracy of physics-informed neural networks by a factor of 50-100x across a range of problems in computational physics. (Received September 17, 2019)

1154-68-2496 **Camille Moyer*** (cmmoyer1@asu.edu), **Hannes Baukmann** and **Tim Conrad.** *Synthetic data augmentation with DCGANs for better diagnoses of osteoarthritis.*

In the medical world, viable data is difficult to come by, particularly when it comes to healthy patients. Osteoarthritis is one of the most common causes of disability in adults, but it is still difficult to obtain enough data to train machine learning processes. In this talk, we explore the use of deep convolutional generative adversarial networks (DCGAN) to supplement the low amount of data for healthy patients. We use tandem mass spectrometry data in the form of heat maps to train the DCGAN and produce more images that can later be used in classification networks for diagnostic purposes. The use of DCGANs to construct more image data is promising in its ability to incorporate the natural variability found in biological systems. (Received September 17, 2019)

1154-68-2599 **Parmida Davarmanesh***, 2009 Huron Pkwy, apt 8, Ann Arbor, MI 48104, and **Kuanhao Jiang, Tingting Ou, Artem Vysogorets, Shantanu Joshi and Nicholas Malaya.** *Automating Artifact Detection in Video Games.* Preliminary report.

The video game industry is the most influential form of entertainment in America and the world. Despite optimized and specialized gaming hardware and software, gameplay can be impaired with graphics errors, screen artifacts, and other forms of corruption which are labor intensive to detect. This research has explored methods to automate anomaly detection and classification. This was accomplished by machine learning models to classify each frame of a video game as glitched or normal. The primary challenge is the lack of labeled and catalogued gaming data. To circumvent this bottleneck, a database was generated by extracting images from gameplay videos and adding artifacts modeled after observed corruption to the images. This work also explored several ways to extract features from the images, such as Fourier spectra, the histogram of gradients, and graph Laplacians. Using the extracted features, multiple classifiers were built to detect different types of glitches. Finally, an ensemble model was constructed by combining the individual classifiers using logistic regression. The results were able to accurately predict real corrupted images with a high degree of accuracy, indicating this approach may be applicable to a wide range of corruption in visual image processing. (Received September 17, 2019)

1154-68-2613 **Swanand Kadhe*** (swanand.kadhe@berkeley.edu) and **Kannan Ramchandran** (kannanr@berkeley.edu). *How can a coding theorist make blockchains more efficient and scalable?*

In this talk, we will highlight how coding theory can play an instrumental role in addressing some of the fundamental challenges in the emerging field of blockchains, and in turn, demonstrate how blockchains open up novel code design problems. In the first half, we will focus on blockchain storage requirements that are

growing near-exponentially. We will describe a ‘Secure Fountain (SeF)’ architecture, based on fountain codes, that enables users to encode blocks into a small number of coded blocks, thereby reducing storage costs by orders of magnitude. We demonstrate that the peeling decoder admitted by fountain codes turns out to be crucial for security against adversarial users that can provide maliciously formed coded blocks. Specifically, it enables us to introduce error-resiliency by leveraging the hash-chain structure of the blockchain. Further, the rateless property of fountain codes helps in achieving high decentralization and scalability. In the second half, we describe how codes can be used to mitigate data withholding attacks in blockchains. We analyze the requirements that a code must satisfy to ensure high availability guarantees, and demonstrate that generalized low density parity check (GLDPC) codes are excellent candidates. (Received September 17, 2019)

1154-68-2798 **Mustafa Hajij***, mustafahajij@gmail.com. *Manifold Learning Using Persistent Homology on the Laplacian Eigenfunctions*. Preliminary report.

We use persistent homology along with the eigenfunctions of the Laplacian to study similarity amongst triangulated 2-manifolds. Our method relies on studying the lower-star filtration induced by the eigenfunctions of the Laplacian. This gives us a shape descriptor that inherits the rich information encoded in the eigenfunctions of the Laplacian. Moreover, the similarity between these descriptors can be easily computed using tools that are readily available in Topological Data Analysis. We provide experiments to illustrate the effectiveness of the proposed method. (Received September 18, 2019)

70 ► *Mechanics of particles and systems*

1154-70-140 **Corey Shanbrom*** (corey.shanbrom@csus.edu) and **Victor Dods**. *Self-similarity in the Kepler-Heisenberg problem*. Preliminary report.

The Kepler-Heisenberg problem is that of determining the motion of a planet around a sun in the Heisenberg group, thought of as a three-dimensional sub-Riemannian manifold. The sub-Riemannian Hamiltonian provides the kinetic energy, and the gravitational potential is given by the fundamental solution to the sub-Laplacian. The dynamics are at least partially integrable, possessing two first integrals as well as a dilational momentum which is conserved by orbits with zero energy. The system is known to admit closed orbits of any rational rotation number, which all lie within the fundamental zero energy integrable subsystem. Here, we demonstrate that all zero energy orbits are self-similar. (Received August 14, 2019)

1154-70-165 **Stefan Steinerberger*** (stefan.steinerberger@gmail.com), 10 Hillhouse Avenue, New Haven, CT 06511. *Dimensionality Reduction via tSNE: an analysis via large-scale ODE systems*.

We study the problem of dimensionality reduction: given a set of high-dimensional points, we would like to get a 2D map of these points preserving essential structures so we can look at it. One of the most widely used algorithms is t-SNE (t-distributed stochastic neighborhood embedding); until recently, no mathematical analysis was available. We show how it can be interpreted as a large coupled system of ODEs; this interpretation allowed us to rigorously prove that the algorithm works in certain cases (indeed, this was the first rigorous argument for tSNE), to establish a connection to classical Laplacian eigenmaps and it also suggests a series of open problems. (Received August 17, 2019)

1154-70-474 **Susan Jane Colley** and **Gary Kennedy*** (kennedy.28@osu.edu). *The stationary train and the runaway train*. Preliminary report.

In work which we finished nearly 30 years ago, we used a tower of compactified curvilinear data spaces introduced in 1954 by Semple. The construction of this tower was independently rediscovered by Montgomery and Zhitomirskii, who called it the monster tower. They pointed out that if one applies the oriented version of the construction beginning with \mathbb{R}^2 , one obtains the configuration space for a standard system in mechanics: a truck pulling multiple trailers.

Within this system, one can pose two complementary problems: (1) Given the path of the truck, what will be the path of the last trailer? (2) Conversely, given the desired path of the last trailer, what should be the path of the truck? Of particular interest are what might be called stationary solutions, i.e., configurations of the train in which it moves as if it were a single rigid object. These stationary trains can be related to natural strata on the monster tower.

These same strata also arise when one studies the runaway train. Here one sets an initial velocity and angular velocity for the truck, and ask what happens as it moves according to Newtonian mechanics. The strata appear as bifurcation loci.

We will recount this story in detail in a book now being written with Corey Shanbrom. (Received September 04, 2019)

1154-70-605 **Connor Jackman*** (connor.jackman@cimat.mx). *Lie point symmetries of the planar Kepler problem.*

The Kepler problem is the attractive central force problem with a force proportional to the inverse square of the distance. In this talk we will find diffeomorphisms of the plane with the property that they preserve orbits of the Kepler problem – conics with focus at the origin are sent to conics with focus at the origin. When one fixes a non-zero energy and asks to preserve this energy as well, we will show that these symmetry groups are $SL_2(\mathbb{R})$. Moreover, we will see that this Lie point symmetry group $SL_2(\mathbb{R})$ is exceptional among central force problems with fixed energy. (Received September 08, 2019)

1154-70-1631 **Maria Emelianenko***, memelian@gmu.edu. *Microstructural entropy and stored energy in modeling recrystallization and microstructure-property relationships in polycrystals.*

While empirical laws like Hall-Petch have long been known to give inverse relationship between average grain size and polycrystalline material strength, complete understanding of the role microstructure geometry and topology play in coarsening and mechanical deformations is still missing. This talk compares different types of entropy measures that may be used to characterize microstructure disorder based on their ability to capture deviations in certain mechanical and kinetic properties estimated via large scale numerical simulations. GPU-based simulations of recrystallization based on the stored energy formalism are also discussed. (Received September 16, 2019)

1154-70-2024 **Nathan Duignan*** (nathan.duignan@sydney.edu.au). *On the Simultaneous Binary Collision and the Curious $8/3$.*

Of central importance in the n -body problem is the fact that isolated binary collisions can be regularised; a singular change of space and time variables allows trajectories to pass analytically through binary collisions unscathed. This so called Levi-Civita regularisation provides a flow smooth with respect to initial conditions. Curiously, when two binary collisions occur simultaneously, we are not so fortunate. In 1999, Martinez and Simó gave strong evidence to conjecture the regularised flow, in a neighbourhood of the simultaneous binary collision, is at best $C^{8/3}$. Remarkably, the conjecture has been shown for some sub-problems of the 4-body problem, including the collinear and trapezoidal problems.

In this talk we provide a proof for conjecture in the planar 4-body problem. Some notable components of the proof are the use of an uncommon normal form procedure, a type of projective blow-up which produces a collision manifold foliated by invariant $\mathbb{R}P^3$, and the study of transitions near manifolds of normally hyperbolic fixed points. Notably, through the proof, a link is established between the inability to construct a set of integrals local to simultaneous binary collisions and the curious loss of differentiability. (Received September 17, 2019)

1154-70-2178 **Gabriel Martins*** (g.martins@csus.edu). *Confining Charged Particles in 3 Dimensions Using Bounded Magnetic Fields.*

We study the problem of confining a charged particle to the interior of a bounded region in 3-space with smooth boundary by using magnetic fields. We show that when considering bounded magnetic fields one may confine particles of low enough energy. This simple model is inspired by the dynamics in the interior of particle accelerators and fusion reactor devices. (Received September 17, 2019)

1154-70-2201 **Patrycja A Przewoznik*** (paprze21@g.holycross.edu), 1 College Street, College of the Holy Cross, Worcester, MA 01610, and **David B Damiano**. *Structural Analysis of the force chains within communities of particles.* Preliminary report.

Granular materials are collections of macroscopic particles in contact with each other which exhibit a variety of behaviors. Granular materials can act as solids, liquids or gases, depending on the physical circumstances and can exhibit complex behaviors which have not yet been sufficiently explained. In order to improve the understanding of interactions between the particles, we provided a mathematical analysis of the structures created by the forces between the particles from an experiment performed on a set of photoelastic disks by Dr. Eli Owens at North Carolina State University. Randomly distributed disks placed in a flat upright container were subject to consecutively increased pressure in each trial caused by weight applied on top of the container. In each case, the forces due to pressure create unique structures called force chains which combine into force networks. To study the changes in those force networks, we tracked individual disks along the pressures. We grouped the disks into communities to study the changes in the force chains across pressure levels. We applied two different versions of the community detection algorithm based on the Newman-Girvan and geographical null models to obtain two sets of communities along all pressures. (Received September 17, 2019)

1154-70-2645 **Jiequn Han***, Fine Hall 1007, Washington Road, Princeton, NJ 08544, and **Chao Ma, Zheng Ma and Weinan E.** *Uniformly Accurate Machine Learning Based Hydrodynamic Models for Kinetic Equations.*

A new framework is introduced for constructing interpretable and truly reliable reduced models for multi-scale problems in situations without scale separation. Hydrodynamic approximation to the kinetic equation is used as an example to illustrate the main steps and issues involved. To this end, a set of generalized moments are constructed first to optimally represent the underlying velocity distribution. The well-known closure problem is then solved with the aim of best capturing the associated dynamics of the kinetic equation. The issue of physical constraints such as Galilean invariance is addressed and an active learning procedure is introduced to help ensuring that the data set used is representative enough. The reduced system takes the form of a conventional moment system and works regardless of the numerical discretization used. Numerical results are presented for the BGK (Bhatnagar–Gross–Krook) model and binary collision of Maxwell molecules. We demonstrate that the reduced model achieves a uniform accuracy in a wide range of Knudsen numbers spanning from the hydrodynamic limit to free molecular flow. (Received September 17, 2019)

74 ► *Mechanics of deformable solids*

1154-74-570 **Robert P Lipton*** (lipton@lsu.edu), Mathematics Department, Room 256 Lockett Hall, Baton Rouge, LA 70803, and **Prashant K Jha** (pjha.sci@gmail.com), 6.328 POB, University of Texas at Austin, Austin, TX 78712. *Classic dynamic fracture recovered as the limit of a nonlocal peridynamic model: The single edge notch in tension.*

A simple nonlocal field theory of peridynamic type is applied to model brittle fracture. The fracture evolution is shown to converge in the limit of vanishing nonlocality to classic plane elastodynamics with a running crack. The kinetic relation for the crack is recovered directly from the nonlocal model in the limit of vanishing nonlocality. We carry out our analysis for a single crack in a plate subject to mode one loading. The convergence is corroborated by numerical experiments. (Received September 07, 2019)

1154-74-1977 **Peter D Dragnev*** (dragnevp@pfw.edu), Department of Mathematical Sciences, Purdue University Fort Wayne, 2101 E. Coliseum Blvd., Fort Wayne, IN 46805, and **Oleg R Musin** (oleg.musin@utrgv.edu), School Mathematical and Statistical Sciences, University of Texas Rio Grande Valley, One West University Boulevard, Brownsville, TX 78520. *Log-optimal $(d + 2)$ -configurations in d -dimensions.*

We enumerate and classify all stationary logarithmic configurations of $d + 2$ points on the unit sphere in d -dimensions. In particular, we show that the logarithmic energy attains its relative minima at configurations that consist of two orthogonal to each other regular simplexes of cardinality m and n . The global minimum occurs when $m = n$ if d is even and $m = n + 1$ otherwise. This characterizes a new class of configurations that minimize the logarithmic energy on \mathbb{S}^{d-1} for all d . The other two classes known in the literature, the regular simplex ($d + 1$ points on \mathbb{S}^d) and the cross polytope ($2d$ points on \mathbb{S}^d), are both universally optimal configurations. (Received September 16, 2019)

1154-74-2607 **Michael C. Barg***, mbarg@niagara.edu. *Numerical solutions in a self-organizing inhibitory system on a torus.* Preliminary report.

We conduct numerical investigations into the size and shape of equilibrium patches on a torus. Such patches arise as solutions to a constrained minimization problem in the context of self-organizing inhibitory systems. The functional to be minimized includes contributions from both local and non-local energy terms. Our results for a torus extend and complement recent work completed by others on ellipsoids. Solutions with a small inhibitory parameter and a small conservation parameter are typically single droplet equilibrium patches centered near a location of maximum Gauss curvature. If such patches are sufficiently small, they will assume a geodesic disk-like shape. In the case of a small inhibitory parameter and a larger conservation parameter, a variety of solutions may be obtained. In this regime, equilibrium solutions no longer take the geodesic disk-like shape. When the inhibitory parameter is sufficiently large, single droplet solutions appear to become unstable and droplet assemblies become the preferred configuration instead. In addition to presenting representative solutions, we discuss improvements to previously used finite element models. Such modifications include the use of different types of finite elements and alternative forms for the conservation constraint. (Received September 17, 2019)

76 ► Fluid mechanics

- 1154-76-645 **Z M Mbugua*** (mburum46@gmail.com), p.o.box private bag X01, scotsville, pietermaritzburg, Johannesburg, kzn 3209, South Africa, **P - Sibanda** (sibandap@ukzn.ac.za), p.o.box private bag X01, Scotsville, Pietermaritzburg, Johannesburg, 3209, South Africa, and **S - Mondal** (sabya.mondal.2007@gmail.com), p.o.box 700135, Newtown, India, Bengal, 700052, India. *A Numerical Study of Entropy Generation in Nanofluid Flow Over an Inclined Cylinder.*

In this study we present a numerical investigation of entropy generation, heat and mass transfer in an unsteady nanofluid flow over an inclined cylinder using Buongiorno's model. A numerical scheme with overlapping grids and bivariate spectral collocation is used to solve the highly non-linear partial differential equations that model the flow. We compute the entropy generation rate and determine the influence of the flow parameters on entropy generation and the fluid properties. The results are presented in both qualitative and quantitative forms. An analysis of the convergence of the numerical method is presented to show the accuracy and stability of the new numerical scheme.

Keywords: Numerical, Entropy generation, Overlapping grid, Nanofluid. (Received September 09, 2019)

- 1154-76-1120 **Lihua Zuo*** (lihua.zuo@tamuk.edu), MSC 172, 700 University Blvd, Kingsville, TX 78363, and **Kan Wu**. *Robust Analysis of Different Models for Fracture Propagation in Formations with Bedlayers.*

Because of the development of hydraulic fracturing in unconventional reservoirs and the singularity issues existing in the numerical computation, accurately modeling the propagation of the hydraulic fractures has been both a necessity and big challenge for the researchers and companies. Among all the techniques for fracture propagation modeling, equilibrium height growth model is the most sufficient methods due to the balance of accuracy and computation efficiency. In this study, three different equilibrium height growth models (Analytical Method, Reference Pressure Method and Direct Minimum Method) are rigorously analyzed and compared for a three layer reservoir zone. Their advantages and disadvantages are summarized and illustrated. For the best knowledge of the authors, this is the first time all these three methods are analyzed and compared so rigorously. The results in this study will provide the researchers a good reference to decide which method is applied. (Received September 13, 2019)

- 1154-76-1249 **Weifan Liu*** (wliu07@syr.edu), 215 Carnegie Building, Syracuse, NY 13244-1150, and **Thomas Witelski** (witelski@math.duke.edu), 295 Physics Bldg, Box 90320, Durham, NC 27708-0320. *Steady-states of thin film droplets on chemically heterogeneous substrates.*

The spreading and dewetting of thin liquid films subject to van der Waals interactions can be described by a nonlinear fourth-order parabolic PDE. Subject to no-flux boundary condition, the steady-state thin film is governed by a second order ODE. The bifurcation of steady-state thin films on homogeneous substrates has been previously studied by Bertozzi et al (2001). We extend the previous studies by presenting results on the steady-state thin fluid films on chemically heterogeneous substrates. Specifically, we use phase planes to study the bifurcation of thin films on stepwise-patterned substrates and develop asymptotic approximation for the steady-state solutions. We find a new bifurcation branch of solutions, characterizing droplets pinned at the interface of heterogeneity, which arises as a consequence of wettability contrast of the substrate. In addition, we discuss an effective measure of the fluid leakage for films in presence of an increasing heterogeneity contrast and show through asymptotic analysis that the leakage is inversely proportional to the heterogeneity contrast. Last, we show all of the analysis in 1-D can be easily extended to axisymmetric solutions in 2-D. (Received September 14, 2019)

- 1154-76-1337 **Guoyi Ke*** (gke@lsua.edu), 8100 Hwy 71 S, Alexandria, LA 71302, and **Sara Calandrini** and **Eugenio Aulisa**. *A novel preconditioning technique for fluid-structure interaction with applications in biomechanics.*

We present a novel preconditioning technique for Krylov subspace algorithms to solve fluid-structure interaction (FSI) linearized systems arising from finite element discretizations. An outer Krylov subspace solver preconditioned with a geometric multigrid (GMG) algorithm is used, where for the multigrid level sub-solvers, a field-split (FS) preconditioner is proposed. The block structure of the FS preconditioner is derived using the physical variables as splitting strategy. To solve the subsystems originated by the FS preconditioning, an additive Schwarz (AS) block strategy is employed. The proposed field-split preconditioner is tested on biomedical FSI applications. Both 2D and 3D simulations are carried out considering aneurysm and venous valve geometries.

The performances of the FS preconditioner are compared with those of a second preconditioner of pure domain decomposition type. (Received September 15, 2019)

1154-76-1403 **A P Hoover***, ahoover1@uakron.edu, and **R Cortez**. *A Regularized Stokeslet Approach to the Motion and Behavior of Microscopic Swimmers.*

In the world of bacterial locomotion, motile organisms generate wakes and eddies that affect their local fluid environment. When many of them are present around each other, they interact with the flows generated by other organisms and form active suspensions whose complex flow structures play a significant role in fluid transport and mixing. In this talk, we present a reduced model for bacterial locomotion that describes their self-propelled motion in a low Reynolds number viscous incompressible fluid. The model is based on a particular limit of regularized Stokeslets with built-in asymmetry in order to produce a swimming direction. The result is a single-particle model of a swimmer that does not require special treatment of the self velocity due to the regularization, while allowing us to efficiently study the collective motion of bacteria. With this model, we are able to model pusher and puller organisms in a straightforward manner for both free-space and periodic domains. We will characterize the particle dynamics and discuss the diffusion of these particles as a function of the concentration density. We will then take advantage of the regularized Stokeslets framework to understand how active suspensions interact with viscoelastic structures, such as biofilms. (Received September 15, 2019)

1154-76-1590 **K Chad Sockwell*** (kcsocw@sandia.gov), **Konstantin Pieper** and **Max Gunzburger**. *Mass Conserving, Hamiltonian-Structure-Preserving Reduced Order Modeling for the Rotating Shallow Water Equations.*

Ocean modeling, in a climate-modeling context, requires long time horizons over global scales, which when combined with accurate resolution in time and space makes simulations very computationally expensive. While high-resolution ocean-modeling simulations are still feasible on large HPC machines, performing uncertainty quantification or other many-query applications at these resolutions is not feasible. Reduced Order Modeling (ROM) techniques utilize existing simulation data to construct more efficient models. Models produced by these techniques provide a tremendous speed up at the cost of reduced accuracy. To offset this trade-off, novel strategies have been developed to retain as much accuracy as possible while still achieving tremendous speedups. Some of these methods improve accuracy by incorporating physical properties into the reduced model, leading to better solution quality. In this talk, a novel reduced order modeling method, the mass-conserving Hamiltonian-structure-preserving reduced order modeling method will be presented and applied to the rotating shallow water equations. (Received September 16, 2019)

1154-76-1607 **Yining Cao**, **Chuck Jia*** (jiac@iu.edu) and **Roger Temam**. *Time-stepping Method for Primitive Equations Modeling the Geophysical Flows In Presence of Topography.*

In this talk, we present a time-stepping method for solving systems of primitive equations modeling the atmosphere with humidity and saturation in presence of topography. Although dealing with the atmosphere, we encounter many difficulties that are common with ocean modeling as well. To overcome the difficulties caused by the nonlinear constraints in the equations, we introduce a fractional time-stepping scheme with a type of projection method. Physically plausible boundary conditions are used in our models. A compatibility condition is introduced and used in the projection method to enforce the boundary conditions and the incompressibility condition. We will demonstrate our time-stepping method by showing the results of numerical experiments with realistic parameters. This is joint work with Yining Cao and Roger Temam. (Received September 16, 2019)

1154-76-2331 **Roseanna Gossmann*** (rgossmann@aopsacademy.org). *The elastohydrodynamics of a simplified model of human birth.*

With a view towards reducing the incidence of unnecessary surgical intervention during birth by reaching a better understanding of the mechanics of human birth, we investigate how amniotic fluid transfers forces from the birth canal onto the fetus during birth. Motivated by a simplified physical model which represents the fetus moving through the birth canal using a rigid cylinder (fetus) that moves through the center of a passive elastic tube (birth canal) immersed in highly viscous fluid (amniotic fluid), we utilize numerical methods to explore the forces on the fetus, and their relationship to the successful progression of birth and to strain in the birth canal. The time-evolving geometry of the elastic tube includes buckling behavior, which also occurs in other systems of flow through collapsible tubes, and is explored by varying properties of the rigid cylinder, elastic tube, and surrounding fluid. Peristaltic contraction of the elastic tube is also included in the model to provide additional insight into the force and velocity of the fetus and strain in the birth canal during human birth, when the birth canal is highly active. (Received September 17, 2019)

- 1154-76-2396 **Lynn Schreyer*** (lynn.schreyer@wsu.edu), Pullman, WA 99164, **Nikolas Voulgarakis** (n.voulgarakis@wsu.edu), Pullman, WA 99164, **Zachary Hilliard** (zachary.hilliard@wsu.edu), Pullman, WA 99164, **Sergey Lapin** (slapin@wsu.edu), Pullman, WA 99164, and **Loren Cobb** (loren.cobb@ucdenver.edu), Denver, CO 80217. *A Deterministic Model for Mammal Migration based on a Continuum Mechanics Phase-Field Approach of Fluid Flow through Porous Media*. Preliminary report.

Here we discuss how a new deterministic model for mammal migration was developed by using a methodical approach heretofore used to develop models for fluid flow through porous media. Mammal migration characteristics that are captured by the proposed model include terrain features and that herds prefer to stay at an “equilibrium density”, i.e. that animals (and people) prefer to be not too far nor too close to each other. To develop such a model, we use hybrid mixture theory (a combination of upscaling via averaging and exploiting an entropy inequality) incorporating concepts from phase field modeling to obtain a generalized Cahn-Hilliard equation. In this talk we go through the evolution of our model and present some numerical simulations. (Received September 17, 2019)

- 1154-76-2723 **Scott David Kelly*** (scott@kellyfish.net). *Hamiltonian vortex-body interactions, inviscid vortex shedding, and models for aquatic locomotion*.

The dynamic interaction of a free solid body with a collection of point vortices in a planar ideal fluid, or with a collection of closed vortex filaments in a three-dimensional ideal fluid, can be modeled by a system of ordinary differential equations exhibiting a noncanonical Hamiltonian structure reflecting the simultaneous conservation of momentum and circulation. Diverse nonlinear control problems can be obtained by allowing the shape of the body or its interior mass distribution to change over time in response to actuation. Idealized mechanisms for vortex shedding can be introduced to such systems through localized velocity constraints like the Kutta condition from classical aerodynamics, enabling the realization of reduced-order models for the self-propulsion of animals and robots in viscous fluids. This talk will survey the construction of such models and present numerical results representing a variety of problems in aquatic locomotion. (Received September 17, 2019)

78 ► *Optics, electromagnetic theory*

- 1154-78-630 **Jichun Li***, Department of Mathematical Sciences, University of Nevada Las Vegas, Las Vegas, NV 89154-4020. *Finite element analysis and simulation of surface plasmon polaritons in graphene*.

Graphene was invented in 2004. Due to its outstanding electrical, mechanical, magnetic, and thermal properties, graphene has gained significant interest among scientists as evidenced by the award of Nobel Prize in Physics 2010 to two graphene experts. In this talk, I will focus on the development and analysis of a mathematical model for simulating the surface plasmon polaritons. Finite element method for solving this model will be presented. Finally, I will show some numerical simulations. (Received September 09, 2019)

- 1154-78-2495 **Alexey Sukhinin*** (alexey.sukhinin@ndsu.edu). *Propagation of light in multi-frequency moving focus model*. Preliminary report.

The moving focus model describes the propagation of intense optical pulses under the effect of Kerr nonlinearity. In this work, we study the model extension to the multi-frequency regime. Spatial solitons with mixed topological charges were obtained numerically for various frequencies in both resonant and non-resonant regimes. These results could lead to a better understanding of critical power for multi-color optical collapse and potentially novel types of filament propagation. (Received September 17, 2019)

81 ► *Quantum theory*

- 1154-81-112 **Michael G. Dombroski*** (dombroskistm11@verizon.net). *A step toward the T:O:E: with Fermion and Boson matrices; using both Transpose(\) and Cispose(/) operations*. Preliminary report.

Dr. Don Lincoln, a senior physicist at Fermilab, wrote an article in SA Nov2012. In it he referred to “a theory of sublime simplicity”. He was talking about a straight-forward model of “preons” proposed independently in 1979 by Haim Harari, Michael A. Shupe, and Nathan Seiberg. In this paper we extend the work of HSS by using two sets of nine real 3x3 Universal Base Matrices (UBS and ubS). The “average” of the elements of one set is (0), the other (+1/3), both the same as HSS. This is hypothesized to be the relation of mathematics to physics.

The Transpose(n) with the mate Cispose(/), generate new, real, Fermion and Boson Matrices, whose elements are also matrices. E, S, W, and G are defined to represent forces that are acted on by (\) and (/), giving Et, Ec, St, etc. The result is a large variety of matrices, with multiple symmetries and magnitudes. (Received August 09, 2019)

1154-81-245 **Michel L Lapidus*** (lapidus@math.ucr.edu), University of California, Riverside, Department of Mathematics, Riverside, CA 92521-0135. *Open Problems in Mathematical Physics: Feynman Integrals, Complex Fractal Dimensions, Origins of Fractality, Scaling Laws and Quantized Number Theory.*

We will discuss several long-term open problems in contemporary mathematical physics. These problems may involve the following topics: (i) The Feynman path integral, quantization and Feynman's operational calculus. (ii) Complex fractal dimensions, oscillations, Riemann surfaces and scaling laws (iii) Quantized number theory, the Riemann hypothesis and phase transitions. (iv) Complex dimensions, fractal cohomology and homology. (v) The possible origins of fractality in Nature.

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1154-81-432 **Sonia Mazzucchi*** (sonia.mazzucchi@unitn.it), via Sommarive 14, 38123 Trento, TN, Italy. *Open problems in the mathematical definition of Feynman path integrals.*

Since the '60s several efforts have been devoted to the rigorous mathematical definition of Feynman path integrals. Nevertheless nowadays several problems still remain open. In this talk I shall show some examples of simple physical system where the mathematical definition of Feynman path integrals presents ambiguities and amazingly requires the introduction of renormalization counterterms in the classical action functional. (Received September 04, 2019)

1154-81-466 **Tepper L Gill***, 2300 6th St NW, Room 3010 LKD, Washington, DC 20059. *The Feynman Formulation of Quantum Mechanics.*

In this talk I will introduce a new Hilbert space $KS^2[\mathbb{R}^n]$, that is natural for the Feynman formulation of quantum theory. This space makes it possible to preserve all the physically intuitive and computational advantages of Feynman and to represent the Heisenberg and Schrödinger formulations. It is one of a new family $KS^p[\mathbb{R}^n]$, of Banach spaces which contain the HK-integrable functions and contain the L^p spaces as continuous dense and compact embeddings. These spaces also contain the space of test functions as a continuous embedding. In this talk, I will show in what sense that the space $KS^2[\mathbb{R}^n]$ was designed for the Feynman formulation of quantum theory. (Received September 04, 2019)

1154-81-565 **Thomas Creutzig, Naoki Genra*** (genra@ualberta.ca) and **Shigenori Nakatsuka.** *New Duality in W -algebras.*

We prove duality isomorphisms between Heisenberg cosets of subregular \mathcal{W} -algebras of type A and Heisenberg cosets of principal \mathcal{W} -algebras of type super A for generic level. The simplest case is the isomorphism between the parafermion vertex algebra and the Heisenberg coset of the $N = 2$ superconformal vertex superalgebra. We also prove the similar duality for type B and type super C . These results may be interpreted as generalizations of Feigin-Frenkel duality in the context of S-duality by Creutzig-Gaiotto. (Received September 06, 2019)

1154-81-600 **Emil Prodan***, 245 Lexington Av., New York, NY 10016. *On the non-commutative geometry of topological insulators and superconductors.*

I will review the extraordinary conjectures related to topological insulators and superconductors and how more than half of them were solved within the framework of non-commutative geometry. In particular, I will discuss 1) the C^* -algebras of bulk and boundary physical observables and how the bulk-boundary principle is formulated with K and KK theories and 2) how natural pairings and Kasparov products lead to local index theorems that can be pushed over Sobolev spaces. The latter cover the regimes of so-called strong disorders, where the conjectures are formulated. As such, these index theorems enable one to draw extremely fine conclusions about the quantum dynamics of the electrons in realistic laboratory conditions. (Received September 08, 2019)

1154-81-791 **Chris A Marx*** (cmarx@oberlin.edu), Department of Mathematics, Oberlin College, 10 N Professor Street, Oberlin, OH 44074, and **Peter D Hislop**. *A spectral shift type estimate and its applications to the density of states measure for random Schrödinger operators.*

Using the Helffer-Sjöstrand formula, we prove a spectral shift type estimate for a one-parameter family of self-adjoint operators $H_\lambda = H_0 + \lambda T_1$ which characterizes the parameter (λ)-dependence of the function $\lambda \mapsto \text{Tr}(T_2 f(H_\lambda) T_2)$, for compactly supported Lipschitz functions f . Here, T_1 and T_2 are assumed to be positive and trace-class relative to powers of the resolvent of H_λ and H_0 is self-adjoint and lower semi-bounded. We apply this to random Schrödinger operators both on graphs and on \mathbb{R}^d to prove that, in both cases, the density of states measure is Lipschitz continuous in the underlying probability distribution.

This talk is based on joint work with Peter Hislop (University of Kentucky). (Received September 10, 2019)

1154-81-811 **Chris A Marx*** (cmarx@oberlin.edu), Oberlin College, Department of Mathematics, 10 N Professor Street, Oberlin, OH 44074. *Large coupling asymptotics for the Lyapunov exponent of quasi-periodic Schrödinger operators.*

In solid states physics, quasi-periodic Schrödinger operators capture the response of electrons in a 2d-crystal layer subject to an external magic field. Mathematically, this corresponds to studying the spectral properties of so-called quasi-periodic Schrödinger operators, discrete Schrödinger operators whose potential sequence is generated by evaluating an analytic function on the unit circle along a trajectory of irrational rotations.

In this talk we quantify the behavior of solutions to the Schrödinger equation in the “large potential regime,” by proving an asymptotic formula for the Lyapunov exponent of the dynamical system associated with the Schrödinger equation. Heuristically, the Lyapunov exponent can be viewed as an inverse decay rate of the solutions to the Schrödinger equation.

The talk is based on joint work with Rui Han (Georgia Tech). (Received September 10, 2019)

1154-81-863 **Tori Hudgins*** (tori_hudgins@baylor.edu), One Bear Place #97328, Waco, TX 76798, and **Jon Harrison**. *The Characteristic Polynomial of a Quantum Binary Graph.* Preliminary report.

Quantum graphs provide a simple model of quantum mechanics in systems with complex geometry and can be used to study quantum chaos. A quantum graph has an associated unitary quantum evolution operator. We study the coefficients of the characteristic polynomial of the quantum evolution operator for families of binary graphs and their generalizations. The Bohigas-Giannoni-Schmit conjecture suggests spectral statistics of generic quantum graphs are modeled by those of random matrices, in the limit of large graphs. However, we show that, for families of binary graphs, there is a uniform deviation from random matrix behavior in the statistics of coefficients of the characteristic polynomial. (Received September 13, 2019)

1154-81-903 **Chenliang Huang*** (ch30@iupui.edu), 402 N Blackford St., LD 270, Indianapolis, IN 46202, and **Evgeny Mukhin, Benoit Vicedo and Charles Young**. *The solutions of $\mathfrak{gl}(m|n)$ Bethe ansatz equation and rational pseudodifferential operators.*

We consider the $\mathfrak{gl}(m|n)$ Gaudin Bethe ansatz equation associated to a weight in a tensor product of polynomial modules. To a solution of $\mathfrak{gl}(m|n)$ Gaudin Bethe ansatz equation, we associate a rational pseudodifferential operator. The rational pseudodifferential operator is invariant under the reproduction procedure. We expect that the coefficients of the expansion of the rational pseudodifferential operator are eigenvalues of the higher Gaudin Hamiltonians acting on the corresponding Bethe vector. (Received September 17, 2019)

1154-81-1261 **Daniel S Freed*** (dafre@math.utexas.edu), The University of Texas at Austin, Mathematics Dept. RLM 8.100, 2515 Speedway Stop C1200, Austin, TX 78712. *Gapped boundary theories in three dimensions.* Preliminary report.

In various situations one often says that a particular physical material is forced to have gapless, or propagating, modes on its boundary, and this is an important feature for applications. In joint work with Constantin Teleman we extract a mathematical theorem for certain 2+1 dimensional (interacting) systems which tells conditions under which gapless boundary modes are forced. The setting for our work is topological field theory, and we lean heavily on the algebra of tensor categories. In this talk I will explain the background and a bit about the techniques used in the proof. (Received September 14, 2019)

1154-81-1287 **David Carfi***, Via Canova 32, 98121 Messina, Messina, Italy, and **Alessia Donato**. *Schwartz diagonalizable operators, relativistic Quantum Mechanics and Dirac equation in distribution spaces.*

In our presentation, we begin the journey needed to construct a reasonable foundation of Relativistic Quantum Mechanics in Laurent Schwartz distribution spaces. Schwartz Linear Algebra approach, on one hand, allows to

adopt rigorously and efficiently the concept of continuous eigenbasis of observables (for the entire state space of a Quantum system), going towards a “covariant” formulation of Quantum Mechanics with respect to the change of Schwartz bases, on the other hand, we can propose a pretty manageable definition of “principal square root” of strictly positive and strictly negative Schwartz diagonalizable operators, shading new light upon the classic relativistic Quantum equations: Dirac equation, Klein-Gordon equation, Weil equation and Proca equation. (Received September 14, 2019)

1154-81-1376 **Robert Sims*** (rsims@math.arizona.edu), **Houssam Abdul-Rahman** and **Gunter Stolz**. *On the Regime of Localized Excitations for Disordered Oscillator Systems.*

We study quantum oscillator lattice systems with disorder, in arbitrary dimension, requiring only partial localization of the associated effective one-particle Hamiltonian. This leads to a many-body localized regime of excited states with arbitrarily large energy density. We prove zero-velocity Lieb-Robinson bounds for the dynamics of Weyl operators as well as for position and momentum operators restricted to this regime. Dynamical localization is also shown in the form of quasi-locality of the time evolution of local Weyl operators and through exponential clustering of the dynamic correlations of states with localized excitations. (Received September 15, 2019)

1154-81-1412 **Jacob Shapiro*** (shapiro@math.columbia.edu), New York, NY 10027, and **Michael I Weinstein**, New York, NY 10027. *Topological Equivalence of Continuum Models with Their Discrete Tight-Binding Limits in the IQHE.*

We study the tight-binding regime of a non-interacting electron in a two-dimensional crystal subject to a perpendicular constant magnetic field, and prove that the Fermi projection of the scaled continuum Hamiltonian converges in norm to that of a discrete tight-binding model as long as the Fermi energy lies within a spectral gap. A corollary of this is that the topological invariants of the respective systems are equal. The edge system is also studied and an analogous equivalence is proven between continuum and tight-binding reduction as well. (Received September 15, 2019)

1154-81-1504 **Alvin Moon*** (asmoon@math.ucdavis.edu). *An energy-dependent Lieb-Robinson bound for quantum rotors.* Preliminary report.

Lieb-Robinson (LR) bounds represent a finite propagation velocity for information in quantum lattice systems. The spectral flow automorphism, defined in terms of the quasi-local Hastings generator, is an invaluable tool in the analysis of spectral properties of lattice models since it robustly relates the presence of a spectral gap to “quasi-adiabatic” evolution of spectral projectors. In this talk, we present a LR bound for the spectral flow automorphism from the quantum rotor model which depends on both spatial and energy distance of observables. (Received September 16, 2019)

1154-81-1553 **Terry Gannon** and **Corey Jones*** (jones.6457@osu.edu). *Vanishing categorical obstructions for permutation orbifolds.*

The orbifold construction $A \mapsto A^G$ is a fundamental method of producing new rational conformal field theories (vertex algebras, conformal nets) from old ones with symmetries. The construction at the level of modular tensor categories $Rep(A) \mapsto Rep(A^G)$ is called gauging. While this construction is always possible for modular tensor categories and symmetries arising from RCFT, it is not always possible for an abstract modular tensor category with symmetry. There is a cohomological obstruction for the existence of a suitable G -crossed extension (which in the abstract setting plays the role of twisted modules).

A major question is whether all modular tensor categories arise from RCFT. It was asked by Muger whether these obstructions always vanish when G is a group acting on $\mathcal{C}^{\otimes n}$ by permuting the tensor factors. If there was an example where they failed to vanish \mathcal{C} could not arise from a RCFT since permutation orbifolds always exist. However, we will sketch a proof that these obstructions always vanish, hence permutation extensions also exist in the abstract setting. An algorithm for computing the fusion rules of these extensions will be presented in a talk by Colleen Delaney. Based on joint work with Terry Gannon. (Received September 16, 2019)

1154-81-1823 **Erica K Swindle*** (eswindle@wlu.edu), **Jon Harrison** and **Mark Sepanski**. *Quantum Cayley Graphs of Finite Groups.*

Quantum graphs are often used to model quantum mechanics in systems with complex geometry. We are interested in investigating the spectra of quantum Cayley graphs of finitely generated groups which incorporated an irreducible representation of the symmetry group in the vertex matching conditions. A secular equation, which encodes the spectrum, is derived for an arbitrary irreducible representation of a general finite group. (Received September 16, 2019)

1154-81-1833 **Alexander Elgart*** (aelgart@vt.edu) and **Abel Klein**. *Localization at the bottom of the spectrum of a disordered XXZ spin chain*. Preliminary report.

Quantum spin chains provide some of the mathematically most accessible examples of quantum many-body systems. However, even these toy models pose considerable analytical and numerical challenges, due to the fact that the number of degrees of freedom involved grows exponentially fast with the system's size. We will discuss the recent progress in establishing many body localization at the bottom of the spectrum of a disordered XXZ chain. (Received September 16, 2019)

1154-81-1838 **Antonino Travia*** (atravia@mail.usf.edu) and **Razvan Teodorescu** (razvan@usf.edu). *A Yang-Mills Approach for Conformal Invariance in Turbulence*.

The relationship between conformal symmetry and stochastic processes has been a rapidly growing field over the last twenty years. In particular, such symmetry of Schramm-Loewner Evolution (SLE) curves in the plane has found many successes with a particular application being numerical evidence for conformal invariance's presence in the turbulence of a two-dimensional incompressible fluid. We present a theoretical approach to realize this correlation using techniques from Yang-Mills theory with a non-Abelian gauge group in order to explain interactions near zero-vorticity lines as well as its correspondence with the probabilistic approach to boundary conformal field theory. (Received September 16, 2019)

1154-81-1839 **Michael Hermele***, Department of Physics, University of Colorado Boulder, Boulder, CO 80309-0390. *Gapped phases of quantum matter and topological quantum field theories*.

In condensed matter physics, one often considers physical systems of quantum mechanical degrees of freedom on a spatial lattice. Such systems are considered to belong to equivalence classes referred to as phases of matter, and the simplest such phases are those possessing a gap in the energy spectrum. It has often been assumed that the low-energy properties of any such gapped phase of quantum matter can be approximated by a suitable topological quantum field theory (TQFT). Indeed, there are many examples of gapped phases where such a TQFT description is known. However, there also exist gapped phases, known as fracton phases, that are incompatible with a low-energy TQFT description. This talk will introduce the notion of gapped phases of quantum matter and discuss what is known about how and to what extent gapped phases are related to TQFTs. (Received September 16, 2019)

1154-81-1889 **Ralph M Kaufmann**, **Sergei Khlebnikov** and **Erika Birgit Wehefritz-Kaufmann*** (ebkaufma@purdue.edu), 150 N University St, West Lafayette IN, IN 47907. *Quantum materials and symmetries*.

We describe condensed matter systems in terms of non-commutative geometry. Their band structure can be analyzed in the classical limit in terms of quantum graph symmetries. This leads to interesting properties of the materials, such as those of exhibiting Dirac points and higher order level crossings, which can lead to Weyl semi-metals. (Received September 16, 2019)

1154-81-1891 **Ralph M Kaufmann*** (rkaufman@purdue.edu), 150 N University St, West Lafayette IN, IN 47907, and **Dan Li** and **Erika Birgit Wehefritz-Kaufmann**. *Topological invariants and insulators*.

We discuss the origin of topological invariants and their interpretation in terms of K-theory. This includes integer as well as $Z/2Z$ valued invariants. The former are known to transfer to the non-commutative side and we give a possible answer for the non-commutative analogue in the $Z/2Z$ case. (Received September 16, 2019)

1154-81-2067 **Michael A Maroun*** (marounm@gmail.com). *A Generalized Nonlinear Schrödinger Equation*.

From the perspective of physics using only first principles, one arrives at a generalization of the nonlinear Schrödinger equation, which is notably different from the equation in the mathematical literature known as the nonlinear Schrödinger equation. The equation arrived at is also not the Gross-Pitaevskii equation. The two equations only coinciding if the Riesz transform were convolution with the identity, which it is not. Some possible distributional and L^p solutions are investigated. The full facts about L^p solutions is believed to be open. Of particular interest are rotationally invariant solutions. The physical significance of the equation is that it in principle captures the self-energy of a bound electron, and may also provide a context for the Uehling potential. (Received September 17, 2019)

1154-81-2245 **Lyda Pamela Urresta*** (lurresta@nd.edu). *Overview of the Geometric Model of Propagating Strings in String Theory and Possible Generalizations.*

String theory is a physics theory that models particles as one-dimensional strings propagating through space-time. In this picture, the strings sweep out a surface, so we can use tools in topology and complex analysis to study these strings and their interactions. This yields an algebraic structure on the space of particle states, called a vertex operator algebra (VOA). In the 1990s, Yi-Zhi Huang constructed a geometric model designed to provide a rigorous geometric underpinning of the notion of a VOA. Building upon Huang's work, we present some possible extensions and generalizations of his geometric model and the algebraic structures that arise as a consequence. (Received September 17, 2019)

1154-81-2251 **Evgeny Mukhin***, 202 N. Blackford St, LD270, Department of Mathematics, IUPUI, Indianapolis, IN 46278. *Deformed W-algebras.*

We present a way to construct deformations of W-algebras associated to all classical series using the representation theory of quantum toroidal algebra of type $\mathfrak{gl}(1)$. This uniform construction recovers all known examples and produces some new ones. Time permitting we will discuss the "local" integrals of motion for all such deformed W-algebras. This is a joint work with B. Feigin and M. Jimbo. (Received September 17, 2019)

1154-81-2277 **Lyda Pamela Urresta*** (lurresta@nd.edu). *On Generalizations of Geometric Vertex Operator Algebras and Related Structures.*

We study the underlying geometries of genus-zero two-dimensional conformal field theory and their corresponding algebraic structures. In particular, we consider generalizations of aspects of Yi-Zhi Huang's definition of a geometric vertex operator algebra that arise from relaxing the grading and meromorphicity restrictions in this notion. We further examine how such choices are tied to the underlying worldsheet geometry. (Received September 17, 2019)

1154-81-2412 **Rachel Lash Maitra*** (maitrar@wit.edu), 550 Huntington Ave, Boston, MA 02015, and **John E Haga** (hagaj@wit.edu), 550 Huntington Ave, Boston, MA 02015. *Time-Sliced Path-Integral Propagator for (1+1) Quantum Gravity with Factor Ordering Ambiguity.* Preliminary report.

We obtain a path integral representation of the time evolution operator in a model of (1+1) quantum gravity, that incorporates factor ordering ambiguity, as an extension of previous results to factor orderings which generate more sharply singular behavior near the Big Bang / Big Crunch singularity. In obtaining a suitable integral kernel for the time-evolution operator, one requires that the corresponding Hamiltonian is self-adjoint; the plurality of self-adjoint extensions for a particular category of orderings leads to a variety of behaviors for the quantized system near the singularity. An appropriate reparametrization allows the use of Hankel transforms to derive an explicit time-sliced formula for the propagator. (Received September 17, 2019)

1154-81-2415 **John E Haga*** (hagaj@wit.edu), 550 Huntington Ave, Department of Applied Mathematics, Wentworth Institute of Technology, Boston, MA 02115, and **Rachel Lash Maitra** (maitrar@wit.edu), 550 Huntington Ave, Department of Applied Mathematics, Wentworth Institute of Technology, Boston, MA 02115. *Factoring ordering ambiguity in (1+1) gravity manifests as the index of a Bessel process.* Preliminary report.

In developing a rigorous path integral representation of the time evolution operator for a model of (1+1) quantum gravity that incorporates factor ordering ambiguity we show that the kinetic part of the Hamiltonian generates a Bessel process with index depending on the factor ordering. We observe that the Laplace-Beltrami ordering arises when the corresponding Bessel path integrals appear as the radial path integrals of 3-dimensional Wiener path integral. (Received September 17, 2019)

1154-81-2497 **Jamal Benbourenane*** (jamal.benbourenane@adu.ac.ae), Abu Dhabi University, College of Art and Science, Abu Dhabi, United Arab Emirates. *PT-Symmetric Systems with Real Valued Complex Potential and their Exactly Solvable wave functions and Positive Energies.*

We propose a new technique based on double scaling and using supersymmetry quantum mechanics (susyqm) in order to study PT-Symmetric systems described by one dimensional Schrodinger equation with some specific complex potential. PT-symmetric systems are known to be used as models for quantum open systems where the gain and loss are balanced. Even though the potential is complex in such systems, the eigenenergies obtained are real. We determine more specifically the exact closed forms of the wave functions and the energies of the bounded states for the complex valued potential with its superpotential given as a linear combination of hyperbolic functions. (Received September 17, 2019)

1154-81-2711 **Luan Bezerra*** (luanpere@iupui.edu) and **Evgeny Mukhin**. *Quantum toroidal superalgebras associated with $\mathfrak{gl}_{m|n}$* .

We introduce the quantum toroidal superalgebras $\mathcal{E}_{m|n}(q_1, q_2, q_3)$, which are affinizations of the quantum affine algebras $U_q \widehat{\mathfrak{gl}}_{m|n}$. We give an evaluation map from $\mathcal{E}_{m|n}(q_1, q_2, q_3)$ to the quantum affine algebra $U_q \widehat{\mathfrak{gl}}_{m|n}$ with $q^2 = q_2$ at level $c = q_3^{(m-n)/2}$, and a bosonic realization of level one $\mathcal{E}_{m|n}(q_1, q_2, q_3)$ -modules. (Received September 17, 2019)

1154-81-2822 **Filipp Uvarov*** (filuvaro@iu.edu), @iu.edu. *Duality for Bethe algebras acting on polynomials in anticommutative variables*. Preliminary report.

We consider the actions of the current Lie algebras $\mathfrak{gl}(n)[t]$ and $\mathfrak{gl}(k)[t]$ on the space of polynomials in kn anticommuting variables. We show that the images of the Bethe algebras under these actions coincide. To prove the statement we use the Bethe ansatz description of eigenvalues of the actions of the Bethe algebras via spaces of quasi-exponentials and establish explicit correspondence between these spaces. (Received September 18, 2019)

82 ► *Statistical mechanics, structure of matter*

1154-82-267 **Pavel Bleher*** (pbleher@iupui.edu), Department of Mathematical Sciences, IUPUI, Indianapolis, IN 46202. *Dimer Model: Full Asymptotic Expansion of the Partition Function*.

We give a complete rigorous proof of the full asymptotic expansion of the partition function of the dimer model on a square lattice on a torus for general weights z_h, z_v of the dimer model and arbitrary dimensions of the lattice m, n . We assume m is even and we show that the asymptotic expansion depends on the parity of n . We review and extend the results of Ivashkevich, Izmailian, and Hu [E. V. Ivashkevich, N. Sh. Izmailian, and Chin-Kun Hu, J. Phys. A: Math. Gen., 35, 5543 (2002)] on the full asymptotic expansion of the partition function of the dimer model. The coefficients of the asymptotic expansion are given in terms of the Jacobi theta functions, Dedekind eta function, and the Kronecker double series. We give a rigorous estimate of the error term in the asymptotic expansion of the partition function.

This is a joint work with Brad Elwood, and Dražen Petrović. (Received August 27, 2019)

1154-82-577 **Matthew Cha, Pieter Naaijken, Bruno Nachtergaele*** (bxn@math.ucdavis.edu) and **Nicholas Sherman**. *A dynamical Toric Code model and stability of the superselection sectors of two-dimensional quantum lattice models*.

Kitaev's quantum double models provide a rich class of examples of two-dimensional lattice systems with topological order in the ground states and a spectrum described by anyonic elementary excitations. The infinite volume ground states of the abelian quantum double models come in a number of equivalence classes called superselection sectors. We prove that the superselection structure of these and similar models remains unchanged under uniformly small perturbations of the Hamiltonians. We introduce a Dynamical Toric Code Model and discuss some of its features. (Received September 07, 2019)

1154-82-590 **Anton Kapustin***, California Institute of Technology, 1200 E California Blvd, Pasadena, CA 91125. *Coarse geometry and family invariants of gapped phases of quantum matter*.

Topological methods have had a dramatic effect on our understanding of gapped phases of quantum matter. In particular, gapped phases of free fermions have been classified using K-theory. In the case of systems without translational invariance, the relevant K-theory has been defined by J. Roe in the context of coarse geometry. After reviewing the connection between Roe's K-theory and free fermions, I discuss possible extensions to interacting gapped systems. I outline a construction of invariants of families of gapped systems which generalize the cohomology class of the Berry curvature. (Received September 07, 2019)

1154-82-1327 **Kang Lu*** (lukang@iupui.edu), LD 270 402 N Blackford St., Indianapolis, IN 46202, and **Evgeny Mukhin**. *On the $\mathfrak{gl}_{1|1}$ supersymmetric XXX spin chains*. Preliminary report.

We study the $\mathfrak{gl}_{1|1}$ supersymmetric XXX spin chains. We show that there exists a bijective correspondence between common eigenvectors (up to proportionality) of Bethe subalgebra in any cyclic tensor product of polynomial evaluation modules of the $\mathfrak{gl}_{1|1}$ Yangian and monic divisors of an explicit polynomial written in terms of the Drinfeld polynomials. In particular our result implies that each common eigenspace has dimension 1. We also show that when the tensor product is irreducible, then all eigenvectors can be constructed using Bethe ansatz and express the transfer matrices associated to symmetrizers and anti-symmetrizers in terms of the first transfer matrix and the center of the Yangian. (Received September 14, 2019)

1154-82-1414 **Houssam Abdul-Rahman, Marius Lemm, Angelo Lucia, Bruno Nachtergaele** and **Amanda Young*** (young@ma.tum.de). *A gapped family of two-dimensional AKLT models.*

The one-dimensional AKLT spin chain is the prototypical example of a frustration-free quantum spin system with a spectral gap above its ground state. Affleck, Kennedy, Lieb, and Tasaki conjectured that the two-dimensional version of their model on the hexagonal lattice also exhibits a spectral gap. We introduce a family of variants of the hexagonal AKLT model, defined by decorating each edge of the lattice with an AKLT chain of length n , and prove that these decorated models are gapped for all $n \geq 3$. (Received September 15, 2019)

1154-82-2111 **Charles W. Monroe** and **M. Zyskin*** (maxim.zyskin@eng.ox.ac.uk). *Continuum and microscopic models of electrochemical transport.*

Recently a thermodynamics-based continuum model of multi-species electrochemical transport has been proposed by Goyal and Monroe. For a low-density gas, we will derive those continuum balance laws and a dissipation function expressing the rate of entropy production from a microscopic statistical theory based in the Boltzmann gas equation. The approach allows direct computation of parameters for the macroscopic continuum model in the case where particle/particle interactions are described by a microscopic pair potential of the Lennard–Jones type. In addition to this illustrative example, we will discuss the linkage between microscopic and macroscopic models in other cases where present knowledge is incomplete. (Received September 17, 2019)

1154-82-2565 **Michael Hermele** and **Marvin Qi*** (marvin.qi@colorado.edu). *Higher form symmetry, topological order, and fractons.*

Global symmetries and their spontaneous breaking provide a powerful framework for understanding and classifying phases of matter. It has been realized in recent years that certain classes of topological phases fall under this framework for a generalization of global symmetry known as higher form symmetry. Motivated by this, we study the role that higher form symmetry plays in the context of fracton phases of matter, which share some qualitative features with topological phases but cannot be described by traditional TQFT. We find the symmetry of the paradigmatic fracton model to be a quotient of the one-form symmetry group. The breaking of this symmetry characterizes the fracton phase of the model. (Received September 17, 2019)

1154-82-2566 **Benjamin Brodie*** (benjamin.brodie@uky.edu) and **Peter Hislop**. *Density of States and Eigenvalue Statistics for Fixed Width Random Band Matrix.* Preliminary report.

We prove that the local eigenvalue statistics for $d = 1$ random band matrices with fixed bandwidth and Gaussian entries is given by a Poisson point process. The proof relies on localization bounds of Schenker and the Wegner and Minami estimates in Peled, Schenker, Shamis, and Sodin. The new component is a proof of the pointwise convergence of the density of states. The method of proof simplifies and extends some ideas used by Dolai, Krishna, and Mallick, who proved regularity results for the density of states for random Schrodinger operators. (Received September 17, 2019)

83 ► *Relativity and gravitational theory*

1154-83-1305 **Arthur E Fischer*** (aef@ucsc.edu), Department of Mathematics, University of California, Santa Cruz, CA 95064. *Friedmann's Equation and the Creation of the Universe.*

We model the universe by the spatially flat Λ CDM (Lambda Cold Dark Matter) dimensionless scale factor

$$a_{\Lambda\text{CDM}}(t) = \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}} \right)^{1/3} \left(\sinh \left(\frac{3}{2} \sqrt{\Omega_{\Lambda,0}} H_0 t \right) \right)^{2/3}$$

which we *time-globalize* to all $t \in (\infty, \infty)$. This scale factor is C^∞ and solves *Friedmann's equation* for all $t \neq 0$ and is continuous with a cusp singularity at the big bang at $t = 0$. The resulting model is an *all-time time-symmetrical zero-energy single-bounce model of the universe*, which shows that encoded in Friedmann's equation is the prediction that the universe (1) existed *before* the big bang during the negative time epoch $(-\infty, 0)$; (2) asymptotically approaches de Sitter vacuum spacetime dS_4 as $t \rightarrow \pm\infty$; and (3) was created asymptotically from nothing at $t = -\infty$ and dies asymptotically to nothing at $t = +\infty$, with the time-globalized Λ CDM model interpolating between the initial and final asymptotic infinitely diluted vacuum states. Our results show that much can be said *classically* about the birth, big bang, and death of the universe *before* one needs to reach for *quantum* gravitational effects. (Received September 17, 2019)

1154-83-1313 **Deborah Ann Konkowski*** (dak@usna.edu). *Open problems in understanding singularities in general relativistic spacetimes.*

I will discuss the status of our understanding of singularities in general relativistic spacetimes. I will cover their definition, location, existence and nature, and I will emphasize what we know, and what we do not know. (Received September 14, 2019)

85 ► *Astronomy and astrophysics*

1154-85-766 **Wendy K. Caldwell***, wkcaldwell@lanl.gov, and **Abigail Hunter, Catherine S. Plesko and Stephen Wirkus.** *Understanding Asteroid 16 Psyche's Composition through 3D Hydrocode Impact Crater Models.*

Asteroid 16 Psyche is the largest M-type (metallic) Main Belt Asteroid, and an upcoming NASA mission will be the first of its kind to visit a metallic body rather than one composed of rock or ice. Psyche is likely the remnant of a differentiated planet core. However, because of its distant location and the limits of available measuring techniques, basic information about Psyche is under debate, including density and diameter. In this work, we study the composition of Psyche by modeling its largest impact craters. To our knowledge, this work is the first of its kind to use impact crater simulations to determine likely material compositions as well as the first 3D models of Psyche's craters. We study various possible material compositions and porosity levels in 2D and 3D, and our 3D simulations test oblique impact angles. From these simulations, we predict that Psyche is indeed likely mostly metallic with a porosity of about 50%. These predictions are consistent with the idea that M-type asteroids are differentiated planet cores. We will show additional simulation images and animations from 3D simulations with final crater dimensions within the expected uncertainties. (Received September 10, 2019)

1154-85-814 **Demond C. Adams*** (demond_adams@yahoo.com). *Hyper-Dimensional Space-Time Topology and the Riemann Zeta Function: Applications in a Dimensional Field Theory.*

Mathematics describes the logic of numbers while physics explicitly explains the laws of nature, yet somehow these separate disciplines appear eternally and intrinsically related. We can not describe nature without referencing physics and we cannot describe physics without introducing mathematics. Are these distinct constructs derived from a similar study "the logic of nature", or is this a coincidental aspect of nature's mystery? Mysteries in Physics and Mathematics - Run away galaxies caused by Dark Energy, clusters of celestial objects held together by Dark Matter, and the force of gravity persists as an unsolved mystery riddling the best minds mankind has ever known. Furthermore, unanswered truths regarding the fundamental mysteries of the Riemann Zeta Function remain elusive. Are these unsolved mysteries found in physics and mathematics related? In this talk, I will discuss the motivation of applying mathematics to physics and argue its cohesive interplay in guiding our investigations of nature. I will use the Riemann Zeta Function as an independent distant ladder to measure the topology of space-time and determine the red-shift divergence of galaxies described as Dark Energy. (Received September 10, 2019)

1154-85-2156 **Reena Tandon*** (reena.tandon2008@gmail.com), Lovely Professional University, Pggagwar144411, Jalandhar, 144411, India. *Bulk Viscous Cosmological model in General Relatively.* Preliminary report.

In this paper, we analyze a cosmological situation proposing a variation law in which the deceleration parameter is assumed to be a simple linear function of Hubble's parameter, which yields scale factor a ad $a = e^{\frac{1}{\beta} \sqrt{2\beta t + k}}$ (where β, k are constants). We have gotten the cosmological models in which the Universe begins from a non-solitary state and grows exponentially with infinite time t till late occasions. The deceleration parameter in the model is found to be time dependent. It is seen that this parameter shows a transition from initial decelerating phase to the present accelerating phase of expansion and supplies the largest value and the fastest rate at which the universe is expanding. Same is also observed by the researchers. The cosmological term Λ approaches to zero as t tends to infinite also shown by recent observations. (Received September 17, 2019)

86 ► Geophysics

1154-86-1323 **Andrei Bourchtein*** (andburstein@gmail.com) and **Ludmila Bourchtein** (ludmila.bourchtein@gmail.com). *On ellipticity of balance equations for atmospheric dynamics.*

The governing equations of atmospheric dynamics represent a system of time-dependent nonlinear partial differential equations (PDEs), which support both relatively slow dominant processes and fast gravity and acoustic waves of small amplitude. Solution of such a stiff system requires definition of initial conditions, which do not give rise to fast oscillations of large amplitude not observed in the real atmosphere.

To determine the required initial conditions, the balance relations in the form of nonlinear PDEs are imposed on the initial data for the atmospheric models. It is well known that some of these PDEs are non-elliptic which makes impossible formulation of well-posed boundary value problem. For example, the classic nonlinear balance equation is of the Monge-Ampere type and as such is non-elliptic for a given pressure function in the regions with anticyclonic activity.

In this study, we derive ellipticity conditions for more complex differential systems of nonlinear adjustment and present a hierarchy of such conditions with respect to the complexity of the adjustment equations. Based on these results, we analyze a distribution of non-elliptic regions in the actual atmospheric fields for different forms of the balance equations. (Received September 14, 2019)

1154-86-2267 **Amanda Ziemann*** (ziemann@lanl.gov), **Christopher X. Ren** and **James Theiler**. *Detecting anomalous changes in remote sensing imagery from different sensors.*

Combining multiple satellite remote sensing sources can provide a far richer, more frequent view of the earth than that of any single source; the challenge is in distilling this large volume of heterogeneous sensor imagery into meaningful characterizations of the imaged areas. The traditional approach to change detection involves difference-based techniques, but these do not naturally extend to image pairs captured by sensors with different designs and phenomenologies. To leverage imagery in this multi-sensor context, algorithms are being developed to effectively combine different kinds of sensor imagery that can identify subtle but important changes among the intrinsic data variation, e.g., multispectral to synthetic aperture radar. Here, we implement a joint-distribution framework for anomalous change detection that can effectively “normalize” for these changes in modality. Results are shown using satellite imagery from different sensor platforms over time. (Received September 17, 2019)

1154-86-2290 **Colleen Collins*** (colleencollins89@gmail.com), **Sarah Glidden**, **Katherine Johnston**, **Grace Rogers**, **Hasan Hamdan** and **Celes Woodruff**. *Automated Identification of Landslides*. Preliminary report.

Landslides occur worldwide and pose threats to property, transportation, and people. Identification of known landslide sites can aid in the creation of hazard maps to inform land-use policies. Digital elevation models (DEMs) created from lidar data make it possible to identify geologic or geomorphic features without the need to visit sites directly. Geologists can use these DEMS to visually identify landslides, but this is a very time-consuming process.

Our project lays the foundation for the creation and implementation of an automated landslide identification tool. We use elevation data collected from three regions in Virginia to create logistic models that identify the presence of a debris flow landslide. In this presentation, we will cover the process of collecting and cleaning the data, identifying variables of interest, and creating and testing logistic models. We will then discuss the limitations of our work and the next steps for this project. (Received September 17, 2019)

1154-86-2401 **Darrell Jarrett Valenti*** (dvalenti984@r.wu.edu), 139 Forest Street, New Canaan, CT 06840, and **Christopher Horvat**. *Tracking the Evolution of a Large Ice Disk in Westbrook, Maine, USA*. Preliminary report.

In winter 2018, a large circular ice disk formed in the Presumpscot river in Westbrook, ME. The disk had a diameter of approximately 102 meters and an area of 8283 m². The disk persisted in the river for at least 7 days before being frozen in. Ice disks such as this one are comparable to much larger sea ice floes that exist in polar regions. Developing methods to study and track such floes is difficult because of the challenge of obtaining repeated measurements of floes in such remote areas. Using three days of continuous time-series observations of the Westbrook disk and surrounding climate, we developed an algorithm that tracks the perimeter, radius, and motion of the disk and analyzes these descriptors with respect to differing climate conditions such as temperature, wind speed, and precipitation. We use this information, along with meteorological data from a nearby weather station, to constrain what causes changes in the disk over time. (Received September 17, 2019)

90 ► Operations research, mathematical programming

1154-90-133 **Jon Lee*** (jonxlee@umich.edu). *Aspects of Symmetry for Sparse Reflexive Generalized Inverses.*

A generalized inverse of a real matrix A is a matrix H that satisfies the Moore-Penrose (M-P) property $AHA=A$. If H also satisfies the M-P property, $HAH=H$, it is reflexive. When A is symmetric, we may desire a symmetric H ; while generally such a restriction on H may not lead to a 1-norm minimizing reflexive generalized inverse. Letting the rank of A be r , and seeking a 1-norm minimizing symmetric reflexive generalized inverse H , we give: a closed form when $r=1$ and when $r=2$ and A is non-negative; an approximation algorithm for general r . Additionally, our symmetric reflexive generalized inverse is structured and has guaranteed sparsity. H is ah-symmetric if AH is symmetric, and ha-symmetric if HA is symmetric. Seeking a 1-norm minimizing ah-symmetric (or ha-symmetric) reflexive generalized inverse H , we give: a closed form when $r=1$ and when $r=2$ and A satisfies a technical condition; an approximation algorithm for general r . Additionally, our ah-symmetric (ha-symmetric) reflexive generalized inverse is structured and has better guaranteed sparsity than obtained via linear programming. This is joint work with Marcia Fampa and Luze Xu. (Received August 13, 2019)

1154-90-471 **Andrea Boskovic*** (aboskovic21@amherst.edu), **Qinyi Chen** (qinyichen@ucla.edu), **Dominik Kufel** (dominic.kufel@gmail.com) and **Zijie Zhou** (zhou759@gmail.com). *Online Approaches for Allocating Products to Users.* Preliminary report.

In order for an e-commerce platform to maximize its revenue, it must recommend customers items they are most likely to buy. However, the company has business constraints on these items, such as the number of each item in stock. In this problem, our goal is to recommend items to users as they arrive on a webpage in an online manner, in a way that maximizes reward for a company, in terms of clicks on an item or purchases, but also satisfies budget constraints. To have a performance comparison for our online customer arrival algorithms, we first approach the problem in an offline manner, where the sequence of customer arrival is known in advance. Then, based on the offline problem, we tackle the problem as an online optimization problem, modeling the customer arrival process as a stationary Poisson process. After creating this algorithm, we make the model more complicated but more realistic, treating the arrival process as a non-stationary Poisson process. Finally, we consider the amount of time the user spends looking at a certain item: we maximize the average dwell time, providing a collection of recommended items to each customer. (Received September 04, 2019)

1154-90-492 **Nicholas Dwork*** (nicholas.dwork@ucsf.edu), nicholas.dwork@ucsf.edu, CA, **Daniel O'Connor**, CA, and **Corey A. Baron** and **Ethan M. I. Johnson**, IL, **John M. Pauly**, CA, and **Peder Larson**, CA. *utilizing the sparsity structure generated by the wavelet transform with compressed sensing.* Preliminary report.

Compressed sensing (CS) relies on the property that a linear transformation of the result is sparse. Fourier Sampling is often imposed by the problem (e.g. MRI), and the Discrete Daubechies Wavelet Transform (DDWT) is a common sparsifying transformation used in practice. Natural images are sparse after a wavelet transform; however, the vector elements corresponding to low frequency components of the data are not. This observation led to multi-level sampling schemes. Previously, the size and shape of the different levels have been determined heuristically.

In this work, we show that the DDWT imposes a specific structure on the sparsity pattern. The square region corresponding to low frequencies is not sparse, and the bins corresponding to high frequencies in any direction are sparse. When the DDWT is applied recursively, the pattern continues, but the average cardinality of the vector increases with iteration.

Thus, for improved accuracy, the CS problem should be altered in the following ways. 1) The structure of the multi-level sampling scheme should be squares with sizes corresponding to the different resolutions of the sub-images of the DDWT. 2) The objective function should be modified to impose an L1 norm on the sparse regions and an L2 norm on the low-frequency region. (Received September 16, 2019)

1154-90-554 **Jon Lee**, **Daphne Skipper** and **Emily Speakman***, University of Colorado Denver, Department of Math. and Stat. Sciences, 1201 Larimer Street, Denver, CO 80204, and **Luze Xu**. *A perspective on piecewise linear convex under-estimators.*

We study a well-known MINLO (mixed-integer nonlinear optimization) formulation of the disjunction $x \in \{0\} \cup [l, u]$, where z is a binary indicator of $x \in [l, u]$, and y “captures” x^p , for $p > 1$. This model is useful when activities have operating ranges, we pay a fixed cost for carrying out each activity, and costs on the levels of activities are strictly convex. The ‘perspective reformulation’ is a well-known method for tightening the convex relaxation of the obvious MINLO formulation. Using volume as a measure to compare convex bodies,

we investigate optimal placement of a fixed number of linearization points for building a best piecewise-linear convex under-estimator (in the context of the perspective reformulation). (Received September 06, 2019)

1154-90-564 **Samuel C Gutekunst*** (scg94@cornell.edu) and **David P Williamson**. *Semidefinite Programming Relaxations of the Traveling Salesman Problem and Their Integrality Gaps*.

The traveling salesman problem (TSP) is a fundamental problem in combinatorial optimization. Several semidefinite programming relaxations have been proposed recently that exploit a variety of mathematical structures including, e.g., algebraic connectivity, permutation matrices, and association schemes. The main results of this paper are twofold. First, de Klerk and Sotirov present an SDP based on permutation matrices and symmetry reduction; they show that it is incomparable to the subtour elimination linear program, but generally dominates it on small instances. We provide a family of simplicial TSP instances that shows that the integrality gap of this SDP is unbounded. Second, we show that these simplicial TSP instances imply the unbounded integrality gap of every SDP relaxation of the TSP mentioned in a survey on SDP relaxations of the TSP of Sotirov. In contrast, the subtour LP performs perfectly on simplicial instances. The simplicial instances thus form a natural litmus test for future SDP relaxations of the TSP. (Received September 06, 2019)

1154-90-582 **Jillian Cannons** and **Jeremy J. Lin***, jlinardi@cpp.edu, and **Thuy Lu**. *Particle Swarm Optimization-Based Source Seeking with Obstacle Avoidance*.

In this work, we consider finding an optimal solution to a source-seeking problem where an electromagnetic source is to be located by a group of robots. We do not assume knowledge of a theoretical model for the source signal and study the case when significant noise may be present in the environment. Thus, robot movement is controlled based upon measurements of the strongest signals received. In particular, we implement an existing algorithm that uses Inertia Weight Particle Swarm Optimization, which incorporates randomness and is intended to mimic behaviors observed in nature. We extend current static and dynamic obstacle avoidance strategies to prohibit mid-path collisions and to improve performance. Simulation results are given to demonstrate the capabilities of the proposed techniques. (Received September 07, 2019)

1154-90-717 **Stefan Sremac** and **Hugo J Woerdeman*** (hjw27@drexel.edu), 3141 Chestnut Street, Philadelphia, PA 19066, and **Henry Wolkowicz**. *Error Bounds and Singularity Degree in Semidefinite Programming*. Preliminary report.

For certain pathological instances of semidefinite programming, state-of-the-art algorithms, while theoretically guaranteed to converge to a solution, do so very slowly or can fail to converge entirely. This issue is exacerbated in that it is generally undetectable. In this paper we propose a method to detect this type of slow convergence by lower bounding forward error, i.e., distance to the solution set. This bound is obtained by analyzing a class of parametric curves that are proven to converge to a solution of maximum rank and then upper bounding that rank. (Received September 09, 2019)

1154-90-805 **Alberto Del Pia** and **Carla Michini*** (michini@wisc.edu). *A simplex-like algorithm for linear programs on lattice polytopes*.

We consider a simple lattice polytope P contained in $[0, k]^n$ and defined via m linear inequalities. The diameter of such lattice polytopes is in $\Omega(kn)$, as proven in Deza et al. Given an integer objective function c to maximize over P and an initial vertex x_0 of P , we compute a path along the edges of P , that starts in x_0 and ends at an optimal vertex of P with respect to c . The runtime of the algorithm is polynomial in n , m and k . (Received September 10, 2019)

1154-90-961 **Robert A Bridges*** (bridgesra@ornl.gov), 1 Bethel Valley Road, PO Box 2008, MS 6418, Bldg 6012, office 208, Oak Ridge, TN 37831. *Recent mathematical developments for vehicle security, analyzing host logs, and other applications*.

I plan to discuss a few projects involving applications of math and machine learning.

First, I'll introduce how modern vehicles use controller area networks (CANs), their inherent security flaws, and our efforts to pioneer a vehicle-agnostic intrusion detection system. Machine learning to reverse engineer proprietary signals from the CAN data in route to a detection capability will be discussed. Next, we consider security operation centers (SOCs), which now collect, store, and analyze an enormous amount of logging data from. Because host logging data is semi-structured and non-uniform, automated use of the logs is difficult. Our approach defines and embeds log sequences into a metric space structure that preserves semantic meaning of the data. We show initial results using the metric space for ransomware detection, user classification, and visualization of user activity.

Finally, I'll consider the more general problem of regression and sensitivity analysis of a function in $C^1(\mathbb{R}^n)$, where we assume the given observation set includes gradient information, $\{(x_i, f(x_i), \nabla f(x_i))\}$. Our proposed

approach is to reduce the analysis of the function to a 1-D manifold, the “Active Manifold” by exploiting a gradient / tangent space decomposition. (Received September 12, 2019)

1154-90-1022 **Yu Du*** (duyu197@gmail.com), 1475 Lawrence Street, R 5021, Denver, CO 80202. *Quantum Bridge Analytics: A Tutorial on Formulating and Using QUBO Models.*

By its association with the Ising problem in physics, the Quadratic Unconstrained Binary Optimization (QUBO) model has emerged as an underpinning of the quantum computing area known as quantum annealing and has become a subject of study in neuromorphic computing. Computational experience is being amassed by both the classical and the quantum computing communities that highlight not only the potential of the QUBO model but also its effectiveness as an alternative to traditional modeling and solution methodologies. This tutorial discloses the basic features of the QUBO model that give it the power and flexibility to encompass the range of applications that have thrust it onto the center stage of the optimization field. We show how many different types of constraining relationships arising in practice can be embodied within the “unconstrained” QUBO formulation in a very natural manner using penalty functions, yielding exact model representations in contrast to the approximate representations produced by customary uses of penalty functions. We also describe recent innovations for solving QUBO models that offer a fertile avenue for integrating classical and quantum computing and for applying these models in machine learning. (Received September 12, 2019)

1154-90-1232 **Edward D Kim*** (ekim@uwlax.edu), 17225 State Street, La Crosse, WI 54601. *Proving a variant of a polynomial positivity conjecture.*

The non-negativity of a multivariate polynomial can be shown via sum of squares optimization, solved by semidefinite programming. This talk will revisit a result of Stahl of polynomial positivity. Approaches to the proof using sum of squares techniques had been considered: in this talk, we prove a slight variant result using sums of squares. (Received September 14, 2019)

1154-90-1259 **Soledad Villar*** (soledad.villar@nyu.edu), 60 5th ave, New York, NY 10011. *Monte Carlo approximation certificates for k-means clustering.*

Efficient algorithms for k-means clustering frequently converge to suboptimal partitions, and given a partition, it is difficult to detect k-means optimality. In this paper, we develop an a posteriori certifier of approximate optimality for k-means clustering. The certifier is a sub-linear Monte Carlo algorithm based on Peng and Wei’s semidefinite relaxation of k-means. In particular, solving the relaxation for small random samples of the dataset produces a high-confidence lower bound on the k-means objective, and being sub-linear, our algorithm is faster than k-means++ when the number of data points is large. (Received September 14, 2019)

1154-90-1354 **Matt Menickelly*** (mmenickelly@anl.gov), **Jed Brown, Yunhui He, Scott MacLachlan** and **Stefan Wild**. *Tuning Multigrid Methods with Robust Optimization.*

In the national laboratory system, a significant scientifically-motivated driver of research in mathematics and large-scale computing has historically been the numerical solution of discretized partial differential equations (PDEs). Popular classes of methods for solving such problems include multigrid methods and domain decomposition algorithms. Local Fourier analysis (LFA) is a useful analytical tool to predict and analyze the actual performance of these algorithms. Here, we view LFA as a means to optimize algorithmic parameters (e.g., relaxation weights), to achieve good convergence properties.

This problem can be posed as one of nonlinear (nonconvex) robust optimization, an NP-hard problem. In many settings, there are many algorithmic parameters that must be determined to obtain efficient algorithms - in these cases, an analytical minimizer cannot be computed and approximations from brute-force sampling are intractable. In this talk, we consider using disciplined robust optimization approaches to solve these minimax problems to yield efficient methods that can outperform brute-force sampling. Different examples, with known and unknown analytical solutions, are presented to show the effectiveness of our approach. (Received September 15, 2019)

1154-90-1707 **Laura Sanità*** (lsanita@uwaterloo.ca). *On the hardness of computing the diameter of a polytope.*

The diameter of a polytope P is the maximum length of a shortest path between a pair of vertices on the 1-skeleton of P , which is the graph where the vertices correspond to the 0-dimensional faces of P , and the edges are given by the 1-dimensional faces of P . In this talk we will discuss some hardness results on the complexity of computing the diameter of a polytope, and their connection with a generalized notion of diameter, called circuit-diameter, that has recently gained a lot of attention in the literature. (Received September 16, 2019)

1154-90-1716 **Steffen Borgwardt*** (steffen.borgwardt@ucdenver.edu), University of Colorado Denver, Dep. of Mathematical and Statistical Sciences, 1201 Larimer Street, Suite 4000, Denver, CO 80204, and **Charles Viss**. *Constructing Clustering Transformations*.

Clustering is one of the fundamental tasks in data analytics and machine learning. In many situations, different clusterings of the same data set become relevant. For example, different algorithms for the same clustering task may return dramatically different solutions. We are interested in applications in which one clustering has to be transformed into another; e.g., when a gradual transition from an old solution to a new one is required.

We devise methods for constructing such a transition based on linear programming and network theory. We use a so-called clustering-difference graph to model the desired transformation and provide methods for decomposing the graph into a sequence of elementary moves that accomplishes the transformation. These moves are equivalent to the edge directions, or circuits, of the underlying partition polytopes. Therefore, in addition to a conceptually new metric for measuring the distance between clusterings, we provide new bounds on the circuit diameter of these polytopes. (Received September 16, 2019)

1154-90-1945 **Yun Lu*** (lu@kutztown.edu) and **Francis Vasko**. *An Empirical Quantification of the Impact of Choice Constraints on Generalizations of the 0-1 Knapsack Problem using CPLEX®*.

It has been well-known for some time that adding choice constraints to certain types of knapsack formulations improves the solution time for these problems when using integer programming solvers, but by how much? In this paper, by using the integer programming option of CPLEX, we provide comprehensive empirical and analytical evidence of the impact of choice constraints on two important categories of knapsack problems. Specifically, we show using multidimensional knapsack problems (MKP) and multi-demand multidimensional knapsack problems from Beasley's OR-Library that adding choice constraints reduces solution time by more than 99.9%. Additionally, using these same problem instances, we show that even if only some of the variables have choice constraints imposed on them, the CPLEX solution times are drastically reduced. These results provide motivation for operations research practitioners to check if choice constraints are applicable when solving real-world problems involving generalizations of the 0-1 knapsack problem. (Received September 16, 2019)

1154-90-2170 **Aniekan A. Ebiefung*** (aniekan-ebiefung@utc.edu), 615 McCallie Ave, Chattanooga, TN 37403. *A Block Principal Pivoting Algorithm for the Vertical Generalized Linear Complementarity Problem (VGLCP) with a Vertical Block P-matrix*.

A block principal pivoting algorithm for the VGLCP, when the associated matrix is a P-matrix, is provided. We are motivated by the efficiency of this method when applied to large scale problems in linear complementarity problems (LCPs), especially that problems involving the VGLCP are likely to be larger and more complex than what we have for the LCP. The algorithm is very efficient and a numerical example shows that it reduces the number of iterations needed to solve the VGLCP when compared to other available algorithms. (Received September 17, 2019)

1154-90-2591 **Eric Chirtel***, emc4tg@virginia.edu, and **Juergen Kritschgau** and **Kota Kageie**. *An Application of Quantum Annealing to the Bike Share Re-balancing Problem*.

Quantum annealing has been used to find good solutions to hard combinatorial optimization problems (Lucas 2014). In this talk, we combine hybrid quantum annealing algorithms implemented on a D-Wave machine with greedy heuristics to solve the bike share rebalancing problem, also known as BSRP (Del amico et al 2014). We compare the accuracy and timing of our method to solutions found via classical methods for a set of benchmark BSRP problems. In particular, we compare classical and quantum implementations of clustering heuristics that are used in capacitated vehicle routing problems (Feld et al 2019). (Received September 17, 2019)

91 ► *Game theory, economics, social and behavioral sciences*

1154-91-47 **Mason A Porter*** (mason@math.ucla.edu). *Opinion and Spreading Models on Social Networks*.

From the spreading of diseases and memes to the development of opinions and social influence, dynamical processes are affected significantly by the networks on which they occur. In this talk, I'll review recent work by my collaborators and me on social influence and opinion models on networks. I'll discuss diverse flavors of models — including threshold models of social contagions, voter models that coevolve with network structure, and bounded-confidence models with continuous opinions — and present how such processes are affected by the social

structures on which they occur. I'll also connect these models to opinion polarization, the "majority illusion" in social networks, and the development of echo chambers in online social networks. (Received September 04, 2019)

1154-91-62

Trevor Lax* (tl106@evansville.edu), Department of Mathematics, University of Evansville, Evansville, IN 47722, and **Gabrielle Wolf** (gmw3144@g.rit.edu), School of Mathematical Sciences, Rochester Institute of Technology, Rochester, NY 14623.

Coevolving Network Model for the Diffusion of Opioid Dependence in a Population.

We study the spread of opioid dependence across a population by employing the framework of coevolving network systems: which are highly pragmatic dynamical systems where the structure of the network coevolves with and is influenced by the dynamics on the underlying network. We aim to study how the interrelation between social network structure and the behavior of individuals affects the diffusion of opioid dependence in a population. We also identify possible social networks based intervention strategies to slow or stop the spread of opioid dependence in communities. We create a simulation to analyze results under various parameter settings highlighting transitions due to the distribution of risk factors and the probability of social selection. Furthermore, we develop a system of differential equations using pair approximations, which takes into account the zeroth and first-order moments (nodes and edges) and approximate the second-order moments (triplets) of the network. The results show that the effect of Social Selection is minimal compared to Social Influence and so we conclude that efforts should heavily focus on reduction of risk factors, as opposed to limiting interaction with drugs or drug-dependent individuals. (Received July 27, 2019)

1154-91-98

Marion Campisi* (marion.campisi@sjsu.edu), **Thomas C Ratliff** and **Ellen Veomett**. *Analysis of partisan gerrymandering tools in advance of the US 2020 census.* Preliminary report.

Over the last decade, mapmakers have precisely gerrymandered political districts for the benefit of their party. In response, political scientists and mathematicians have more extensively investigated tools to quantify and understand the mathematical structure of redistricting problems. Two primary tools for determining whether a particular redistricting plan is fair are partisan-gerrymandering metrics and stochastic sampling algorithms. In this work we explore the Declination, a new metric intended to detect partisan gerrymandering. Within out analyses, we show that Declination cannot detect all forms of packing and cracking, and we compare the Declination to the Efficiency Gap. We show that these two metrics can behave quite differently, and give explicit examples where that occurs. (Received August 05, 2019)

1154-91-209

Heather Zinn Brooks* (hbrosks@math.ucla.edu) and **Mason A Porter**. *A Model for the Influence of Media on the Ideology of Content in Online Social Networks.*

Many people rely on online social networks for news, and the spread of media content influences online discussions and impacts actions offline. To examine such phenomena, we generalize bounded-confidence models of opinion dynamics on a social network by incorporating media accounts as influencers. We quantify partisanship of content as a continuous parameter, and we present higher-dimensional generalizations to incorporate nuanced political positions and content quality (a key novel contribution of our work). We use simulations to quantify the entrainment of non-media content to the ideologies of media accounts. We maximize media impact over a social network by tuning the number of media accounts that promote the content and the number of followers per account. We find that entrainment of the ideology of content spread by non-media accounts to media ideology depends on structural features of the network. Finally, we incorporate multiple media sources with ideological biases and quality-level estimates drawn from real media sources to demonstrate that our model can produce communities that are polarized in both ideology and quality. Our model provides a step toward understanding content spreading dynamics, with ramifications mitigating the spread of undesired content. (Received August 23, 2019)

1154-91-742

Timmy Ma* (timmy.ma@dartmouth.edu), Mathematics Department, Dartmouth College, HB 6188, Hanover, NH 03755. *Object-Label-Order effect when learning from an inconsistent source.*

Learning in natural environments is often characterized by a degree of inconsistency from an input. We focus on the setting where a learner receives and processes a sequence of utterances to master associations between objects and their labels, where the source is inconsistent: it uses both "correct" and "incorrect" object-label pairings. We hypothesize that depending on the order of presentation, the result of the learning may be different. We consider two types of symbolic learning procedures: the Object-Label (OL) and the Label-Object (LO) process. With OL, the learner is first exposed to the object then the label. This order is reversed with LO. It is observed

experimentally that OL learners are generally better at processing inconsistent input compared to LO learners. We show that the patterns observed in the learning experiments can be reproduced in the simulations if the mathematical model includes: (1) an ability to regularize the input (and also to do the opposite, i.e. undermatch) and (2) an ability to take account of implicit negative evidence (i.e. interactions among different objects/labels). The model suggests that there is a difference in regularization patterns: OL learners regularize the input and thus form a consistent system, whereas LO learners undermatch. (Received September 12, 2019)

1154-91-904 **Rene A. Carmona*** (rcarmona@princeton.edu), 210 Sherrerd Hall, Princeton University, Princeton, NJ 08544. *Stochastic Graphon Game Models as Extensions of Mean Field Games.*

We introduce a class of stochastic games with a continuum of players as limits of finite player stochastic games for which the players get idiosyncratic independent and identically distributed random signals, and interact according to a network structure underpinned by a graph. We analyze the limits as graphon games and we emphasize the similarities and the differences with mean field games. (Received September 11, 2019)

1154-91-2367 **Rajkumar Verma*** (rkver83@gmail.com), Department of Management Control and, information, University of Chile, Av. Diagonal Paraguay 257, 8330015 Santiago, Chile, **Abha Aggarwal** (abhaaggarwal127@gmail.com), University School of Basic and Applied Scien, Guru Gobind Singh Indraprastha University, Delhi, 110078, India, and **José M. Merigó** (jmerigo@fen.uchile.cl), Department of Management Control and, information, University of Chile, Av. Diagonal Paraguay 257, 8330015 Santiago, Chile. *A new approach to solve 2-tuple linguistic matrix games based on linguistic scale functions.*

Game theory has been found successful applications in a variety of decision-making areas. To model uncertainty and vagueness in real-world decision problems, many studies have been carried out to solve matrix game problems with interval, fuzzy, and intuitionistic fuzzy numbers payoffs. Fuzzy linguistic variables are very useful and efficient tools to represent uncertain or vague information. To avoid information loss during the linguistic operational process, the 2-tuple linguistic (2-TL) model was introduced for computing with words. In recent years, a wide range of decision-making approaches has been developed with 2-TL information. The objective of this work is to study a 2-player constant-sum matrix game with 2-TL payoffs based on linguistic scale functions. For doing so, first, we propose a new aggregation operator for aggregating 2-TL information by utilizing the linguistic scale functions and prove their basic properties. Then, the paper formulates a linguistic linear programming (LPP) model to solve a constant-sum matrix game problem with 2-TL payoffs based on the proposed aggregation operator. Finally, a real-life matrix game problem is considered to demonstrate the process and the applicability of the proposed method in solving matrix games under the 2-TL environment. (Received September 17, 2019)

1154-91-2378 **Olivia J. Chu*** (ochu@princeton.edu), Lewis-Sigler Institute, Princeton University, Princeton, NJ 08544, and **Vitor V. Vasconcelos** and **Corina E. Tarnita.** *Evolutionary Dynamics in a Group Population Structure with Barriers to Group Entry.* Preliminary report.

The evolution of cooperation has been studied in many systems, from bacterial communities to human populations. It is well known that population structure is crucial to a system's dynamics. In human populations, group memberships are critical. Humans often meet and interact with each other due to common group memberships. There exist network-based models to study human dynamics, but they generally do not allow for multiple group affiliations or incorporate barriers to group entry. In this work, we present a framework in which individuals in a group-structured population interact, through an evolutionary game, with those who share their groups. Individuals update stochastically, with strategy and group memberships subject to evolutionary updating. We impose realistic barriers to group entry based on group size. We find that with barriers, cooperation can emerge, but that it is most favored when we allow for the existence of "loners": a changing subset of individuals who spend a temporary "time-out" period not interacting with others. This work provides an analytical framework in which behavior in realistic population structures can be studied, and adds to a growing body of literature that recognizes the existence of loners as vital parts of systems. (Received September 17, 2019)

1154-91-2429 **Elizabeth Tripp*** (elizabeth.a.tripp.gr@dartmouth.edu), Department of Mathematics, 27 North Main Street, Dartmouth College, Hanover, NH 03755, and **Feng Fu** and **Scott Pauls.** *Evolutionary Kuramoto Dynamics.* Preliminary report.

Understanding the synchronization of systems of coupled oscillators has a rich history in the study of dynamical systems and applications in numerous fields, where decades of work have demonstrated the complex interplay between the properties of these systems and the oscillators' ability to synchronize. We bring the tools of evolutionary game theory (EGT) to the study of coupled oscillatory systems, with the neurons of the suprachiasmatic

nucleus as a motivating example. We cast the oscillatory system as a collection of agents, one for each oscillator, that play a game with one another where the payoffs are based on their phases. We find that even one of the simplest models of these systems yields a rich diversity of outcomes linked to classical games - the prisoner's dilemma, the snowdrift game, etc. - based on the relative costs and benefits of synchronization in the organism. Within this complexity lies a simplicity as well, as we consistently find a simple condition, under various assumptions, between the cost and benefit parameters which leads to the overall synchronization within the population of agents. This simple framework opens the door to a plethora of mathematical and biological questions for future study. (Received September 17, 2019)

1154-91-2452 **Daewa Kim*** (daewa.kim@mail.wvu.edu), 332 Bent Tree Court, APT 328, Morgantown, WV 26505, and **Annalisa Quaini**. *A Kinetic Theory Approach to Pedestrian Motion and Onset of Disease Spreading*. Preliminary report.

We present a kinetic approach for crowd dynamics. First, we model the crowd evacuation from bounded domains. The interactions of a person with other pedestrians and the environment, which includes walls, exits, and obstacles, are modeled by using tools of game theory and are transferred to the crowd dynamics. The model allows to weight between two competing behaviors: the search for less congested areas and the tendency to follow the stream unconsciously in a panic situation. For the numerical approximation of the solution to our model, we apply an operator splitting scheme which breaks the problem into two pure advection problems and a problem involving the interactions. Through a series of numerical tests, we also show that our approach is capable of handling evacuation from a room with one or more exits with variable size, with and without obstacles, and can reproduce lane formation in bidirectional flow in a corridor. Next, we consider a crowd model known as ASCRIBE that can track the level of emotional contagion in evacuation scenarios. We propose a modification of this model to track disease contagion. Finally, we couple the disease contagion model with the one-dimension kinetic approach for pedestrian dynamics to simulate the initial spreading of an infectious disease. (Received September 17, 2019)

1154-91-2609 **Joanna R. Wares***, jwares@richmond.edu, and **Jana L. Gevertz, Ami Radunskaya, Sara Solomon and Douglas Wiebe**. *Overdose Prevention Site Placement Informed by Agent-Based Modeling*. Preliminary report.

An Overdose Prevention Site (OPS) is a well-lit hygienic space where people can inject illicit drugs under medical supervision. OPSs also provide advice to clients about injection-related harms and how to avoid them, provide clean injecting supplies, provide linkages to medical and social services, and reverse overdoses that occur on site. As Philadelphia prepares to launch the first comprehensive overdose prevention site (OPS) in the nation, the Penn Injury Science Center has joined forces with local organizers and community stakeholders to help find the most suitable location for the first OPS. In this talk, we describe our part in these efforts. We are building an agent-based model that will incorporate usage, overdose, and demographic data from Philadelphia to help determine best placement of OPSs. We will also utilize the model to predict reduction in overdose deaths, disease spread, public usage, needle litter, and various other outcome measures in the particular areas around where the OPSs could be located. (Received September 17, 2019)

1154-91-2838 **Julia E Vasile*** (julia.vasile@stonybrook.edu), 329 Miller Place Road, Miller Place, NY 11764. *Implementation of Toyota's e-Palette Mobility System to Develop Data-driven Optimization of Transportation Networks*.

Toyota Motor Corporation is one of the world leaders in sales of hybrid electric vehicles. In 2018, in an effort to shift toward becoming a provider of mobility services, they revealed the e-Palette mobility model, a fully autonomous electric vehicle that could include various functionalities, such as transportation, delivery services, etc. The University of Tsukuba has a large campus where students and faculty may opt to use different modes of transportation. Due to traffic congestion, buses have been inefficient, resulting in longer and inconsistent waiting times. We propose that the introduction of e-Palettes could help reduce student waiting times and congestion. We introduce an e-Palette bus hybrid model, which outputs an optimal number of e-Palettes and buses that correspond to an optimal schedule. We propose that this hybrid model improves the current bus system at Tsukuba University by reducing waiting and travel times, particularly during on-peak hours. We compare waiting times and costs between a bus model, e-Palette model and hybrid model to compare efficiency in waiting times, as well as cost effectiveness. Furthermore, incentive analysis will show that our hybrid model is a beneficial alternative for consumers. (Received September 18, 2019)

92 ► *Biology and other natural sciences*

1154-92-49 **James Keener*** (keener@math.utah.edu). *The Mathematics of Life: Making Diffusion your Friend.*

Diffusion is the enemy of life. This is because diffusion is a ubiquitous feature of molecular motion that is constantly spreading things out, destroying molecular aggregates. However, all living organisms, whether single cell or multicellular have ways to use the reality of molecular diffusion to their advantage. That is, they expend energy to concentrate molecules and then use the fact that molecules move down their concentration gradient to do useful things.

In this talk, I will show some of the ways that cells use diffusion to their advantage, to signal, to form structures and aggregates, and to make measurements of length and size of populations. Among the examples I will describe are signalling by nerves, cell polarization, bacterial quorum sensing, length measurement of flagella and cilia, and cell size measurement. In this way, I hope to convince you that living organisms have made diffusion their friend, not their enemy. (Received September 18, 2019)

1154-92-77 **Jun Liu** (juliu@siue.edu), Department of Mathematics and Statistics, Southern Illinois University Edwardsville, Edwardsville, IL 62026, and **Xiang-Sheng Wang*** (xswang@louisiana.edu), Department of Mathematics, University of Louisiana at Lafayette, Lafayette, LA 70503. *Optimal control of a size-structured model for metastatic cancer treatment.*

We propose a unified size-structured PDE model for the growth of metastatic tumors, which extends a well-known coupled ODE-PDE dynamical model developed and studied in the literature. A treatment model based on the proposed unified PDE model is investigated via optimal control theory, where its first-order necessary optimality system characterizing the optimal control is derived. We prove that the uniqueness of the optimal control depends on the chosen objective functional, and the optimal control is of bang-bang type when it is unique. For obtaining its efficient numerical solutions, a projection gradient descent algorithm based on the characteristic scheme is developed for solving the established optimal treatment model. Our study reveals that: (i) only the total drug dosage matters if one just cares about the final treatment output; (ii) given the same total drug dosage, the optimal bang-bang treatment plan outperforms the others in the sense that it maximally reduces the total tumor sizes during the whole period of treatment, although their final tumor sizes are the same. (Received July 30, 2019)

1154-92-87 **Tiffany N Kolba*** (tiffany.kolba@valpo.edu). *The Mathematics Behind Twin Motherhood.*

The phenomenon of twins, triplets, quadruplets, and other higher order multiples has fascinated humans for centuries and has even captured the attention of mathematicians who have sought to model the probabilities of multiple births. This talk gives an overview of the previous mathematical research on multiple births, as well as my own contributions to the field, which were motivated by my personal experience of twin motherhood. In particular, this talk presents novel estimates for the zygosity type frequencies of twins, triplets, and quadruplets, where the zygosity of a set of multiples is defined as the number of eggs from which they derived. Twins can be either monozygotic (identical) or dizygotic (fraternal), while triplets can be mono-, di-, or trizygotic and quadruplets can be mono-, di-, tri-, or quadrazygotic. The zygosity type estimates rely upon modeling the relative rates of the two distinct biological mechanisms that produce multiple births, namely, division and polyovulation. (Received July 31, 2019)

1154-92-116 **Lauren R Dickman** (lrjohns7@asu.edu), **Evan Milliken*** (evan.milliken@asu.edu) and **Yang Kuang** (kuang@asu.edu). *Global analysis of a model of dendritic cell therapy for melanoma.* Preliminary report.

Melanoma, the deadliest form of skin cancer, is regularly treated by surgery in conjunction with a targeted therapy or immunotherapy. Dendritic cell (DC) therapy is an immunotherapy that capitalizes on the critical role dendritic cells play in shaping the immune response. Previous models of DC therapy were too complex to allow for extensive mathematical analysis. In this talk, a reduced model of DC therapy is presented. This model is simple enough to allow for mathematical analysis. At the same time, the model retains important interactions and remains complex enough to provide a good fit to murine data. Mathematical analysis and simulation reveals rich dynamics including backward bifurcation and Hopf bifurcation, which are both likely artifacts of a Bogdenov-Takens bifurcation that occurs on the boundary of the biologically relevant parameters space. The model, bifurcation analysis, and conditions for tumor persistence are presented along with simulations and discussion. (Received August 09, 2019)

1154-92-132 **Brittany Bannish*** (bbannish@uco.edu) and **Sean Lavery*** (slavery@uco.edu). *Exploring modeling by programming: a technique for involving undergraduates in mathematical biology research.*

Undergraduate research projects can be incredibly rewarding for both the students and the faculty mentors. A challenge encountered by faculty hoping to work with students is finding someone with the appropriate prerequisite knowledge, who is not graduating too soon. In this talk, we present a few tips for getting students of almost any mathematical background started in research, and a few examples of successful projects. We emphasize the use of technology early on, building up from smaller, simpler problems to the question of interest. This helps students gain intuition about the appropriate mathematical topics and details of their particular system, think critically about what results mean biologically, and consider possible model modifications or directions for more rigorous mathematical analysis. Using specific examples like parasite transmission in cats, the role of protein regulation in establishing circadian rhythms, and the cost of treatment in an influenza outbreak, we show how we approach undergraduate research, from picking a project topic, to learning to program, to analyzing the model, to making biological hypotheses based on model results. (Received August 13, 2019)

1154-92-135 **Burt Simon*** (burt.simon@ucdenver.edu). *Group Selection.*

There are many misconceptions about how group selection works. The best way to understand group selection is to study a good mathematical model. I will describe a Markovian model of group-structured populations that features individual-level birth, death, and migration events, and group-level fission and extinction events, and three techniques for studying the resulting population dynamics. From the model we can see how group selection works, and its efficacy as an evolutionary force. Unsurprisingly, group-level events are the key to group selection, but standard characterizations of group selection, e.g., the Price equation, do not account for group-level events, resulting in the misconceptions. Examples from genetic and cultural evolution will be discussed. (Received August 17, 2019)

1154-92-178 **Garri Davydyan*** (garri.davydyan@gmail.com). *Split-quaternions and diverging mechanisms of biologic evolution.*

A biologic evolution is an ability of a biologic matter to develop its functional and structural properties. It is based on reproduction of its own elements. Only core features are reproducible in the following generations. This is determined by steadiness and stability of functional organization of biological units (systems). Based on the existing data, previously it was proposed that positive feedback, negative feedback and reciprocal links (PNR) being represented as elements of imaginary part of split-quaternions, form a functional basis of internal structure of biologic systems. Formation of these basis elements corresponds with a 3D pseudo-Euclid space of functional regulatory elements of autonomous biologic systems. There is a correspondence between two opposite directions in 3D space and a direct sum of two operators with opposite entries. This intrinsic property of basis functional elements of biologic systems determines splitting mechanism of evolution. (Received August 19, 2019)

1154-92-181 **Olusegun Michael Otunuga*** (otunuga@marshall.edu), Department of Mathematics, Marshall University, SH 523, One John Marshall Dr, Huntington, WV 25755. *Closed form probability distribution of number of infections at a given time in a stochastic SIS epidemic model.*

The objective of this work is to derive and analyze the closed form probability distribution of the number of infections at a given time in a stochastic SIS epidemic model. We study the effects of external fluctuations in the transmission rate of certain diseases and how these affect the distribution of the number of infections over time. The properties of the distribution of the number of infections, together with the effect of noise intensity, are analyzed. The distribution is demonstrated using parameter values relevant to the transmission dynamics of influenza in the United States. (Received September 12, 2019)

1154-92-223 **David Sankoff*** (sankoff@uottawa.ca). *Fractionation after whole genome doubling.*

A basic tool for studying the polyploidization history of a genome, especially in plants, is the distribution of duplicate gene similarities in syntenically aligned regions of a genome. This distribution can usually be decomposed into two or more components identifiable by peaks, or local maxima, each representing a different polyploidization event. The distributions may be generated by means of a discrete time branching process, followed by a sequence divergence model. The branching process, as well as the inference of fractionation rates based on it, requires knowledge of the ploidy level of each event, which cannot be directly inferred from the pair similarity distribution. We have developed a way of inferring the ploidy of up to three successive WGD and/or WGT events by estimating the time of origin of each of the similarities in triples of genes. This may be generalized to a larger number of events and to higher ploidies. (Received August 25, 2019)

1154-92-249 **Robert Gatenby*** (robert.gatenby@moffitt.org), 12902 Magnolia Dr, Tampa, FL 33612. *Ecology and evolution in control and cure of metastatic cancers.*

Increasing numbers of effective therapies are available for metastatic cancers. For example, there are currently 52 drugs approved for use in metastatic prostate cancer. However, of the 35,000 men diagnosed this year, none will be cured. The reason for treatment failure is evolution. With access to the information stored in the human genome, cancer cells can deploy strategies to counter all current therapies. A new treatment approach focuses on the evolutionary dynamics that allow resistant cells to proliferate sufficiently to repopulate the tumor. Resistance usually comes with phenotypic costs from operation of the associated molecular machinery which can be exploited to reduce cellular proliferation in the substrate-poor environments of clinical cancers. Evolution-based cancer treatment is designed to delay or prevent proliferation of resistant cells. Developed through mathematical models, this approach is now in clinical trials. A new initiative focuses on the eco-evolutionary dynamics of Anthropocene extinctions to develop strategies for curing metastatic cancers with available drugs. Mathematical models and computer simulations have demonstrated this to be a feasible approach. Initial pre-clinical studies and pilot clinical trials are underway. (Received August 27, 2019)

1154-92-270 **Paulina Volosov*** (volosp@rpi.edu) and **Gregor Kovacic.** *Linking Functional and Structural Neuronal Network Connectivity using Incomplete Observation Data and Leveraging the Spectral Properties to Characterize Network Structure.*

The extent of the relation between architectural and functional connectivity in the cerebral cortex is a question which has attracted much attention in recent years. Neuroscientists frequently use the functional connectivity of neurons, i.e. the measures of causality or correlations between the neuronal activities of certain parts of a network, to infer the architectural connectivity of the network, which indicates the locations of underlying synaptic connections between neurons. Architectural connectivity can be used in the modeling of neuronal processing and in the forming of conjectures about the nature of the neural code.

We begin by reconstructing the entire network using time-delayed spike-train correlation, and we determine the time required before an adequate reconstruction becomes possible and compare this to time spans employed by experimentalists. To be more experimentally valid, we next examine a small “slice” or submatrix of the network and determine how much information we can deduce about the whole network from this small piece. Finally, we develop a metric using the spectrum of the adjacency matrix which enables us to classify small-world networks. (Received August 27, 2019)

1154-92-308 **Ronald E Mickens*** (rmickens@cau.edu), Dept of Physics, Clark Atlanta University, Atlanta, GA 30314-3773, and **Kale Oyedeji** (kaleoyedeji@comcast.net), 5343 Fox Valley Trace, Stone Mountain, GA 30088. *A Three Tropic-Level Model of Interacting Populations.*

We construct and analyze a three-tropic level model of interacting populations expressed as a system of coupled, nonlinear, first-order differential equations. This model might provide a particular representation of the long running experiment on Isle Royale involving pine trees, moose and wolves [1]. Our model differs from standard mathematical models in its characterization of the birth, death, and interaction terms as products of non-integer powers of the three populations. An interesting and important feature is that the exact analytical solution can be calculated in terms of elementary functions. Since our particular model is representative of the whole class of models for three tropic levels systems, this allows us to make a number of general conclusions holding for all of them. In addition to the location of the fixed-points, and their stability properties, both local and globally, we will discuss the possibility of finite-time dynamics, i.e., conditions under which one or more populations can go to zero in a finite time.

[1] B.E. McLaren and R.O. Peterson, Wolves, Moose, and Tree Rings on Isle Royale, AAS,260, 5190 (Dec. 2, 1994) 1555-1558: www.jsor.org/Stable/2885186 (Received August 30, 2019)

1154-92-319 **Daniel Fairbanks*** (daniel.fairbanks@uvu.edu) and **Bob Palais** (bob.palais@uvu.edu). *How Good is the χ^2 Goodness-of-Fit test? Revisiting a controversy from the birth of modern statistics and genetics.*

In 1900, the same year that Gregor Mendel’s paper that laid the foundations for the science of genetics was rediscovered, Karl Pearson published what is now known as the chi-squared goodness-of-fit test. Two years later, Pearson’s colleague, W.F.R. Weldon applied the test to Mendel’s data and came to the startling conclusion that they appeared suspiciously close to expectation. Weldon’s paper set off a firestorm that pitted the biometricians against the Mendelians. Ronald Fisher entered the fray in 1911, but it was his 1936 paper, based on the same mathematics, that ignited what is still known as the Mendel-Fisher Controversy.

We will describe elementary mathematical transformations and efficient algorithms that eliminate the need for often crude chi-squared approximations of cumulative probability. The ease of achieving greater accuracy offers new perspective on this colorful debate. Our methods extend to larger sample sizes, multiple degrees of freedom, and sampling without replacement. We will also discuss recently published evidence that Darwin's Origin of Species influenced the sections of Mendel's paper that address issues of evolution. (Received August 31, 2019)

1154-92-353 **Suzanne L Weekes*** (sweekes@wpi.edu), Department of Mathematical Sciences, WPI, 100 Institute Road, Worcester, MA 01609. *A multicompartment mathematical model of cancer stem cell-driven tumor growth dynamics.*

Tumors are appreciated to be an intrinsically heterogeneous population of cells with varying proliferation capacities and tumorigenic potentials. As a central tenet of the so-called cancer stem cell hypothesis, most cancer cells have only a limited lifespan, and thus cannot initiate or reinitiate tumors. Longevity and clonogenicity are properties unique to the subpopulation of cancer stem cells. To understand the implications of the population structure suggested by this hypothesis, we develop a mathematical model for the development of the aggregate population. We show that overall tumor progression rate during the exponential growth phase is identical to the growth rate of the cancer stem cell compartment. Tumors with identical stem cell proportions, however, can have different growth rates, dependent on the proliferation kinetics of all participating cell populations. Analysis of the model reveals that the proliferation potential of non-stem cancer cells is likely to be small to reproduce biologic observations. Furthermore, a single compartment of non-stem cancer cell population may adequately represent population growth dynamics only when the compartment proliferation rate is scaled with the generational hierarchy depth. (Received September 02, 2019)

1154-92-392 **Renee Brady** and **Heiko Enderling*** (heiko.enderling@moffitt.org), 12902 Magnolia Dr, Tampa, FL 33612. *Learning from prostate-specific antigen dynamics early during androgen deprivation to predict subsequent individual responses.*

Intermittent androgen deprivation therapy (IADT) is an attractive treatment for biochemically recurrent prostate cancer (PCa), whereby cycling treatment on and off can reduce cumulative dose and limit toxicities. It has also been shown to delay the development of treatment resistance in mice. We follow a recent modeling pipeline [Brady & Enderling, *Bull. Math. Biol.*, 2019] to calibrate, validate and evaluate a mathematical model that simulate enrichment of prostate cancer stem cell (PCaSC) dynamics during treatment as a plausible mechanism of resistance evolution. Simulated PCa stem and non-stem cells dynamics demonstrate that PCaSC proliferation patterns correlate with longitudinal serum prostate-specific antigen (PSA) measurements in 70 PCa patients undergoing IADT. By learning dynamics from each treatment cycle, individual model simulations predict evolution of resistance with an overall accuracy of 90%. Model simulations based on response dynamics from the first IADT cycle identify patients who would benefit from concurrent docetaxel in subsequent cycles. Our results demonstrate the feasibility and potential value of adaptive clinical trials guided by patient-specific mathematical models of intratumoral evolutionary dynamics. (Received September 03, 2019)

1154-92-418 **Maria-Veronica Ciocanel*** (ciocanel.1@osu.edu), **Riley Juenemann**, **Adriana Dawes** and **Scott McKinley**. *Topological Data Analysis for Ring Channels in Intracellular Transport.*

Contractile rings are cellular structures made of actin filaments that are important in development, wound healing, and cell division. In the reproductive system of the worm *C. elegans*, ring channels allow nutrient exchange between developing egg cells and the worm and are regulated by forces exerted by myosin motor proteins. In this talk, I will present an agent-based modeling and data analysis framework for the interactions between actin filaments and myosin motors inside cells. This approach may provide key insights for the mechanistic differences between two motors that are believed to maintain the rings at a constant diameter. In particular, we propose tools from topological data analysis to analyze time-series data of filamentous network interactions. Our proposed methods clearly reveal the impact of key parameters on significant topological circle formation, thus giving insight into ring channel formation and maintenance. (Received September 03, 2019)

1154-92-446 **Kyeongah Nah***, kyeongah.nah@gmail.com. *The potential impact of climate change on the transmission risk of tick-borne encephalitis: insights from dynamical modeling.*

To design the immunization program to mitigate the tick-borne encephalitis (TBE) incidence adapting to climate change, it is essential to assess the transmission risk of tick-borne encephalitis virus (TBEV) in the enzootic cycle and further predict its risk in a future using projected climate conditions. In this talk, we introduce a mathematical model for retroactive analysis of weather fluctuation on TBE prevalence in Hungary. The TBE transmission-human case reporting cascade model couples a TBE virus transmission dynamics among ticks with

multiple development stages, animal hosts and humans, with the stochastic observation process of human TBE reporting given infection. By fitting human incidence data in Hungary to the model, we estimate key parameters relevant to the tick-host interaction and tick-human transmission. Then we compute the basic reproduction number which determines the long-term behaviors of the periodic system of integro-differential equations - the TBE transmission dynamics. We then show that the developed model provides an effective tool for projecting TBEV transmission risk in the enzootic cycle by integrating climate projection with emerging knowledge about the region-specific tick ecological and pathogen epidemiological processes. (Received September 04, 2019)

1154-92-447 **James Watmough***, watmough@unb.ca, and **Lin Wang, Myriam Barbeau and Ali Gharouni**. *Larval versus adult dispersal: implications of two modes of dispersal on the spread rate of an invasion.*

Aquatic invasive species often have larvae or propagules as the most dispersive stage. However, other stages, such as the adult stage, can naturally disperse too. We use a structured integro-difference equation model of the the spread of the green crab up the northwest coast of the Atlantic is used as a case study to examine the dependence of both adult and larval dispersal on invasion dynamics. Adding an additional dispersive stage will increase invasion spread rates. However, it is unclear how the sensitivity of spread rate to underlying parameters might change with additional dispersive stages. Knowledge on the sensitivity of spread rate to demographic and dispersal parameters helps inform management strategies. This is joint work with Lin Wang, Myriam Barbeau and Ali Gharouni. (Received September 04, 2019)

1154-92-460 **Patrick Shipman***, shipman@math.colostate.edu. *Continuous Natural Language Vowel Change.*

The Great English Vowel Shift of 16th–19th centuries and the current Northern Cities Vowel Shift are two examples of collective language processes characterized by regular phonetic changes, that is, gradual changes in vowel pronunciation over time. We develop a structured population approach to modeling such regular changes in the vowel systems of natural languages, taking into account learning patterns and effects such as social trends. We treat vowel pronunciation as a continuous variable in vowel space and allow for a continuous dependence of vowel pronunciation in time and age of the speaker. The theory of mixtures with continuous diversity provides a framework for the model, which extends the McKendrick–von Foerster equation to populations with age and phonetic structures. (Received September 04, 2019)

1154-92-463 **Gail Wolkowicz*** (wolkowicz@math.mcmaster.ca), Department of Math. & Stats., McMaster, Ontario L8S 4K1, Canada, and **Chiu-Ju Lin and Lin Wang**, lwang2@unb.ca. *An alternative formulation for a delayed logistic equation.*

An alternative single species logistic distributed delay differential equation with decay-consistent delay in growth is derived. We prove that the model does not permit sustained oscillations and that when the delay is sufficiently long the population dies out. Such dynamics are more biologically realistic compared to the dynamics predicted by the classical delayed logistic model. We establish a threshold for survival and extinction: in the former case, it is confirmed using Lyapunov functionals that the population approaches the delay modified carrying capacity; in the later case the extinction is proved using the fluctuation lemma. We further use adaptive dynamics to conclude that the evolutionary trend is to make the mean delay in growth as short as possible. This confirms Hutchinson’s conjecture and fits biological evidence. (Received September 04, 2019)

1154-92-480 **Sherli Koshy-Chenthittayil*** (koshychenthittayil@uchc.edu), 263 Farmington Avenue, Farmington, CT 06030, and **Reinhard Laubenbacher, Pedro Mendes and Anna Dongari-Bagtzoglou**. *Agent-based modeling of multi-species biofilm and its optimization.* Preliminary report.

Mixed fungal-bacterial biofilms are a major source of infections on medical implants and in mucosa, such as the mouth. The common fungal pathogen being *Candida albicans*. There is strong evidence that different bacteria like *Lactobacillus* and *Streptococcus* can either inhibit or promote growth and virulence of *C. albicans* biofilms. To fully understand the interactions between these different species, an agent based model (ABM) is constructed using data from experiments. In the ABM, the different bacteria and fungi are the agents with characteristics of growth, decay, death and mechanical interactions. The ABM simulations are validated using images of the biofilm. To reduce the virulence of *C.albicans*, the biofilm composition has to be optimized. This involves minimizing the species diversity and minimizing the biofilm thickness. This multi-objective optimization of the ABM will be addressed in two steps-dimension reduction and then optimization using genetic algorithms. This involves searching for the Pareto frontier (set of solutions where there is a cost associated with improving a solution) for the original and reduced model and then comparing the two. (Received September 05, 2019)

1154-92-485 **Tricia Phillips*** (tphill130@vols.utk.edu), **Suzanne Lenhart** (slenhart@utk.edu) and **Christopher Strickland** (cstric12@utk.edu). *A Data-Driven Approach to Modeling the Heroin and Fentanyl Epidemic*. Preliminary report.

A preliminary report will be given on the formulation and analysis of a heroin/fentanyl epidemic model. This model, consisting of a system of ordinary differential equations, aims to better understand the dynamics between prescription opioid use, prescription opioid addiction, heroin/fentanyl addiction, and recovery from opioid addiction. (Received September 05, 2019)

1154-92-501 **Vincent J. Jodoin*** (vjodoinjr@gmail.com), University of Tennessee, Knoxville, TN 37996, and **M. Todd, S. Payne, J. Bintz, S. Lenhart** and **C. Strickland**. *Modeling the Effect of Social Networks on the Spread of Opioid Use*. Preliminary report.

The opioid crisis has recently become classified as a national health emergency, and we investigate the effect of social networks on the spread of opioid use. We are modeling the impact the presence of a highly connected and highly positive individual has on the network to see if they can decrease the number of addicts in the overall network, or the rate of users becoming addicts. We built an agent-based model via NetLogo to create our spatially clustered network to represent how individuals remain non-users or become users, addicts, or rehabilitated. Our model examines the premise of a mechanism to encourage users or addicts to end the use of opioids by surrounding themselves with a sub-network of positive individuals. This sub-network will encourage a user to have less negative influences. (Received September 05, 2019)

1154-92-502 **Suzanne Lenhart***, University of Tennessee, Knoxville, TN 37996. *Optimal Control of the Flow Rate in a PDE Model of an Invasive Species in a River*. Preliminary report.

Managing invasive species in rivers can be assisted by appropriate adjustment of flow rates. Using a partial differential equation (PDE) model representing an invasive population in a river, we investigate controlling the water discharge rate as a management strategy. Our goal is to see how controlling the water discharge rate will affect the invasive population, and more specifically how water discharges may force the invasive population downstream. We complete the analysis of a flow control problem, which seeks minimize the invasive population upstream while minimizing the cost of this management. Using an optimality system, consisting of our population PDE, an adjoint PDE and corresponding optimal control characterization, we illustrate some numerical simulations in which parameters are varied to determine how far upstream the invasive population reaches. We also change the river's cross-sectional area to investigate the impacts of this on the optimal control. This work is in collaboration with Rebecca Pettit. (Received September 05, 2019)

1154-92-541 **Lisette dePillis*** (depilllis@hmc.edu), Harvey Mudd College, Department of Mathematics, 301 Platt Blvd., Claremont, CA 91711. *Mathematical Models of Immune System Dynamics and Cancer*.

The role of immune system dynamics in the context of cancer progression has become increasingly central in the development of new treatment strategies. The importance of the immune system in combating cancer has been verified clinically, as well as through mathematical models, yet open questions remain. For example, how do we think about non-uniform patient responses to treatments, and how might we personalize therapy protocols? In this talk, we will present a selection of mathematical models that yield some interesting insights. (Received September 06, 2019)

1154-92-544 **John G. Alford***, jalford@shsu.edu, and **Abigail Miller, William I. Lutterschmidt, William Godwin** and **Justin Williams**. *Modeling Agricultural Timber Risks and Southern Pine Beetle Aggregation*. Preliminary report.

In Texas, the southern pine beetle (SPB) is recognized as the most destructive pest of commercial pine. In 1985 an outbreak killed over 50,000 acres. Over the last two decades the landscape of pine timberlands in Texas and across the southern United States has changed in dramatic ways that affect our current understanding of SPB outbreaks. For example, vast acreage has changed hands from pine timber managers to conservation managers. Conservation managers are expected to be less likely to cut-out beetle outbreaks or use chemical control, possibly increasing risk of outbreaks. Furthermore, conservation minded managers may be more likely to convert susceptible loblolly to resistant longleaf pine, decreasing risk of outbreaks. In this research, we simulate forest growth with a spatially explicit model as well as a non-spatial model (or gap model) to better understand how spatial heterogeneities affect SPB risk assessment. We incorporate updated ecological and life history parameters selected from a query of a broad spectrum of land management strategies and use these models to examine how contrasting land management strategies affect the risk to infestation by SPB. (Received September 06, 2019)

1154-92-573 **James M Greene** and **Cynthia H Sanchez-Tapia*** (hixacyn@gmail.com), 110 Frelinghuysen Road, 1st floor, Hill Center. Department of Mathematics, Rutgers University, Piscataway, NJ 08854, and **Eduardo D Sontag**. *Control Structures of a Cancer Drug Resistance Model*.

The more recent experimental discovery of epigenetic and phenotype plasticity suggests that chemotherapy can produce drug-resistant clones. In this work, we seek a treatment protocol which maximizes the time to reach a critical tumor size. We utilize both the Pontryagin Maximum Principle and differential-geometric techniques to characterize solutions that maximize the time until treatment failure. The necessary conditions then imply that the optimal control can be synthesized as a combination of bang-bang and path-constrained arcs. We also investigate the dependence of the control structure and treatment efficacy as a function of both the chemotherapeutic cytotoxicity and the rate at which resistance is induced by the drug. Our results suggest that the latter may significantly alter the outcome of treatment, and may in fact be more important than drug toxicity in certain parameter ranges. Hence, the propensity of a treatment to promote resistance is clinically significant, demonstrating the need for further experimental and mathematical research. (Received September 07, 2019)

1154-92-608 **Anita Layton*** (anita.layton@uwaterloo.ca), , Canada. *A system biology approach for understanding sex differences in hypertension*.

Hypertension is a global health challenge: it affects one billion people worldwide and is estimated to account for > 60% of all cases or types of cardiovascular disease. In part because sex differences in blood pressure regulation mechanisms are not sufficiently well understood, fewer hypertensive women achieve blood pressure control compared to men, even though compliance and treatment rates are generally higher in women. Thus, we seek to identify which factors contribute to the sexual dimorphism in response to anti-hypertensive therapies targeting the renin angiotensin system. To accomplish that goal, we develop sex-specific blood pressure regulation models. Sex differences in the renin angiotensin system, baseline aldosterone level, and the reactivity of renal sympathetic nervous activity are represented. We conduct simulations to explain females' resistance to developing hypertension, and the higher effectiveness of angiotensin receptor blockers in treating hypertensive women (but not men), compared to angiotensin converting enzyme inhibitors. These sex-specific models are a major step towards personalized medicine. (Received September 08, 2019)

1154-92-658 **W Christopher Strickland*** (cstric12@utk.edu), 227 Ayres Hall, 1403 Circle Drive, University of Tennessee, Knoxville, Knoxville, TN 37996, and **Andrew Bernoff, Michael Culshaw-Maurer, Rebecca Everett, Maryann E. Hohn and Jasper Weinburd**. *Agent-based and Continuous Models of Locust Hopper Bands*.

Locusts gather by the millions to feed on crops, destroying fields of agricultural produce. As juveniles, wingless locusts march together and form a variety of patterns including wave fronts. We examine this collective propagation through two models: an agent-based model and a set of partial differential equations. The agent-based model is directly linked to individual behavior with respect to resource availability and differs from previous such models in that direct locust-to-locust interactions are not involved. The PDE model yields insight into the collective, analytic behavior of the aggregate group. In this talk, I will introduce both models and some of the conclusions we have been able to draw in the context of data from the biological literature and parameter sensitivity analysis. (Received September 09, 2019)

1154-92-678 **Rebecca Segal***, rasegal@vcu.edu, and **Wanda Strychalski, Sarah Bryant, Baasansuren Jadamba, Eirini Kilikian, Xiulan Lai, Leili Shahriyari, Ning Wei and Laura Miller**. *Fluid Dynamics of Nematocyst Prey Capture*.

A nematocyst is a specialized organelle within cells of jellyfish and other Cnidarians that sting. Nematocysts are also present in some single-celled protists. They contain a barbed, venomous thread that accelerates faster than almost anything else in the animal kingdom. Here we simulate the fluid-structure interaction of the barbed thread accelerating through water to puncture its prey using the 2D immersed boundary method. For simplicity, our model describes the discharge of a single barb harpooning a single-celled organism, as in the case of dinoflagellates. One aspect of this project that is particularly interesting is that the micron-sized barbed thread reaches Reynolds numbers above one, where inertial effects become important. At this scale, even small changes in speed and shape can have dramatic effects on the local flow field. This suggests that the large variety of sizes and shapes of nematocysts may have important fluid dynamic consequences. We find that reaching the inertial regime is critical for hitting prey over short distances since the large boundary layers surrounding the barb characteristic of viscous dominated flows effectively push the prey out of the way. (Received September 09, 2019)

- 1154-92-699 **Jemal Mohammed-Awel*** (jmohammedawel@valdosta.edu), 1500 North Patterson Street, Valdosta, GA 31698, and **Abba Gumel**, Arizona State University, Tempe, Tempe, AZ. *Mathematics of an epidemiology-genetics model for assessing the role of insecticides resistance on malaria transmission dynamics.*

The widespread use of indoors residual spraying (IRS) and insecticides-treated bednets (ITNs) has led to a dramatic reduction of malaria burden in endemic areas. Unfortunately, such usage has also resulted in the challenging problem associated with the evolution of insecticide resistance in the mosquito population in those areas. Thus, it is imperative to design malaria control strategies, based on using these (IRS- and ITNs-based) interventions, that reduce malaria burden while effectively managing insecticide resistance in the mosquito population. This talk is based on using a mathematical model, which couples malaria epidemiology with mosquito population genetics, to explore control scenarios. (Received September 09, 2019)

- 1154-92-750 **Marissa Renardy*** (renardy@umich.edu), **Denise Kirschner** and **Marisa Eisenberg**. *Structural parameter identifiability in age-structured epidemic models.* Preliminary report. Identifiability analysis addresses the question of whether the input parameters of a mathematical model can be uniquely identified, given observed data. This analysis is crucial in interpreting biologically relevant predictions from computational and mathematical models that rely on parameter values that are estimated from experimental datasets. In the context of epidemic models, parameter identifiability gives increased confidence in model predictions that are based on estimated parameters. While structural identifiability analysis has been extensively applied in the context of ODE epidemic models, it has not yet been widely explored for age-structured PDE models. These models present additional difficulties due to increased number of variables and derivatives as well as the presence of boundary conditions. In this work, we derive analytical identifiability results for age-structured epidemic models using a differential algebra framework. We focus on an SEIR model and explore the effects of age-dependent parameters on identifiability, as well as compare identifiability results to the corresponding ODE systems. (Received September 16, 2019)

- 1154-92-759 **Azmy S. Ackleh*** (azmy.ackleh@louisiana.edu), Department of Mathematics, University of Louisiana at Lafayette, Lafayette, LA 70504-1010, and **Md Istiaq Hossain**, **Amy Veprauskas** and **Aijun Zhang**. *A discrete-time predator-prey model: the long-term dynamics with and without evolution to resist a toxicant.* Long term exposure to a toxicant may result in evolutionary responses in a population to develop resistance for this toxicant. These responses may impact other species with which the evolving species interacts. In this talk, we present a discrete-time model describing the interaction of predator and prey populations. We then extend this model to an evolutionary model which couples the population dynamics with the dynamics of an evolving phenotypic trait that represents the mean toxicant resistance attained by the prey population. We provide results concerning the long term dynamics of this model including persistence, stability of boundary and interior equilibria and existence of cycles. We show that the evolution to develop toxicant resistance in the prey may allow both the predator and prey to persist when, without the evolution, both may go extinct. (Received September 10, 2019)

- 1154-92-772 **Wei Lin***, wlin@fudan.edu.cn. *Time series analytics: Networks reconstruction and dynamics prediction.* In this talk, I will introduce two model-free frameworks of dynamical time series analytics. One framework is to detect the causation interactions among a large group of dynamical variables, which probably recovers a network hidden in a real-world system we are concerned. The second framework is to make a forecast or future prediction based only on short-term and high-dimensional time series, which is usually believed to be a challenging task. Both frameworks use the advantages of Taken's embedding techniques, which reveals that utilization of dynamical system theory is more likely to exploit useful information from time series not only from the models but also from the real-world systems. (Received September 10, 2019)

- 1154-92-776 **Shelby M Scott*** (sscott41@vols.utk.edu), 569 Dabney Hall, Knoxville, TN 37996. *Handguns and Hotspots: Cellular Automata Models of Gun Crime in Chicago, Illinois.* Preliminary report. Gun violence annually leads to the death of 31,000 individuals and the non-fatal injury of 78,000 others. It also costs the United States \$229 billion each year. Chicago, Illinois, experienced the highest number of citywide murders in 2012 and has seen inexplicable spikes in gun crime during the past four years. Gun crime in Chicago shows temporal variation in intensity, specifically related to seasonal changes. As a city, Chicago is divided into 77 discrete community areas. Statistical analyses of available data find that gun crime in Chicago shows contagious

behavior and that socio-economic conditions within a community area contribute significantly to the number of gun crimes that occur in each region. With this information, we create a discrete-time and discrete-space cellular automata model. Each cell represents a community area of Chicago and the internal states represent the amount of crime present at a given time period. Cell states are updated at each time step based on community area characteristics and local interactions. This presentation discusses how methods from ecology, epidemiology, and mathematics can be used to observe, predict, and reduce gun crime in Chicago. (Received September 10, 2019)

1154-92-789 **Tuan A Phan*** (tuanpa86@nmsu.edu), New Mexico State University, Department of Mathematical Sciences, 1290 Frenger Mall, MSC 3MB / Science Hall 236, Las Cruces, NM 88003-8001, and **Jianjun P Tian**. *A basic model of stochastic type for tumor virotherapy.*

In this paper, we propose a common basic model of stochastic type for oncolytic viral therapy that is able to predict the success of viral treatment based on the burst size of free virus particles and noise intensities. We found a threshold that provides a sufficient and almost necessary condition for the extinction and persistence of the stochastic model. This threshold gives a complete classification for the model. Numerical simulations are also carried out to demonstrate our results. (Received September 10, 2019)

1154-92-846 **Lauren Johnson Dickman*** (lrjohns7@asu.edu), 900 S. Palm Walk, Tempe, AZ 85281. *Tumor control, elimination, and escape through a compartmental model of dendritic cell therapy for melanoma.*

Melanoma, the deadliest form of skin cancer, is regularly treated by surgery in conjunction with a targeted therapy or immunotherapy. Dendritic cell therapy is an immunotherapy that capitalizes on the critical role dendritic cells play in shaping the immune response. We formulate a mathematical model employing ordinary differential and delay-differential equations to understand the effectiveness of dendritic cell vaccines, accounting for cell trafficking with a blood and tumor compartment. We reduce our model to a system of ordinary differential equations. Both models are validated using experimental data from melanoma-induced mice. The simplicity of our reduced model allows for mathematical analysis and admits rich dynamics observed in a clinical setting, such as periodic solutions and bistability. We give thresholds for \mathcal{R}_0 that ensure tumor elimination. Bistability emphasizes a need for more aggressive treatment strategies, since the reproduction number below unity is no longer sufficient for elimination. A sensitivity analysis exhibits the parameters most significantly impacting the reproduction number, thereby suggesting the most efficacious treatments to use together with a dendritic cell vaccine. (Received September 11, 2019)

1154-92-923 **Thomas G Stojsavljevic*** (tgs59@drexel.edu). *Theory of Resource Competition Incorporating Preferential Uptake in a Water Column.* Preliminary report.

The theory of resource competition in spatially extended systems incorporating resources and biomass fluxes is far from trivial. Here, we analyze the competition between two phytoplankton species for light and two nutrients, one of which is assumed to be preferred. First, a game theoretic approximation is considered, where the depth of the phytoplankton layer is treated as the strategy the phytoplankton adopt. The evolutionary stable strategy (ESS) is the depth at which the phytoplankton are equally limited by both resources. We analytically derive the ESS of the proposed preferential uptake model. Next, we extend the game theoretic approximation by extending the classic R^* rule to allow for spatial resource gradients. A theoretical framework outlining the competition between the two phytoplankton species is then presented. (Received September 11, 2019)

1154-92-964 **Scott Greenhalgh***, 515 Loudon Road, Loudonville, NY 12211, and **Carly Rozins**. *Novel compartmental models of infectious disease transmission.*

Many methodologies in disease modeling are invaluable in the evaluation of health interventions. Of these methodologies, one of most fundamental is compartmental modeling. Compartmental models have many different forms with one of the most general characterizations occurring from the description of disease dynamics with nonlinear Volterra integral equations. Despite this generality, the vast majority of disease modellers prefer the special case where nonlinear Volterra integral equations reduce to systems of differential equations through the traditional assumptions that 1) the infectiousness of a disease corresponds to incidence, and 2) the duration of infection follows either an exponential or Erlang distribution. However, these assumptions are not the only ones that simplify nonlinear Volterra integral equations in such a way. In this talk, we illustrate new assumptions that reduce systems of nonlinear Volterra integral equations to a class of novel compartmental models. We demonstrate the consistency of these novel compartmental models to their traditional counterparts, and provide a novel compartmental model for a Pearson distributed duration of infection. (Received September 12, 2019)

1154-92-1006 **Jim Michael Cushing*** (cushing@math.arizona.edu), Department of Mathematics, University of Arizona, 617 N Santa Rita, Tucson, AZ 85721. *Survival or extinction: difference equation models of evolutionary adaptation*. Preliminary report.

Darwinian dynamic versions of difference equation population models are used to investigate the following questions: under what conditions will a population threatened with extinction be able to evolutionarily adapt to survive and what life history strategies (traits) will evolution select? Key components in the model equations are fertility versus survival trade-offs and trait dependent nonlinear density effects (including possible Allee effects). (Received September 12, 2019)

1154-92-1008 **Jim Michael Cushing*** (cushing@math.arizona.edu), Department of Mathematics, 617 N Santa Rita, University of Arizona, Tucson, AZ 85721. *Darwinian dynamic models of adaptation in changing environments*. Preliminary report.

Under what conditions can a population threatened with extinction due to a temporally degrading environment adapt by Darwinian principles so as to survive? And what life history strategies will evolution choose to accomplish this? I will use Darwinian dynamic versions of some nonautonomous difference equation population models to investigate these questions. Key components in the model equations are fertility versus survival trade-offs and trait dependent nonlinear density effects (including possible Allee effects). (Received September 12, 2019)

1154-92-1018 **Margaret Knight*** (m_knight@coloradocollege.edu), **Priscilla Cho** (priscilla.cho@emory.edu), **Lucas Fiet** (lauefiet@vols.utk.edu), **Aaron Lin** (aaron.lin@emory.edu), **Audra Hinson** (ahinson3@utk.edu) and **David Talmy** (dtalmy@utk.edu). *Viral infection rates of *Micromonas pusilla* under contrasting nutrient and light conditions*. Preliminary report.

Phytoplankton are central to oceanic photosynthesis and form the basis of the marine food web. In recent years, an increased understanding of the prevalence of viruses in marine ecosystems revealed a need to study the role of viruses in the microbial loop. Existing systems model the viral infection dynamics of phytoplankton, but the viability of these models requires assessment through comparison to data. We used the Metropolis-Hastings algorithm with experimental data to fit transfer affinity, lysis rate, host growth rates (susceptible, infected and control) and burst size in our model system of ordinary differential equations. We examined datasets which consider a host *Micromonas pusilla* (Mp-LAC38) and a virus (MpV-08T) populations under contrasting nutrient and light conditions. Host and virus exposure to both nutrient limited and low light conditions resulted in increased transfer affinities and decreased lysis rates. However, limited nutrients induced higher susceptible growth rates and burst sizes, while low light showed opposite effects. This implies that the virus has greater success infecting the host under limited conditions, while the production of free viruses is slower. These findings can be used to assess nutrient cycling and carbon fixation within ecosystem models. (Received September 12, 2019)

1154-92-1023 **Helen Byrne, Heyrim Cho, Rachel Jennings, Allison Lewis, Angela Reynolds, Blerta Shtylla and Kathleen Storey*** (storeyk@umich.edu). *Data-driven Modeling of Tumor Growth and Response to Radiotherapy*.

Despite recent technological advances that make it possible to collect detailed tumor information, clinical assessments about treatment responses are typically based on sparse datasets. In this work, we compare tumor growth models of varying complexity, in an effort to determine the level of complexity needed to accurately predict tumor growth dynamics and response to radiotherapy. We start by considering a simple, one-compartment ordinary differential equation model which tracks tumor volume and a two-compartment model that accounts for tumor volume and the fraction of necrotic cells. We investigate the structural and practical identifiability of these models, and the impact of noise on identifiability. We also generate synthetic data from a spatially-resolved, agent-based model (ABM) that simulates tumor growth and response to radiotherapy. We investigate the fit of the ODE models to ABM-generated volume data and use sequential model calibration to determine how much data is necessary to accurately infer model parameters. Our results suggest that a tumor with a large necrotic volume is the most challenging case to fit, but supplementing total volume data with additional necrotic information enables the two compartment model to significantly outperform the one compartment model. (Received September 12, 2019)

- 1154-92-1042 **Hwayeon Ryu*** (hryu@elon.edu) and **Sue Ann Campbell** (sacampbell@uwaterloo.ca). *Stability, Bifurcation, and the Emergence of Synchronization and Clusters in Time Delayed Neural Networks.*

We study synaptically coupled neuronal networks to identify the role of coupling delays in network's synchronized behaviors. We consider a network of coupled neurons where two distinct populations, each of which consists of a pair of excitatory-inhibitory neurons, interact with each other. Two pairs are coupled via excitatory neurons, while the inhibitory neuron is communicating only with its respective excitatory neuron in the same population. Linear stability analysis of the equilibrium solution of this system is conducted to derive a parameter space consisting of the coupling delays between the populations and the coupling strengths. It is shown that, when coupling delays are present, the equilibrium point may lose stability via a Hopf bifurcation from which a synchronous periodic solution emerges. Qualitatively different behaviors such as clusters between two pairs could also emerge via other types of bifurcations. Numerical simulations are conducted to confirm and supplement our analysis. (Received September 12, 2019)

- 1154-92-1162 **Lauren M Childs*** (lchilds@vt.edu), **Samantha Erwin** and **Stanca M Ciupe**. *Mathematical model of broadly reactive plasma cell production.* Preliminary report.

Strain-specific plasma cells are capable of producing neutralizing antibodies that are essential for clearance of challenging pathogens. These neutralizing antibodies also function as a main defense against disease establishment in a host. However, when a rapidly mutating pathogen infects a host, successful control of the invasion requires shifting the production of plasma cells from strain-specific to broadly-reactive. In this study, we develop a mathematical model of germinal center dynamics and use it to predict the events that lead to improved breadth of the plasma cell response. We examine scenarios that lead to germinal centers that are composed of B-cells that come from a single strain-specific clone, a single broadly-reactive clone or both types of clones. We find that the initial B-cell clonal composition, T-follicular helper cell signaling, increased rounds of somatic hypermutation, and B-cell selection strength are among the mechanisms differentiating between strain-specific and broadly-reactive plasma cell production during acute and chronic infections. Understanding the contribution of these factors to emergence of breadth may assist in boosting broadly reactive plasma cells production. (Received September 13, 2019)

- 1154-92-1184 **Yu Jin*** (yjjin6@unl.edu). *Population Dynamics in River Networks.*

Natural rivers connect to each other to form river networks. The geometric structure of a river network can significantly influence spatial dynamics of populations in the system. We consider a process-oriented model to describe population dynamics in river networks of trees, establish the fundamental theories of the corresponding parabolic problems and elliptic problems, derive the persistence threshold by using the principal eigenvalue of the corresponding eigenvalue problem, and define the net reproductive rate to describe population persistence or extinction. By virtue of numerical simulations, we investigate the effects of hydraulic, physical, and biological factors, especially the structure of the river network, on population persistence. (Received September 13, 2019)

- 1154-92-1187 **Julie C Blackwood***, 33 Stetson Ct, Williamstown, MA 01267, and **Junyan Duan**, **Mykhaylo Malakhov**, **Jordan Pellett**, **Ishan Phadke**, **Suzanne Lenhart** and **Katriona Shea**. *Transboundary Management Under Alternative Objectives with Applications to Diseases.*

Public health policy is inextricably linked with the allocation of regulatory authority between different levels of government. While infectious disease dynamics are in general well-understood, few modeling studies have considered spatially heterogeneous populations that fall under multiple administrative jurisdictions and hence under levels of government with potentially differing objectives. We pose and numerically analyze a two-patch SIRS-type model that explicitly incorporates migration and allows managers to choose between vaccination, quarantine, medication, border closure, and a travel ban on infected individuals while aiming to minimize either the number of patients or the number of deaths. In particular, we consider three classes of manager: a central government that acts equitably, local governments that act selfishly, and a non-governmental organization that seeks to maximize the overall good. We establish general guidelines for optimal governance and demonstrate several anomalous cases of interest. (Received September 13, 2019)

- 1154-92-1245 **Suzanne Sindi*** (ssindi@ucmerced.edu), 5200 North Lake Road, Merced, CA 95340, and **Fabian Santiago**. *Estimating Kinetic Rates of Prion Replication from a Structured Population Model.*

Prion proteins cause a variety of fatal neurodegenerative diseases in mammals but are harmless to yeast, making it an ideal model organism for these diseases. Determining kinetic parameters of prion replication in yeast

are complicated because experiments reflect both the disease and yeast population dynamics. We present a structured population model describing the distribution and replication of yeast prions in a population of cells. We then develop a likelihood based approach for estimating kinetic rates on simulated data and six different yeast prion strains. (Received September 14, 2019)

1154-92-1246 **Abdul-Aziz Yakubu*** (ayakubu@howard.edu), Department of Mathematics, Washington, DC 20509. *A viral dispersal linked farmed — wild salmon infectious salmon anemia virus discrete — time model.*

We introduce a discrete-time viral dispersal-linked farmed and wild salmon infectious salmon anemia virus (ISAv) disease model with intrinsically generated Ricker demographic population cycles. For the model, we use an extension of the next generation matrix approach for calculating the basic reproduction number, R_0 , to explore the effects of viral migration on the persistence or extinction of ISAv disease infection, where the salmon demographic dynamics is periodic. When $R_0 > 1$, we use simulations to study the relationship between the period of the demographic Ricker period k population cycles and the period of the endemic period k population cycles of the viral dispersal-linked farmed-wild salmon ISAv model. (Received September 14, 2019)

1154-92-1247 **Najat Ziyadi*** (najat.ziyadi@morgan.edu), Morgan State University, 1700 East Cold Spring Lane, Baltimore, MD 21251, and **Abdul-Aziz Yakubu**. *Persistence or Extinction in a Discrete-Time Nutrients-Phytoplankton-Oysters (NPO) Model of a bay ecosystem.*

In this talk, we introduce a discrete-time NPO model that describes the interactions of nutrients (N), phytoplankton (P) and oysters (O) in a bay ecosystem. Using stability analysis, we derive verifiable conditions for the persistence and extinction of phytoplankton and oysters in a bay ecosystem. Furthermore, we use examples to illustrate how human activities such as increased nutrients inflow can generate phytoplankton bloom via Neimark-Sacker bifurcation oscillations in the oyster biomass and nutrients level in the bay ecosystem. (Received September 14, 2019)

1154-92-1315 **Andre Khalil*** (andre.khalil@maine.edu), Chemical and Biomedical Engineering, 5737 Jenness Hall, Orono, ME 04469-5737. *Wavelet based multifractal analysis of loss of tissue homeostasis in mammographic breast tissue.*

When compared to normal tissue environment, the tissue in the microenvironment of tumors is disrupted, as quantified via a wavelet-based multifractal method. The density fluctuations in healthy mammographic breast tissue, characterized by their surface roughness by the Hurst exponent, H , are either anti-persistent ($H < 1/2$) for fatty tissue or long-range correlated ($H > 1/2$) for dense tissue. However, tissue regions with $H \approx 1/2$ are found predominantly in tumorous breasts. The underlying physical processes associated with a $H \approx 1/2$ signature are randomness, lack of spatial correlation, and free diffusion, which we associate with loss of homeostasis in the breast tumor microenvironment. (Received September 14, 2019)

1154-92-1316 **Shanshan Chen** (jxshix@wm.edu) and **Junping Shi***, Department of Mathematics, College of William & Mary, Williamsburg, VA 23187-8795. *Global dynamics of the diffusive Lotka-Volterra competition model with stage structure.*

The global asymptotic behavior of the classical diffusive Lotka-Volterra competition model with stage structure is studied. A complete classification of the global dynamics is given for the weak competition case. It is shown that under otherwise same conditions, the species with shorter maturation time prevails. The method is also applied to the global dynamics of another delayed competition models. (Received September 14, 2019)

1154-92-1319 **Yan Wang** and **Junping Shi***, Department of Mathematics, College of William & Mary, Williamsburg, VA 23187-8795. *Dynamics of a Reaction-Diffusion Benthic-Drift Model with Strong Allee Effect Growth.*

The dynamics of a reaction-diffusion-advection benthic-drift population model that links changes in the flow regime and habitat availability with population dynamics is studied. In the model, the stream is divided into drift zone and benthic zone, and the population is divided into two interacting compartments, individuals residing in the benthic zone and individuals dispersing in the drift zone. The benthic population growth is assumed to be of strong Allee effect type. The influence of flow speed and individual transfer rates between zones on the population persistence and extinction is considered, and the criteria of population persistence or extinction are formulated and proved. (Received September 14, 2019)

1154-92-1442 **Rithika Narayan*** (rithikanarayan2019@gmail.com), Department of Mathematics, Elwood John Glenn High School, 478 Elwood Rd, Elwood, NY 11731. *A Mixed Lattice Approach for Determination of the Areas of Cellular and Subcellular Structures.* Preliminary report.

Determination of even subtle changes in cellular / subcellular dimensions inform a number of disease processes. Hypertrophy of cardiomyocytes informs ventricular remodeling, glomerular expansion precedes glomerular scarring and an increasing cystic index is prognosticative of worsening polycystic kidney disease. The majority of shapes occurring in biology are not defined. Many of these structures are often irregular and curvilinear in nature and accurate determination of their areas is challenging. Absent set formulas to define the area of curvilinear irregular planar regions, the method of counting unit squares is used. Nonetheless, infinitesimally small that the unit square may be this method still underestimates the area of this region and can therefore misrepresent subtle changes. Developed herein is a mixed unit lattice approach that represents an improvement over the unit square method in approximating the area bounded by an irregular curvilinear planar region. In test cases comprising glomeruli and renal cysts, this method outperformed the conventional unit squares method. This work forms the foundation for refining the method used by most software today for estimating the area of irregular curvilinear cellular and subcellular structures. (Received September 15, 2019)

1154-92-1454 **Joan Ponce*** (ponce0@purdue.edu), West Lafayette, IN 47906, and **Zhilan Feng** (fengz@purdue.edu), West Lafayette, IN 47906. *Dynamics of a childhood disease model with isolation.*

Epidemiological models with exponentially distributed disease stages, although simpler to analyze, have been shown to have limitations in many cases. The model results can be improved by considering more realistic distributions. In this talk, I will present a model with gamma distributions for the exposed and infectious stages to study the impact of isolation on sustained oscillations observed in many childhood diseases. This model is an extension of the model considered in Feng and Thieme (Math Biosc. 1994), in which exponential distributions are assumed for disease stages and it is shown that the threshold value for isolation to generate sustained oscillations is very long for most childhood diseases. By analyzing the stability of the endemic equilibrium and the threshold for Hopf bifurcation of our model, we show that the minimum value for the isolation period required for Hopf bifurcation can be reduced significantly so that the model can be applicable to many childhood diseases. (Received September 15, 2019)

1154-92-1479 **Aijun Zhang*** (zhangai@tigermail.auburn.edu), Department of Mathematics, 205 Maxim Doucet Hall, University of Louisiana at Lafayette, Lafayette, LA 70504, and **Erik S. Van Vleck.** *Transition Fronts of Monostable Lattice Differential Equations in Locally Spatially Inhomogeneous Patchy Environments: Existence and Non-Existence.*

Front propagation occurs in many applied fields such as population dispersals in biology, combustion in chemistry, neuronal waves in neuroscience, fluid dynamics in physics and more. This talk is devoted to the study of spatial propagation dynamics of species in locally spatially inhomogeneous patchy environments or media. For a lattice differential equation with monostable nonlinearity in a discrete homogeneous media, it is well-known that there exists a minimal wave speed such that a traveling front exists if and only if the wave speed is not slower than this minimal wave speed. We shall show that strongly localized spatial inhomogeneous patchy environments may prevent the existence of transition fronts (generalized traveling fronts). Transition fronts may exist in weakly localized spatial inhomogeneous patchy environments but only in a finite range of speeds, which implies that it is plausible to obtain a maximal wave speed of existence of transition fronts. (Received September 15, 2019)

1154-92-1480 **Owen L Lewis*** (olewis@math.fsu.edu) and **James P Keener.** *Enhanced electrodiffusive transport across a mucus layer.*

Diffusive transport of small ionic species through mucus layers is a ubiquitous phenomenon in physiology. However, some debate remains regarding how the various characteristics of mucus (charge of the polymers themselves, binding affinity of ions with mucus) impact the rate at which small ions may diffuse through a hydrated mucus gel. Indeed it is not even clear if small ionic species diffuse through mucus gel at an appreciably different rate than they do in aqueous solution. Here, we present a mathematical description of the transport of two ionic species through a mucus layer based on the Nernst-Planck equations of electrodiffusion. Through novel application of classical techniques, steady state fluxes of ionic species are quantified, as are their dependencies on the chemical properties of the mucus gel and the composition of the bath solution. We outline a mechanism for generating enhanced diffusive flux of hydrogen across the gel region, and hypothesize how this mechanism may be relevant to the apparently contradictory experimental data in the literature. (Received September 15, 2019)

1154-92-1481 **Paulina Czarnecki*** (pczarn@umich.edu), **Jennifer Crodelle** and **Calina Copos**. *Building a Mathematical Model of the Merkel Cell*. Preliminary report.

Indentation of the epidermis is communicated to the brain through the electrical activity of nerve cells. Recent experiments suggest that cells located in the upper layer of the skin, called Merkel cells, may work together with neurons to sense light touch, but their exact role is not well defined. Mechanosensitive channels on the Merkel cell membrane are found to be activated by the indentation of the skin and are responsible for the generation of calcium-dependent action potentials. Experimental observations suggest that communication between the Merkel cell and the nerve cell occurs through synaptic transmission. We present a mathematical model that describes the action potential of a Merkel cell in response to an applied current and indentation. We use a Hodgkin Huxley formulation to model the behavior of ion channels found on the Merkel cell membrane. Additionally, we incorporate a model of the mechanosensitive nonspecific cation channel and analyze the resulting electrical activity of the cell. We show that our model qualitatively matches the experimentally measured behavior of a Merkel cell in response to electrical and physical stimuli. (Received September 15, 2019)

1154-92-1577 **Md Rafiul Islam*** (rafiul.islam@ttu.edu), **Mathew Gray** (mgray11@utk.edu) and **Angela Peace** (a.peace@ttu.edu). *Mathematical model of the Emerging Fungal Pathogen Batrachochytrium Salamandrivorans transmission on the Eastern Newt with multiple stages of infections*.

We developed compartmental host-pathogen models to examine the transmission dynamics of an emerging fungal pathogen (*Batrachochytrium salamandrivorans*, Bsal) on a North American salamander population. Multiple stages of infection are incorporated into the model, allowing disease-induced mortality and zoospore shedding rates to vary as the disease progresses. Parameter sensitivity analysis shows that the recovery and disease induced mortality rates, the length of the incubation period, and environmental zoospore degradation rates are influential parameters. Calculation of the basic reproductive number ($R_0 > 1.15$) highlights the invasion potential of this pathogen and was used to determine that direct transmission via host contact was the dominant transmission pathway for small population densities, while environmental transmission dominated in large populations. Collectively, these results suggest strategies that reduce host contacts at small densities or reduce environmental persistence of zoospores at high host densities may be effective Bsal management strategies. (Received September 16, 2019)

1154-92-1592 **Niels Halama*** (niels.halama@nct-heidelberg.de), INF 460, 69120 Heidelberg, Germany, and **Pornpimol Charoentong**, **Nek Valous** and **Rodrigo Rojas Moraleda**. *Systematic exploration strategies in heterogeneous biological and medical datasets for cancer therapy: from image analysis to multi-agent modelling*.

Cancer treatment is being changed by new types of information being available through high-throughput sequencing, proteomics and other -omics technologies. Fundamental problems arise as soon as one has to deal with $n=1$ datasets and individual patient predictions. Beyond small or complex datasets, spatial distributions within tissues govern the interaction and can have major impact on the clinical outcome especially for immunotherapy. In contrast to population based models or differential equation models, spatial interactivity is preserved in multi-agent modelling. Together with new tactics for data acquisition and analysis, a new coherent workflow allows to create a quantitative mathematical framework to allow practical guidance for clinical assessment of immunotherapy and other modalities. (Received September 16, 2019)

1154-92-1610 **Zhilan Feng**, Department of Mathematics, West Lafayette, IN 47907, **Katharine F. Gurski**, Department of Mathematics, Washington, DC, DC 20059, **Olivia Prosper**, Department of Mathematics, Knoxville, TN 37996, **Miranda Ijang Teboh-Ewungkem*** (mit703@lehigh.edu), Department of Mathematics, 210 Chandler-Ullmann Hall, 17 Memorial Drive East, Bethlehem, PA 18015, and **Margaret Grogan**, Department of Mathematics, Knoxville, TN 37996. *A vector-borne disease model with non-exponentially distributed infection and treatment stages*.

The assumption of exponentially distributed residence times in disease stages in epidemiological models is common. This assumption, which simplifies the model formulation and analysis, do not in some instances, allow for the accurate descriptions of the interactions between pathogen loads and time to reach a certain load size, and also the interaction between pathogen load size and drug levels within a human host. Here, we illustrate that through a vector-borne disease model in which arbitrarily distributed sojourn times for disease stages are considered. The model formulation is presented using two proven equivalent approaches: integral equations and partial differential equations. Analysis of the model includes the existence of equilibrium solutions and stability, which are shown to be dependent on whether the obtained control reproduction number is less than or greater

than 1. It is also shown that, when the general distributions are replaced by gamma distributions, the system of integral equations can be reduced to a system of ordinary differential equations, which has some non-trivial characteristics, only captured by non-exponential distributions for disease stages. (Received September 16, 2019)

1154-92-1617 **Chris McCarthy*** (cmccarthy@bmcc.cuny.edu), BMCC City University of New York, 199 Chambers Street, New York, NY 10007, and **Johannes Familton** (jffamilton@bmcc.cuny.edu), BMCC City University of New York, 199 Chambers Street, New York, NY 10007. *Quasispecies and error catastrophe*.

Quasispecies refers to the paradigm of viewing a species as a distribution of mutating genotypes. Error catastrophe refers to deleterious effects due to excessive mutation rates. We will discuss some of the interesting mathematics, implications, and history of the quasispecies concept and the error catastrophe theory. (Received September 16, 2019)

1154-92-1623 **Keisha Cook*** (kcook7@tulane.edu), New Orleans, LA 70125. *Analysis of Active Transport in Human Lung Cells using Stochastic Modeling*. Preliminary report.

Live cell imaging and single particle tracking techniques have become increasingly popular amongst the mathematical biology community. We study endocytosis, the cellular internalization and transport of bioparticles. We are specifically interested in titanium dioxide nanoparticles (TiO₂) in human lung cells (A549), observed locally in enlarged lysosomes. Using fluorescence microscopy, we track, classify, and analyze the movement in the cells. Single particle tracking techniques allow us to collect data in order to develop statistical methods for analyzing the movement in the cells. We classify the movement as active, diffusive, sub-diffusive, or stuck. The question becomes, how does the change in the size of the lysosomes alter transport type? The larger the lysosomes, the more obstacles present themselves inside the cell. We want to ensure that the classification of active movement is really active, given short path trajectories. We employ stochastic analysis techniques, including Bayesian inference methods, to analyze and determine the best method to classify active transport. (Received September 16, 2019)

1154-92-1627 **Anna Nelson*** (anelson@math.utah.edu), **Aaron Fogelson** and **James Keener**. *Understanding the role of fibrinogen in fibrin gel formation: Kinetic models of two monomer polymerization*.

Motivated by fibrin-fibrinogen interactions in blood coagulation, a two monomer polymerization system is presented with monomers that participate in different reaction types and have varying functionality. Using a framework pioneered by Ziff and Stell [*J. Chem. Phys.*, 73 (1980), pp. 3492 - 3499], the proposed two monomer kinetic model is studied up until gelation, which is defined as the emergence of an oligomer of infinite size in finite time. Given certain initial conditions and functionalities, we determine if the addition of a second monomer inhibits or enhances gelation with analytical and numerical results presented. (Received September 16, 2019)

1154-92-1642 **Kaitlyn E Johnson*** (k.john@utexas.edu), k.john@utexas.edu, and **Aziz Al'Khafaji**, **Eric Brenner**, **Grant Howard**, **Daylin Morgan**, **Andrea L Gardner**, **Angela Jarrett**, **Thomas E Yankeelov** and **Amy Brock**. *Integration of multimodal experimental data sets for parameter inference of a mathematical model of therapy-induced resistance in cancer*.

The ability to quantify the dynamics of drug resistance from data has direct consequences for the optimization of cancer treatments. Recent technological advances have enabled the ability to capture high-throughput “omics” data and longitudinal population dynamics, which can in theory be used to disentangle the role of differential growth and death rates from the effects of directed transitions. However, integrating these data types into a comprehensive mathematical modeling framework has remained a challenge in the field. In this work, we develop a mathematical-experimental approach to calibrate and validate a model of drug-induced resistance from multimodal data sets. We utilize single cell RNA sequencing and lineage tracing to characterize gene expression states associated with chemotherapy resistance and quantify the phenotypic composition. We combine the phenotypic frequencies with longitudinal drug response data to jointly calibrate a mathematical model of drug-induced resistance. The inferred parameters are then used to make predictions about the effect of different treatment regimens. This is the first work to our knowledge that combines single cell RNA sequencing with longitudinal data into a mathematical model to reveal the dynamics of drug-induced resistance. (Received September 17, 2019)

- 1154-92-1657 **Kaniz Fatema Nipa*** (kaniz.fatema.nipa@ttu.edu), 2500 Broadway, Lubbock, TX 79409, and **Linda J.S. Allen**. *Disease Emergence and Persistence in a Multi-Patch Stochastic Epidemic Model with Demographic, Environmental and Periodic Variability*.

Seasonality and contact patterns are subject to changes in the environment and population demography. These in turn affect the dynamics of disease spread. We investigate the effects of demographic, environmental and periodic variability on disease emergence and persistence in continuous-time, nonhomogeneous stochastic epidemic models, where the disease is spread between several regions or patches. The continuous-time nonhomogeneous stochastic processes have either discrete or continuous random variables. A multitype branching process approximation is used to estimate the probability of a disease outbreak for various patch connectivities and periodicity assumptions in transmission and dispersal. In addition, a system of stochastic differential equations is used to investigate the effect of the three types of variability near the endemic state. The implications of these results for disease control are also discussed. (Received September 16, 2019)

- 1154-92-1689 **Margaret A. Grogan*** (mgrogan1@vols.utk.edu) and **Olivia Prosper** (oprosp@utk.edu). *Non-exponentially distributed waiting times in a two-strain SITR vector-borne disease model*. Preliminary report.

Many epidemiological models assume that the waiting times for each of the disease stages are exponentially distributed in order to simplify the model formulation and its analysis. However, this is not always the correct assumption and many methods have been developed to account for the potential variability in waiting time distributions for each stage. It is important to create a more accurate representation of these waiting times when dealing with pharmacological responses due to disease treatment in order to capture the interaction between drug concentration and pathogen load within hosts. This interaction can help us model within-human pharmacodynamics and pharmacokinetics, which can help inform drug development and treatment in human populations. It is especially important to understand this interaction in order to reduce the spread and prevalence of drug-resistant strains. We use malaria as a guiding example and formulate a two-strain SITR model with general waiting time distributions in order to more accurately capture the within-human and between-human disease dynamics with treatment. (Received September 16, 2019)

- 1154-92-1751 **Faina S Berezovskaya*** (berezovskaya@howard.edu), Washington, DC 20059, and **Georgy P Karev**. *Conceptual Model for “Replicator – Genetic Parasite” Dynamics: Bifurcation Approach*.

Genetic parasites are ubiquitous satellites of cellular life forms (Koonin et.al, Biol Direct 2017). A series of Volterra type models of replicator-parasite coevolution was studied in (Berezovskaya et.al. 2018). The models show that replicator-genetic parasite coexistence is possible but not inevitable. Here we construct and study the model assuming the ratio-dependence of host-parasite interactions and parasite fertility. The model depends on 4 parameters: the carrying capacity of total system, the “damage” of replicators, the mortality rate and “effectiveness” of the parasites. The model has up to 4 equilibria including two non-trivial: a node and a saddle. Bifurcation analysis reveals the parametric area of the node stability. Loss of the node stability (in supercritical Hopf bifurcation) precedes of a conjugation and disappearance of non-trivial equilibria. We construct a phase-parametric portrait of the model and trace a sequence of dynamic behaviors of the system under variation of the model parameters. The results can be interpreted as follows. There exists a large area of model parameters and initial values in which both populations coexist or both populations go to extinction simultaneously; here replicator-genetic parasite coexistence is inevitable. (Received September 16, 2019)

- 1154-92-1761 **Iordanka Panayotova*** (iordanka.panayotova@cnu.edu), 1 Avenue of the Arts, Department of Mathematics, Newport News, VA 23606, and **Nicole Rychagov** (nrychagov@college.harvard.edu), Cambridge, MA 02138. *Modeling the effects of a novel HIV drug on human immune system with differential equations*. Preliminary report.

Human immunodeficiency virus (HIV) destroys a person’s immune system by attacking T-cells of the immune system and using them as viral replicating hosts until the immune system is no longer able to fight the infection; therefore, a person infected with HIV is more likely to get other infections that the immune system can’t fight efficiently. Currently, there is no medication that allows a person to be cured of HIV, but it can be controlled through antiretroviral therapy (ART). A novel drug didehydro-Cortistain A (dCA) was introduced recently, which targets an HIV protein; blocks reactivation of the viral genome in the cells and locks the HIV virus into a state of latency. The purpose of this study is to model the effects of the novel drug on the HIV infected immune system dynamics in humans by the means of a mathematical model. Using differential equations, different treatment strategies for the disease are investigated and compared to traditional ART. The model shows that with the novel medication, the virus rebounds much slower and to a level that can be managed by the immune

system than in the traditional ART treatment. The results of this study support the hypothesis that the novel "block and lock" therapy may be the first step on the road to a 'functional cure' of AIDS. (Received September 16, 2019)

1154-92-1792 **Damilola O Olabode*** (damilola.olabode@wsu.edu) and **Xueying Wang**. *Stochastic models for the emergence of HIV-1 drug resistance*. Preliminary report.

Drug-resistant HIV1 has caused growing concern in public health. Although combination antiretroviral therapy can contribute massively to the suppression of viral loads in patients with HIV-1. Continuing viral replication during therapy leads to the accumulation of drug-resistance mutations, resulting in increased viral load and a greater risk of disease progression. In this work, we investigate the dynamics of the emergence of HIV-1 drug resistance using stochastic models. A continuous-time Markov chain model and a stochastic differential equation model are formulated based on an ODE model for the within-host dynamics of two HIV-1 strains that include both forward and backward mutations. An estimate for the probability of disease extinction during the early stage of the drug resistance is computed by approximating the Markov chain with a multitype branching process. Accordingly, analytic estimates are validated with numerical simulations. Unlike the deterministic dynamics where the basic reproduction number serves as a sharp threshold parameter, the stochastic model indicates that there is always a positive probability of disease extinction in patients. Numerical examples illustrate the differences between the stochastic and the deterministic model. (Received September 16, 2019)

1154-92-1810 **Caitlin S. Hult*** (cshult@umich.edu), **Jennifer J. Linderman** and **Denise E. Kirschner**. *Agent-based multi-scale modeling enhances understanding of the immune response to *M. tuberculosis* infection*. Preliminary report.

Human infection with the bacteria *M. tuberculosis* (Mtb) produces a complex immune response that results in the formation of unique, emergent lung structures called granulomas. Due to the duration and dynamic nature of this immune response, as well as the involvement of processes that occur over tissue, cellular, and molecular scales, we take a multi-scale and mechanistic computational modeling approach. We build a hybrid agent-based model at the cellular scale which produces output at a tissue scale that incorporates mathematical elements including diffusion and recruitment, and we use a middle out approach to make this model multi-scale by adding molecular scale dynamics. We generate simulated granulomas whose range of spatial configurations reflects the heterogeneity observed experimentally, and through the use of uncertainty and sensitivity analyses, we identify parameters that drive such heterogeneous outcomes. We are particularly interested in how the behavior of neutrophils, a newly added model cell type, contributes to Mtb protection versus pathology. Through the parallel development of a 3D model, we analyze simulated output in 2D and 3D environments, and we present novel 3D visualization techniques to enhance conceptual understanding of the immune environment. (Received September 17, 2019)

1154-92-1907 **Erica M. Rutter*** (erutter2@ucmerced.edu), **H.T. Banks** (htbanks@ncsu.edu) and **Kevin B. Flores** (kbflores@ncsu.edu). *Non-Parametric Estimation of Intratumoral Heterogeneity from Spatiotemporal Data*.

Cancer is an extremely heterogeneous system. Intratumoral heterogeneity can be modeled by assuming that some parameters in an underlying dynamical system are not constants, but are probabilistically distributed across the population. We model Glioblastoma Multiforme (GBM), a primary brain tumor, using a random differential equation version of the reaction-diffusion equation in which the parameters describing diffusion and proliferation are random variables with an underlying distribution. The underlying distribution informs the frequency of individuals that exhibit each growth rate or diffusion rate along the set of possible values.

We present techniques for determining the underlying distribution, quantifying uncertainty in those distributions, and discussing the effects that including heterogeneity have on predictions of treatment efficacy. (Received September 16, 2019)

1154-92-1933 **Marianne C DeBrito** (debrito@umich.edu), 39711 Deepwood St., Canton, MI 48188, and **Nicolas Toumbacaris*** (nicolas.toumbacaris1@marist.edu), 728 Frankford Rd., West Babylon, NY 11704. *The Role of Utility in the Optimal Allocation Strategy of Annual Plants*.

Annual plants must allocate resources to growth of vegetative mass (roots and shoots) or reproductive mass (flowers and fruits) throughout a season of unpredictable length. We modeled this allocation strategy as an optimal control problem, maximizing the average utility of the plant's reproductive yield at the end of the season as in the seminal model of King and Roughgarden (1982). To model strategies of plants with varying reproduction priorities and growth rates, we solved for optimal strategies using general power utility functions

and production functions. With a combination of analytic and numerical computation, we found that a period of mixed allocation to vegetative and reproductive growth in the optimal strategy arises for a range of different utility functions. We also compared solutions of varying production functions on the end-of-season payoff and generated a complete characterization of the optimal control strategy. These models provided insights to the way plants allocate their resources in natural environments. (Received September 16, 2019)

1154-92-1963 **Manuchehr Aminian*** (aminian@colostate.edu), Department of Mathematics, 1874 Campus Delivery, Fort Collins, CO 80523, and **Helene Andrews-Polymenis, David Threadgill and Michael Kirby.** *Application of anomaly detection tools to mice telemetry data to infer health state.*

We discuss the application of anomaly detection tools for time series, such as Multivariate State Estimation Technique, to identify the onset of disease in mice. The data comes from an exploratory study to identify the broad range of immune responses to infection of Collaborative Cross mice to an inoculation of *S. typhimurium*. The mice are embedded with devices which continuously track basic vital signs such as core temperature and activity. These signals are approximately circadian while healthy, but exhibit a wide variety of behaviors post-inoculation. These signals are embedded using a time-delayed embedding which reveals the underlying geometry of the problem, from which we show how anomalies can be successfully detected. Finally, we explore how features built from these anomalies relate to corresponding clinical classifications of the mice's response to infection. (Received September 16, 2019)

1154-92-1973 **Colin T Barker*** and **Naveen K Vaidya.** *Modeling HIV-1 Infection in the Brain.*

While highly active antiretroviral therapy (HAART) is successful in controlling replication of Human Immunodeficiency Virus (HIV-1) in many patients, currently there is no cure for HIV-1 due to the presence of reservoirs of the virus. One of the least studied viral reservoir is the brain, which the virus enters by crossing the blood-brain barrier (BBB). The presence of HIV-1 in the brain often leads to HIV associated neurocognitive disorders (HAND), such as encephalitis and early-onset dementia. In this study we develop a novel mathematical model that describes HIV-1 infection in the brain and in the plasma coupled via the blood-brain barrier. The model predictions are consistent with data from macaques infected with a mixture of simian immunodeficiency virus (SIV) and simian-human immunodeficiency virus (SHIV). Using our model we estimate the rate of transport of the virus across the blood-brain barrier as well as viral replication inside the brain, and compute the basic reproduction number. Our model provides useful insight into viral dynamics within the brain and predicts that the brain can be an important reservoir causing long-term viral persistence. (Received September 16, 2019)

1154-92-1982 **Zahid Mondal Hasan*** (mdmondal.zahid@mavs.uta.edu). *Ebola: Impact of hospital's admission policy in an overwhelmed scenario.*

Infectious disease outbreaks sometimes overwhelm healthcare facilities. The case of West Africa in 2014 when an Ebola virus outbreak overwhelmed facilities in Sierra Leone, Guinea and Liberia. In such scenarios, how many patients can hospitals admit to minimize disease burden? This study considers what type of hospital admission policy during a hypothetical Ebola outbreak can better serve the community, if overcrowding degrades the hospital setting. Our result shows that which policy minimizes loss to the community depends on the initial estimation of the control reproduction number, R_0 . When the outbreak grows extremely fast ($R_0 \gg 1$) it is better (in terms of total disease burden) to stop admitting patients after reaching the carrying capacity because overcrowding in the hospital makes the hospital setting ineffective at containing infection, but when the outbreak grows only a little faster than the system's ability to contain it ($R_0 \gtrsim 1$), it is better to admit patients beyond the carrying capacity because limited overcrowding still reduces infection more in the community. However, when R_0 is no more than a little greater than 1 (for our parameter values, 1.012), both policies result the same because the number of patients never exceeds the maximum capacity. (Received September 17, 2019)

1154-92-2064 **Greg Dwyer*** (gdwyer@uchicago.edu), 1101 East 57th Street, Department of Ecology and Evolution, University of Chicago, Chicago, IL 60637-1573. *Using epidemiological theory to guide the use of pathogens in pest control.* Preliminary report.

Outbreaks of forest insects can cause severe defoliation, leading to widespread tree mortality, with severe impacts on timber production and recreation. Epidemics of naturally occurring viral diseases mitigate these effects, but often do not occur until defoliation has already caused high tree mortality. The ability to predict in advance when natural epidemics will occur would save the USDA Forest Service millions of dollars that would otherwise be spent on pest control. Models from human epidemiology can provide useful descriptions of the dynamics of insect viruses, but their parameters must be estimated from disease spread in nature, which in turn requires the application of advanced nonlinear fitting algorithms and high-performance computing. We used such an

algorithm to fit an epidemiological model to data for the spread of a virus of the Douglas-fir tussock moth, a pest of fir trees in western North America. In 2019, we successfully used this model to help the Forest Service identify outbreaking populations that would collapse naturally, and to identify populations that should instead be subject to insecticidal spray. Our work demonstrates the usefulness of statistical computing in guiding resource management. (Received September 17, 2019)

1154-92-2093 **Katharine Gurski*** (kgurski@howard.edu), Department of Mathematics, Howard University, Washington, DC 20059. *Analysis of the Seasonal Threshold Number Calculation for Malaria*. Preliminary report.

Mathematical epidemiological models exhibit threshold dynamics characterized by the basic reproduction number, R_0 . However, when the model is time-periodic due to seasonality, the dynamics of the system are described by a threshold number, T_0 that only reduces to the biological reproduction number when time dependence is removed. In this talk we will discuss to Posny and Wang's 2014 model that transforms the operator eigenvalue problem for the threshold number into a matrix eigenvalue problem. We will present modifications to the threshold number calculation and bounds on the error. The threshold number calculations will be presented in the context of a seasonal malaria problem with parasites sensitive and resistant to drug intervention. (Received September 17, 2019)

1154-92-2112 **Robert A Gatenby*** (robert.gatenby@moffitt.org), Tampa, FL 33612. *The ecology and evolution of intratumoral heterogeneity*.

Classic models of intratumoral heterogeneity assume a primary role for accumulating genetic mutations. However, these models typically assume that the intratumoral environment is stable. In fact, because angiogenesis is disordered, intratumoral blood flow can vary over both time and space. Thus, an alternative model of intratumoral heterogeneity proposes temporal and spatial variations in micro-environmental characteristics as a result of disordered blood flow select for different adaptive strategies with corresponding changes in genotype and phenotype. The cancer cells can affect some of these dynamics through niche construction strategies such as angiogenesis and micro-environmental acidosis which create both ecological and genetic heritages. Modeling studies find these properties can be modified to select for slower growing tumor subpopulations – a prediction confirmed through in vivo studies. (Received September 17, 2019)

1154-92-2118 **Brody H Foy*** (bfoy1@mgh.harvard.edu), Simches Research Building, Massachusetts General Hospital, 185 Cambridge Street, Cambridge, MA 02114, and **John M Higgins**. *Estimating the age of red blood cells through mathematical simulation: a validated, population dynamics approach*.

Understanding how red blood cells (RBCs) develop, grow, and respond to pathological scenarios is of immense importance to haematology, and more broadly to medicine. However, due to the challenges of in vivo experimentation, precise measurement of the age distribution of RBCs is extremely difficult.

To address this issue, we use a partial differential equation model to simulate the growth of red blood cells from initial release into the bloodstream through to senescence (death). The model uses patient-specific parameters, estimated from clinical blood sample measurements, and produces realistic estimates of RBC age distributions.

To validate the model these estimates are compared to mean RBC age estimates derived from a previously published haemoglobin glycation model (Malka et al. 2016, *Sci Trans Med*). For 52 diabetic subjects, model-derived mean red cell ages show strong concordance with glycation-model derived estimates ($\rho = 0.55$, $R^2 = 0.33$). However, unlike the glycation model, the proposed dynamics model does not require accurate estimates of average glucose, and thus can be applied to a much wider array of scenarios.

Patient data was collected through an Institutional Review Board approved study at Massachusetts General Hospital. (Received September 17, 2019)

1154-92-2133 **Daniel B Cooney*** (dcooney@math.princeton.edu), Program in Applied and Computational Math, Fine Hall, Washington Road, Floor 2, Princeton, NJ 08540. *Ecology and Infectious Disease on Large Metapopulation Graphs*.

In this presentation, we discuss reaction-diffusion models for predator-prey or SIS disease interactions in patch-structured populations with between-patch dispersal on large graphs. For these problems, we aim to unify two classical approaches for studying spatial dynamics in ecological systems: spatially-continuous models where dispersal typically follows a local diffusion operator and spatially-discrete patch models with more general network

connectivity between the patches. Making use of the recently-developed formalism of graph limits, or graphons, we derive a continuum analogue of patch reaction-diffusion models which can describe the role of dispersal in the presence of non-local connectivity schemes like small-world or power law networks. A useful feature of these continuum limits is that one can find threshold quantities for the onset of pattern formation in predator-prey models and for persistence of a disease outbreak in terms of the largest eigenvalue of the graphon's Laplacian operator, and therefore the qualitative behavior of these metapopulation dynamics is intricately linked to the topology of the dispersal network. (Received September 17, 2019)

1154-92-2142 **Leah Mitchell*** (lmitchell@wpi.edu) and **Andrea Arnold** (anarnold@wpi.edu).

Analyzing the Effects of Observation Functions in Nonlinear Filtering for the SIR Model.

Nonlinear filtering is an approach to solving the inverse problem of estimating unknown states and/or parameters of a system. The ensemble Kalman filter (EnKF) is one such algorithm that can be used for nonlinear, non-Gaussian systems within a Bayesian inference framework. One component of the EnKF is the observation function, which relates the discrete, noisy data back to the system model. The observation function can take different forms based on assumptions relating to the available data and relevant system parameters. The goal of this research is to explore the effects of selecting different observation functions in the EnKF framework, for both parameter and state predictions in epidemic models. In particular, four different observation functions, of different forms and various levels of complexity, are examined in the SIR model. Results discuss the effects of the observation function selection on the filter outputs. (Received September 17, 2019)

1154-92-2145 **Zerotti L Woods*** (zerotti.woods@jhuapl.edu), Laurel, MD 20794. *A New*

Regularization Term for Deep Neural Networks With Applications to Biological Data.

In this work, a new regularization term that penalizes the conditioning of the weight matrices in a deep neural network is presented. We give a mathematical argument that suggests that in certain situations, the conditioning number of the weight matrices have a direct impact on the error in classification. Empirical evidence suggests that improving the weight matrix associated with the output layer of a matrix improves generalizability when classifying ECG data from a benchmark data-set, and also a novel malaria infection data-set. (Received September 17, 2019)

1154-92-2146 **Paul Macklin*** (macklinp@iu.edu), 4128 Luddy Hall, 700 N Woodlawn Ave, Bloomington, IN 47408. *Exploring agent-based models of complex multicellular cancer systems with HPC.*

Cancer is complex multiscale dynamical system: dysfunctional molecular-scale signaling in cancer cells lead to increased proliferation, decreased death, migration, reduced adhesion, and aberrant metabolism. Cancer cells invade and disrupt local tissues and ultimately metastasize other sites. Chemical and mechanical interactions with other cell types can both promote and inhibit this progression. In this talk, we introduce open source models that combine PDE models of signals and substrates with discrete cell agents to understand complex multicellular cancer systems. We show work to use high performance computing to massively explore a 3-D cancer immunotherapy model, and we show how machine learning can aid model interpretation. (Received September 17, 2019)

1154-92-2155 **A. Miller*** (arm087@shsu.edu), **C. King**, **W. Godwin**, **J. Williams**, **W.**

Lutterschmidt and **J. Alford**. *A mathematical model to investigate the risk and*

prevention of future Southern Pine Beetle infestations in East Texas. Preliminary report.

Land management practices have an effect on Southern Pine Beetle (SPB) outbreaks across the United States. A particular land management strategy affects the age distribution and population density of pine trees as well as tree species diversity. Currently, land management practices are non-uniform across the pine forests of East Texas and therefore the response to potential SPB infestation is variable. We created a simulation model in the object-oriented computer software platform NetLogo to mimic the diversity of timberland across East Texas. Our model utilizes natural growth, mortality, and competition to simulate forest growth through time. We investigate how the different land management objectives increase the vulnerability to attack from SPB and use the model to help mitigate the damage to pine forest from SPB infestations. (Received September 17, 2019)

1154-92-2248 **Seth T Cowall*** (cowall_st@mercer.edu), Mercer University, Department of Mathematics, 1501 Mercer University Drive, Macon, GA 31207, and **Matthew J Oliver** and **L Pamela Cook**. *A saddle point phytoplankton bloom mechanism in a reaction-diffusion NPZ model.* Preliminary report.

Phytoplankton are the base of the marine food web. They are also responsible for much of the oxygen we breathe, and they remove carbon dioxide from the atmosphere. The mechanisms that govern the timing of seasonal phytoplankton blooms is one of the most debated topics in oceanography. Here, we present a macroscale

plankton ecology model consisting of coupled, nonlinear reaction-diffusion equations with spatially and temporally changing coefficients to offer insight into the causes of phytoplankton blooms. This model simulates biological interactions between nutrients, phytoplankton and zooplankton. It also incorporates seasonally varying solar radiation, diffusion and depth of the ocean's upper mixed layer because of their impact on phytoplankton growth. The model's predictions are dependent on the dynamical behavior of the model. The model is analyzed using seasonal oceanic data with the goals of understanding the model's dependence on its parameters and of understanding seasonal changes in plankton biomass. A study of varying parameter values and the resulting effects on the solutions, the stability of the steady-states, and the timing of phytoplankton blooms is carried out. The model's simulated blooms result from a temporary attraction to one of the model's steady-states. (Received September 17, 2019)

1154-92-2299 **Sebastian J Schreiber*** (sschreiber@ucdavis.edu), **Alexandru Hening** and **Dang H. Nguyen**. *Coevolution of Habitat Use in Stochastic Environments*.

Species live and interact in landscapes where environmental conditions vary both in time and space. In the face of this spatial-temporal heterogeneity, species may co-evolve their habitat choices which determine their spatial distributions. To understand this coevolution, we analyze a general class of stochastic Lotka-Volterra models that account for space implicitly. We define a (stochastic) coevolutionarily stable strategy (coESS) as a set of habitat choice strategies for each species that, with high probability, resists invasion attempts from mutant subpopulations utilizing other habitat choice strategies. We show that the coESS is characterized by a system of second-order equations. This characterization shows that the stochastic per-capita growth rates are negative in all occupied patches despite the species persisting. Applying this characterization to the coevolution of habitat-choice of competitors and predator-prey systems identifies under what environmental conditions, natural selection excorciates "the ghost of competition past" and banishes some predators to prey-free habitats. Collectively, our results highlight the importance of temporal fluctuations, spatial heterogeneity and species interactions on the evolution of species' spatial distributions. (Received September 17, 2019)

1154-92-2384 **Baoling Ma*** (baoling.ma@millersville.edu) and **Megan A McGee** (mamegee@millersville.edu). *Study of the recovery of nesting bald eagle population in New Jersey by statistical and mathematical models*. Preliminary report.

By the 1970's New Jersey's eagle population diminished to just one known nest because of persistent pesticides, primarily DDT. Due to the ban of DDT and tremendous restoration efforts, New Jersey's eagle population has steadily increased to 185 active pairs by 2018. The Division of Fish and Wildlife's Endangered and Nongame Species Program (ENSP) have monitored nesting bald eagles since 1982. We used data collected over the monitoring period to quantify the recovery of the nesting population in New Jersey. Mathematical and Statistical models were built to analyze the growth of nesting population and productivity rate. The population's asymptotic growth rate was calculated and the carrying capacity was predicted. Our results indicate that bald eagle population in New Jersey grows on average by approximately 10.74% per year. However, human disturbance, habitat loss, and contaminants in the food web may negatively affect the continued stability and growth of the nesting population. (Received September 17, 2019)

1154-92-2395 **Dilini Fonseka*** (dilini.fonseka@ttu.edu), 1108 Memorial Circle, Lubbock, TX 79409, and **Angela Peace**. *The number of pollinators really drives the stability of a stoichiometric plant-pollinator-herbivore model*.

Plant-pollinator interactions play an important role in the maintenance of the balance of nature. All organisms living in the environment are composed of different ratios of chemical elements. By considering the balance of essential chemical elements in nature, we can formulate mathematical models to study their role in the dynamics of the system as well as nature. We formulate and analyze a stoichiometric herbivore-plant-pollinator model. Our model includes four-dimensional systems of ordinary differential equations to represent the plant, pollinator, herbivore populations, as well as the varying nutrient levels of the plant. We analyze the dynamics of the systems such as non-negativeness and boundedness of solutions, as well as the existence and stability of boundary equilibria. We perform numerical simulations and bifurcation analysis of the model. Bifurcation analysis shows the existence of critical thresholds of number of pollinators for plants to survive and for herbivores to die. (Received September 17, 2019)

- 1154-92-2398 **Md Masud Rana*** (md-masud.rana@ttu.edu), Lubbock, TX 79409, and **K. R. Long** and **Angela Peace**. *Reduced Order Modeling in Analyzing the Dynamics of a Spatio-Temporal Stoichiometric Producer-Grazer System*.

Simulations of spatiotemporal behavior of biological systems produce large data sets that can be difficult to analyze. We use reduced-order modeling to interpret simulations of a stoichiometric producer-grazer system in terms of an underlying low-dimensional dynamical system. A well-known property of the singular-value decomposition (SVD) is that it can produce optimal low-rank approximations to a matrix. This idea can be generalized to find low-dimensional models that approximate the behavior of a dynamical system with many (perhaps infinitely many) degrees of freedom. This technique is known as reduced-order modeling. The purpose of this approach is not to reduce the computational complexity of the system, rather gain some biological insight of a large number of variables in space. We obtain and record a set of 'snapshot' results from the numerical simulations of our model to produce a reduced-order basis. Then we project our current simulation into this basis and use phase plane and bifurcation analysis to analyze the dynamics of the system. (Received September 17, 2019)

- 1154-92-2410 **Yair Castillo**, **Nora Culik**, **Leah Darwin**, **Doménica Garzón** and **Abigail Hardin***, ahardin@ou.edu, and **Anji Yang**, **Gabriela Navas**, **Carlos Castillo-Garsow** and **Karen Ríos-Soto**. *Dynamics of Prion Proliferation Under Combined Treatment of Pharmacological Chaperones and Interferons*.

Diseases such as mad cow disease in bovines, chronic wasting disease in cervids, and Creutzfeldt-Jakob disease in humans are incurable illnesses caused by prions. Prion diseases are caused when the prion protein PrP^C misfolds into PrP^{Sc}, which is capable of inducing further misfolding in healthy PrP^C proteins. Recent *in vivo* experimental results have shown that pharmacological chaperone treatment can be used to prevent this conversion, where the pharmacological chaperones act as a short-term "vaccine" against the PrP^{Sc} proteins. A second strategic approach uses interferons to decrease the concentration of PrP^{Sc}. In this work, a non-linear system of ordinary differential equations is constructed to model how these two treatments slow the proliferation of prions in the brain. Through this work it was found that interferons have a greater effect on the prion population over time, but that the pharmacological chaperones begin to effect the system earlier. This information can guide future prion experiments and inform potential treatment protocols. (Received September 17, 2019)

- 1154-92-2434 **Dana Paquin*** (dpaquin@calpoly.edu), **Brady Berg**, **Conor Carroll**, **Weston Grewe**, **Brian Knight** and **Tuyen Pham**. *Multiscale Registration-Based Image Segmentation*. Preliminary report.

Image segmentation, the process of partitioning an image into separate components, has many applications in image processing and analysis, especially as related to medical imaging. Although numerous image segmentation algorithms have been published and analyzed, many existing methods fail to produce accurate segmentation results when the images to be outlined contain high levels of noise. In this talk, we present a novel Iterative Multi-Scale Registration-Based Segmentation Algorithm (IMMRSA), which allows for the successful segmentation of noisy images via multi scale image registration. Image registration is the process of determining the optimal spatial transformation that maps one image to another. The IMMRSA algorithm is based on the multi-scale registration algorithm of D. Paquin, D. Levy, E. Schreibmann, and L. Xing (Multiscale Image Registration, Mathematical Biosciences and Engineering, Volume 3, Number 2, April 2006), which uses the hierarchical multiscale image decomposition of E. Tadmor, S. Nezzar, and L. Vese (A multiscale image representation using hierarchical (BV, L^2) decompositions, Multiscale Modeling and Simulations, vol. 2, no.4, pp. 554- 579, 2004) to accurately register highly noisy images. (Received September 17, 2019)

- 1154-92-2447 **Anh Phong Tran*** (tran.anh@husky.neu.edu), **Muhammad Ali Al-Radhawi**, **Irina Kareva** and **Eduardo D Sontag**. *Finding the optimal metronomic chemotherapy regimen: a delicate balance between immune recruitment, cancer resistance, and drug cytotoxicity*.

It has been shown experimentally that administration of low doses of chemotherapy at frequent time intervals can lead to improved and sustained anti-tumor immune responses. However, the trade-offs and mechanisms underlying the interplay between dose regimens, immune recruitment, and tumor burden reduction remain poorly understood and quantified. Based on detailed experimental data of a glioma mouse model treated with cyclophosphamide (CPA) on several high frequency low dose schedules, we developed a semi-mechanistic mathematical model that captures the delicate balance between the immunostimulatory and immunosuppressive effects of the drug, as well as its effects on the emergence of therapeutic resistance. We then used optimal control to identify therapeutic regimens that can maximize tumor reduction, lead to sustained anti-tumor immune activity or

minimize immune-based resistance. The semi-mechanistic nature of the model allows testing hypotheses about whether and what aspects of our current understanding of tumor-immune interactions drive experimentally observed results. (Received September 17, 2019)

1154-92-2450 **Jeffae Schroff*** (jeffae@schroff.com), **Sergiy Koshkin** (koshkins@uhd.edu) and **Michael Tobin** (tobinm@uhd.edu). *Optimal water flow through plants and the segmentation hypothesis.*

We study a model of water transport through plants that optimizes the water flow given the pressure-dependent conductance functions (vulnerability curves) of stem and leaves. The model predicts bottleneck behavior where the leaf segments limit the overall optimal flow through the plant in agreement with the hydraulic vulnerability segmentation hypothesis. We derive explicit conditions of such segmentation and test vulnerability curves of a number of plants for displaying it. (Received September 17, 2019)

1154-92-2451 **Naveen K. Vaidya*** (nvaidya@sdsu.edu), 5500 Campanile Drive, San Diego, CA 92182. *Modeling Zika Virus Transmission Dynamics: Parameter Estimates, Disease Characteristics, and Prevention.*

The recent devastating spread of Zika virus (ZIKV) across Americas has posed a public health emergency of international concern. Because of limited data, much remains uncertain about parameters related to transmission dynamics of ZIKV. In this talk, I will present a method for parameter estimation that utilizes mathematical models and a recently investigated complex-step derivative approximation. Applying our method to epidemic data from the ZIKV outbreaks in French Polynesia and Yap Island, we found that the parameters that can be estimated vary from Island to Island, suggesting that the same set of parameters cannot be estimated from every data set, and thus the parameter estimation based on standard techniques may provide misinformation about the ZIKV transmission dynamics. Our method allowed us to estimate ZIKV related parameters with substantially reliable confidence intervals. I will also provide the basic reproduction number estimated by our method, and explore the effectiveness of potential prevention strategies for controlling zika epidemics. (Received September 17, 2019)

1154-92-2471 **Dilruba A Sofia*** (dsafia@uamssd.edu), 1165 Pleasant St., Fall River, MA 02723, and **Nida Obatake** and **Anne Shiu**. *Classifying the Maximum Number of Steady States of Chemical Reaction Networks through Mixed Volume.* Preliminary report.

Many chemical processes can be modeled mathematically by chemical reaction networks. A chemical reaction network can give some ordinary differential equations that model the concentrations of the chemical species as time varies and conservation laws that constrain the total concentrations. Under mild hypotheses, this resulting system is a polynomial system. We can use this acquired system to solve for steady states of the network by setting the equations equal to zero and finding the values of the variables. For a generic polynomial system, the number of nonzero complex solutions is called the mixed volume. Since the concentrations of chemicals can only be described as positive real numbers, we are interested in the instances when the mixed volume equals to the maximum number of positive real solutions, because this would allow us to efficiently compute the maximum number of steady states of a network. We classify the monomolecular networks whose maximum number of steady states is equal to the mixed volume of the corresponding polynomial system. We also compute the maximum number of positive steady states and mixed volume for the genuine network with up to two reactions. (Received September 17, 2019)

1154-92-2486 **Swati Patel*** (spatel120@tulane.edu) and **Scott McKinley**. *Proteins with random effects become evolutionary capacitors.*

Evolution plays a fundamental role in the persistence of natural populations, including favorable ones, such as keystone species, or harmful ones, such as pathogens. Understanding how populations evolve has the potential to shape how we think about conservation as well as combating diseases. Classically, we have thought populations evolve through novel mutations, which may be beneficial and allow a population to adapt to new environments. More recently, scientists have suggested that certain proteins mask the function of mutations, allowing for them to accumulate and be “stored”, for when they are needed. These proteins are called “evolutionary capacitors”. While this seems extraordinary, some have argued that such a phenomenon of hiding genetic variation should be common amongst regulatory proteins. They suggest this pattern is purely the outcome of complex epistasis, i.e., nonlinear effects of proteins and mutations on traits, and the selection process. In this talk, I will give examples of proteins hypothesized to be evolutionary capacitors and then present a stochastic model that accounts for complex epistasis to test this hypothesis. Finally, I will discuss how structure in the randomness may impact whether a focal protein evolves to be an evolutionary capacitor. (Received September 17, 2019)

- 1154-92-2498 **Kayleigh Campbell** (kacampbell@wpi.edu), **Laura Staugler*** (lstaugler@wpi.edu) and **Andrea Arnold** (anarnold@wpi.edu). *Estimating Time-Varying Applied Current in the Hodgkin-Huxley Model.*

The classical Hodgkin-Huxley model is widely-used for understanding the electrophysiological dynamics of a single neuron. While applying a constant current to the system results in a single voltage spike, it is possible to produce more interesting dynamics by applying time-varying currents, which may not be experimentally measurable. The aim of this work is to estimate time-varying applied currents of different forms given voltage data. In particular, we utilize an augmented ensemble Kalman filter with parameter tracking to estimate three different applied currents, analyzing how the model dynamics change in each case. We further test the efficiency of the parameter tracking algorithm in this setting by exploring the effects of changing the parameter drift variance and the frequency of data available on the resulting applied current estimates. (Received September 17, 2019)

- 1154-92-2501 **Lydia M Bilinsky*** (lydia.m.bilinsky@gmail.com). *Modeling liver glutathione redox balance metabolism and the action of agents which promote oxidative stress in liver: applications to arsenical poisoning and acetaminophen overdose.*

Reactive oxygen species such as hydrogen peroxide (H₂O₂) are a byproduct of normal cellular metabolism, but when present at high concentrations, a situation referred to as “oxidative stress,” cause cytotoxicity. A major line of defense against oxidative stress is the glutathione redox cycle, in which H₂O₂ combines with two molecules of glutathione (GSH, the reduced form) to form glutathione disulfide (GSSG, the oxidized form); GSH is then regenerated from GSSG. The relative levels of GSSG and GSH (the “glutathione redox balance”) provide a measure of intracellular oxidative stress. I present an ODE model of the biochemical reactions which determine glutathione redox balance in liver. I then show how this model can be used as a starting point in the creation of models of liver injury induced by drugs or toxins thought to act via the induction of extreme oxidative stress. The examples of acetaminophen overdose and exposure to the arsenical dimethylarsinous acid (DMAIII) are presented; a major result is the prediction that extreme GSH depletion is the immediate cause of cytotoxic oxidative stress. I also describe a tipping-point phenomenon in which agents which only modestly increase endogenous H₂O₂ production can cause lethal oxidative stress after a delay period. (Received September 17, 2019)

- 1154-92-2505 **Anne E. Yust*** (yusta@newschool.edu), 65 W. 11th St., New York, NY 10011, and **David S. Smyth**. *Simulating Antibiotic Resistance in the Computer and Biology Labs: Ideas for Undergraduate Research.*

Antibiotic resistance is a capacious and global problem, considered to be one of the most important public health threats of the 21st century. In this talk, we aim to outline ideas for mathematics and computer science faculty to work in conjunction with biology faculty to provide a multidisciplinary approach to undergraduate research projects. We will introduce integrated computer simulation and laboratory-based course modules designed for student exploration within the curriculum, as well as independent co-curricular research projects that will deepen student understanding of antibiotic resistance and its effect on planetary health. Prerequisite knowledge for the proposed projects and activities ranges so that the biological concepts, epidemiological problems, and social justice issues of antibiotic resistance can be accessible to and pondered by all undergraduate students. (Received September 17, 2019)

- 1154-92-2511 **David F. Snyder*** (ds08@txstate.edu), MCS 470, Department of Mathematics, Texas State University, 601 University Drive, San Marcos, TX 78666. *On the Dynamics of Solvent-excluded Surface Topology under Protein Flexing.* Preliminary report.

The solvent-excluded surface (SES) of a protein has a significant role in a protein’s solvation, folding, and function. Accurate quantitative relationships between these features remain mostly unknown. The Protein Data Bank (PDB) protein database contains over 100,000 sets of coordinates, but only slightly over 10,000 unique protein chains. The PDBFlex database (<http://pdbflex.org>) provides free public information on the flexibility of protein structures as revealed by the analysis of variations between depositions of different structural models of the same protein in the PDB. Using protein clusters as identified in PDBFlex, we compute the persistent homology of the solvent-accessible surface of each conformation within a given protein cluster, to investigate the dynamics of the surface topology as the structure flexes through its different conformations. Mathematically, we are computing the Leray sheaf $\mathcal{H}^*[\pi]$ of a map π which is (generically) a surface bundle over a space of rigid embeddings of a simplicial 1-complex in euclidean 3-space. We demonstrate the results for a couple of protein clusters of biomedical interest and discuss potential applications. (Received September 17, 2019)

1154-92-2516 **Angelica Bloomquist*** (abloomquist@sdsu.edu) and **Naveen K Vaidya**. *Modeling the risk of HIV infection for drug abusers.*

It has been well established that drugs of abuse, such as opiates, are one of the leading causes for transmission of HIV in the United States and many parts of the world. Drug abusers often face a higher risk of acquiring HIV infection because target cell (CD4+ T-cell) receptor expression differs in response to morphine, a metabolite of common opiates, exposure as shown in previous studies. In this study, we use a viral dynamics model that incorporates the T-cell expression difference to formulate the increased probability of infection among drug abusers. With a more in depth understanding of the dynamics and the increased risk for these individuals, we further evaluate how preventive therapies, including pre- and post-exposure prophylaxis, affect the infection risk in drug abusers. These results are useful to devise ideal treatment protocols to combat the several obstacles those under drugs of abuse face. (Received September 17, 2019)

1154-92-2523 **Kathryn G. Link***, Department of Mathematics, University of Utah, 155 South 1400 East, JWB 233, Salt Lake City, UT 84112, and **Nicholas Danes, Matthew Sorrells, Karin Leiderman, Keith B. Neeves and Aaron L. Fogelson**. *Platelet Plug This Hole: A mathematical model of flow-mediated platelet accumulation in an extravascular injury.* Preliminary report.

Mathematical models of flow-mediated blood clotting, a highly localized biochemical and biophysical phenomenon, have been used to predict modifiers of both bleeding and thrombosis. These conditions are results of dysregulation of two intertwined processes of hemostasis; the function of platelets, a blood cell that circulates in the human vasculature, and the complex network of reactions known as coagulation. We have developed a temporally varying mathematical model of flow-mediated primary hemostasis in an extravascular injury. The model consists of a system of ordinary differential equations that describe platelet accumulation and blood flow through the injury. We coupled the contribution of increased resistance due to growth of the platelet aggregate to the flow through the injury using a Brinkman-Stokes-Brinkman calculation. Calculations were calibrated using an analogous partial differential equation (PDE) model and partial validation was performed using occlusion times and flow rates from microfluidic flow assays. Lastly, we investigated the effects of inhibition of soluble agonist-mediated platelet activation on two metrics of bleeding: occlusion times of the injury channel and the flow rates through the platelet aggregate. (Received September 17, 2019)

1154-92-2525 **Andrea Arnold*** (anarnold@wpi.edu). *Analyzing Uncertainty in Time-Varying Parameter Estimates for Biological Systems.*

Estimating and quantifying uncertainty in system parameters remains a big challenge in many biological applications. In particular, many biological systems involve parameters that are known to vary with time but have unknown dynamics and cannot be measured. Examples include the seasonal transmission parameter in modeling the spread of infectious diseases and the external voltage in modeling the spiking dynamics of neurons. This talk will address aspects of uncertainty in sequential Monte Carlo nonlinear filtering estimates of time-varying parameters, with particular emphasis on how uncertainty in the parameter estimates affects the corresponding model output predictions. Results will be demonstrated on biological applications. (Received September 17, 2019)

1154-92-2546 **Zhisheng Shuai*** (zhisheng.shuai@ucf.edu), University of Central Florida, Orlando, FL 32816. *Rigorous Analysis of Edge-Based Network Disease Models.* Preliminary report.

Edge-based network disease models, in comparison to classical compartmental epidemiological models, can better capture heterogeneity in various aspects of disease transmission processes. Rigorous analysis will be provided to show that disease dynamics of such models are completely determined by the basic reproduction number. Specifically, if the basic reproduction number is below 1, then the disease dies out; while if the basic reproduction number is above 1, then there is an epidemic. (Received September 17, 2019)

1154-92-2569 **Gary K Nave*** (gary.nave@colorado.edu), **Hadley Tallackson and Orit Peleg**. *Modeling decision-making in honey bee swarm formation.*

While a western honey bee (*Apis mellifera*) colony searches for a new nest site, the individuals will aggregate together into a swarm. Honey bee swarms, comprising roughly 10,000 individual bees, attach to tree branches or other surfaces and hang freely while scouts decide on a new nest site. Through agent-based modeling methods, we explore what individual decisions may lead to swarm formation and compare these candidate models to experimental observations of honey bee swarm formation. This exploration will connect local interactions with the resulting emergent behavior for various systems in order to address how a large number of individuals with limited cognitive abilities achieve large-scale results. This work informs both our basic understanding of collective

behavior in biological systems and potential applications to multi-agent robotic systems. (Received September 17, 2019)

1154-92-2598 **Yanyu Xiao***, 2815 Commons Way, FHW4199, Cincinnati, OH 45221. *Dynamics of an HIV-1 virus model with both virus-to-cell and cell-to-cell transmissions.*

In this paper, we incorporate both virus-to-cell and cell-to-cell transmissions into an HIV-1 virus model with nonlinear general incidence rate, intracellular delay, and cytotoxic T lymphocyte (CTL) immune responses. By Lyapunov functionals and LaSalle invariance principle, it is verified that global threshold dynamics of three equilibria, infection-free, CTL-inactivated and CTL-activated, can be explicitly described by the basic reproduction numbers. (Received September 17, 2019)

1154-92-2629 **Jasper Weinburd*** (jweinburd@hmc.edu), Department of Mathematics, Harvey Mudd College, 301 Platt Blvd, Claremont, CA 91711, and **Andrew J Bernoff, Michael Culshaw-Mauer, Rebecca A Everett, Maryann E Hohn and W. Christopher Strickland.** *Agent-based and continuous models of locusts form a traveling pulse mediated by resource consumption.* Preliminary report.

Juvenile locusts aggregate in swarms that march and forage through fields. These groups display collective behavior by forming coherent structures such as a distinct traveling pulse. As the swarm eats its way across a field it forms a clear line perpendicular to the direction of motion. We study this swarming behavior from two perspectives, an individual-based (microscopic) and a collective (macroscopic). Our agent-based model (ABM) encodes the behavior of individuals while our PDE model describes the collective behavior. In this talk we discuss how resource-dependent foraging drives the formation of the traveling swarm of locusts. (Received September 17, 2019)

1154-92-2636 **Cynthia Melissa Ramirez*** (cramir98@calstatela.edu), 5151 State University Dr, Los Angeles, CA 90032, Los Angeles, CA 90022. *Continuous Mathematical Model of Blood Vessel Formation.*

Cancer is the second leading cause of death in the United States. Tumors occur due to mutation that allow the cells to grow out of control. As nutrients and oxygen supplies are limited, growing tumors undergo metabolic stress and release Vascular Endothelial Growth Factor (VEGF). VEGF triggers the formation of new blood vessels from the existing capillaries which provide the tumor system with new supply of oxygen and nutrients. This process, called angiogenesis, plays a key role in tumor growth and its progression into cancer. The objective of this project is to investigate the effect of Avastin drug in suppressing angiogenesis and tumor growth through mathematical model and computer simulation. We develop a continuous model for tumor growth and angiogenesis. This model consists of a system of ordinary differential equations (ODEs) that govern the changes in the tumor cell density, VEGF, and oxygen concentration, as well as endothelial cell density that forms the capillaries. The least square method is employed to estimate the model parameters. This model is extended to include the effect of Avastin on angiogenesis and tumor growth. (Received September 17, 2019)

1154-92-2664 **Jordan Michael Culp*** (jculp@math.wsu.edu), Department of Mathematics and Statistics, Washington State University, PO Box 643113, Pullman, WA 99164. *A simple ODE model relating BDNF content and circadian rhythms.* Preliminary report.

The suprachiasmatic nucleus (SCN) of the hypothalamus serves as the master clock and coordinator of circadian rhythms in many living creatures. The 20,000 neurons in the SCN are entrained to the day-night light cycle by retinal ganglion cells that project directly to the SCN along the retinohypothalamic tract (RHT). Evidence suggests that brain-derived neurotrophic factor (BDNF) and its receptor tropomyosin-related kinase B (TrkB) are implicated in the gating of photic input to the SCN along the RHT. Adapting a positive-feedback model for circadian rhythms based on the dimerization and proteolysis of PER and TIM in *Drosophila melanogaster*, I aim to reproduce the results of BDNF and TrkB mediated signalling on the circadian oscillator. (Received September 17, 2019)

1154-92-2728 **Eric J Puttock*** (eputtoc@uci.edu). *Patterned heterogeneity in Colon Cancer: Xenografts, scRNAseq and Mathematical Modeling.*

Spatial patterns of metabolism have been observed in colon cancer, where clusters of cells show higher glycolytic activity than their surrounding neighbors. There is evidence that Wnt signaling regulates this pattern of metabolism. Recently, we performed single cell RNA sequencing (scRNAseq) on xenograft tumors that indicates the interactions between cell types. Here, we develop a multispecies mathematical model that incorporates cellular interactions informed by scRNAseq analysis. The model recapitulates the spotted patterns, stromal content,

morphologies and their spatial variations in the tumor. In addition, the cellular interactions indicated by scRNAseq are found to enhance the robustness of the patterns. The model predicts that blocking positive feedback signaling will alter population heterogeneity and the spatial patterning of Wnt signaling, indicating that there is a possible link between heterogeneity and drug resistance. (Received September 17, 2019)

1154-92-2756 **Shelby R Stanhope*** (shelby.stanhope@usafa.edu) and **Isaac Klapper**. *Using Stochastic Differential Equations to Model the Immune System Response to Bacterial Infection on the Surface of a Medical Implant Device*.

The occurrence of biofilms on the surface of implant devices in the human body is a problem of great concern in the medical community. In this study, we use mathematical modeling in conjunction with data collected in laboratory studies, to model the immune response to bacterial infection on the surface of implant devices. We focus on the early stage of the initiation of infection and aim to understand which factors determine whether the infection will be eliminated or will eventually lead to the development of a biofilm on the device. One of the first responders in the innate immune response are neutrophils, which follow chemical signals to locate and phagocytose bacteria. We model the growth of bacterial populations and their release of chemoattractants (which initiate the immune response) with a system of partial differential equations. The chemotactic movement of neutrophils to the sites of infection occurs through directed random motion, which is modeled using stochastic differential equations. By varying the amount of bacteria present in the initial infection and the strength of the immune response, we observe cases where the infection is completely eradicated and others where the infection persists. (Received September 17, 2019)

1154-92-2771 **Megan J Chambers*** (mjchambe@ncsu.edu). *Topological Data Analysis on Murine Pulmonary Arterial Networks Under Hypoxia-Induced Pulmonary Hypertension*.

From micro-CT images of the lungs of mice, one can observe that the pulmonary arterial network forms a rapidly branching structure. Using image analysis software, such as 3D Slicer and DGTal, spatial graphs $G = (N, E)$ were extracted from images of control mice and mice with hypoxia-induced pulmonary hypertension. These graphs include the (x, y, z) coordinates of terminal and junction nodes (N) and edges between nodes (E), as well as radii of the vessels. While it is apparent that these graphs are branching trees, their exact topological and geometric structure varies widely due to experimental conditions, parameters set during the segmentation process, and presence of hypertension. Recently, topological data analysis (TDA) has emerged as a useful tool for detecting structural differences in data. By viewing a data set as a topological object, persistent homology can be computed and provide insight into the structure of the data. Regarding the networks in this study, we ask two questions: can TDA on pulmonary arterial networks distinguish control and hypertensive mice? And what, if anything, can persistent homology tell us about the space-filling properties of the vasculature? (Received September 17, 2019)

1154-92-2782 **Amy Veprauskas** and **Tingting Tang*** (ttang2@sdsu.edu). *A stochastic discrete time population model with multiple independent environmental factors*. Preliminary report.

We develop a mathematical framework for examining the impact of multiple types of (independent) disturbances on a population. We assume that the occurrence and duration of these disturbances are independent of each other, but how these disturbances collectively impact the population is not. In particular, we allow for the combined effect of the disturbances to be greater or less than their additive effects, corresponding to synergistic or antagonistic interactions, respectively. This model uses a discrete-time Markov chain to describe the environment, with the impact of the environment on the population incorporated into the vital rates of a structured matrix model. We use this model to examine how persistence of a single or interacting species is impacted by multiple, interacting disturbance (Received September 17, 2019)

1154-92-2784 **Amanda N Laubmeier*** (laubmeier@unl.edu), **Richard Rebarber** and **Brigitte Tenhumberg**. *Applying observers to track Astragalus dynamics with reduced population counts*. Preliminary report.

Ecological data are costly to collect, particularly considering the long timescales over which we might want to observe a study system. We are therefore interested in methods which minimize the burden of data collection, or maximize the information obtained from smaller data sets. In this work, we consider the use of observers to supplement data sets in which we lack population counts for some subpopulations. The benefit to such an approach is that some subpopulations may require time-consuming samples, and only observing select subpopulations can reduce the burden of data collection. Specifically, we utilize a data set tracking Astragalus plants over several years, in which large flowering plants are easily counted but small or dormant plants are difficult to count.

We demonstrate the utility of an observer based on a discrete-time annual model for *Astragalus* reproduction. (Received September 17, 2019)

1154-92-2796 **Rebecca E. Gasper*** (rebeccagasper@creighton.edu), Creighton University Dept. of Mathematics, 2500 California Plaza, Omaha, NE 68178-0610. *Can you hear me now? A PDE Model for Stochastic Properties of the Auditory Neuron.*

We create and analyze a PDE model for electrophysiological properties of the Auditory Nerve Fiber. This elegantly illustrates the stochastic nature of neurons with small diameter. Our probability density function lives in a state space of neurons which do or do not fire; we see how an independent set of neurons behave or a probabilistic understanding of a particular frequency being heard at the other end of a single neuron. From a 2D ODE to a multi-dimensional PDE with parallel computing, we can "gain" layers of understanding and "adapt" our understanding of signal in cochlear implants. (Received September 18, 2019)

93 ► *Systems theory; control*

1154-93-2687 **Lance Dengelegi*** (ldengelegi467@r.wu.edu) and **Hasala Gallolu Kankanamalage** (hgallolu@r.wu.edu). *Physical characterization of string stability with applications to automobile platoons.*

Safety analysis of automobile platoons plays a significant role in the design of adaptive cruise control systems. The notion of String stability characterizes the longitudinal safety margins of automobile platoons. In this work, we provide two characterizations of string stability. One characterization offers a physical interpretation of string stability. The second characterization provides a simplified mathematical framework to analyze the safety margins of automobile platoons. We illustrate the practical and theoretical significance of each characterization. Additionally, we provide how these ideas support the design of Adaptive Cruise Control (ACC) systems and Cooperative Adaptive Cruise Control (CACC) systems. (Received September 17, 2019)

1154-93-2718 **Hasala Gallolu Kankanamalage*** (hgallolu@r.wu.edu). *Lyapunov Characterizations of String Stability of Interconnected Systems with Delays.*

String Stability plays a significant role in the design of adaptive cruise control systems. This notion provides an important safety measure of the longitudinal dynamics of automobile platoons. Even though string stability has a rich literature in adaptive control design, string stability of interconnected systems with delays needs careful consideration. In this work, we present a Lyapunov-Krasovskii characterization of String Stability. We will extend this characterization to applications of the safety of automobile platoons. We further provide an idea of the practical importance of the adaptive control design process. (Received September 17, 2019)

94 ► *Information and communication, circuits*

1154-94-11 **Ingrid Daubechies***, Duke University, Durham, NC 27708. *Mathematical Frameworks for Signal and Image Analysis. Lecture I: Multiresolution analysis, wavelets and beyond.* Preliminary report.

We will visit several examples in signal and image analysis, every time building an appropriate mathematical framework, ranging from functional and harmonic analysis to differential geometry. Although there will be cross-referencing, the lectures will be sufficiently independent that each can stand on its own, so that JMM participants could follow any of the lectures even if they missed one or more preceding it.

Lecture I: A review of some existing techniques in mathematics underlying image analysis, as well as some very recent developments. (Received September 3, 2019)

1154-94-12 **Ingrid Daubechies***, Duke University, Durham, NC 27708. *Mathematical Frameworks for Signal and Image Analysis. Lecture II: Diffusion methods in manifold and fibre bundle learning.*

We will visit several examples in signal and image analysis, every time building an appropriate mathematical framework, ranging from functional and harmonic analysis to differential geometry. Although there will be cross-referencing, the lectures will be sufficiently independent that each can stand on its own, so that JMM participants could follow any of the lectures even if they missed one or more preceding it.

Lecture II: Diffusion methods help understand and denoise data sets; when there is additional structure (as is often the case), one can use (and get additional benefit from) a fiber bundle model. (Received September 3, 2019)

1154-94-13 **Ingrid Daubechies***, Duke University, Durham, NC 27708. *Mathematical Frameworks for Signal and Image Analysis. Lecture III: Adaptive time-frequency methods.*

We will visit several examples in signal and image analysis, every time building an appropriate mathematical framework, ranging from functional and harmonic analysis to differential geometry. Although there will be cross-referencing, the lectures will be sufficiently independent that each can stand on its own, so that JMM participants could follow any of the lectures even if they missed one or more preceding it.

Lecture III: Many audio and medical signals (such as ECG) depending exhibit frequency profiles that “change with time”. To even define this notion, one needs time-frequency (a.k.a. microlocal) analysis. Extracting precise components can be quite challenging; this talk reviews some recent advances. (Received September 3, 2019)

1154-94-204 **Hiram H. Lopez*** (h.lopezvaldez@csuohio.edu), **Gretchen L. Matthews** and **Ivan Soprunov**. *Monomial-Cartesian codes and their duals, with applications to LCD codes, quantum codes, and locally recoverable codes.*

A monomial-Cartesian code is an evaluation code defined by evaluating a set of monomials over a Cartesian product. It is a generalization of some families of codes in the literature, for instance toric codes, affine Cartesian codes and J -affine variety codes. In this talk we use the vanishing ideal of the Cartesian product to give a description of the dual of a monomial-Cartesian code. Then we use such description of the dual to prove the existence of quantum error correcting codes and MDS quantum error correcting codes. Finally we show that the direct product of monomial-Cartesian codes is a locally recoverable code with t -availability if at least t of the components are locally recoverable codes. (Received August 22, 2019)

1154-94-414 **W. Cary Huffman*** (whuffman@luc.edu), Department of Mathematics and Statistics, Loyola University, Chicago, IL 60660. *On the Theory of Cyclic \mathbb{F}_q -linear \mathbb{F}_{q^t} Codes.*

We will discuss the general theory of cyclic \mathbb{F}_q -linear \mathbb{F}_{q^t} codes including their count and methods to construct them. We will also construct and count these codes that are self-orthogonal or self-dual under various appropriate inner products. (Received September 03, 2019)

1154-94-567 **Faisal Shah Khan*** (faisal.khan@ku.ac.ae), PO Box 127788, Abu Dhabi, 127788, United Arab Emirates. *Nash embedding: a road map to realizing quantum hardware.*

The non-Euclidean nature of the state-space of qubits (and qudits in general) gives rise to the problem of practically implementing quantum circuits in physical hardware which necessarily resides in the Euclidean space \mathbb{R}^3 . On the other hand, the Euclidean nature of bits (and dits in general) makes the implementation of reversible circuits in physical hardware relatively straight forward. I offer here a road-map to solving this problem in which the Nash embedding theorem isometrically maps qubits into bits and a quantum circuit into an equivalent reversible one, followed by the embedding of the resulting reversible circuit into \mathbb{R}^3 as a hardware graph. (Received September 07, 2019)

1154-94-807 **Doel A Rivera*** (driveralaboy@pucpr.edu), HC 08 Box 278, Ponce, PR 00731-9704. *Codes from Cayley Graphs*. Preliminary report.

Low Density Parity Check Codes (LDPC Codes) are constructed from sparse binary matrices. Most binary matrices chosen at random offer good decoding performance and coding gains. However, if LDPC codes are constructed from richly structured mathematical objects such as Euclidean finite geometries, then they offer better decoding performance due to the code’s underlying structure. Our aim is to construct LDPC codes from different Cayley graphs in order to improve upon the performance of current codes, like those used in the ATSC 3.0. In our work, we study LDPC codes from Cayley graphs and their line graphs. In the case of line graphs, we use the classical LDPC code construction. As the edge vertices of a line graph are adjacent to two vertices, we also take advantage of this fact to develop a path-based decoder. We compare our LDPC code performance with other LDPC codes. We also consider applications of codes from our Cayley graphs to Locally Recoverable Codes. This work was conducted under the supervision of Prof. Fernando Piñero, of the PR-TN REU in Probabilistic Combinatorics and Algebraic Coding Theory. (Received September 10, 2019)

1154-94-848 **Allison Beemer*** (anbeemer@njit.edu), **Sanghamitra Dutta**, **Haewon Jeong**, **Esther Lamken**, **Emina Soljanin** and **Mansi Sood**. *Longevity of User Anonymity.*

In our data-driven world, ensuring user privacy is paramount and takes many forms. One aspect is guaranteeing that the existence of communication links between particular users be kept private. In other words, safeguarding the anonymity of users. The widespread use of communication platforms may be leveraged to ensure that users are “unlinkable” by an adversary: anonymity mixes are message routers that use various techniques to disguise the identities of pairs of communicating parties. A batch mix, for example, receives and holds packets from message sources, forwarding all messages to their respective destinations in a single batch only once some large

number of messages have been accumulated. However, protecting anonymity using this strategy comes at the cost of communication delays for users. This talk explores the trade-offs between the amount of time a user is guaranteed anonymity using a mix and the incurred latency. (Received September 11, 2019)

1154-94-936 **Jon-Lark Kim*** (jlkim@sogang.ac.kr), Department of Math (Building R 1401), Sogang University, 04107, South Korea. *Recent results on Choi Seok-Jeong's orthogonal Latin squares.*

The concept of orthogonal Latin squares was known to be found first by Euler. However, Choi Seok-Jeong, a Korean mathematician, studied Latin squares at least 60 years earlier than Euler. He constructed a pair of orthogonal Latin squares of order 9 in his book called Koo-Soo-Ryak. These two orthogonal non-diagonal Latin squares produce a magic square of order 9. In this talk, we introduce some new recent results on Choi Seok-Jeong's orthogonal Latin squares such as their generalizations and other interesting properties. We also describe how to construct error-correcting codes from orthogonal Latin squares. (Received September 12, 2019)

1154-94-1074 **Qiyu Sun*** (qiyu.sun@ucf.edu), Department of Mathematics, Orlando, FL 32765. *Some Inverse Problems in Graph Signal Processing.* Preliminary report.

Graphs signal processing offers a powerful tool to handle massive data set having complicated structure and interactions. In this talk, I will present some distributed approach to some inverse problems, such as signal reconstruction, inverse filtering and phase retrieval, in graph signal processing. (Received September 13, 2019)

1154-94-1115 **Emily McMillon***, emily.mcmillon@huskers.unl.edu, and **Allison Beemer** and **Christine A Kelley.** *Graph-Based Codes & Window Decoding.* Preliminary report.

Graph-based codes have been shown to be capacity-achieving over many channels using low-complexity graph-based iterative decoders. It is well known that certain graph substructures, such as absorbing sets, can prohibit the decoder from succeeding. In this talk, we will present results on the existence of absorbing sets in LDPC code graphs, and the implication of these results for spatially-coupled LDPC codes under window decoding. (Received September 13, 2019)

1154-94-1148 **Heide Gluesing-Luerssen** and **Hunter Lehmann*** (hunter.lehmann@uky.edu). *Distance Distributions of Cyclic Orbit Codes.*

Subspace codes are collections of subspaces of the finite vector space \mathbb{F}_q^n under the subspace metric. The distance distribution of such a code is the vector whose i^{th} entry counts the number of pairs of codewords with subspace distance i . Constant dimension cyclic orbit codes, which are contained in a Grassmannian $G_q(n, k)$ and are the orbit of a subspace under an action of $\mathbb{F}_{q^n}^*$ on $G_q(n, k)$, are of particular interest. We show that for optimal such codes, the distance distribution depends only on q, n , and k . For more general codes, we can relate the distance distribution to the number of orbits which contain intersections between different codewords. (Received September 13, 2019)

1154-94-1161 **Steve Szabo*** (steve.szabo@eku.edu), Lexington, KY 40475. *On Codes over Noncommutative Rings.* Preliminary report.

In this talk we explore the considerations on the structure of the ring alphabet and the properties of the bilinear form used in defining duality needed to have the traditional tools necessary for coding theoretic study. For instance, in what settings 1) do we have MacWilliams identities, 2) does the size condition on codes hold 3) can we consider self dual codes? (Received September 13, 2019)

1154-94-1192 **Manabu Hagiwara*** (hagiwara@math.s.chiba-u.ac.jp), 1-33 Yayoi-cho, Inage-ku, Chiba City, Chiba Pref 2630022, Japan. *An Introduction to How to Relate Coding Theory and d-complete Posets.*

This is an introductory talk on the relation between coding theory, d-complete posets, minuscule elements of Weyl groups. Here coding theory means theory for deletion correcting codes. (Received September 13, 2019)

1154-94-1239 **Gilad Lerman*** (lerman@umn.edu), 127 Vincent Hall 206 Church St. SE, Minneapolis, MN 55455, and **Yunpeng Shi** (shixx517@umn.edu), 127 Vincent Hall 206 Church St. SE, Minneapolis, MN 55455. *Robust Synchronization via Cycle Consistency Inference.* Preliminary report.

We propose a strategy for improving the existing methods for solving synchronization problems that arise from various computer vision tasks. Our strategy identifies severely corrupted relative measurements based on cycle consistency information. We provide exact recovery guarantees as the ratio of corrupted cycles per edge is sufficiently small. We further guarantee linear convergence of the proposed iterative solution. We also establish stability of the proposed algorithm to sub-Gaussian noise. (Received September 14, 2019)

1154-94-1469 **R Oliver Vandenberg*** (vandenberg1@kenyon.edu), 1910 El Dorado Dr SE, Grand Rapids, MI 49506, and **Nuh Aydin** (aydinn@kenyon.edu). *A New Algorithm for Equivalence of Cyclic Codes and Its Applications.*

Cyclic codes are among the most important families of codes in algebraic coding theory for both theoretical and practical reasons. They can be viewed as building blocks of other useful codes such as Quasi-cyclic (QC) codes that contain many codes with best known parameters. Another fundamental notion in coding theory is the notion of code equivalence. Some results on equivalence of cyclic codes were obtained by previous researchers. In this work we obtain new results on this topic with useful practical implications for computer search algorithms. We prove that existence of certain maps between the roots of the generating polynomials (or cyclotomic cosets) of cyclic codes is sufficient for the codes being equivalent. The existence of this map can be checked very quickly in contrast to the difficulty of checking code equivalence in general. Therefore, it can be used to build a partitioning algorithm for equivalence classes of cyclic codes, which can greatly reduce redundancy in certain search algorithms for new codes. Implementing this idea, we already found new linear codes that are cyclic. These results may also be useful for other applications such as code-based cryptography. (Received September 15, 2019)

1154-94-1678 **Kathryn Haymaker*** (kathryn.haymaker@villanova.edu), **Jessalyn Bolkema**, **Katherine Benson**, **Christine A Kelley**, **Sandra Kingan**, **Gretchen Matthews** and **Esmeralda Natase**. *On graph substructures that impede the interval passing algorithm in compressed sensing.* Preliminary report.

In compressed sensing, sparse measurement matrices are used to recover k -sparse signals. The Interval Passing Algorithm (IPA) is an iterative algorithm that operates on the bipartite incidence graph of the measurement matrix. Sparse Tanner graphs that are used to define error-correcting codes can also be used as measurement matrices. Yakimenka and Rosnes (2016) showed that graph structures called termatiko sets characterize the IPA failure in this context. In this talk we determine the minimum size of termatiko sets for classes of combinatorial codes, including codes from finite incidence structures. Using observations from this analysis, we construct matrices that avoid small termatiko sets. (Received September 16, 2019)

1154-94-1777 **Jessalyn Bolkema*** (jbolkema@hmc.edu), **Khodakhast Bibak**, **Robert Calderbank**, **Neville Fogarty** and **Felice Manganiello**. *Families of quantum error-correcting codes from Reed-Muller codes.* Preliminary report.

Quantum error-correcting codes play an essential role in the further development of fault-tolerant quantum computation. In this talk, we construct families of CSS codes with a particular property: for every X -stabilizer x , the Y -stabilizers supported on x contain a self-dual code. This property is both powerful and useful, with implications for the transversal realization of a certain logical gate, as shown by Rengaswamy and Calderbank. We consider algebraic and computational properties of the constructed families. (Received September 16, 2019)

1154-94-1837 **Maria Bras-Amorós**, **Iwan Duursma** and **Euijin Hong*** (euijin.hong@colorado.edu). *Isometry-Dual Flags of AG Codes.*

Consider a complete flag $\{0\} = C_0 < C_1 < \dots < C_n = \mathbb{F}^n$ of one-point AG codes of length n over a finite field \mathbb{F} . The codes are defined by evaluating functions with poles at a given point Q in points P_1, \dots, P_n distinct from Q . A flag is isometry-dual if the given flag and its dual flag are the same up to isometry. For curves, including the projective line, Hermitian curves, Suzuki curves, Ree curves, and the Klein curve over the field of eight elements, the maximal flag, obtained by evaluation in all rational points different from the point Q , is self-dual. More generally, we ask whether a flag obtained by evaluation in a proper subset of rational points is isometry-dual. For a curve of genus g , a flag of one-point AG codes defined with a subset of $n > 2g + 2$ rational points is isometry-dual if and only if the last code C_n in the flag is defined with functions of pole order at most $n + 2g - 1$. We extend this characterization to all subsets of size $n \geq 2g + 2$ and show that this is best possible. We also prove a necessary condition, formulated in terms of maximum sparse ideals of the Weierstrass semigroup of Q , under which a flag of punctured one-point AG codes inherits the isometry-dual property from the original unpunctured one. (Received September 16, 2019)

1154-94-2085 **Carolyn Mayer*** (cdmayer@wpi.edu). *Distributions for Luby Transform Codes with Low Encoding Node Degrees.* Preliminary report.

Luby Transform (LT) codes are class of rateless erasure codes. The encoded symbols of an LT code are generated by dynamically constructing a bipartite graph according to a degree distribution. The performance of an LT code is tied to the degree distribution chosen. We investigate degree distributions in which nodes corresponding to encoded symbols are restricted to low degrees. (Received September 17, 2019)

- 1154-94-2476 **Parshuram Budhathoki***, Department of Mathematical Sciences, Cameron University, Lawton, OK. *Efficiency of Quantum Circuits for Multiplication Operation using different algorithms*. Preliminary report.

Theoretically, quantum computers are faster than classical computers. Practically, one of the requirements for the faster quantum computer is efficient quantum circuits for basic mathematical operations, such as multiplication. Efficiency of quantum circuits can be compared by measuring its use of computational time and space. Multiplication is one of the expensive operations in quantum computing. In this presentation, we are going to talk about quantum circuits for the multiplication operation. We will look over different algorithms and their contribution for efficient circuits. (Received September 17, 2019)

- 1154-94-2558 **Jed A. Duersch*** (jaduers@sandia.gov) and **Thomas A. Catanach** (tacatan@sandia.gov). *A Generalized Theory of Information to Improve Training and Predictions*. Preliminary report.

We present a theoretical framework in which present belief serves a critical role in how we measure information gained by a past change in belief. Thus, as evidence grows, information assessed in a previous belief update evolves as we reevaluate its importance. This framework allows us to reconstruct all information measures that admit an expectation-based interpretation from first principles and rigorously examine emergent properties. Moreover, this theory yields a new measure of information; we can compute the information contained in a model prediction in the view of additional evidence. We examine implications of this theory with three applications. (Received September 17, 2019)

97 ► Mathematics education

- 1154-97-102 **Audrey E Hendricks*** (audrey.hendricks@ucdenver.edu), University of Colorado Denver, Campus Box 170, PO Box 173364, Denver, CO 80217-3364. *Successful and sustainable undergraduate research in statistics through vertical integration of experience and horizontal integration of disciplines*.

Undergraduate research is a powerful tool for exposing students to statistics and preparing them for data science careers. Scaling statistics undergraduate research programs to more than a handful of students can be a challenging investment of time and resources, especially for junior faculty. Here, I discuss how I developed a successful and sustainable program as an Assistant Professor including structuring meetings, examples of student self-assessment, progress reports, and mentorship. To increase sustainability, I built a team with varied research experience and use tiered and peer mentorship which encourages a cohesive group and enables students to gain leadership experience. PhD students' experiences in such a program enables them to port this model and promote undergraduate research at their future institutions. We use technology to enable virtual chats, sharing code, and tracking of action items. Vertical integration protects faculty time and enables a larger team, which engages students from a wide array of academic backgrounds. This variety of expertise leads to greater creativity and helps students learn to communicate across disciplines, an in-demand skill for careers in academic and industry where multidisciplinary collaboration is common. (Received August 07, 2019)

- 1154-97-104 **Stephan Ramon Garcia*** (stephan.garcia@pomona.edu), 610 N College Ave, Claremont, CA 91711. *Guiding principles for supervising undergraduate research*.

We propose a list of general principles for developing a sustainable undergraduate research pipeline. (Received August 07, 2019)

- 1154-97-269 **Shoo Seto*** (shoos@uci.edu), 419 Rowland Hall, UC Irvine, Irvine, CA 92697. *UC Irvine Math CEO and After-School Math Programs*. Preliminary report.

I will discuss my experience as a curriculum coordinator for the UC Irvine Math CEO (Community Educational Outreach) and as an instructor at an after school math program called the Russian School of Math. (Received August 27, 2019)

- 1154-97-289 **Chad Estabrooks*** (chad.estabrooks@westpoint.edu) and **David McArdle**. *The Design and Implementation of a Course in Mathematical Research and Communication*. Preliminary report.

The design and implementation of a project-based, student-focused course in mathematical research and communication is described. Each student in the course chose an exercise from their assigned textbook from a short approved list, and were asked to write a paper containing all of the background necessary to solve this problem, as well as the final solution. This was intended to expose students to the foundation of mathematical research,

as they were asked to approach their chosen problem as though it were an open problem. At the end of the semester, students were also required to prepare and deliver a presentation on the work that they did throughout the semester. All of the students involved had taken a course in real analysis the previous semester, and the underlying material for the course was topology. None of the students had experience with the subject outside of the topology of the real line and basic metric spaces from their real analysis course. (Received August 29, 2019)

1154-97-306 **Avijit Kar***, avijitkar@gmail.com, Atlanta, GA 30324. *College Algebra Students' Understanding of Rational Functions Using Computer-Aided-Instruction (CAI).*

Postsecondary institutions continue to search for effective instructional methods to meet the needs of and retain college students who require mathematics remediation. Using MyMathLab learning aids, the study describes college students' understanding of the rational function, changes in their understanding, and ways of knowing they use to investigate problems. Four categories of evidence (conception, representation, connection, and application) were used to make conjectures about the students' understanding over three instructional phases. Case studies were developed based on evidence gathered from background interviews, clinical interviews, and participant task observations. The students' understanding contained a central theme: the rational function as a collection of problems to do. Three categories of beliefs are described in the report based on the study. (Received August 30, 2019)

1154-97-309 **Ellina Grigorieva*** (egrigorieva@twu.edu), PO BOX 425464, Denton, TX 76204. *Teaching Methods of Solving Nonstandard Problems.*

It is known that it is difficult for students to solve nonstandard mathematical problems - those that are not directly solved by standard methods, but instead rely on understanding and synthesis of various mathematical ideas. In this talk, numerous strategies and methods that can be used to solve such problems will be presented. A friendly, non-intimidating teaching approach emphasizes the connections between different areas of mathematics and offers several different ways to solve the same problem. Topics covered include functions and their properties, polynomials, trigonometric and transcendental equations and inequalities, optimization, and differential equations. This presentation will appeal to college students, whether preparing for the exam or improving their math skills, as well as anyone who loves math. University professors will be able to use the presented ideas as an additional resource in the classroom to renew the traditional curriculum, stimulate abstract thinking and inspire original thought. (Received August 30, 2019)

1154-97-321 **Michael A Radin*** (michael.radin@rit.edu), Rochester Institute of Technology, School of Mathematical Sciences, College of Science, Rochester, NY 14623, and **Olga A Orlova** (olga.a.orlova@gmail.com), Munich Technical University, Department of Mathematics, Munich, Germany. *University level teaching styles with high school students and international pedagogical innovations.*

The primary goal is to implement university level material and teaching styles with high school students. The first question to address is if it can be done? The second question is how to implement it? This will lead to several challenges that will emerge and to new pedagogical and especially international pedagogical innovations in order to handle the challenges. The new ideas can work in the American system but additional challenges will arise while implementing them overseas. (Received August 31, 2019)

1154-97-334 **Jasmine H. Hocutt*** (jhhocutt@ua.edu). *Email Correspondence: Reaching Out Individually for Student Success.* Preliminary report.

Today's university classes are growing by leaps and bounds – as such, students can easily get lost in the shuffle. It is getting harder to identify struggling students early on, and to encourage students who are performing well. Students in large lecture classes feel that they are part of a faceless mass, where their instructors do not care for their individual success.

To counter this feeling, we will explore the effect of reaching out individually to students, both in traditional and large lecture classrooms. We have experimented with emailing students on a personalized basis regarding their performance on assignments and exams, and expressing availability for help. We will then analyze data collected on the resulting overall performance of the students so contacted. It is hoped that this data will provide valuable insight into the effect of personal communication in an impersonal class setting. (Received August 31, 2019)

1154-97-388 **Brian P Katz*** (briankatz@augustana.edu). *Mathematics is Not Neutral.*

Mathematics is not a neutral worldview, and some of the common aspects of our disciplinary epistemologies hinder justice and rehumanizing efforts or actively support problematic systems. However, the power of these

worldviews and epistemologies makes these issues hard to see and resists their critique and change. In this session, I will discuss some of my efforts to have these conversations and make these issues visible with students and faculty, attending to the impact and responsibility coming from my identities. (Received September 03, 2019)

1154-97-397 **Ricela Feliciano-Semidei*** (ricela@niu.edu). *Use of Computer Software to Do Mathematics and the Mathematics Achievement of Students in Puerto Rico Using Restricted 2015 NAEP Data.*

This quantitative study explored the relationship between the mathematics achievement patterns of eighth grade students in Puerto Rico and their use of computer software application programs for doing mathematics. The theoretical framework used is the educational production function. The researcher analyzed 2015 restricted National Assessment of Educational Progress (NAEP) mathematics data. Data analysis consisted of descriptive statistical analysis and multilevel modeling analysis. Control variables to measure socioeconomic status and absenteeism were included in the multilevel model. Results of this study showed that average scores on NAEP 2015 were higher for students who use computer programs to do mathematics with less frequency than students who use it with more frequency. Understanding the relationship between the use of computer programs to do mathematics and the mathematics achievement of these students help the mathematics education community to cautiously create policies that do not focused on frequency of using technology. The researcher provided a discussion of the results and implications for researchers, administrators and teachers that would help them to target on the improvement of mathematics achievement of these students. (Received September 03, 2019)

1154-97-453 **Sarah Patterson*** (pattersonse@vmi.edu) and **Blain Patterson** (pattersonba@vmi.edu). *Thanos Population Dynamics.*

In the end of the “Avengers Infinity War,” the villain Thanos snaps his fingers and turns half of all living creatures to dust with the hope of restoring balance to the natural world. How does this affect the long term behavior of various species? Investigate the validity of his claim by modeling various population dynamics such as unconstrained and constrained growth, competing species, and predator-prey. In this talk, we will provide instructors with an engaging way for students to analyze the behavior of various population models. (Received September 04, 2019)

1154-97-530 **Vlajko L Kocic*** (vkocic@xula.edu), 4912 Elmwood Pkwy., Metairie, LA 70003. *The course “Mathematical Modeling in Life Sciences” at Xavier University of Louisiana.*

The course “Mathematical Modeling in Life Sciences” was introduced several years ago at Xavier University of Louisiana. It is a sophomore level course designed for mathematics and science majors interested in applied mathematics with emphasis to applications in life sciences. The only mathematical prerequisite for the course is completion of Calculus I which present major challenge. In this talk we discuss several challenges and ways to overcome them such as limited mathematical background, mathematically diverse student population, interdisciplinary aspect of the course, as well as the selection of topics, students’ projects, and materials specifically developed for this course. (Received September 06, 2019)

1154-97-558 **Karen M. Bliss*** (blisskm@vmi.edu). *Preparing for the workforce through integration of technology, modeling, and writing.*

In recognition of the fact that Virginia Military Institute has very few math majors moving on to graduate school, we have shifted our attention away from a pure curriculum to an applied mathematics program. This talk will focus on changes to our differential equations course and how we use modeling competitions. Both are designed to encourage students to think beyond the academic environment and both incorporate components of technology and writing with a focus on audience. (Received September 06, 2019)

1154-97-646 **Katharine Ott*** (kott@bates.edu). *GirlsGetMath: Building and expanding a day program for high school students.*

GirlsGetMath is a weeklong summer program for high school students held annually at the Institute for Computational and Experimental Research in Mathematics (ICERM) since 2014. The content of the program emphasizes experimental and computational aspects of mathematics through interactive presentations and lab activities in MATLAB. Efforts are currently underway to share the GirlsGetMath curriculum and to train other faculty to lead a GirlsGetMath program at their home institution. This talk will describe the development of the curricular materials, recruitment of participants and TAs, and logistics of the GirlsGetMath Train-the-Trainer program. (Received September 09, 2019)

- 1154-97-671 **Chad M. Topaz*** (cmt6@williams.edu). *Data science for the liberal arts.* Politics, art, science, commerce, and numerous other domains increasingly rely on the large quantities of data enabled by a revolution in computing over the past 30 years. The field required to support work in these domains is data science. While data availability has grown rapidly, there is a severe shortage of data skills in the national job pool. Students graduating with a skill set that combines data literacy with a liberal arts background will be prepared to not only gather and manage big data, but also to think critically, ask questions, communicate findings, and raise ethical concerns. In this talk, we present some the opportunities and challenges associated with designing data science pathways for undergraduate non-majors. (Received September 09, 2019)
- 1154-97-740 **Vishal Arul** (varul.math@gmail.com) and **Gweneth McKinley*** (gweneth@mit.edu). *Program design and student outcomes at MIT $\sqrt{\text{mathroots}}$.* $\sqrt{\text{mathroots}}$ is a 14-day summer program hosted by the MIT math department since 2015 for high-potential high school students from underrepresented backgrounds or underserved communities who are interested in exploring creative topics in mathematics and problem solving. Students discover new mathematical ideas and learn problem solving skills through a series of classes, group activities, and invited lectures led by a team of MIT graduate students and instructors with diverse experiences teaching both research and competition math. We will talk about our experiences with $\sqrt{\text{mathroots}}$, including outreach strategies, organizational practices, and curriculum development. We will also discuss program outcomes and feedback we have received from students. (Received September 10, 2019)
- 1154-97-941 **Mohamed Omar*** (omar@g.hmc.edu), 301 Platt Boulevard, Claremont, CA 91711. *The Art and Craft of Problem Design: For Undergraduate Research.* Following the speaker's JMM Invited Address "The Art and Craft of Problem Design", the speaker will address the question "How do I best design problems for undergraduate research?" (Received September 12, 2019)
- 1154-97-1043 **Steven J Miller*** (sjm1@williams.edu), Department of Mathematics and Statistics, Williamstown, MA 01267. *YouTube University II: Shared upper level math courses across schools.* For the past several years I've recorded my lectures and uploaded them to YouTube. At last year's JMM I discussed technology issues and benefits. In this talk I'll report on progress over the past year, in particular running courses with remote students, and co-teaching a class with colleagues at multiple institutions. (Received September 12, 2019)
- 1154-97-1136 **Luis Saldivia*** (lsaldivia@ets.org) and **Michelle Worthington** (mworthington@ets.org). *Integrating Learning, Cognitive and Measurement Theories for Identifying Gifted Mathematics Students.* Preliminary report. Identifying and building an instrument to reliably and validly identify gifted mathematical students requires clearly defining the constructs associated with several complex skills. In this section, we will discuss advances in cognitive and learning theories towards specifying the skills and abilities of gifted mathematical students, and the mental processes they engage in when solving complex mathematical problems. Additionally, recent advances in measurement theories will be discussed with the goal of providing a framework for developing assessments that can elicit sufficient evidence of the degree to which students exhibit the constructs associated with gifted mathematical students, and how to use these assessments to identify and train these students. (Received September 13, 2019)
- 1154-97-1145 **Emily Cilli-Turner*** (ecilli-turner@laverne.edu), **Robin Wilson** (robinwilson@cpp.edu) and **Gail Tang** (gtang@laverne.edu). *Lessons from A Multi-Tiered Approach to Mathematics Enrichment.* In this talk, we will report on lessons learned during one and a half years of an after-school mathematics enrichment program for middle and high school students. Our program is run at a community program called Pomona Hope, which is a free after-school program for students in Pomona, CA and has a high number of low-income and underrepresented students. We aim to provide exposure to mathematics outside of the normal curriculum for these students and have successfully taught material in cryptography and mathematical modeling. The program is run using a multi-tiered approach to teaching and learning; we, university faculty, bring together college students from our respective institutions, and high school students from Pomona Hope, to work on mathematics and teaching methods, and co-facilitate a mathematics lesson for middle school students. This model, borrowed from the Young People's ProjectTM (YPP) to teach mathematics to youth outside of school time, has been a good way to not only teach mathematics, but also to develop leadership and teaching skills for the college and high school students. (Received September 13, 2019)

1154-97-1153 **Scott Annin*** (sannin@fullerton.edu), 800 N. State College Blvd, Department of Mathematics, Fullerton, CA 92831, and **Anael Verdugo** (averdugo@fullerton.edu), 800 N. State College Blvd., Department of Mathematics, Fullerton, CA 92831. *A Comprehensive Model for Preparing Underrepresented Students for Graduate Programs in Mathematics.*

Many enrichment programs for undergraduates in mathematics are aimed at providing high quality research experiences for students aspiring to enter graduate programs in the mathematical sciences (REUs, for example). The goal of our National Science Foundation program – Graduate Readiness and Access in Mathematics (GRAM) – is to provide a more comprehensive preparation program that includes several additional components. These components include rigorous content preparation, training for the general and subject mathematics GRE, training and development of problem solving skills, coaching and mentoring on professional presentation skills, training on time and stress management, assistance with graduate school applications and decisions, participation and attendance in a variety of professional conferences, and cultural and family support.

Our institution hosts a large population of underrepresented mathematics majors, from which we have mentored 18 GRAM Scholars since 2016. In this presentation, we will look at some of the outcomes from our project thus far, and we will share stories of the challenges and successes of our program. We hope that our experience in directing GRAM will be useful to others wishing to organize similar programs in the future. (Received September 13, 2019)

1154-97-1309 **sarah-marie belcastro*** (director@mathily.org). *Identifying very strong high-school students + An early-college REU.*

We will describe two aspects (one relevant to discovery, and one relevant to training) of the {MathILy, MathILy-Er, MathILy-EST} ecosystem. All three programs are very selective intensive inquiry-based summer mathematics experiences.

- (1) MathILy and MathILy-Er use an admissions instrument, the Exam Assessing Readiness (EAR), to select high-school participants. The EAR also functions as a tool for discovering students who have untapped potential. We will use the EAR as a context to discuss the creation of instruments to identify mathematically capable students, and describe *why* the attributes we highlight are important.
- (2) MathILy-EST is a new early-college NSF-funded Research Experience for Undergraduates (REU) that is co-sited with MathILy. We will explain how the MathILy-EST REU is structured and focus on the aspects designed to train students with less formal classwork background to approach open problems quickly.

(Received September 14, 2019)

1154-97-1356 **Megan Olivia Powell*** (mpowell14@unca.edu), One University Heights, Department of Mathematics, University of North Carolina Asheville, Asheville, NC 28804, and **Angela Antonou** and **Sara Liesman**. *Impacts of a cross-institutional undergraduate research experience workshop on student understanding of and self-efficacy for research.*

There are many perceived benefits to undergraduate student research; however, students may not have a full understanding of the research process prior to engaging in a project. The Intercollegiate Biomathematics Alliance Cross-Institutional Research Experience (IBA-CURE) workshop seeks to help students understand more about the research process before embarking on a full research project. In this talk, we discuss an analysis of students' understanding of academic research as well as their self-efficacy for conducting research. Students were surveyed before and after the workshop in 2018 and 2019 to determine changes in their understanding of research, effective collaboration, their role, and their perception of their own skills specific to biomathematics research. Examples of specific results that will be discussed are faculty encouragement being the highest motivator for participation and a shift in perception of research from solving a particular problem to contributing to a field by discovering or creating new knowledge. (Received September 15, 2019)

1154-97-1426 **Aris Benjamin Winger*** (aris.winger@gmail.com), 1012 Jonathan Lane, Tucker, GA 30084. *Reaching Inside before Reaching Out: Acknowledgement before attempting to provide Access.*

The challenges in reaching out and strengthening the pipeline are numerous but are oftentimes viewed in one direction. What are the types of activities, personal and professional, in which mathematicians should participate before reaching out? What admissions must be made about ourselves before we encourage others who do not navigate this world the way we do to join us? What must we acknowledge about our discipline prior to visiting groups that have been potentially left out by the discipline itself? Drawing upon some strategies and exercises in

the area of Equity in Mathematics, this talk will seek to challenge each mathematician to look within to examine their own beliefs and thoughts about mathematics and its community before looking outward to ask anyone else to join. (Received September 15, 2019)

1154-97-1478 **Jennifer A Czocher*** (czocher.1@txstate.edu), San Marcos, TX 78666, and **Elizabeth Roan**, San Marcos, TX 78666. *SCUDEM Update: Students' Expected and Measured Gains.*

We present an update on student gains from participation in SCUDEM. We will first offer a synthesis of pre- and post-competition survey data from 3 rounds of the competition to describe the expectations students held going into the competition, the extent to which they were met, and how their expectations compared to those of other stakeholders. Results show that students, researchers, and designers held differing expectations. We explore implications of this finding for broadening participation. In addition, we will give preliminary results of a study linking gains in student self-efficacy to gains in modeling competencies. (Received September 15, 2019)

1154-97-1478 **Wisam Bukaita*** (wbukaita@ltu.edu), wbukaita@ltu.edu, Southfield, MI, 4807, MI 48075. *The students from knowledge receiver to the pivot of the group leader.* Preliminary report.

Implementing project-based modeling in differential equation class reflects an amazing improvement in understanding differential equations concepts and how to utilize this tool in a real-life problem. Students explore through differential equations modeling a virtual experience can be formed by launching a new journey based on differential equations modeling. Instead of limiting differential equations knowledge within the classroom walls, students discover that the real differential equations are outside the classroom, and the classroom's lectures are just a window to see that. After utilizing differential equations tools in a real-life problem, the students beginning a valuable journey to read, to search, to discover, to analyze, and to design math solving a real problem. After performing the project, the students become a researcher, a designer, a problem solver or a decision-maker. (Received September 15, 2019)

1154-97-1483 **Max Warshauer*** (max@txstate.edu), Math Dept, Texas State University, 601 University Dr., San Marcos, TX 78666, and **Hiroko K Warshauer** (hw02@txstate.edu), Math Dept, Texas State University, 601 University Dr., San Marcos, TX 78666. *Research about Math Outreach Programs.*

Math Outreach Programs present a unique opportunity to research, develop and test new ideas in the teaching and learning of mathematics. By systematically studying what we do in these programs, we can improve what we do while connecting educational theory and research to practice. This talk will describe several research projects that use Math Camps as a laboratory for educational research. In addition, a list of proposed areas for future work will be described. The areas that we will address include: how do programs identify and recruit participants; what special efforts are needed to reach out to and include disadvantaged students; what do programs do to address the needs of all participants; what is the goal of the program –is it limited to mathematics, or are other factors considered; how is the curriculum developed; what is the role of the teachers; are there counselors and what is their role; how do we measure our success in each of the areas above? Every program has its own unique collection of participants. What are the features of the unique “communities of practice” that are created, and how does each component support the other parts? Finally, how does one connect the program to ongoing research at a university? (Received September 15, 2019)

1154-97-1554 **Michael Ferrara*** (michael.ferrara@ucdenver.edu). *Promoting Equity in Undergraduate Research with Work Study.*

Undergraduate research is a transformative opportunity for undergraduates in a breadth of disciplines. Furthermore, graduate programs in the mathematical sciences increasingly look for undergraduate research as one indicator of students' fitness for research. At some institutions, many (or all) opportunities for undergraduate research may be “extra-curricular” and are often unpaid. This creates a troubling equity gap, particularly for students with significant financial need who may need to maximize their out-of-class time working.

One remedy for this issue that has been growing in popularity nationwide is the use of federal and state work-study funds to provide paid undergraduate research opportunities for qualifying students. The structure of work-study dictates that about 75% of students' wages covered by financial aid, making such programs “force multipliers” for institutional funds. We present our experiences creating the Education through Undergraduate Research and Creative Activities (EURECA!) program at the University of Colorado Denver. Following our path from a small pilot in 2016-17 to a program supporting over 70 scholars, we will share successes, challenges and lessons learned that may be of interest to others hoping to start such a program. (Received September 16, 2019)

1154-97-1594 **Kristin M. Kurianski*** (kmdett@mit.edu) and **Jennifer French**. *Two models for blended learning in mathematics courses using online content*. Preliminary report.

Success in calculus and differential equations courses has a large effect on success in later engineering courses at MIT. We created a set of interactive online resources that we have used as a free MOOC, as the main course material for a flipped classroom on campus, and as supplementary reading and online homework (blended learning) for a traditional lecture course. Each course is designed to provide motivation, encourage transfer to later engineering courses, and keep students actively engaged with the content. In this talk, we describe the motivation for creating these online resources, an overview of content type and structure, and provide evidence of student engagement via data collected from end-of-term surveys. We also outline two examples of ways in which this online content is utilized in teaching on-campus calculus and differential courses. (Received September 16, 2019)

1154-97-1613 **Gulden Karakok*** (gulden.karakok@unco.edu). *Mathematics Education Research Ideas for Undergraduate Research*.

Mathematics education research is an area that is accessible to many undergraduate students to learn more about the existing studies and tools for research. As undergraduate students gain interest in educational questions focusing on mathematics teaching and learning at various levels, it is important to develop an understanding what it means to conduct research and ask researchable questions. In this talk, I will compare and contrast mathematics education research to other areas of research, share some examples of researchable questions and study designs. Furthermore, I will share a brief summary of existing knowledge and possible “open” research questions. (Received September 16, 2019)

1154-97-1875 **Scott Jeremy Baldrige*** (sbaldrid@math.lsu.edu), Department of Mathematics, Baton Rouge, LA 70803. *Psychological Drivers of Mathematically Gifted Students*. Preliminary report.

Common psychological drivers for gifted students include the need for achievement, belongingness, or status. They help students develop the right mixture of talent, interest, commitment, and persistence needed to become researchers in mathematics. But there are other psychological drivers as well, some that are possibly unique to mathematics as a field, others that emerge from early childhood trauma. Searching for and identifying these drivers can help the math community find gifted students that may otherwise be overlooked. In this talk we explore some of these other types of psychological drivers and discuss ways to work with students that have them. (Received September 16, 2019)

1154-97-1961 **Shea Swauger*** (shea.swauger@ucdenver.edu), 1100 Lawrence Street, Denver, CO 80204, and **Michael Ferrara** (michael.ferrara@ucdenver.edu), **Dennis DeBay** (dennis.debay@ucdenver.edu), **Diane Fritz** (diane.fritz@ucdenver.edu) and **Matthew Mariner** (matthew.mariner@ucdenver.edu). *The Data to Policy Project: A Framework for Social Justice Advocacy in Mathematics Education*.

In 2015, students at the University of Colorado Denver were deeply upset by several high-profile police shootings of Black people in communities throughout the United States. During the following weeks, campus-wide dialogues coupled with students’ desires to find ways to effect real change led to the formation of the Data to Policy (D2P) project. D2P challenges students to explore public data and use quantitative and policy-making tools to ask difficult questions related to social justice and propose policies to their local, regional and national communities. From the initial focus on issues in policing, D2P has grown to include gentrification and affordable housing. Over four semesters, more than 250 students have presented nearly 100 projects developed through courses in mathematics and statistics, public affairs, education, and other disciplines. We will outline the development of D2P and highlight successes and challenges for mathematics educators to consider. Our model is scalable for many different institutions and policy areas, so we will focus on the crucial development of support mechanisms for participating students, faculty and courses. We’ll also share examples of student projects and discuss the outcomes of some new developments for Fall 2019. (Received September 16, 2019)

1154-97-1976 **Cezar Lupu*** (cezar.lupu@ttu.edu), 2500 Broadway, Lubbock, TX 79409. *How to coordinate a problem at the International Mathematical Olympiad?* Preliminary report.

In this talk, I shall describe my experience as a coordinator at the 2018 International Mathematical Olympiad (IMO 2018) which was held in Cluj-Napoca, Romania.

The problem I coordinated with other Romanian colleagues was N4 from the IMO Shortlist. We discuss several alternative approaches to this problem where p-adic valuation plays an important role.

Also, we present the social aspect of the Olympiad in which the predominant factor was the interaction with other mathematicians around the world. (Received September 16, 2019)

1154-97-2002 **Brandy S Wieggers*** (brandy.wieggers@cwu.edu), 400 E University Way, Ellensburg, WA 98926. *Creating Community-Responsive Mathematical Outreach Programs.*

For more than two decades, K-12 students have been gathering in classrooms after school to explore problem solving and non-curriculum based mathematics in programs run by universities across the United States called Math Circles. While there have been many successful variations of these programs, recent efforts have focused on programs that went beyond existing models and strived to create more engaged community-responsive projects that encourage student participants in an effort to reduce community mathematical illiteracy while encouraging these same students to envision long-term careers in science, technology, engineering and mathematics (STEM). This presentation will discuss how unique such efforts are in the context of the broader national Math Circle movement, will discuss one group's experience in creating such a program, will provide some analysis of what made us successful, and end with some thoughts about where that leaves the national movement for future successful projects. (Received September 17, 2019)

1154-97-2074 **Po-Shen Loh*** (ploh@cmu.edu), 6113 Wean Hall, Carnegie Mellon University, Pittsburgh, PA 15213. *Developing interest and extraordinary ability in mathematics.*

Suppose that your objective is to develop strong mathematical reasoning ability among as many people as possible. What is a reasonable expectation for "strong?" What is a reasonable expectation for "many?" Is this even important? The speaker will share his experiences from working towards these questions over the past 10 years. He has approached these questions from several different angles, ranging from personally developing talent face-to-face in small groups meeting weekly, to collaborating with non-mathematical YouTube influencers, to working with the Mathematical Association of America to lead the USA Math Olympiad team as National Coach. To build perspective, he has also traveled extensively between the United States and China, giving math talks to students in over 80 schools in each country in their respective native languages, and discussing mathematics with students, parents, and teachers. (Received September 17, 2019)

1154-97-2086 **Li-Mei Lim*** (mei121@bu.edu), Department of Mathematics and Statistics, 111 Cummington Mall, Boston, MA 02215, and **Max Warshauer, Dan Zaharopol, Jacob Castaneda** and **Cory Colbert**. *Panel on Reaching Underrepresented Students at the Pre-College Level.*

This panel will include undergraduate and graduate students in mathematics from underrepresented backgrounds. They will share their experiences getting into math with a focus on pre-college experiences that prepared them for college and graduate school. The panel will showcase both successes in preparing the panelists as well as areas where the mathematical community can improve its work. (Received September 17, 2019)

1154-97-2103 **Joe Skufca*** (jskufca@clarkson.edu), 8 Clarkson Ave, Potsdam, NY 13699. *Ecosystem Development: Supporting new programs for the next generation of applied mathematician.*

The science of data has emerged as a novel new tool, with clear implications for the future of quantitative methods in business, industry, and government. The growth of this science has been rapid, disruptive, and with full appreciation that the set of underlying main concepts, skills, and ethics are not well developed. To produce the next generation of tool users and tool developers in the quantities required to meet the need necessitates new infrastructure to support that preparation.

For many institutions, making the transition from traditional math programs to these new programs seems like a major undertaking. In this talk, we describe such problems might be constructed from existing faculty capacities through innovative, interdisciplinary cooperation. We describe (as case study) the implementation of a Data Analytics Master's Program and a Data Science Undergraduate program using the framework of existing coursework and faculty.

We attempt to provide a template that can allow other universities to answer ... "how could we do that here." (Received September 17, 2019)

1154-97-2109 **Zeynep Akcay*** (zakcay@qcc.cuny.edu), 222-05 56th Ave, Bayside, NY 11364, and **Dona Boccio** (dboccio@qcc.cuny.edu), 222-05 56th Ave, Bayside, 11364. *Use of VoiceThread Audio-Visual Commenting in Partially Online Algebra Classes at a Community College.* Preliminary report.

We study the use of VoiceThread audio-visual commenting technology in partially online algebra classes at Queensborough Community College, CUNY. Algebra is considered to be an obstacle to graduation, as it is a course with high enrollment and low success rates. We utilize VoiceThread, a cloud-based, multi-media

collaborative space that incorporates voice and video commenting with image and text. This technology is compared to traditional collaborative tools such as discussion boards. It is expected that the enhanced methods of communication will lead to increased student participation, retention, and test scores. A broader goal of this study is to improve the quality of mathematics classes in all modalities as these tools can be used in traditional face-to-face classes as well. We aim to contribute to the enhancement of mathematics learning with the methods and materials that we develop in this project. Preliminary findings from the pilot study will be shared, including sample VoiceThread and Discussion Board assignments with students' responses and results of student satisfaction surveys. (Received September 17, 2019)

1154-97-2195 **Cory H Colbert*** (ccolbert@wlu.edu). *Growth Through Outreach: How I Became A Better Mathematician & Educator Through Outreach Programs.*

My journey to becoming a professional mathematician was heavily influenced by my work with outreach programs, such as Bridge to Enter Advanced Mathematics (BEAM). I will speak about how I grew as a mathematician and educator by working at BEAM, and how BEAM and my academic life came to complement each other in a very useful way. (Received September 17, 2019)

1154-97-2205 **Volkan Sevim*** (vsevim@uscb.edu), One University Blvd., Bluffton, SC. *Investigating college algebra students' current pre-requisite understandings and testing the effects of an alternative pre-requisite algebra curriculum: A mixed-methods study.* Preliminary report.

The aim of this session is to share the results of a mathematics education project designed to help improve student learning in College Algebra, which uses students' current pre-requisite understandings to develop an alternative teaching module. This project stemmed from work conducted during the summer months of 2017. In August 2017, the author ran a free, one-week, 23 hours long math enrichment program called "Math Bootcamp," which was designed to strengthen the pre-requisite algebra skills of incoming freshmen. Since then, with the help of other faculty, the author has offered the program every summer. The author and other faculty have discussed the need for finding effective ways to scale-up the pre-requisite algebra instruction to reach more students taking College Algebra. The project has two objectives: 1) To investigate a small sample of newly admitted freshmen students' current understandings of pre-requisite algebra topics: fraction operations, evaluating expressions, rules of exponents, rules of radicals, and solving equations and inequalities, and 2) To develop and test, based on the information obtained through the interviews, an alternative curricular module on the listed pre-requisites that can be used across the university. (Received September 17, 2019)

1154-97-2244 **Daniel Zaharopol*** (dan_zaharopol@artofproblemsolving.org). *Trends in pre-college advanced mathematics preparation and access for historically marginalized groups.*

Over the past few decades, access to advanced topics in mathematics has exploded among high school students. Well-connected students can access resources and training (online, through summer programs, and through specialized schools) that go well beyond a typical high school curriculum. However, Black, Latinx, and many low-income communities have not been able to take advantage of the same resources and have not had the same level of achievement in mathematics during high school. It is likely that pre-college access also contributes to underrepresentation in college and graduate school.

In this talk, we will examine data surrounding student access at the pre-college level. Data will be presented demonstrating both the overall trend of growth in access and the limited reach of that access for varying communities. The presentation will provide a context for work addressing disparate access, as well as data to support the need that can be shared with university administrators, funders, and others. (Received September 17, 2019)

1154-97-2316 **Monte S Meyerink*** (montemeyerink@montana.edu), Montana State University, 442 Reid Hall, Bozeman, MT 59717, **Fenqjen Luo** (fluo@montana.edu), Montana State University, 118 Reid Hall, Bozeman, MT 59717, and **Ciara D Burgal** (ciaraburgal@montana.edu), Montana State University, 205 Reid Hall, Bozeman, MT 59717. *Montana Math Circle.* Preliminary report.

The Montana Math Circle is an outreach program currently focused on middle and high school students. The goal of our math circle is to give rural students exposure to higher level mathematical experiences and competition style math. This presentation will share the activities we have done as well as give an insight to the experiences of the students. (Received September 17, 2019)

1154-97-2327 **James R. Valles, Jr.*** (jrvalles@pvamu.edu), Department of Mathematics, P. O. Box 519 – Mailstop 2225, Prairie View, TX 77446-0519, and **Natali Hritonenko.** *Doing Calculus to Learn Calculus.* Preliminary report.

According to *A Common Vision for Undergraduate Mathematical Sciences Programs in 2025*, instructional delivery via active learning models, such as simulations, reading, writing, or problem solving, should be the primary method used (as opposed to lecturing). With this in mind, we worked on a project to create calculus lessons designed to provide real-world applications and modeling for calculus students. The project was based on current trends in science and technology to make calculus more engaging and more relevant, thus inspiring deeper critical thinking and active learning. Some lessons created will be discussed as will future directions that can be pursued. (Received September 17, 2019)

1154-97-2414 **Dylan P Noack*** (dnoack@yccd.edu) and **David Weisbart.** *Using Manipulables to Teach Non-linear Scaling.* Preliminary report.

Students in precalculus classes tend to have difficulty in using spacial reasoning to answer questions involving non-linear scaling. Even after significant prompting, our experience has shown that significantly fewer than five percent of the students in our precalculus classes can answer the following question.

If it takes 15 grams of paint to spray paint a ball that has a volume of 10 cubic inches, then under the assumption that the paint is negligibly thin, how many grams of paint does it take to spray paint a ball that has a volume of 20 cubic inches?

To encourage critical thinking and spacial reasoning as opposed to memorization of specific formulae, we have introduced wooden blocks as manipulables in collaborative learning workshops that cover non-linear scaling. Students are put into groups and given worksheets that walk them through the use of blocks as tools for visualizing problems. Manipulating the blocks increases students' willingness to draw pictures, visualize problems and engage their physical intuition. (Received September 17, 2019)

1154-97-2419 **Uri Treisman*** (uri@austin.utexas.edu), **Michael O'Sullivan** and **Borbala Mazzag.** *When Novel Instructional Practices Go To Scale: A Case Study.* Preliminary report.

Corequisite Instruction is an evolving hybrid of instructional intensification practices with roots in the Emerging Scholars Programs of the 1980s and administrative practices designed to narrow the gap between campus student support services and classroom instruction. In a growing number of states, it has become the dominant approach to serving students deemed to need remediation in introductory college math courses. It has been the focus of large scale studies using a wide range of methodologies including randomized controlled trials and regression discontinuity studies. In this session, we'll describe the evolution of corequisite instruction and summarize the results of studies of its effectiveness. We'll then look at the the implementation of corequisite instruction in the California State University System. We describe the complex challenges implementation at scale creates for mathematics chairs and other campus instructional leaders and the lesson learned to date in one large university system. (Received September 17, 2019)

1154-97-2425 **Erica Winterer*** (ewinterer@utexas.edu). *Attention Is the Beginning of Devotion: Noticing and Acculturative Routines in a Large Freshman Calculus Course.* Preliminary report.

What does it mean to know one's freshman students when they number 120 and are diverse in their ethnic backgrounds, economic circumstances, and intellectual and career interests? Does knowing one's students in some personal sense really matter to their learning in a large introductory course? And, if it does matter, how in the crucible of our classroom practice can we come to know our students in meaningful ways? I describe a set of instructional practices developed collaboratively with my teaching partner, Uri Treisman, at UT Austin, and discuss the effects of these practices on students' course performance and academic trajectories. I conclude the talk with video interviews of students' recollections of these practices and their impact after two years have passed. (Received September 17, 2019)

1154-97-2514 **Sarah Trebat-Leder*** (strebat-leder@aops.com). *Art of Problem Solving's Pathway Program: Connecting Underserved Students to the New Ecosystem of Mathematics Enrichment.*

While the United States is nowhere near the top of the pack when it comes to mathematics standardized test scores, the recent International Mathematics Olympiad results tell a different story. In the past five years, the United States has won the elite competition three times and tied for first place once. These outstanding students have pulled ahead largely through extracurricular programs and resources like those offered by Art of Problem Solving. However, many promising students, especially those from low-income and minority backgrounds, are

not connected into this new ecosystem of math enrichment. In this talk, we'll discuss one of Art of Problem Solving's recent initiatives to combat this issue. The Pathway Program started out as a partnership between our Gaithersburg AoPS Academy learning center and the Montgomery Blair High School Magnet Foundation with the goal of increasing diversity in their prestigious magnet program by helping to prepare promising students from underserved communities for the admissions test. The foundation identified a cohort of 3rd graders to participate in AoPS Academy enrichment math classes at greatly reduced rates. Now in its third year, the program is expanding to other AoPS Academy locations with the support of the Jack Kent Cooke Foundation. (Received September 17, 2019)

1154-97-2528 **Marisa Debowsky*** (marisa@mathcamp.org). *Building and Supporting Gender Diversity at Canada/USA Mathcamp.*

Over the last decade, Mathcamp has gradually and significantly shifted the demographics of both our applicant pool and our student body, bringing us to a gender-balanced population at camp, including a strong contingent of trans* and non-binary students and staff. In this talk, we'll discuss the tools that have made the program increasingly accessible and welcoming to women and gender minorities, as well as specifics about workshops and activities we run during the program designed to support our diverse population. (Received September 17, 2019)

1154-97-2545 **Jacob Castaneda*** (jacob@artofproblemsolving.org). *Early Access to Advanced Mathematics for Underrepresented Students.*

Across almost every field in the mathematical sciences, people from low-income and minority backgrounds are dramatically underrepresented, a gap that traces back to K-12 education. A wealth of programs exist to help students study mathematics deeply during these formative years - from summer programs to math circles to after-school programs - but most programs have limited outreach and support for reaching underserved students. In an attempt to shed light on strategies that foster inclusion in mathematics at the pre-collegiate level, we will share the experiences of Bridge to Enter Advanced Mathematics (BEAM), a program that has been operating in New York City since 2011 and Los Angeles since 2018. BEAM reaches hundreds of students each year through summer programs, weekend programs, and mentoring from 6th grade through 12th grade. Curriculum includes math ranging from logical reasoning through number theory, combinatorics, and group theory, in addition to college access work and educational advising. We will share further information about our program's strategies and outcomes. It is our hope that others can find replicable program elements, and that our experiences can strengthen the broader ecosystem of support for marginalized students. (Received September 17, 2019)

1154-97-2627 **John F McClain*** (john.mcclain@unh.edu). *Sometimes Predicting the Future is Easier than Deciphering the Past and Other Aspects of Modeling the Draining of a Bottle.* Preliminary report.

I will share an activity focused on modeling the height of a fluid being drained from a bottle, adapted from one available on Simiode.org to work in my class of roughly 200 students, partly in the large lecture and partly in smaller recitations (20 students). I will also share the results of a student survey on modeling activities. This activity includes data collection from video (from Simiode), development of a regression model, derivation of a differential equation, solution of the differential equation, and interpretation of results. It ends with a common sense model verification that leads to a discussion of piecewise-defined solutions and lack of uniqueness for initial value problems corresponding to empty bottles. I emphasize the robustness of the differential equation to changing parameters in contrast to the regression model. I have found that the students appreciate the use of physical principles and geometric analysis in deriving the equation, however, they struggle with the use of the limit definition of derivative. Also, this model gives a physical example of an initial value problem without a unique solution: if the initial condition has height zero at time t_0 , any solution describing a bottle emptying before t_0 also satisfies the initial condition. (Received September 17, 2019)

1154-97-2677 **Gangadhar R Hiremath*** (gangadhar.hiremath@uncp.edu), Department of Mathematics and Comp Science, P.O. Box 1510, One University Drive, Pembroke, NC 28372. *A Note on New Separation Axioms and Their Implications in the Theory of Generalized Metrizable Spaces.* Preliminary report.

New separation axioms introduced in this article are at the fundamental level and generalize both regular spaces and normal spaces. These axioms are used in the characterization of regular spaces and normal spaces. How these axioms enrich the class of compact, countably compact, or paracompact spaces are studied. Generalizations of some classical theorems of metrizability or pseudo-metrizability are established. Inheritance properties and preservations under nice maps like perfect maps are established. (Received September 17, 2019)

- 1154-97-2692 **Demi Dubois*** (demid@bu.edu), Boston University, Department of Mathematics & Statistics, 111 Cummington Mall, Boston, MA 02215, and **Li-Mei Lim**. *PROMYS Math Circle: A Case Study in Organizing and Sustaining an Outreach Program*.

What does it take to launch an extracurricular math circle that serves underrepresented high school students? We will look at how PROMYS launched its outreach program in 2016, focusing on the successes and challenges of organization, funding, recruitment, retention, and more. (Received September 17, 2019)

- 1154-97-2705 **Ekaterina (Katya) Yurasovskaya*** (yurasove@seattleu.edu), Department of Mathematics, Seattle University, 901 12th Ave, Seattle, WA 98122. *Supportive environment in a mathematically rigorous co-requisite lab course ensures student success in the Introduction to Proofs*.

The first course in mathematical proofs serves as a gateway for student entry into mathematics major. The course presents many difficulties to students, who learn to write rigorous formal proofs, encounter challenging and abstract new material. In order to help students succeed, Seattle university has created a co-requisite problem-solving lab course that we called Mathematical Communication. Combining elements of Math Circles and the Treisman model, the lab course features problem-solving, group work, formative feedback, no grades, and a carefully and deliberately constructed warm and supportive environment for all students. As a result, we have been able to cut non-passing rates in half and increase the number of A's and B's for at-risk students. The lab course is straightforward and inexpensive to implement, and we are very excited to share the model with you in this talk. (Received September 17, 2019)

- 1154-97-2719 **Cameisha Clark***, 223 James P. Brawley SW, Atlanta, GA 30314, and **Daniel Teodorescu** and **Torina Lewis**. *Gender Differences in the Use and Effectiveness of Adaptive Learning in Calculus I at a Private HBCU*. Preliminary report.

The study established whether the implementation of Assessment and Learning in Knowledge Spaces (ALEKS) - a web-based, artificially intelligent adaptive learning and assessment system improved male student mastery of learning outcomes, retention, and persistence rates in undergraduate Science Technology Engineering and Mathematics (STEM) degree programs at a higher rate than female students at a private Historically Black Colleges and Universities HBCU. The system was implemented as a corequisite model to assist students that struggled with precalculus concepts. Ten sections of Calculus I averaging 30 students, were exposed to the intervention from spring 2018 - spring 2019. Students navigated ALEKS precalculus content while simultaneously studying Calculus I concepts. We examined whether the adaptive learning technology was more effective with helping male students gain the necessary knowledge to be successful in Calculus I. (Received September 17, 2019)

- 1154-97-2734 **Tatiana Shubin*** (tatiana.shubin@sjsu.edu), San Jose State University, Department of Mathematics and Statistics, San Jose, CA 95192-0103, and **Bob Klein**. *Alliance of Indigenous Math Circles (AIMC) as a study in network building*.

AIMC was founded three years ago in order to spread the model obtained through the work on the Navajo Nation to other communities. In the talk we will share our experience in building networks among mathematicians and Native American communities of several states as far-flung as Alaska and Oklahoma. (Received September 17, 2019)

- 1154-97-2737 **Reimi Hicks***, Reimi Hicks, 77 Massachusetts Avenue, E17-139, Cambridge, MA 02139, and **Eboney Hearn** and **Kristin Kurianski**. *Prepare and Inspire: Broadening participation in STEM through outreach programs for pre-college students at MIT*.

This presentation will provide an overview of a proven model for broadening participation of students from historically underrepresented backgrounds in STEM fields. For 45 years, the MIT Office of Engineering Outreach Programs (OEOP), housed in the School of Engineering, has offered highly motivated and diverse precollege students — 4,400 and counting — opportunities to pursue their passion for STEM at MIT. The OEOP provides students challenging learning environments with high expectations, an innovative hands-on curriculum that strengthens their foundational skills in STEM, and access to positive role models in STEM. The OEOP offers three programs, (residential, online, and Saturdays-only), free-of-charge, which serve approximately 335 students, annually. OEOP programs have proven to influence college application and enrollment behavior, as well as, choice of college major and aspirations for careers in STEM. As supporting the academic success of students goes beyond what happens in the classroom, this talk will explore the importance of intentional strategies for recruitment, selection, staffing/mentoring, and community building in programs that serve students from underrepresented and underserved backgrounds. (Received September 17, 2019)

1154-97-2742 **Boyan S. Kostadinov*** (bkostadinov@citytech.cuny.edu). *A Project-based Promulgation of Computational Thinking in Mathematics.*

In this talk, we present several projects designed to mix programming, mathematics and experimentation. The mathematics behind the projects makes use of finite sums of complex exponentials, out of phase logarithmic spirals to visualize spiral galaxies, fractal systems, and other approaches for creating complex mathematical art. We also illustrate a sample workflow for implementing data science projects using RStudio, which now offers the ability to combine the power of R and Python in the same source R Markdown document, along with TeX expressions, allowing us to create publication quality project reports in several different formats, including interactive dashboards, which can deliver visually rich storytelling about the data being explored. (Received September 17, 2019)

1154-97-2777 **Boyan S. Kostadinov*** (bkostadinov@citytech.cuny.edu). *Exploring Linear Algebra Concepts Through the Lights Out Game Using SageMath.*

In this talk, we present a computational approach to exploring linear algebra concepts through the Lights Out game and its variants, using SageMath as the computational engine. More specifically, we provide concrete explorations for students to investigate these games, from simple exercises to open problems, suitable for student research projects. We showcase sample SageMath code for numerical explorations of Lights Out, which can supplement successful proof-writing by offering insight and building intuition for deeper mathematical understanding. (Received September 17, 2019)

1154-97-2778 **Adrian B. Mims*** (abmims20@gmail.com), 168 Pond Street, Randolph, MA 02368. *The Calculus Project: Re-envisioning the Brilliance of Black and Latino students in Mathematics.*

The Calculus Project (TCP) was conceived in 2009 in one schoolhouse – Brookline High School in Massachusetts – in response to:

- 1) the historical and ongoing dismal data at Brookline High, replicated nationally, on the math achievement of African American students;
- 2) the predictive power of success in high-level mathematics in high school – particularly calculus – on overall graduation rates from college, and increased enrollment and success in the STEM disciplines at college;
- 3) the very positive prospects for employment in the STEM professions in our nation where millions of STEM positions are going unfilled, and where there are continued low numbers of STEM professionals of color.

TCP is defined by its comprehensiveness, high expectations, cultural sensitivity and commitment to sustainability. Calculus Project programs begin in the middle schools and expand to high schools over a five-year period. TCP provides the kind of services and supports to historically underachieving students that more advantaged families and communities provide to their children. These include: 1) The Summer Academy; 2) The After-School Academic Center; 3) The Pride Curriculum; 4) Field trips to STEM worksites; 5) Peer-teaching; and 6) A class grouping strategy. (Received September 17, 2019)

1154-97-2811 **Ben Galluzzo*** (bgalluzz@clarkson.edu), Clarkson University, 8 Clarkson Avenue, Potsdam, NY 13699. *Computational Thinking in the Secondary Mathematics Classroom.*

The U.S. Bureau of Labor Statistics estimates that more than half of all STEM jobs next decade will require computational thinking skills; however, less than 40% of public U.S. high schools provide Computer Science (CS) offerings. In this talk, we'll discuss the NSF funded Computing with R for Mathematical Modeling (CodeR4MATH) project that develops activities focused on shrinking the CS supply and demand gap by infusing computational thinking into the high school mathematics curriculum. In particular, we'll show how the project's mathematical modeling focused modules align with mathematics standards, discuss how teachers have used CodeR4MATH modules in classrooms, and provide examples of student work. (Received September 18, 2019)

1154-97-2817 **Roselyn Williams*** (roselyn.williams@fam.u.edu). *Bridging the Gaps in Undergraduate Mathematics Education.*

The mathematical courses of today contain students from a large spectrum of backgrounds and skills. In a classroom setting there are students who are well prepared to master the subject matter, as well as those who are under prepared to do so. The question is how to provide instruction that challenges the best students and with enough detail that benefits under prepared students in a manner in which all students can be successful in their course work. This talk explores strategies to complement classroom lectures and text books in a manner that empowers and inspires students to be independent learners. For all students, learning should be an enjoyable

experience of building confidence, acquiring knowledge, and developing the skills necessary to become effective problem solvers. (Received September 18, 2019)

1154-97-2832 **Rhonda D Fitzgerald*** (rdfitzgerald@nsu.edu), Department of Mathematics, 700 Park Ave, Norfolk, VA 23504, and **Anne Fernando**. *Creating Interactive Classrooms Using the Flipped Model with Supplemental Instructors*. Preliminary report.

Successful mastery of concepts taught in mathematics courses proves essential to the pursuit of any STEM degree. Post-Secondary institutions play an important role in producing STEM professionals and Historically Black Colleges and Universities (HBCUs) play a major role in producing minority STEM graduates. Today, many students enter institutions with deficits in mathematical skills and concepts. In addition to the deficit, students lack the knowledge of how to be successful in a college math course. Key patterns noticed in the students who struggle are 1) not consistently and earnestly completing assigned work 2) not utilize resources (i.e. instructor's office hours and tutoring services) and 3) failing to appropriate effective means of study. Here we discuss engaging students through the use of flipped classrooms and Supplemental Instructors to not only address the mentioned issues but to also enhance students' meta-cognitive skills. (Received September 18, 2019)

Table of Contents for MAA Abstracts of Invited Addresses, Presentations by Teaching Award Winners, SIGMAA Guest Lecturers, and Session Papers

MAA Invited Addresses, Presentations by Teaching Award Winners, and SIGMAA Guest Lecturers	354
Abstract Algebra: Teaching, Topics, and Techniques	357
Active Learning in Introductory Courses: Insights from Math Departments in the Process of Change	359
Creating Spaces for Mathematics	366
Research in Undergraduate Mathematics Education: Highlights from the Annual SIGMAA on RUME	367
COMAP Modeling Contests	368
Can Mathematics Help Us Trust Our Elections Again?	369
Modernizing the Introductory Statistics Course	370
Experiencing Geometric Transformations and Evaluating Learning Progressions: Celebrating the Work of David W. Henderson	371
Inspiring Diversity in Mathematics: Culture, Community, and Collaboration	372
Best Practices and Considerations in Designing and Developing Online Math Courses ...	374
Combining Technological Tools and Innovative Practices to Improve Student Learning Outcomes	375
Data Across the Curriculum	378
Developments, Directions, and Experiences in Open Educational Resources	380
Discrete Mathematics in the Undergraduate Curriculum — Ideas and Innovations in Teaching	383
The EDGE (Enhancing Diversity in Graduate Education) Program: Pure and Applied talks by Women Math Warriors	385
Effective Assessment Practices in Large-enrollment Classes or Non-traditional Classrooms	387
Fostering Creativity in Undergraduate Mathematics Courses	388
A History of Mathematics in the United States and Canada: A Session in Honor of Math Historian David Zitarelli	391
Inclusive Excellence for the Mathematical Sciences	392

Incorporating Realistic Applications of Mathematics Through Interdisciplinary Collaborations	397
Innovations in Complex Analysis	400
Innovative and Effective Ways to Teach Linear Algebra	401
Inquiry-Based Learning and Teaching	402
Integrating Research into the Undergraduate Classroom	407
Making Business Calculus Relevant	409
Mathematical Experiences and Projects in Business, Industry, and Government (BIG) ..	410
The Mathematical Knowledge of Teachers as an Integrated Application in Core Mathematics Courses	411
Mathematics and the Arts: In the Classroom and Beyond	413
Mathematics and Sports	418
Modeling-First Inquiry-Based Course Activities	420
Quantitative Literacy and Social Justice: An Ongoing Dialogue	423
Recreational Mathematics: Puzzles, Card Tricks, Games, and Gambling	425
Re-Envisioning the Calculus Sequence	427
Research in Undergraduate Mathematics Education (RUME)	431
Role of Explanation in Mathematical Proofs	439
The Scholarship of Teaching and Learning in Collegiate Mathematics	440
Scholarship on Teaching and Learning Statistics	443
Tangents to Math Circles	445
The Teaching and Learning of Undergraduate Ordinary Differential Equations	447
Teaching Mathematics with Humor or Pop Culture	449
Tell Me a Story: Connections between Mathematics and Performed or Print Narrative ..	450
Interdisciplinary Topics in Mathematics	452
Mathematics and Technology	454
Mentoring	457

Modeling and Applications	460
Outreach	465
Teaching and Learning Introductory Mathematics	468
Algebra	473
Analysis	475
Applied Mathematics	478
Geometry	488
Graph Theory	489
Number Theory	493
Probability and Statistics	498
Topology	500
Other Topics	502

The presenter of each talk is indicated by an asterisk (*) in the abstract.

MAA Invited Addresses, Presentations by Teaching Award Winners, and SIGMAA Guest Lecturers

1154-A0-71 **Karin Leiderman***. *Recent challenges and successes in mathematical modeling of blood clotting*

Blood clot formation is a complex and nonlinear process that occurs under flow and on multiple spatial and temporal scales. Defects and perturbations in the clotting system can result in serious bleeding or pathological clot formation, but due its complexity, the responses and their underlying mechanisms are challenging to predict. Mechanistic mathematical models of blood clot formation and coagulation can elucidate biochemical and biophysical mechanisms, help interpret experimental data, and guide experimental design. In this talk I will briefly describe such models and show how our integrated mathematical and experimental approach has facilitated discovery of previously unrecognized interactions within the clotting system. I will also discuss a recent study to determine how the major clotting enzyme, thrombin, is strongly sequestered by the polymer that thrombin itself produces. Using Bayesian inference, we learned model parameter distributions from experimental data, but for the model to best fit the data, we had to make an additional assumption that thrombin could become irreversibly sequestered; this led to a new hypothesis that thrombin becomes physically trapped during polymerization. (Received July 30, 2019)

1154-A0-74 **Scott Adamson***, Chandler-Gilbert Community College, 2626 E Pecos Rd, Chandler, AZ 85225. *Mazes, riddles, zombies, and unicorns!*

Can mathematics be an engaging endeavor worthy of academic pursuit? Can students be involved in meaningful learning experiences? Having observed many classrooms, it seems that the answers are a heartbreaking “No.” Reflect on what often happens in classrooms. Teachers may unwittingly communicate that mathematics is so mind numbing and senseless, that they must “jazz it up,” tricking students to engage in mathematical activity; hijack thinking by telling students what to know without allowing them opportunities to grapple with ideas; write notes while students passively copy. There is no expectation or opportunity to make sense, to reason, to understand, or to engage in authentic mathematical thinking and discovery. Reflect on what could happen in classrooms. Imagine experiences where students are using manipulatives to represent real-world situations; building procedural fluency with algorithms by developing conceptual understanding; struggling productively to make sense of ideas while modeling with mathematics; collaborating as they make their reasoning visible while solving challenging problems. I will highlight these issues and provide effective strategies for involving students in authentic mathematical thinking and discovery as they joyfully engage in joyful mathematics! (Received July 30, 2019)

1154-A0-75 **Mohamed Omar***, Harvey Mudd College, 301 Platt Blvd, Claremont, CA 91711. *The art and craft of problem design.*

How does one pick the right research problem to work on? How can we create assignment problems that are not decent nor good, but great? How does one make innovative problems for math competitions? These are questions that have been central throughout the speaker’s career, and common threads between them have had surprising influences on each other. Come hear how the art and craft of problem design plays a key role in a mathematical career. (Received July 30, 2019)

1154-A0-76 **Michael Huemer*** (bmsjrqrna@snkmail.com), CU Boulder Philosophy, Hellems 169 UCB 232, Boulder, CO 80309-0232. *Possible and impossible infinities.* Preliminary report.

The infinite gives rise to many paradoxes. Some are aptly resolved by declaring certain infinite quantities impossible. But which infinities are possible, and which are impossible? On an Aristotelian view, there can be no “actual infinities”, only “potential infinities”. This view is wrong; there are many obvious examples of actual infinities. I draw three distinctions: between cardinal numbers and magnitudes, between intensive and extensive magnitudes, and between natural and artificial magnitudes. I then propose a new theory of the impossible infinite: there are infinite cardinal numbers, extensive magnitudes, and artificial magnitudes, but there can be no infinite natural, intensive magnitudes. This view rules out most of the scenarios appearing in the paradoxes of the infinite. (Received July 30, 2019)

1154-A0-79 **Della Dumbaugh***, University of Richmond, 28 Westhampton Way, Richmond, VA 23173. *Prospering through mathematics.*

Solomon Lefschetz, Emil Artin, Gertrude Cox and Rudy Horne all enjoyed successful careers in mathematics. These mathematicians, separated by mathematical discipline, nationality, time, institution and background also all faced—and overcame—hardships. They met the challenges of their human lives, in part, through their mathematical work at particular institutions. But what if we turn this lens around? What if we consider how mathematics created the space for these mathematicians to find success? Mathematics in particular allowed them to prosper, to the point where they had plenty to give to the next generation. Their lives prompt us to take up the natural next question of “how?” How can we use the space of mathematics to help shape students and colleagues into better human beings? And how can we accomplish this lofty goal with increasing demands on our time and rosters of students who may only take a single math class? This talk explores the professional experiences and personal lives of mathematicians to underscore the power of mathematics, not just as a career path, but as a place to grow into a full human being. It also outlines effective strategies for intentionally identifying the power of the discipline to direct students and colleagues toward meaningful lives. (Received July 31, 2019)

1154-A0-80 **Vilma Mesa***, University of Michigan, 500 S State St, Ann Arbor, MI 48109. *Instruction and resources in post-secondary mathematics: How their interplay shape what we do in the classroom.*

In this presentation I describe studies I have conducted to investigate how instructors, students, and resources interact in classrooms in order to create opportunities for mathematics learning in post-secondary settings. I showcase the evolution of two apparently independent research strands that together have helped me understand first, the centrality of resource use by instructors and students and its implications for student learning and, second, the complexity of the work that faculty do. (Received July 31, 2019)

1154-A0-81 **aBa Mbirika***, University of Wisconsin-Eau Claire, 105 Garfield Ave, Eau Claire, WI 54701. *Two research projects birthed from curiosity, recreation, and joy.*

This talk will center around two undergraduate research projects that were born from two specific recreational math topics. These topics brought me joy and then suddenly turned into full-blown research. The first topic emerged from a connection between the Fibonacci sequence modulo 10 and astrology. Oh No! Does the speaker believe in astrology!? Don't worry, this topic will be strictly number theory with, of course, a foundation that gives its connection to astrology (in particular, the zodiac). The second topic arose from noticing the magical and mystic golden ratio appearing as an eigenvalue of a certain tridiagonal real symmetric matrix. Generalizing this matrix to ever-increasing sizes, a wondrous joy is born from the corresponding sequence of characteristic polynomials that emerge. And lo and behold the diagonal entries in Pascal's triangle appear as the coefficients of these polynomials in an attractively inviting manner. Though the first project is one of number theory, and the second is one of combinatorial linear algebra, a cute connection between the two topics will be revealed at the end of the talk. (Received July 31, 2019)

1154-A0-82 **Ami Radunskaya***, Pomona College, 333 N College Way, Claremont, CA 91711. *On the scales of one to infinity: Learning to listen to your mathematics.*

Many mathematical constructs can be manifested as sounds! The visual palette is three dimensional, or four if you include color; in some ways, the sonic palette is richer. Our ears can perceive along many axes, including pitch, loudness, timbre, harmonic complexity and time. How could you better understand your mathematical problem by hearing it, as well as seeing it? For example, why is the “harmonic” series called by this name? Can we hear that the harmonic series diverges? Did you know we can “listen” to a dynamical system in order to understand its structure? Some features of common functions are better heard than seen! Together, we will explore the “route to chaos” via graphs and sounds, with live demonstrations. (Received July 31, 2019)

1154-A0-84 **Carrie Diaz Eaton***, Bates College. *Calling bull (with R)*

We are in a new age of digital information. Information is a form of power an our students consume and produce more unfiltered information than ever. They need agency as individuals and tools as future workforce to ethically and responsibly process this information. I talk about my journey to developing an introductory information literacy course using Open Educational Resources, including Calling Bull, Figure of the Day, and RStudio. Students use these tools to explore their own agency as a digital citizen. This course serves as a forum to think meaningfully about probability, data analysis, and data visualization, while providing a gentle introduction to programming and a context to examine the interplay of information, power, and social justice. (Received July 31, 2019)

1154-A0-99 **Trevor Patch***. *The importance of statisticians in a development-driven world.*
Giving new professionals the skills of understanding model building assumptions, rigor in statistical inferences, and the importance of communicating conclusions to key stakeholders will be paramount to the professionals' success and the success of the products that they touch. As technology grows even more pervasive in our daily lives, the nuts and bolts of the underlying models and the statistical integrity of the people building them becomes increasingly important. This talk will highlight the lessons that have stuck with me that I learned through my statistical education and career with the Colorado Rockies, along with some of the projects such lessons have impacted. The talk will also discuss the skills that candidates must possess to succeed in the sports analytics industry. (Received August 5, 2019)

1154-A0-108 **Suzanne L Weekes***, Department of Mathematical Sciences, Worcester Polytechnic Institute, Worcester, MA 01602. *Taking your place, making more space, and keeping it real.*
In this talk, the speaker will share her experiences engaging in partnerships between universities and industry, and her efforts towards ensuring that a more diverse generation of people with a diverse range of interests are able to find their home in our mathematics communities. (Received August 07, 2019)

1154-A0-242 **Federico Ardila-Mantilla***, San Francisco State University, San Francisco, CA. *Todxs Cuentan: Difference, humanity, and belonging in the mathematics classroom.*
Everyone can have joyful, meaningful, and empowering mathematical experiences; but no single mathematical experience is joyful, meaningful, and empowering to everyone. How do we build mathematical spaces where every participant can thrive? Audre Lorde advises us to use our differences to our advantage. Bell Hooks highlights the key role of building community while addressing power dynamics. Rochelle Gutierrez emphasizes the importance of welcoming students' full humanity. This talk will discuss some efforts to implement these ideas in mathematical contexts, and some lessons learned along the way. (Received August 26, 2019)

1154-A0-358 **June Barrow-Green*** (june.barrow-green@open.ac.uk), School of Mathematics & Statistics, The Open University, Milton Keynes, MK7 6AA, United Kingdom. *The historical representation of women in mathematics.*
From medieval times to the modern day, women in mathematics have been represented in a myriad of ways, whether pictorially or by the written word. In my talk, I shall look at a variety of these representations from different periods and in different contexts, and I shall reflect on what they can tell us about women in mathematics and the attitudes prevailing at the time. (Received September 02, 2019)

1154-A0-1824 **Mark Tomforde*** (tomforde@math.uh.edu). *Lessons Learned from Lessons Taught.*
I will discuss how my personal views on the teaching and learning of mathematics have evolved over the past 20 years. In particular, I wish to share how certain beliefs I once held as a beginning instructor have changed and been replaced by new viewpoints or more nuanced ways of thinking about students. (Received September 16, 2019)

1154-A0-2162 **Federico Ardila-Mantilla*** (federico@sfsu.edu), San Francisco State University, 1600 Holloway Ave., San Francisco, CA 94110. *The joyful community practice of doing mathematics.*

I firmly believe that:

- 1 - Mathematical potential is distributed equally among different groups, irrespective of geographic, demographic, and economic boundaries.
- 2 - Everyone can have joyful, meaningful, and empowering mathematical experiences.
- 3 - Mathematics is a powerful, malleable tool that can be shaped and used differently by various communities to serve their needs.
- 4 - Every student deserves to be treated with dignity and respect.

I will discuss some efforts to make these principles a reality. (Received September 17, 2019)

1154-A0-2323 **Matthew Boelkins***, Grand Valley State University, Allendale, MI. *The future of textbooks.*

In 1952, George Thomas published a text that grew quickly in popularity and ended up establishing expectations for the content and style of calculus texts. Today, Thomas's Calculus is in its 14th edition and remains widely used. It's remarkable that for nearly 70 years, the broad content and format of Thomas's text (and others) have remained remarkably consistent.

In 2014, Rob Beezer released PreTeXt (née Mathbook XML), a free authoring platform for creating the next generation of textbooks. PreTeXt offers many features, perhaps most importantly appealing HTML output that

looks great in any browser (including on a smartphone) and allows in-place interactive elements such as Sage cells, Geogebra applets, JavaScript figures, WeBWorK exercises, and more.

At present, not only do we have amazing and modern technologies that offer new options for textbooks, but we also understand better than ever how students learn. What should we expect of textbooks in 2020? 2030? 2050? I'll share some examples of the next generation of textbooks being written in PreTeXt, give an overview of formats and interactive features that are currently possible, dream a bit about what else might be possible, and cast a vision for what we should expect of all mathematics textbooks in the near future. (Received September 17, 2019)

1154-A0-2812 **Maria Mercedes Franco***, Queensborough Community College, CUNY, NY. *Reflections on diversity and undergraduate research.*

The speaker will share insights gained from a wide range of experiences mentoring undergraduate research students. These experiences include roles as GRA, faculty mentor, and program director; working with students from 2- and 4-year colleges/universities; and facilitating or personally engaging them in summer, course-based or academic year research. What these experiences have in common is also what attracted the speaker to them: they all address issues of underrepresentation in the mathematical sciences or, more broadly, in STEM. (Received September 18, 2019)

Abstract Algebra: Teaching, Topics, and Techniques

1154-A1-139 **Faun C. C. Doherty*** (fdoherty@washjeff.edu), 135 Melody Lane, Washington, PA 15301. *Bringing back the take-home exam: an experiment to assess ownership in an Abstract Algebra course.* Preliminary report.

Most of my colleagues have given up the task of determining whether or not their students have cheated on take-home exams. It is almost to be taken for granted that some form of help has been found during homework and take-home exam writing. To avoid this, our exams take place in-class. I believe that this situation leaves a hole in my assessment of upper level math skills, and I wanted to find a solution. Would including an oral element to a take-home exam reduce the practice of cheating? It may not, but could it push students to a different level of understanding of their solutions despite the procurement of help? Ultimately, I want students to learn. If the threat of needing to show oral "ownership" of their solutions gives the motivation to really understand what they are writing, then we both win. I test this idea in an upper level Algebraic Structures course in the fall in which I have scheduled three take-home exams, each with an oral component occurring after the exam has been turned in. I will tailor my oral questions to ideally assess the real understanding behind the written solutions. This talk will include the details of my reasoning and course structure, along with my reflections on the process, and student reactions. (Received August 14, 2019)

1154-A1-901 **Violeta Vasilevska*** (violeta.vasilevska@uvu.edu). *Enhancing Abstract Algebra Classes with Various Teaching Techniques and Activities.* Preliminary report.

In this talk, we describe different methods/activities that the presenter has used in Abstract Algebra classes. The methods/activities, which promote active in-class participation and encourage proof writing, include: modified Inquiry Based Learning, active in-class participation, weekly study sessions, writing projects, in-class presentations, technology enhanced projects, etc. In this presentation we discuss what did/did not work well with each of these methods/activities. In addition, students' feedback and useful suggestions will be shared about each method/activity used. (Received September 11, 2019)

1154-A1-965 **Mike Janssen*** (mike.janssen@dordt.edu), Dordt University, Math/Stats Department, Sioux Center, IA 51250. *An Open Source Inquiry-Based Abstract Algebra Text.*

In this talk, a new open source, inquiry-based abstract algebra textbook will be introduced. The text is currently designed for students who have taken at least one proof-based course. After a review of properties of the integers, the text explores ring theory through the lens of factorization, seeking to find conditions sufficient to guarantee unique factorization. The book is authored in PreTeXt, and so will be available in both HTML and PDF forms, and the source code is also available for download. (Received September 12, 2019)

1154-A1-1288 **Myron Minn-Thu-Aye*** (myron.minn-thu-aye@uconn.edu). *Incorporating Student Choice in Assessment in Abstract Algebra.* Preliminary report.

Students of mathematics are generally accustomed to being assessed via some combination of homework, quizzes, and in-class exams. In Abstract Algebra, I have been employing assessments to challenge students in different

ways and broaden definitions of and pathways to success. We will discuss a grading scheme where each student completes midterm assessments in three formats (a take-home exam, a project, and an oral exam) then chooses one of these formats for their final assessment in the course. We will consider student responses via surveys and interviews to examine the impact that varied assessment and choice of assessment have on students' motivation, approaches to learning, and learning outcomes in Abstract Algebra. (Received September 14, 2019)

1154-A1-1351 **Vicky Klima*** (klimavw@apstate.edu). *Connecting Course Content to Professional Publications*. Preliminary report.

The College Math Journal, The American Mathematical Monthly, The AMS Feature Column, and many more outlets regularly present interesting and exciting applications of abstract algebra at a level accessible to students finishing their first abstract algebra course. For many years the presenter has asked students to review such articles as part of her abstract algebra course. When simply prompted to compose a review, students typically acknowledged some interesting applications but often failed to truly own the relationship between the mathematics they had been learning in class and the mathematics represented in the paper they were reviewing. In this presentation we will discuss how a small change to the prompt has resulted in more meaningful connections by the students. We will discuss the assignment's goals and outcomes, student submissions and impressions, as well as strategies for assessment. (Received September 15, 2019)

1154-A1-1419 **Kristi Meyer*** (kristi.meyer@wlc.edu), 8800 W. Bluemound Rd, Milwaukee, WI 53051. *From Small to Large: Adapting Techniques in the Face of Increasing Class Sizes*.

When I first started teaching, I had the luxury of very small Abstract Algebra classes (generally 3-6 students per semester). This small class size let me use teaching techniques and assessment methods that would have been impossible to implement in a larger classroom. Over the past 14 years, my class sizes have gradually increased to the point where using these same teaching techniques and assessment methods would be prohibitively time-consuming. This presentation will discuss the evolution of a variety of aspects of my Abstract Algebra classroom in the face of increasing class sizes, focusing especially on homework and oral final exams. (Received September 15, 2019)

1154-A1-1676 **Jill E. Jordan*** (jill.jordan@houghton.edu). *An odd (or even?) inquiry-based approach to quotient groups*.

Many undergraduate algebra texts introduce quotient groups by first defining cosets and normal subgroups, after which quotient groups are presented as a way to use normal subgroups of a known group to produce a "smaller" group. In this talk I will discuss a classroom activity taken from an inquiry-based abstract algebra curriculum in which students use their existing knowledge of odd and even integers to develop an understanding of quotient groups. As students explore ways in which to break up a group into sets that exhibit a group structure similar to that of the sets of odd and even integers, they develop the concepts of cosets and normal subgroups. I will share student responses to the inquiry-based approach as time allows. (Received September 16, 2019)

1154-A1-1935 **Robert Kelvey*** (rkelvey@wooster.edu) and **Jennifer Bowen** (jbowen@wooster.edu). *Project-based abstract algebra using Office Hours with a Geometric Group Theorist*.

In the Spring of 2018 the authors co-taught an abstract algebra course using the text *Office Hours with a Geometric Group Theorist*, edited by Matt Clay and Dan Margalit. The chapters of *Office Hours with a Geometric Group Theorist* are each written by a different mathematician. As the editors state in their introduction, each author was tasked with writing a chapter that aims to answer one of the following questions, potentially posed by an undergraduate or beginning graduate student: *will you tell me about your research? Will you help me find a topic for a senior project?* As such, this text provides advanced undergraduate students with a plethora of example rich topics that can culminate in semester or year-long projects.

In this talk, we will discuss how we leveraged the structure of this textbook to design and implement a project-based second course in abstract algebra. We will discuss the course structure, presentation formats, our assessment practices, and overall take-aways from the experience from both the student and instructor point-of-view. (Received September 16, 2019)

1154-A1-1958 **Sara Jensen*** (sjensen1@carthage.edu), 2001 Alford Park Dr., Kenosha, WI 53140. *Teaching Abstract Algebra: Lessons from my Children*.

Students first learning Abstract Algebra often find the material unfamiliar, difficult to remember, and unintuitive. Although performing computations with many different examples of groups can help alleviate these issues, this practice seems to justify rules rather than motivate them. This talk will discuss the solution I came up with to combat these issues: to motivate as many group theory topics as possible with concepts familiar to my 4 year old.

We will discuss how books, colors, candy, and even princess dolls can secretly teach us about exponentiation, cosets, factor groups, and permutation groups. (Received September 16, 2019)

1154-A1-2164 **Peri Shereen*** (pshereen@csumb.edu). *A blended approach to active learning in an abstract algebra course.*

The premise that active learning classrooms improve student performance in mathematics is supported in the research literature. This talk will share an implementation of a first semester, abstract algebra course which blended into an inquiry based learning model the tools found within the reading apprenticeship (RA) model. RA is a pedagogical framework which makes visible to the student their disciplinary reading and develops students' metacognitive skills by addressing the four dimensions: personal, cognitive, social and knowledge-building. We will discuss the course structure, more specifically, how student reading was brought into an inquiry based model. In addition, we will discuss tools from RA that were used to develop a classroom community that encourages students to engage in rich mathematical activities while collaborating with their peers. The talk will also share preliminary reports on the effectiveness of the first implementation and welcomes any feedback for future revisions. (Received September 17, 2019)

1154-A1-2329 **John K Osoinach*** (josoinach@dallas.edu), 1845 E. Northgate Dr., Irving, TX 75062. *A Theorem Sequence Approach to Abstract Algebra for Undergraduates.* Preliminary report.

This talk will describe successes teaching with a theorem sequence in an undergraduate Abstract Algebra course at The University of Dallas over the past four years. The sequence leads students through the traditional undergraduate topics of groups and rings, following the 2015 CUPM Curriculum Guide released by the MAA. We will report on student achievement and struggles using the theorem sequence, as well as how students responded to final exam questions in courses taught with a textbook and those taught using the theorem sequence. Sample sections from the theorem sequence will be provided. (Received September 17, 2019)

1154-A1-2335 **Jeremy Muskat*** (jmuskat@western.edu), Hurst Hall 112, 1 Western Way, Gunnison, CO 81231. *Including Applications in a Theoretical Course.*

In an effort to engage students in undergraduate mathematics courses, I have slowly added applications in all of the courses that I teach. I will discuss a lengthy example involving error-correcting codes that can be continually revisited through a semester of Abstract Algebra. (Received September 17, 2019)

1154-A1-2709 **Melinda Schulteis*** (melinda.schulteis@cui.edu), 1530 Concordia West, Irvine, CA 92612. *An Active Learning Approach to Abstract Algebra.* Preliminary report.

Would you like to hear your math majors say, "Wow, we just came up with the definition of a ring. . . all on our own!?" This talk will showcase a Concept Attainment Lesson, a type of active learning teaching model, which helps students who have previously covered group theory, be able to successfully come up with the definition of a ring. . . all on their own. You will be shown how to revisit the activity to let students also create the definition of an integral domain and a field. Additionally, the talk will include ideas and examples from active learning worksheets that accompany the lesson and have students experiment with, and extend, the definitions for a deeper understanding of the concepts. (Received September 17, 2019)

Active Learning in Introductory Courses: Insights from Math Departments in the Process of Change

1154-A5-36 **Aleksei Talonov*** (aleksei.talonov@unlv.edu) and **Viktoriya Savatorova** (vs1445@ccsu.edu). *Transformative Teaching Redesign: Development of High-Impact Strategies to Maximize Learning and Minimize Failure in Introductory Courses of Mathematics.*

We will present teaching and learning practices aimed at improving the success and retention rates in introductory courses of mathematics. In our transformative course redesign we integrate student-centered active learning activities with transparent assignments to increase learning options for students. The key component in our teaching redesign is to state unit objectives for students in the beginning of each course module and to develop a set of in class activities for each of these objectives. Our interactive exercises are based on peer learning and learning by doing and aimed to cultivate students' sense of belonging and build their confidence in problem solving. A coordinated team work of all the instructors is another key feature of our teaching redesign. Systematic data collection has made teaching and learning more effective revealing student's major difficulties in learning and helping to resolve problems as they are discovered. Students have noticed instructor's attention to their

success, stopped being passive observers and became active learners. Students indicated that they started to enjoy doing mathematics after they saw that they can succeed. We will provide examples of active learning activities and analysis of the data on student performance. (Received July 06, 2019)

1154-A5-332 **Gina Monks*** (monks@psu.edu). *Establishing Learning Assistant Programs at Branch Campuses.*

Over the past two years we successfully implemented the first Learning Assistant Program at our small two-year branch campus of Penn State University. In this talk we discuss the unique challenges we faced and how we overcame them. We will also explain how our initial successes have led to the participation in the program of several additional faculty members at our own campus, and how networking with others through an NSF grant has led to the formation of Learning Assistant Programs at other branch campuses of Penn State as well. (Received August 31, 2019)

1154-A5-477 **Lake R Ritter***, 1100 S. Marietta Pkwy SE MD 9085, Marietta, GA 30060, and **Jennifer Vandebussche, Kadian Callahan** and **Erik Westlund**. *Using Strand Committees to Build Faculty Support for Departmental Change.*

An important facet of an institution's efforts to create sustainable instructional change is providing mechanisms for that change while fostering the necessary faculty buy-in. In this talk, we will describe the implementation of curricular Strand Committees as an instrument to facilitate significant course changes while providing oversight. A key feature of the Strand Committee process is to enact department level changes to improve student success while continuing to uphold and honor the philosophy of shared governance. The talk will include a brief history of strand committees at Kennesaw State University, highlight examples of beneficial changes resulting from their work, and provide recommendations for academic departments interested in adopting a strand committee structure. (Received September 05, 2019)

1154-A5-490 **Jacqueline Jensen-Vallin*** (jjensenvalli@lamar.edu), Dept of Mathematics, Lamar University, PO Box 10047, Beaumont, TX 77706. *Coordinating First-Year Courses.* Preliminary report.

As part of the State of Texas' requirement that state colleges and universities provide co-requisite options for most underprepared students (75% by Fall 2020), Lamar University has begun coordinating our first-year mathematics courses. For most of these first-year courses, college-ready and corequisite students are comingled in the college-level courses, and corequisite students are comingled from all college-level sections into corequisite support courses. For the curriculum to run smoothly, we are coordinating these courses on a very large scale. Weekly coordination meetings are being held to discuss not only content, but also pedagogy in those courses. We are working to create a community of practice, where we share information on content and pedagogy and search for ways to provide better support for all students in a first-year mathematics course. This talk will discuss our model and results we've seen in creating our community of practice. (Received September 05, 2019)

1154-A5-849 **Emilie Hancock*** (emilie.hancock@cwu.edu), **Spencer Bagley, Lorraine Franco** and **Gulden Karakok**. *A Holistic Approach to Supporting Student-centered Pedagogy: Navigating Co-requisite Calculus I.*

As calculus instruction moves further from traditional practice toward student-centered pedagogy, support services must align with the changing goals for the course. This talk describes the work of a department as they collectively design and redesign a co-requisite Calculus I course, in an effort to better promote students' academic and social integration into active, inquiry-based calculus classrooms. We highlight the inclusion of more cognitively demanding and group-worthy tasks, re-negotiation of support for procedures and fundamental concepts, and promotion of collaborative participation. Our discussion is supplemented with reflections from faculty, graduate student instructors, and students. (Received September 11, 2019)

1154-A5-861 **Jesus R Oliver*** (jesus.oliver@csueastbay.edu), 25800 Carlos Bee Boulevard, Hayward, CA 94542, and **Julia Olkin** (julia.olkin@csueastbay.edu), 25800 Carlos Bee Boulevard, Hayward, CA 94542. *A Community of Practice Model for Infusing Active Learning in the Classroom.*

This work outlines a model for creating momentum for active learning at the departmental level through a Mathematical Community of Practice (MCoP). This approach focuses on empowering members of the community by providing easy access to active learning tasks, pacing guide via a dynamic calendar, and teaching tips for the course.

The dynamic calendar, housed in an electronic repository for easy access, incorporates a weekly pacing guide, live links to group-worthy tasks, and helpful comments on content emphasis and coverage. The active learning

tasks and pacing designated in the dynamic calendar are discussed and shared in monthly MCoP meetings. The instructors at these meetings discuss ideas about the course, check in on their pacing, and share insights. Our instructors report that they feel heard, respected and supported as a result of being part of the MCoP.

The SEMINAL project at CSUEB has seen some early success in lowering the departmental DFW rates for Calculus 1. Specifically, the strategic infusion of active learning brought about by the SEMINAL effort has coincided with a lower DFW rate for all sections of Calculus 1. This lower DFW rate has persisted across all sections taught. In particular, the historical DFW for this course has shifted from 33% to about 19%. (Received September 11, 2019)

1154-A5-974 **Steve Bennoun*** (s.bennoun@cornell.edu) and **Tara Holm**. *Establishing Consistent Active Learning in Calculus I Courses.*

Each year, over 700 students enroll in one of the two introductory Calculus courses offered at Cornell University. In 2017 the Mathematics Department secured an internal grant to implement systemic change in these courses and establish active learning as a major teaching component. In this talk we present the modifications we have made to reach this goal. We describe the elements that have been pivotal to the success of the initiative for each course, as well as the hurdles we have faced and how we have overcome them. We also report on our ongoing efforts to assure these changes are sustained. (Received September 12, 2019)

1154-A5-1151 **Marion Campisi*** (marion.campisi@sjsu.edu), **Wes Maciejewski**, **John Bragelman**, **Tim Hsu**, **Andrea Gottlieb**, **Jordan Schettler**, **Trisha Bergthold** and **Bem Cayco**. *Change comes from without: lessons learned in a chaotic year.*

San Jose State University is one campus in a larger, multi-campus system. In 2017, the Chancellor of the university system discontinued the developmental mathematics programs at the 23 campuses. This caught us by surprise and compelled us to act quickly. This article gives an overview of our response, highlighting the change our department underwent to improve our pre-calculus stream and general education courses, making them more interactive, supportive, and student-centered. (Received September 13, 2019)

1154-A5-1405 **David M McClendon*** (mccle2@ferris.edu), 820 Campus Drive, ASC 2021, Big Rapids, MI 49307. *Transforming tradition in calculus instruction at Ferris State University.*

In this talk, we describe efforts to improve calculus instruction at Ferris State University, a mid-sized, career-oriented, comprehensive public university located in Big Rapids, MI. We will tell the story of how an initial effort at reform led by one faculty member has led to a collaborative effort to comprehensively assess our calculus courses. The data we have collected is helping drive changes in course content and teaching practice. Specifically, we are more greatly incorporating technology in the classroom as well as inquiry-based and other student-centered modes of instruction. During this presentation, we will highlight our achievements thus far, and discuss challenges we still face. (Received September 15, 2019)

1154-A5-1596 **Mike Weimerskirch** (weim0024@umn.edu) and **Shelley Kandola*** (kandola2@msu.edu). *Flipping the Formula in Active Learning Classrooms.*

The University of Minnesota has shifted its PreCalculus sequence from a lecture-style course to an active learning course that focuses on problem-solving skills, communication skills, and an understanding of algebra as the language of abstraction. Instead of asking students to find numerical answers to specific problems based on a general formula presented by the instructor, we instead ‘flip the formula’. We begin with specific problems which lead students to build to a generalization. Students find solutions to specific problems until they can find a pattern, then describe an algorithm for solving problems of that type, and use that algorithm to solve the general case. The program is now in its seventh year, with over 1000 students annually in three courses, taught by a staff of 40 instructors including professors, post-docs, grad students, and undergrad teaching assistants. Early measures of success include a decline in the withdrawal rate, increased retention of students in the PreCalculus sequence, and an increased likelihood of completing Calculus. The biggest challenge is finding appropriate ways to measure student proficiency in higher-order thinking skills that translates to a valid determination of course grades at the end of the semester. (Received September 16, 2019)

1154-A5-1651 **Sarah K. Bleiler-Baxter*** (sarah.bleiler@mtsu.edu), Middle Tennessee State University, 1301 E. Main Street, MTSU Box 34, Murfreesboro, TN 37132, and **Jeremy F. Strayer** and **Jordan E. Kirby**. *Teaching TRIOs: A Peer Observation Model for Sharing Teaching Practice and Initiating Department Change.* Preliminary report.

An important step to achieving a sustainable department culture that encourages active-learning models of instruction is providing faculty with opportunities to share teaching practice and learn from one another. This

sharing of teaching practice should be accomplished in a manner that respects and values the individuality of all instructors while simultaneously exposing faculty to new ideas in order to facilitate continued growth. In this session, we describe the initial design and evolution of Teaching TRIOs—a Time-sensitive, Reciprocal, Inclusive, and Operative process of peer observation intended to respect all teaching practice while promoting dialogue and innovation. The Teaching TRIOs model has been used across 5 semesters with 31 distinct faculty members from our Department of Mathematical Sciences at MTSU. We share some of the early successes of this model that include (a) providing a safe space for faculty to discuss teaching and active learning, (b) allowing exchange of ideas by faculty who typically come from different content areas (particularly useful in large departments), (c) fostering teaching relationships that persist beyond the TRIO initiative, and (d) fostering a more open and inclusive climate for reflection on teaching across the department. (Received September 16, 2019)

1154-A5-1682 **Vesna Kilibarda*** (vkilibar@iun.edu), Indiana University Northwest, Department of Mathematics and Actuarial Sc., 3400 Broadway, Gary, IN 46408, and **Yuanying Guan** and **Xiaofang Wang**. *Role of Active Collaborative Learning Workshops in Changing the Introductory Mathematics Course.*

Literature have shown that active learning increases student performance in science, engineering, and mathematics courses, irrelevant of the field or the course type [Freeman, S. et al, 2014]. In addition, active learning has a disproportionately beneficial effect on members of minority groups in STEM fields [Treisman, U., 2009].

As a part of a larger project, Reimagining the First Year, our faculty developed active collaborative learning (ACL) activities for the mathematics general education course with a focus on increasing student success. After a preliminary run in several sections, our results have shown an increase of 6% in scores on common midterm and final exams for ACL compared to control sections and a 17% increase in retention.

After such encouraging results, we involved the whole department. We obtained a Teaching and Learning Prototype Grant, which awarded stipends for associate faculty to attend ACL workshops and become familiar with the ACL activities and course improvements. All instructors of the course added ACL activities. Our efforts resulted in improvements in both GPA and DFW rates. The overall class GPA for the course in fall 2018 was 2.15 with a DFW rate of 31% as compared to the baseline class GPA of 1.91 with a 44% DRW rate. (Received September 16, 2019)

1154-A5-2089 **Deborah Moore-Russo*** (dmr@ou.edu), **Keri Kornelson** (kkornelson@ou.edu), **Milos Savic** and **Noel Brady** (nbrady@ou.edu). *First Year Mathematics Program at the University of Oklahoma.*

Over the last 5 years, the Department of Mathematics at the University of Oklahoma has developed its First Year Mathematics (FYM) Program. This effort aimed to improve students' experiences and learning in early math courses. The first wave of change was structured around the 7 Characteristics of Successful Calculus Programs identified by the MAA (Bressoud et al., 2015). Crucial to the success of FYM efforts was the transition from extensive use of adjuncts to renewable term faculty positions and the selection of an experienced director in 2017. Additional efforts include piloting corequisite course offerings to help borderline students avoid no-credit remedial courses and leveraging partnerships with the College of Engineering to help fund fast-track sequences to offer catch-up opportunities for students needing to complete the calculus sequence. Recently, FYM turned to Calculus I incorporating coordination and using grassroots efforts to create active learning activities that allow for more student-centered instruction. Change has been steady, and FYM has had to be responsive to consistently increasing enrollment pressures and to challenges faced due to placement decisions made at administrative levels. In this talk, we describe the status and goals of the FYM program at OU. (Received September 17, 2019)

1154-A5-2394 **Sean Gruber*** (sgruber9@umd.edu), University of Maryland, 3942 Campus Drive, Benjamin Building, College Park, MD 20742, and **Kasso Okoudjou** and **Raluca Rosca**. *Active Learning in an Undergraduate Precalculus Course: Insights from a Course Redesign.* Preliminary report.

As a part of the Student Engagement in Mathematics through an Institutional Network for Active Learning, or SEMINAL network, the Math Department at UMCP has continued to work towards implementing active learning in a redesign of its Precalculus course. As a gatekeeper course, a major aim of the redesign of PC has been to evaluate how active learning can better support all students in being successful in PC. The overall goals of the redesign include: building on existing active learning resources; phasing in active learning materials/strategies into different versions of the course; coordinating the instruction and development of active learning materials/strategies; adapting active learning materials/strategies based on local data; and receiving support from on-campus resources and the SEMINAL network. The central features of our redesign include: regrouping/reordering of syllabus content; regularly using formative assessments; using small group activities

involving student-to-student interaction; and incorporating metacognitive strategies for students to reflect on their studying practices. In our presentation, we will share practical examples of our implementation and success with active learning. (Received September 17, 2019)

1154-A5-2499 **Martina Bode*** (bodem@uic.edu), **Jenny Ross** (jross19@uic.edu) and **Matthew Lee** (mattlee@uic.edu). *Moving large lectures to collaborative learning in newly designed classrooms*. Preliminary report.

Introductory math classes are often taught in large lecture halls. What happens when you move these classes into newly designed active learning spaces?

In response to low passing rates in introductory math classes, and 4-year graduation rates of 37%, we have been actively integrating active learning methods into our Introductory Calculus and Pre-Calculus classes.

Spring 2019, we opened a newly designed classroom designed to foster active learning in the classroom. This room seats 120 students at round tables. Each round table has a white board and an LCD screen. We moved all of our introductory Calculus I classes to the new classroom. Fall 2019, more active learning classrooms opened on campus. The designs are slightly different, but they are built with the idea of fostering active learning; some instructors even prefer the newer rooms.

This presentation will be a hands-on overview on designing and utilizing new learning spaces that allow moving away from traditional lecturing; best practices, benefits, challenges, and first outcomes. We saw improved pass rates, in particular impressive increases in students receiving an A or B in these courses, and significant higher pass rates for latinx students. (Received September 17, 2019)

1154-A5-2534 **Gary A Olson*** (gary.olson@ucdenver.edu), **Michael Ferrara**, **Michael Jacobson** and **RaKissa Manzanares**. *Active Learning Infusion Through a Comprehensive Graduate Teaching Assistant (GTA) Training Program*. Preliminary report.

In this talk we discuss how a comprehensive GTA training program was utilized to infuse active learning throughout our introductory courses and shift the culture of the department with respect to views around teaching and learning. We will discuss how a pre-semester Excellence in Teaching Symposium is used to gather active learning buy in and provide GTAs with examples of how activities and pedagogy could be facilitated and used in the classroom. We will also illustrate the use of a multi-level mentoring program involving a TA Coach to support new GTAs use of active learning in their classroom during their first year of teaching. We will also share how activity development and organization has increased the use of active learning across all faculty in our introductory courses. Finally, we will present results on how our active learning emphasis has improved student success and retention rates in select introductory courses. This work is sponsored in part by NSF Grant 1821454. (Received September 17, 2019)

1154-A5-2547 **Roberto C Soto*** (rcsoto@fullerton.edu), California State University, Fullerton, CA 92831, and **Alison Marzocchi** (amarzocchi@fullerton.edu), California State University, Fullerton, CA 92831. *Learning about Active Learning While Actively Learning: Best Practices for Faculty Professional Development*.

Many colleagues at our university have accepted that active learning can improve student outcomes yet having little to no formal training in education. They desire professional development that is both effective and meets their needs. Thus, our department embarked on an initial year of active learning professional development for mathematics faculty, implementing and researching a variety of professional development formats. We conjectured that effective faculty professional development, like effective mathematics instruction of students, would engage faculty in active learning experiences. Initial results indicate that effective professional development for faculty is tightly aligned with effective classroom instruction of students. This talk provides a framework of professional development practices for faculty, informed by a subset of mathematics instructional practices. (Received September 17, 2019)

1154-A5-2586 **Darryl J Chamberlain Jr.** (dchamberlain31@uf1.edu), **Kevin Knudson**, **Melissa Shabazz**, **Willard Keeran**, **Alexander York**, **Amy Grady*** (a.grady@uf1.edu) and **Corey Stone**. *Active Calculus at the University of Florida*. Preliminary report.

This talk will present how faculty at the University of Florida flipped half of their Calculus I courses. This was a large-scale endeavor as a typical fall semester enrolls approximately 1,800 students. In addition to providing an overview of how active learning was incorporated into the flipped classes, we also detail early successes (e.g., student affect) and challenges faced (e.g., students' mathematical writing ability). (Received September 17, 2019)

1154-A5-2590 **Jennifer Clinkenbeard, Alana Unfried and Jeffrey O Wand*** (jwand@csumb.edu). *Coordinating Active Learning Across First-Year Mathematics and Statistics.*

Consistent opportunities for active learning across coordinated courses are a challenge for a variety of reasons. Faculty need support both in terms of teaching materials and professional development. Transparent expectations and regular communication are also an important consideration when approaching this goal. In this session, we share California State University Monterey Bay's approach to coordinated active learning in introductory mathematics and statistics courses. Active learning is implemented through the framework of Complex Instruction, which aims to address social inequalities in the classroom. Undergoing a major course redesign, including the elimination of remedial math and the introduction of corequisite structures in response to an executive order from the California State University Chancellor's office, was the catalyst for the design and implementation of coordinated active pedagogy across four introductory courses. We discuss questions of design, support, and buy-in at all levels. In addition, we share results of student and faculty perceptions of the change, and consider how these data inform future work in this area. (Received September 17, 2019)

1154-A5-2641 **Joanna G. Jauchen*** (jjauchen@gmu.edu), Department of Mathematical Sciences, 4400 University Drive, MS 3F2, Fairfax, VA 22030, and **Bob Sachs and Catherine A Sausville.** *Ready for Change - Institutional and departmental factors supporting active learning recitations.* Preliminary report.

In Fall of 2019, the math department at George Mason University initiated a series of systemic changes in order to implement active learning strategies in all Calculus I and II recitations. In addition to funding from an NSF IUSE grant, our initiative benefits from several institutional and departmental contextual factors which predated this change. We began departmental planning in January of 2019, which continued during the year and included conversations about the nature of calculus recitations and the data needed before making decisions. One early measure of success is that faculty seemed to be genuinely interested in the initiative and we received little-to-no resistance to enacting this sweeping change to the Calculus sequence. The main changes underway are: (1) recitations in classrooms with whiteboards on all 4 walls, led by teams of GTAs and Undergraduate Learning Assistants; (2) the adaptation of curricular materials from SDSU for these recitations; (3) an improved orientation for GTAs and more consistent follow-up during the term; (4) creation of a mathematics teaching and learning seminar; (5) data gathering on placement testing and student backgrounds; and (6) more consistent communication with Calculus I and II lecture faculty. (Received September 17, 2019)

1154-A5-2749 **Topaz H Wiscons*** (topaz.wiscons@csus.edu). *Designing, Coordinating, and Supporting Active Learning for Multi-Section College Algebra Courses.*

In August 2017 The California State University (CSU) issued Executive Order 1110, which effectively eliminated all pre-baccalaureate developmental mathematics courses across all campuses of the CSU. In response to this, CSU, Sacramento (Sacramento State) introduced two new entry-level baccalaureate credit-bearing college algebra courses in Fall 2018: one developed for STEM majors and one developed for non-STEM majors. Both courses were designed to be taught using inquiry-based methods to elicit student engagement, strengthen analytical ability, and increase student confidence and self-reliance. To uphold the objectives of the course design and to support the course instructors, these courses are coordinated and course materials are provided to instructors. In this talk, I will share how these two courses are coordinated across 39 sections and 28 instructors (primarily part-time instructors and graduate students) in Fall 2019 and the parameters that informed our coordinating decisions. I will discuss what we have learned from our efforts to support both instructors and students, including professional development for instructors. Finally, I will report student anecdotes and the impact this redesign has had on student success in algebra at Sacramento State. (Received September 17, 2019)

1154-A5-2773 **Lisa Driskell*** (ldriskel@coloradomesa.edu), Department of Mathematics and Statistics, 1100 North Avenue, Grand Junction, CO 81501, and **Tracii Friedman** (tfriedma@coloradomesa.edu), Department of Mathematics and Statistics, 1100 North Avenue, Grand Junction, CO 81501. *Shifting the Culture: Actively Engaging Students in Introductory Service Courses.*

Colorado Mesa is a regional public university with over 10,000 students and more than 3,200 taking in introductory mathematics and statistics courses each year. The math department has recently made a commitment to re-evaluate each of our introductory course offerings with the goal of making them more meaningful to students enrolled while also addressing new state guidelines for placement. Our early efforts include implementing new placement practices, piloting supplemental instruction courses, assessing current active learning course activities, and meeting with department leaders across campus to determine how to better serve the needs of their

programs. Our aim is to be intentional about actively engaging students in content relevant to their disciplines and future careers.

In this talk, we will discuss these early efforts as well as our initiative to facilitate a culture shift in our department by creating venues for sharing experiences and expertise for actively engaging students. Such venues include Faculty Communities of Practice, regular Faculty Professional Development activities, and community electronic resource collections. We describe our experiences navigating the successes and challenges while in the early stages of change. (Received September 17, 2019)

1154-A5-2791 **Rebecca Swanson***, 1500 Illinois st., Applied Mathematics and Statistics, Golden, CO 80401, and **Debra Carney**, **Megan Sanders**, **Samuel Spiegel** and **Ariana Vasquez**. *Calculus II Course Redesign: Supporting Faculty in Pedagogical Change*. Preliminary report.

We describe a partially flipped course design for Calculus II which has been successfully implemented by a varied instructional team. The main factors which have contributed to the success of the implementation are (1) institutional support for teaching, (2) use of evidence-based practices, and (3) creation of faculty community. There are demonstrations of both student and faculty success through student assessment data and faculty interviews. (Received September 18, 2019)

1154-A5-2799 **Cristina Villalobos*** (cristina.villalobos@utrgv.edu), **Tim Huber** (timothy.huber@utrgv.edu), **Hyung Kim** (hyung.kim@utrgv.edu), **Roger Knobel** (roger.knobel@utrgv.edu), **Shaghayegh Setayesh** (shaghayegh.setayesh@utrgv.edu), **Andras Balogh** (andras.balogh@utrgv.edu) and **Anahit Galstyan** (anahit.galstyan@utrgv.edu). *Active Learning and Coordination in Precalculus to Calculus 2 classes*.

In this talk, we discuss the implementation of recitation labs in Precalculus, Calculus 1, and Calculus 2 classes to increase student success via active learning. Given the large number of sections, undergraduate Learning Assistants play a critical role in providing guidance and feedback during recitation labs. In addition, we discuss the coordination of assessments and syllabi in each of the three courses. We present data including a comparison of student success metrics. (Received September 18, 2019)

1154-A5-2803 **Megan E Selbach-Allen*** (mseibach@stanford.edu) and **Amy Ksir** (ksir@usna.edu). *Activating Calculus to Command the Seas: Reflecting on ten years of active and inquiry-based learning at the US Naval Academy*.

Over the past ten years, the authors of this paper have implemented a series of project-based and inquiry-based learning initiatives in the calculus sequence at our institution. Each one had successes, and each had failures from which we learned and improved; we now have a thriving community of instructors successfully implementing IBL in calculus. At the same time, the department culture has gradually shifted, and active learning in many different forms has become increasingly common. In a recent survey of our department, the majority of faculty members who responded indicated that they have increased their use of active learning and/or IBL, for various reasons. In this presentation, we will describe how we built on an existing community of practice around teaching to create an environment where active learning can flourish. (Received September 18, 2019)

1154-A5-2816 **Daniel James*** (jamesdw@udel.edu), Ewing Hall, 15 Orchard Rd, Newark, DE 19716, and **Christopher Raymond**. *Our Experiences in a Graduate Teaching Assistant Training Pilot*. Preliminary report.

During the 2018-2019 academic year, we partnered with a colleague from the College of Education & Human Development to train our graduate students assigned to first-semester Calculus to effectively lead their discussion sections. The pilot included: discussions of cognitive demand, learning goals, and lesson plan construction around those learning goals; intentional use of active learning in the classroom; anticipating students' needs through reflecting on pathways to solution; weekly lesson plan critiques; a recording of interactions with students during office hours and a video/audio recording of each gTA's class paired with a reflective essay for each. Effects on undergraduate students' engagement, demonstration of knowledge, and ownership of knowledge were measured as a way to determine what effect - if any - the training program was having on the gTAs' classrooms. We found these metrics as well as the frequency with which the gTAs used active learning in their classrooms increased with the frequency of our meetings. The pilot was met with great resistance, and many adjustments to our ongoing training program for the 2019 - 2020 academic year have been made based on the results of the pilot program. (Received September 18, 2019)

1154-A5-2818 **Kendra E Pleasant*** (kendra.pleasant@morgan.edu) and **Ahlam Tannouri**. *What Active Learning Looks Like at Morgan State University*.

Culturally responsive teaching (CRT) is a teaching pedagogy that involves using the students' personal experiences and backgrounds to influence the course design. At Morgan State University, we are using this idea to construct a set of five principles that governs the classroom atmosphere and influences the problem selection. We incorporated CRT into the department through a pilot program including five faculty members and one graduate student. We are now in the process of expanding CRT to the entire mathematics department. We will share a set of examples or study cases used in the classroom. We will also discuss the response CRT has gotten from the faculty and students. (Received September 18, 2019)

Creating Spaces for Mathematics

1154-AA-48 **Madeline E. Polhill*** (madeline.polhill@richmond.edu). *Bryn Mawr College, 1885–1940: "Mathematics on its mind"*.

With the 100th anniversary of the passage of the 19th amendment arriving later this year, and as women continue working to break through glass ceilings in academia, government, and industry, looking to the past helps provide inspiration for members of today's mathematical community to support female students and colleagues. Though its yearly total enrollment never surpassed 640 students in its first 55 years of existence, Bryn Mawr College awarded more mathematics PhDs to women from 1885–1940 than all but two other American institutions. Together, administrators M. Carey Thomas and Marion Edwards Park and mathematicians Charlotte Angas Scott and Anna Pell Wheeler, in their efforts to support women, advanced mathematics and advanced the women around them. An exploration of the existing scholarly body and archival work at Bryn Mawr College and the Library of Congress revealed several common, timeless ways in which these four women helped create a haven for mathematics at Bryn Mawr. These women's efforts remain relevant today as we continue working towards gender equity in mathematics. (Received July 24, 2019)

1154-AA-399 **Sloan Evans Despeaux*** (despeaux@wcu.edu), Department of Math and CS, Western Carolina University, Cullowhee, NC 28723. *Prospering Through Questions and Answers: Almanacs as a space for English mathematicians to prosper, 1775-1850*.

English almanacs created a space for mathematicians outside of the university sphere. These mathematicians used this space to display mathematical prowess and achieve renown both locally and nationally. This attention in turn opened the doors to employment opportunities, in particular the Royal Military institutions. (Received September 03, 2019)

1154-AA-1494 **Patti W Hunter*** (phunter@westmont.edu), 955 La Paz Road, Santa Barbara, CA 93108. *Spreading the Wealth: The Ford Foundation and Eugene Northrop's Advancement of Mathematics and Science at Home and Abroad*.

In 1934, Eugene Northrop finished his Ph.D. in mathematics at Yale, and was competing for scarce jobs with his classmate, Saunders Mac Lane. He settled for a position at the private boarding school, Hotchkiss, where he stayed until 1943. That year, Northrop took up a post at the newly founded College of the University of Chicago. Partnering with the dean at the time, F. Champion Ward, Northrop developed and taught in the college's mathematics program. A year spent as an education consultant for the NSF and the Ford Foundation's Fund for the Advancement of Education in 1955, along with his connections to Ward, who had joined the foundation a few years earlier, positioned Northrop to move from the world of American liberal arts education to a role in which he helped shape the funding activities of what had become the richest philanthropic foundation in the world. This talk will consider Northrop's work at the University of Chicago, and his contributions to the development of university science in Turkey through his position at the Ford Foundation. (Received September 15, 2019)

1154-AA-1579 **Brit Shields*** (bshields@seas.upenn.edu). *Emergency Mathematics for the Engineer: US Regional Training Programs in the Second World War*.

During the Second World War, the US Office of Education embarked upon a massive effort to train – and retrain – engineers for the war effort. Under the Engineering, Science and Management War Training Program, courses were developed at hundreds of universities and colleges spanning the country. An important component of the scientific training of engineers was a strong mathematical foundation. This talk will examine how mathematics was seen as a vital part of the war effort through the lens of training engineers in various regions of the country and tailored for various war related industries. (Received September 16, 2019)

- 1154-AA-1987 **Deborah A. Kent*** (deborah.kent@drake.edu), Drake University, Department of Mathematics and Computer Science, 2702 Forest Avenue, Des Moines, IA 50311. *19th-century eclipse expeditions: A space for mathematical greatness?*

The second half of the nineteenth century brought the golden age of eclipse expeditions. Equipped with the latest technology and dreams of glory, international observing parties trekked around the globe to find astronomical insight in the solar darkness. North American eclipse paths in 1860, 1869, and 1878 especially played into the scientific agenda articulated by mid-century mathematical practitioners in the United States. In pursuit of national scientific achievement, legions of government scientists were deployed in increasingly coordinated efforts.

Their successes and attendant publicity boosted American science and laid a foundation for American space science. (Received September 18, 2019)

Research in Undergraduate Mathematics Education: Highlights from the Annual SIGMAA on RUME

- 1154-AB-467 **John Paul Cook*** (cookjp@okstate.edu), 401 Mathematical Sciences Building, Oklahoma State University, Stillwater, OK 74078. *Monster-barring as a catalyst for connecting secondary algebra to abstract algebra.*

Abstract algebra is a course that most future secondary teachers are required to take as undergraduates, yet many - both students and educators - see abstract algebra as unrelated to secondary algebra. In this talk I offer a new avenue for discussion on this issue: instead of a cumbersome difficulty to be overcome, the substantial differences between secondary algebra and abstract algebra can (and should) be (1) acknowledged, and (2) reframed as productive opportunities for learning. I defend these claims by reporting on a teaching experiment in which a preservice secondary mathematics teacher leveraged his intuitive knowledge of secondary algebra in order to develop productive, foundational understandings in abstract algebra. A critical step in the learning trajectory involved his outright rejection of zero-divisors, an idea in abstract algebra that has few, if any, traces in secondary algebra. This activity - an example of what Lakatos (1976) called monster-barring - was then productively repurposed as a meaningful way for him to classify types of abstract algebraic structures. I conclude by proposing that these kinds of learning experiences could be beneficial for all undergraduate students and, specifically, prospective teachers. (Received September 04, 2019)

- 1154-AB-524 **Elise Lockwood***, elise.lockwood@oregonstate.edu. *Using a computational context to investigate student reasoning about whether "order matters" in counting problems.*

When solving counting problems, students often struggle with issues of order - that is, with distinguishing between permutations and combinations. There is a need to explore potential interventions to help students conceptually understand whether "order matters" and to differentiate meaningfully between these operations. In this talk, I investigate students' understanding of the issue of order in the context of Python computer programming. I show that some of the program commands seemed to reinforce important conceptual understandings of permutations and combinations and issues of order. These results suggest that even elementary programming activity can illuminate important combinatorial distinctions for students. I suggest that this is one example of a way in which a computational setting may facilitate mathematical learning. (Received September 06, 2019)

- 1154-AB-556 **George Kuster*** (george.kuster@cnu.edu), 1 Avenue of the Arts, Newport News, VA 23606, and **Steven Jones** (sjones@mathed.byu.edu), Brigham Young University, 167 TMCB, Provo, UT 84602. *Variational Reasoning Used by Students While Discussing Differential Equations.*

In this study we investigated how a small sample of students used variational reasoning while discussing ordinary differential equations. We found that students had flexibility in thinking of rate as an object, while simultaneously unpacking it in the same reasoning instance. We also saw that many elements of covariational reasoning and multivariational reasoning already discussed in the literature were used by the students. However, and importantly, new aspects of variational reasoning were identified in this study, including: (a) a type of variational reasoning not yet reported in the literature that we call "feedback variation" and (b) new types of objects, different from numeric-quantities, that the students covaried. (Received September 06, 2019)

- 1154-AB-1160 **Irma E. Stevens*** (istevens@umich.edu). *The Role of Multiplicative Objects in a Formula.*

One of the upcoming avenues of research in the quantitative reasoning literature is studying the role the construction of a multiplicative object has in a meaning for a graph "as a continuum of states of covarying quantities"

(Saldanha & Thompson, 1998) (e.g., Frank, 2016, 2017, in press; Stevens & Moore, 2017; Thompson, 2011; Thompson & Carlson, 2017). In this paper, I build on the research done with graphical representations by discussing the role constructing a multiplicative object has in a meaning for a symbolic representation (namely, a formula) that represents the varying measures of attributes identified in a situation. I propose a way to conceive of a multiplicative object with a formula. I then demonstrate the role of conceiving of a multiplicative object when constructing a formula to represent quantities in a situation. To do so, I will use the results of a four-month long individual teaching experiment designed to support a preservice secondary mathematics teacher's covariational reasoning and construction of formulas through dynamic geometric environments. (Received September 13, 2019)

1154-AB-2110 **Jessica Gehrtz*** (jessica.gehrtz@uga.edu). *Responsiveness as a Disposition and Its Impact on Instruction.*

There is evidence that instructors who are responsive to students' thinking tend to provide more positive learning experience for students. Additionally, effective instruction relies on an instructor's ability to respond to student thinking, which is especially relevant due to the increased attention on improving college mathematics instruction. In order to investigate instructor responsiveness to student thinking as a disposition (that guides action) and responsiveness to student thinking as an action (the enacted evidence of the underlying disposition), eight college Calculus instructors were interviewed three times over the course of one academic year. A thematic analysis of the task-based interviews indicated that instructors who exhibited a responsive disposition to their students' thinking enact this through eliciting student thinking, reflecting on student thinking, and responding to student thinking. Further, these instructors view themselves as decision-makers, and thus feel empowered to act on their responsive disposition. (Received September 17, 2019)

COMAP Modeling Contests

1154-AC-993 **Jessica M. Libertini***, libertinijm@vmi.edu. *Simple Models for the Complex Problem of Measuring and Improving Sustainable Development.*

In 2015, the Interdisciplinary Contest in Modeling (ICM) asked students to develop a way to measure and compare countries' relative levels of sustainable development, present a targeted 20-year plan to help improve the sustainable development for one underdeveloped nation, and model the impact of their plan. The issue of sustainable development is a complex one, as we must look for a balance between the preservation of natural resources for future generations and the use of those resources today to promote improvements in a society's well-being. Despite the complexity of the problem, some of the strongest submissions leveraged relatively simple mathematics in powerful ways. In this talk, we will present some of these contest-winning approaches. (Received September 12, 2019)

1154-AC-1390 **Amanda Beecher*** (abeecher@ramapo.edu) and **Chris Arney**. *Who should we send to colonize Mars?*

The new space race aims to colonize Mars, which provides an opportunity to design a productive and thriving workforce. The goal is to maximize the output of the society, while simultaneously meeting the needs of the individuals. Policies governing income, education, and social equality must be considered to ensure a flourishing society, just as on Earth. This talk will showcase models used, by teams of students answering this 2017 ICM Problem F, to set priorities to determine the best mix of 10,000 persons to establish this initial colony and how the demographics chosen may need to change as more colonists reach Mars. (Received September 15, 2019)

1154-AC-1587 **Elizabeth W Schott*** (eschott@fsw.edu). *The Mathematics of the COMAP Modeling Contests - Problem A.*

Over the last five years, the students involved in the COMAP modeling contest have explored the challenges a variety of topics ranging from eradicating Ebola, managing the Zambezi River, optimizing a hot bath, multi-hop HF radio propagation to exploring the ecology of dragons. This talk will discuss the Problem A problems posed and look at the math behind the proposed solutions. (Received September 17, 2019)

1154-AC-2163 **Rodney X Sturdivant*** (rxsturdy@gmail.com) and **Robert E Burks**. *Data Driven Mathematical Modeling.*

The "Data Insights" problem became part of the Mathematical Contest in Modeling (MCM) in 2016. This problem is meant to include mathematical modeling challenges specifically associated with the use of data in addressing difficult open ended problems. While not necessarily a 'big data' problem, the teams are presented

with real-world data with interesting complicating factors that impact their modeling efforts. We will provide an overview of the first few years of “Data Insights” problems and then discuss the types of data challenges the problems posed. We will include examples of how some of the outstanding teams handled these problems and highlight techniques used in data cleaning, visualizations, and ultimately producing data-based models to solve difficult problems. (Received September 17, 2019)

1154-AC-2255 **William C. Bauldry*** (bauldrywc@appstate.edu), Dept of Mathematical Sciences, Appalachian State University, 121 Bodenheimer Dr, Boone, NC 28608. *Perspectives on Student’s Mathematical Approaches for the “B Problem” (Discrete) of COMAP’s Mathematical Modeling Contest.*

COMAP’s Mathematical Contest in Modeling (MCM) always includes a problem focused on discrete mathematics — the “B Problem.” After summarizing recent contest problems, we will discuss student approaches that have been used and offer insights from Final Judges. (Received September 17, 2019)

1154-AC-2417 **Michelle Lynn Isenhour*** (mlisenho@nps.edu), 1411 Cunningham Rd., GL-263, Monterey, CA 93943. *Time to Leave the Louvre: Student Modeling Approaches.*

Imagine it’s Thursday at 5:00 pm. Your boss walks into the office, concerned about the recent terror attacks in Paris, and tells you that he needs you to develop a mathematical model representing the evacuation of the Louvre Museum. You know, just develop the model, analyze the dynamics, and then write a 20-page report complete with policy recommendations. And as if that isn’t enough, it needs to be completed before Monday at 8:00 pm! You might think this is crazy and unrealistic, but more than 5,200 teams of undergraduate students provided solutions to this exact problem as part of the 2019 ICM! In this talk, we’ll look at the mathematics behind some of these student solutions. We’ll showcase at the wide variety of evacuation models used and some of the novel ways in which teams implemented the models. Additionally, we’ll see how they applied optimization algorithms, such as Dijkstra, ant colony and particle swarm optimization algorithms, and even how many teams implemented simulation software in the development of their solutions. (Received September 17, 2019)

Can Mathematics Help Us Trust Our Elections Again?

1154-AD-377 **Karen Saxe*** (kxs@ams.org). *Redistricting for the Next Decade.*

Every ten years each state redraws its congressional district maps. Many map-makers are accused of partisan gerrymandering, and these challenges have been considered in our courts, including in the Supreme Court. I will give background on how redistricting is done by the states, an update on how mathematics and statistics are being called on by the courts in their deliberations, and what changes to look for when the states start drawing in 2020. (Received September 02, 2019)

1154-AD-372 **Philip B. Stark*** (stark@stat.berkeley.edu). *Risk-Limiting Audits.*

Risk-limiting audits (RLAs) check whether the reported winner(s) of an election really won. They have been piloted in more than 10 states and are currently mandated by law in four. RLAs require a trustworthy paper record of voter intent. A procedure is a RLA with risk limit α iff the chance it will correct the reported outcome if the reported outcome is wrong is at least $1 - \alpha$, and the chance it alters a correct outcome is zero. RLAs can be formulated as tests of the hypothesis that one or more reported winners did not actually win. The most efficient method checks how the voting system interpreted randomly selected ballots and uses sequential hypothesis tests based on martingale inequalities.

RLAs have been developed for many social choice functions (e.g., plurality, multi-winner plurality, majority, super-majority, and IRV) using a variety of sampling schemes tailored to individual voting systems, to how jurisdictions handle ballots, and to how states coordinate audits. RLAs are recommended by NASEM, the Presidential Commission on Election Administration, the American Statistical Association, and the League of Women Voters, among others. (Received September 02, 2019)

1154-AD-819 **Stephanie Frank Singer*** (sfsinger@campaignscientific.com). *Data Science for Election Oversight.* Preliminary report.

Like many mathematical systems, elections are complex and sensitive to detail. Unlike mathematical systems, elections affect and are affected by their messy real-world context. How can mathematicians effectively and realistically contribute to efforts to make elections both fair and trusted?

One approach is to apply data science and predictive analytics to real-time election oversight by building tools to identify anomalies in the preliminary election results available on and shortly after Election Day. The

analysis, if done quickly enough, can allow for timely investigation and appropriate consequences before election results are finalized. We will discuss progress and challenges in carrying out this program. (Received September 11, 2019)

1154-AD-2577 **Ronald L. Rivest*** (rivest@mit.edu). *Auditing elections in "other ways"*.

We show how to use mathematics to audit elections in "other ways" (other than the usual "risk-limiting audit" way, that is). We explore auditing with Bayesian methods, sampling using "k-cuts" (like cutting a deck of cards repeatedly), and even designing vote-counting methods that are quite efficiently auditable without statistical techniques. (Received September 17, 2019)

Modernizing the Introductory Statistics Course

1154-AE-2849 **Beverly Wood***. *GAISE 2016 in action*.

A brief overview of the updated GAISE College Report followed by the results from a mixed method study of instructor experiences with implementation in various classroom environments. Survey and interview data about the awareness of and implementation efforts made by instructors since the American Statistical Association's endorsement of the revised College Report in July 2016. New and experienced instructors from two-year, four-year, and graduate level institutions were included in the study. Particular attention was paid to the new emphasis on multivariable thinking. (Received September 17, 2019)

1154-AE-2850 **Patti Frazer Lock***. *How technology facilitates modernizing intro stats*.

Technology can dramatically change how and what we teach in Introductory Statistics. Some traditional topics may be no longer necessary or relevant, while other topics that used to be inaccessible in an introductory course are now easily accessible and increasingly important. We discuss what might be in, what might be out, and how making this switch to a technology-rich course can dramatically increase student engagement and student success. (Received September 17, 2019)

1154-AE-2851 **Albert Y. Kim***. *Statistical inference via data science: A "tidy" approach*.

We present a pathway for students to learn statistical inference using data science tools widely used in industry, academia, and government. We first introduce the tidyverse suite of R packages, including the ggplot2 package for data visualization, and the dplyr package for data wrangling. After equipping students with just enough of these data science tools to perform effective exploratory data analysis, we cover traditional introductory statistics topics like 1) multiple regression modeling and 2) confidence intervals and hypothesis testing using simulation-based inference. This approach centers on the use of data visualization and real-world multivariate datasets all throughout. (Received September 17, 2019)

1154-AE-2852 **Daniel Kaplan***. *Stats for data science*.

The canonical topics of college-level statistics are not the result of thoughtful examination of conceptual roots of the field. They are historically contingent, reflecting the needs, resources, and understanding of the era in which they were developed. Almost all of the topics of introductory statistics stem from the period from 1830 to 1925. They reflect the establishment of sociology, early genetics, and experiments on the bench-top or in agricultural field stations. Methods were tailored to very small amounts of data and calculation by hand.

Needless to say, needs and opportunities are different today. Data are abundant and multivariate; hypotheses are investigated by the dozens (nutrition research) or hundreds of thousands (genomics); an underlying methodology is machine learning; data are used to inform decisions, necessitating responsible inference about causation by adjusting for covariates. This new situation has resulted in a merging of components of computer science and statistics into "data science." In my presentation, I'll examine the appropriate statistical underpinnings for a meaningful engagement with data science, pointing out how they differ and sometimes contradict the topics of the century-old canon.

(Received September 17, 2019)

1154-AE-2853 **Kari Lock Morgan***. *The p-value: Replacing 0.05 with understanding*.

The American Statistical Association's 2016 P-Value Statement and The American Statistician's 2019 Special Issue "Statistical Inference in the 21st Century: A World Beyond $p < 0.05$ " both provide a call for action and dramatic change regarding how p-values are taught and used. In particular, they discourage the focus on whether a p-value is less than a specific threshold (e.g. 0.05), and encourage a better understanding of what the p-value

actually represents. This talk will discuss ways to replace “ $p < 0.05$ ” with conceptual understanding of a p-value in introductory statistics courses. (Received September 17, 2019)

1154-AE-2854 **Beth Chance***. *Conceptual approaches to teaching multivariable statistical thinking: Using simulation methods and visualization.*

Use of simulations to motivate and improve student understanding of formal inference procedures has shown promising results. In particular, students can more quickly focus on the entire statistical investigation process by leveraging tactile experiences, a conceptual focus that stays close to real data, and visualizing the simulation process. However, additional topics and concepts are also important. In particular, multivariable thinking continues to grow in importance as a fundamental part of a student’s statistical training. In this presentation, we will share ideas for using simulation methods and visualization techniques in order to explore multivariable reasoning (e.g., randomized block designs, interactions, conditional associations) with students in first and second algebra-based statistics courses. (Received September 17, 2019)

Experiencing Geometric Transformations and Evaluating Learning Progressions: Celebrating the Work of David W. Henderson

1154-AF-2843 **Daina Taimina***. *In search of meaning.*

A retrospective look at David Henderson’s career as a research mathematician and his profound impact on mathematics education, especially in the area of experiential pedagogy and the search for geometric meaning, will be presented by his colleague, collaborator, and partner Daina Taimina. (Received September 17, 2019)

1154-AF-2844 **Richard Leher*** and **Daina Taimina**. *What stays the same? Children’s exploration of Line and Polygon on plane, cylinder, sphere and hyperbolic plane.*

We report a multi-week study in which a class of fifth-grade students investigated “straight” curves and polygons on plane, cylinder, sphere and hyperbolic plane. Most of the participating students were from low-income families with diverse cultural and linguistic backgrounds, including Pacific Islander and Latino immigrant families, and rural white families. Student investigations featured literal and imagined motion to create curves and polygons on each surface from intrinsic and extrinsic perspectives. For example, we invited students to create tension on a rope, and after they agreed that the resulting path was straight, students wore video head cams and “baby crawled” without raising their heads, to experience straight paths from an intrinsic perspective. They also stood up and walked the same path to experience an extrinsic perspective. The same process was repeated for curves that they agreed were not straight. We propose to report some of the highlights of students’ investigations and the longer-term sense they made of them as indicated by their responses to the interview questions and tasks. (Received September 17, 2019)

1154-AF-2845 **Megan Wongkamalasai***. *Young children constructing understandings of 3D symmetries.*

Contemporary studies of STEM professions and of mathematical development consistently highlight the critical contribution to each made by understanding and visualizing space. Yet, current K-12 mathematics instruction largely ignores the systematic development of children’s mathematical understandings of space. In partnership with David and a team of first grade teachers, we explored how children’s construction of 3-D structures can serve as a context to foster initial understandings of isometry and symmetry transformations and engagement in mathematical defining. The instructional design was grounded in David’s commitment to fostering understandings of geometry through tangible experiences with space that extend to art and design. Findings presented in this talk address how the instructional design opened up new mathematical resources and forms of mathematical engagement to young children that expanded both their mathematical understanding of space and their senses of the nature of mathematics. (Received September 17, 2019)

1154-AF-2846 **Gregory Budzban***. *Geometric transformations and the grounding of proof.*

Much of David Henderson’s career involved arguing strongly against the notion that mathematics is only a study of formal deductive systems. In particular, Henderson argued for a particular definition of the concept of a “living proof” as a “A convincing communication that answers the question—Why?” This talk will explore the ideas at the heart of Henderson’s perspective of the concept of proof and will use the area of geometric transformations to compare and contrast the interplay of intuition and formalism. (Received September 17, 2019)

1154-AF-2847 **Edith Aurora Graf***. *Geometric transformations as functions: A qualitative analysis of student responses.*

The work we will discuss in this talk represents a collaborative effort among four organizations: Educational Testing Service, The Algebra Project, The Young People's Project, and Southern Illinois University Edwardsville. The goal of the project is to validate the interpretation of a learning progression-based assessment for the concept of function. A learning progression (LP) is a theory of how student thinking evolves from more intuitive ideas to more in-depth and nuanced understanding. The LP that provides the guiding theory for the design of the assessment tasks consists of three strands: the traditional strand, the finite-to-finite strand, and the geometry strand. As the lead developer of the Algebra Project's Symmetry, Shape, and Groups curriculum module, David played a key role in the conceptualization of the LP and the review of the tasks. He believed strongly in the importance of attending to student thinking in task design. Cognitive interviews were conducted with students as they solved geometry strand tasks. In May 2019, project staff met for a working meeting to analyze student responses to these tasks. Highlights from the meeting, which honor David's legacy of attending to student thinking in the development of learning theory and tasks, will be discussed. (Received September 17, 2019)

1154-AF-2848 **Kate Belin*** and **Fannie Lou Hamer**. *Experiencing high school geometry.*

David Henderson taught me that mathematics emerges from our personal (and shared?) human experience. High school geometry curricula can feel fraught with terminology, definitions, and algebraic manipulations, but David created materials that started with and built upon the experience of the individual learner. His notion that a proof is "a convincing communication that answers the question: why?" allows students to break away from the narrow idea of a two-column proof and to actually argue for what they see as true. In his work with teachers, he made us feel respected as educators. He helped us grow mathematically, in his gentle way that did not make us feel intimidated. In this talk, we will explore some of the ways that David's high school materials approached symmetries, straight lines and how considering multiple surfaces such as the sphere and hyperbolic plane can highlight certain features that are unique to the plane. (Received September 17, 2019)

Inspiring Diversity in Mathematics: Culture, Community, and Collaboration

1154-AG-1039 **Brittany Stephenson***, bstephenson@lewisu.edu, and **Cristina Lanzas, Suzanne Lenhart, Jason Bintz, Eduardo Ponce** and **Judy Day**. *Comparing Intervention Strategies for Reducing Clostridioides difficile Transmission: An Agent-Based Modeling Study*. Preliminary report.

In recent years, healthcare facilities have experienced an increasing substantial burden from the toxin-producing bacteria *Clostridioides difficile*, which can cause severe intestinal disease. This bacteria can survive for extended periods of time on hospital surfaces. In this talk, I will discuss the development of an agent-based model that simulates the transmission of *C. difficile* in a healthcare setting and considers contributions of the pathogen from environmental surfaces. This model explicitly incorporates healthcare workers (HCWs) as vectors of transmission, tracks individual patient antibiotic histories, incorporates varying risk levels of antibiotics with respect to *C. difficile* infection, and tracks contamination levels of ward rooms by *C. difficile*. I will also discuss how we used the model to evaluate the efficacy of a variety of control interventions and combinations of interventions on reducing *C. difficile* nosocomial colonizations and infections. The control techniques include two forms of antimicrobial stewardship, increased environmental decontamination through room cleaning, improved HCW compliance, and a preliminary assessment of vaccination. (Received September 12, 2019)

1154-AG-1222 **Azmy S. Ackleh** (ackleh@louisiana.edu), University of Louisiana at Lafayette, Lafayette, LA 70504, **Md Istiaq Hossain*** (hossain.istiaq@louisiana.edu), University of Louisiana at Lafayette, Lafayette, LA 70504, **Amy Veprauskas** (aveprauskas@louisiana.edu), University of Louisiana at Lafayette, Lafayette, LA 70504, and **Aijun Zhang** (aijun.zhang@louisiana.edu), University of Louisiana at Lafayette, Lafayette, LA 70504. *Persistence of a discrete-time predator-prey model with stage-structure in the predator*. Preliminary report.

We propose and investigate a discrete-time predator-prey model with a structured predator population. We describe the predator population using two stages, juveniles, and adults and assume that only the adult stage consumes the prey species. The unit of time is taken to be the maturation period so that all juveniles mature after one time unit. Meanwhile, prey population growth, in the absence of an adult predator, is assumed to

follow the Beverton-Holt nonlinearity. For this model, we discuss conditions for the existence and global stability of the extinction and predator-free equilibria as well as conditions for the existence and uniqueness of an interior equilibrium. We also find the conditions for the persistence of both prey and predator. Finally, we use numerical simulation to demonstrate various dynamical scenarios. (Received September 15, 2019)

1154-AG-1856 **Forest O Mannan*** (fmannan@western.edu), **Karin Leiderman** and **Miika Jarvela**.
Volvox: Modeling the Synchronization of Flagella on the Exterior of a Sphere.

Flagella are hair-like appendages attached to microorganisms that allow the organisms to traverse their fluid environment. The algae *Volvox* are spherical swimmers with thousands of individual flagella on their surface and their coordination is not fully understood. In this work, a previously developed minimal model of flagella synchronization is extended to the outer surface of a sphere submerged in a fluid. Each beating flagella tip is modelled as a small rotating sphere, elastically attached to a point just above the spherical surface and a regularized image system for Stokes flow outside of a sphere is used to enforce the no-slip condition. Biologically relevant distributions of rotors results in a rapidly developing and robust symplectic metachronal wave traveling from the anterior to the posterior of the spherical *Volvox* body. (Received September 16, 2019)

1154-AG-2138 **Laura A Miller*** (lam9@email.unc.edu), CB 3280 Coker Hall, Department of Biology, Chapel Hill, NC 27510. *Using computational fluid dynamics to understand muscle driven movement: Case studies in tubular hearts and jellyfish.*

Recent advancements in computational fluid dynamics have enabled researchers to efficiently explore problems that involve moving elastic boundaries immersed in fluids for problems such as cardiac fluid dynamics and animal swimming. These advances have also made modeling both nutrient exchange in a fluid and the muscle driven motion of a flexible organ or organism through a fluid feasible. The work presented here focuses on the development and implementation of such methods and models for the pumping and pulsation of tubular hearts and jellyfish bells. We leverage existing computational algorithms for fluid-structure interactions and extend this technology to “living” boundaries. Muscle models integrate feedback between the conduction of action potentials, the contraction of muscles, the movement of tissues, and fluid motion. These models are then used to resolve pumping mechanisms in tubular hearts and resonant swimming in jellyfish. (Received September 17, 2019)

1154-AG-2626 **Sebastian J. Schreiber*** (sschreiber@ucdavis.edu), **Shuo Huang**, **Jifa Jiang** and **Hao Wang**. *Reproductive numbers, metastability and extinction in discrete-time models of endemic diseases.*

For epidemiological models, the basic reproductive number R_0 corresponds to the expected number of individuals infected by an infected individual in a mostly susceptible population. For deterministic models of endemic diseases, $R_0 > 1$ often implies that the disease will persist indefinitely. In contrast, $R_0 < 1$ often implies that the disease goes extinct asymptotically. In sharp contrast, the stochastic counterparts of these deterministic models predict that the diseases go extinct in *finite* time whether or not $R_0 > 1$. To understand this discrepancy, we analyzed the quasi-stationary distributions (QSDs) for discrete-time Markov chain models of endemic diseases. Mathematically, QSDs correspond a left eigenvector of the transition matrix associated with dominant eigenvalue $\lambda < 1$. Biologically, QSDs describe the long-term statistical behavior of the disease conditioned on non-extinction and $1/(1 - \lambda)$ is the mean time extinction (MTE) when following the QSD. We show that (i) $R_0 > 1$ implies that the MTE increases exponentially with the population size N , while (ii) $R_0 < 1$ implies that the MTE increases and saturates with N . We also prove the QSDs are concentrated on the attractors of the corresponding deterministic models when N is large. (Received September 17, 2019)

1154-AG-2650 **Erika Tatiana Camacho*** (erika.camacho@asu.edu), 4701 W. Thunderbird Rd., Glendale, AZ 8530, and **Thierry Leveillard** and **Danielle Brager**. *Modeling Cone Aerobic Glycolysis.*

Patients affected by the inherited retinal disease retinitis pigmentosa, experience loss of vision due to photoreceptor degeneration of the rods followed by the cones, leading to irreversible blindness. Photoreceptors rely on aerobic glycolysis to supply the metabolites necessary for outer segment renewal and maintenance. In this work, we develop a mathematical model to investigate the biochemical processes in the cones triggered by glucose catabolism and by Rod-derived cone viability factor (RdCVF), in order to better understand the mediated survival exerted by the rods on the cones. We demonstrate via mathematical analysis the mediated effects of RdCVF on cone survival, with regard to carbohydrate metabolism, antioxidant lipid synthesis, and energy production. Our findings demonstrate the utility of a mathematical model of aerobic glycolysis for exploration

of the roles of various pathways. Additionally, our model illustrates the relevance of quantitative models to fully understand the mechanisms driving cone death in RP. (Received September 17, 2019)

Best Practices and Considerations in Designing and Developing Online Math Courses

1154-B1-977 **Nathaniel Miller*** (nathaniel.miller@unco.edu). *An Online IBL Geometry Class.*

During the summers of 2015, 2017 and 2019, I developed and taught an online IBL class for Master's students who were in-service secondary teachers. The class met synchronously via videoconferencing software for 3 hours a day over the course of 16 days. The class time was divided between two strands. In one strand, students worked on a problem sequence from David Clark's IBL textbook "Euclidean Geometry: A Guided Inquiry Approach," presenting their solutions to the class on a daily basis, just as they would have in a traditional classroom. In the other strand, the students worked on inquiry projects in groups using open-source applications like GeoGebra and Spherical Easel. In this talk, I will discuss the course, how it was structured, and how IBL teaching methods can be successfully adapted to online classes. (Received September 12, 2019)

1154-B1-1172 **Kate McGivney*** (kgmcki@ship.edu), 1871 Old Main Dr., Shippensburg University, Shippensburg, PA 17257. *Tools, tips, and techniques: Lessons learned from a decade of teaching statistics online.*

For the past ten summers I have taught an online introductory statistics course at Shippensburg University. During this time, I have continued to modify the course to ensure that the I am offering a high-quality course that meets the needs of all students. In this session, I will highlight practical strategies including: connecting early (and often) with students; clearly articulating course expectations; organizing the course in the learning management system so that it is clear and intuitive; presenting course material through a variety of modalities including videos, class notes, applets, graded worksheets, and statistical software guides; setting up effective avenues to communicate using email, discussion boards, and video office hours; providing feedback on graded assessments; and requiring a proctored final exam. I will discuss lessons learned and the challenges and benefits that I have experienced through teaching online. (Received September 13, 2019)

1154-B1-1871 **Susanna S Epp*** (sepp@depaul.edu). *Online Exercises for a Proofs Course.*

Developing and using online interactive exercises for a course emphasizing logical reasoning and proof has special and significant challenges. My involvement began when I used WebWork to write exercises for my classes. More recently, I have been working with a publisher to adapt exercises from my discrete math book for use with WebAssign. This talk will offer examples of online exercises for a proofs course, and it will discuss advantages and disadvantages of various types of exercises and issues involved in their construction. (Received September 16, 2019)

1154-B1-1968 **Erika Wittenborn*** (erika.wittenborn@wgu.edu), **Heather Rosenblatt** (heather.rosenblatt@wgu.edu), **Hilda Black** (hilda.black@wgu.edu) and **Neil Starr** (neil.starr@wgu.edu). *Addressing Mindset, Pacing, Comprehension, and Persistence in Online Self-Paced Mathematics Classes.* Preliminary report.

How can we design online mathematics courses to assist students in getting started, developing good study strategies, staying engaged, and persisting through setbacks? What resources need to be developed for students and faculty members to address the fixed mindset and/or math anxiety with which many students, particularly non-traditional learners, enter online mathematics courses? This session seeks to answer these questions by presenting the findings of a year-long research project conducted by several mathematics faculty at Western Governors University. The study collected qualitative data from students, faculty mentors, enrollment counselors, and mathematics faculty on barriers to student success and ideas on how to address these barriers. The findings and proposed solutions will be presented in this session. (Received September 16, 2019)

1154-B1-2253 **Neil Starr*** (neil.starr@wgu.edu), **Hilda Black** (hilda.black@wgu.edu), **Heather Rosenblatt** (heather.rosenblatt@wgu.edu) and **Erika Wittenborn** (erika.wittenborn@wgu.edu). *Addressing Mindset and Math Anxiety with Online Math Students.* Preliminary report.

Students at Western Governors University are primarily adults returning to school, most with some college experience but no degree. Many come to WGU with concerns about their abilities to succeed, especially in math classes, and their math anxiety/fear of math is one reason they have not completed their degrees. Addressing

this anxiety at the start of their academic careers can help set students up for success. In this session, we will talk about the live webinars we have for students that 1) address their fears of math, including discussing their automatic (bio) reactions; 2) teach them to recognize their reactions and to control them; 3) explore students' educational histories – to help them to understand that it is not usually math itself that produced the anxiety but the situation in which the math was taught that did so; and 4) provide effective study strategies. We will also discuss how these strategies can be built into courses using asynchronous tools. (Received September 17, 2019)

1154-B1-2608 **Sharmila Sivalingam*** (ssivalingam@maryville.edu). *Designing and Developing an Online Math Course – Creating a student-centered model.*

Over the last decade online education has grown tremendously. It is becoming important that the institutions provide online courses or degree programs in the disciplines to survive the competition. As simple the idea seems to be, it has different components from course development, delivery, adaptive learning, engagement, assessment, and student satisfaction. This presentation talks about online courses in math discipline, developing the content of the course, adaptive learning tools, what is meaningful for students, online apps that could be used to interact with students. Keeping students active and on track could be an issue in online courses. This presentation addresses the issue and strategies to keep students motivated and on track. (Received September 17, 2019)

1154-B1-2633 **Kirsten K Meymaris*** (mheeren@purdueglobal.edu), kmeymaris@purdueglobal.edu, and **Michael Heeren** (mheeren@purdueglobal.edu). *Hands-On-Online Real Life Data Collection.*

Teaching statistics in an online format has many of the same inherent challenges as a large lecture class. Students are intimidated by the remoteness or the instructors and their classmates. Incorporating hands-on activities is a research-supported practice that often engages and motivates the learner (ref). Though computer simulation is a feasible option, there is a “black-box” feel that is unfortunately commonplace for math, furthermore statistics, and even more so with Sampling Distributions (Pfaff, 2009). Online education has unique challenges when it comes to implementing physical hands-on activities. This session will share the implementation of “Cents and the Central Limit Theorem” (Scheaffer, 2014) to engage online students with tangible, real time/life data collection using Google Suite apps. Best practices for modifying hands-on activities for the online and traditional large lecture classes will be discussed.

Thomas J. Pfaff & Aaron Weinberg. (2009) Do Hands-on Activities Increase Student Understanding?: A Case Study, *Journal of Statistics Education*, 17:3, DOI: 10.1080/10691898.2009.11889536

Scheaffer, Richard L. (2004). *Activity-based statistics : Student Guide* (2nd ed). Key College Publishing, Emeryville, CA. (Received September 17, 2019)

Combining Technological Tools and Innovative Practices to Improve Student Learning Outcomes

1154-B5-542 **Gregory A Varner*** (gvarner@jbu.edu), 2000 W University St, Siloam Springs, AR 72761. *The Use of a Flipped Classroom to Strengthen Problem Solving Skills.*

The flipped or hybrid classroom has been a topic of much interest in Mathematics education over the last several years. Unfortunately, despite this interest, data on results has been mixed. However, one advantage is the ability to work with the students on problems in the classroom, which could increase problem solving skills. Results will be presented on the success of using a flipped style of teaching in a small classroom setting, which was done over the last few years at John Brown University, including results on student performance, student perception of the course, and the effect on student's ability to problem solve.. (Received September 06, 2019)

1154-B5-593 **Marilyn A. Reba*** (mreba@gsu.edu). *The Creative Use of Technology in Dynamic Classrooms, Interactive Textbooks, and Independent Remediation.*

Achieving active learning in the classroom today involves more than interspersing short lectures with problems for students to work individually or in groups. It involves finding creative ways to use a variety of technological tools to engage students fully inside and outside the classroom. We will discuss how we attempt to do this. Inside the classroom, the instructor projects digital ink showing each step of his (or the student's) solution and students use their cell phones to record their attendance and to answer questions projected to the entire class. Core content worksheets focus student attention on the objectives in each class, and application group projects encourage students to work together on a problem that expands their understanding. Outside the classroom,

an online interactive text, via Top Hat, motivates students to read and to view short videos, to answer graded Quick Check questions, and to complete graded homework problems. Our course management system houses not only the typical documents, but also optional remediation videos and worksheets. The Discussion Board stimulates discussions, not only on math concepts, but also on time-management, study, and test-taking tips. WebEx enables some online office hours. (Received September 07, 2019)

1154-B5-741 **Bart Snapp*** (snapp.14@osu.edu), 100 Math Tower, 231 West 18th Avenue, Columbus, OH 43210-1174, and **Chris Porter** (porter.284@osu.edu), 191 West Woodruff Ave, Columbus, OH 43210. *Virtual reality in the calculus 3 classroom.*

For the past several semesters at The Ohio State University researchers from the Mathematics and Physics departments have been experimenting with Virtual Reality in the Calculus 3 classroom. In this talk we will discuss two interventions that were done across all sections of Calculus 3 (with sections for control) and the results of these interventions. (Received September 10, 2019)

1154-B5-954 **Daniel Moseley** (dmosele@ju.edu), 2800 University Blvd N, Jacksonville, FL 32211, **Daniel Franz*** (df Franz1@ju.edu), 2800 University Blvd N, Jacksonville, FL 32211, and **Aaron M. Montgomery** (amontgom@bw.edu), 275 Eastland Rd, Berea, OH 44017. *Transitioning to Free Mathematical Software in Freshman Calculus.* Preliminary report.

Students are becoming increasingly comfortable using personal technology as part of their academic experience. We have decided to leverage this fact to give our students more options in how they discover the beauty of calculus. Traditionally, we used Maple, an expensive program that our students could only access in a few locations on campus. By using the free online graphing calculator Desmos instead, our students can complete our newly modified assignments anywhere, including on a tablet or smartphone. In this talk we will give a few examples of assignments that we have transitioned to incorporating Desmos in place of Maple. (Received September 12, 2019)

1154-B5-976 **Dan McGee*** (mcgeed4@nku.edu), **Axel Brandt** (brandta2@nku.edu), **Steven Wilkinson** (wilkinson@nku.edu) and **Bethany Noblitt** (noblittb@nku.edu). *Using traditional online content, innovative online assignments and back-to-basic pen-and-paper in class student projects to address the needs of a diverse precalculus body.*

In order to address the diverse needs of students at different skill sets, Northern Kentucky University has launched a precalculus project to promote growth mindsets in students, provide effective contexts for students to develop a “figure it out” attitude towards mathematics and incorporate pen-and-paper tables that students fill out in order to remedy numerical deficiencies while learning precalculus concepts. These objectives were addressed with the following materials: (A) Online course content with links to prerequisite tutorials that can be accessed at the discretion of the student and instructor; (B) In-Class activities that address numeric deficiencies while developing pre-calculus content through back-to-basics formula/table/graph connections; and (C) WeBWorK assignments that encourage reflective generalizations of ideas presented in the classroom. This presentation will present an overview of the needs of the project and the philosophy used to address these needs, samples of the online and in-class materials that were used and preliminary results associated with the project. (Received September 12, 2019)

1154-B5-1156 **Hasan*** (ah2409@humboldt.edu), 1441 Merganser Dr., McKinelyville, CA 95519. *A Technology-Enhanced, Flipped Course Design to Foster Students' Understanding of Statistics.*

In this paper, we compare an introductory statistics course taught using flipped pedagogy in a blended learning environment enhanced with simulation-based activities, interactive video tutorials, and online assessments to our traditional class. In the traditional course, students listened to lectures, took notes, and completed homework assignments. In the flipped course, students had to watch video tutorials with imbedded short answer questions before class and completed workbook activities in class. Our student data suggest that the use of interactive video tutorials and simulation activities improves students' mastery of statistical concepts, reduces the failure rates, and boost their overall course performance. (Received September 13, 2019)

1154-B5-1291 **Stephen M. Walk*** (smwalk@stcloudstate.edu), Department of Mathematics and Statistics, ECC 139, St. Cloud State University, 720 4th Ave. S., St. Cloud, MN 56301-4498. *General exponential functions in calculus.*

Many calculus textbooks don't look closely at general exponential functions (of the form $f(x) = b^x$ for an arbitrary positive base b). Instead, they devote attention to the natural exponential function alone, seeing other

exponential functions only as altered versions of e^x and differentiating them via the chain rule—if they see and differentiate them at all.

We present a simple investigation with Geometer's Sketchpad and Excel that allows students to discover and remember the derivative formulas for general exponential functions and, perhaps more importantly, reminds students why exponential functions are useful in the first place. (Received September 14, 2019)

1154-B5-1664 **Matthew Jonathan Peeples*** (peeples@naps.edu), NAPS Mathematics Division, 440 Meyerkord Ave., Newport, RI 02841. *Flipping the Classroom: An updated approach.*

At the Naval Academy Preparatory School (NAPS), located in Newport, RI, students come from across the United States with a very diverse range of abilities. A flipped classroom offers a means to reach a wide range of students within each classroom with a differentiated approach, rather than a traditional, one-size-fits-all lecture approach. I have been using a flipped classroom for several years. My approach, previously presented to JMM in 2015, involved the use of video lessons uploaded to You Tube and embedded in forms I created using Google Docs as a means to follow up each lesson. In light of research using the book by Robert Talbert and in my own experience, I found that simply flipping the script so that students watching videos or reading outside of class was not as effective an approach. In my presentation, I detail my results from the flipped classroom approach I presented earlier and how I have updated my flipped approach so that students are more motivated to read and do work prior to coming to class. As a result, students seem more prepared based on evidence such as students asking better questions during class and are more willing to engage with harder material in the classroom. (Received September 16, 2019)

1154-B5-1739 **Katiuscia C. B. Teixeira*** (katiuscia.teixeira@ucf.edu) and **Eduardo V. O. Teixeira.** *Integrating technology and pedagogical strategies for learning and teaching calculus.*

The Department of Mathematics at the University of Central Florida (UCF) has recently undertaken a major redesign project for its first course of Calculus, historically taught in very-large class environments and with an over 50% of DFW rate. The pedagogical solution proposed involves active teaching strategies, an online adaptive homework platform, and a perceptive assessment system focusing on higher level skills (conceptual understanding). Adaptive assignments are due before corresponding lecture, which yield initial knowledge background for meaningful learning. While adaptive assignment only covers procedural skills, they efficient emulate one-to-one tutoring. We will detail the components of the course and discuss the data and the results obtained. (Received September 16, 2019)

1154-B5-1762 **Nina Bailey*** (nbaile15@uncc.edu) and **Samuel D Reed** (sdr4m@mtmail.mtsu.edu). *Which One Doesn't Belong? Examining Graphs with Dynamic Representations.*

Dynagraphs (sometimes called dynamaps) dynamically represent functions affording the exploration of the dynamic aspects of functions (e.g., covariation) in ways that paper and pencil tasks do not. For example, instead of attending to intercepts, students are able to focus on relative location, and instead of attending to the steepness of graph, students are able to focus on relative rate of change. Dynagraphs allow students to describe rate of change and covariation without being limited by typical, static representations of graphs. We paired this representation with a pedagogical routine, "Which one doesn't belong?" as a way to elicit rich mathematical discussion regarding covariation and relative rate of change. In this presentation we will share the Dynagraphs task paired with the "Which one doesn't belong?" pedagogical strategy. We will also highlight typical student responses when engaging with the task, discuss the power of using such a representation of functions, and provide pedagogical strategies to enact similar tasks in class. (Received September 16, 2019)

1154-B5-1990 **Jing Wang*** (jwang@cbu.edu) and **Sandra Davis** (sadavis@cbu.edu). *Integrating Technology and Project Based Learning in a Finite Mathematics Class.* Preliminary report.

Finite Mathematics is one of the most popular freshman courses required for business and arts majors at Christian Brothers University. A traditional finite math curriculum covers topics such as linear systems, matrices, linear programming, modeling, and financial math. We will discuss the technology we use in and out of classroom for this course. We will also share some project ideas throughout the course to help students understand the math concepts and see their connections to the real world. (Received September 17, 2019)

1154-B5-2139 **David G Taylor*** (taylor@roanoke.edu), 221 College Lane, MCSP Department, Salem, VA 24153, and **Adam F Childers.** *Real Data is Important: Live, Data Collection and Analysis for Statistics Classrooms Using The Free Classroom Stats Platform.*

The MAA's CUPM guidelines and ASA's GAISE report are clear and unified in their belief that effectively teaching statistics requires constant exposure to real data. Research has shown that students are more invested

in data that is relevant to them and that they learn better in an active classroom. Classroom Stats is a fun, free, flexible mobile and web-based platform that lets instructors quickly collect, visualize, and analyze student data in real time. In under a minute, instructors can create a list of questions, both quantitative and categorical, that students respond to using the free Classroom Stats mobile app. The results can be shared live with the students in an instant via the web-based instructor portal using a suite of descriptive and inferential statistical tools, or the data can be downloaded to analyze in any statistical software. Our students report that they enjoy the spontaneity of being able to quickly survey the class and the instant analysis of the data, and are more engaged in the statistics content. (Received September 17, 2019)

1154-B5-2400 **Marianna Bonanome*** (mbonanome@citytech.cuny.edu), 300 Jay Street, Brooklyn, NY 11201, and **Ariane Masuda** (amasuda@citytech.cuny.edu), 300 Jay Street, Brooklyn, NY 11201. *OERs Harnessing Technological Tools and Innovative Pedagogies to Support Student Learning and Engagement in Gateway Mathematics Courses*. Preliminary report.

The New York City College of Technology and Borough of Manhattan Community College of CUNY are collaborating to develop free, high-quality OERs supporting student learning in gateway mathematics courses. This work is part of a 5 year Title V collaborative grant where cohorts of full-time and part-time faculty fellows from both campuses are incorporating cutting-edge pedagogies with technology such as WeBWorK and Desmos to create classroom activities and STEM applications. Join us as we present some of the exciting OERs developed and utilized by our grant participants along with lessons learned. (Received September 17, 2019)

1154-B5-2657 **G. Gustave Greivel*** (ggreivel@mines.edu), Applied Mathematics and Statistics, 1500 Illinois St., Golden, CO 80401, and **Holly Eklund** and **Scott A. Strong**. *Engineering Learning in Studio Calculus III Honors*.

The department of Applied Mathematics and Statistics at the Colorado School of Mines (Mines) is exploring a variety of pedagogical strategies to (i) encourage active learning in multi-section core mathematics courses and (ii) make the content of these courses more engaging and relevant to our students. With support from the Office of Naval Research, we have piloted a Student-Centered Active Learning Environment for Undergraduate Programs (SCALE-UP) version of our honors course in Multivariate Calculus. We have reinforced our instructional activities with undergraduate teaching assistants (UTAs) and Mathematica notebooks designed around the particulars of the weekly assignments for the SCALE-UP environment. With four-years of delivery, we take a moment to look back on the course and its accomplishments. In particular, we discuss our initial pilot, student feedback, lessons learned, content design (and re-design), and the impacts of these efforts on the students, UTAs and faculty who have participated in this model. (Received September 17, 2019)

Data Across the Curriculum

1154-C1-632 **Johanna N.Y. Franklin*** (johanna.n.franklin@hofstra.edu), Department of Mathematics, Room 306, Roosevelt Hall, Hofstra University, Hempstead, NY 11549-0114. *Adding data analysis to a mathematical statistics course*.

When I began teaching our upper division probability/statistics sequence four years ago, I added two new components: an introduction to R and a final project requiring my students to find questions that interest them and design studies to answer these questions using authentic data sets. I will discuss the way in which I integrate these components into my course and scaffold my students' assignments to help them develop the necessary fluency in R to carry out their projects over the course of the term. (Received September 09, 2019)

1154-C1-1010 **Rachel Grotheer*** (rachel.grotheer@goucher.edu). *Calculus Through a Data Lens: Broadening Scope Through Data and Modeling*.

Recently, Goucher College went through a campus-wide curriculum change that included replacing, our "Mathematical Reasoning" general education requirement with a Data Analytics requirement that requires students to take one semester-long course learning the foundations of data analytics and then another semester-long course learning data analytics techniques in the context of another discipline, usually their major. Additionally, Goucher has decided to discontinue the traditional mathematics major and start a new Integrative Data Analytics (IDA) major. As a result of these changes, our first semester calculus class has been redesigned to fulfill the first part of the data analytics requirement, and to weather its transition from a foundation of the mathematics major to a foundation of the IDA major. This talk will focus on the development and evolution of this course, Calculus Through Data and Modeling, which covers topics in both single and multivariable calculus with an applied focus,

while also addressing data literacy, and computational proficiency in RStudio or Excel. (Received September 12, 2019)

1154-C1-1662 **Albert W Schueller*** (schuellow@whitman.edu), 345 Boyer Avenue, Walla Walla, WA 99362. *Using Smartphone Sensor Data in the Mathematics Classroom.*

We demonstrate the use of smartphone sensor data in the undergraduate mathematics curriculum. We show how students can gather their own sensor data. We employ tools like jupyter/python to visualize and analyze the data. We also stress the importance of accompanying the data analysis with abstract mathematical concepts when in the mathematics classroom. We will discuss two projects: a GPS project using raw GPS data and free mapping software to find the fastest mile in a longer exercise run and an accelerometer project using raw accelerometer data and Fourier analysis to determine how fast a washing machine spins during its spin cycle. Students are excited to find novel uses of their own phones as well as novel uses of the mathematics they are learning in the classroom. (Received September 16, 2019)

1154-C1-2072 **Chris Oehrlein*** (coehrlein@occc.edu). *Using Atmospheric Data to Determine How Well a Separable ODE Models the Vertical Motion of a Dry Air Parcel.*

Using basic thermodynamic principles as the foundation, Atmospheric Physics contains many relationships that students taking Introductory Differential Equations, Integral Calculus, and Multivariable Calculus students can model. Assuming no exchange of heat as a dry air parcel rises leads to a separable, first-order, ordinary differential equation relating the parcel's temperature to its pressure. How well does NOAA data fit the analytic solutions to the separable ODE? Students can sample NOAA datasets from a variety of years and world locations and plot this data against the analytic model. By assigning each student or small group of students a different data set from which to sample, opportunities for the investigation of many different data analysis topics arise – topics that our math, physics, and engineering majors do not formally encounter until later in their programs, if ever. (Received September 17, 2019)

1154-C1-2217 **Annela R Kelly*** (annela.kelly@regiscollege.edu). *Calculus and Epidemics.* Preliminary report.

My talk will discuss the use of calculus to determine the trends of opioid epidemic. Using statistics applet, we will discuss the properties of polynomial and exponential functions that best fit the data from health indicators database. The students determine the rate of change of these functions and predict its future values. The talk features multi-variable calculus tools to determine the gradient vectors and the quickest areas of increase of the epidemic. (Received September 17, 2019)

1154-C1-2298 **McKenzie Lamb*** (lambm@ripon.edu), 642 Woodside Ave, Ripon, WI 54971. *Data, History, and Design: Finding Common Ground in Contested Landscapes.* Preliminary report.

One of the required courses in the Ripon College curriculum emphasizes integrating tools and perspectives from multiple disciplines. One incarnation of this course, co-taught by three faculty members, brings elements of history, design, and data science to bear on environmental controversies. The data science component focuses on using graphical representations of quantitative data to tell a compelling story. The design component emphasizes the use of formal design elements to convey a particular perspective. The history component helps students to develop a nuanced understanding of various contemporary environmental controversies by analyzing and articulating their historical context.

This talk will describe how the data science component interacts with and is complemented by the other components of the course. I will also describe some of the challenges associated with teaching a general audience of students to produce complex graphical representations of data (such as animations and shaded maps). (Received September 17, 2019)

1154-C1-2324 **Tyler J Jarvis*** (jarvis@math.byu.edu), 302 TMCB, Department of Mathematics, Brigham Young University, Provo, UT 84602. *Teaching all the Math for Data Science, AI, and Beyond.*

The Brigham Young University undergraduate Applied and Computational Mathematics Emphasis (ACME) began in 2013 and has been very successful at attracting students into mathematics and preparing students for rewarding careers in data science, machine learning, scientific computing, and other industries using applied and computational mathematics. ACME has also placed many students in excellent graduate programs in both pure and applied mathematics and in many other disciplines, including biology, economics, physics, engineering, computer science, and statistics.

This talk is a report on some of the important lessons I have learned in setting up, teaching, and administering the BYU ACME program. (Received September 17, 2019)

Developments, Directions, and Experiences in Open Educational Resources

1154-C5-367 **Al P. Maneki*** (apmaneki@earthlink.net), 9013 Nelson Way, Columbia, MD 21045. *The PreTeXt-Nemeth Connection: Enabling Sighted and Blind People to Share the Mathematical Experience.*

The primary problem for exchanging mathematical concepts between blind and sighted persons has more to do with the channels of communication than the inability of blind people to comprehend and do math. For sighted people, this channel is visual. For blind people, it must be tactile, primarily Braille.

The development of an economical, automated and reliable means of converting printed math to Braille and the inverse process has long been a stumbling block. Today, with PreTeXt, an uncomplicated XML vocabulary that uses LaTeX syntax for math, and the Nemeth Braille code, we are optimistic that we can have this economical, automated and reliable process.

The Nemeth Braille code is a rich form of mathematical expression. Any statement that can be rendered in PreTeXt can also be rendered in Nemeth.

The nonlinear forms of mathematics (subscripts, superscripts, etc.) can already be translated into the linear codes of LaTeX, ASCIIMath, and PreTeXt. There are many technical details to overcome when translating PreTeXt to Nemeth. Initial results in this effort to date have been encouraging. Our goal is to develop software that can transcribe a PreTeXt book into a Nemeth Braille book accurately enough to pass the proofreading prowess of a certified Nemeth Braille proofreader. (Received September 02, 2019)

1154-C5-1141 **Virgil U Pierce*** (virgil.pierce@unco.edu), School of Mathematical Sciences, Campus Box 122, Greeley, CO 80639. *Teaching Mathematics with Jupyter and Pretext.* Preliminary report.

For a mathematics course that is built heavily around the use of Jupyter as an in-class tool, such as a Data Science, Mathematical Modeling, or even Linear Algebra or Calculus courses it makes sense to use Jupyter to compose the course notes. The prose surrounding the computations depends on the results of the computation for example, or the figures being produced are dependent on choices made in the computations. Jupyter can even be used as an interactive presentation during these classes.

The resulting notebook files can be easily published via Github or similar services, or exported to PDF or HTML. However, these are unsatisfactory ways of sharing the material for some uses, and one might prefer to bundle the notes into something more like an online textbook. In this presentation we will give an overview of the use of Jupyter as an in-class tool in these kinds of mathematics courses, and then discuss a conversion script for turning Jupyter notebooks into PreTeXt XML files for publishing as an online textbook (nicely formatted for mobile devices). (Received September 13, 2019)

1154-C5-1178 **Michael E. Gage*** (gage@math.rochester.edu), Department of Mathematics, University of Rochester, Rochester, NY 14627. *Embedding WeBWorK problems anywhere.* Preliminary report.

The open source online math homework system WeBWorK has been used for many years to deliver homework sets with individually generated problems and immediate feedback to students at all levels of mathematics from algebra through calculus, differential equations and matrix linear algebra.

An old feature of WeBWorK, that the problems in the homework sets are also available individually via a webservice, has recently become more prominent as authors of OER materials seek to include 'live' homework problems not just as end of the chapter exercises, but embedded in the text. LibreTexts (libretexts.org), PreTeXt (<https://pretextbook.org/>) and ActiveCalculus (<https://activecalculus.org/>) are a few of the texts already taking advantage of this feature.

This webservice is available to anyone who has access to a course on a WeBWorK site. I'll illustrate the basic capability of embedding these in a WeBWorK question in an ordinary webpage. This could be used in writing up class notes to present 'live' homework examples to reinforce the concepts or to create a worksheet to guide open exploration. With this basic example I'm hoping to encourage discussion and further exploration of innovative ways to use embedded online homework to further student learning. (Received September 13, 2019)

1154-C5-1194 **Alexei Kolesnikov*** (akolesnikov@towson.edu), Department of Mathematics, Towson University, 8000 York Rd., Towson, MD 21252. *Automated transcription of a mathematics textbook into Nemeth Braille.*

Blind students studying mathematics have to request a Braille copy of their mathematics textbook at least one semester in advance. The limitation is due to the fact that the process of producing Nemeth Braille textbook from the source files is not automated. Even commercial products are not sufficiently robust to produce reliably accurate results.

The goal of the project described in this talk is to build an open-source automated system that would be good enough to translate an advanced undergraduate-level mathematics textbook into Nemeth Braille. I will describe the process used in the project and will talk about progress and challenges. (Received September 13, 2019)

1154-C5-1473 **Jim Fowler*** (fowler@math.osu.edu), Columbus, OH 43210, and **Bart Snapp**, Columbus, OH 43210. *T_EX in the browser.*

The author has written an open-source Pascal compiler which targets WebAssembly, meaning that Knuth's T_EX source can be compiled to something which runs in a web browser. Combining this with dvi2html, the resulting DVI can be rendered to HTML, with PGF specials rendered as SVG. (This permits us to build TikzJax, i.e., a tool akin to MathJax but for rendering figures built with TikZ).

Making T_EX a first-class citizen on the web yields additional benefits to OER. With "isomorphic git" the usual git commands can also be run from within a web browser, so T_EX worksheets stored on GitHub can be cloned to a web browser, compiled in the web browser, and viewed and edited by students, all without involving a server. This talk will demonstrate this technology and some of the interactive Ximera features built around it. (Received September 15, 2019)

1154-C5-1477 **Mike Weimerskirch*** (weim0024@umn.edu) and **Shelley Kandola** (kandola2@msu.edu). *Open Educational Resources in Active Learning Classrooms.*

Modern technology allows practice of routine computational skills to happen outside of class time freeing instructors to focus on higher-order thinking skills. In turn, technology can enhance active learning instruction by providing interactive tools students can use for experimentation. This two-pronged approach, offered through openly-licensed videos, homework and software is having a profound effect on student learning at the University of Minnesota. The first advancement was to replace printed textbooks with video as the primary source of information. Students are able to begin working on homework as soon as the semester starts, perhaps even before the first class meeting. Lacking from the traditional curriculum have been rich exercises that require students to engage with mathematics in a deep and meaningful way. Creation of group projects worthy of having three people thinking and working together over an entire class period is necessary to make the active learning experience worthwhile. The current goal is to collaborate with the greater mathematical community to expand the material available for institutions at all levels. (Received September 15, 2019)

1154-C5-1756 **Duane Q. Nykamp*** (nykamp@umn.edu). *A preview of the Distributed Open Education Network (Doenet).*

The Distributed Open Education Network (Doenet) is an open data-driven educational technology platform currently in development that is being designed for measuring and sharing student interactions with web pages and storing anonymized data in an open distributed data warehouse. When fully deployed, Doenet will allow instructors to assign activities published on outside web pages and collect the resulting data to evaluate student performance. Educational researchers will be able to analyze the resulting anonymized data to identify effective content, helping future instructors to select the best content.

We will preview Doenet by demonstrating one tool from the upcoming Doenet ecosystem: an approach for developing interactive content with DoenetML (which is based on the PreTeXt XML vocabulary). (Received September 16, 2019)

1154-C5-1811 **Kent E Morrison*** (morrison@aimath.org), American Institute of Mathematics, 600 E. Brokaw Rd., San Jose, CA 95112. *Open Textbooks in Mathematics: Progress and Challenges.* Preliminary report.

The AIM Open Textbook Initiative began in 2010 and now after 10 years it is a good time to assess what has been accomplished in the effort to promote high quality open textbooks in mathematics and to survey the challenges we face in making further progress. (Received September 16, 2019)

1154-C5-1930 **Volker Sorge*** (v.sorge@cs.bham.ac.uk), School of Computer Science, University of Birmingham, Birmingham, B146BE, United Kingdom, and **Davide P Cervone** (dpvc@union.edu), Department of Mathematics, Union College, Schenectady, NY 12308. *Transforming Math Documents with MathJax Version 3.*

MathJax is a Javascript library for typesetting Mathematics on the Web. We have recently released version 3, which is a complete re-implementation of the original system that takes advantage of modern web tools and programming paradigms. MathJax version 3 not only provides fast and reliable typesetting, but offers a number of additional features, such as copy and paste of formulas, responsive re-flow, as well as advanced accessibility support such as speech generation, tactile Braille output, magnification and interaction.

In this presentation, we will discuss some of MathJax's main features on the Web as well as its usage on a server to transform LaTeX documents into Web-ready content that can also be used in eBook readers or translated into specialist formats. In the context of the latter we will in particular emphasise recent efforts on turning mathematics textbooks into tactile Braille volumes. (Received September 16, 2019)

1154-C5-2181 **Oscar Levin*** (oscar.levin@unco.edu), School of Mathematical Sciences, University of Northern Colorado, Campus Box 122, Greeley, CO 80634. *PreTeXt for day-to-day teaching.* PreTeXt is an open source markup language, especially useful for writing open source math textbooks, that can be converted to pdf, highly accessible html, and other formats. The interactive webpages that PreTeXt produces are particularly enticing for educational resources beyond textbooks: course notes, homework sets, in-class activities and other handouts, and even exams and other summative assessments. For this to be practical, instructors need to be able to quickly and easily create course materials inside PreTeXt in the midst of busy teaching schedules. In this talk, I will share my experience doing just this for two of my mathematics courses. We will consider a workflow, open source tools, and a few simple customizations that allow an instructor to write (almost) all their course materials in PreTeXt and create pdf handouts and a free, always up-to-date course website including all materials (almost) automatically. (Received September 17, 2019)

1154-C5-2218 **Kenneth M Monks*** (kenneth.monks@frontrange.edu). *Building a Differential Equations course on collaborative Overleaf projects: better instructor feedback, active learning benefits during homework, and zero cost!*

Sometimes an OER consists of a free or low-cost textbook; other times it is a total departure from the traditional model of centering a course around a textbook. Here, we look at a recent redesign of an undergraduate Differential Equations course, replacing the textbook with Overleaf projects. Overleaf is a free shared-editing platform for LaTeX projects. We created one Overleaf project per topic, each of which houses definitions, theorems, examples, homework, links to external resources, and discussion boards. The key is that the students are not only responsible for writing up solutions to homework problems in Overleaf, but are also responsible for peer-reviewing each other's work. This creates a component of active learning traditionally not present in homework! This has also enabled the instructor to provide better feedback in less time and incorporate far more student writing into the course. (Received September 17, 2019)

1154-C5-2445 **K. Andrew Parker*** (kparker@citytech.cuny.edu). *Making WeBWorK Fun: Using Game Design to Increase Student Engagement.*

The WeBWorK platform has an under-utilized feature that we at NYC College of Technology have harnessed to improve measures of student engagement, and to some extent, overall course performance. Our initial experimentation with WeBWorK's "Achievement Point" system showed significant gains in student engagement and final exam performance. This early round of positive evidence led us to expand WeBWorK's achievement system by developing an automated leaderboard that would allow students to compare their achievement scores within their own class and even across other sections of the same course. Join us as we discuss the evolution of this project, the results of our experiments with achievements in WeBWorK so far, and future directions for this new leaderboard feature. (Received September 17, 2019)

1154-C5-2774 **Paul E Seeburger*** (pseeburger@monroecc.edu), 1000 E. Henrietta Rd., Rochester, NY 14623. *Using the LibreTexts Platform to Customize OER Textbooks for Calculus II and III.* Preliminary report.

The presenter will share his experiences using the LibreTexts platform to customize OpenStax textbooks for his Calculus II and III courses. LibreTexts includes a WYSIWYG content editor to seamlessly edit the textbook content, using LaTeX only where needed to format math content. You can add your own sections, subsections, examples, and exercises using a consistent numbering system to form a textbook that looks professional and is customized for your course. Using CalcPlot3D, rotatable 3D figures can be added to bring the figures in

the textbook to life. Anyone can use these textbooks on the LibreTexts platform or customize them for their own courses. See https://math.libretexts.org/Courses/Monroe_Community_College. (Received September 17, 2019)

Discrete Mathematics in the Undergraduate Curriculum - Ideas and Innovations in Teaching

1154-D1-42 **Jason Lee*** (jason.lee@montgomerycollege.edu). *Counting Problems with Python.*

A standard combinatorics problem in a Discrete Math class is a so-called “stars and bars” problem. For example, “How many ways are there to distribute 7 indistinguishable candies to 4 distinguishable kids?” The students learn that the answer is $\binom{7+4-1}{4} = \binom{10}{4} = 210$. Although sometimes it is difficult for students to really understand this solution, it’s possible to make this answer completely transparent by concretely enumerating all 210 ways. On paper, this is tedious to do, but the built-in Python package `itertools` can do this easily. `itertools` is useful for making combinatorics questions and answers more concrete, and for allowing us to easily loop through combinatorial structures. This talk will show how to apply `itertools` to this and other combinatorics problems. (Received July 15, 2019)

1154-D1-211 **Filippo Posta*** (filippo.posta@estrellamountain.edu), Avondale, AZ 85392. *Discrete Math and Pseudocode: a pedagogical endeavor to improve persistence and achievement of non-Math majors.*

Discrete Mathematics is a common requirement for most Computer Science and Computer Science-related Engineering degrees. The ever-expanding need for programming skills in the workplace has created an increase in student pursuing CS-related degrees, such as cybersecurity. This shift in curricular requirements has led to many students taking Discrete Math courses earlier in their studies. These students are very motivated, but often lack the foundational knowledge to timely achieve a good understanding of the theoretical subtleties of competencies such as counting and np-completeness. To reduce attrition and promote academic achievement, we added pseudocode-based programming activities throughout the discrete math course curriculum. Due to our location in a poor metropolitan area, we also implemented these activities using OER technologies, as well as creating our own material. We present the details and outcomes of our efforts to improve understanding and persistence through a co-curricular use of basic pseudocode. (Received August 23, 2019)

1154-D1-566 **Elise Lockwood***, elise.lockwood@oregonstate.edu, and **Zackery Reed**. *Using a Categorization Activity to Develop Students’ Reasoning about Fundamental Counting Formulas.*

In this talk, we describe students’ engagement with a set of tasks, which we call the Categorization Activity, that we used to help students to reason productively about four basic counting problem types and their formulas. In these tasks, students engaged in three kinds of activities: first, they initial solved 10-14 counting problems, then they sorted those problems into categories, and then they characterized those categories, expressing general formulas for each group of problems. We suggest that this instructional intervention is an effective alternative to other ways of introducing students to basic counting problems and formulas. We present data from both a paired and a small group teaching experiment with undergraduate students who were novice counters, and we discuss the ways that the students’ engagement in specific generalizing activities influenced their thinking. (Received September 07, 2019)

1154-D1-894 **Edmund A Lamagna*** (ea1@cs.uri.edu), Department of Computer Science & Statistics, Kingston, RI 02881. *Puzzling Through Discrete Mathematics.*

The presenter teaches an applied, yet mathematically rigorous, course on combinatorial problem solving. Algorithmic thinking is emphasized throughout, and the class provides a solid foundation for a follow-on course on the design and analysis of algorithms. Major topics include sets, logic, probability, proofs by induction and contradiction, the pigeonhole principle, arrangements, selections, distributions, binomial identities, inclusion-exclusion, recurrence relations and recursion, and graphs and trees.

Each class begins with a set of puzzles (typically four) that introduce and begin to stimulate thinking about the topic for the day. Students work on the puzzles in small groups for about one-third of the period. When puzzles were introduced, it was thought that less material could be covered but this would be outweighed by an increase in student interest and participation, and the course would be more fun. Unexpectedly, all the original

material can still be covered since students are now better prepared and motivated for the more traditional presentation that follows “puzzle time.”

The key to this approach is selecting relevant, intriguing puzzles for each topic. Examples covering a variety of subjects that have been utilized successfully are presented in the talk. (Received September 11, 2019)

1154-D1-1070 **Satish C. Bhatnagar*** (bhatnaga@unlv.nevada.edu). *Making Proofs Palatable!*

This paper is about the following three pedagogical features pertaining to the teaching of proofs: 1. VIDEO. In the first week, students are encouraged to watch the Mathematical Mystery Tour and submit its critique within a week. The video has beautiful narratives on what proof in mathematics is, verification not being a proof, and the types of problems mathematicians tackle. Not even a single student has ever felt that his/her time was wasted in watching it. On the contrary, they are wowed and inspired. 2. LET US DO IT. In the class, students are actively engaged in the writing of short proofs of the statements. The Document Camera lets me write my proof in a way that students cannot see it. After a few minutes, I project my proof on the screen for the students to see and compare. I check the proofs of a couple of students. They are urged them to re-do these proofs without looking at the class notes - in the comfort of their study, preferably on the same day. 3. PEER LEARNING. The i-gadgets have been creating both physical and psychological isolation amongst the humans. Therefore, before any peer learning begins, the students have to speak with each other! This is inculcated by team quizzes given once a week. Also, study groups are encouraged for doing homework too. (Received September 17, 2019)

1154-D1-1369 **Mohamed Jamalodeen*** (mjmaloo@ggc.edu), Georgia Gwinnett College, Lawrenceville, 30043-6982. *Using tools like Mathematica and Wolframalpha to develop exercise problems with solutions in a Discrete Math Course.*

We discuss how we use freely available Mathematica notebooks such as on the Wolfram Demonstrations Project and the Wolframalpha capabilities in Mathematica to generate exercise problems with solutions for a discrete math course. At our institution we are moving to using open source materials and textbooks for our discrete math course. The challenge is finding enough exercise problems for topics across the course. Toward solving this we use the Mathematica/WolframAlpha tools to develop pools and banks of exercise problems across most of the topics in the course. We will demonstrate how we use these to write both questions and complete solutions to the following partial set of topics: sets (producing Venn diagrams, determining if a set is a subset of another, determining if two sets are equivalent), logic (obtaining and evaluating truth tables, producing circuits from propositions and vice versa, analyzing statements with quantifiers), induction proofs (including summation formulas, and inequalities with complete solutions), number theory (converting between number representation systems and the division algorithm). (Received September 15, 2019)

1154-D1-1715 **William Ted Mahavier*** (ted.mahavier@lamar.edu) and **David M. Clark.** *Discrete Mathematics – Where to Start the Undergraduates?*

Over the years, various mathematicians have proposed replacing Calculus with Discrete Mathematics as a gateway course to collegiate mathematics. When our CS department asked our Mathematics Department to address the content of Discrete Mathematics, they wanted more mathematical maturity. They wanted their students to be able to understand and possibly even produce proofs! Our course was redesigned to emphasize depth over breadth. After a decade of refining our course I would argue that if this emphasis is retained, then Discrete Mathematics can be as valid a starting point as Calculus. In this talk we will discuss the notes, and some surprises I have experienced teaching from them. These notes are free, written in PreTeXt, viewable on any mobile device, and available on-line at www.jiblm.org/mahavier/discrete/html/index.html. (Received September 16, 2019)

1154-D1-1789 **Adaline E De Chenne*** (dechenna@oregonstate.edu). *Connecting Sets of Outcomes with Counting Processes: What is the mth element?*

There is much evidence that students struggle when solving enumerative combinatorics problems. Some documented errors suggest there is a misunderstanding between the objects being counted and the process used to count them, such as ignoring order when order matters, or using order when it does not. We suspect that the frequency of such errors can be reduced when students focus more explicitly on the relationship between the objects they are counting and their counting process. In this talk, we focus on instances where students were asked to develop an algorithmic procedure to enumerate and count the objects in a combinatorics problem using the Python computer coding environment, or to develop a listing procedure by hand. In these cases, we discuss how students solved questions of the form “what is the mth outcome,” which have been particularly successful in illustrating the relationships between the objects being counted and the counting process. (Received September 17, 2019)

1154-D1-2262 **(Rachel) Shelley Stahl*** (rstahl@bridgew.edu), Bridgewater State University, Department of Mathematics, 131 Summer Street, Bridgewater, MA 02324. *Leveling the Playing Field in Discrete Math with Inquiry-Based Learning*. Preliminary report.

Students enrolling in a Discrete Math course intended for Computer Science Majors will enter the classroom with a wide spectrum of comfort with and interest in mathematics. While some may be pursuing a minor or double-major in math, others will be less interested and may even suffer from math anxiety. Math anxiety tends to disproportionately affect women and people of color - groups that are already underrepresented in STEM. Thus it falls upon the course instructor to ensure that the classroom environment supports and encourages students at all levels. This presentation will discuss methods that allow instructors to guide students toward the confidence to engage in active problem solving by utilizing an Inquiry-Based Learning model. We also cover strategies and tips to create a classroom dynamic in which IBL can be successfully implemented. (Received September 17, 2019)

1154-D1-2386 **Miriam Harris-Botzum*** (mharrisbotzum@lccc.edu), 4525 Education Park Drive, SH 7D, Schnecksville, PA 18078. *Teaching Discrete Mathematics as a Writing-Intensive Course*.

At many institutions, Discrete Mathematics often serves as the introductory proofs course, as well as a content course. As such, teaching students what constitutes a well-written proof, and how to write good proofs, is a core part of this course. With this in mind, I examined some of the best practices used in writing-intensive courses in English, Social Science, and the Humanities. These courses do not merely involve a lot of writing; rather, they focus intently on the writing process, and on developing a student's skill in writing. Over the past few semesters, I have been adapting these practices and applying them in my Discrete Mathematics classes. This talk focuses on my experience implementing these practices, and what has or has not worked. (Received September 17, 2019)

1154-D1-2807 **John S. Caughman***, caughman@pdx.edu, and **Elise Lockwood**. *The Power of Nested Loops*.

Nested loops are a fundamental structures found in many programming languages. But did you know they can offer a novel and effective introduction to some of the most basic combinatorial objects, like permutations and combinations? Indeed, teachers of discrete mathematics are likely familiar with the classic 2×2 grid of counting formulas enumerating selections (with/without) repetition and where order (does/does not) matter. However, recent research (Lockwood & De Chenne, in press) indicates that students can leverage their understanding of simple python code to reason coherently about each of these four central cases, and their work suggests there may be pedagogical merit in the incorporation of such a computational approach. In this talk, we explore several possible extensions of their ideas to Stirling numbers, set partitions, and a number of other related concepts. (Received September 18, 2019)

The EDGE (Enhancing Diversity in Graduate Education) program: Pure and Applied talks by Women Math Warriors

1154-D5-896 **Keisha Cook*** (kcook7@tulane.edu), New Orleans, LA 70125. *Stochastic modeling in cellular internalization and transport*. Preliminary report.

Live cell imaging and single particle tracking techniques have become increasingly popular amongst the mathematical biology community. We study endocytosis, the cellular internalization and transport of bioparticles. We are specifically interested in titanium dioxide nanoparticles (TiO_2) in human lung cells (A549), observed locally in enlarged lysosomes. We want to determine if the change in the size of the lysosomes alters transport type. Using fluorescence microscopy, we track, classify, and analyze the movement in the cells. Single particle tracking techniques allow us to collect data in order to develop statistical methods for analyzing the movement in the cells. We classify the movement as active, diffusive, sub-diffusive, or stuck. The standard method to compute the diffusivity and velocity involves the mean squared displacement formula. Mathematically, we use Bayesian inference methods to infer a credible region of values for the diffusivity and velocity. Specifically, we develop a Gamma distribution for diffusivity and a Normal distribution for velocity, then sample from both using the Gibbs Sampler technique. Our application shows short path trajectories. Do short particle tracks provide enough information to tell us if they exhibit the properties of Brownian Motion? (Received September 11, 2019)

1154-D5-1458 **Carolyn Otto*** (ottoa@uwec.edu), **Griffen Potrykus** and **Bethany Schueller**. *A Class of Virtual Knots With Crossing Weight Zero*. Preliminary report.

While trying to distinguish if two or more virtual knots are the same, we generally invoke the use of certain invariants. In this presentation, a particular virtual knot invariant known as the crossing weight number will be discussed. We will focus on its properties as they pertain to the Whitehead doubling operator. More precisely, we will show that the weight of each crossing in a Whitehead double of a virtual knot vanishes. (Received September 15, 2019)

1154-D5-1548 **Michelle Craddock Guinn*** (michelle.guinn@belmont.edu), michelle.guinn@belmont.edu, and **Amy Buchmann**, **Sarah Bryant**, **Susan D'Agostino** and **Leona Harris**. *Highlights From the EDGE Book: The EDGE Program, Inclusivity and Outreach, Math Teaching, Math Research, and Mathematical Lives*.

Over more than two decades, the Enhancing Diversity in Graduate Education (EDGE) Program has offered the mathematics community substantial insights about best practices for providing ongoing support and academic enrichment for women entering and persisting through mathematical sciences doctoral programs. Now, A Celebration of the EDGE Program's Impact on the Mathematics Community and Beyond (Springer's AWM Series, 2019) offers evidence of the many reasons the White House Office of Science and Technology Policy, with the National Science Foundation, awarded EDGE the 2018 Presidential Award for Excellence in Science, Mathematics, and Engineering Mentoring. All of the peer-reviewed papers included in the book have at least one co-author who is a member of the EDGE community, including participants, mentors, instructors, and directors. In this talk, we highlight some of the book's papers focused on the EDGE Program, other inclusivity and outreach program in mathematics, mathematics teaching, mathematics research, and mathematical lives. Be sure to attend for a comprehensive understanding of why the American Mathematical Society recognized the EDGE Program as a "program that makes a difference" in fostering inclusivity in the US mathematics community. (Received September 17, 2019)

1154-D5-1862 **Katharine Ahrens*** (kaahrens@ncsu.edu). *Beyond Cyclotomics: Polynomial Roots, Vandermonde Matrices, and RLWE Cryptography*. Preliminary report.

Hard problems arising from ideal lattices are a leading candidate for implementation in future public key cryptographic schemes due to their conjectured quantum resilience. One notable example is the Ring Learning with Errors (RLWE) problem, which is used in many submissions to the 2017 NIST Post-Quantum Cryptography challenge. A version of RLWE is known to be weak for certain classes of polynomials. In the process of widening the attack, we have found fascinating connections to analytic polynomial theory, matrix analysis, and enumerative combinatorics. We present our methodology and some of our most recent results. (Received September 16, 2019)

1154-D5-2397 **Mary Leah Karker*** (mkarker@providence.edu), Dept of Mathematics and Computer Science, Providence College, 1 Cunningham Square, Providence, RI 02918, and **Ryan Alvarado**, **Maia Averett**, **Benjamin Gaines**, **Christopher Jackson**, **Malgorzata Aneta Marciniak**, **Francis Edward Su** and **Shanise Walker**. *The Game of Cycles and the Filled Board Theorem*.

This talk concerns the Game of Cycles as described in Su's 2020 book, *Mathematics for Human Flourishing*. In this game, two opponents take turns marking edges on a planar graph with a direction. No sinks or sources are allowed. The game ends in a win when one player completes a directed cycle on the boundary of a single cell or when the last playable move is made. In this talk we will discuss general gameplay and present a proof that every finite, connected game board in which all edges have been marked with a direction must contain a directed cycle surrounding a single cell. (Received September 17, 2019)

1154-D5-2575 **Talitha Washington*** (talitha.washington@howard.edu), Howard University, Washington, DC. *Dr. Roselyn Williams: The Work and Influence of a Woman Math Warrior*.

Dr. Roselyn Williams has greatly influenced many women in the mathematical sciences. She has done research with women as a faculty member at Florida A&M University, she has hosted the EDGE Program, she was a founder of the Math Alliance, she was an original founding organizer of the Research Experiences for Undergraduate Faculty, and she has served as Secretary-Treasurer of the National Association of Mathematicians for more than 20 years. In this talk, we review the work of this incredible mathematician and celebrate her achievements. (Received September 17, 2019)

1154-D5-2605 **Melanie R. King*** (melanie-king@uiowa.edu). *Length Space Skeletonization in 2-Dimensions*.

The collapse problem in image registration is the decrease in object area to an area of nearly measure zero resulting from applying a diffeomorphism to an image. In some cases, we can detect that collapse has occurred by studying the skeletal curve of the object. In his formulation, Harry Blum offered a “grassfire analogy” as way of visualizing the reduction of area from the object boundary towards its skeleton. Set the boundary on fire and track the advancement of the fire front at uniform speed. Skeletal points are the quench points of opposing fire fronts. In order to study the collapse problem, consider an object to be a bounded 2D region G with some overlapping bounded region W . We call W an obstruction to G and so the non-overlapping region $D = \overline{G} - \overline{W}$ is the unobstructed portion of G . We apply the grassfire analogy to the unobstructed boundary of D . A length skeleton is comprised of the quench points of opposing fire fronts in a uniformly advancing unobstructed boundary. We will formally define the length skeleton for two overlapping 2D regions. Moreover, we discuss some results regarding the characterization of the level sets describing area reduction and the characterization of the skeleton for specific cases of the unobstructed boundary. (Received September 17, 2019)

1154-D5-2761 **Ashlee Keolaulani Kalauli*** (akalauli@math.ucsb.edu). *Solving the Word Problem for Euclidean Artin Groups*. Preliminary report.

A finitely presented group G has a solvable word problem if there exists an algorithm to decide if a product of generators and their inverses represents the identity. In 1926, Artin showed that braid groups—Spherical Artin groups—have such an algorithm. The computation time of this algorithm, however, was exponential in the length of the word. In this talk, we will discuss how Frank Garside’s solution provides a more efficient algorithm for solving the word problem for braid groups. Though Euclidean Artin groups are not Garside groups, we discuss how to modify Garside’s solution to produce a new solution to the word problem for the Euclidean Artin group \tilde{A}_2 . (Received September 17, 2019)

1154-D5-2766 **Bonita V Saunders***, 100 Bureau Drive, Stop 8910, Gaithersburg, MD 20899. *Celebrating the Career of Dr. Fern Hunt*.

Dr. Fern Hunt has achieved outstanding success in careers in both academia and the federal government. After being awarded a PhD in mathematics from the Courant Institute of Mathematical Sciences in 1978, she worked at Howard University and later at the National Institute of Standards and Technology. She has skillfully navigated a career that includes research and publications in ergodic theory, dynamical systems, and applied probability as well as interdisciplinary work on a variety of projects with numerous collaborators. We look at her life and work, not just to celebrate her accomplishments, but to gain some insight into the remarkable experiences and influences that inspired her to excel. (Received September 17, 2019)

Effective Assessment Practices in Large-enrollment Classes or Non-traditional Classrooms

1154-E1-1966 **Darryl J Chamberlain Jr.*** (dchamberlain31@ufl.edu). *Mastery-based assessment in a large-enrollment online College Algebra course*. Preliminary report.

Mastery-based grading is an assessment scheme that encourages students to learn from their mistakes and develop an understanding of material before moving on. For an entry-level course with large enrollment like College Algebra, this could allow some students to move at an appropriate pace for themselves, improving their learning gains throughout the semester, while simultaneously allowing stronger students to push on toward new concepts. In this talk, I’ll describe the changes I made to make College Algebra at the University of Florida an asynchronous mastery-based course for the 1000+ students who take the course per year and describe how I addressed the numerous pitfalls that appeared along the way, focusing on how to effectively implement a mastery-based course for a large-enrollment hybrid or fully online course. (Received September 16, 2019)

1154-E1-1975 **Hannah Mae Lewis*** (hannah.lewis@usu.edu), Mathematics and Statistics C/O Erin Rowley, 451 E 400 N, Price, UT 84501, and **Kady Schneiter**. *Implementation and Effects of University College Algebra Growth Mindset Structured Assessments in Large Lectures*.

Recent scientific evidence shows the incredible potential of the brain to grow and change. Students with a growth mindset view errors and obstacles as opportunities for growth. These students welcome challenges and the opportunity to learn from their mistakes. Although some university instructors are incorporating growth mindset into their lectures, attitudes, and exams in small classes, the traditional exam method used in large lecture undergraduate mathematics classrooms is a fixed mindset model.

The purpose of this study was to understand the relationship between (1) large lecture college algebra undergraduate growth mindset structured assessments and (2) students' achievement, drop/fail/withdraw rates, mindsets, and anxiety. No statistically significant difference in mean final exam scores was found, however, withdrawal and fail rates were lower for the class participating in the growth mindset structured assessments than the control classes. Lower levels of math test anxiety and higher levels of growth mindset were demonstrated in the class participating in the growth mindset structured assessments. (Received September 16, 2019)

1154-E1-2096 **Kelly MacArthur*** (macarthur@math.utah.edu), Salt Lake City, UT 84112. *Re-Humanizing Assessments in Large-enrollment Calculus 2 Courses*. Preliminary report. Answering a call to change, given by Rochelle Gutiérrez, to rehumanize math (Gutiérrez, 2018) and in line with the MAA Instructional Practices that "teaching and learning are forces for social change" (MAA, 2018), I changed assessments in my large-enrollment university calculus II courses, to include small group discussions between students for a portion of the exam. There is ample evidence to suggest active learning classrooms, that include discussions and mathematical exploration among students, produce better learning results for our mathematics students than traditional lecture format (Freeman et al, 2014). However, our exam structure has typically remained unchanged from the standard silent-solo testing model. The assessment change implemented here, along with a consistently enacted classroom mission statement geared at both engaging students and creating a humane learning environment, produced higher exam scores and improved student confidence. (Received September 17, 2019)

1154-E1-2697 **Jeffrey J. King*** (jking@western.edu) and **Michelle Morgan** (mmorgan@western.edu). *Alternate Scoring Practices in Collegiate Mathematics Courses*. Preliminary report. Scoring practices have been shown to affect the goal orientations of students in mathematics classrooms. This qualitative case study explored the use of alternative scoring practices in undergraduate mathematics. We focused on two alternative scoring practices: circle scoring in an upper-level Modern Geometry course, and bullseye scoring in an entry-level College Algebra course. For each course, we interviewed the course instructor, a developer of the scoring practice, and students about their experiences with the alternative scoring practices. In addition to interviews, we observed each class for a period of four weeks. We analyzed the transcriptions of each interview to explore emergent themes within and across the two cases. The results of this analysis offer several key themes based on students' experiences, including: the emphasis on non-numerical scores de-emphasizing the punitive role of grades, feedback being an essential component for students to be able to improve their score, revisions as a motivator and being "stuck in a grade book" as a de-motivator. From the instructors' perspective, we argue that the use of alternative scoring practices benefited the instructors by removing point values as the focus of grades. (Received September 17, 2019)

Fostering Creativity in Undergraduate Mathematics Courses

1154-E5-43 **Emerald Tatiana Stacy*** (estacy2@washcoll.edu), Chestertown, MD 21620. *Leveraging Wonder to Increase Accessibility in Calculus*.

Last year, I started including a term-long project into my Differential Calculus courses. This project was such a hit with the students - particularly students who self-identified as "not math people," that this year I am piloting a new two-semester version of Differential Calculus that includes as-needed algebra and trigonometry. In order to register for the course, the student must not have met the requirements to register for Differential Calculus.

Each semester includes a term-long project that connects Calculus to a field they are passionate about. In Spring, the prompt will be the same used last year. For Fall, however, their project will need to include some form of optimization or curve analysis.

The goal of this project is for students to recognize that mathematics, and problem solving in general, is a creative endeavor. This talk will include samples of student projects, and student feedback on the process. (Received July 16, 2019)

1154-E5-579 **Margaret Smolinka Adams*** (margaret.adams@sngsc.edu), South Georgia State College, 2001 South Georgia Parkway, Waycross, GA 31503. *Flipping the elementary math education classroom to cultivate creativity within each student through the design and presentation of unique, imaginative Common Core lesson plans*.

Students enrolled in elementary education math concepts courses fear teaching math in any class beyond 2nd grade. Struggling with unfamiliar Common Core methods, they don't feel confident explaining math concepts to others. Traditional lecturing neither sparked interest nor held attention, so the classroom was flipped to

foster creativity using Rhode's (1961) 4-P's framework. Students created and presented their own lesson plans associated with the chapter sections in the textbook. A PLOPPPS lesson plan model, designed by the author, was implemented (prior learning, objective, pre-test, presentation, post-test and summary). Collaborative group activities, differentiated instruction, problem-solving up at the board and opportunities for questions were required. Each student created novel engaging tasks, activities, and original assessments for their slide presentations, including a short math video to hook the audience into learning. Calling upon classmates to solve problems at the board was required. Designing a lesson required motivation, content knowledge and all 4 components of creativity. Peer feedback ratings, open ended peer-critique and a videotape of the session were conducted to prepare for ultimate real classroom scenarios. (Received September 14, 2019)

1154-E5-736 **Houssein El Turkey*** (hel turkey@newhaven.edu), 300 Boston Post Road, West Haven, CT 06516, and **Gulden Karakok, Gail Tang, Paul Regier, Milos Savic and Emily Cilli-Turner**. *Tasks to Foster Mathematical Creativity in Calculus I*.

Fostering students' mathematical creativity necessitates certain instructional actions - one of which is designing and implementing tasks that foster creativity. Drawing on the literature on mathematical creativity, we describe existing research-based features of tasks for eliciting student creativity, or creativity-based tasks, and provide suggestions for implementation of such tasks. Based on these features, we analyzed two instructors' first experiences designing and implementing creativity-based tasks in Calculus I. Both instructors' frequent use of the multiple-solutions feature suggests that this feature could be an entry-point for designing and implementing creativity-based tasks for other instructors seeking to foster creativity. (Received September 10, 2019)

1154-E5-1077 **Lori Carmack and Veera Holdai*** (vxholdai@salisbury.edu). *Students Co-Authoring Problems for Class Activities and Assessments*. Preliminary report.

Many researchers strongly advocate active learning as an essential part of the educational process. Numerous studies have concluded that active learning improves student retention, understanding, performance, and attitudes toward the overall learning experience. In the mathematics classroom, requiring students to create problems of their own is a simple and fun way to introduce active learning. This paper presents some examples and ideas for engaging students in this way, illustrating the ease with which active learning can be implemented in mathematics instruction. (Received September 13, 2019)

1154-E5-1099 **Kathy Tomlinson*** (kathy.tomlinson@uwrf.edu), Mathematics Department, 410 S. Third Street, River Falls, WI 54022, and **Keith Nabb** (keith.nabb@uwrf.edu). *Employing the Number Talk concept in the Undergraduate Classroom*.

Number Talks are a popular way to promote efficiency and flexibility in mathematical thinking at the elementary level (Parrish, 2011, 2014). By focusing on a wide variety of strategies for solving a single task, this pedagogical tool blends computational fluency and conceptual understanding, while simultaneously engaging students. In recent sections of Calculus II and Advanced Differential Equations, we applied the Number Talk idea through student-led discussions of multiple approaches to a calculus or differential equations task. Both as leaders and as participants in the talks, students juxtaposed ideas from different content areas of the course, resulting in imaginative and innovative approaches. We will share examples of Number Talks at the elementary level, how the idea came to fruition in our undergraduate mathematics courses, examples of student-led Number Talks, and the insights we gained from implementation. (Received September 13, 2019)

1154-E5-1238 **Rick Cleary*** (rcleary@babson.edu), Math/Science Division, Babson College, 231 Forest St., Wellesley, MA 02482. *Relaxing Constraints on Creativity*.

Some researchers have proposed that an individual's creativity is constrained by a fear of failure, and by a lack of tolerance for ambiguity. In this presentation we discuss the results of a recent experiment by Aylesworth and Cleary that suggests that creativity can be encouraged by approaches that relax these constraints. The study was conducted on MBA students, and we now turn our attention to how the results might be applied to the teaching and learning of undergraduate mathematics. (Received September 14, 2019)

1154-E5-1776 **Manmohan Kaur*** (mkaur@ben.edu), 5700 College Road, Lisle, IL 60532. *Inquiry and Creative Problem Solving in a General Education Mathematics Course*. Preliminary report.

Over the centuries, mathematics has played an important role in almost every aspect of human development, and in many cases has changed the course of history. Mathematical problem solving intrinsically requires finding patterns, building models, and creative thinking. However, to most of our students, mathematics just means crunching numbers and following rules. In this presentation, we will discuss a general education mathematics course that seeks to foster inquiry and creative problem solving in our students. In this course, with less focus on algebra and more on discovery, we pose classical problems, and then discuss the intriguing and ground breaking

mathematical concepts that help solve those problems. In a culminating final project, the students research a topic of their choice, and create their own magazine article and oral presentation to share an important aspect of mathematics. We will share student comments and feedback to show that this approach leads to a more rewarding student experience, and a shift in student mindsets. (Received September 16, 2019)

1154-E5-2147 **Erin E Martin*** (martine@william.jewell.edu), 500 College Hill, Liberty, MO 64068. *Play Dough and Dumpsters.*

It is important to foster creativity in undergraduate mathematics courses to help students fully understand and embrace concepts. I will discuss two activities I have used when teaching Calculus III that incorporate modeling with play dough and designing objects like dumpsters. (Received September 17, 2019)

1154-E5-2215 **Aaron D Wangberg*** (awangberg@winona.edu), Winona, MN 55987. *Unleashing student creativity and mathematics in Math 100: Survey of Mathematics.*

Introductory math courses, like Math 100: Survey of Mathematics, often take the viewpoint "Mathematics is all around us" and proceed to show students math topics which they can identify in their lives. The course takes on a very different feel if the term 'mathematics' is treated as a verb, or process, rather than as a noun or as a list of content. The former case provides space for students to be creative - to play, make conjectures, engage in discourse, and arrive at justified conclusions or contradictions. As "Mathematics is all around us", this viewpoint provides the flexibility for the instructor to help students extract mathematics from their own lives. In this talk, we will describe how the course structure and activities in Math 100 helped students do mathematics in their lives. The talk will include the innovative, creative final projects generated by students and a short summary of the impact this course had on student perceptions of mathematics. (Received September 17, 2019)

1154-E5-2270 **Lucy Spardy Oremland*** (loremlan@skidmore.edu) and **Csilla Szabo** (cszabo@skidmore.edu), Skidmore College, Department of Mathematics and Statistics, 815 North Broadway, Saratoga Springs, NY 12866. *Design - 3D Print - Calculate: Student Created 3D Printed Solids of Revolution.*

In mathematics, it is often necessary to build, visualize, and manipulate complex multi-dimensional structures. Instructors and students must become makers - through drawing, simulating, and crafting - in order to engage deeply with key concepts. In this talk we present a solids of revolution making activity for Calculus II (Integral Calculus). As part of the activity, students designed and 3D printed volumes of solids of revolution using our college's maker space. They also created a poster detailing the steps to calculate the volume analytically using integration methods. The printed solids and posters were exhibited in our department for a competition judged by students, faculty, and staff. (Received September 17, 2019)

1154-E5-2520 **Kayla K Blyman*** (kayla.blyman@westpoint.edu), United States Military Academy, Department of Mathematical Sciences, 646 Swift Road, West Point, NY 10996, and **Kristin M Arney** (kristin.arney@westpoint.edu), United States Military Academy, Department of Mathematical Sciences, 646 Swift Road, West Point, NY 10996. *Developing and Evaluating a Course that Prepares Students as Creative Problem Solvers.*

We live in a world full of complex and ill-defined problems. As educators, we are tasked with the vital role of preparing the next generation to solve unforeseeable future problems. While we cannot know what these problems will be, we can be almost certain that solving them will require creativity.

Our course, a first-year undergraduate mathematical modeling course, prepares students as creative problem solvers. Creative problem solving cannot be learned through a single experience, so we provide our students with a blend of experiences. Our course structure enables creative problem solving through class instruction, during class activities, during out of class assessments, and during in-class assessments. We have found that the combination of these elements within our course structure increases student's creative problem-solving abilities. We believe that each of these components contributes to students' gains in comfort with solving open-ended and ill-defined problems like those they will encounter in the real world.

Each of the components of our course, the overarching structure of the course, and our rubric for evaluating creative problem solving will be discussed. (Received September 17, 2019)

1154-E5-2624 **Lucas Castle*** (castlelc@vmi.edu). *Art Gallery Design: Where's the Math in That???* Preliminary report.

As part of a general education math course, we task students with designing a large art exhibit. This assignment, developed in conjunction with an art historian in our English, Rhetoric, and Humanistic Studies department, challenges students to use the math modeling process to navigate arranging 40 pieces of art in a 3D space. In addition to math modeling, this project addresses the mathematical skills of ratios and spatial reasoning, as well

as the transferable skills of verbal, written, and visual communication. Rather than prescribe a set format for the submission, students are encouraged to be creative in how they communicate their design concept. As a result, we have seen students submit models made from cardboard, in Minecraft, using AutoCAD, and more! In this talk, we will explain the project, give examples of student submissions, and share feedback from our students. (Received September 17, 2019)

1154-E5-2714 **Nancy Emerson Kress*** (nancy.kress@colorado.edu). *A Framework for Equitable Mathematics Instruction: Broadening What is Considered Mathematical.*

Rehumanizing mathematics requires that we cultivate the development of mathematics communities in which the unique and varied lived relationships and experiences that individual people have with mathematics are fully valued. This research report will describe a theoretical framework for Equitable Mathematics Instruction by first briefly describing important aspects of the foundational concepts of mathematics identity, sense of belonging in mathematics, and robust mathematics identity, and then describing the core features of the framework including the nature, role and impact of active and student-centered instructional practices, asset orientations and sociopolitical awareness. The central focus of this report will be on the ways this framework supports and calls for a broadening of what is considered mathematical including (but not limited to) increased opportunities for students to be creative when doing mathematics. Firsthand experience from using instructional practices aligned with this framework will be shared. These experiences suggest that valuing creativity in mathematics could contribute to the creation of a rehumanized mathematics with expanded views of what mathematics is, how mathematics is done and what mathematics is used for. (Received September 17, 2019)

A History of Mathematics in the United States and Canada: A session in Honor of Math Historian David Zitarelli

1154-F1-31 **Amy Ackerberg-Hastings*** (aackerbe@verizon.net). *David Zitarelli and Historical Methods: Mathematics in Early 19th-Century American Colleges.*

I knew David for nearly 20 years, having met him sometime during my transition from nearly-finished to newly-minted PhD. His cheerful welcome to the history of mathematics was especially valuable since I was just beginning to learn to navigate the awkward waters between mathematics, the discipline of most of my colleagues, and the history of science, the discipline of my degree. We also shared an interest in the history of American mathematics and a love for that subject's primary sources, although we often differed in our methods for analyzing the evidence found in those sources. I will comment on how historians define theoretical approaches to historical interpretation, list some characteristics of David's and my methodologies, and suggest what we learn about American college mathematics education in the early 19th century from combining our approaches. (Received July 01, 2019)

1154-F1-67 **Lawrence A. D'Antonio*** (ldant@ramapo.edu). *Noether in Cambridge.* Preliminary report.

In 2011 David Zitarelli published an interesting and well-researched article entitled "Hilbert in Missouri". The title was something of a misdirection. Hilbert never visited Missouri, but he had three American students, O.D. Kellogg, E.R. Hedrick and W.D.A. Westfall, who played fundamental roles in the development of the mathematics department at the University of Missouri.

To honor David's work, this talk will consider a similar connection. Max Noether had two American students, H.W. Tyler and W.F. Osgood, who played significant roles in the development of the mathematics departments at M.I.T. and Harvard, respectively. We will examine the life and work of these two mathematicians and the role they played in establishing the reputation of their departments. (Received July 29, 2019)

1154-F1-148 **Cynthia J Huffman*** (cjhuffman@pittstate.edu), Math Dept., Pittsburg State University, 1701 S. Broadway, Pittsburg, KS 66762. *A Brief History of the Mathematics Honor Society Kappa Mu Epsilon.*

From the end of the 19th into the early 20th century, professional societies began appearing in many disciplines, including mathematics. Kappa Mu Epsilon was founded in 1931 to meet the need for a national mathematics honor society which focused on students at primarily undergraduate institutions. In this presentation, we will take a brief look at the history of KME and its impact as it has developed into the honor society that it is today with over 200 chapters across America and nearly 90,000 members. (Received August 15, 2019)

1154-F1-327 **Patricia R. Allaire*** (patalaire@gmail.com) and **Antonella Cupillari** (axc5@psu.edu). *Helping to build the American mathematical community - two early journals.*

Apart from the “big name” journals, smaller lesser-known journals played an important role in the growth of the American mathematics community toward the end of the 19th century. In this talk, we look at two of the more successful publications, *The Mathematical Visitor* and *The Mathematical Magazine*, both founded and edited by Artemas Martin, a self-taught mathematician. (Received August 31, 2019)

1154-F1-385 **Della Dumbaugh*** (ddumbaugh@richmond.edu), Jepson Hall, Room 202, University of Richmond, Richmond, VA 23173, and **Stephen Kennedy**. *Preserving the Legacy: An Update and Preview of the Zitarelli Volumes.*

With the first volume of David Zitarelli’s “A History of Mathematics in the United States and Canada” in press at the time of his death, the mathematical community is naturally interested in the future of this project that meant so much to him. In this talk, we offer an update on this important work along with a peek at the upcoming second volume. (Received September 02, 2019)

1154-F1-765 **Leon M. Hall*** (lmhall@mst.edu), 812 Sycamore Dr., Rolla, MO 65401. *The best mathematician in the state of Missouri (in 1897)?*

Every mathematics department probably has had one or more faculty members in its history who stood out and who became something of a local (at least) legend. David Zitarelli was such a person. Keeping the memories of these people alive is a worthwhile endeavor. Time passes and interesting stories can be lost. The most memorable mathematics faculty legend at the Missouri School of Mines and Metallurgy (now Missouri S&T) was George R. Dean. Dean was at MSM as Professor of Mathematics for the first third of the 20th century and then some: 1897-1935. He had a research career that would be respectable today and that was not matched or even approached in the department until the 1960s. His favorite teaching method was to send students to the board to work problems, and he would throw erasers at the ones who made mistakes to get their attention. Active learning is not a new idea. This talk will give an overview of Dean’s career. Other names dropped will include W.H. Echols, J.M. Greenwood, B.F. Finkel, C.P. Steinmetz, and E.R. Hedrick. (Received September 10, 2019)

1154-F1-1155 **Amy Shell-Gellasch*** (ashellge@emich.edu), 4279 Lilac Lane, Ypsilanti, MI 48197. *David Zitarelli: A Short Introduction.* Preliminary report.

This talk will give an all-to-brief overview of David and his work in the history of mathematics, and why he is sorely missed. (Received September 13, 2019)

1154-F1-2221 **David Lindsay Roberts*** (robertsd1@aol.com). *In Search of Lillian Lieber: Google, Serendipity, and Ancestry.com.*

In attempting to learn more about the notable twentieth century American mathematics popularizer, Lillian Lieber (1886-1986), I discovered key facts in expected and unexpected places, by both digital and old-fashioned methods. In this talk I will describe some of my finds, and will offer some reflections on the changing nature of historical research. (Received September 17, 2019)

1154-F1-2340 **John McCleary*** (mccleary@vassar.edu), Box 69 Vassar College, 124 Raymond Avenue, Poughkeepsie, NY 12604. *Hassler Whitney and Fire Control in WWII.*

There were many mathematicians that worked for the US war effort during World War II outside the atomic bomb project. Hassler Whitney threw himself into these efforts and was assigned to work on fire control, the mathematics of aiming weapons for accuracy. This talk will consider how he got into this work and what sort of successes he had. (Received September 17, 2019)

Inclusive Excellence for the Mathematical Sciences

1154-F5-118 **Michaela Kubacki*** (mkubacki@middlebury.edu), Warner Hall, 303 College Street, Middlebury, VT 05753. *Building Community and Improving Equity through Small-Group Collaborative Learning.* Preliminary report.

The Small-Group Collaborative Learning Sessions Program is a new resource that endeavors create a more inclusive and equitable calculus experience for students in their first semester of college. In its pilot implementation, first-semester students in two sections of Calculus II had the option of signing up for Small-Group Collaborative Learning Sessions. In these sessions, students met weekly in small groups of 3-4 classmates, along with an experienced peer leader, for one hour. These small-group meetings served to help build a sense of community

and belonging, while also providing a non-judgmental setting for asking and answering questions. This type of resource promotes a more inclusive classroom by facilitating positive relationships amongst students on a more personal level, while also exposing students to a diverse and excellent set of role models as peer leaders.

At the end of the semester, students filled out an anonymous questionnaire regarding their experience in these sessions. The feedback was overwhelmingly positive. We will discuss the organization and implementation of this pilot program, as well as observations of its impact in the classroom. We will also present the results from the student questionnaire and future plans for this endeavor. (Received August 09, 2019)

1154-F5-179 **John Symms*** (jsymms@carrollu.edu) and **Kathrine Kramer**. *Promoting STEM Retention through Self-Efficacy*.

With funding from the National Science Foundation, the Carroll University Pro-STEM Initiative is providing support to low-income students with demonstrated financial need and academic promise to succeed in STEM disciplines at Carroll University. The project is funding scholarships for students who are pursuing bachelor's degrees in applied physics, biochemistry, chemistry, computer science, and mathematics. To promote academic success and retention within these STEM majors, the program (1) incorporates self-efficacy building strategies in general education courses of Carroll University's liberal arts curriculum, (2) fosters support of project goals through a faculty professional learning community populated by those who teach first-year courses in the targeted fields, and (3) uses peer-led study groups and community building activities that support self-efficacy and retention. This project applies Habits of Mind strategies and practices to build academic skills and self-awareness. Activities of the project contribute to the knowledge base on STEM retention and student academic success. The results further serve as a model to aid other small schools in increasing completion rates and effectively preparing students for success beyond college. (Received August 19, 2019)

1154-F5-429 **Liljana Babinkostova*** (liljanababinkostova@boisestate.edu), 1910 University Drive, Boise, ID 83725, and **Marion Scheepers** (mscheepe@boisestate.edu), 1910 University Drive, Boise, ID 83725. *Research Program in Mathematics for Young Scientists*. Preliminary report.

Research Experience in Mathematics for Young Scientists (REMYs) is an eight week program at Boise State University that engages high school students from Idaho and from underrepresented groups in research in mathematics. The program offers communication development and vertical integration that brings together research mathematicians, graduate and undergraduate students, and high school students. The overarching goal of REMYS is to encourage and promote the flow of diverse high school students towards higher education and careers in mathematics. In this talk we present the REMYS extracurricular scientific activities and outreach efforts towards recruiting students from underrepresented groups. (Received September 03, 2019)

1154-F5-975 **Matthew K Voigt*** (mkvoigt@gmail.com). *Unpacking LGBTQA: Experiences and supports for queer spectrum students in math*. Preliminary report.

Queerness is an entity which often defies categorization and as such it can be hard to unpack notions of queerness; furthermore, this becomes increasingly difficult when one considers queerness in the context of Mathematics. In this study we draw on large scale survey data and iterative categorization to understand and unpack students' self-described sexual identity. Specifically, we present data from students enrolled in introductory math courses at 20 universities across 898 classrooms. Queer spectrum students, those identifying in some way with the sexual minority, represented 10.0% (n=2,454) of the total student responses in the study (n=24,327). Quantitative analysis reveals a pattern that Asexual students report the most positive instructional practices (instructor interactions, peer interactions, math affect, sense of community) while Bisexual and Queer+ (e.g., queer, pansexual, multiple queer identities) students report the lowest levels of positive instructional experiences. Insights from this analysis and follow-up student interviews provide recommendations for programmatic supports for queer spectrum students. (Received September 12, 2019)

1154-F5-992 **James E. Hamblin*** (jehamb@ship.edu), 1871 Old Main Drive, Shippensburg, PA 17257. *Making the Classroom More Inclusive: Ideas and Strategies*.

In this talk, I will outline some ideas and strategies, both specific and general, for making the mathematics classroom more inclusive for LGBTQ+ students. These include helping students communicate their name and pronouns in a confidential way, including inclusive language on handouts and assignments, and advocating for LGBTQ+ students across campus. (Received September 12, 2019)

1154-F5-1065 **Hyunju Oh*** (ohh@triton.uog.edu) and **Leslie Aquino** (aquino18112@triton.uog.edu). *The Young Scholars Research Experiences in Mathematics (YSREM) at University of Guam*

We will present our successful outreach project experiences “The Young Scholars Research Experiences in Mathematics (YSREM) at University of Guam (UOG)”, supported by Tensor-Summa grant. Six enthusiastic and talented math students from tenth to twelfth grades in Guam have participated in research experiences in mathematical thought and effort in Coding theory and Vaccination Game theory during summer 2019. The YSREM program is combined National Research Experiences for Undergraduates Program (NREUP) at UOG, which run concurrently as a single program. UOG’s six undergraduate research students in NREUP serve as one-on-one mentors and work as a collaborators in the team research projects under their faculty mentors for the seven weeks. The YSREM project promotes gifted teens in the Guam area: an opportunity to explore mathematics that is not generally taught in schools, role models, intellectual challenge, true peers, and the confidence to dream bigger dreams. We will report how we mentor students during the joint summer research program. (Received September 18, 2019)

1154-F5-1388 **Tatiana Shubin** (tatiana.shubin@sjsu.edu), **Bob Klein** (kleinr@ohio.edu) and **Amanda Serenevy*** (amanda.serenevy@riverbendmath.org). *The Alliance of Indigenous Math Circles Report on the 2019 Summer Camps for Indigenous Students and K-12 Teachers.*

Refining a nearly-decade’s-long model of partnering mathematicians with teachers and students in underserved Indigenous regions, AIMC operated two residential camps this summer. The third annual summer camp for Indigenous school children was hosted at Navajo Preparatory School in NM. Thirty-five students engaged with mathematicians, undergrads, peer-mentors, K-12 teachers, and tribal elders to grow in their knowledge of math and regional cultures. Students from Navajo, Hopi, Apache, and Pueblo tribes collaborated on engaging problems and grew in their belief in self. Evaluations suggested that students would willingly trade another week of summer to do more camp. Second, a residential immersion workshop for K-12 math teachers was hosted at the Institute of American Indian Arts (IAIA) in Santa Fe. Teachers collaborated with mathematicians in problem solving, developing the capacity to choose “good problems” and to facilitate them in classroom settings, including allowing more think time and valuing struggle (and failure) as part of the process. Teachers gained a network of mathematicians to support them in these efforts. The AIMC forms communities of support and engagement to increase Indigenous students’ access to and participation in post-secondary education and STEM fields. (Received September 15, 2019)

1154-F5-1636 **Natalie LF Hobson*** (natalie.hobson@sonoma.edu). *Girls Tinker Academy- How a summer program in making tries to close the gap.*

An immersive summer program has the capacity to quickly create a group culture through shared experiences, projects, and challenges. The Girls Tinker Academy, a two-week summer program for 24 middle school girls in Sonoma County, sought to do just that as an effort to close the gender gap in the mathematical sciences by helping girls see themselves as members of a community of makers and doers of mathematics. The program took place in a makerspace and provided girls with opportunities to explore their own design interests, empowering them to build and create from what inspired them. A team of female mentors and techs also assisted girls along the way by forming relationships that continued to blossom outside of the program. In this talk, I will share some of the unique qualities of the program and present takeaways from the girls themselves. (Received September 16, 2019)

1154-F5-1795 **Jana Gevertz** (gevertz@tcnj.edu), Ewing, NJ, **Matthew Mizuhara** (mizuharm@tcnj.edu), Ewing, NJ, **Susan Schmoyer*** (schmoyes@tcnj.edu), Ewing, NJ, and **Suriza van der Sandt** (vandersa@tcnj.edu), Ewing, NJ. *Backward Design and Creating a Supportive Culture in Precalculus and First-semester Calculus.* Preliminary report.

This talk will focus on improving inclusive excellence in Pre-Calculus and first semester Calculus. Efforts have been focused on both curriculum design and creating a supportive culture in the department. Curriculum design changes include course modification, by using backward design, to align knowledge and skills required in STEM fields with the learning outcomes of our courses. Emphasis is also being placed on meaningful active student participation in class through redesigned class activities. Given that this work is supported by an HHMI Inclusive Excellence initiative and a FIRST grant from the NSF, we are particularly interested in assessing the impact of such changes on first generation college goers and minority students in STEM. Beyond curricular changes, the department is implementing cultural changes, which include creating self-identifying mechanisms for faculty

being first-generation college goers (or being supportive of first generation college goers), and including senior students in supportive roles. (Received September 16, 2019)

1154-F5-1892 **Kerry M. Luse*** (lusek@trinitydc.edu). *An Interdisciplinary Approach to Inclusive Excellence*. Preliminary report.

The math and science instructors at Trinity Washington University are striving to increase inclusive practices in order to improve retention of STEM students. We have found that introductory math courses and introductory science courses are a point in the curriculum in which many students stop out. In this talk I will describe interdisciplinary interventions developed by our math and science departments to support and retain students in STEM majors. Our approach includes updates to the first year math course, the creation of an embedded tutor program, and the addition of a 1 credit course to foster student belonging and increase skills for college success. Preliminary results show that our revised curriculum enhances student belonging in STEM but does not yet show a statistically significant difference in success in introductory math. I will discuss how this data led to the creation of the tutoring program and changes to the 1 credit course as well as initial results from these additions. Our work is supported by an HHMI Inclusive Excellence grant. (Received September 16, 2019)

1154-F5-2127 **Hanna Bennett*** (hbennett@umich.edu), **Paul Kessenich** (paulkess@umich.edu), **P. Gavin LaRose** (glarose@umich.edu) and **Nina White** (whitenj@umich.edu), Department of Mathematics, University of Michigan, 530 Church St., Ann Arbor, MI 48103-1049.
Programmatic Inclusive Environment Building in the Context of a Large Department.

In this paper we discuss a series of loosely connected initiatives to improve the inclusiveness in a large department, and in the large enrollment courses we teach. We make the case that there is a synergy arising from department members' overlapping engagement in interrelated efforts to increase awareness of issues of inclusion, incorporate themes of inclusiveness in instructor training, and take concrete steps to make courses more welcoming of under-represented groups. We describe a number of these efforts, including development of a learning community on inclusive teaching (engaging faculty, post-doctoral and graduate student instructors), instructor training and support for inclusive teaching, and concrete programmatic changes we are making in the assessment and administrative structure of our large enrollment courses to improve their inclusiveness. In our discussion a primary focus is on how these efforts may be applicable in other departments, and highlight concrete outcomes that have improved our courses and instruction. (Received September 17, 2019)

1154-F5-2348 **Sarah Cassie Burnett*** (burnetts@math.umd.edu). *Girls Talk Math: Providing a Path to Math-related Careers through Early Support and Media.*

Girls Talk Math (GTM) is a low-cost, two-week day camp hosted at the University of Maryland with the goal to promote women in mathematics. The program supports 40 female and non-gender conforming rising high school students residing locally. The objectives of this program are to explore new mathematical concepts with campers, to learn about the role of women in mathematics, and to create a relationship between campers and other GTM participants. Campers work in groups to accomplish the following tasks: (i) complete the packet of material, (ii) write a blog post about what they learned, (iii) research and record a podcast about a famous female mathematician whose research relates to the material, and (iv) present their learnings on the last day of camp. New topics have been tailored to the graduate and undergraduate student volunteer interests including Number Theory, Patterns and Fractals, Cryptography, Finance, Network Science, and Quantum Mechanics. This program has run for two consecutive years with the last year supported by the MAA Tensor Women and Mathematics Grant. It is based on the structure of the original GTM at the University of North Carolina which has run for four years. Please attend to learn more about how you can establish a sister chapter at your institution. (Received September 17, 2019)

1154-F5-2349 **Christina Lee*** (christina.lee@emory.edu). *How Oxford College's Mathematics Center Encourages Inclusivity.*

Oxford College's Mathematics Center supports and promotes the learning of mathematics. A large part of its mission is served by tutoring students in the 100-level mathematics courses. The center, comprised of a Director and student tutors, is housed in a purpose built and dedicated space in the mathematics department. Moreover, the Director is a mathematics faculty member whose primary responsibility at the college is to run it. As the Math Center Director, one of my goals has been to actively recruit women and underrepresented minorities to take more of a leading role in mathematics by becoming student tutors, and eventually head tutors. I have found that by doing so many of my tutors have become more confident in their mathematical abilities, and many have gone on to become Math majors. This talk will discuss the model for our Math Center, our outreach activities,

as well as preliminary data on the retention of women and underrepresented minorities in mathematics and computer science. (Received September 17, 2019)

1154-F5-2402 **Eugene Fiorini*** (eugenefiorini@muhlenberg.edu), 2400 Chew Street, Allentown, PA 18104, and **Gail Marsella** (gaimarsella@muhlenberg.edu), **James Russell**, **Karen Turk** and **Marten Edwards**. *Using New Invasive Species to Engage Minority Students in STEM Research*.

The spotted lanternfly arrived in Berks County, PA, in 2012 from Asia and has spread outward to neighboring counties and states. Spotted lanternflies are known for damaging trees, grapevines and other agricultural goods by feeding on them and excreting a sticky substance that leads to mold. The SLIMES (Spotted Lanternfly Investigated through Mathematical and Environmental Sciences) program is a two-year program designed to engage area high school students, particularly from underrepresented groups, in scientific and mathematical investigation. Participants will contribute to a lean but growing body of research aimed at reducing the spotted lanternfly population. This talk will discuss the progress of the SLIMES program's first year as well as the college-readiness aspects of the program intended to encourage high school students from a background of economic need to consider pursuing a STEM-related career. (Received September 17, 2019)

1154-F5-2420 **Luis Leyva**, **R Taylor McNeill*** (reagin.t.mcneill@vanderbilt.edu) and **Brittany Marshall**. *An analysis of racialized and gendered logics in Black and Latinx women's interpretations of instructional events in undergraduate pre-calculus and calculus classrooms*. Preliminary report.

Inquiry into racialized and gendered aspects of pre-calculus and calculus instruction is underexplored yet vital to improve retention and inclusion in undergraduate mathematics. This report presents an analysis of ideological and institutional logics raised in eight Black and Latinx women's perceptions of instructional events from pre-calculus and calculus classrooms reported as discouraging or marginalizing. Several themes were revealed in the analysis. First, participants assumed instructors to favor students of the same race and gender, especially when the tenor of instructor-student interactions varied consistent with preferential treatment of students with the same identity. Second, participants perceived power dynamics based on race, gender, and professional hierarchy limiting student options for responding to discouraging incidents to staying silent or switching classes. Thirdly, students cited negative racialized and gendered stereotypes surrounding mathematics and intellectual ability to explain invalidating or dismissive instructor attention. These findings give insight into the racialized and gendered logics in terms of ideologies and institutional structures that shape marginalizing experiences of entry-level mathematics instruction among Black and Latinx women. (Received September 17, 2019)

1154-F5-2574 **Daniel Collister*** (dcol1010@ucr.edu), **Tim McEldowney**, **Alex Rowell** and **Po-Ning Chen**. *The Advanced Mathematics Program (AMP) and promoting equity through community*.

The Advanced Mathematics Program (AMP) at UC Riverside is devoted to equity and advancing the careers of underrepresented undergraduate math majors. To accomplish this goal, AMP held preparatory seminars in abstract algebra and real analysis over the summer, using reverse classroom techniques to create a community setting for the students. Additionally, it offered talks from active mathematicians, mentoring, and help applying to conferences. This is the third year of the program, and took place this year from July 1 to July 26. The program was funded by the 2019 MAA Tensor-SUMMA (Strengthening Underrepresented Minority Mathematics Achievement), which funded 5 underrepresented minority participants of the program. I will present information gathered from the participants about the program, which found that participants feel better prepared to pursue a career in mathematics. I will also present our findings about methodology and improvements we plan to make for the upcoming year's program. (Received September 17, 2019)

1154-F5-2594 **Tracii Friedman*** (tfriedma@coloradomesa.edu) and **Cathy Bonan-Hamada**. *GirlsDoMath in Western Colorado*. Preliminary report.

The GirlsDoMath Summer Camp is a week-long mathematics enrichment camp for girls entering 8th or 9th grade. The goals of the GirlsDoMath Camp are to provide mathematics enrichment activities and a "mini-research" experience to girls early in their education; to introduce girls to careers that require a strong background in mathematics; and to improve girls' perceptions of women as mathematicians. The camp uses hands-on activities to introduce exciting topics in mathematics that the girls are not likely to have seen in their traditional school curriculum. We also organize two follow-up events with the camp participants during the academic year that are designed to further involve the girls in our mathematics community. The GirlsDoMath Camp has run the last two summers, both times supported by an MAA Tensor Women and Mathematics Grant. We believe that by giving

the young women who attend our camp the opportunity to engage with and explore interesting mathematics in a supportive environment, they will achieve greater confidence and mathematical growth. In this presentation, we will describe year two of our Camp experience and report on our ten-month post-camp survey from year one. (Received September 17, 2019)

1154-F5-2750 **Vita Borovyk*** (vita.borovyk@uc.edu) and **Christina Therkelsen** (christina.therkelsen@uc.edu). *Math Circle continues: activities, assessment, and campus visit.*

This talk continues the story of the math circle that we run at a low-performing Cincinnati public school with an economically disadvantaged student population. The math circle is supported by an MAA Tensor-SUMMA grant. We will describe new math-related activities the students were engaged in over the past year, assessment measures we employ, and the campus visit tentatively scheduled for Fall 2019. (Received September 17, 2019)

1154-F5-2780 **Bhuvaneswari Sambandham** (buna.sambandham@dixie.edu), **Clare Banks**, **Jie Liu*** and **Vinodh Chellamuthu**. *Dixie Tensor Scholar Program*. Preliminary report.

Through the MAA Tensor Grant for Women in Mathematics, we developed Dixie Tensor Scholar program to provide our undergraduate students and students from local high schools with activities aimed at connecting women STEM majors, high school girls, and successful women STEM professionals to exchange ideas through a year-long mentoring program. We began the academic year by recruiting female STEM majors into the DTSP. Starting October, we will begin our DTSP Lecture Series, which will bring together scholars and successful women in STEM areas. In the spring, we are planning a one day workshop for parents and female students from local high school careers by increasing awareness of career options, demonstrating the benefits of studying mathematics by providing them an environment to exchange ideas, strengthen problem-solving skills, and advance their intellectual confidence. In our presentation, we will share our experiences and progress of our new DTSP program. (Received September 17, 2019)

1154-F5-2781 **Jana Talley*** (jana.r.talley@jsums.edu), 1400 J. R. Lynch, Jackson, MS 39217, and **Carmen Wright** (carmen.m.wright@jsums.edu), 1400 J. R. Lynch, Jackson, MS 39217. *The Impact of the Jackson State University Girls Engaging in the Mathematical Sciences Program on Participant Self-Efficacy and Propensity to Engage in Mathematical Activities*. Preliminary report.

The Jackson State University Girls Engaging in the Mathematical Sciences Program (JSU GEMS) is a one-week summer enrichment program for middle school girls who are members of groups underrepresented in the mathematical sciences. The summer program addresses the underrepresentation of women and minorities by offering participants a variety of rigorous learning experiences and engagement with mathematicians and professionals in mathematically intensive careers.

Both surveys and interviews were conducted with two cohorts of JSU GEMS participants to address the following research questions: RQ1: How has participation in JSU GEMS impacted participants' propensity to engage in mathematics activities? RQ2: How has participation in JSU GEMS impacted participants' propensity to enroll in elective or advanced mathematics courses? RQ3: How has participation in JSU GEMS impacted participants' interest in pursuing a career in mathematics? RQ4: How has participation in JSU GEMS impacted participants' mathematics self-efficacy?

The presentation will specifically compare results from Cohorts 1 and 2 of the research study to identify ongoing themes related to RQ1 and RQ4. (Received September 17, 2019)

Incorporating Realistic Applications of Mathematics Through Interdisciplinary Collaborations

1154-G1-231 **Victor I. Piercey*** (victorpiercey@ferris.edu). *A Hurricane Katrina Case Study: Integrating Social Justice, Health Care, and Business through Mathematics*. Preliminary report.

Ferris State University ran an interdisciplinary faculty learning community in which mathematics was integrated with the curriculum in business, social work, and nursing. The resulting products were infused into a mathematics course for students in these majors as well as into the courses in the programs themselves. In this talk, we will share an example of a product: a case study on Hurricane Katrina. Along the way, we will discuss the processes within the faculty learning community that led to the result as well as student impact. (Received August 26, 2019)

- 1154-G1-343 **Artemis Karaali** (artemis.karaali@gmail.com) and **Gizem Karaali*** (gizem.karaali@pomona.edu). *Delicious Mathematics: Contexts for Mathematical Exercises from the Science and Engineering of Food*. Preliminary report.

We explore the various contexts within food science and food engineering where mathematics plays a significant role. We then offer templates of mathematical exercises for a range of standard mathematics courses where these contexts might be explored (such as calculus, differential equations, and mathematical modelling). We also provide sufficient background information for the instructor to motivate students as well as to prepare them to effectively tackle the problems. Topics explored include processing, packaging, and shelf life of various agricultural and industrial food products, lethal doses and safe zones for specific ingredients, and various related topics in predictive microbiology. Finally we share some notes on the collaboration that led to this work, between a mathematician and a food scientist, as well as a mother and a daughter. (Received September 01, 2019)

- 1154-G1-654 **Rebecca Segal***, rsegal@vcu.edu. *Connecting Partner Disciplines with Mathematics through Applications in Differential Equations*. Preliminary report.

Virginia Commonwealth University teaches an average of 20 sections of Differential Equations per year. The majority of the students in the course are majors in Engineering or Sciences. Anecdotally, students do not transfer knowledge content from the math course into courses within their majors. In order to improve knowledge retention, we have worked to establish conversations between Mathematics and the partner disciplines of Chemistry, Biology, Physics, and Engineering. Using the “Curriculum Foundations Project: Voices of the Partner Disciplines” report as source of discussion questions, a Fishbowl activity was held with faculty from Chemistry, Biology, Physics, and Engineering. There was good consensus about having the students explore a variety of application problems within each differential equation techniques. To further prioritize content for the course, a follow-up online survey was used to compile a topics wish list from the partner disciplines. Using this information, we began to move the instruction of Differential Equations to have a greater inclusion of application focused class activities and projects. Preliminary successes from the classroom will be shared. (Received September 09, 2019)

- 1154-G1-733 **Caroline Maher-Boulis*** (cmaherboulis@leeuniversity.edu), Lee University, 1120 N. Ocoee St., Cleveland, TN 37320. *Collaboration between the College of Education and Department of Mathematical Sciences at Lee University*. Preliminary report.

In this talk we describe the collaboration between the College of Education (COE) and Department of Mathematical Sciences (MTHSC) at Lee University that started 3 years ago. We will describe how the process started, the recommendations voiced by faculty from the COE and the response of the MTHSC department to these recommendations. One such recommendation is the use of manipulatives in the mathematics classroom, since future school teachers are expected to implement them in their own classrooms upon graduation. Education students are also given a Math Manipulatives Proficiency exam which they are expected to pass before they are allowed to student teach. In this talk we will showcase the addition of this requirement to the curriculum. We will also discuss preliminary outcomes, such as student engagement and retention, resulting from this collaboration. (Received September 10, 2019)

- 1154-G1-1558 **Katherine Radler*** (katie.radler@slu.edu), **Kimberly Druschel**, **Michael May** and **Sadita Salihovic**. *Introductory Statistics for nursing, physical therapy and allied health students*.

In 2017, the mathematics department at Saint Louis University revisited the delivery of the Introductory Statistics class, a class mainly taught by adjuncts and teaching assistants, serving nursing, physical therapy and allied health students. Students were doing well but there was not much depth, the topics taught varied widely and much of the course was not focused on the topics that would be useful for future application in their majors. Since there was previously little faculty involvement, Kimberly Druschel taught a pilot course with a semi-flipped classroom and some in-context projects. The pilot course’s success became part of conversations with client disciplines, where it was decided that a better way to serve students was to incorporate in-context projects in a semi-flipped fashion. The nursing department also suggested that we use the Hospital Compare data set to bring context to the in-class projects. The next steps were designing in-context projects and helping instructors implement the new course approach. In this talk, I will talk about the collaboration efforts, some of the Hospital Compare projects that came out of this collaboration as well as some student and instructor feedback from these projects and our continuing efforts to collect more data. (Received September 16, 2019)

- 1154-G1-1658 **Shahrooz Moosavizadeh*** (smoosavizadeh@nsu.edu), Norfolk State University, Department of Mathematics, 700 Park Avenue, Norfolk, VA 23504. *Magnifying the Role of Mathematics in the World of Sciences in Math Courses.*

Mathematics is often viewed by students as a series of concepts and skills unrelated to their respective majors. The lack of adequate quantity and quality of applications presented in a typical math course may have greatly contributed to the existing disconnect. To remedy the problem, the Departments of Biology, Engineering, and Mathematics at Norfolk State University joined forces to create a section of college algebra for biology majors, as well as courses in calculus and differential equations curtailed to the needs of engineering majors. Establishing learning communities, to include the faculty in the mathematics department and its partner disciplines, formed the foundation for such initiatives. The applications in biology and engineering, as final project products, and their presentations in a few preselected math courses are the focus of this paper. The program, sponsored by the SUMMIT-P grant, includes hands-on applications, team-teaching, active learning strategies, and effective presentation formats. The proposed applications are not the usual textbook examples; they highlight the research expertise of the faculty and/or concepts in the upper-level courses. The program, in its fourth year, has generated much interests among both students and faculty alike. (Received September 16, 2019)

- 1154-G1-1721 **Tao Chen*** (tchen@lagcc.cuny.edu), 3110 Thomson Avenue, Long Island City, NY 11101, **Glenn Henshaw**, 3110 Thomson Avenue, Long Island City, NY 11101, **Soloman Kone**, 3110 Thomson Avenue, Long Island City, NY 11101, and **Choon Shan Lai**, 3110 Thomson Avenue, Long Island City, NY 11101. *Contextualize College Algebra with Economics.* Preliminary report.

College Algebra serves a huge population at LaGuardia Community College. As part of SUMMIT-P project, LaGuardia team consisting of math and economics faculty contextualizes College Algebra with the aim of helping students to realize the usefulness and meaningfulness of mathematics learning and facilitating their economics learning. (Received September 16, 2019)

- 1154-G1-2204 **Jody Sorensen*** (sorensj1@augsborg.edu) and **Suzanne Dorée** (doree@augsborg.edu). *Creating Calculus Activities through Interdisciplinary Collaborations.*

As part of the national SUMMIT-P project, Augsburg University has been renovating its calculus sequence to include more authentic applications in order to engage students from across the university. We restructured Calculus I and II to include daily inquiry-based activities as well as weekly labs. We met with partner discipline colleagues to discuss how they use Calculus in their courses, which led to the creation of several activities. This talk will describe our process and detail several examples, including Lynx-Hare differential equations modeling with original sources (Ecology), adoption of trends to study logistic growth (Social Sciences), and turning the Lennard-Jones potential from Chemistry into a love story. We developed other activities from textbook examples, and then had partner discipline colleagues read them for authenticity. We are tracking student interest in and enjoyment of our courses. This material is based upon work supported by the National Science Foundation under NSF award number 1625142. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation. (Received September 17, 2019)

- 1154-G1-2443 **Nicoleta Corcodel** (nicoleta.corcodel@med.uni-heidelberg.de), Department of Prosthodontics, University of Heidelberg, **Kevin Rion** (krion@bridgew.edu), Department of Mathematics, Bridgewater State University, **Irina Seceleanu*** (iseceleanu@bridgew.edu), Department of Mathematics, Bridgewater State University, and **Wanchunzi Yu** (wyu@bridgew.edu), Department of Mathematics, Bridgewater State University. *Incorporating Dentistry Applications from Interdisciplinary Collaborations into the Classroom.*

In this talk, we highlight the research collaboration between faculty in the Department of Mathematics at Bridgewater State University, USA and faculty at the Department of Prosthodontics at the University of Heidelberg, Germany. We will outline the nature of the collaboration including partnerships with industry companies, and describe how the collaboration began and evolved into a successful research partnership. Moreover, we will highlight one particular mathematical model that emerged from the collaboration and describe how it was incorporated in an upper-level class in our mathematics program at Bridgewater State. In particular, we will showcase the use of inverse prediction and their confidence intervals for regression models to help technicians and dentists determine the cut level for zirconia crowns that maintains the color of the restoration withing acceptable parameters. Moreover, we will describe how this application was integrated into the classroom through project-based learning and the impact on student learning for this class. (Received September 17, 2019)

1154-G1-2593 **Kyle Evans*** (kyle.evans@trincoll.edu). *Mathematics and Redistricting: Authentic Learning through Government Partnerships.*

Institutional initiatives and intrinsic motivation led to the creation of a brand-new course on Mathematics and Redistricting which bridged three disciplines (Mathematics, Political Science, and Public Policy) and included a community partnership with government workers in Hartford who have redrawn district maps of Connecticut for 30 years. I will discuss the nature of this unique and beneficial collaboration which includes experiential and authentic learning from a range of experts, the topics covered in the course, and experience of the course from both student and instructor points of view. (Received September 17, 2019)

1154-G1-2755 **Theresa Jorgensen*** (jorgensen@uta.edu), **Elizabeth Griffith** (griffith.906@osu.edu), **W. Ashley Griffith** (griffith.233@osu.edu), **Casey Saup** (saup.4@buckeyemail.osu.edu) and **Rebekah Aduddell** (rebekah.aduddell@mavs.uta.edu). *Integrating Geoscience Contexts into College Algebra.*

Motivated by the goals of (1) improving the success of intended STEM majors in College Algebra and (2) introducing first year students to the geosciences to increase the number and diversity of students who choose a geoscience major, our small team of mathematicians and geoscientists are modifying the curriculum of College Algebra by integrating current geoscience research. For each College Algebra learning objective (five total), our geoscientists chose a corresponding geoscience theme and a geoscientist who does research related to that theme. We developed videos featuring each geoscientist describing how the mathematical skill the students are learning is essential for his/her scientific research. These videos are incorporated in weekly lab meetings in College Algebra. A portion of student homework exercises are cast in the context of that geoscience research, followed by problem-solving activities that synthesize and apply mathematical concepts with problems related to geoscience content. We present pre- and post-course survey results that assess student interest and self-efficacy in science and math as well as career perceptions of geoscience and related sciences. We highlight curricular examples from implementation of each of the geoscience themes. (Received September 17, 2019)

Innovations in Complex Analysis

1154-G5-1132 **Stephen Deterding*** (stephen.deterding@westliberty.edu), West Liberty University, West Liberty, WV 26074. *Complex Hearts: a game of complex numbers.* Preliminary report.

Complex hearts is a variation of the card game hearts that uses complex numbers. In order to play the game, players need to add, subtract, and multiply complex numbers as well as finding the moduli of complex numbers. Thus complex hearts can be a fun way for students to practice operations with complex numbers as part of a complex variables course or even as part of a college algebra or pre-calculus course (Received September 13, 2019)

1154-G5-2130 **Michael A. Brilleslyper*** (mike.brilleslyper@usafa.edu). *Visual Explorations for a 1-Parameter Family of Harmonic Trinomials.* Preliminary report.

Let n and k be integers with $1 \leq k \leq n - 1$, and let c be real with $0 < c \leq 2$. Consider the family of harmonic trinomials $p_c(z) = z^n + c\bar{z}^k - 1$. Unlike analytic trinomials, $p_c(z)$ can have more than n zeros. For fixed values of n and k , we explore how the number of zeros varies with c . Using different visualizations on Mathematica, it is possible to obtain a conceptual understanding of why there is a discrete set of c -values at which new zeros are "born." We introduce the critical circle, which separates the orientation preserving and reversing regions for $p_c(z)$ and show how it plays a fundamental role in finding the discrete set of c -values. Along the way, we will visit intersections of level curves, winding numbers, and even hypocycloids! (Received September 17, 2019)

1154-G5-2148 **Beth Schaubroek*** (beth.schaubroek@edu.usafa.edu). *Finding roots of a class of harmonic polynomials.* Preliminary report.

Harmonic polynomials of a single complex variable provide a rich avenue for research. Here, we focus our attention on the class of harmonic polynomials $p_c(z) = z^n + c\bar{z}^k - 1$, where n and k are natural numbers with $k < n$ and c is a positive real number. The number of roots varies from n to $n + 2k$ as c increases. The discussion of winding number, and what it means for harmonic functions, plays a key role in the results. (Received September 17, 2019)

1154-G5-2152 **Kasey Bray, Jerry Dwyer, Roger W. Barnard, Erin Williams and G. Brock Williams*** (brock.williams@ttu.edu). *Iteration of Newton Maps.*

Newton's method is a well-known process for locating roots of a function f using the iterative formula $z_{n+1} = z_n - \frac{f(z_n)}{f'(z_n)}$. The associated function $F(z) = z - \frac{f(z)}{f'(z)}$ is the Newton map for f .

We will discuss the iteration of Newton maps for various classes of rational and trigonometric functions. (Received September 17, 2019)

1154-G5-2330 **Joseph Previte and Michelle Previte*** (michelleprevite@psu.edu), 1 Prischak Building, 4205 College Dr, Erie, PA 16563. *The Beautiful Chaotic Dynamics of i^z .*

Euler and Eisenstein explored the concept of iterated exponents and their convergence. One particular iterated exponent of interest is $i^{(i^{(i^{\dots})})}$ which can be shown to converge to $z_* \approx 0.4382829366 + 0.3605924718i$

In this talk, we explore the dynamics of the principal branch of the complex map $f(z) = i^z$, which has z_* as a fixed point. This function exhibits interesting dynamics, including chaos, making the analysis of this map an excellent case study in an undergraduate complex analysis course or as a supplemental undergraduate research project in such a course. At the end, we present several open problems that an undergraduate with a solid complex analysis course would be able to investigate. (Received September 17, 2019)

Innovative and Effective Ways to Teach Linear Algebra

1154-H1-948 **David Austin and Paul E. Fishback*** (fishbacp@mail.gvsu.edu), Department of Mathematics, 1 Campus Drive, Allendale, MI 49401. *A Second Linear Algebra Course Emphasizing Data Analysis within a Jupyter Notebook Environment.*

Exposure to contemporary applications is an essential part of the student linear algebra experience and both justifies and motivates the study of more abstract principles. We describe a second linear algebra course, which introduces students to substantial applications they rarely encounter in the prerequisite. Among these are simple and multiple linear regression, principal component analysis, clustering (both k-means and spectral), and numerous singular value decomposition applications. Throughout this course students utilize real-world data sets in a Jupyter notebook environment. (Received September 12, 2019)

1154-H1-1521 **Mark Hughes*** (hughes@mathematics.byu.edu), 324 TMCB, Provo, UT 84602. *Designing a computational linear algebra lab course using Google Colab.* Preliminary report.

In this talk I will discuss the design and implementation of a lab course which introduces students to computational algorithms in linear algebra using Python. This course does not assume students have any prior programming experience. Students work through weekly labs, programming their solutions using Google Colab online notebooks before submitting them to a website for automated grading. I will discuss the results of implementing a similar course at Brigham Young University, as well as some of the issues we encountered. (Received September 16, 2019)

1154-H1-1925 **John Conrad Merkel*** (jmerkel@oglethorpe.edu). *Combining Flipped Linear Algebra Using Open Educational Resources with Concept Questions a la Peer Instruction.* Preliminary report.

I report on three Linear Algebra for Engineers courses taught over three semesters. Two of the courses followed a flipped format and made use of open educational resources. Additionally, the flipped courses incorporated concept questions as per the Peer Instruction pedagogy. The third course followed a more traditional lecture format. I compare exam scores and student response surveys from the three courses. Preliminary results suggest students perform better in the flipped course. (Received September 16, 2019)

1154-H1-2512 **Sepideh Stewart*** (sepidehstewart@ou.edu). *Teaching and learning Linear Algebra proofs.*

While, proof has a significant place in the linear algebra curriculum, its teaching and learning remains a challenge for both instructors and students. Although research on proof in mathematics education is well established, very few empirical studies on teaching and learning of proof in linear algebra exist in the literature. In a recent study, Stewart and Thomas (2019) opened the case for a pedagogy of proof in linear algebra and examined students' reactions to, and voices on, proof in a first-year course in linear algebra. The study addressed student views on understanding of proof, the purpose of a proof, and when and how proofs communicated to them. The study employed Tall's (2013) three worlds of mathematical thinking as well as Harel's (2018) intellectual need to analyze the data from interviews with 16 students. The results showed that understanding a proof

was an important objective for many students. In this talk, I will give some highlights from this study and will discuss some preliminary results from another study on proof with a group of second year linear algebra students. (Received September 17, 2019)

1154-H1-2703 **Inyoung Lee*** (ilee28@asu.edu), Tempe, AZ 871804. *Two perspectives to interpret an object in relation to a coordinate system; coordinates-open and coordinates-closed.*
Preliminary report.

For students to broaden their ways of thinking to non-standard coordinate systems, it is crucial that students get much chance to take into account an object itself that they are dealing with and of how it gets transformed between different coordinate systems. In this sense, I offer two perspectives to interpret an object in relation to a coordinate system; coordinates-open and coordinates-closed. For “coordinates-open” view, we consider any object as composed of points, and any coordinate system describes those points’ locations. The object’s points are “coordinatized” by the system laid atop the object. On the other hand, “coordinates-closed” means that an object is embedded in a particular coordinate system. The object is defined relative to the coordinate system, and the object is transformed into a different shape by a transformation that maps points in the first coordinate system to points in the second coordinate system. The Jacobian matrix as one of the transformation matrices takes its role of relating two representations of an object using local linearity and the concept of differentials. Based on the two perspectives, literature and students’ conceptions will be discussed. (Received September 17, 2019)

1154-H1-2842 **Gilbert Strang***. *Principal component analysis and deep learning.*
Principal Component Analysis depends on the Eckart-Young theorem that the best rank k approximation to a data matrix comes from the first k principal components in the Singular Value Decomposition. We describe a short proof and separately we discuss the success of deep learning in generalizing from training data to test data. (Received September 22, 2019)

Inquiry-Based Learning and Teaching

1154-H5-33 **Krystyna Kuperberg*** (kuperkm@auburn.edu), 1195 Hancock Dr NE, Atlanta, Georgia, Atlanta, GA 30306. *IBL and multiple choice tests.*

Teaching a large undergraduate analysis class can be a challenge. Class presentation time is limited and could be used only for some especially interesting solutions, or the time could be used to give each students a chance for a short presentation. In either case, the grade has to be based to a large extent on tests and exams. Carefully prepared multiple choice tests turned out to be very helpful, educational, and a time saver for the instructor. We plan to share with you examples of problems on such exams and how the test were graded. (Received July 04, 2019)

1154-H5-65 **Ahmad Alhammouri***, 700 Pelham Rd N, Ramona Wood Hall, A-4, Jacksonville, AL 36265. *Understanding Mathematics Matters (Umm...) Camp: Improving School Students’ Perception of Mathematics.*

The Umm... Summer Camp is a week-long mathematics camp for upcoming 7th or 8th grade students. The main goal of the camp is to improve the campers’ perception of mathematics by engaging them in hands-on activities written in the form of “here is a situation, think about it.” The Umm... camp occurred for the first time in July 2019 at Jacksonville State University (JSU) in the State of Alabama. The camp was fully supported by the School of Education at JSU. In this session, I will discuss the Umm... Summer Camp experience and how it might positively change the campers’ perception of mathematics. (Received July 29, 2019)

1154-H5-369 **David Clark** (clarkd@newpaltz.edu) and **Xiao Xiao*** (xixiao@utica.edu). *The Number Line: A Guided Inquiry into Real Analysis.*

In this talk, we will report on a recently completed IBL book on real analysis that is written with secondary school mathematics teachers in mind. The Mathematical Education of Teachers II published by the Conference Board for the Mathematical Sciences in 2012, pointed out that the real number line is assumed to exist and satisfies the same properties as the rational numbers in high school mathematics, and rightly so, without any deep explanation of why. Yet, the teachers need to understand the construction of the real numbers and a proof that they satisfy the properties of operations in order to avoid false simplifications and be able to answer questions from students seeking further understanding. In this book, we choose course content to give students a working knowledge of that part of real analysis that directly underlies the materials on numbers and functions

that is taught in high school. To give one example, this book contains one chapter on examining how infinite decimal expansions can be used to represent real numbers and one chapter on the construction of real numbers using the infinite decimal expansions. (Received September 02, 2019)

1154-H5-509 **Eric Sullivan*** (esullivan@carroll.edu), Helena, MT. *An Open Source Collection of Tasks for Inquiry-Based Numerical Analysis.*

Numerical Analysis is a subject with many clear opportunities for inquiry-based learning. For example, students can build algorithms in their own language, they can test error approximations with numerical experiments, or they can give experimental evidence of convergence rates. We will discuss a free, open source, collection of tasks to foster inquiry-based learning in a first semester numerical analysis course. Several tasks, along with samples of student work, will be highlighted. Access to the materials will be made freely available to interested audience members. (Received September 05, 2019)

1154-H5-869 **Hansun To** (hto1@worchester.edu), 486 Chandler Street, Worcester, MA 01602, and **Eileen B. Perez*** (eperez2@worchester.edu), 486 Chandler Street, Worcester, MA 01602. *How to choose Topics in Liberal Arts Math for an IBL course?* Preliminary report.

We will discuss our usage of Inquiry Based Learning in a course for Liberal Arts Mathematics, including setting the schedule for the course topics. Our notes will be available through JIBLM Open source Inquiry Based Learning materials of Survey of math at Worcester State University. It includes day by day materials under the five chapters(topics) modules. (Received September 11, 2019)

1154-H5-1198 **John Paul Cook***, 401 Mathematical Sciences Building, Oklahoma State University, Stillwater, OK 74078, and **Allison Dorko, William Jaco, Michael Oehrtman and Michael Tallman.** *The Mathematical Inquiry Project: Fostering Sustainable, Widespread Adoption of Inquiry-Based Learning.*

In this talk, we will discuss the Mathematical Inquiry Project, an NSF-funded collaboration of mathematics faculty from all 25 public institutions of higher education in the state of Oklahoma to support inquiry-based learning in Quantitative Reasoning, Modeling, College Algebra, Precalculus, and Calculus courses. The overarching goal of this 5-year project is to effect widespread, sustainable instructional change in these entry-level courses across the 27 public institutions of higher education in Oklahoma using a grassroots model that offers opportunities for ongoing professional development. Our talk will (1) provide an overview of the project, including the project design as well as our research component, and (2) share initial results about themes that emerged amongst the goals of the participating faculty who participated in year 1 of the project. (Received September 13, 2019)

1154-H5-1599 **Erika L Ward*** (eward1@ju.edu). *Reflective Writing for IBL Classrooms.*

Inquiry Based Learning requires students to engage in activities that many find unusual – it's a divergence from what they expect in a classroom, perhaps especially in a mathematics one. Taking an active role in the classroom can be exciting, but some students also find it uncomfortable, confusing, and difficult. Incorporating reflective writing in the classroom requires that students stop and think about their thinking, the activities, and why I ask them to do these things, and what they've learned. Especially in classes with inquiry based tasks and structures, the back-stop of reflection has helped my students integrate what they've learned and see the progress they've made. It also provides instructors with a peek at the perceptions and problems of students, so that the supports can be shifted to help students succeed. Both short, in the moment student reflections, and longer, end-of-semester reflections are illuminating for both students and instructors. (Received September 16, 2019)

1154-H5-1870 **Philip DeOrsey*** (pdeorsey@westfield.ma.edu) and **Christine von Renesse** (cvonrenesse@westfield.ma.edu). *Encouraging Student Buy-In through Islamic Geometry with Technology.*

Despite not being well defined, student buy-in is often noted as a common barrier by facilitators of inquiry-based learning. We connect this informal notion to the study of student resistance and explain our techniques for breaking down this barrier. Specifically we connect these strategies to an Islamic geometry activity. In our experience this activity creates little student resistance making it an ideal choice for an introduction to inquiry-based learning practices. In this talk we hope to provide resources to both new and experienced facilitators of an inquiry-based classroom to help reduce their student resistance. (Received September 16, 2019)

1154-H5-1880 **Ekaterina Lioutikova*** (elioutikova@usj.edu). *Guided student reflections in an inquiry-based mathematics classroom*. Preliminary report.

Self-reflection is an important meta-cognitive skill related to increased student performance and understanding. In this presentation, we will describe an implementation of online guided reflection forms in two inquiry-based college classes: one in discrete mathematics and one in first-semester calculus. Via weekly reflection assignments, students were encouraged to identify learning experiences that they would like to improve upon, select strategies to overcome challenges, and set specific goals for future growth. At the same time, the reflections served as an additional line of communication between the students and the instructor. Implications for students' buy-in to an IBL-style classroom and their attitudes toward learning mathematics will be discussed. (Received September 16, 2019)

1154-H5-2012 **Joshua P. Bowman*** (joshua.bowman@pepperdine.edu). *An inquiry-based approach to Cauchy's Theorem*.

Cauchy's Theorem is a cornerstone in the theory of holomorphic functions. In an undergraduate course on complex analysis, however, it is all too easy to gloss over the proof of Cauchy's Theorem, or to omit its proof altogether. I will outline a sequence of activities I created for an IBL course that guides students through a complete proof of Cauchy's Theorem, while also strengthening their understanding of other important concepts in the course. In addition, this approach reinforces important ideas from earlier calculus classes and points towards a proof of the Poincaré Lemma on differential forms. (Received September 17, 2019)

1154-H5-2108 **Mami T Wentworth*** (wentworthm1@wit.edu), Wentworth Institute of Technology, 550 Huntington Ave, Boston, MA 02016, and **Mel Henriksen** (henriksem@wit.edu), Wentworth Institute of Technology, 550 Huntington Ave, Boston, MA 02016. *Complementing Inquiry-based Learning with Specification Grading in a Differential Equations Course*. Preliminary report.

Our introductory differential equations course is taught using our Active Differential Equations material, which was developed to encourage guided-inquiry in topics pertaining to differential equations. To complement our emphasis on conceptual understanding through an inquiry-based approach, we employ specifications-based grading. Each specification is assessed either as "mastered" or "not yet mastered", and the course grade is based on the combination of the number of specifications mastered and the final exam score. In this talk, we will present our implementation of specifications-based grading and modifications from the previous semester. We will then discuss student feedback on our assessment technique and the effect of specifications-based grading on their experience with the inquiry-based material. Comparison of student performances using different assessment techniques will also be presented. (Received September 17, 2019)

1154-H5-2222 **Andrew Diener*** (adiener@cbru.edu), 36 S. Merton, Memphis, TN 38112. *Modeling Sound Waves in Trigonometry*.

Christian Brothers University has created a standalone Trigonometry class for underprepared Precalculus students. One of the advantages of this class is the ability to take extra time to do things like using technology to model sound waves and then fitting wave functions to these models. This allows students to make connections between the real world, models created in the real world, and the more theoretical functions defined in Trigonometry. It also helps the students understand the various technologies that can be used to create sound waves, record sound waves, and then analyze these sound waves. Here we will discuss the technologies used as well as the type of problems that lend themselves well to modeling. (Received September 17, 2019)

1154-H5-2280 **John D Ross*** (rossjo@southwestern.edu), 1001 E University Avenue, Georgetown, TX 78626. *Creating A Flipped Geometry Course with Room for Inquiry*.

This presentation looks at our efforts to create an upper-level Geometry class (with a focus on Differential Geometry) that contains plenty of room for inquiry-based classroom experiences. The result was a flipped classroom with online video lectures and in-class explorations, many of which utilized Mathematica. We discuss the experience, present aspects of the course and the inquiry opportunities offered, and comment on their observed effectiveness. (Received September 17, 2019)

1154-H5-2342 **Sarah A. Nelson*** (sarah.nelson@lr.edu), 625 7th Ave NE, LR Box 7527, Hickory, NC 28603. *Team Work and Evaluation: Finding the Missing Link*. Preliminary report.

As I learn more about inquiry based learning, I find myself increasingly drawn to finding the most effective ways to facilitate team work during class time. Working together helps level the playing field for all of my students while encouraging them to all learn the material in a deeper way. There are so many benefits to working together. Historically, though, assessments come in the form of exams during which students are completely isolated. As

a result, these traditional exams undermine my teaching practices and send mixed messages to my students. There should be a link between the learning process and how we assess students. On the other hand, having students complete all their assessments together presents too many opportunities for individual students to slip between the cracks. In this talk, we will share the process of finding common ground between our teaching and assessment practices. We will share our observations and what we have learned along the way. (Received September 17, 2019)

1154-H5-2355 **Benjamin Purkis*** (benjamin.purkis@emory.edu). *The Origins of Mathematics: An Inquiry-based Seminar in the History of Mathematics*. Preliminary report.

This talk will examine the structure and learning goals behind a first-year history of mathematics seminar taught in an inquiry-based style. As part of Oxford College's new Discovery Seminar program for incoming first-year students, this seminar seeks to introduce students to mathematical "ways of knowing". Rather than learning about historical developments in mathematics, students instead confront problems from various eras in mathematical history and work in small groups to devise solutions. Groups then present their solutions to each other and compare and contrast not only their solutions, but also the approaches that they took. This problem-based approach allows students to become more critically engaged in their mathematics, as well as discern patterns in what problem solving techniques are fruitful. The desired learning outcome is for students to think about mathematics differently, using these historical problems as a springboard to begin their own mathematical journeys. (Received September 17, 2019)

1154-H5-2407 **Heidi M. Andersen*** (handersen@udallas.edu). *Using IBL Methods and Games to Teach Logic and Proof Writing to Non-Math Majors*.

The math department at the University of Dallas in Irving, Texas, has created the textbook-free logic-and-proof course "Euclidean and Non-Euclidean Geometries" especially for non-science majors. Central to the course is a "Neutral Geometry" conjecture sequence similar to, but different than, Euclid's classic theorem sequence. Students must puzzle out the truth, or lack thereof, of each conjecture and either rigorously prove this conjecture or provide a counterexample to it. During the past year, I made substantial innovations to the curriculum, including the addition of several new features to my praxis. This talk will highlight these topics and will feature new developments from the Fall 2019 semester. (Received September 17, 2019)

1154-H5-2439 **Aviva Halani*** (ahalani@exeter.edu). *Collaborative Writing as Assessment in an Abstract Algebra Course*.

Students in inquiry-based classrooms are expected to uncover mathematics through a sequence of carefully constructed tasks, but they may have trouble synthesizing all of the new ideas. This presentation addresses one method of encouraging student synthesis: a collaborative writing project to create a reference guide. We report on the implementation and grading of such assignments in an Abstract Algebra course which used Teach Abstract Algebra for Understanding (taafu.org) as the core of its tasks. Student feedback to the project will be presented. (Received September 17, 2019)

1154-H5-2519 **Nicholas E Weaver*** (nicholas.weaver@ucdenver.edu), 1201 Larimer Street, Denver, CO 80217, and **Gary Olson**. *The TA Coach: A Peer Mentoring Approach to Support GTAs in the use of Active Learning Pedagogy*. Preliminary report.

In this talk, we introduce a peer mentoring model that utilizes an experienced Graduate Teaching Assistant (GTA) referred to as the TA Coach. This model provides in-class support to GTAs new to using active learning in the classroom. GTAs first create a lesson plan that incorporates active learning and then team teach the lesson in their classroom with guidance and support from the TA coach. TA Coach led conversations both before and after implementation allows GTAs to highlight useful strategies and facilitation practices that can be used to support active learning pedagogy. Insights from past TA Coaches will be provided along with a discussion on the impact to college algebra student success rates. This work is sponsored in part by NSF Grant 1821454. (Received September 17, 2019)

1154-H5-2572 **Elizabeth W Schott** (eschott@fsw.edu) and **Laurice Garrett*** (laurice.garrett@fsw.edu). *From Redesigning a Class Curriculum to Redesigning a Classroom Instead*.

In recent years, inquiry-based learning has risen as an innovative method to teach college mathematics. Personal teaching experience led faculty to be interested in this teaching methodology, and so three years ago faculty incorporated inquiry-based learning into the curriculum for a specific course with over the course of a semester. As part of the pilot program, the classroom itself was also redesigned to support the teaching style by incorporating a layout of group clusters for desks and dry erase boards encircling the entire room, making it a more interactive

classroom. Since the pilot, that specific classroom has been used to teach a variety of math and science classes with interesting results. This talk will detail the lessons learned from the pilot semester as well as the surprising results of teaching in a group and interactive classroom, to include student reactions to the room, reactions from other teachers using the classroom, and examples of interactive activities incorporated into various classes as a result of the classroom and its setup. (Received September 17, 2019)

1154-H5-2600 **Megan Wawro*** (mwawro@vt.edu), **Michelle Zandieh**, **Christine Andrews-Larson** and **David Plaxco**. *Promoting Inquiry in Linear Algebra with Student Videos, Reflections, and Portfolios*. Preliminary report.

The goal of the Inquiry-Oriented Linear Algebra (IOLA) curricular materials is to promote a research-based, student-centered approach to the teaching and learning of introductory linear algebra. IOLA's curricular materials build from a set of experientially real tasks that allow for active student engagement in the guided reinvention of key mathematical ideas. In this talk, I will discuss and share student work for three types of activities that can be used to complement the IOLA tasks in promoting both student and instructor inquiry: end-of-class reflection questions, pre-exam portfolios, and student-generated video presentations. I will conclude by mentioning four upcoming new IOLA instructional units that will integrate into and complement the existing IOLA materials. (Received September 17, 2019)

1154-H5-2622 **Susan B. Crook*** (susan.crook@loras.edu). *Why Aren't We All Using IBL?*

Inquiry-based learning has been a buzzword in college mathematics teaching circles for years. By having students have more ownership over their own learning, IBL has been shown to begin closing the achievement gap for minority students. Anecdotal evidence of its success abounds. . . so, with all this positive data, why isn't every math classroom taught using IBL methods? In this interactive talk, we'll discuss some of the common obstacles and challenges professors experience when they use IBL and brainstorm some ways to overcome these issues. (Received September 17, 2019)

1154-H5-2745 **Amanda Croll** and **Julie Melberg***, julie.melberg@cui.edu. *Inquiry Learning in a Topic Rich General Education Mathematics Class*. Preliminary report.

Concordia University Irvine's general education mathematics course is taught to all freshmen regardless of major, and focuses on Great Works of mathematics (including "Liber Abaci", "Elements" and Cantor's Theorem) and the impact these Great Works have had on how people think. After attending IBL workshops ourselves, we realized the ineffectiveness of teaching these topics through traditional lecture, so have restructured our course to use IBL strategies utilizing active learning and groupwork. These activities allow students working in groups to ask questions, observe and generalize patterns and "re-invent" mathematical concepts on their own. Specifically, students discover the golden ratio from the Fibonacci numbers, generalize patterns using the Fibonacci numbers, investigate non-Euclidean geometries using spheres and hyperbolic surfaces, and determine that different sizes of infinity must exist. These active learning lessons have encouraged deeper discussions of mathematics, and students are more engaged with and in conversations about mathematics both in and out of the classroom. In this talk we will highlight activities we use in our general education mathematics course, and discuss changes we have seen in student involvement, attitude and success in the course. (Received September 17, 2019)

1154-H5-2793 **Jonas D'Andrea*** (jdandrea@westminstercollege.edu). *Who gets an A?* Preliminary report.

Active learning relies heavily on inquiry while hopefully promoting curiosity and risk-taking. Do our assessment and grading practices support these goals? I will present on my experiences with mastery grading in linear algebra, discrete mathematics, and statistics. I have been using specifications standards-based grading for over two years, and will describe practical strategies for implementing successful mastery grading into your course. Student ownership of and agency in their own learning is critical for any inquiry-based learning course. Promoting student buy-in and managing student perceptions of their learning still require attention and time. However the nature of the conversations I now have with students surrounding buy-in are easier and more natural with assessment practices and grading that align to support their curiosity and risk-taking in the face of their anxiety of "not understanding." Further, we discuss what constitutes success, and ultimately how we can provide evidence of student understanding in ways that are meaningful to the students themselves. (Received September 18, 2019)

Integrating Research into the Undergraduate Classroom

1154-I1-35 **Viktoriya Savatorova*** (vs1445@ccsu.edu) and **Aleksei Talonov** (aleksei.talonov@unlv.edu). *Above and beyond traditional instruction through integrating research projects into Calculus classroom.*

We present our practices of using research projects to enhance student's engagement in Calculus classes, which have had traditionally low success rates and retention. Our experience shows that many of the students who struggle are not just underprepared, but also are not motivated. The goal for them is to pass the class with whatever minimum effort they can. The most challenging task for an instructor becomes not just to present the material, but to transform students from passive listeners to active participants and doers. For the semester we choose an actual science or engineering problem. The actuality of the problem we present in the form of the short reading assignment. This assignment is complemented by a short talk given by a STEM faculty working in the corresponding area. This guest speaker emphasizes the importance of the problem and the necessity of the knowledge of mathematics in order to solve it. Throughout the whole semester student's group work in class and at home is guided using a set of transparent assignments. The full task is broken down in subtasks with carefully described instructions. Students learn by doing, and it increases the knowledge. Students also benefit from peer learning and discovering the application of Calculus in an amusing and exciting way. (Received July 06, 2019)

1154-I1-248 **Mahmud Akelbek*** (make1bek@weber.edu), Ogden, UT 84403. *Research Projects in Numerical Analysis.*

Course based undergraduate research project not only has meaningful positive impact on a larger number of students, but it also actively engage students with the learning process throughout the semester. Numerical Analysis course require students to have a solid knowledge on essential mathematics courses, like calculus and linear algebra. It also requires students using computer programming to solve wide range of applied mathematics and engineering problems. In this presentation, I will give a short description of numerical analysis course for Spring 2019 semester that we incorporated research projects. I will present a number of undergraduate research projects from this course. I will also discuss some of the challenges and future direction. (Received August 27, 2019)

1154-I1-649 **Chinenye Ofodile*** (chinenye.ofodile@asurams.edu), 504 College Dr., Albany, GA 31705. *Internationalizing the Undergraduate Calculus Course at Albany State University.* Preliminary report.

Recently, Albany State University (ASU) has embarked in an initiative to incorporate various effective teaching strategies throughout all disciplines. One discipline, Mathematics, was analyzed. Based on the success rate in the service courses (or gate-keeping courses), a recommendation was made to revise the curriculum. In this talk, I will discuss effective ways students were able to learn while engaging in research. (Received September 09, 2019)

1154-I1-850 **Mihhail Berezovsky, Marylesa Howard and Aaron Luttman***, aaron.luttman@pnnl.gov. *Collaborative Classroom Research Projects Motivated by Applications to National Security.*

Among the most difficult challenges to incorporating real research projects in the classroom are (i) finding projects that are tractable but also open-ended without known solutions, (ii) helping students with mathematical or scientific problems well outside of their area of expertise, and (iii) gauging how much to teach and how much to let students learn on their own. Embry-Riddle Aeronautical University (ERAU) and the US Department of Energy's Nevada National Security Site and Pacific Northwest National Laboratory have partnered to actively collaborate on national security projects in the classroom. ERAU offers a course dedicated to industrial research projects, and the industrial partners are engaged throughout the semester via regular communication with the student teams. In this work, we will describe how the partnership was formed, topics of several of the recent projects and our pedagogical approaches to helping the students make meaningful contributions, the students' solutions and results, and some of the longer-term impacts on the students who were involved in the projects. We will also provide an industrial partner perspective on the value of undergraduate mathematics programs' emphasizing students' gaining relevant research experience in the classroom. (Received September 11, 2019)

1154-II-1135 **Minah Oh*** (ohmx@jmu.edu). *How Exposure to Research in Numerical PDEs Influenced Students In My PDE Class.*

In this talk, I will talk about how I included my research in numerical partial differential equations (PDEs) in my undergraduate PDE class. I introduced several past and current research projects to my students at a level that they can understand throughout the semester, and that had a wonderful effect on how motivated they were about learning. In this talk, I will present some of these research projects that I introduced in my class and also include some success stories that resulted from integrating research into my classroom. (Received September 13, 2019)

1154-II-1782 **Mihhail Berezovski*** (mihhail.berezovski@gmail.com), Math Department, 1 Aerospace Boulevard, Daytona Beach, FL 32114. *Bringing Industrial Mathematics Projects into classroom.*

In this talk, we will discuss the challenges of mentoring undergraduate students in data-enabled research project-based course: Research Projects in Industrial Mathematics. We highlight several important aspects and challenges of running multiple research projects within one course. We will discuss getting students involved in these projects and share ideas for successfully designing and mentoring such projects. The outcomes of the projects and lessons learned from these research classroom experiences will also be presented. (Received September 16, 2019)

1154-II-1882 **Ryan Higginbottom*** (rhigginbottom@washjeff.edu). *A Two-Stage Approach to a Research-Based Capstone Experience.* Preliminary report.

At Washington & Jefferson College, the capstone experience for math majors is split between the junior and senior years. Students complete these guided research projects in pairs as juniors and on their own as seniors. In this talk we will discuss the design and goals of these courses along with some lessons learned from several years of experience. (Received September 16, 2019)

1154-II-1922 **Sarah L. Marsh*** (sarah.marsh@okbu.edu). *"When will I ever use this?" Incorporating Student Research into an Introductory Linear Algebra Course.* Preliminary report.

The variety of students' majors in an introductory Linear Algebra course can make presenting a sufficiently diverse, yet engaging and relevant, array of course content applications a formidable task for instructors. In this talk, I will discuss an introductory Linear Algebra student research project that aims to develop mathematical research and presentation skills as well as an appreciation for the relevance of Linear Algebra content in students' personal and/or professional lives. I will share a description of and motivation for the project, as well as some student feedback on the experience of researching and presenting mathematical applications. (Received September 16, 2019)

1154-II-2098 **Kelli M Karcher*** (kkarcher@vt.edu). *Creating Combinatorial Curiosity: Student Projects in an Applied Combinatorics Class.* Preliminary report.

How do we engage a variety of students from diverse academic disciplines in abstract mathematics? We will discuss the benefits of allowing students to research the intersection of Applied Combinatorics with their academic interests to create a research project topic. Furthermore, we will consider the challenges in constructing a research project that is appropriate for a variety of project topics, and is stimulating and motivates students without being overwhelming. (Received September 17, 2019)

1154-II-2630 **Cayla D. McBee*** (cmcbee@providence.edu). *Research in the Discrete Math and Combinatorics Classroom.*

Discrete Mathematics and Combinatorics are courses that lend themselves to the exploration of a wide range of topics. Students from a variety of majors at Providence College enroll in these courses and the vast majority of these students have no prior experience with mathematical research. In an attempt to expose a larger number of students to undergraduate research in mathematics I have incorporated a semester long research project into these classes. This talk will discuss the projects as well as responses from students. I will also provide reflections on my experiences mentoring these research projects. (Received September 17, 2019)

Making Business Calculus Relevant

1154-I5-161 **Leslie Jones*** (ljones@ut.edu), FL 33606, and **Britney Hopkins** (bhopkins3@uco.edu). *Strengthening the Connection between Business Calculus and Business Courses*. Preliminary report.

We present current trends in mathematics requirements for business programs and how mathematical concepts are incorporated into business content. First, we look at undergraduate business programs at our peer institutions, which cover a cross section of public and private colleges and universities. We provide information on the required mathematics course(s) to include title, placement in the curriculum, and minimum grade. Secondly, we present a curriculum map of key topics in business calculus to business courses, such as intermediate economics, based on surveys of business faculty at our institutions. We share how our findings are guiding the restructuring of our business calculus course, informing our recommendations to the business school, and enabling mathematics faculty to speak with more confidence about the relevance of what they are teaching. (Received August 16, 2019)

1154-I5-967 **Stephanie R. Branham*** (sbranham@ut.edu). *Assessing Preparedness for Business Calculus*. Preliminary report.

In order to successfully apply concepts learned in Business Calculus to future business-focused courses, students must possess pre-requisite computational skills. This discussion will examine pre-requisite courses at universities across the country, and will also assess student success rates at the University of Tampa. (Received September 12, 2019)

1154-I5-1253 **Mike May*** (mike.may@slu.edu). *A complete CRAFTY inspired Business Calculus Course*.

Saint Louis University teaches a one-semester calculus course for business students using a locally developed open source text following the recommendations of the MAA Curriculum Project report of 2004. Such a business calculus should:

- Use a spreadsheet as the basic computational engine.
- Give greater emphasis to numerical methods, modeling, and conceptual understanding.
- Make the connections between math and business courses more obvious to the students.
- Business examples, terminology, and notational conventions are used whenever possible.

The text is business centered, available electronically, and assumes students have laptop computers and internet access in the classroom. The business faculty was surveyed early in the process to see how their desires varied from what was presented in the standard texts. There are ongoing discussions between math and business to foster better cooperation. Resources were developed to help instructors, mainly adjuncts, adjust to methods that are not typical in math classes. The text has been used for several years now for about 5 sections each semester. Materials are available for use at other institutions. There is a current effort to convert most homework to WeBWorK format. (Received September 14, 2019)

1154-I5-1710 **Wojciech K Kossek*** (wojciech.kossek@du.edu), Dept of Mathematics, University of Denver, Knudson Hall, Room 300, 2390 S York St, Denver, CO 80208. *Real life projects in Business Calculus*. Preliminary report.

Calculus for Business and Social Sciences was redesigned at University of Denver, following feedback from students and from other departments. Real life projects were introduced. For example, students are expected to conduct surveys and to construct their own demand function for a product, based on the results of their surveys. This brings a rather abstract notion to life and helps our students have a more meaningful experience in the course. They seem to appreciate how mathematics and calculus in particular can be relevant to them. This and other projects shall be discussed along with some other lessons learned along the way. (Received September 16, 2019)

1154-I5-2169 **Steven Hetzler*** (smhetzler@salisbury.edu) and **Robert Barber**. *A Cooperatively Designed Decision-Focused Approach to Business Calculus*.

In this talk, we report on a cooperative effort of a mathematician with 25+ years of experience teaching calculus and a founder and CEO of a large engineering organization, whose career gives him necessary perspective in applying calculus to business decisions. We have developed a new and different holistic approach to teaching Business Calculus, focused on business decisions, with a spiral structure to the content. Highlights include a wealth of business applications including a business model and new elasticities from calculus motivation, as well as early exposure to calculus applications, early introduction to direction/concavity and turning/inflection

points, and spiraling the content so that we review calculus fundamentals often throughout the semester. We will also provide a preview of sonification resources and other interactive technological tools under development, which will be available Spring 2021. (Received September 17, 2019)

1154-I5-2366 **Thomas Milligan*** (tmilligan1@uco.edu), 100 N University Dr, Box 129, Edmond, OK 73034, and **Bradley J Paynter** (bpaynter@uco.edu), 100 N University Dr, Box 129, Edmond, OK 73034. *Using metacognitive techniques to help students learn from their first business calculus exam*. Preliminary report.

Students often arrive in business calculus courses unprepared for what they will face. This unpreparedness includes deficiencies in their algebraic background, but more importantly it includes deficiencies in students' ability to problem solve, to search for answers, and to critically analyze their mistakes. Metacognitive techniques have been shown to help students overcome these "soft skill" deficiencies in many situations. In this presentation we will detail the implementation of some of these techniques in a business calculus class at a large, public, teaching university. We will also present some preliminary results. (Received September 17, 2019)

1154-I5-2468 **Deborah L. Gochenaur***, Mathematics Department, Shippensburg University, 1871 Old Main Drive, Shippensburg PA 1725, PA 17257, and **Ji Young Choi**, Mathematics Department, Shippensburg University, 1871 Old Main Drive, Shippensburg PA 1725, PA 17257. *A Peek into the Minds of Business Calculus Students*.

DFW rates in Applied Calculus were on the rise and students who passed this course had a low graduation rate; students identified as at-risk were more likely to fail and need to re-take the course. Faculty consensus on a number of key components of the course, both with respect to documents and policies, enabled us to modestly lower the DFW rate and greatly improve the overall performance of students through a course redesign. Through this process, we found a strong link between student mindset, first exam performance, and final course grades; this, in turn, informed the course redesign process. A combination of approaches was used to address the overall issues with the course, helping students build better study and learning habits of mind, and working with faculty to include more research based pedagogical methodologies in order to build students' overall conceptual understanding. Resources on growth mindset, teaching students to learn, and active learning were used for the course redesign. The combination of mindset analysis and determining course grade component correlations is crucial to ferreting out the issues in an individual course and/or across an entire department. (Received September 17, 2019)

Mathematical Experiences and Projects in Business, Industry, and Government (BIG)

1154-J1-1003 **James H Fife*** (jfife@ets.org), 660 Rosedale Road, Princeton, NJ 08541. *A Learning Progression for Geometric Transformations: From Research to Validation*.

A learning progression is "a sequence of successively more complex ways of thinking about an idea that might reasonably follow one another in a student's learning" (Smith, Wiser, Anderson, Krajcik, & Coppola, 2004). In this talk, I will present a learning progression for geometric transformations that is based on research that demonstrates the importance of viewing transformations as functions of the plane (Fife, James, & Bauer, 2019). The five levels of the progression reflect a student's evolving understanding of transformations as functions and their evolving understanding of the domain of these transformations as functions. I will also demonstrate how an analysis of cognitive interview data, conducted as part of a larger NSF-supported project, was used to validate (or not) the learning progression. (Received September 12, 2019)

1154-J1-2091 **Julianna Bernardi** (bernardij1@wit.edu), **Vincent Filardi*** (vincent.filardi@gmail.com), **Malorie Finch** (malorief1@gmail.com), **Tanvi Shinkre** (tanvi.shinkre@berkeley.edu) and **Jeffrey Zhang** (jzhan154@jhu.edu). *General Electric Aviation Material Wear Data Analysis*.

The quality control of hundreds of thousands of airplane engines flying around the globe depends on our understanding of the material wear variability in the various engine components. Herein we use data projection techniques and supervised learning algorithms on a GE Aviation dataset to predict material wear on critical engine parts and to determine the key factors that influence wear. Such a model would allow engine designers, material scientists, and GE Aviation to continue to set the standards and design airplane engines to reduce material wear and produce longer-lasting engines. (Received September 17, 2019)

1154-J1-2379 **Julia Eaton*** (jreaton@uw.edu), School of Interdisciplinary Arts & Sciences, 1900 Commerce Street, Box 358436, Tacoma, WA 98402. *Starting small: developing consulting experiences for mathematics students.*

Students often struggle to articulate how their mathematical skills transfer to the real-world setting. An internship is a wonderful way for a student gain real-world experience, but internships are not always easy to line up. At our institution, we focus on identifying, developing and facilitating small-scale mathematical/statistical consulting experiences for students. We give a broad set of principles for developing such experiences as well as specific examples. (Received September 17, 2019)

1154-J1-2393 **Travis Shrontz*** (tshrontz@arete.com), Huntsville, AL, and **Seth Zuckerman, Timothy Klein, Alexander Boxer** and **Amanda Steck**. *The ALIEN Network: An Automatic Target Recognition Algorithm.*

In recent years, it has become increasingly important in the defense and security domains to conduct surveillance, reconnaissance, and intelligence-gathering via powerful camera systems, usually airborne and for extended periods of time. In particular, High Altitude Wide Area Motion Imagery (HA-WAMI) systems have produced a copious amount of data making prediction, but this data has been historically difficult to analyze in real time.

In order to address this, Areté has designed an efficient and robust deep learning algorithm, based on classical detection theory: the Attribute Localization and Instance Extraction Neural (ALIEN) Network. When we pair this network with the novel use of masking functions to remove non-target inferences from the overall network loss, we obtain a deep-learning network that performs each of the three operations of (1) detect, (2) localize, and (3) characterize in a single forward pass with high accuracy in seconds. In this talk, we describe the basic architecture of this network and summarize its performance when applied to publicly available datasets. We also summarize some of the challenges and future work related to this problem, in particular in the domain of national defense. (Received September 17, 2019)

The Mathematical Knowledge of Teachers as an Integrated Application in Core Mathematics Courses

1154-J5-1312 **James Stuart Tanton*** (stanton.math@gmail.com), 5033 E Turquoise Ave, Paradise Valley, AZ 85253. *Seriously: Why is negative times negative positive? Reflections on a META-Math annotated lesson plan.*

Here's an age-old question: Why is negative times negative deemed positive?

The universal “inner workings” of arithmetic in school mathematics are often overlooked. School students are usually presented with different ad hoc models to motivate various arithmetical operations—the product of negative numbers, the distributive property of multiplication over addition, for instance—and the idea that there are fundamental common structures to these models is buried. It is not until an advanced undergraduate course—perhaps an Abstract Algebra or a Proofs course—that students are invited to explore the questions of why arithmetic works the way it does.

But can our undergraduates, masters of ring and field operations, explain why the product of two negative numbers is sure to be positive in the real number system? Can they themselves answer the age-old question?

In this presentation we outline one annotated lesson plan developed MAA's META Math, the “Mathematical Education of Teachers as an Application of Undergraduate Mathematics” project. This project works to create undergraduate curriculum materials that highlight secondary-level mathematics teaching as a valuable, if not vital, application of undergraduate mathematics. (Received September 14, 2019)

1154-J5-1330 **James A. Mendoza Álvarez*** (james.alvarez@uta.edu), **Andrew Kercher** (andrew.kercher@mavs.uta.edu) and **Kyle Turner** (kyle.turner2@mavs.uta.edu). *On the Development and Effectiveness of Tasks Focused on Analyzing Student Thinking as an Application for Teaching in Abstract Algebra.*

The Mathematical Education of Teachers as an Application of Undergraduate Mathematics (META Math) project developed lessons connecting content from an undergraduate abstract algebra course to related secondary school content that prospective secondary mathematics teachers (PSMTs) may eventually teach. This explicit integration of applications to teaching on par with other applications (e.g. physics) responds to recommendations from the Conference Board of the Mathematical Sciences in its publications on the mathematical education of teachers. These recommendations call for PSMTs to experience and understand the underlying mathematical connections between advanced mathematical content required for their undergraduate degrees and content they

will teach. META Math lessons integrate student-thinking tasks (STT) that require undergraduates to analyze and explain student work presented in the task. The STT have been field tested at several universities. We will outline the development and refinement of these types of tasks for an undergraduate abstract algebra course based upon participants' work and instructor and participant interviews. We will also discuss the effectiveness of the STTs as perceived by undergraduates who engaged in these tasks. (Received September 17, 2019)

1154-J5-1562 **Tamas Szabo*** (szabot@uww.edu), 800 W Main St, Dept of Mathematics, Whitewater, WI 53190. *Meaningful content for high school teachers.*

Excerpts will be shared from a textbook in preparation, written for a two semester capstone mathematics course to connect the undergraduate curriculum to high school teaching. The book aims to connect different mathematical areas, and introduce new content that is useful for teachers but not covered in any other courses. The presented sample activities and problems from Discrete Mathematics, Geometry, and Number Theory are both challenging and motivating for future teachers. They can easily be integrated into standard content courses. (Received September 16, 2019)

1154-J5-1765 **Scott Zinzer*** (szinzer@aurora.edu). *Explicit Teaching Connections in Elementary Number Theory.* Preliminary report.

A course in Elementary Number Theory offers many rich connections to the K-12 mathematics curriculum. In this talk, I will describe the inclusion of explicit teaching tasks, reflections, and assignments newly added to my Elementary Number Theory course for mathematics majors. These activities are designed to highlight applications to the work of high school mathematics teachers and to encourage future secondary teachers to seek connections to the elementary mathematics curriculum. At the same time, the activities require an added depth of mathematical thinking that benefits every student. In addition to sharing sample activities, I will give an overview of student reactions to the activities and the impacts on student learning and engagement. (Received September 16, 2019)

1154-J5-1852 **Elizabeth G. Arnold, Elizabeth A. Burroughs*** (burroughs@montana.edu) and **Elizabeth W. Fulton.** *Five types of connections addressing mathematical knowledge for teaching secondary mathematics.*

Robust preparation of future secondary mathematics teachers requires attention to the acquisition of mathematical knowledge for teaching. Many future teachers learn mathematics content primarily through mathematics major courses that are taught by mathematics professors who do not specialize in teacher preparation. What curricular supports could exist to assist mathematics professors in making connections between the content of undergraduate mathematics courses and the content of secondary mathematics? We present an articulation of five types of connections that can be used as part of secondary mathematics teacher preparation. (Received September 16, 2019)

1154-J5-2047 **Yvonne Lai*** (yvonnexlai@unl.edu), **Jeremy Strayer** (jeremy.strayer@mtsu.edu) and **Cynthia Anhalt** (canhalt@math.arizona.edu). *Purposeful use of simulations of teaching practice to uncover Mathematical Knowledge for Teaching.*

Our project (MODULES²) has developed curriculum materials that incorporate simulations of secondary teaching practice (SoPs) for use in university mathematics courses. (MODULES² stands for Mathematics of Doing, Understanding, Learning, and Educating for Secondary Schools.) Our purpose is to address teachers' perceived disconnect between undergraduate mathematics and their future teaching practice. SoPs are written and video assignments where pre-service secondary teachers respond to a description of a teaching situation, including sample secondary student work. In this presentation, we provide examples of these SoPs, the kind of mathematical work that the teachers have done prior to being assigned these simulations, the feedback that they receive, and how these fit together to help the course instructor see opportunities for teachers to refine their MKT. We also discuss how lessons learned about designing SoPs and the curriculum materials via 2 years of Plan-Do-Study-Act cycles, a heuristic for documenting tests of change. Through these cycles, we as project developers examine how instructors have used the materials, what pre-service teachers have produced, and what this tells us about improving and clarifying curriculum materials, including SoPs. (Received September 17, 2019)

1154-J5-2542 **Cody L Patterson*** (codypatterson@txstate.edu), Department of Mathematics, Texas State University, 601 University Drive, San Marcos, TX 78666, and **Christopher Duffer** (christopher.duffer@utsa.edu), Department of Mathematics, University of Texas at San Antonio, One UTSA Circle, San Antonio, TX 78249. *Building connections to secondary algebra teaching in a course on ring and field theory.*

The study of ring and field theory at the undergraduate level provides rich opportunities for students to reflect on procedures and problem solving strategies often taken for granted, such as solving polynomial equations by factoring, searching for rational roots of polynomials, and identifying and using conjugates of algebraic numbers to rewrite expressions. In this talk, we will discuss some key opportunities to develop secondary algebra teaching as an application of abstract algebra. We will also share some pedagogical strategies we have used to encourage students to make connections between secondary and abstract algebra, and some tensions we have uncovered along the way. (Received September 17, 2019)

Mathematics and the Arts: in the Classroom and Beyond

1154-K1-122 **Anduriel Widmark*** (anduriel@andurielstudios.com), Denver, CO 80211. *Design Principles of Hexastix and Related Homogeneous Cubic Cylinder Rod Packings.*

Hexastix and Tetrastix are periodic non-intersecting arrangements of cylinder packings. Some basic arrangements of congruent cylinder packings that are restricted to only three and four directions are described. The design and construction of intricate models is illustrated using simple and economic tubular building materials. Complex structures are produced from line segments that are tensioned into parallel groups with rubber bands. Many of these packing constructions provide a variety of options in the way of coloration, shape, and structure, that allow for creativity and innovation. Interesting combinations can be made and used to highlight a variety of geometric problems such as; calculating packing density, intersecting prisms to create polyhedrons, and relationships to crystal structures. These intricate geometric forms present a challenging assembly, even after you understand the space filling structures. Building these sculptures and models is a fun way to get creative and use mathematics to explore symmetry and build spatial intuition. (Received August 10, 2019)

1154-K1-445 **Susan Happersett*** (fibonaccisusan@icloud.com). *Mapping Cubic Lace Drawings.*

This presentation will illustrate my process for generating 2-D projections of 3-D cubic lace patterns. Algorithms guide the connections between sets of points around the four sides of a rhombus. I have generalized the algorithms to adapt to rhombi with varying numbers of perimeter points. This allows me to create the illusion of stacking the concentric cubes to create more intricate lattices. These cubic forms become the building blocks to produce a tiled flat surface that seems to both protrude off the plane and recede into the plane. These Cubic Lace Drawings are a continuation of my exploration into the aesthetic qualities of geometric figures using bijective and non-bijective mapping processes. (Received September 04, 2019)

1154-K1-607 **Margaret Kepner*** (renpek1010@gmail.com). *Visualizing Integer Sequences – Part 2.* Preliminary report.

The patterns within integer sequences can be revealed by displaying them in various visual formats. One example is the Ulam Spiral, where the prime numbers are plotted on a square spiral, producing unexpected patterns. This approach can be generalized to other sequences and formats. In the past, I have explored various grid layouts, relying primarily on color to plot the integers belonging to a particular sequence. More recently, I have expanded my vocabulary to include variations in orientation, shape, and texture. I will show how several of the resulting images relate to pieces of modern art. Finally, I will explain how I have used patterns generated by integer sequences in my own artistic work. (Received September 08, 2019)

1154-K1-777 **Ingrid Daubechies***, ingrid@math.duke.edu, and **Dominique Ehrmann**. *Collaborative sculptural art installation using textile and other media to illustrate mathematical creativity.* Preliminary report.

The sculptural art installation *Time to Break Free*, by one of us (DE), inspired the other (ID) to explore a possible similar installation, with a mathematical theme. In the narrative arc of *Time to Break Free*, a steampunk machine enables characters featured in a quilt to break free and become three-dimensional. The proposed new sculptural installation would illustrate how mathematical concepts get abstracted, starting from observations of the world around us, then get processed, and ultimately can give rise to completely different objects, constructions, adventures. The presentation will explain our joint vision of this proposed endeavor, and present a first tentative model. We hope that many mathematicians interested in art will join us in this project

– in its conceptual phase (determining in greater detail all the aspects of what will be built), and in its concrete realization, during the calendar year 2020.

The completed installation is expected to contain many components, organized into one narrative theme. The total finished size would be (approximately) 14 ft. wide, 30 inches deep (or more), and 7 to 10 feet high. (Received September 10, 2019)

1154-K1-854 **Rosanna Iembo*** (rosannaiembo@libero.it), **Irene Iaccarino** (irene.iaccarino@hotmail.it) and **Maria Rosaria Iaccarino** (miriamiaccarino@hotmail.it). *A NEW DAWN By 'narrative voice' of Leonardo da Vinci.*

In this paper the authors want to tell the harmony of art and science in the precious union of the friar and mathematician Luca Pacioli, and of the genius of the Italian Renaissance, Leonardo da Vinci, whose collaboration found its maximum expression in the "Divine Proportion". But they emphasize that art and science have ancient roots to which all drew: the ever-living roots of Pythagorean thought (Received September 11, 2019)

1154-K1-944 **Randall E. Cone*** (recone@salisbury.edu), 128 Henson Hall, Salisbury University, Salisbury, MD 21801. *Mathematics, Consistency, and Beethoven.*

Mathematics may be seen as an attempt to investigate the nature and content of certain types of self-consistent systems. In this session, we explore self-consistency within the art of Beethoven by examining a series of related mathematical visualizations. (Received September 12, 2019)

1154-K1-1199 **Douglas Dunham*** (ddunham@d.umn.edu), Department of Computer Science 320 HH, 1114 Kirby Drive, Duluth, MN 55812-3036, and **Lisa Shier** (kwajshier@yahoo.com), 102 Lazy Oak Dr., New Market, AL 35761. *Using Papercrafting to Create Hyperbolic Patterns.* Preliminary report.

We have been experimenting with different technologies for creating hyperbolic art, concentrating on patterns in the Poincaré circle mode of hyperbolic geometry. In the past we have used a computer controlled embroidery machine to good effect.

We are currently investigating the use of a computer controlled plotter/paper-cutter to create a pattern of shells inspired by M.C. Escher's Regular Division Drawing number 42. This drawing is mathematically interesting in that the apparent 4-fold rotation centers of scallop shells really only have 108 degree rotational symmetry, and there are two kinds of 4-fold rotations at the meeting points of the conchs. In our hyperbolic version, we preserve the 2-fold scallop meeting points and have two kinds of 5-fold conch rotation centers, which we will show.

Some schools have computer controlled plotters and/or paper cutters, so similar projects would be accessible to those students. Digital cutters are both less expensive and easier to use than computerized embroidery machines. Papercrafting is a way for primary and secondary school students to experience the creation of mathematical art. The current marketing of papercraft to females suggests that it could be an excellent way to engage girls in both math and mathematical art. (Received September 13, 2019)

1154-K1-1300 **Douglas McKenna***, doug@mathemaesthetics.com. *The Art of New Generalized Hilbert Curves: Pinwheel, Mandala, Meta-Hilbert, and Tendril-Based Half-Domino Curves.*

Hilbert-style space-filling curves, based on an $n \times n$ recursive subdivision of a square, continuously fill that square, corner to adjacent corner, yielding a linear border. Using a perturbation technique based on these square-filling, order- n generalizations, we can generalize further to create infinite sequences of eye-catching pinwheel or mandala curves that are tiles whose borders' fractal dimension can, in theory, be made arbitrarily close to 2.0. After confronting several interesting combinatorial constraints that arise, we show a new meta-Hilbert construction: an infinite sequence of rotationally symmetric pinwheel curves whose borders converge to a set of fourteen Hilbert Curves. We then show how the Hilbert Curve generalizations that fill a unit square are actually degenerate cases of "half-domino" curves, which fill exactly half a 1×2 domino. Their self-negative borders are self-similar, infinitely detailed, yet almost-everywhere linear (i.e., non-fractal). Many are visually beautiful and some are reminiscent of indigenous folk art designs. These new results will be presented visually and interactively using the speaker's recently published, combined app/eBook *Hilbert Curves: Outside-In and Inside Gone*. (Received September 14, 2019)

1154-K1-1438 **Maria C. Mannone*** (mariacaterina.mannone@unipa.it). *Klein Concert: A Report from a Math-Musical Classroom Experience.*

Can music help mathematical education? Can some shapes, that challenge visual imagination, be translated into music? I present a pedagogical application of my interdisciplinary research. This concerns a cycle of seminars about mathematics and music I gave in Italy at the Music Conservatory of Palermo and that, in a shortened version, I also gave at Ca' Foscari University in Venice, at the Conservatory of Livorno, and in the UK, at the University of Greenwich. Seminars included a theoretical section and a workshop. Mathematical concepts helped students formalize their musical knowledge, and music provided an intuitive understanding of mathematical concepts, such as the dualism continuous/discrete, the concept of envelope, and the abstraction of categories. Then, the students were given the Klein bottle, and they translated it into music. The resulting musical compositions were different but they shared general structures highlighting the bottle's features: e.g., counterpoint represented multiple simultaneous paths on the bottle, and quadrphony recreated a feeling of an additional dimension through spatialization and time variations. Musical cyclicity and inversions rendered the non-orientability of the Klein surface. The final output was a string orchestra concert. (Received September 16, 2019)

1154-K1-1522 **Matthew Lehman*** (matthew.lehman@umb.edu), University of Massachusetts Boston, 100 William T. Morrissey Blvd, Boston, MA 02125. *Resolving a Logical Inconsistency in a Framework to Model Musical Rhythms.*

We define a simple combinatorial framework called a 'rhythm' to model music, consisting of 0s, 1s and -s. The number of rhythms of length n , $r_n = F_{2n+1}$, where F_n is the n th Fibonacci number. The linear recursion defines $r_0 = 1$ which we can interpret as \emptyset . Some interesting subsets are defined and enumerated. The number of palindromic rhythms is F_{n+2} . The number of rhythms that begin or end with a 1 is F_{2n} . The number that begin and end with a 1 is F_{2n-1} . The number of cyclic rhythms $R_n = L_{2n}$ where L_n is the n th Lucas number. The linear recursion defines $R_0 = 2$ which is an apparent logical inconsistency. Reframing rhythms as a formal language resolves this issue. (Received September 16, 2019)

1154-K1-1561 **Jennifer Wilson*** (wilsonj@newschool.edu). *Dynamic Symmetry and Art.*

This talk will examine the mathematical ideas and history of Dynamic Symmetry, a geometric approach to design invented by Jay Hambridge. Dynamic symmetry was popular in the 1920s and 30s, inspiring artists and art educators, as well as math teachers and student math clubs. We will discuss Hambridge's fascination with root rectangles and their relationship to the golden ratio. We will look at the works of George Bellows and Jose Orozco whose paintings were organized using Hambridge's ideas. And we will see how Hambridge's geometric constructions lead to interesting mathematical problems for the contemporary mathematics classroom. (Received September 16, 2019)

1154-K1-1698 **Gary R. Greenfield*** (ggreenfi@richmond.edu), Math & Comp Sci, University of Richmond, Richmond, VA 23173. *Another cellular automaton for generative art.* Preliminary report.

Consider a toroidal grid where each cell has m rotors that can cycle from 0 to $n_i - 1$ for $i = 1, \dots, m$. Randomly place m painting objects on the grid with the property that the i -th object has a sequence of length n_i whose entries lie in the set $\{R, D, L, U\}$ representing (R)ight, (D)own, (L)eft, (U)p. At each time step, color the cell containing the i -th object with the i -th color, use the value of the cell's i -th rotor as an index into the object's sequence to determine where the object moves to, and then advance the cell's i -th rotor. This rotor-router model can be used to deterministically simulate random walks in the plane. We show how to formulate this process as a 2D cellular automaton and describe how to apply a genetic algorithm to this scheme to evolve drawings that achieve aesthetic goals. (Received September 16, 2019)

1154-K1-1812 **Federico Favali*** (federicofavali@gmail.com), Universidad Nacional de Tres de Febrero (Buenos Aires), and **Maria C. Mannone** (mariacaterina.mannone@unipa.it). *Mathematics and Musical Entropy.*

Arts can provide intuitive examples to enhance the understanding of complex mathematical concepts. Also, mathematics can give precise answers to musical questions that can be raised during the analysis of works of art including musical scores. If musical parameters and mathematical objects are seen as belonging to categories, analysis becomes a dialogue and a mapping between them. This paper develops an idea of "open" musicology that exploits suggestions and contaminations with other research areas, first of all from mathematics and the mathematical formalism behind physics. The focus is on the concept of entropy joined with Discrete Fourier Transforms (DFT), that can be extended to the definition of a musical entropy, as an abstract concept, as well as

a computational paradigm. The entropy can be seen as the quantification of the degree of disorder throughout the temporal evolution of musical structures of an entire musical piece. Entropy can also be considered with respect to one or more musical parameters. Their temporal evolution acquires an artistic meaning in itself, as well as the variation of its degree. A method to quantitatively evaluate the degree of entropy is presented: a new approach to the topic of entropy, that can also open new pedagogical scenarios. (Received September 16, 2019)

1154-K1-1883 **Larry G. Blaine*** (lblaine@plymouth.edu). *Music, Mathematics, and Mr. Euler.*

Like many great scientists and mathematicians, from the time of Pythagoras to the present day, Leonard Euler was intrigued by the mathematical structure of music. His writings on the subject include one major treatise (the *Novum Tentamen*) and several long papers. The purpose of this talk is to describe and demonstrate some of Euler's theories on this subject, and to relate them to the wild controversies about the nature of music that were raging in his time. (Received September 16, 2019)

1154-K1-1909 **Robert A Bosch*** (rbosch@oberlin.edu), 291 Oak Street, Oberlin, OH 44074. *Recent Progress in Opt Art.* Preliminary report.

My ongoing Opt Art project includes TSP Art (continuous line drawings produced by solving large-scale instances of the traveling salesman problem), domino portraits (mosaics made out of complete sets of dominos), and Game-of-Life mosaics. In this talk I will present numerous additional examples of Opt Art. (Received September 16, 2019)

1154-K1-2045 **Vincent J. Matsko*** (vince.matsko@gmail.com) and **Stephen M Campbell** (stephen@smcampbell.eu). *Image Processing: Sgraffito Simulation.*

Sgraffito is a technique whereby differently colored layers of some material, such as plaster or paint, are overlaid, and then an etching tool is used to reveal colors in layers underneath the surface. This process may be digitally simulated and used to posterize images. Details of the simulation will be presented; the simulation is written in Processing, and will be made available to participants. (Received September 17, 2019)

1154-K1-2050 **Carolyn Yackel*** (yackel_ca@mercer.edu), 1500 Mercer University Dr., Macon, GA 31207-0001. *Orbifolds Attainable through Itajime Shibori.* Preliminary report.

A collection of traditional forms of Japanese cloth dyeing known as shibori have recently become popular in the West. The itajime technique relies on folding the cloth and clamping it between mirrored shapes which create a resist to the dye. This investigation tackles the question: Which wallpaper patterns are achievable through itajime shibori techniques? (Received September 17, 2019)

1154-K1-2233 **Annalisa Crannell*** (annalisa.crannell@fandm.edu), F&M Mathematics, Box 3003, Lancaster, PA 17604-3003. *Anamorphic distortion: it's closer than you think.*

What is the difference between Trompe L'Oeil and anamorphic art? One difference is that we think of the former as being surprisingly realistic, and as the latter as a distortion that can morph into being realistic if we stand in the "correct" spot. But both of these perspective art genres employ the same underlying mathematics, and in that sense, one might conclude they are both equally realistic. In this talk, we offer a measure of anamorphic distortion and describe its dependence on the standard measure of viewing distance. We conclude that Trompe L'Oeil and anamorphic art are not truly different in kind, but are rather variations along a spectrum, and that the spectrum varies with relative distances of the image to the viewing target of the particular piece. (Received September 17, 2019)

1154-K1-2304 **Ciara D Bursal*** (ciarabursal@montana.edu), Montana State University, 205 Reid Hall, Bozeman, MT 59717, **Monte S Meyerink** (montemeyerink@montana.edu), Montana State University, 442 Reid Hall, Bozeman, MT 59717, and **Fenqjen Luo** (fluo@montana.edu), Montana State University, 118 Reid Hall, Bozeman, MT 59717. *An Outlet for Creativity in Mathematics.* Preliminary report.

The purpose of this activity is to see if middle and high school students can connect their knowledge of polygons and art. Students are asked to create unique pieces of art, comparable to quilts, that contain several different shapes, mainly polygons. Students are encouraged to create shapes whose areas can be found without using a measuring tool. We are interested in seeing how many students choose to create a design that needs a measurement tool versus a design whose area could be found without a measuring tool. (Received September 17, 2019)

1154-K1-2374 **Susan Goldstine*** (sgoldstine@smcm.edu) and **Carolyn Yackel**. *Color Swaps in Mosaic Knitting*.

Mosaic knitting, a form of two-color knitting developed by designer Barbara Walker in the 1970's, has gained popularity because it is easier for the knitter than most traditional forms of color work. The price of this ease is an unusual set of restrictions on color placement, and the core of our work is a classification of the color-swapping symmetries that are possible within these constraints. (Received September 17, 2019)

1154-K1-2438 **Zachary Treisman*** (ztreisman@western.edu), Department of Mathematics, Western Colorado University, 1 Western Way, Gunnison, CO 81231. *The vague boundary between geometric models and mathematical art: Examples from the geometry classroom to the sculpture gallery*.

Making a model is a useful tool in understanding many geometric ideas. Many of the models we create are interesting even to non mathematicians. Depending on the materials used and other design choices, the line between what is a geometric model and what is art is not always distinct. I present and discuss a selection of examples from my ongoing collection of student projects in geometry classes, from the instructional models that I use in teaching, and from my collaborations with the artist London Tsai. (Received September 17, 2019)

1154-K1-2533 **John Nicholson*** (nicholsonja@apsu.edu). *Curve stitching variations in 2D and 3D*.

Curve stitching was invented by Mary Boole in the 1800s when she used thread to create new types of patterns on sewing cards. Over 100 years later, curve stitching images are now often rendered using software, which allows for renderings of both 2D and 3D images. Even after all this time, though, and even with the addition of computational tools, curve stitching images are still almost exclusively created using short line segments. In this talk, I demonstrate variations that extend the basic principles of curve stitching in which line segments are used to connect two points moving at different rates along a path or shape such as a circle. Rather than using line segments to connect the moving points, this new method uses curves, shapes, and surfaces to connect the points. The resulting images from these variations are visually rooted in the traditional curve stitching approach while simultaneously increasing its possibilities. (Received September 17, 2019)

1154-K1-2559 **Samaneh G. Hamidi*** (shamidi@rider.edu), Department of Mathematics, Rider University, Lawrenceville, NJ 08648. *Mathematics in Persian Art*.

Persian left innumerable marks on the civilizations and cultures of human beings, dating back to ancient times. From the first scientific notions of astronomy and the measurement of the celestial sphere and determination of the new year according to the vernal equinox, to the mathematics of shapes, solids, and numbers. Also, Persian arts have played important roles. Persian workers have combined skills in theoretical mathematics and practical techniques such as ceramics, along with artistic ideas from their own history and around the world. Putting these together, they have expressed several appealing forms, which bridge mathematics and the arts. The purpose of this talk is to get a glimpse of Mathematics in Persian arts in different eras and also to present a more detailed visual analysis of a ceramic design. (Received September 17, 2019)

1154-K1-2585 **Karl H. Schaffer*** (karl_schaffer@yahoo.com), 325 Lucinda St., Scotts Valley, CA 95066. *Dancing the Quaternions*.

Swirling movements, popular among contemporary dancers and choreographers, often embody double rotations of the limbs similar to the Dirac plate or belt trick, in which an object attached to a stationary body by a flexible cord returns to its original state after a rotation of not 360 degrees but 720 degrees. This phenomenon is also seen in the Balinese candle dance, baton twirling, poi, and other performance forms. It is efficiently modeled by the quaternions and illustrates the mathematical theorem that the group $SU(2)$ double covers the rotation group $SO(3)$. We will look at how this plays out in dance and other performing arts, and how comprehending the embodiment of the quaternions helps us better understand both the mathematics and the relevant movement arts. (Received September 17, 2019)

1154-K1-2743 **David A. Reimann*** (dreimann@albion.edu), Math/CS Department, 611 E. Porter St., Albion, MI 49224. *A retrospective of creating five years of cover art for Mathematics Magazine*.

For the years 2015 through 2019 I was invited to create cover art for the MAA journal *Mathematics Magazine*. For each issue, an original artwork was created to reflect some aspect of the issue. With a publishing schedule of five times a year, this resulted in a body of twenty-five artworks. Challenges of creating artwork included the small 5 inch by 5 inch space and the limited color palette of grayscale plus the background color. A discussion of a selection of these artworks will be made, including mathematical background and other inspirations. (Received September 17, 2019)

1154-K1-2775 **Mara Alagic*** (mara.alagic@wichita.edu). *Learning mathematics through mathematical modeling of crafts.*

An ongoing interdisciplinary longitudinal study focused on developing math-related pedagogical content knowledge (Ball, 2000) in the context of visual and creative digital environments resulted in a collection of projects clustered around different themes. The cluster that this abstract is focusing on is a result of preservice elementary teachers creating mathematical representations/models of everyday crafts, actually crafts from a culture different from their own.

The resulting research presentation will take you through a gallery of bridges between mathematics and the arts/crafts in three ways: First, a selected set of students' constructed mathematical models of crafts will be presented; Second, the sample of students' interpretations/reflections will be included; and Finally, data analysis of students' mathematical thinking about their own learning processes and changing (or not) perceptions about ways in which they intend to teach while nurturing creative ways of understanding/doing math. (Received September 17, 2019)

Mathematics and Sports

1154-L1-45 **Janet Liu*** (janl@microsoft.com), **Garrett Lucash** (garrett@acskating.com) and **Diana S Cheng** (dcheng@towson.edu). *Envisioning possibilities: Visual analytics in figure skating.*

This presentation describes how mathematical modeling and computer software can be used to solve key problems in figure skating to inform athletes, coaches, and policy makers, including: 1) Based on past competitors' scores, what jumps were needed at certain levels for athletes to qualify for Nationals? 2) Based on previous competition entries, what geographic boundaries should be set for future collegiate competitions? (Received July 17, 2019)

1154-L1-497 **Reza O Abbasian** (rabbasian@tlu.edu), Texas Lutheran University, Dept of Math-CS, 1000 W. Court St., Seguin, TX 78155, **John T Sieben*** (jsieben@tlu.edu), Texas Lutheran University, Dept of Math-CS, 1000 W. Court st., Seguin, TX 78155, and **Oakley Saint-Vincent**. *Application of Grey Model to Predict the Results and Detect the Anomalies of Olympic Track and Field Events*. Preliminary report.

In this presentation, we will use the first Grey method to model the results of Olympic track and field events. Grey method relies on certain combination of elements of a discrete time series to create a smoother sequence. Specifically, we will be utilizing our model to predict and compare the winning results for the Seoul Olympics and show that the actual results were highly improbable which in turn led to the establishment of the World Anti-Doping Agency to enhance the reliability of tests for detection of performance enhancing drugs for the subsequent Olympics. Five years ago, we attempted this project using a biased least squares approach to develop exponential regression models to predict the winning results. In this talk we will give a brief description of the Grey method, compare our results based on the Grey model to the least squares approach and show that the Grey method compares favorably with our previous approach. We will also use the known results of the Rio Olympics to check the validity of the Grey method and then will use the method to predict the results of the Tokyo Olympics. (Received September 05, 2019)

1154-L1-664 **John A David*** (davidja@vmi.edu). *Decision Forest, Partial Dependency and Matching Method Analysis of NFL Play-by-Play Data in 2018.*

There have been a series of articles and studies in both mainstream media and analytics sources questioning the play calling of NFL coaches, particularly their choice to run versus pass. In this talk, we will use all individual plays in 2018 and decision forest and partial dependency approaches to analyze this question in more detail. Combining these approaches with the expected points added in each play will allow us to quantify the value of run versus pass in a variety of situations, including down, distance to go, game margin, etc. To compliment this analysis we will use a similar approach to evaluate the factors that influence this decision to run versus pass and compare their current decisions with the expected benefits. Finally, we will use a matching approach to further investigate key situations discovered by the partial dependency analysis, including goal to go from inside the opponents' 3 yard line, first and ten and inside the opponent's red zone. (Received September 09, 2019)

1154-L1-703 **Andrew B Perry*** (aperry@springfieldcollege.edu). *Pythagorean Expectation As Predictor in Major League Baseball.*

In its original form, Bill James' Pythagorean Model posits that a major league baseball team's winning percentage will be approximately equal to its "Pythagorean Winning Percentage" (PWP), which is defined PWP=

$RS^2/(RS^2+RA^2)$, where RS = runs scored and RA = runs allowed. Considered at some point during a baseball season (such as the All Star Break) taking a weight, PWP tends to be a slightly better predictor of future winning percentage than ordinary winning percentage (OWP). Might it be possible to obtain a more accurate predictor of late-season success by taking a weighted average of PWP and OWP? Or perhaps by weighting the wins, losses, runs scored and runs allowed of more recent games more heavily? We consider the general question of how best to predict a baseball team's future success based solely on the scores of the games played to date in the current season. (Received September 09, 2019)

1154-L1-711 **Chris Watkins*** (watki115@mail.chapman.edu), **Cyril Rakovski** and **Vincent Berardi**.
Pitcher Effectiveness: A step towards in game analytics and pitcher evaluation.

With the introduction of Statcast in 2015, baseball analytics have become more precise. Statcast allows every play to be precisely tracked and the data it generates is easily accessible through Baseball Savant, which opens the opportunity for improved performance statistics to be developed. In this paper we propose a new metric, Pitcher Effectiveness, that uses Statcast data to evaluate starting pitchers dynamically, based on the results of in-game outcomes after each pitch. The Pitcher Effectiveness metric successfully predicts instances where starting pitchers give up several runs, which we believe make it a candidate for the in-game evaluation of starting pitchers. In addition, Pitcher Effectiveness is consistent with other established metrics (e.g., ERA), which suggests that it can also be used to evaluate a pitcher's start as a whole. (Received September 09, 2019)

1154-L1-910 **Carly Berry*** (cberry6@students.towson.edu) and **Diana S Cheng**
(dcheng@towson.edu). *Jumping Circles: Developing fraction sense through kinesthetic activities.*

We present a set of elementary level kinesthetic activities about fractions. Using jumping within hula-hoop circles marked with fraction partitions, students in grades K-4 solved problems related to equivalent fractions, fraction comparisons, and area representations of fractions. Students' work on these problems is discussed. (Received September 18, 2019)

1154-L1-1468 **Isabelle Pardew***, iparde1@students.towson.edu. *MLB: Decision Making on the Corners.*
Preliminary report.

Pitching with runners on base poses a problem to some pitchers' concentration, while others are unaffected. However, when there are runners on first and third, and the runner on first takes off, there is a unique situation that warrants extra consideration: where should the catcher and pitcher direct their thought?

If they decide to target the runner on **first**, and make an errant throw, the runner on third could score. On the other hand, if they target the runner on **third** and make an error, they will still score and the runner on first will still advance. Then comes the question: should there even be a throw, and instead just focus on the batter at the plate? Such an in-game decision requires thorough research to determine the situations in which throwing through is required, optional, or entirely unnecessary. I will use Statcast, PITCHf/x, and Retrosheet data to answer several questions evaluating the risk of throwing through or holding back, and thus determine guidelines for where and when to target the baserunners.

This analysis will investigate and attempt to solidify a model describing the impact of the relationship between a catcher's throw and a pitcher's delivery with runners on the corners. (Received September 15, 2019)

1154-L1-1694 **Derek L Margulies*** (dmargu2@students.towson.edu) and **Christopher R Cornwell**.
An Application of the Mapper Algorithm to Sports Analytics in College Basketball.
Preliminary report.

We use a novel method to analyze data from college basketball games. The data are obtained from the play-by-play logs from games played by Towson University's Men's Basketball team. After initial preprocessing, a set of data points is created, one for each of several time intervals during each game, and for each on-court player. The data points come from Oliver's Four Factors for that player during the time interval. A plus/minus statistic is also calculated and associated to each point.

We visualize this data set with a network of nodes and edges, i.e. a *graph*, by using the Mapper algorithm (Singh, Mémoli, and Carlsson 2007). The development of Mapper was inspired by a construction in topology and has produced useful insights in data exploration.

In our use of Mapper, we consider points with similar Four Factor stats as part of the same node. A metric is used to add in edges as Four Factor stats vary, and the variation of a plus/minus statistic is seen as you traverse the graph. The goal is to find Four Factor profiles that consistently appear together on the court and in conjunction with a very positive or very negative plus/minus statistic. Such knowledge may be applied by the coaching staff to identify optimal lineups at key moments in games. (Received September 16, 2019)

- 1154-L1-1944 **Elizabeth L. Bouzarth*** (liz.bouzarth@furman.edu), **Andrew W. Cromer**, **William J. Fravel**, **Benjamin C. Grannan** and **Kevin R. Hutson**. *Dynamically Scheduling National Football League Games to Reduce Strength of Schedule Variability.*

The National Football League schedules games where some matchups are based on the previous year's results. Since team composition changes from year to year, this scheduling policy sometimes benefits teams unfairly, allowing some an easier path to the playoffs than others. Thus, strength of schedules vary between teams and arguments have to be made why some teams make the playoffs and others do not. We propose methods to produce an NFL schedule that combines some of its traditional elements with dynamically-scheduled games aimed at optimizing different objectives, such as reducing the variability of teams' strengths of schedules or minimizing the number of pairwise comparisons needed to differentiate team quality. (Received September 16, 2019)

- 1154-L1-2125 **Adam F Childers*** (childers@roanoke.edu), 2521 Robin Hood Rd SE, Roanoke, VA 24014. *Analyzing Shot Expectation Through Spatial Data Collection Applications.*

Understanding the quality of a shot, be it in basketball, lacrosse, field hockey, volleyball, tennis, or soccer, is a critical component of describing the performance of a team. The challenge, however, is to efficiently assess this in real-time to inform coaches' decisions. Over the last three years, the Roanoke College Stat Crew, has been creating and refining spatial data collection apps to give coaches and players in-game and post-game analysis of their performance. Through a combination of statistical modeling and discussions with coaches, players, and Stat Crew members, the apps have been constantly changing to be as efficient and illuminating as possible. This talk will detail the development of the applications, the partnership with athletic teams, the shot expectation models, and the future of customizable applications. (Received September 17, 2019)

- 1154-L1-2141 **Nicholas Gorgievski*** (nick.gorgievski@nichols.edu), Nichols College, 123 Center Road, Dudley, MA 01571, and **Brianna T Doherty** (brianna.doherty@nichols.edu), Nichols College, 123 Center Road, Dudley, MA 01571. *A Logistic Regression/Markov Chain Model for the NFL.* Preliminary report.

Paul Kvam and Joel Sokol developed a combined Logistic Regression/Markov Chain (LRMC) model for predicting the outcome of NCAA tournament basketball games. Their model was shown to be significantly more successful than other common methods, such as tournament seedings, the RPI, and Sagarin ratings. We have attempted to replicate the results by applying the LRMC method to NFL football games. In this talk, we will present our findings when applying the LRMC method to predict NFL playoff games. (Received September 17, 2019)

- 1154-L1-2682 **Jeffrey W Heath*** (jeffrey.heath@centre.edu), 600 W. Walnut St., Danville, KY 40422, and **Will Britt**, **Shelby Montgomery** and **Jackson Norris**. *Creating a High School Sports Rating System.* Preliminary report.

Sports rating systems are an important factor in the selection and ranking of teams for postseason play in major college sports, notably NCAA basketball and football. There is considerable fan interest in seeing how their team rankings rise and fall throughout the course of the season. Further, these ratings can provide predictive power in future matchups. In this talk we will discuss the development of a rating system for Kentucky high school sports. We will highlight the importance of using historical data to fit our model to this particular set of teams, and discuss the challenges of a statewide rating system on a weakly connected schedule of matchups. (Received September 17, 2019)

Modeling-First Inquiry-Based Course Activities

- 1154-L5-233 **Michael A. Karls***, Department of Mathematical Sciences, Muncie, IN 47306. *Dynamic Groundwater Flow Modeling Investigations with Excel.*

We provide an example of a groundwater flow model that can be investigated dynamically with Excel. This illustrates a means to study differential equations in a fashion similar to using commands such as Mathematica's Manipulate to vary parameters in a model. (Received August 26, 2019)

- 1154-L5-1011 **Carl Lienert*** (lienert_c@fortlewis.edu). *Using SIMIODE Scenarios to Drive a Differential Equations Course.*

On the first day of my Differential Equations course in Fall 2019 I presented students with a SIMIODE scenario, i.e. an open ended "story problem." This scenario and subsequent ones, and their solutions, determined the

trajectory of the course. Rather than follow a standard textbook curriculum, we discussed and developed in class the tools needed to model and solve the problems.

I'll discuss the results:

- Did students have a better idea of the value of differential equations?
- Were they confused by the non-textbook path?
- Were standard DE curricular goals compromised?
- Could the students propose models on their own?
- Did my colleagues in Engineering notice?
- etc.

(Received September 12, 2019)

1154-L5-1281 **Corban Harwood*** (rharwood@georgefox.edu). *Comparing Symmetrical, Asymmetrical, and Parallel Modeling Projects.*

In this talk, we compare the implementation, group dynamics, and student learning benefits for symmetrical, asymmetrical, and parallel modeling projects. For symmetrical projects, each group investigates the same problem but arrives at independent conclusions, whereas with parallel projects each group investigates a different problem. We define asymmetrical projects as having each group investigate a different problem in a common theme where each perspective is needed to finish the project. These comparisons will be fleshed out with example projects from courses in liberal arts mathematics, differential equations, numerical analysis, linear algebra, and partial differential equations. We will share best practices on implementation in the curriculum, supporting group dynamics, and benefiting student learning which come through pedagogical experiences, assessments, and student evaluations from these courses. (Received September 14, 2019)

1154-L5-1374 **John Z. Hossler*** (hosslerj@spu.edu), Seattle Pacific University, 3307 Third Ave W, Seattle, WA 98133. *What does he \bar{X} ? Thinking about statistics, not just hearing about statistics: Getting students to engage in the process of learning and doing statistics.* Preliminary report.

Drawing students into the material is important! Simply telling students about a model or statistical fact is not the best way to foster a long-term learning environment. Instead, students should be actively engaged in creating models and thinking through the underlying concepts. A focus on the big picture and reasoning behind the formulas helps develop statistical intuition. This talk will discuss examples of how I get my undergraduate introductory statistics students involved in the process of thinking about concepts and developing statistical tools, rather than simply hearing me talk about them. (Received September 15, 2019)

1154-L5-1684 **Arati Nanda Pati*** (patia@stthom.edu), Mathematics Dept, 3800 Montrose Blvd, Houston, TX 77006. *Teaching Differential Equations with Modeling First.*

Teaching methodology is evolving with modern time. Gone are those days when students were learning procedural differential equations from the analytical perspective. There is a great need to update the pedagogies to introduce modeling first to appreciate the differential equations techniques. Inspired by MINDE workshop, I introduced the modeling first approach in my classroom teaching to welcome inquiry oriented learning. Data collection, data visualization, and parameter estimation using technology has gained better understanding of math modeling using differential equations not only for Math major but also all STEM students who take differential equations as a core course. In this talk, I will present my effort to incorporate SIMIODE modeling scenarios in my differential equations and mathematical modeling classes. (Received September 16, 2019)

1154-L5-1691 **Brian J Winkel*** (brianwinkel@simiode.org). *Be sure to step out of the room – good advice for teaching modeling.*

When teaching modeling to students it is important to stand aside, to get out of the way so students can explore and let their minds wander, dream, and inquire. To be sure this happens it is appropriate to leave the room or at the very least stop “professing” and be quite, perhaps just to listen. It is also important to be accepting, never saying, “Yes, but . . .”, and be ever ready to go on a long strange trip with student ideas as they wander, perhaps (almost certainly) far off the track YOU thought they would take. It is also important to know what to do AND what not to do when strange things happen in class and to be prepared to come out of class tired, very tired, almost exhausted at times. We give examples from our 40+ years of getting out of the way of students doing modeling in the classroom. Indeed, modeling is the ultimate inquiry-based mathematics experience if we but let it happen more often in our classrooms. (Received September 16, 2019)

- 1154-L5-1843 **Bonnie S Moon*** (moonb@byui.edu) and **Paul Cox** (coxp@byui.edu). *Using Modeling in Introductory Differential Equations to Motivate Relevance, Positive Attitudes Toward Mathematics and the Transfer of Knowledge to Other Courses*. Preliminary report.

We have redesigned our introduction to ordinary differential equations course using modeling scenarios to provide context for the content. We have incorporated a Modeling, then Visualizing and finally Solving pattern in all elements of the course. The course has been divided into smaller modules which begin with students studying a modeling scenario. They work on modeling the scenario both individually and in groups. After the groups determine a model, they then apply visualizing and solving techniques in the context of the scenario. We are studying the impact this redesign has on student attitudes toward mathematics and on their ability to transfer understanding to courses in other disciplines. (Received September 16, 2019)

- 1154-L5-1894 **Terrance L Pendleton*** (terrance.pendleton@drake.edu). *Empirical Models for Tropical Storm Windspeeds After Landfall—A Modeling Activity using Separable Equations*.

We model the decay of tropical cyclone winds once a storm makes landfall. We use data from two recent storms from the National Hurricane Center to estimate parameters emanating from a differential equation using a first order exponential decay model. (Received September 16, 2019)

- 1154-L5-2021 **Eli E Goldwyn*** (goldwyn@up.edu), University of Portland, 5000 N Willamette Blvd, Portland, OR 97203. *Two Inquiry Based Learning Modeling Activities: The Warming of Cold Water in a Bottle and Using Phase Models to Describing Oscillating Populations*.

We start by introducing a modeling scenario that should be appropriate for the first week of an introductory undergraduate differential equations course. This scenario steps through the creation and understanding of a differential equation by describing how fluid in a water bottle will change temperature and what factors influence the rate of temperature change. The second modeling scenario walks the student through the process of modeling and analyzing equilibria behavior of an oscillatory system through the use of phase variables with a focus on predator-prey oscillators. (Received September 17, 2019)

- 1154-L5-2196 **Chris McCarthy*** (cmccarthy@bmcc.cuny.edu), BMCC City University of New York, 199 Chambers Street, New York, NY 10007. *Throwing a ball can be such a drag*.

If a tennis ball is thrown through the air it will eventually hit the ground due to gravity. It is common and fairly easy to model this if we neglect drag (air resistance). In this talk we discuss how to include drag. The resulting model is solved using Euler's method for higher order differential equations. This talk is based on work done at the 2019 SIMIODE DEMARC summer workshop. (Received September 17, 2019)

- 1154-L5-2502 **Reza R Ahangar*** (reza.ahangar@tamuk.edu), 729 Alexander Ave., Kingsville, TX 78363. *Stability of random Perturbed Logistic Model*.

We are going to study a model that represents the rate of changes of the population with a limited environmental resources described by, $p'(t) = p(a - b \cdot p) + g(t, p)$ where a measures the growth rate in the absence of the restriction force and a/b represents the carrying capacity of the environment and b represent restricted factor b . The random perturbation $g(t, p)$ is generated by random changes in the environment. The behavior of the solution of this model for continuous and discrete case when $g(t, p) = r \cdot p$ with a random change factor r will be studied. The stability and the behavior of the equilibrium point will also be investigated. A computational approach to simulate the solution of this random differential equation and also a regression method will be developed. (Received September 17, 2019)

- 1154-L5-2735 **Jennifer L. Garbett*** (jennifer.garbett@lr.edu). *SIR and Beyond: An Inquiry Based Approach to Modeling the Spread of Disease in an Introductory Differential Equations Course*.

In this talk, we will discuss an inquiry based modeling activity I have used in my introductory Differential Equations course. First, with some guidance, students develop an SIR (susceptible, infected, recovered) model to model the potential spread of the measles virus at Lenoir-Rhyne University. Students are then asked to consider assumptions they have made and to come up with possible modifications to their basic SIR model which could be used to improve the model. Finally, the activity culminates with a project where each student is asked to choose and implement an appropriate addition or modification to the basic model from class in order to model the spread of a disease of the student's choice among a population of the student's choice and to analyze the predictions made by their model as well as its limitations. Students present their work to the class. I will also address where this activity fits in my course and the response from students to this and other inquiry based modeling activities I have used, and I will share lessons I have learned. (Received September 17, 2019)

Quantitative Literacy and Social Justice: An Ongoing Dialogue

1154-M1-155 **Kenan Ince*** (kince@westminstercollege.edu), Westminster College, Foster Hall 311, 1840 South 1300 East, Salt Lake City, UT 84105. *Developing a social justice math course using publicly-available materials.*

Many resources exist for K-12 math educators interested in incorporating social justice into their curricula, but resources are comparatively scarce for college and university instructors. We describe the design and implementation of a course entitled Social Justice by the Numbers at Westminster College in Salt Lake City. We will describe how we developed a course structure based on student interests, synthesized social justice math modules from various sources, promoted community outreach within the course, and designed materials to create this course and promote student buy-in and learning. Special emphasis will be placed on how educators can design their own social justice mathematics courses using publicly-available materials. (Received August 16, 2019)

1154-M1-784 **Carrie Muir*** (cmuir@whatcom.edu). *When Equity is Conditional: Social Justice Topics and Contingency Tables for Teaching Probability.*

Conditional probability provides a natural lens for examining equity issues, but is often a challenging topic for quantitative literacy students. We will consider some social justice topics suitable for use with a quantitative reasoning approach to conditional probability, and look at using contingency tables to help students organize and explore probabilities. (Received September 10, 2019)

1154-M1-888 **Maria G Fung*** (mfung@worchester.edu), 486 Chandler Street, Worcester, MA 01602. *Quantitative Literacy for Pre-service Elementary Teachers in a Remedial Pilot Program.* Preliminary report.

Worcester State University has developed different pathways to help under-prepared students for success in their mathematics courses. One of these groups involves pre-service elementary teachers who are not ready for the three-semester sequence of mathematics content courses required for their major. In this talk, I will discuss the change of a general first-year seminar "What the Numbers Say—an Introduction to Quantitative Literacy" now taken concurrently with an accelerated remedial mathematics course. The capstone project in the course now focuses on social justice issues, with the hope that quantitative understanding of these issues in Worcester, MA as they relate to a variety of problems including suspension and drop-out rates in the public schools, voting rates per district, trafficking, and lack of educational opportunities for certain groups, will help my students become leaders in the needed change in elementary schools. (Received September 11, 2019)

1154-M1-930 **Aris B Winger*** (aris.winger@gmail.com), 1012 Jonathan Lane, Tucker, GA 30084. *The Possibility, Necessity and Urgency for Bayes' Rule in Elementary Statistics classes.*

Through a series of examples, some standard, some radical, some general, some deeply personal, and yet all uniformly accessible to freshman statistics students, we will present the power of Conditional probability and Bayes' theorem to be transformational. It is the goal of the presentation, through demonstrating the impact of the theorem on the nature of how we see ourselves as citizens, and the ease of alternative demonstrations of its application, to compel listeners to make the theorem and conditional probability more than just a section in an advanced probability class for math and science majors. The goal of the talk is to make the bold claim that if the purpose of college is to enlighten, and if statistics is so important that thousands of freshman and sophomores take the course every year around the country, then this theorem and topic are both accessible enough and mind altering enough for their inclusion in the elementary statistics curriculum to be demanded. (Received September 12, 2019)

1154-M1-1146 **Gary Hagerty*** (garyhagerty@boisestate.edu), Gary Hagerty, 1910 University Dr. MS 1556, Boise, ID 83725-1556. *Creating a Social Justice Pathway.*

Social Justice is defined as the fair distribution of wealth, opportunities, and privileges in society. The math community recognizes the role the community can play in improving Social Justice, when they make statements such as, "Mathematical models and structures can yield powerful insights in exploring, understanding, and even resolving a range of complex social issues and challenges."

In today's world of creating student success opportunities through developing pathways for success, the question arises, "What is a Social Justice Pathway?" When we consider underrepresented populations arriving at the University, how do we provide a successful pathway for fair distribution of wealth, opportunities, and privileges in society? (Received September 13, 2019)

1154-M1-1171 **E Cabral Balreira*** (ebalreir@trinity.edu), One Trinity Place, Dept. of Mathematics, San Antonio, TX 78201. *The mathematics of Gerrymandering in the classroom?* Preliminary report.

In the past year, we have been developing techniques to teach about Gerrymandering and fair elections in our introduction to Mathematics course. Students are asked to think about fairness and how to quantitatively characterize fairness in an apparent gerrymandered election. In this talk, we will share some of the lessons and activities we have developed to address the problem of redistricting from apportionment to metrics to detect gerrymandered districts. We will also present some projects with undergraduate students that have continued beyond the classroom. (Received September 13, 2019)

1154-M1-2256 **Patti Frazer Lock*** (plock@stlawu.edu), Dept of Math, CS, and Stat, St. Lawrence University, Canton, NY 13617. *Math and Social Justice: Some Great Class Activities*. Preliminary report.

The presenter taught a senior seminar on Math and Social Justice last year, and will teach a similar course as a sophomore seminar this year. She will discuss topics covered, lessons learned, and some in-class activities that worked particularly well. (Received September 17, 2019)

1154-M1-2296 **Lisa A Oberbroeckling*** (loberbroeckling@loyola.edu), Mathematics and Statistics Department, 4501 N Charles St, Baltimore, MD 21210. *Redlining in an Applied Calculus Course*. Preliminary report.

It has always been encouraged to incorporate the Common Text in some way into our courses at Loyola University Maryland. The recent Common Texts have led to discussions across campus on the impact of systemic racism and/or poverty. Within an Applied Calculus course, the practice of redlining, in particular within Baltimore, was introduced. Data from the Baltimore Neighborhood Indicator Alliance were explored. The issue of subprime loans along with basic compound interest calculations were incorporated into the course. This unit can be modified and incorporated into many different quantitative or financial literacy courses. (Received September 17, 2019)

1154-M1-2521 **Anne E. Yust*** (yusta@newschool.edu), 65 W 11th St, New York, NY 10011, and **Debasmita Basu** and **Carol Overby**. *Teaching Social Justice and Responsibility through a Quantitative Reasoning Framework in Two Classroom Modules*.

The conjunction of QL and social justice has long been discussed. Recently the journal Numeracy noted QR/QL as a useful tool for understanding and potentially changing injustice, and that QR/QL moves math from abstractions to tools for engagement, and inserts the personal into social issues. Our university provides unusual support for social justice issues, with a vision statement asserting, "Social engagement should orient students' academic experiences to help them become critically engaged citizens, dedicated to solving problems and contributing to the public good." With this support, we have developed numerous modules to promote social engagement in our first-year QR course, and will share two modules: one comparing crime rates in two cities, focusing on making disparate situations more comparable, and encouraging student critique of data, statements, or implications; and a second module quantifying behaviors and outcomes around plastic consumption, encouraging students to reflect critically on their own practices and on those of larger entities. We conclude with some qualitative measures of these modules' effectiveness, and hope this will inspire our colleagues to help students critically engage with the world through quantitative frameworks. (Received September 17, 2019)

1154-M1-2656 **Erin R Moss*** (erin.moss@millersville.edu), **Allison Toney** and **Caitlin L Ryan**. *When All of the Above Isn't the Answer: Examining Mathematics Pedagogy through a Lens of Social Justice*.

In our geometry courses for future teachers, we implemented an interdisciplinary literature, mathematics, and social justice project that was marked by good intentions and unintended consequences. Our students read Shelley Pearsall's novel *All of the Above*, then engaged in a related class discussion and writing assignment. The book, based on a true story, describes the attempt of inner-city middle school students to construct the world's largest tetrahedron. Beyond exploring connections between the tetrahedron and the structure of the text, our goal was for students to experience a counternarrative to traditional views of students of color by studying a novel that positions the young characters as mathematicians. Our students' verbal and written responses suggest that we fell woefully short on our second goal. Upon further reflection, we realized the traditional approach to pedagogy presented in the novel served to disempower the characters mathematically, preventing it from providing a compelling counternarrative. Where there is a challenge though, there is opportunity. As we discuss the inherent complexities in trying to teach for social justice, we share the humbling lessons we learned in

the process and propose modifications to bring outcomes more in line with our intentions. (Received September 17, 2019)

1154-M1-2809 **Samuel Luke Tunstall*** (stunstal@trinity.edu), **Victor Piercey**, **Gizem Karaali**, **Catherine Crockett**, **Mark A Branson** and **Jason Douma**. *Mathematics, Quantitative Literacy, and Social Justice: Continuing the Conversation*. Preliminary report.

Over the past year, the SIGMAA on QL has had dialogue internally and with the broader MAA community on the relationship between mathematics, quantitative literacy, and social justice. Of particular interest to us have been questions such as: What is the relationship between quantitative literacy and social justice? How do questions about ethics and data intertwine with quantitative literacy? What role should mathematicians play in attending to issues of social justice? In this presentation, we highlight interactions and synergies between these themes, as well as models and ways to forward for the SIGMAA itself. (Received September 18, 2019)

Recreational Mathematics: Puzzles, Card Tricks, Games, and Gambling

1154-M5-294 **Tricia Muldoon Brown*** (tmbrown@georgiasouthern.edu). *A bijection on maximum arrangements of nonattacking pawns*.

The problem of enumerating nonattacking arrangements of chess pieces has a long history. We briefly discuss classical results for traditional pieces such as bishops, knights, and rooks, and, using a bijective proof, we show the number of ways to arrange a maximum number of nonattacking pawns on a $2m \times 2m$ chessboard is $\binom{2m}{m}^2$. (Received August 29, 2019)

1154-M5-673 **John M. Harris*** (john.harris@furman.edu), Department of Mathematics, Furman University, 3300 Poinsett Highway, Greenville, SC 29613. *In Search of a Card Trick*.

The speaker will present an original card trick — one whose motivation was a commercially available trick called “Detection.” The work in creating this trick was as much fun as the trick itself, and some parts of this journey will be described. As is the case with much of mathematics, the investigation led to some interesting questions and some ideas for future work. (Received September 09, 2019)

1154-M5-761 **Andrew Niedermaier*** (aniedermaier@janestreet.com). *Colonel Blotto, 10 Years Later*. Preliminary report.

Remember Colonel Blotto? A master tactician of the battlefield, his problem is a doozy: apportion 100 soldiers to fight at 10 different castles. The opposition also has 100 soldiers, and the larger army at each castle will claim victory. Should Blotto send 10 soldiers to each castle, or perhaps should he focus his efforts on a subset of the 10?

In 2009 I first brought the Blotto problem to the Joint Meetings, highlighting its value as a fun classroom activity that touches on several game theoretic concepts.

In 2013 I spoke about how the Blotto problem could be used to model certain situations in quantitative finance, and discussed results from the first Jane Street company-wide Blotto tournaments.

Now, in 2020, I intend to present a survey of the first 10 years’ worth of company-wide Blotto tournaments, highlighting interesting trends, surprising strategies, epic fails, and other miscellanea from across more than 60 variations of the “basic” Blotto game. (Received September 10, 2019)

1154-M5-1014 **Daniel C Florentino*** (florentinodc@jay.washjeff.edu) and **Ryan Higginbottom** (rhigginbottom@washjeff.edu). *More on Equilibrium Patterns in the Three-Person Candy-Sharing Circle*. Preliminary report.

A well-known game called the candy-sharing circle is played this way: Position n players in a circle, each with a positive, even number of pieces of candy. Each player passes half their pile to their left and takes a piece from a common pile only if they end the round with an odd number of pieces. It is easily shown that a candy equilibrium is reached. In this talk, we present results for $n = 3$ when the initial distribution forms an arithmetic progression. (Received September 12, 2019)

1154-M5-1054 **Robert W Vallin*** (robert.vallin@lamar.edu), Department of Mathematics, Lamar University, P.O. Box 10047, Beaumont, TX 77710. *Waiting for Penney’s to Turn Up*.

Penney’s Game, a coin-flipping example of a non-transitive game, has a spinoff question: If a person chooses a three-outcome sequence of coin flips (e.g. THT) and an impartial judge flips a coin until the chosen sequence

appears, what is the expected number of tosses (wait time)? The answer is known for a fair coin. In this talk we look at what happens if the coin is not fair. (Received September 12, 2019)

1154-M5-1105 **Jathan Austin, Brian Kronenthal*** (kronenthal@kutztown.edu) and **Susanna Molitoris Miller**. *Counting Catan Configurations*.

Catan is a dynamic property-building and trading board game in which players build a new board every time they play. In this talk, we will discuss a strategy for counting the number of non-equivalent boards that utilizes techniques from abstract algebra. (Received September 13, 2019)

1154-M5-1147 **R. Teal Witter*** (rwitter@middlebury.edu) and **Alex Lyford** (alyford@middlebury.edu). *Applications of Graph Theory and Probability in the Board Game Ticket to Ride*.

In the board game Ticket to Ride, players race to connect cities and build railroads on a map of the U.S. Unfortunately, the scoring system is arbitrary and simplistic. The reward for building a railroad between a pair of cities is simply the fewest number of trains needed to do so, ignoring the number of paths and the ease of building them. The points for owning a railroad increase exponentially relative to its size, even though train cards are collected at a constant rate. Luckily, we can enhance our approach by using math, thinking of the board as a graph where the cities are vertices and the railroads connecting them are edges. We explore the underlying graph theory and probability to improve players' strategies and the game itself. First, we calculate the effective resistance between pairs of cities—taking into account the difficulty and number of paths—so that players can connect the most cost-effective pair. We then apply a martingale to the railroad-building process and argue that the points for a railroad should be linear, not exponential, in relation to its size. Finally, based on graph-theoretic measures like the number of incident paths and betweenness centrality, we identify the most strategic railroads for players to build. (Received September 13, 2019)

1154-M5-1608 **David A Nash*** (nashd@lemoyne.edu), Le Moyne College, Department of Math, Stats, and CS, 1419 Salt Springs Road, Syracuse, NY 13214. *A New Take on Classic Pen Problems*. Preliminary report.

A classic optimization problem in differential calculus is the so-called "Pen Problem" in which a farmer attempts to maximize the area enclosed by rectangular pens of equal area while using a fixed amount of fencing under various conditions (a shared wall, using a river as one side, etc.) As it turns out, many generalizations of this problem (including into higher dimensions) all have the same basic solution. We explore these generalizations as well as additional related problems which are *not* quite so nice. (Received September 16, 2019)

1154-M5-1764 **Hossein Behforooz*** (hbehforooz@utica.edu), Utica, NY 13501. *New Developments on the Benjamin Franklin Magic Squares*.

Yes it is puzzle and magic time. When I ask my students about the next number after 1, 2, 4, 8, 16, all of them yell and say 32. But when I give them a little hint and ask them to think about the next number after 1, 2, 4, 8, 16, 23, then dead silence is there. We have faced with this situation when we wanted to extend the orders of Benjamin Franklin Magic Squares after 4, 8 and 16. I have fixed 32 by 32 Franklin magic square and I expected the next one will be of order 64 by 64 but it became totally different. In this presentation I will mention some of the recent studies and developments on Franklin magic squares from original orders 4, 8, 16, to 32, 49 and 64. Then I will present many interesting properties of the Franklin magic squares that we can see them in higher orders but not in orders 4 or 8. General audience and undergraduate students can easily follow my talk. Come and join us and have fun. Math is FUN. References [1] Hossein Behforooz. 32 by 32 Behforooz-Franklin Magic Square, *Journal of Recreational Mathematics*, Vol. 33, No 4, 2005. [2] Seymour S. Block and Santiago A. Tavares: *Before Sudoku, the World of Magic Squares*, Oxford University Press, UK, 2009. [3] Ronald P. Nordgren: *How Franklin Made His Squares*, *Mathematics Magazine*, Vol. 91, No 1, 2018. (Received September 16, 2019)

1154-M5-1770 **Michael Czekanski*** (mczekanski@middlebury.edu) and **Alex Lyford** (alyford@middlebury.edu). *Making Money on Scratch Tickets—The Impact of Prize Structures*.

Scratch tickets are ubiquitous, and the consensus stands that these games are not worth playing to make money. From the outset, the expected value of a ticket is less than the cost to play the game. Thus, in the long run, we will lose money if we play this game repeatedly. However, for lotteries that publish prize structures and the number of unsold tickets and grand prizes remaining, we can accurately estimate the expected value of a ticket. For these games, we show that we can accurately bound the expected value of the game through simulation—the lower bound of which frequently exceeds the cost of a ticket. The point and frequency at which this occurs is

highly dependent upon the prize structure of the game. In this work, we examine Vermont's scratch tickets, since the Vermont Lottery both publishes this data and has prize structures that vary substantially. For example, some games have just a few grand prizes worth upwards of \$20,000, and others have more prizes, each worth much less, such as the game "Win Either \$25 or \$50". Finally, we analyze the impact of different prize structures to see how they affect the behavior of the expected value of a ticket over the course of the game. (Received September 16, 2019)

1154-M5-2041 **Hideo Nagahashi*** (hnagahashi@triton.uog.edu). *Magic Tricks on Codes over $GF(4)$* . We present magic tricks based on codes over the field with 4 elements. (Received September 17, 2019)

1154-M5-2188 **David Richeson*** (richesod@dickinson.edu), Dickinson College, Department of Mathematics and Comp. Sci., Carlisle, PA 17013. *Squaring the Circle in a Mirror*. In 2013 the collaborative contemporary art group Troika had an exhibit entitled "The Far Side of Reason." One of the sculptures, "Squaring the Circle," hung from the ceiling. Standing in front of the sculpture, the viewer saw a circle, but its reflection in a mirror behind it appeared to be a square. From the side, the sculpture was clearly neither a circle nor a square but four curved arcs joined together at their ends. In this talk, we discuss the mathematics behind this illusion and give instructions on how to make your own version out of paper. (Received September 17, 2019)

1154-M5-2510 **L. Kerry Mitchell*** (1kmitch@gmail.com). *Sequences and Patterns Arising from Mancala on an Infinite Board*.

The present work considers the game of Mancala, played on an infinite board instead of the regular six-pit board. In particular, sequences of moves are investigated wherein the player's last stone lands in the home pit, allowing for another turn. The distribution of stones that can be cleared in one series of turns exhibits many interesting patterns, including locally linear and other polynomial arrangements. And the sequence of pits that are played displays periodic patterns of increasing length and complexity. It is speculated that, given enough pits, arbitrarily complex structures can be found. (Received September 17, 2019)

1154-M5-2576 **Therese Aglialoro and Robert Hochberg*** (hochberg@udallas.edu), University of Dallas, Department of Mathematics, 1845 E. Northgate Drive, Irving, TX 75062. *Snakes: Legal, Illegal and Dodecahedral*.

Solved Rubik's cubes are all alike; every unsolved cube is unsolved in its own way. Among these unsolved positions lies a wonderful array of beautiful patterns, and discovering them can be the occupation of many an enjoyable hour. One of the oldest of these patterns is the *snake*, which is formed by putting an "L" or "I" on each face so that they connect to form a contiguous shape winding around the cube, entering and leaving each face once. The authors wondered whether the Megaminx puzzle (dodecahedral variant of the Rubik's cube) also had snakes, and if so, how many. Finding the answer required a pleasant tour of graphs, groups and geometry which this talk aims to share. And *our* answer to *how many* is 21, but arguments could be made for 6, or 700. (Received September 17, 2019)

1154-M5-2686 **Robert A. Cohen*** (rcohen@western.edu), 1 Western Way, Gunnison, CO 81231. *Proofs by Board Games*. Preliminary report.

An interesting, simply stated, and easily accessible problem can serve as a springboard into a wide variety of mathematical discoveries at many levels. This talk will highlight one such problem and share several avenues of mathematical exploration that may follow. The problem begins with a simple game board that, while analyzing from a probabilistic standpoint, leads to a fascinating infinite sum involving the Fibonacci sequence. It has been shared with and studied by students from precalculus courses to calculus-based probability courses, and has even been the topic of an undergraduate research project. We will generalize the results to an extended class of similar game boards while also changing the rules of play, which will ultimately give us the ability to generate infinite sums that, without the game boards, would be very difficult to justify. (Received September 17, 2019)

Re-Envisioning the Calculus Sequence

1154-N1-93 **Girija Sarada Nair-Hart*** (nairhaga@ucmail.uc.edu), 4200, Clermont College Drive, Batavia, OH 45103. *Re-Envisioning Calculus Sequence - Example of a Challenge Based Unit*. Preliminary report.

The majority of college calculus students aim at perusing STEM careers. However, they encounter many barriers to success in these courses and some eventually change their major to peruse alternate career paths. These hurdles

that students face partly stem from inadequate mathematics preparation and incompatible prior learning styles that do not allow a smoother transition from high school to college. While reconfiguring calculus curriculum is a complex task, individual faculty members can lead the way to the transition by incorporating innovative instructional components. Presenting their steps with its rewards and detriments benefit peer faculty, and the institution in their efforts in equipping students with the 21st-century skills necessary for their success in the real-world setting. Adding the Challenge Based Learning (CBL) component to the calculus course could encourage student persistence due to the supportive culture that CBL establishes to emphasize their success. Besides, the hands-on design experience of CBL provides students with a more fulfilling learning experiences and help create stronger connections with other disciplines. During this talk, I will provide a brief introduction of CBL, then describe a unit on optimization piloted in a university calculus course. (Received August 02, 2019)

1154-N1-174 **Katrina Palmer*** (palmerk@appstate.edu), 121 Bodenheimer Dr, Boone, NC 28608, and **Katie Mawhinney** and **Greg Rhoads**. *Co-requisite and Fall-back Courses for Calculus Students*. Preliminary report.

The Department of Mathematical Sciences at Appalachian State University has developed several support options for students who are underprepared for Calculus I. This talk will describe the Department's calculus placement, then share results from the 1-hour co-requisite support course and the design of a fall-back course that starts about a month into the semester. This fall-back course is intended for students who realize early on that they are in over their head in Calculus I. Without penalty students can drop Calculus I and enroll in the fall-back course to help prepare them for calculus in the next term. (Received August 19, 2019)

1154-N1-549 **Troy D. Riggs*** (triggs@uu.edu), 1050 Union University Drive, Jackson, TN 38305. *Pedagogical Advantages of Teaching Calculus Using the Hyperreal Numbers*.

In this talk, I will describe the advantages in using the hyperreals to teach the essential concepts introduced in first semester calculus. We will examine specific ways that students engage concepts like "infinitely close," "continuous at a point" and "relative growth rates" by using functions evaluated at hyperreal numbers along with Bryan Dawson's formulation of 'approximation' (an equivalence relation) on the hyperreals.

In brief, students immediately recognize that they can bring their imaginations into play and build directly on their existing algebraic skills. In the typical calculus class, we see students struggling to manage the awkward limit concept. In contrast, with hyperreals the entire process has proven to encourage continual engagement and confident questioning on the part of the student. (Received September 06, 2019)

1154-N1-1681 **Maura A. Murray***, 352 Lafayette Street, Mathematics Department, Salem State University, Salem, MA 01970. *Re-envisioning the Math Pathways in Massachusetts Public Higher Education*.

A subcommittee of Massachusetts public higher education faculty was recently charged with challenging the status quo of traditional mathematical sequences and increasing student success. The subcommittee, which I co-chaired, came up with recommendations that were adopted by the Massachusetts Board of Higher Education in 2018. I will share our recommendations regarding math pathways, removing barriers by encouraging co-requisite models for remediation, and aligning math pathways with our K-12 public schools. I will also present challenges faced in our K-12 work based on college admissions messaging regarding Calculus. (Received September 16, 2019)

1154-N1-1931 **Tenchita Alzaga Elizondo*** (halzaga@pdx.edu), **Kristen Vroom** and **Matt Voigt**. *An Overview of the Motivation, Enactment, and Perceived Success of Course Variations*.

Precalculus to single variable calculus (P2C2) courses are often seen as a barrier for many students intending to pursue a science, technology, engineering, or mathematics (STEM) degree, motivating multiple calls to re-envision the sequence. Previous research has shown that course variations are prevalent at institutions across this country, which is one way that mathematics departments can challenge the status quo and begin transforming the sequence. What remains to be understood is departments' motivations for creating the variations, how the different variations are enacted, and what's the perceived success of the variation. This study focuses on investigating these aspects of different P2C2 course variations at ten institutions across the county. In this presentation, we will present illustrative and comparative examples of the different variations related to their motivation, enactment, and perceived success. (Received September 16, 2019)

1154-N1-2119 **Hanna Bennett** (hbennett@umich.edu), **Paul Kessenich*** (paulkess@umich.edu) and **P. Gavin LaRose** (glarose@umich.edu), Department of Mathematics, University of Michigan, 530 Church St., Ann Arbor, MI 48109-1043. *Re-envisioning Assessment at Scale: Curricular and Programmatic Design to Improve Outcomes, Inclusiveness, and Retention.*

In the early 1990s, reform calculus movements funded by the NSF sought to transform calculus from a "filter" to a "pump," to increase student persistence and success through multiple courses, and to address barriers presented by differences in preparation and background. We consider how the re-envisioning of our calculus sequence in the course of this calculus reform informs and underpins a second reform movement that we are undertaking now to incorporate mastery assessment, improve instruction and instructor support, and increase the inclusiveness of our introductory course sequence. We discuss the use of active learning at scale in a program that teaches 95 sections of calculus I in the fall semester, specific efforts to improve instruction across those sections, the development of a mastery assessment structure, and concrete administrative changes we have made to make our courses and their assessment more inclusive of groups traditionally under-represented in mathematics. (Received September 17, 2019)

1154-N1-2129 **Jessie K Lenarz*** (jklenarz@stkate.edu), 2004 Randolph Ave, Box 4091, St. Paul, MN 55105. *Pairing Mastery-based Testing and Active Based Learning to Improve Student Performance and Persistence in the Calculus Sequence.*

Mathematics education research has provided data supporting the claim that student engagement is correlated with persistence through STEM courses. The effect is greater for students with test anxiety and groups historically under-represented in mathematics. Mastery-based exams provided one tool to combat test anxiety in mathematics and increase a student's ownership of her own learning. Active based learning strategies have been shown to increase student performance in STEM disciplines. We discuss our implementation of mastery-based testing in the calculus sequence with active based learning. In particular, the logistics of organizing a course for sufficient student feedback and meeting the needs of students with accommodations will be addressed. We will share the successes and pitfalls we encountered and propose solutions to some of these issues in addition to providing preliminary data to support the claim that the pairing of mastery based testing and active learning result in better student performance and persistence through the calculus sequence. (Received September 17, 2019)

1154-N1-2158 **Suzanne I. Dorée*** (doree@augsborg.edu) and **Jody Sorensen** (sorensj1@augsborg.edu). *Active and Applied Calculus: a SUMMIT-P collaboration.*

What calculus concepts, skills, and habits of mind do students need to continue in mathematics, economics, and the sciences? As part of the national SUMMIT-P project, faculty from mathematics, economics, and chemistry renovated the calculus sequence to meet the needs of the partner disciplines. As a result, we shifted limits/continuity to Calculus 2, some differential equations/partial derivatives to Calculus 1, and built a path directly from Calculus 1 to Multivariable. We focused on key tools and narrowed the range of techniques and algebraic complexity. Weekly labs now feature partner discipline contexts/questions and are placed later to make it less obvious what calculus ideas and tools are needed in the hopes of improving transference. Class time was reimaged as 20-20-20: 20 minutes each on an inquiry-based activity, a "barely-enough" lecture, and practice problems – many with applications. Our findings include a high level of student engagement and persistence in STEM. This material is based upon work supported by the National Science Foundation under NSF award number 1625142. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation. (Received September 17, 2019)

1154-N1-2210 **Naneh Apkarian*** (naneh.apkarian@gmail.com), **Tenchita Alzaga Elizondo**, **Brittney Ellis**, **Brigitte Sánchez Robayo**, **Claire K. Robbins** and **Estrella Johnson**. *Departmental Change in Reaction to the Threat of Losing Calculus: Three Cases.*

In this talk, we present narratives of change at three university mathematics departments in response to administrative insistence on improvements in the face of pressure from engineering colleges. Specifically, these three departments were faced with the prospect of losing calculus teaching to another unit on campus and reacted to prevent that outcome. All three departments initiated change efforts related to calculus revision, including the appointing new leaders to oversee improvements, and all have successfully maintained control of calculus because of perceived program improvements. Drawing on sociological perspectives, we discuss the ways in which change was navigated and negotiated at each site. We found common characterizations of change leadership limited these descriptions of complex change processes, suggesting a need to refine such characterizations in ways

that capture more nuanced situations. Implications for future change initiatives in other contexts are discussed. (Received September 17, 2019)

1154-N1-2317 **Sarah Hews*** (shews@amherst.edu) and **Geremias Polanco** (gpolanco@amherst.edu). *Calculus in Context: An Innovative Approach to Calculus.*

The Calculus in Context (CiC) sequence at Hampshire College uses differential equations as a fundamental object of study. In these inquiry based courses, students learn how to engage in mathematics, take risks, and work in groups. By focusing on concepts, geometry, graphing, and numerical solutions before techniques, algebra, and closed form solutions, students who have already taken a class or classes in calculus are challenged immediately, and students who enroll in the courses with no calculus experience and unpleasant experiences in algebra courses are able to engage with mathematics in a fresh way. Since students are discovering the concepts of calculus through group explorations, advanced students are able to engage deeper with the material and novice students learn how to fill gaps in their mathematical background. These explorations are motivated by real life applications that serve as a natural context to learning calculus. For the last few weeks of the semester, the students work in groups on projects that could be novel research, applied problems, or historical explorations. This talk will describe the CiC courses, how they are broadly accessible and challenging, and present techniques and topics that are transferable to calculus courses at other institutions. (Received September 17, 2019)

1154-N1-2446 **Frank Savina***, 3925 W. Braker Ln., Suite 3.801, Austin, TX 78759. *Rethinking Calculus Pre-requisites—What does it take for Students to be Successful in Calculus?*

We often identify algebraic fluency as one of the core mathematical skills necessary for success in Calculus. However, it is important to recognize that in addition to fundamental algebraic fluency, students in programs that require Calculus also need to develop reasoning skills that allow them to grapple with deep and meaningful problems associated with change. How do we as math faculty create preparatory courses that develop these skills while simultaneously developing algebraic fluency? This session will share insights from a 2-year long project at the Charles A. Dana Center at UT Austin that attempted to answer this question, and culminated in a re-envisioned sequence of preparatory courses for Calculus. (Received September 17, 2019)

1154-N1-2479 **Mary Nelson*** (mnelso15@gmu.edu), Department of Mathematical Sciences, MSN 3F2, George Mason University, Fairfax, VA 22030*4444, and **Robert Sachs** (rsachs@gmu.edu) and **Joanna Jauchen** (jjauchen@gmu.edu). *Re-envisioning Calculus with Constraints: A Diverse Research State University's Ongoing Work.*

Currently, 70-80% of our Calculus I students have had some calculus before entering our course (in line with MAA Calculus Study figure for a Research University). This often causes anxiety among the other students. For students from urban and rural schools, calculus in high school was often not available. We have set in place several alternative routes to help more students succeed in Calculus I, thus opening the possibility of STEM majors even for at-risk students. These include: a summer preview, a two-semester Calculus I course, revised active learning recitations, a pathway for students who earned an AP3, and a variety of support mechanisms. We do have a large transfer population so we are not changing content within the sequence. (Received September 17, 2019)

1154-N1-2685 **Joel Kilty*** (joel.kilty@centre.edu), 600 W. Walnut Street, Centre College, Danville, KY 40422, and **Alex M McAllister**, **Alison Marr** and **John Ross**. *A Re-Envisioning of the Calculus Sequence for the Modern Student.* Preliminary report.

The mathematics faculty of Centre College and Southwestern University collaboratively engaged in a thoughtful re-envisioning of the calculus sequence for the modern student. Our goals were incorporating more modeling and “real-life” applications, including “new” ideas in each course for students who studied calculus in high school, incrementally increasing the challenge from one course to the next, and encouraging persistence. Toward achieving these goals, we identified new topics, alternative themes for grouping and sequencing familiar topics, and different approaches to scaffolding complexity. In this talk, we discuss this re-envisioned calculus as an example of a successful curricular reform project, to include: designing the courses, developing the courses independent of current textbooks, making the hard decisions on what context stays and what remains, and winning widespread buy-in from multiple constituencies. (Received September 17, 2019)

1154-N1-2698 **Alex M. McAllister*** (alex.mcallister@centre.edu), 600 W. Walnut Street, Centre College, Danville, KY 40422, and **Robin Cruz**, **Tom Halverson**, **Joel Kilty**, **Alison Marr** and **Chad Topaz**. *Calculus: Origins, Reforms, and New Directions.*

As a jumping off point to learning about new approaches to how we teach calculus, we begin by considering where we have come from and what might call us to change. How did calculus shift from cutting edge research

to a common element of high school and college curricula? What topics have we “always” studied? How has our approach to the teaching and learning of calculus evolved over time? What modern-day realities call us to re-examine and re-envision how we teach calculus at this moment in time? How do current calculus reform efforts build upon past work? Drawing on the tables of contents and examples of calculus textbooks across the centuries, this talk explores what has been preserved and what has evolved in our teaching of calculus. We also highlight the persistent calls to action in the curriculum guides of mathematical societies and the problematic outcomes that generate such a sense of urgency. (Received September 17, 2019)

Research in Undergraduate Mathematics Education (RUME)

1154-N5-152 **Kevin C. Moore*** (kvcmoore@uga.edu), 105 Aderhold Hall, Dept. of Math & Science Education, UGA, Athens, GA 30602. *Variation, Covariation, and Conceptual or Experiential Time.*

A growing number of researchers have illustrated the importance of students’ covariational reasoning—the mental actions involved in constructing measurable attributes changing in tandem across a range of K-12 and undergraduate concepts. Due to the pervasive importance of covariational reasoning, these researchers have called for more descriptive models of students’ covariational reasoning that identify marked differences in their reasoning and the implications of such differences. I respond to this call by describing students’ covariational reasoning in terms of the extent it is with respect to experiential time or conceptual time. Previous researchers have argued (co)variational reasoning is parameterized with respect to time; an image of variation necessarily involves conceiving different temporal states. Yet, there is a difference between (co)variational reasoning that is over experiential time—reasoning constrained to a situation as experienced—and conceptual time—reasoning in which a situation can be re-presented in ways different than its initial experience. Using data collected during teaching experiments with undergraduate students, I illustrate differences in students’ reasoning with experiential or conceptual time, as well as research and instructional implications. (Received August 16, 2019)

1154-N5-189 **Qingxia Li*** (qli@fisk.edu), 1000 17th Ave N, Dubois Hall 110, Nashville, TN 37208, **Thomas Gross** (thomas.gross@wku.edu), Western Kentucky University, Bowling Green, KY 42101, and **Patricia McCarroll** (pmccarro@fisk.edu), 1000 17th Ave North, Nashville, TN 37208. *Integrated College Algebra into General Biology by Interdisciplinary Course Projects.*

Low participation and retention of African-American STEM students remains prevalent. However, integrated approaches have increased participation and retention in STEM programs. We created an intrusive Learning Community (LC) based on the Performance Pyramid Model that included peer-led weekly sessions to reinforce biology course content and connect biology with mathematics concepts through interdisciplinary projects. We compared LC students to separate biology and mathematics control groups on pre-post biology and mathematics quizzes, respectively. We compared all groups on post-test assessments of Performance Pyramid supports and student perceptions. The LC group had greater improvements on biology quiz scores than the biology control group. The LC group reported higher rates of feedback compared to both control groups ($d = .79$ to $.81$), and greater confidence ($d = .62$) and science knowledge ($d = .72$) compared to the math control group. Implications for applying this model will be discussed. (Received August 21, 2019)

1154-N5-236 **Candice M. Quinn*** (cmq2b@mtmail.mtsu.edu). *Purposeful placement: How to group a room full of students.*

There is an increasing use of group work in undergraduate mathematics classrooms, however group composition can have differential effects on group performance based on gender, ethnicity, and achievement. There are different ways to organize a room of students into groups, however there is still the concern that not all students in groups are given the same opportunity to learn. I have drawn upon ideas from cognitive psychology and mathematics education to develop The Working Group Survey which can be used to arrange students in equitable working groups. This survey consists of 12 Likert-scale questions, five of which were adapted from the Attitude Survey created by Brookstein, Hegedus, Dalton, Moniz, & Tapper (2011), and five open-ended questions. In this presentation I will demonstrate how the survey was used to arrange groups, and discuss students’ perception of their groups, and how the groups’ interactions differed during a group exam for three contrasting group compositions. (Received August 26, 2019)

1154-N5-599 **Sayonita Ghosh Hajra*** (sayonita.ghoshhajra@csus.edu) and **Yajenny Pacheco Enamorado** (yajennypatricia@csus.edu). *Examining area of a rectangular region with area-unit comparison*. Preliminary report.

In this talk, we will share the data contrasting responses of four pre-service teachers (PTs) to problems that were part of an ongoing study of PTs' conceptions of area of a rectangle at a public university in the United States. We asked PTs to find the areas of the two rectangular regions with specific dimensions, one with whole number dimensions and other with fractional dimensions, using 3" by 5"-index cards. In the other tasks, we asked them to respond to some imaginary child's confusion on finding the area of a rectangular region. We will discuss various strategies, including covering, area formula, comparison of quantities using arithmetic operations, such as multiplication and division, that PTs used to solve the tasks. We will discuss the commonalities of PTs' struggles on specific parts of the tasks and the specific questions raised by the researcher to guide the PTs in making sense of their thinking on understanding of the area concept. (Received September 08, 2019)

1154-N5-870 **Ciera N Street*** (ciera.street@unco.edu), 4750 West 29th St., Greeley, CO 80634, and **Andrea C Alt, Lida M.L. Bentz** and **Hortensia Soto**. *Moving towards equitable teaching: A continuum of transforming collegiate mathematics teaching*.

In this study, we describe how the participants of a professional development program for collegiate mathematics faculty, implemented practices in their classrooms developed within the PD program. This funded professional development program focused on the themes of rich tasks, student-centered learning, and sense of community all with an eye toward equity. Video recordings of five of the participants' teaching at different points in the academic year served as evidence of the participants' incorporation of the professional development program themes in their classroom. We found that each of the five participants implemented the themes to different degrees and our analysis suggests that these levels of implementation may be related to how the participants attended to equity in their classroom. We argue that creating a sense of community in the classroom supports integration of the PD program themes and promotes equitable teaching practices. (Received September 11, 2019)

1154-N5-899 **Tim Archie*** (tim.archie@colorado.edu), **Sandra Laursen** (sandra.laursen@colorado.edu) and **Charles N Hayward** (chuck.hayward@colorado.edu). *For whom do workshops work? Personal and contextual factors associated with college instructors' implementation of inquiry-based learning after professional development*.

Student-centered, research-based instructional strategies (RBIS), such as inquiry-based learning (IBL), have been shown to improve learning and persistence in US undergraduate STEM education. Professional development (PD) of college instructors has been identified as the most influential factor in the adoption of RBIS. This research uses longitudinal survey data at 3 points from 580 participants in one of three sets of broadly similar, intensive IBL workshop offerings from 2010-2019. Because these workshops sought to increase classroom implementation of IBL, the focus of this research is to identify the factors most strongly associated with IBL implementation. We first used descriptive statistics to develop profiles of IBL implementers and non-implementers, then used regression analyses to identify personal and contextual factors that were strongly associated with IBL implementation and non-implementation. For example, we find a strong relationship between IBL implementation and department chair support (a contextual factor), and weak relationships with personal factors such as prior teaching experience. By identifying factors that can help or hinder IBL implementation, the study helps to clarify instructors' decision-making about teaching and inform future PD offerings. (Received September 11, 2019)

1154-N5-912 **Ralph E. Chikhany*** (ralph.chikhany@wsu.edu). *Towards a Hypothetical Learning Trajectory for business and marginal functions*.

For my doctoral dissertation, I conducted a pilot and two iterations of a design based research study in order to generate a Hypothetical Learning Trajectory (HLT) for business calculus students being introduced to business functions and their marginals. The effectiveness of the proposed guided reinvention tasks was measured by comparing the hypothesized learning to the observations from recordings of students working in pairs, their written solutions, and a project they worked on individually. The data analysis suggests that, while not perfect, the HLT and tasks allowed students to meet most of the learning goals. I also suggest subsequent refinements to the latest version of the materials for a potential future iteration. (Received September 11, 2019)

1154-N5-981 **Janessa M Beach*** (janesa.beach@mavs.uta.edu). *Investigating Prospective Secondary Mathematics Teachers' Evolving Function Concept Images.*

Whereas a strong conception of function is important to the study of much of undergraduate mathematics including calculus, it is particularly significant for prospective secondary mathematics teachers (PSMTs). Research studies indicate that practicing teachers' conceptions about and experiences with function influence how they teach function. Although various studies describe function conceptions of undergraduate students and teachers, there is limited research on changes in PSMTs' function conceptions. This study draws on Tall and Vinner's (1981) concept definition and concept image as a theoretical framework to investigate how seven PSMTs' function concept images evolved after ten weeks of engaging in research-based tasks designed to elicit function related cognitive conflicts. We collected individual, open-ended, task-based, pre- and post-interviews designed to evoke components of PSMTs' function concept images. Using thematic analysis methods on pre- and post-interviews, we identify and compare aspects of their function concept images. We discuss identified aspects including beliefs about function and equation, non-numerical functions, function and graphs, and the vertical line test. We also discuss how their course experiences may have influenced their concept images. (Received September 12, 2019)

1154-N5-1020 **Paul Regier*** (paulregier@ou.edu). *Creativity-fostering instruction's impact on student motivation and self-efficacy in upper-level undergraduate mathematics courses.*

There is a need to better understand the role creativity can play in facilitating cognitive and affective aspects of students' mathematical development. To study the impact of creativity on student motivation and self-efficacy in upper-level mathematics classrooms, a large-scale ($n=300$) quantitative research study was designed utilizing: (1) a new instrument for measuring student perception of creativity-fostering mathematics instruction, (2) an adaptation of the academic motivation scale toward mathematics, and (3) the self-efficacy for proving scale. Analysis of these three instruments will be presented to discuss their validity and reliability, as well as the results of hierarchical linear modeling to study multi-level influences of creativity-fostering instruction on pre/post semester changes in student motivation and self-efficacy for proving. (Received September 12, 2019)

1154-N5-1207 **Natalie LF Hobson** (natalie.hobson@sonoma.edu) and **Martha Byrne***. *Mathematics Stretch Courses as an Alternative to Remedial Mathematics Classes.*

There is a nationwide discussion about the status of remedial college courses. The emerging consensus on this topic is that students are not served well by a system that requires they take remedial mathematics classes before being able to obtain college credit. An alternative model to remedial math classes, is that of year-long stretch courses. In this talk, we describe a newly designed and implemented stretch course model at Sonoma State University. We present challenges and successes during the first year of full implementation. We share our initial assessment methods and findings on student learning outcomes compared with traditional courses and teaching methods. (Received September 13, 2019)

1154-N5-1484 **Yufeng Ying*** (yy16186@uga.edu), 1000 Lakeside Drive Apt351B, Athens, GA 30605, and **Kevin C Moore**. *A conceptual analysis of students' abstracted structural reasoning.*

"Structural reasoning" is a term that has been widely used in mathematics education but loosely defined. Built from previous research on Piaget's reflective abstraction, Arcavi's symbol sense, Hoch's structure sense, and Harel's structural reasoning, we introduce the notion of "abstracted structural reasoning" to model students' mental operations in abstracted arithmetic structures including analyzing arithmetic structures, using arithmetic structural properties in problem solving, and associating between different arithmetic structures. For example, equation $ab/(a+b)=4$ quantifies the ratio between "ab" and "a+b," and also enables inferring information on the sum " $1/a+1/b=1/4$ ". To use the rich inferences that one can gain from an arithmetic structural, students are required to abstractedly reflect on their various meanings for such a structure. Building on data from clinical interviews with several pre-service teachers, we illustrate examples of students' abstracted structural reasoning along with conceptual analysis, highlighting both affordances and constraints of such reasoning. Against this backdrop, we illustrate that students' abstracted structural reasoning supports solving general algebraic questions and having a dynamic understanding of an algebraic expression. (Received September 15, 2019)

1154-N5-1485 **Rachel Tremaine*** (rjean@rams.colostate.edu), **Jessica Ellis Hagman**, **Matthew Voigt** and **Jessica Gehrtz**. *You Don't Want to Come Into a Broken System: Critical and Dominant Perspectives for Increasing Diversity in STEM among Undergraduate Mathematics Program Stakeholders.*

As attention grows towards the disparities between majority groups and underrepresented minorities within undergraduate STEM education, there is a need for understanding where different university stakeholders stand on the subject of increasing diversity. This paper aims to juxtapose categorizations of stakeholder motivations

and dominant and critical perspectives to propose a framework for analyzing stakeholder attitudes and informing increasingly productive conversations on eliminating inequity in STEM fields. (Received September 15, 2019)

1154-N5-1492 **Kedar M Nepal*** (nepal_k@mercer.edu), 1501 Merver University Drive, Macon, GA 31015, and **Krishna Pokharel, Deepak Basyal, Debendra Banjade and Manoj Lamichhane.** *Students' Non-attendance to the Mathematical Meanings of the Symbols and their Common Errors in Calculus Courses.* Preliminary report.

Research shows that students make common and persistent errors in Algebra and Calculus courses. We designed a research to examine whether there is an association between such errors and students' non-attendance to the mathematical meanings of the symbols. We gave a carefully written Precalculus and a Calculus test to students in four different undergraduate courses for which these courses were a prerequisite. We were interested in examining why students perceive incorrect mathematical statements, logics, justifications, or answers as correct and vice-versa. Therefore, the tests included only True or False questions; however, if students' selected a given statement, process or solution as False, they were required to justify why it was not True, and vice-versa. The preliminary results suggest that many of the student errors are the direct consequence of their non-attendance to the mathematical meanings of the symbols. We will provide examples describing how common errors originated as a result of this student behavior. (Received September 15, 2019)

1154-N5-1567 **Mollee C. Shultz*** (mollee@umich.edu) and **Patricio Herbst.** *The levers beyond personal beliefs that enable inquiry-oriented instruction: Professional obligations in instructor decision-making.*

When instructors say they use or do not use inquiry-oriented instruction (IOI), it is not immediately obvious why. We know that beliefs often influence instructors' decision-making, but what else plays a role? We designed and administered a set of surveys to a national sample of college undergraduate instructors to discover what specific inquiry-oriented practices are being used, and what might be contributing to those decisions. We use structural equation modeling to test the relationships between the use of IOI-practices, individual factors (beliefs), and professional obligations such as the responsibilities that instructors have toward individual students or their institution. Professional obligations significantly predicted the use of open problems, having students work with each other in group work, having students make presentations, inviting students to construct and critique work, and using interactive lecture. A closer inspection of the professional obligations (disciplinary, interpersonal) that tend to significantly predict higher uses of particular practices (student presentations, interactive lecture) could inform how advocates of IOI could architect decisions to use IOI. (Received September 16, 2019)

1154-N5-1630 **William Hall*** (w.hall@wsu.edu) and **Ashley Whitehead.** *Using autoethnographies in qualitative research concerning the evolution of preservice mathematics teachers' professional identity.*

We report on one case, Krista's, experiences leading up to and during a regional mathematics education conference, and how these experiences affected the evolution of her identity as a preservice teacher of mathematics. Reflections on the conference, focus group data, and autoethnographies written by the students were collected. Autoethnographies are simultaneously a research method and a product in which individuals analyze their own experiences in light of academic research on a given social group, in this case, teachers of mathematics. Here, we present our analysis with the overall goal of discussing the issues/strengths of using autoethnographies as a research methodology in mathematics education. We will also present our findings regarding using Krista's experiences to help recreate similar conference experiences for other preservice teachers as hers was a transformative one. (Received September 16, 2019)

1154-N5-1728 **Kristen Vroom*** (vroom@pdx.edu), Portland, OR. *A Case of Defining as a Way to Learn about Mathematical Language.* Preliminary report.

There is mounting evidence that students have difficulties with mathematical language, and particularly multiply quantified statements. In response, there have been several studies that propose interventions for supporting students' interpretation of these statements. Here, I propose an alternative instructional approach which engages students in defining several concepts that can be articulated using quantified variables as a way to learn about and gain fluency with mathematical language. Using Realistic Mathematics Education's emergent models, I will present how relationships between quantified variables first emerged for a pair of students as a model-of their defining activity and then these relationships became more explicit as they reflected on their definitions. (Received September 16, 2019)

1154-N5-1807 **Keri Kornelson*** (kkornelson@ou.edu), **Deborah Moore-Russo, Stacy Reeder, Matthew Grover, Justin Wollenberg and Randa Shehab.** *A Study of Skills and Weakness of Incoming Mathematics Students.*

Prior knowledge plays an important role in student learning in and completion of undergraduate mathematics courses. The transition to university mathematics is difficult for many reasons, and gaps in students' foundational skill sets have the potential to hamper students' progress not only in their initial mathematics courses but also in subsequent mathematics courses. In this talk, we build on previous research of Reeder and Stewart (2017, 2019) related to the algebra skills of calculus students. Specifically, we use output from a proctored, widely-used, commercial mathematics placement instrument to identify the demonstrated skills and weakness (in algebra as well as other mathematical areas preceding calculus) of incoming freshman at the University of Oklahoma. We then connect these assumed prerequisite skills to student performance on course exams and to overall success in their initial and subsequent mathematics courses. We also look at the continuation of STEM majors as they persist on to further STEM courses based on the results of the placement instrument. We discuss results of the student, consider implications for this particular university as well as others in light of study results, and conclude by sharing potential areas for future research related to this topic. (Received September 16, 2019)

1154-N5-1822 **Younggon Bae*** (younggon.bae@utrgv.edu), 1 W University Blvd., BLHSB 2.526, Brownsville, TX 78520. *Understanding students' uses of dynamic geometry environments in screencast presentation.*

In this talk, I present an empirical study on students' enactment of dynamic geometry tasks in an axiomatic geometry course. Students constructed a Klein model in a Dynamic Geometry Environment (DGE) and investigated angle measurements and parallel lines in this model. In submissions of their work in class, students produced screencast presentations demonstrating how they created and examined constructions in the DGE. Given the observable changes on the screen of the recordings, I generated codes to identify students' purposeful uses of the DGE in association with their verbal responses. The analysis showed how the DGE was used to support students' mathematical communication and inquiry involved in the task enactment. First, students referred to diagrams on the screen when specifying certain figures or describing particular movements of the figures as they present their work or discuss with peers. Also, students used the drag feature to inquire about invariant properties of figures or particular examples. In particular, students switched their dragging strategies as they generated, specified, and tested conjectures using empirical evidence obtained from the responsive diagram of the DGEs. Further implications for researchers, educators, and task designers will be discussed. (Received September 16, 2019)

1154-N5-2080 **Patricio G Herbst** (pgherbst@umich.edu), 610 East University Ave #4117, Ann Arbor, MI 48109-1259, **Irma E. Stevens*** (istevens@umich.edu), 610 East University Ave #2412, Ann Arbor, MI 48109-1259, and **Amanda M. Milewski, Michael G Ion and Inah Ko.** *State of the undergraduate Geometry courses for secondary Teachers: Curriculum, Instructional Practices, and Student achievement.*

The GeT Support project is gathering data from undergraduate geometry courses for future secondary teachers. We offer descriptives documented with instruments developed by our project and completed by up to 40 instructors and their 300 students. Based on syllabus analysis and instructor responses to a questionnaire, we document the variations in the curriculum, showing the incidence in the sample of various archetypes of geometry for teachers courses. We add gleanings from an instructional log, completed 3 times a semester by each instructor, where we observe variations in self-reported practice. The data shows considerable evidence that some student-centered instruction is present but this involvement is limited in some possible areas (e.g., only a few instructors ask students to propose or critique definitions). Finally, our comparison of students' responses to the MKT-G (Herbst & Kosko, 2014) test—administered at the beginning and end of the course—shows that students make significant gains in mathematical knowledge for teaching geometry over the course of one semester, with this difference being in the order of .16 standard deviations. We will elaborate on what these findings mean in terms of improving the course. (Received September 17, 2019)

1154-N5-2292 **Andrew Kercher*** (andrew.kercher@mavs.uta.edu). *Factors Influencing the Development of Mathematical Problem Solving of Upper-division Undergraduate and First Year Graduate Mathematics Students.* Preliminary report.

Much extant research already attempts to describe and classify the problem solving methods of both novice and experienced mathematical problem solvers. While such taxonomies may be critical for descriptive and comparative purposes, more research is needed in identifying and examining factors that may influence mathematical problem solving development and to which different problem solving behaviors may be attributed. During an

approximately hour long and semi-structured task-based interview, upper-division undergraduate mathematics students and first-year mathematics graduate students were asked to reflect on how their experiences affected the growth of their emergent problem solving behavior. Preliminary analysis using grounded theory techniques indicate that while not all participants felt that their problem solving behavior had been significantly affected by an influential event, others made reference to their involvement in teaching, an unexpectedly difficult or novel classroom experience, or a particular professor's instructional style. In this talk, I provide examples of each category and compare the self-reported problem solving strategies of the participants with their observed problem solving tendencies. (Received September 17, 2019)

1154-N5-2303 **Heather Lynn Johnson***, Campus Box 106, P.O. Box 173364, Denver, CO 80217-3364, and **Gary Olson, Amber Gardner, Amy Smith and Xin Wang**. *Cartesian Graphs, Covariational Reasoning, and Powerful Positioning in College Algebra*.

Cartesian graphs are ubiquitous, yet students can face challenges conceiving of those graphs as representing relationships between attributes that are capable of varying and possible to measure (covariational reasoning). If students experience math instruction as a compliance driven pursuit of answers, they may miss opportunities to engage in covariational reasoning. We report results of a longitudinal study, conducted at an urban public university in the Midwest, across all sections of College Algebra (500 students) during three consecutive semesters. We implemented a three part intervention: Provide students with innovative online graphing activities (Techtivities); Foster instructors' examination of their power to impact students' learning opportunities; Measure students' covariational reasoning via a fully online, validated assessment. Instructors volunteered to participate in the study. Students in treatment sections received both the Techtivities and the assessment; students in comparison sections received the assessment only. Our findings revealed statistically significant differences between treatment and comparison students' covariational reasoning. We discuss implications for systemic efforts to promote students' covariational reasoning in introductory math courses. (Received September 17, 2019)

1154-N5-2371 **Nolisa S. Malluwawadu*** (nolisa@rams.colostate.edu) and **Jess Ellis Hagman**. *As asset oriented approach to studying students of color who thrive and survive in calculus*.

There are many studies that address differences in achievement in STEM between students of color and white students. It is potentially detrimental to continue to emphasize only those who are not succeeding and why, or in other words, coming from a deficit orientation. In order to improve both the success and the experience of students of color in STEM, we need to figure out what have been the factors that have helped students of color who have succeeded and how we can learn from them, in other words, looking at the problem at hand from an asset orientation. In this study, we seek to understand more about the population of students of color who persist in Calculus, and use descriptive analysis to answer the following questions: (1) Among students of color who persist through Calculus I to Calculus II, what are the characteristics of students who thrive versus those who survived?, (2) What enabled the thrivers to thrive?, and (3) What supports the survivors to survive? (Received September 17, 2019)

1154-N5-2389 **Franklin H Yu*** (fhyu1@asu.edu), 8426 E. Stella Lane, Scottsdale, AZ 85250. *Students' meanings for the Derivative at a Point in the context of Linear Approximation*.

The purpose of this study is to examine students' meanings for the derivative at a point. While students may associate rate of change with the derivative, this does not mean that the meaning they have for it is productive. This study explores students' responses to a typical calculus 1 problem that requires use of derivative to determine a linear approximation. Despite most students writing a correct response, the meanings students attributed to their work vastly differed. These results indicate that typical linear approximation problems that are used in standard Calculus 1 courses do not sufficiently assess what students actually understand about the derivative at a point. (Received September 17, 2019)

1154-N5-2426 **Melissa Newell*** (mnewell@iona.edu) and **Fabiana Cardetti**. *Investigating Mathematical Needs and Helpful Support Strategies through the Perspectives of Students and Tutors*.

Peer tutors and students spend hours together at quantitative learning centers (QLCs) on college campuses each week. The hours they spend at QLCs are filled with questions about various mathematics courses and topics, with homework assistance and with test preparation. Within these hours, peer tutors gain insights into the students' experiences with mathematics that instructors are not always privy to. In this talk, we will present the results of a study that addressed the questions (1) What are the mathematical needs of the students who visit a QLC, and (2) What tutor strategies help students understand the mathematical topics they seek help with? We were particularly interested in considering the perspectives of the tutors and students at a QLC. Our

talk will give particular focus to interesting comparisons we found between these two perspectives. The study followed a mixed methods design. Quantitative data was collected through two surveys, one sent to students who visited the university's QLC and one that was sent to students enrolled in calculus 1, 2, and two business mathematics courses. Qualitative data was collected through the two surveys, observations of tutoring sessions and semi-structured interviews with seven tutors and four students. (Received September 17, 2019)

1154-N5-2475 **Erika David Parr*** (davide@rhodes.edu), 2000 North Parkway, Memphis, TN 38112. *Distinctions in Undergraduate Students' Interpretations of Differences from Calculus Statements in the Graphical Register.*

Key ideas in Calculus are often stated using expressions involving differences and illustrated with graphs. The ways in which students interpret these expressions within the graphical register may influence their understanding of such concepts. This presentation describes an investigation of students' interpretations of differences from Calculus statements within the graphical register. As part of a larger study, I conducted clinical interviews with 13 undergraduate math students. In the interviews, I asked the students to evaluate propositional statements related to Calculus concepts using graphs of various functions. The related concepts included continuity at a point, the difference quotient, and the Mean Value Theorem. I describe two ways students interpreted differences, which emerged from this study. One way students interpreted differences, which I refer to as a cardinal interpretation, is characterized by an additive measure of units along an axis or along the trace of a graph. In contrast, other students interpreted differences using a magnitude interpretation, characterized by a multiplicative measurement of distance between two reference points. I discuss the implications of each of these interpretations for teaching and research in Calculus and other areas. (Received September 17, 2019)

1154-N5-2537 **Anna Marie Bergman*** (a.bergman@pdx.edu). *What group is it? Well it depends on who you know. Students' varying use of group isomorphisms while investigating molecular structures.*

This presentation will show how mathematics students engaged with the concept of group isomorphisms in the context of chemistry. Three pairs of students, graduate students with extensive group theory experience, undergraduates who had completed an introductory group theory course, and undergraduates with no prior group theory exposure; all participated in a series of teaching experiments. For each experiment students were given ball and stick models of various molecules and through a series of tasks each pair successfully developed a classification algorithm for most molecular structures. However, during their mathematical activity each of the pairs engaged with group isomorphisms quite differently. The graduate students established isomorphisms verbally considering various observable group properties such as number of elements, self-inverses, etc., but rarely produced explicit maps. In contrast those who had taken an introductory course focused on defining explicit bijections between groups and then argued they were isomorphic through their extensive use of Cayley tables. Lastly, the undergraduates with no previous group theory experience, who also had no formal knowledge of isomorphism, quickly noticed isomorphic groups by attending to similarities in their group presentations. (Received September 17, 2019)

1154-N5-2625 **Wesley K. Martsching*** (wesley.martsching@unco.edu), 2330 South Colorado Ave., Unit C, Loveland, CO 80537. *Students' Gaze Patterns and In-the-Moment Shape Thinking.*

Previous research has revealed that rote memorization and a weak understanding of concepts foundational to calculus attribute to students' difficulties in visualizing (co)variation and an inability to extrapolate mathematical concepts to solve novel problems. There has been a shift toward dynamic imagery and multimedia usage in task construction to alleviate these issues; however, little is known about how students look at images in calculus. The purpose of this study is to identify potential themes that emerge in students' gaze patterns relative to their ways of thinking about a novel calculus-based task. Two one-hour semi-structured interviews were conducted with eleven undergraduate calculus students. Audio and visual recordings of the interviews were used to code participants' in-the-moment ways of reasoning for this task. Gaze patterns were observed and commonalities in eye movements that emerged within similarly coded instances were recorded. Individuals' in-the-moment static shape thinking appears to coincide with fewer switch counts between relevant areas of interest of the task while their in-the-moment emergent shape thinking appears to coincide with higher switch counts and greater attention to these areas of interest. This study focuses on the results from one task. (Received September 17, 2019)

- 1154-N5-2673 **Ray DeWitt*** (rdewitt2@lssu.edu) and **Soofia Malik** (smalik@unoh.edu). *Correlations and Predictability of Undergraduates' Attitude toward Mathematics Scores and Mathematics Class Placement.*

In this two-semester study, we aim at investigating correlation between undergraduates' attitude toward mathematics scores and their mathematics class placement. In other terms, our goal is to examine how the undergraduates' attitude toward mathematics scores compare between their ending scores of one class and the beginning scores of another, for example, end of Pre-Algebra and start of Intermediate Algebra. Students enrolled in mathematics classes ranging between Pre Algebra and Calculus 1 are part of this study. Tapia's (1996) Attitudes Toward Mathematics Inventory (ATMI) will be used to obtain attitudes toward mathematics scores. The ATMI forms include information about the last mathematics class student completed, either in high school or college and their grade they earned in their last math class. The data obtained from the ATMI will be used to answer the following research question: Does a student's ATMI score correlate to mathematics course they are enrolled in? If so, how do grades factor into this relationship? This presentation will expand upon the preliminary findings and the future direction of this project. (Received September 17, 2019)

- 1154-N5-2675 **J. Victoria Truman*** (jtruman@vt.edu). *Affect Toward Unusual Mathematical Tasks in Mathematically Experienced Adults on the Autism Spectrum.*

In this report, I examine the responses by mathematically experienced adults on the autism spectrum given paradoxical or otherwise unusual mathematical problems. I compare their interest and overall perspective on such tasks against results from students in the general population. Interview results combined with previous data suggest that adults on the autism spectrum are more likely to have positive views and enthusiasm for paradoxical mathematical tasks, and less likely to show rejection of the tasks or their results. Analysis of possible reasons for showing these differences and implications for teaching and further autism-related research are presented. (Received September 17, 2019)

- 1154-N5-2689 **Susan L Addington*** (saddingt@csusb.edu), Math Dept., CSUSB, California State University, San Bernardino, San Bernardino, CA 92407, and **Lamies Nazzal, MaDonna McCabe, Pilar Olid and Kelli Wasserman.** *Detecting and Filling Gaps in Preparation for College Mathematics.* Preliminary report.

Contrary to accepted belief, gaps in college students' mathematical preparation are often in elementary topics: measurement, place value, and multiplicative thinking, in addition to higher-level mathematics. These topics are introduced in elementary school, but students need experience using these concepts at a college level: real applications, problems with "hard" numbers, including decimals and very large and small numbers; in short, number sense. This project is attempting to identify what number sense encompasses, and how it differs from proficiency in skills. A three-question pretest was given to 250 incoming freshmen in a 6-week Early Start program. The questions addressed measurement and units (How many square inches in a square foot?), a mental approximate multiplicative comparison with real-life numbers, and decimal percent (express 0.05% as a decimal without using a % sign.) The course included college-level material on functions and a support class that included conceptual instruction, with practice, on the topics mentioned above. Correct responses to the pretest were in the 20% range; on the posttest, in the 50% range. This presentation will analyze results and student work, and describe work on a current project to produce an online unit on these topics. (Received September 17, 2019)

- 1154-N5-2691 **Benjamin David Sencindiver*** (sencindi@rams.colostate.edu). *Calculus Students' Understandings of Graphical Representations of Functions and Differences of Outputs.* Preliminary report.

Graphs of functions are commonplace in mathematics. They can be used as a source of reasoning for student problem solving and can support student understanding of mathematical concepts. Recent research at the postsecondary level has found that students have a variety of understandings of graphs of functions. My research aims to understand how students' different meanings for the graph of a function may impact the meanings they develop for various calculus concepts, such as derivative. In this presentation, I draw on the location-thinking and value-thinking constructs of David, Roh, and Sellers (2018) to describe students' thinking about outputs of functions and connect these forms of thinking to students' understanding of differences of outputs. Theoretical and inductive thematic analyses were conducted to code clinical student interviews. This presentation will include preliminary themes and potential sources of variation of students' thinking about outputs of functions and differences of outputs. (Received September 17, 2019)

1154-N5-2720 **Abigail Higgins*** (abigail.higgins@csus.edu), 6016 J St, Sacramento, CA 95819, and **Jessamyn Minners**. *Student-Use of Internet Resources in Undergraduate Mathematics Courses*.

Internet resources to support the learning of mathematics and the completion of mathematics homework have developed significantly in the last decade; students today are studying and completing homework in entirely new ways. Most instructors teaching today learned mathematics without access to these resources. Reflection on one's own experience with learning mathematics is an important component of instructional decision-making. Without experiences using these resources as students, instructors lack important data to inform instructional design. Research in this area is sparse, primarily because these resources are so new and are constantly developing. This talk will present results from a study conducted in Summer 2019 which examined which resources students are using and how they are using these resources for mathematics learning and homework completion. Data informing results from this study include student surveys, video- and audio-recordings of semi-structured, task-based interviews, and Livescribe pen-casts collected from Calculus II and College Algebra students. This talk will conclude with a discussion of implications from these results for instructional design, assessment, and best practices for student-use. (Received September 17, 2019)

1154-N5-2731 **Erin D Glover***, erin.glover@pcc.edu, OR. *Managing Conditions, Assumptions, and Properties: Explicating a Disciplinary Practice of Mathematics*.

Educators and policymakers have called attention to mathematics disciplinary practices (MDPs) in the last decade because they are a fundamental aspect of learning and doing mathematics. Much of this work is focused in K12 and there is a need to articulate these practices in the post-secondary context. MDP describe the mathematical activity that mathematics is done across mathematical content. These practices include conjecturing, proving, and modeling. Much of the research in undergraduate mathematics education has focused on student cognition across a variety of content and highlight issues related to teaching and learning of this content. With the exception of the MDP of rigorous proof, only a handful of studies explicitly view mathematical practices as the focal object of interest. Few national documents outline the mathematical disciplinary practices undergraduate students should be learning to engage in (i.e., cognitive recommendations 2015 CUPM Guidelines). In this talk I will articulate the Conditions, Assumptions, and Properties (CAPs) MDP which emerged in an interview study with pure and applied mathematicians. I will briefly describe the relationship between CAPs and other MDPs and offer a framework for the CAPs so support the teaching and learning of this practice. (Received September 17, 2019)

Role of Explanation in Mathematical Proofs

1154-O1-487 **James R Henderson*** (jrh66@psu.edu), 402 W Main Street, Titusville, PA 16354. *Explanatory Proofs*.

When investigating the character of mathematical explanation, it is not unreasonable to begin with the literature concerning scientific explanation. If this approach is taken, the discussion must begin with Hempel and Oppenheim's 1948 paper, "Studies in the Logic of Explanation." Here a template for explanation is laid out, and a body of writing, some critical and some supportive, develops. The pump thus primed, new formulations of "scientific explanation" were produced in the ensuing years. Not surprisingly, some explanatory schemes designed for a scientific setting are a better fit for a mathematical context than others. I will stick to definitions of 'proposition', 'argument', and 'proof' used in this literature to assure a sound analysis. Study of equivalent theorems (for instance, the Mean Value Theorem and Rolle's Theorem) will offer interesting insights into this undertaking. It may also be useful to try to address the question of whether it is coherent to describe an explanation as successful and, if so, whether the success of an explanation is subject to degree. (Received September 05, 2019)

1154-O1-716 **Bonnie Gold*** (bgold@monmouth.edu). *What makes proofs explanatory? Let's look at some examples*. Preliminary report.

Both in mathematics and in philosophy, a few good examples can help clarify a concept. So, to try to make some progress understanding the idea of explanatory proofs, I will look at several proofs that the sum of the first n positive integers is $n(n+1)/2$, and examine to what extent, and why, some of them seem more explanatory than others. (Received September 09, 2019)

1154-O1-1421 **Jeff Buechner*** (buechner@newark.rutgers.edu). *Are mathematical explanations interest-relative?* Preliminary report.

Hilary Putnam introduced a wrinkle in the philosophical literature on explanation when he argued that explanations are interest-relative. What counts as an explanation for one set of interests might not count as an explanation for another set of interests. Suppose that some mathematical proofs do provide an explanation of what is proved. Are such explanations interest-relative, or are mathematical explanations via proofs immune to the interest-relativity of explanations? Certainly there can be different explanations of the same theorem—because there are different mathematical proofs of that theorem. For example, the interests of a topologist are satisfied by a topological proof of theorem A, while the interests of a number-theorist are satisfied by a number-theoretic-proof of theorem A. Can there be a topological proof of theorem A which explains A for, say, one topologist but not for another topologist (where both topologists are equally competent)? (Received September 15, 2019)

1154-O1-2227 **Jeremy Case*** (jrcase@taylor.edu) and **Sam Stueckle** (sstueckle@trevecca.edu). *Mathematical Explanation as an Aesthetic*. Preliminary report.

A traditional view of knowledge is that knowledge is justified true belief. Assuming a mathematical result is true, a person may not believe, or feel satisfied, with a valid argument. Beginning mathematicians often want examples to be convinced. Seasoned mathematicians may need explanations since most have seen counterexamples of seemingly solid proofs. Furthermore, a skeptic could challenge every deduction or claim. When does a proof become accepted? We propose that Mathematics at its core is a creative act, and every creative act has at its core an aesthetic. Mathematical aesthetics provides a necessary guide of mathematical knowledge or at least the acceptance of mathematical knowledge. (Received September 17, 2019)

1154-O1-2672 **Paul Zorn*** (zorn@stolaf.edu). *Proofs that explain, proofs that don't, and proofs of the obvious*.

Proof and explanation are fundamentally different mathematical and psychological activities. For example, careful mathematicians (e.g., journal referees) will mainly agree with each other that a proposed proof is or isn't valid, but they may disagree strongly on its explanatory value. Proofs are rule-bound and formal; explanations may appeal mainly to psychology, or even to aesthetics.

Yet proof and explanation are closely tied in practice, especially in pedagogy. Standard proofs mathematics majors encounter may explain a lot, a little, or almost nothing. And, surprisingly, proofs of ostensibly obvious facts can illuminate unexpected and deeper properties of ostensibly familiar objects. I'll illustrate with simple examples, mainly from set theory. (Received September 17, 2019)

1154-O1-2716 **Susan Ruff*** (ruff@mit.edu). *How logic is presented may obscure or enlighten*.

Whether a proof enables readers to recognize why a statement is true may depend on both the proof's logic and how that logic is presented. This talk focuses on the latter.

If a proof's logic is sufficiently amenable, a somewhat obvious strategy for making the proof explanatory is to add text that explicitly draws readers' attention to why the statement is true or to salient aspects of the proof. This strategy may increase the length of the proof or of the surrounding exposition. A less obvious strategy is to use Known→New structure to craft the proof so it flows well, thus revealing the flow of the underlying logic while keeping the proof concise. These two strategies are complementary. I will explain Known→New structure and provide examples to illustrate how combining the two strategies can help readers to follow a proof and, assuming the underlying logic is sufficiently amenable, to recognize why a statement is true. (Received September 17, 2019)

The Scholarship of Teaching and Learning in Collegiate Mathematics

1154-O5-29 **Sarah J Greenwald***, 121 Bodenheimer Drive, 326 Walker Hall, Boone, NC 28608. *Maximizing Inclusion and Engagement while Raising the Bar with Second Chance Homework*. Preliminary report.

The CBMS statement “Active Learning in Post-Secondary Mathematics Education” (2016) and the MAA Instructional Practices Guide (2018) include strong recommendations for active learning. Active learning works very well for many students. But what about the students who are left behind because of sickness, weaker

backgrounds, or various other reasons? What works to help struggling students engage in active learning, what are their demographics, and how can we still encourage on-time work for most?

In linear algebra, I am investigating the use of technology to cultivate multiple access points in interactive homework that I have designed. This includes a second chance homework that is repeatable and builds in my hints to common errors. This may concern some instructors, since it could be seen as a pathway to procrastination. However, most students still meet initial due dates because the second chance homework has a higher bar. We'll compare the participation of groups underrepresented in mathematics to their classmates and discuss survey data and student comments. Appalachian State University's Institutional Review Board (IRB) has determined that this research is exempt from IRB oversight. (Received June 29, 2019)

1154-O5-78 **Shanda R Hood*** (hoods@uark.edu) and **Joshua Girshner**. *The Impact of Frequent Student-Faculty Interactions on Repeater Students*. Preliminary report.

Data show that half of all students who have dropped/failed Survey of Calculus or Finite Mathematics at the University of Arkansas will drop/fail again. These students face a lack of motivation and a fair amount of anxiety toward mathematics. To make connections and create an environment in which they are comfortable discussing any issues with the professor, repeater students were asked to meet with the professor to complete a personalized academic improvement plan. This plan establishes the need for regular contact with the instructor and should increase the student's level of comfort with the instructor. Utilizing the "Academic Improvement Plan" serves as a catalyst for students to meet face to face with their professor and provides a roadmap for continuing that contact on a consistent and regular basis, regardless of the course. We believe that frequent and regular faculty/student interactions will result in increased academic success for this group of at-risk students while allowing these students to develop a deeper understanding of course materials, improve mathematical self-efficacy, and cultivate skills applicable to other courses/situations. (Received July 31, 2019)

1154-O5-627 **Jimmy Edward Miller*** (jmille54@utk.edu), 1317 Chatam Ridge Lane, Knoxville, TN 37932. *An Analysis of Post-Secondary Mathematics Instructors*. Preliminary report.

The purpose of this study was to determine significant similarities and differences among post-secondary mathematics instructors with a focus on how they teach and engage students in learning mathematics. Prior research confirms that some of the similarities and differences among post-secondary mathematics education practices fit within categories that include teaching preparation, academic background, and cultural issues. Therefore, the study sought to explore these characteristics and to possibly identify others, with the intention of improving the overall effectiveness of mathematics teaching at the post-secondary level. Data were collected from a sample of post-secondary mathematics instructors through a screening survey and narrative interviews in order to answer the following questions: What are the similarities and differences among the teaching philosophies of post-secondary mathematics instructors? What are the specific teaching practices identified by post-secondary mathematics instructors as critical to the teaching and support of students learning abstract mathematics? How do the teaching philosophies of post-secondary mathematics instructors influence their teaching practices which are used to teach and support students learning abstract mathematics? (Received September 08, 2019)

1154-O5-1037 **Marie Meyer*** (mmeyer2@lewisu.edu), **Amanda Harsy**, **Michael Smith** and **Brittany Stephenson**. *Preliminary Analysis of the Impact of Active Learning in General Education Mathematics Courses*. Preliminary report.

In this talk, we share the preliminary results of a study that explores the overall perceptions and attitudes of students in general education mathematics courses. Our work includes an analysis of survey data collected from several different general education mathematics courses using pre and post surveys. We compare students' responses in courses taught using primarily active learning-based methods such as group work, projects, and discovery learning to the responses of those in general education courses taught using a more traditional, lecture-based method. The surveys explore students' disposition, mindset, mathematical confidence, mathematics anxiety, and perceptions of effective pedagogical methods. By comparing pre-survey and post-survey responses, our analysis also explores how these perceptions and dispositions evolved over the duration of each course. (Received September 12, 2019)

1154-O5-1455 **Rita Patel*** (patelr35@cod.edu), **Amanda Harsy** (harsyram@lewisu.edu) and **Angela Antonou**. *Using inquiry-based learning as a form of professional development to assess teachers' dispositions towards mathematics*. Preliminary report.

Providing high-quality and effective professional development for K-12 teachers is a critical need for both teachers and their students. In order to meet the need for teachers to provide more engaging and powerful learning opportunities for their students, researchers suggest that we should provide similar active learning opportunities

for teachers. That is, professional development for teachers should model high-impact instructional strategies in a way that allows teachers to experience being the learners in the classroom. Math Teachers' Circle activities provide one such model for this type of professional development. In this presentation, we discuss the impact on teachers who were participants of two professional development workshops (one which lasted three days, the other lasting one day) administered as Math Teachers' Circle events. By comparing pre-survey and post-survey responses, we explore the impact of inquiry-based learning on teachers' disposition towards the teaching of mathematics. (Received September 16, 2019)

1154-O5-1544 **Jean McGivney-Burelle*** (burelle@hartford.edu), 200 Bloomfield Avenue, West Hartford, CT 01077, and **Larissa Schroeder, Mako Haruta and Fei Xue.** *S³MT: Supporting and Sustaining Scholarly Mathematics Teaching*. Preliminary report.

Supporting and Sustaining Scholarly Mathematics Teaching (S3MT) is a three-year multi-institutional project supported by the National Science Foundation (IUSE 1725952). This project seeks to develop a cadre of scholarly teachers who implement research-based instructional strategies, with an emphasis on active learning, and who are committed to studying the effectiveness of these pedagogical innovations. More specifically, we aim to: (1) create a multi-institutional network comprised of math faculty interested in becoming scholarly teachers who conduct and publish the Scholarship of Teaching and Learning (SoTL) on their teaching and their students' learning, and (2) to identify the challenges, opportunities, and structures necessary for supporting mathematics faculty who are at different stages of implementing SoTL. Ultimately, we want to develop an understanding of how to best leverage SoTL to support a more widespread adoption of engaged teaching and learning strategies. Beginning in 2018, 16 mathematics faculty from seven colleges and universities, with diverse student populations, have been participating in a faculty development program around conducting SoTL. In this session we will describe the professional development program and lessons learned during the past two years. (Received September 16, 2019)

1154-O5-1695 **Ashley D. Cherry*** (ashley.cherry@luc.edu). *Utilizing Exam Wrappers to Improve Student Metacognitive Skills in College Algebra*. Preliminary report.

College algebra courses have a high number of freshmen who are still adjusting to the college classroom. One strategy applied in many disciplines to help students regulate their learning is the exam wrapper. An exam wrapper is a task typically assigned when graded exams are returned. The goal is to help students think critically about their learning and to help them diagnose their academic strengths and weaknesses. While literature shows that students respond positively to the use of exam wrappers (Andaya, Hrabak, Reyes, Diaz, & McDonald, 2017), there are still unanswered questions. For example, in what ways do students change their study habits and strategies over the course of a semester after using exam wrappers, and do exam wrappers actually help students narrow the gap between perceived knowledge and actual knowledge?

To attempt to answer these questions, 54 college algebra students completed an exam wrapper after each of the three major exams. We also administered a survey called the Metacognitive Awareness Inventory (MAI) at the beginning and at the end of the semester to see if students' perceptions of their metacognitive skills had changed. Quantitative and qualitative results from this preliminary study will be discussed as well as plans for future research. (Received September 16, 2019)

1154-O5-1740 **Rochy Flint*** (flint@tc.columbia.edu) and **Baldwin Mei.** *Perspectives of MathChavrusa in Collegiate Classrooms*.

MathChavrusa is a novel application of Talmudic study techniques into the context of mathematics education, emphasizing long-term, partnered text study and problem solving. This study implemented MathChavrusa in semester long graduate mathematics courses, collecting data from both instructor and student perspectives on the effectiveness of MathChavrusa in facilitating student understanding of course material and engagement in communicating mathematics. Results indicated that students largely had a positive impression of MathChavrusa, highlighting its impact on student engagement, creating a conducive environment for asking peer-peer and peer-instructor questions, and was a factor in better understanding course materials. This study model has the potential to enhance student engagement in mathematics classrooms and could be adopted more broadly. (Received September 16, 2019)

1154-O5-1910 **Kristin A. Camenga*** (camenga@juniata.edu). *What kinds of homework more effective for Calculus I students?* Preliminary report.

How do students learn most effectively in Calculus? Research suggests that the time students spend practicing problems and the feedback that they receive have a positive impact on student learning. However, there are many ways to accomplish this practice and give feedback, so what is most effective? In two sections of Calculus I one

instructor used three different treatments for practice and feedback in different orders: teacher created homework with a student grader, textbook homework, and self-check homework, some supplemented by online homework problems. In this talk, we will consider the effect of the treatments on student achievement as measured by exams and a common final as well as student confidence and preferences as self-reported on surveys. (Received September 16, 2019)

1154-O5-2126 **Spencer Bagley*** (sbagley@westminstercollege.edu) and **Anil Venkatesh**. *Using Primary Sources to Improve Classroom Climate and Promote Shared Responsibility.*

To address a deteriorating classroom climate at the midpoint of a two-semester upper-division mathematics course sequence, we employed a novel instructor-led intervention: reading a mathematics education manuscript together with students as an invitation to legitimate peripheral participation in the scholarship of teaching and learning (SoTL). This intervention resolved many student complaints about the structure and level of the course, and the manuscript's discussion of the didactical contract promoted the idea of shared responsibility. We propose that reading mathematics education literature (and perhaps especially SoTL literature) with students can be an effective tool for improving the climate of the classroom, and that using the didactical contract in this way can particularly help students claim their share of responsibility for their own learning. (Received September 17, 2019)

1154-O5-2441 **Daniel L Kern*** (dkern@fgcu.edu), Department of Mathematics, Florida Gulf Coast University, 10501 FGCU Boulevard South, Fort Myers, FL 33965-6565, and **Menaka Navaratna** (mnavarat@fgcu.edu). *Retrieving, Interleaving, and Intervention in Calculus.*

Calculus students with a weaker mathematical background or who initially fail to understand key topics may find themselves getting left behind. Several techniques were implemented to address this, resulting in a more consistent intervention model. During the first week, students were given review assignments to strengthen their knowledge of basic techniques and concepts. Explicit retrieval techniques (including more frequent quizzing), and increased interleaving of topics were added during regular class meetings. Additionally, varied remediation opportunities were implemented if a student displayed a poor performance on assignments or a test. Test data is statistically compared to previous groups; where possible, the analysis will adjust for results from a review test. (Received September 17, 2019)

1154-O5-2702 **Kathleen Michelle Clark*** (kclark@fsu.edu), 1114 West Call Street, School of Teacher Education, Tallahassee, FL 32306-4459, and **Seyda Uysal** (su17b@my.fsu.edu), 1114 West Call Street, School of Teacher Education, Tallahassee, FL 32306-4459. *Examining the Secondary-Tertiary Transition from Multiple Perspectives.* Preliminary report.

The secondary-tertiary transition (STT) has received increased attention in recent decades. We conducted a small pilot study in 2019 (with 12 participants), in which we sought to gain an understanding of salient features of the STT experienced by pure mathematics majors (including students preparing to teach mathematics in secondary school). The research question guiding the pilot study was: In what ways are students' perceptions on the nature of mathematics related to their affective and emotional response to the transition from school to university mathematics? We first interviewed students to capture their experiences along three dimensions of interest (emotional disposition towards mathematics, vision of mathematics, and perceived competence in mathematics). Next, an analysis of the interviews aided our design of a supportive intervention, which confronted aspects of the STT via an examination of the (historical) changes in the nature of mathematics (in this case, related to the concept of derivative). Data collected during the short (4-day) seminar include survey responses, video recordings of seminar sessions, student reflection diaries, and student work samples. In our talk we will present initial findings and discuss how this work informs future STT research. (Received September 17, 2019)

Scholarship on Teaching and Learning Statistics

1154-P1-277 **Leon Kaganovskiy*** (leonkag@gmail.com), 1233 East 19, apt 6J, Brooklyn, NY, NY 11230. *Modeling and Statistical Research with R.*

In this talk, I will discuss the use of R for modeling and visualizations in a Mathematical Statistics and intermediate Applied Statistics courses. I will review R textbooks at various levels and discuss students projects with R. In the remaining time, I will discuss the most recent joint project with Ecologist Dr. Lowman which was made possible by R. (Received August 28, 2019)

1154-P1-416 **Reza O Abbasian*** (rabbasian@tlu.edu), Texas Lutheran University, Dept of Math-CS, 1000 W. Court St., Seguin, TX 78155, and **Michael L Czuchry** (mczuchry@tlu.edu), Texas Lutheran University, Dept of Psychology, 1000 W. Court St., Seguin, TX 78155. *Study of Student Performance and Perceptions in Inverted Introductory Statistics.*

In this presentation we will introduce our results from the three-year NSF funded grant titled “Inverted and Active Learning Pedagogies (IALP) for Student Success” for the introductory statistics classes, taught by faculty from mathematics, information system and data analytics, at Texas Lutheran University. Our results are based on four semesters of data. We will present our results comparing student achievement and retention between inverted (flipped) classrooms and traditional lecture formats. We will discuss the study design, data gathering, faculty and students’ surveys, and the methodology used for the study. We will share results from instruments that were developed to measure different levels of cognitive understanding across multiple sections of the same course. We will also discuss various factors affecting the results including use of technology, classroom activities, class size, students’ attitudes, type and lengths of the instructional videos, and the instructor. (Received September 03, 2019)

1154-P1-1273 **Vinodh Kumar Chellamuthu***, 225, S University Avenue, Saint George, UT. *Improving Student Ownership in Introductory Statistics Class through a Project-Based Approach.*

The presentation will highlight a semester-long project-based approach in Introductory Statistics class that deepens on critical thinking and conceptual understanding, rather than mere knowledge of procedures. In this presentation, a project-based approach that improves students’ ability to use the techniques and skills learned in Introductory Statistics class in real-world applications will be discussed. The primary goal of this project-based approach is to foster students thinking by creating an experience where students choose a mini-research project based on their passion/major study. During this process, students created their own hypothesis for their chosen project, designed a study, conducted the study, described the data, and made conclusions using the data. The presentation will also highlight feedback from the student’s and their experiences with this project-based approach pedagogy. (Received September 14, 2019)

1154-P1-1802 **Cathy M. Frey*** (frey@norwich.edu), Department of Mathematics, Norwich University, 150 Harmon Drive, Northfield, VT 05663. *Videos Lectures to Enhance Student Learning and Performance in an Online Elementary Statistics Course.* Preliminary report.

Our Elementary statistics course covers the study of frequency distributions, averages and standard deviations, normal curve, probability, decision-making, sampling techniques, testing hypotheses, students-t -distributions, correlation and linear regression. We offer multiple sections of this course online each semester in our adult degree completion programs. This study compares student performance for two groups over six semesters with the same course content, assignments and exams. One group also had benefit of video lectures to supplement the lecture notes in the course. Analysis of the student performance with the video lectures showed a significant increase in average grade, from 2.38 to 2.71 quality points (p-value = 0.0326), as well as a significant decrease in the failure/withdrawal rate, from 16.4% to 4.3% (p-value = 0.0010). (Received September 16, 2019)

1154-P1-2311 **J. Todd Lee*** (tlee@elon.edu). *SOTL- driven evolution of a university probability course.*

The growth of STEM disciplines (especially in data sciences) in middle-sized universities creates a new reality for the purpose and clients in an introductory probability course. In the initially investigated class, the instructor faced the challenge of providing a conceptual framework that is productive for the learning and application of probability to a broad range of majors (Lee, ICOTS10, 2018). The course serves as an upper-level elective for various STEM majors, a required course for math education majors, and a prerequisite for most concentrations in statistics. As with many data science classes, there is an ever-growing variation in the corridors of both prior-knowledge and future aspirations of the students. The granular density metaphor (Lee & Lee, 2014) was used as a core content element to address the needed multi-purpose structure. There is a need for reliable, primary intuitions about what probabilities “represent” in application and on how density functions work. The initial research was on the viability of using a metaphor that blends classical and subjective views on probability distributions. This paper reviews the initial findings and then turns to focus on the resulting reflections and modifications used in teaching the next iteration of the course. (Received September 17, 2019)

1154-P1-2587 **Heather Pierce*** (pierceh@emmanuel.edu). *Using GeoGebra for Statistics.*

Last year, I stopped using statistical tables, and started using calculators in my Introductory Statistics course. I will discuss how I used GeoGebra for statistical calculations and visualizations in class, as well as on exams. I will also discuss how this lead to a deeper understanding of statistics in my introductory course. (Received September 17, 2019)

Tangents to Math Circles

1154-P5-540 **James C Taylor*** (jtaylor@mathcirclesnm.org). *MathAmigos: A Community Math Initiative.*

I present an overview of a broad, and fairly novel, community mathematics initiative in its first two years in Santa Fe, New Mexico. This was the subject of an article in the new Journal of Math Circles published this summer, and in this talk I will summarize our program, including the latest developments from summer and fall of 2019.

At every level, our program embraces community-wide collaboration—from the leadership team, to the elements of the mathematics being implemented (primarily math circles and the Global Math Project’s Exploding Dots), to the funding model. Our MathAmigos program now includes three categories of math circle-related programs: professional development (PD), classroom/after-school weekly student math circles (in which we hope to include math wrangle practice), and math festivals. In our outreach with professional development, student math circles and festivals, we work closely with the Santa Fe Public School district and some charter schools. I will further touch on one of the more novel components of our PD: the use of retired master teachers as classroom coaches. (Received September 06, 2019)

1154-P5-604 **Liz Lane-Harvard*** (elaneharvard@uco.edu), Edmond, OK 73034. *From Math Circles to Science Fairs.*

Math Circle activities are generally low-threshold, high-ceiling. Hence, many problems can be utilized in Science and Engineering Fair competitions. In this talk, potential projects will be presented. Additionally, the structure of a particular Science Fair will be addressed, along with how it incorporates other Math Circle activities. (Received September 08, 2019)

1154-P5-700 **Sharon Lanaghan*** (slanaghan@csudh.edu) and **Carolyn Yarnall** (cyarnall@csudh.edu). *Circles Tangent to Calculus.*

How can a mathematics department provide a meaningful experience to incoming first-year STEM students with a wide variety of high school experiences and place them appropriately on a path to calculus? At CSUDH, we offer a 1-unit problem-solving course using problems from our Math Teachers Circle. The problems involve all students in doing mathematics, and allow them each to contribute and grow at their own level. Meanwhile, algebra review is provided by ALEKS software done primarily outside of class.

How is it working? In this session, we will describe our program and curriculum and share preliminary data collected from the program. (Received September 09, 2019)

1154-P5-753 **Anne M. Ho*** (aho5@utk.edu), 227 Ayres Hall, 1403 Circle Drive, Knoxville, TN 37996, and **W. Christopher Strickland**. *Werewolves and Addicts: An Exploration of Math Modeling and Stigma.* Preliminary report.

Games have been used in classrooms and math circles as a fun way to actively engage students. They can also be used as a more comfortable way to ease into a conversation about a difficult topic. In this talk, we will present the first iteration of a new game which invites participants to think about the effects of stigma on the opioid crisis. This game was first run at a Math Teachers’ Circle and is based on Bezier Games’ One Night Ultimate Werewolf. We will describe the details of the design to connect mathematical thinking with a sociology model of stigma (Goffman 1975) as well as SIAM’s guidelines for teaching math modeling (GAIMME 2019). In addition, we will discuss participant reactions and comments. Lastly, we will present ideas for future work to modify this game for nursing education. (Received September 10, 2019)

1154-P5-1183 **Sarah Cobb*** (sarah.cobb@msutexas.edu) and **Marcos Lopez**. *(MC)²: Math Club as a Math Circle.*

The Midwestern State University Math Club has become a program that allows students to engage with mathematics not usually seen in their undergraduate curriculum. Instead of following a prescribed list of examples and problems, meetings now mimic the structure of a math circle. As a result of this transformation, students have grown in mathematical creativity and independence, and several students have joined our department. We will discuss how we have used math circle topics in this setting and the ways that transitioning to a math circle model has helped us achieve our goals for the MSU math club. (Received September 13, 2019)

1154-P5-1188 **Michael E Matthews*** (michaelmatthews@unomaha.edu), 6001 Dodge Street, University of Nebraska at Omaha, Department of Mathematics/DSC 231, Omaha, NE 68182, and **Greg Lawson** (greglawson@unomaha.edu), **Hannah Seidl** (hseidl@unomaha.edu) and **Rachel Pugh** (rachelpugh@unomaha.edu). *Cold War Escape Rooms (and other Student to Student Math nights)*. Preliminary report.

At the University of Nebraska at Omaha the Noyce Scholars have created a student-run set of Math Nights for local middle school students that we hold several times a year. The events have been very popular and tend to fill up quickly. The most successful math circles of late have taken the form of Escape Rooms - especially themed Escape Rooms. In Spring 2019, the Noyce Scholars hosted a Cold War themed Escape Room. The Scholars were able to successfully integrate a bit of history into their room while creating fun math-related puzzles for the middle -school-aged students to explore. We will share some of our best puzzles and discuss best practices that we have learned about how to run effective Math Nights and Themed Escape Rooms in which students engage in fun and meaningful mathematics. (Received September 13, 2019)

1154-P5-1411 **Maria S. Nogin*** (mnogin@csufresno.edu), 5245 North Backer Avenue M/S PB 108, California State University, Fresno, Fresno, CA 93740, and **Adnan H. Sabuwala** (asabuwala@csufresno.edu), 5245 North Backer Avenue M/S PB 108, California State University, Fresno, Fresno, 93740. *Fresno Math Circle Program: Much More Than Just a Math Circle*.

The Fresno Math Circle program includes a traditional math circle for students as well as local and national competitions and preparation sessions for them, and presentations at local schools. Undergraduate, credential, and graduate students are involved in leading or co-leading various sessions under the guidance of the university faculty. We will describe multiple benefits of this format and how we are building the mathematical community in our area. (Received September 15, 2019)

1154-P5-1598 **Jane H Long***, longjh@sfasu.edu. *Widening the East Texas Math Teachers' Circle*.

The East Texas Math Teachers' Circle has been operating since 2013. In that time, we've partnered with federal grant programs (GEAR UP, NSF Noyce), incorporated a Math Circle demonstration at our MAA section meeting, offered professional development and G/T credits for area teachers, hosted Julia Robinson Mathematics Festivals, incorporated Math Circle problems in seminar courses, and strengthened our departmental connections by welcoming pre-service teachers and faculty. In this talk, we will discuss details of these endeavors. (Received September 16, 2019)

1154-P5-1645 **Corey Pooler*** (cpooler6698@westfield.ma.edu) and **Philip DeOrsey** (pdeorsey@westfield.ma.edu). *Mathematical Zendo*.

Mathematical Zendo is a logic game that actively engages participants in problem solving and critical thinking. While originally created for math circle sessions, we have adapted this game to be used in the mathematics classroom. The goal for participants is to guess a secret rule that has been chosen by the leader of the game. The rules are guided by chosen mathematical topics, and teams of participants compete against each other in order to guess the correct rule first. During this talk we will demonstrate the game, and discuss best practices for its implementation. A teacher guide will also be provided for all participants. (Received September 16, 2019)

1154-P5-1890 **Gabriella Pinter*** (gapinter@uwm.edu), Department of Mathematical Sciences, P.O.Box 413, Milwaukee, WI 53201-0413. *Math Circle-type Activities in K-5 Intervention Classes*.

Multiple research studies show that early math (K-3) is critical for students and is a better predictor of later school success than early literacy or social skills. The UWM Strong Start in Mathematics program was a three-year program for a cohort of K-3 teachers in public schools in Milwaukee supported by a grant from the US Department of Education through the Wisconsin Department of Public Instruction. Throughout the program teachers participated in Math Circle type activities and games connected to number sense, number and operations in base ten and geometry. This program catalyzed a healthy university-school-community partnership that has promising plans to bring the excitement of fun mathematical activities, puzzles and games to the families, schools and communities in Milwaukee County. In this talk we will present some activities that were used successfully in K-5 math intervention classes and that would work well with young children in general. We believe that if children grow up with more exposure to games and puzzles in the family, daycare and kindergarten then they develop better number sense and reasoning skills. Our aim is to spread math excitement in the community to generate curiosity and bring about change in the perception of mathematics. (Received September 16, 2019)

1154-P5-2318 **Chiru Bhattacharya*** (cbhattacharya@rmc.edu), 114 College Avenue, Ashland, VA 23005. *Math Circle problems to undergraduate capstone projects.*

Puzzles and modular arithmetic are popular and accessible math circle topics. We describe two problems with beginnings in math circles which were adapted to undergraduate capstone projects. (Received September 17, 2019)

1154-P5-2721 **Tatiana Shubin*** (tatiana.shubin@sjsu.edu), San Jose State University, Department of Mathematics and Statistics, San Jose, CA 95192-0103. *Math Circles Library as a source of thought-provoking problems for math circles and classrooms.*

MSRI MCL is an AMS book series founded in 2008. By now, 24 volumes have been published and several more are getting ready. These volumes are incredibly versatile, ranging from books intended for use with preschoolers to texts suitable for undergraduate college courses. In the talk we will share a number of gem problems found in the MCL books and will discuss possible future developments for the series. (Received September 17, 2019)

The Teaching and Learning of Undergraduate Ordinary Differential Equations

1154-Q1-127 **Lorelei Koss*** (koss@dickinson.edu). *SIR Models: Differential Equations that Support the Common Good.*

In 2015, UNESCO argued that “education and knowledge are common goods and represent a collective societal endeavor in a complex world based on respect for life and human dignity, equal rights, social justice, cultural diversity, international solidarity and shared responsibility for a sustainable future.” In this talk, we explore how SIR models have been extended beyond investigations of biologically infectious diseases to other topics that contribute to social inequality and environmental sustainability. We present models that have been used to study sustainable agriculture, drug and alcohol use, the spread of violent ideologies on the internet, criminal activity, and health issues such as bulimia and obesity. Teaching these models in an undergraduate differential equations class contributes to the common good as proposed by UNESCO. (Received August 12, 2019)

1154-Q1-214 **Samer Habre***, P.O. Box 13-5053, Chouran, Beirut, 1102 2801, Lebanon. *Students’ Comprehension of Qualitative Techniques in an Inquiry-Oriented Differential Equations Course.*

In Spring 2018, an inquiry-oriented differential equations course was offered at the Lebanese American University. This paper highlights students’ comprehension of qualitative techniques for solving differential equations, such as what constitutes a solution, slope fields, phase lines, and phase planes. The paper also discusses students’ performance on the final exam; more specifically, I present a comparison between their performance on procedural vs. non-procedural problems. (Received August 24, 2019)

1154-Q1-618 **Glenn Ledder*** (gledder@unl.edu). *Modeling the Historical Effects of Overhunting and the Current Effects of Conservation on Whale Populations.*

I will describe a case study that focuses on development and qualitative analysis of a model consisting of a single differential equation for population of a prey species, given a fixed population of predators. The behavior of the model, which can be determined using a nonstandard method for phase line analysis, depends in a critical way on the overall hunting capacity (number of predators times efficiency), a number that has gradually changed through technology and international agreements. The stable equilibrium population can change dramatically as this parameter moves through a bifurcation value, leading to sudden changes in population levels corresponding to small changes in hunting effort. The various scenarios can be strung together to create a historical narrative of whale populations along with a prescription for restoring a sustainable hunting capacity. (Received September 08, 2019)

1154-Q1-747 **Timothy A Lucas*** (timothy.lucas@pepperdine.edu). *Mobile Apps for Exploring Ordinary and Partial Differential Equations.*

I introduce two mobile apps developed by faculty and students at Pepperdine University that allow users to explore numerical methods and graphical solutions to ordinary and partial differential equations. *Slopes* contains activities for investigating slopefields, phase planes, oscillations and explicit numerical methods. *Waves* allows users to plot and animate Fourier series as well as solutions to the heat and wave equations. The name of the app originates from the technique of expressing solutions as a linear combination of sine and cosine waves. Both apps are currently available for iPhone and iPad. One advantage of using these apps is that iPhones and iPads

are highly portable and feature larger touch screens that allow students to view and manipulate content easily. Research based on observations of mathematics courses at Pepperdine University has shown that students are more willing to collaborate and share their results when using tablets such as the iPad (Fisher, Lucas et al. 2013). The intuitive interfaces of *Slopes* and *Waves* invite students to fully immerse themselves in the world of differential equations so that they can understand the concepts from not only algebraic, but also graphical and numerical perspectives. (Received September 10, 2019)

1154-Q1-2322 **Mel Henriksen*** (henriksem@wit.edu), Wentworth Institute of Technology, 550 Huntington Ave, Boston, MA 02016, and **Mami Wentworth**, Wentworth Institute of Technology, 550 Huntington Ave, Boston, MA 02016. *Active Differential Equations, Whiteboard Pedagogy and Slope Fields.*

In our differential equations course, we employ an inquiry-based approach in which students work in groups at the whiteboard for most of the class time. For most lessons, students write their ideas and solutions directly on the whiteboard. We find that standing and working at the whiteboard has these advantages: it facilitates student interaction, it ensures that each person in the group is on the same page, it allows students to view their peers' work and compare it to their own, and it allows the instructor and TAs to see easily what each group is doing. First, we will discuss our Active Differential Equations curriculum, our classroom approach, as well as student feedback to group work in general. Secondly, we will focus on our implementation of this whiteboard pedagogy when covering Slope Fields and Euler's method. Previously, in our traditional classrooms, we had students use applications such as Dfield to draw slope fields and Desmos to draw piece-wise linear approximations to solutions functions. But we have found that using large, poster-size printed slope fields allows students to learn effectively while maintaining the advantages of the whiteboard pedagogy. Software applications are introduced to reinforce students' understanding after the initial lesson with posters. (Received September 17, 2019)

1154-Q1-2596 **Beverly Henderson West*** (bhw2@cornell.edu). *Discussion: The global role of the CODEE Journal.*

The CODEE Journal for the teaching of ordinary differential equations reaches out around the world – consistently 2/3 of the downloads are outside the United States, spread all over the world.. The highest numbers of downloads are in Europe, but Asia, Australia, Africa, North and South America, and the Middle East are well represented. Unique countries pop up too, such as Mauritius.. Commercial as well as educational institutions are listed. The resources of the CODEE Journal are sought by colleagues and students all over the globe. Our 2018 Special Issue Linking Differential Equations to Social Justice and Environmental Concerns has been especially successful, and despite lack of publicity, followed this trend of international service. CODEE (Consortium of Ordinary Differential Equations Educators) would like to continue its long history (started with NSF support in 1992) and incorporate other themed special issues. This will be an occasion to discuss the future of the CODEE Journal. We invite all interested parties to contribute. Can you suggest a theme for another special issue? Would you like the 2018 issue to have a follow-up? Is there a particular topic you would like to see addressed in article? (Received September 17, 2019)

1154-Q1-2637 **Scott A. Strong*** (sstrong@mines.edu), Applied Mathematics and Statistics, 1500 Illinois St., Golden, CO 80401, and **Allison R. Palmer** and **G. Gustave Greivel**. *Engaging STEM Students with Ordinary Differential Equations through Modern Models.* Preliminary report.

Why hasn't differential equations suffered the demise predicted by Rota in his 1997 address to the MAA at Simmons College where he brought into question the typical narrative of introductory courses? Perhaps, contrary to his provocative prediction, its fundamental importance in STEM education may allow the course to persist without any serious reformation into a modern form. Currently, blends with linear algebra and computational environments are common but often at the cost of developing strong connections to modern qualitative analysis. At the same time, emphasizing a dynamical systems perspective often results in a weakened motivation for practicing classical techniques. Of the modern perspectives, where should priorities be placed? In this talk, we describe a pedagogy that leverages cooperative learning engagements to introduce first-year STEM students to ordinary differential equations by threading technical content with mathematical models and modeling. Doing so provides the instructor flexibility in the use of quantitative, qualitative and approximation procedures while motivating learning across a diverse STEM audience. The talk will describe the course's context-rich assignments, supporting student growth in higher-order cognitive domains, and preliminary assessments. (Received September 17, 2019)

1154-Q1-2713 **Itai Seggev*** (is+research@cs.hmc.edu), Urbana, IL. *Using Complex Visualization to Enhance Understanding of DE Solution Techniques.*

Many solution techniques, especially those popular in engineering, involve complex analysis. These include Laplace transforms, Fourier transforms, and complex potentials. Complex numbers are less familiar to students, and in general are hard to visualize. In this talk we survey techniques for visualization of complex-valued functions and functions of complex variables from the view of how they can bring insight to differential equations solution techniques. (Received September 17, 2019)

1154-Q1-2821 **Douglas B Meade*** (meade@math.sc.edu), Department of Mathematics, University of South Carolina - Columbia, Columbia, SC 29208. *Using Interactive Visualizations to Stimulate Conceptual Understanding in Differential Equations.*

The phenomenon of beats in the solution of a periodically-forced spring-mass problem can be difficult for students to grasp because they become bogged down in the algebra. In this talk I will present my approach to this classic topic that starts with an interactive visualization of solutions to develop a conceptual understanding of the phenomenon before embarking on the algebraic, and trigonometric, manipulations to prove the conjectures made based on the graphical evidence. This approach is also effective in many other situations in differential equations, including the appreciation for multiple representations of solutions to first-order (linear and nonlinear) systems of differential equations. (Received September 18, 2019)

Teaching Mathematics with Humor or Pop Culture

1154-Q5-939 **Adrienne Chu*** (chua@sunysuffolk.edu). *From Star Wars Lightsabers to the Broadway Stage Door: Infusing Pop Culture in Statistics, Pre-Calculus and Beyond.*

This presentation will discuss varied examples of how pop culture and contemporary data are infused into mathematics classes at a community college. The presenter will discuss how the Broadway stage door and Google Maps tie into an intermediate algebra class. The presenter will share how Thanksgiving cranberry sauce and Star Wars lightsabers are incorporated into a calculus class. The talk will include other examples of pop culture such as prix fixe restaurant menus, the Avengers: Infinity War movie, Taylor Swift's birthday, and more! (Received September 12, 2019)

1154-Q5-1263 **Jennifer Firkins Nordstrom*** (jnordstrom@linfield.edu) and **Jessica K. Sklar.** *Cinematic Chicken: A Friendly Introduction to Game Theory.*

Many popular films use the game of Chicken to demonstrate conflict between characters. Examining the use of Chicken in films such as *Rebel Without a Cause* (1955), *Footloose* (1984), and *Crazy Rich Asians* (2018) provides students with an introduction to two-player game theoretic concepts such as payoff matrices, Nash equilibria, strategy, cooperation, and defection, while serving as a springboard for discussions of topics such as power, privilege, and gender. Using Linfield's first-year seminar Game Theory in Popular Culture as a model, we discuss the ways in which films can be employed in a general education course that combines introductory game theory with a study of popular culture, allowing students with a wide range of backgrounds and interests to productively engage with mathematics. (Received September 14, 2019)

1154-Q5-2729 **Susan B. Crook*** (susan.crook@loras.edu). *Stories and Jokes that Worked At Least Once.*

All the humor and stories shared in this talk have worked on at least one student - me. I will recount the ways in which my teachers used humor and how I continue to use their stories to help my students relax, enjoy, and remember mathematical concepts. Come find out what 50 the cat has in common with calculus concepts! (Received September 17, 2019)

1154-Q5-2739 **Cesar Martínez-Garza*** (cxm58@psu.edu), Penn State Berks, Tulpehocken RD, Reading, PA 19610. *Vector Calculus Imagery in Science Fiction and Fantasy.*

The use of computer graphics in science fiction and fantasy offers applied and visual examples of Vector Calculus. Whether watching a documentary detailing how a Balrog was created for "Lord of the Rings" or, looking at holograms of gravity-assisted trajectories in "The Expanse", one gets immediate access to visual references of vector fields, parametric curves, etc., in an applied setting that quickly illustrates the mathematical concepts. Students also become more alert to mathematical images embedded in movies and TV shows. A side of mathematical cartoons and the iconic cheeseburger wind-tunnel test will accompany this presentation. (Received September 17, 2019)

Tell Me a Story: Connections between Mathematics and Performed or Print Narrative

1154-R1-46 **Susan D'Agostino***, sdagost2@jhu.edu. *Free Your Inner Mathematician Through Stories*. The best math stories, like the best math teaching, artfully balance technical details with engaging narratives to make the math accessible and compelling. Mathematician writers accomplish this by offering an enticing lead, introducing memorable characters, ensuring dramatic tension, including inviting sketches, and delivering practical and personal lessons. In this talk, participants will hear some of the 46 stories—complete with pictures—included in the speaker's forthcoming book, *How to Free Your Inner Mathematician: Notes on Mathematics and Life* (Oxford University Press, March 2020). These tales weave math content with lessons for mathematical success. Come prepared to hear about Archimedes' pursuits, the Pigeonhole Principle, Fermat's Last Theorem, Penrose patterns, game theory, bacteriophages with icosahedral symmetry, and more. Leave with lessons that encourage you and all aspiring mathematicians to embrace change, proceed at one's own pace, resist comparison, have faith, fail more often, look for beauty, exercise imagination, define success for oneself, and more. Both this talk and the book on which it is based not only deliver engaging mathematical content but provide reassurance that mathematical success has more to do with curiosity and drive than innate aptitude. (Received July 19, 2019)

1154-R1-532 **Kim Regnier Jongerius*** (kimj@nwciova.edu). *Pattern and Structure: "The Curious Incident of the Dog in the Night-Time" and Understanding a Mathematical Mind*. Mathematically-minded people can focus (almost?) obsessively on pattern and structure, while theatrically-minded people tend to focus on emotion. This presentation explores a meeting of the two minds as senior math and actuarial science majors work with actors and stage crew to understand and visually represent the patterned thinking of a teen-aged character who is on the spectrum. (Received September 06, 2019)

1154-R1-902 **Sharon K. Robbert*** (sharon.robbertr@trnty.edu), Trinity Christian College, 6601 W. College Drive, Palos Heights, IL 60463. *Novels with a Geometric Twist*. Preliminary report. Geometry is enhanced by images. Images of polygons and polyhedra, images with symmetry and images illustrating the hyperbolic plane, and images constructed with compass and straight-edge. But what if the images you constructed had the power to protect from nefarious forces—but only if they were drawn perfectly. Or what if your ability was enhanced by a special magical connection to a multi-dimensional graph?

These questions are answered in part by two particular novels: B. Sanderson's "The Rithmatist" and L. Correia's "Grimnoir Chronicles." In this presentation, the novels will be described and how the stories can support understanding of modern geometry. (Received September 11, 2019)

1154-R1-926 **Beverly L. Wood*** (woodb14@erau.edu) and **Debra T. Bourdeau**. *Reaching STEM Students Through Storytelling*.

The presenters will discuss their team-developed and team-taught humanities course, *How Fiction, Film and Popular Culture Represent Science and Mathematics*, a class that both focuses on how the stories of math and science are told and compels students to create their own STEM stories. Texts in the class include an edition of Mary Shelley's *Frankenstein*, annotated for "scientists, engineers and creators of all kinds" and Ray Bradbury's short story, "A Sound of Thunder." Students watch and discuss the film *Hidden Figures* and numerous clips from other movies and television shows such as BBC's "Sherlock" to "The Big Bang Theory" as they explore how mathematical and scientific concepts are portrayed in various media. The course includes blogs, web comics, TEDTalks, and even music videos. Most importantly, students are asked to create their own stories. They do so first through media journals where they—via a podcast or a blend of audio and visuals of their choice—examine representations of science and math in such everyday settings as music, television, film, museum visits, newspaper/magazine articles, books or scenic locations. Students finish the course by creating their own three-act trailer for an imagined project that blends STEM and the humanities. (Received September 11, 2019)

1154-R1-1490 **Andy R. Magid***, Department of Mathematics, University of Oklahoma, 601 Elm Room 423, Norman, OK 73019. *The Simplest Equation – Mathematical Science Fiction of Nicky Drayden*.

We discuss two remarkable metaphors for mathematics from Nicky Drayden's science fiction short story *The Simplest Equation*. The story concerns two young women in an advanced math class at a future Stanford: the narrator, an Earthling Mariah (like the author, a woman of color), and an interstellar extraterrestrial, Kwalla, who become study partners, and then friends. Kwalla teaches Mariah how to understand equations as stories and vice versa (the first metaphor) and then shows her how to use a device from her home planet that turns spoken

equations/stories into evanescent crystalline displays (the second metaphor). As a consequence, Mariah, who has burned out as a math major, rediscovers her love of mathematics. The climax of the story involves Mariah creating the equation that tells the story of her relationship with Kwalla and then producing its crystalline realization.

No mathematical background is required to appreciate this charming tale of doing mathematics. It is suitable for readers from young adult on up. The published version of *The Simplest Equation* may be hard to locate. But an audio version read by Levar Burton is available at <https://player.fm/series/levar-burton-reads/ep-49-the-simplest-equation-by-nicky-drayden>. (Received September 15, 2019)

1154-R1-1774 **Bill Linderman*** (wclinder@king.edu). *A Novel Idea: Teaching Mathematics Using Apostolos Doxiadis's Uncle Petros and Goldbach's Conjecture.*

We discuss how Apostolos Doxiadis's gem of a novel, *Uncle Petros and Goldbach's Conjecture*, can be used as required reading in an upper-level mathematics course as a way to explore several important themes in higher mathematics - the nature of proof, creativity, truth, and obsession. A work of fiction about a mathematical genius, the novel is rich with historical mathematical references and includes cameos by Ramanujan, Gödel, and Turing. (Received September 16, 2019)

1154-R1-2065 **Rebekah Yates*** (rebekah.yates@houghton.edu). *Oh, the Books You Can Read!*

From liberal arts math courses to linear algebra, children's books offer a wealth of mathematical connections. In this talk, I will discuss several children's books that I use in various courses to inspire discussion and deepen understanding. (Received September 17, 2019)

1154-R1-2202 **Paul R Coe** and **Jeanette Mokry*** (jolli@dom.edu). *Discussing the Accuracy of Mathematics in Film.*

In our Senior Capstone, one of the course goals is to think about mathematics in a context beyond the classroom. We spend one class showing and discussing mathematics in film. The film clips fall into one of two categories. Some films have mathematical content and we discuss the correctness (or not) of the content. An example of this is the scarecrow's statement near the end of *The Wizard of Oz*. Other films portray characters doing mathematics and we discuss the social implications of being good (or bad) at math. We often show a clip from *Mean Girls* during this part of the class. In our presentation we will show a couple of short clips that are representative of these two threads and give examples of the types of discussion that usually ensues. (Received September 17, 2019)

1154-R1-2477 **Alison M. Marr*** (marra@southwestern.edu), Southwestern University, 1001 E. University Ave., Georgetown, TX 78626, and **Daniela Beckelhymer** and **D'Andre Adams**. *Choose Your Own Adventure: An Analysis of Interactive Gamebooks Using Graph Theory.*

"BEWARE and WARNING! This book is different from other books. You and YOU ALONE are in charge of what happens in this story." This is the captivating introduction to every book in the interactive novel series, Choose Your Own Adventure (CYOA). Our project uses the mathematical field of graph theory to analyze forty books from the CYOA book series for ages 9-12. We first began by drawing the digraphs of each book. Then we analyzed these digraphs by collecting structural data such as longest path length (i.e. longest story length) and number of vertices with outdegree zero (i.e. number of endings). In this paper we discuss the results of statistical analyses we used to compare books by author, year, and reader preference. We also discuss numerous errors we found in the description of certain books and the publication of others. (Received September 17, 2019)

1154-R1-2725 **Elizabeth A. DeWitt*** (elizabeth.dewitt@trnty.edu). *The Search for Truth in A Certain Ambiguity*. Preliminary report.

Gaurav Suri and Hartosh Singh Bal in their text *A Certain Ambiguity: A Mathematical Novel* invite us to peer over the shoulders of Ravi Kapoor and his classmates as they explore infinity, our pencils in hand to beat them to the completion of several famous mathematical arguments. Juxtaposed is Ravi's discovery of the secret story of his grandfather, who as a young mathematician was jailed awaiting certain conviction for his confidence in Euclid and the axiomatic method. In both the real and fictional classrooms, a key purpose of grappling with infinity, geometry, and their development is, as the fictional Dr. Nico Aliprantis says, "understanding how humans think and understanding the limits of what we can think." By creating an accessible context through the mathematical examples, the struggles of the characters, and even the imagined commentaries of the great mathematicians themselves, the novel exposed students in a capstone course to the philosophical impact of several modern mathematical discoveries and challenged them to consider what we know and how we know it. (Received September 17, 2019)

Interdisciplinary Topics in Mathematics

1154-VC-101 **Brianna Gambacini*** (brianna.gambacini@uconn.edu) and **Sam Macdonald**. *Open and Closed Convexity in 3-Sparse Neural Codes*. Preliminary report.

Neural codes are mathematical models of neural activity. In the early 1970s, neuroscientists discovered neurons called place cells, which fire when animals are in specific (and usually convex) regions in space. Through monitoring these place cells and recording data on when they fire, we can construct neural codes, which tell us what neurons fire together. Of particular interest to the mathematical community is identifying which codes can be represented by open or closed convex sets. Recently, authors have posed several conjectures regarding conditions that distinguish open convex codes from closed convex codes. In this talk, we discuss the difference between open and closed convexity, and examine counterexamples for two conjectures regarding closed convex neural codes. (Received August 06, 2019)

1154-VC-484 **Sarah Ritchey Patterson*** (pattersonse@vmi.edu), Applied Mathematics, 402 Mallory Hall, Virginia Military Institute, Lexington, VA 24450. *Modeling Vascular Congestion in the Microcirculation*.

Blood flow can be modeled as a fluid-structure interaction problem in which the vessel is represented as an infinitely thin, elastic interface that exerts a singular force on the internal and surrounding fluid. The immersed interface method was created to solve this type of immersed boundary problem with second-order accuracy in space and time. However, the interface must be a closed shape, which is not conducive to modeling flow in a vessel.

An extension of the immersed interface method is presented to numerically solve immersed boundary problems where the interface is shaped like an open tube that transverses the fluid domain. Additionally, two mathematical models for simulating renal blood flow under physiological and pathophysiological conditions are presented. The first model simulates the effect of pericyte contractions on vascular congestion in the descending vasa recta. The second model simulates the auto-regulatory myogenic response to changes in systolic blood pressure in the afferent arteriole. (Received September 05, 2019)

1154-VC-710 **Dan Han*** (dan.han@louisville.edu), 2720 Hounz Ln, Louisville, KY 40223, and **Rajib Paul**. *Analysis of Social Network Structure Using Bayesian Exponential Random Graph Model with Heavy Tail Priors*.

Bayesian Exponential Random Graph Models (BERGM) are popular in accounting uncertainties in social networks. Usually, Gaussian priors are used for model parameters in BERGM. We develop a set of heavy tail priors using scale mixtures of Gaussians and show that this prior outperformed the previously existing priors in terms of model fitting and predictions. The proposed method is highly flexible because users can control the shapes of the distributions by carefully selecting the smoothness parameters. This new prior can be implemented using population Markov Chain Monte Carlo algorithms and adaptive direction sampling techniques. Through a series of simulations studies, we calculated the acceptance rate of the model that is the probability of accepting the proposed change in the network based on the resulting posterior distribution. Higher acceptance rates indicate a better fit of the model to the data. Further, we applied our method on friendship network data collected from Goodreau's Faux Magnolia High School and demonstrated that our prior has higher acceptance rates than existing methods. (Received September 10, 2019)

1154-VC-1177 **Steven M Deckelman*** (deckelmans@uwstout.edu), Department of MSCS, University of Wisconsin-Stout, Menomonie, WI 54751. *The Quantum Mechanical Approach to the Riemann Hypothesis*. Preliminary report.

Is there a physical reason the Riemann hypothesis should be true? *Interdisciplinary mathematics* is often thought of in terms of applications of mathematics to other disciplines. This talk will go in the converse direction highlighting an example where quantum physics has the potential to inform the Riemann hypothesis. In 1914 George Pólya suggested that one possible physical approach to the Riemann hypothesis would be to find a physical problem for which the imaginary parts of the nontrivial zeros of the zeta function were so connected with the physical problem that the Riemann hypothesis would be equivalent to the fact that all the eigenvalues of the physical problem be real. In the decades that followed, quantum mechanics became a natural reservoir to search for such physical problems because it is replete with (unbounded) self-adjoint operators (whose eigenvalues are always real) which model quantum observables. Over time this problem became known as the Hilbert-Pólya conjecture. In recent years, a flurry of popular media and blog articles have appeared, some sensationalizing quantum mechanics as a breakthrough approach to proving the Riemann hypothesis, with others being more cautiously optimistic. This talk will explain some of the ideas to non-experts. (Received September 13, 2019)

- 1154-VC-1293 **James D Martin*** (james.martin@hsutx.edu), HSU Department of Mathematics, 2200 Hickory St, #16060, Abilene, TX 79698. *A naive mathematician's approach to finding asteroids.*

Three years ago I moved into the office next to the director of the International Astronomical Search Collaboration (IASC). Every month, IASC distributes images taken from telescopes on mountaintops in Hawaii to high school and college classrooms throughout the world to search for asteroids. The reason is that, even with automated sky surveys, faint asteroids are often lost in the noise of the images. By blinking a sequence of photos, the human eye can recognize the motion of these faint asteroids across the sky when computers cannot.

As a professional mathematician, very amateur astronomer, and recovering software developer, I wondered if there was a way to help guide the eyes of thousands of schoolchildren toward undiscovered objects. My approach was naive, but there are some nice results to share. (Received September 16, 2019)

- 1154-VC-1314 **Michael Jones** and **David McCune***, mccuned@william.jewell.edu, and **Jennifer Wilson**. *A Survey of Proportional Apportionment Methods Used in the Republican Presidential Primary.*

We survey the apportionment methods used by the Republican Party in their 2012 and 2016 state presidential primaries, with a focus on the seven methods that are proportional. All of the proportional methods are quota-based, and all but one are new (or at least previously unstudied). After comparing the apportionment methods for three candidates using simplicial geometry, we evaluate how they differ in bias toward the top and bottom vote-getting candidates. We use the bias comparisons to suggest which methods should be used at different junctures in the primary season. We discuss how these new methods were implemented in practice and summarize how successful these methods were in making the Republican delegate process more proportional. (Received September 14, 2019)

- 1154-VC-1444 **Ho-Chun Herbert Chang*** (hochunhe@usc.edu), 3502 Watt Way, Suites G4 & G6, Los Angeles, CA 90089-0281, and **Feng Fu**. *Social Network of Mathematical Concepts: A Study on Wikipedia.* Preliminary report.

Collaborative encyclopedias like Wikipedia provide a unique opportunity to understand the temporal, social and historical dynamics of how knowledge is organized. In mathematics, a theorem is any true statement generated by a set of axioms by logical consequence. While there are multitudes of "theorems," there is some notion of importance attributed to key theorems. Wikipedia only lists 1,063 theorems, many of which do not have their own page. In other words, mathematical concepts and their curation arise from collective and social consensus. This paper "maps" how mathematical concepts are popularly organized on Wikipedia, its editorial behavior and mathematicians in history. A multi-layer social network of mathematical objects (axioms, theorems, and conjectures) is constructed using graph-theoretic metrics. Results reveal a core-periphery topology on each layer, including a giant components. Raw categories (224 total) follow a power-law distribution, which is typically consistent with human curation and consensus. Furthermore, many theorems spill over to other fields in the natural and social sciences, for which we provide preliminary results, which yields insight into historical collaborations between pure and applied mathematics and also potential future synergies. (Received September 15, 2019)

- 1154-VC-2336 **Chris Fagin*** (scfagin@yahoo.com), 650 E Parkway S, Memphis, TN 38104, and **Yudong Tao, Yeojin Amy Ahn, Laura Vitale** and **Daniel Messinger**. *Analyzing Facial Expression Data in Autism Research.*

The Autism Diagnostic Observation Schedule, 2nd version (ADOS-2) is a 40-60 minute Autism Spectrum Disorder (ASD) diagnostic assessment in which a child's behavioral symptoms are observed. In the ADOS-2, the child's parent and examiner wear Pivothead[®] glasses with an embedded video camera while the child interacts with the parent and the examiner. The goal in this study is to utilize objective measurements available through commercial software. This is done by manipulating raw facial expression data matrices using eigenvalues to find latent factors in facial expressions in a statistical technique known as Exploratory Factor Analysis (EFA). The latent factors are then correlated with ADOS-2 Calibrated Severity Score (CSS) Social Affect scores. This talk will describe the linear algebra methods employed in the study. (Received September 17, 2019)

- 1154-VC-2544 **Jonathan Fischer*** (jrfischer@berkeley.edu), jrfischer@berkeley.edu, and **Dan D. Erdmann-Pham** and **Yun S. Song**. *A generative model-based deconvolution of bulk expression data using single-cell references.* Preliminary report.

Bulk gene expression experiments estimate thousands of transcript levels averaged over myriad cells. Unfortunately, direct comparison of different bulk expression profiles is complicated by the mixtures of distinct cell

types in each sample, obscuring whether perceived differences are actually due to changes in expression or cell type composition. Recent advances make it possible to measure gene expression in individual cells, achieving higher resolution in exchange for increased noise. If carefully incorporated, such data can be used as references for the supervised deconvolution of bulk samples to yield cell type proportions. This permits us to disentangle the effects of differential expression and cell type mixtures, both of which are relevant to our understanding of aging and disease. Previous solutions prioritize the inference of cell type proportions at the expense of error quantification or the ability to identify when cell types are missing from the reference. We thus propose a generative model which uses asymptotic theory and a robust estimation procedure to supplement cell type proportion estimates with confidence intervals and a hypothesis test for reference cell type missingness. We demonstrate the effectiveness of our approach on real and simulated data. (Received September 17, 2019)

1154-VC-2635 **David McCune, Lori McCune*** (lmcune@missouriwestern.edu) and **Dalton Nelson**. *The Cutoff Paradox in the Kansas Presidential Caucuses*.

The Kansas Republican Party uses an apportionment method of its own invention to apportion delegates to candidates in its presidential caucus. This method includes a threshold that eliminates candidates (and their votes) who receive less than 10% of the vote. Eliminating candidates can lead to a *Cutoff paradox*, a paradox in which a surviving candidate receives fewer delegates as a result of the elimination of the candidates that fall beneath the threshold. We compute the proportion of elections that are susceptible to this paradox for three candidate elections using the method of the Kansas Republican Party and compare this to the same proportions for Hamilton's method, the method used by the Democratic Party in all of its presidential primaries and caucuses. (Received September 17, 2019)

1154-VC-2688 **Boyan S. Kostadinov*** (bkostadinov@citytech.cuny.edu). *Iterated Circular Convolutions in the Binomial Options Pricing Model*.

This work was inspired by the observation that iterated circular convolutions can be used to express the martingale pricing algorithm in the binomial options pricing model. We use discrete Fourier analysis in this context of iterated circular convolutions to derive analytical results that could prove useful for fast real-time options pricing on algorithmic trading platforms, as well as for a deeper theoretical understanding of no-arbitrage options pricing in terms of the inverse Fourier transform of the terminal option payoff, and the Fourier transform of a special weight vector built from the risk-neutral probabilities. This work is a direct application of a surprisingly similar problem in geometry that we have investigated in the context of limiting forms of iterated circular convolutions of random skew polygons in higher-dimensional Euclidean spaces. (Received September 17, 2019)

1154-VC-2783 **Edward Dougherty***, Roger Williams University, One Old Ferry Road, Bristol, RI 02809, and **James Turner** and **Frank Vogel**. *Iterative Methods and Preconditioners for Finite Element Method Based Simulations of Transcranial Direct Current Stimulation*.

Mathematical modeling and numerical simulation of neurostimulation treatments has provided a bridge between practicing clinicians and in silico experimentation, enabling this field to investigate transcranial direct current stimulation (tDCS) with patient-specific, computer-based solutions. A drawback of this approach is the burden in solving the partial differential equations that model tDCS, given the need by the medical community to utilize patient-specific head geometries and finely discretized computational grids. To address this issue, we compare the performance of distinct numerical approaches when solving the linear system of equations produced from tDCS based finite element discretizations. Computer experiments incorporate medically-based tDCS electrode configurations on actual MRI-generated head geometries with biological brain tissue conductivity values. In addition, convergence performances of each solution approach are lined to theoretical estimates. Results show that a properly configured geometric multigrid preconditioner for Krylov subspace based linear solvers achieve superior convergence rates when compared to alternative approaches. In addition, we demonstrate that real-world aspects of tDCS yield multigrid as a stand-alone solver highly inappropriate. (Received September 17, 2019)

Mathematics and Technology

1154-VD-330 **Brittany Anne Carlson*** (bcar1005@ucr.edu). *Mediating Victorian Geometry Anxiety Through Curve Stitching*.

The turbulent mathematical scene of the nineteenth century was ridden with anxieties regarding the crumbling formal axiomatic foundations of Euclidean geometry, which was previously considered a gold standard in

mathematics in the early 19th century. Consequently, many anxieties regarding its validity and utilitarian purpose emerged in mathematical and non-mathematical communities alike. To learn Euclidean geometry in this anxiety-ridden climate, geometry required mediation through less-threatening media. In this paper, I assert that everyone ranging from the young men studying geometry for the first time to professionals – including fictional ones such as Sherlock Holmes – mediated geometry anxieties by blurring the boundary between mathematics and art/play with common ephemera, including string, curve-stitching (as popularized by Mary Boole), and cigarette cards. By doing so, not only did people confront their anxieties and learn geometry, but their practices also contributed to the field of mathematical knowledge as a whole, thus turning the geometrically anxious into geometers in their own right. (Received August 31, 2019)

1154-VD-1475 **Nilakantha Paudel** (nilu.paudel@gmail.com), , Italy, and **Ram C Neupane*** (ram.neupane@tamut.edu), 7101 University Avenue, BASS 221, Texarkana, TX 75503. A *Standard Architecture for a Real-Time Monitoring System Based on the Internet of Things*.

The recent use of the real-time monitoring system (RTMS) based on the Internet of Things (IoT) is growing exponentially. It has some specific challenges according to its architectural design and functional domain. In this paper, we study the architectural design of various real-time monitoring systems that are designed to monitor the different real-world environments. After that, we have combined the common architectural concepts of those designs, and then we have proposed a standard architectural design for RTMS based on the IoT. Our architecture combines various technological tools and innovative practices. We also explain how to implement and extend our proposed architecture to monitor the specific field, and hence we believe that this architecture can be a standard architecture for real-time monitoring systems. Moreover, it can be extended to develop a system that helps to improve the traditional students' learning paradigm to the modern virtual technological environment (VTE) model. The VTE facilitates students to enhance their knowledge from different perspectives. (Received September 15, 2019)

1154-VD-1515 **G. Bezhanishvili, N. Bezhanishvili** and **J. Lucero-Bryan*** (joel.bryan@ku.ac.ae), Khalifa University of Science and Technology, Department of Mathematics, PO Box 127788, Abu Dhabi, United Arab Emirates, and **J. van Mill**. *Existence of Measurable Cardinals and Modal Logic*.

Over the years several links between set theory and modal logic have been discovered. There is an interesting connection between non-well-founded set theory and infinitary modal logic. The modal logic of forcing extensions of ZFC is S4.2. The only proof that the modal logic of the Čech-Stone compactification $\beta\omega$ of the discrete space ω is S4.1.2 uses that each MAD family has cardinality 2^ω ; which is not provable in ZFC and whose necessity remains open.

Our contribution concerns measurable cardinals and topological semantics of modal logic. We show the existence of a measurable cardinal is equivalent to the existence of a normal space whose modal logic is the modal logic L of the Kripke frame isomorphic to the powerset of $\{a, b\}$.

Assume κ is a measurable cardinal. A countably complete ultrafilter on κ yields a normal P -space Y which embeds into the remainder of $\beta\mu$ of a discrete space μ . Then $Y \cup \mu$ is a normal subspace whose logic is L. Conversely, assume Z is a normal space whose logic is L. Then there is a subspace of Z containing a P -point. From this we build an Ulam-measurable family of subsets of Z ; implying the existence of a measurable cardinal. (Received September 16, 2019)

1154-VD-2229 **Gerald Kruse*** (kruse@juniata.edu), Juniata College, 1700 Moore St., Huntingdon, PA 16652. *Using a Mathematical Software Package in a Class Exercise to Demonstrate Google's Simple PageRank Algorithm*. Preliminary report.

In many cases, the effectiveness of demonstrating applications of underlying mathematical theories is hampered by the accessibility or specificity of the tools used. An in-class exercise will be presented which demonstrate Google's PageRank algorithm for search results. In particular, a mathematics software package (in this case, Maple), will be used to highlight applications of topics in Linear Algebra and Graph Theory. A sample network of web pages and their forward facing links will be represented as a graph, this graph will be used to construct the corresponding PageRank matrix in Maple, and then the eigenvectors of this matrix will be calculated and used to provide a search ranking. While the PageRank algorithm is deprecated, this example is still an elegant introduction to Google's first search technique. (Received September 17, 2019)

1154-VD-2236 **Yevgeniy Galperin*** (egalperin@esu.edu), 200 Prospect St, East Stroudsburg, PA 18301. *Digital Image Processing in Undergraduate Math.*

We discuss the use of digital image processing as a vehicle to review the fundamental concepts and techniques taught in the undergraduate mathematics curriculum. We also demonstrate that digital image processing is the right framework for introducing Complex Analysis, Fourier Analysis, and Wavelet Theory at the level accessible to undergraduate students. This approach also allows us to introduce basic computer programming techniques in a fun and engaging context. (Received September 17, 2019)

1154-VD-2362 **John Robert Botzum*** (botzum@kutztown.edu), 5528 Heather Lane, Orefield, PA 18069, and **Andrew Martin.** *Conflicting Sines.*

Three hundred plus years since Leibniz first published *Acta Eruditorum* (1693) and Newton sent Leibniz the "Epistola Posterior" (1676) the debate as to who merits the honor of having created Calculus rages on. There are those who would attribute the creation to Newton, those who would attribute it to Leibniz, those who would give the nod to both, and even those who maintain that Leibniz stole Newton's work. We are not so presumptuous as to claim we have settled the debate but evidence is presented, via the infinite series for sine, that strongly supports one of the above. We also unabashedly dangle a result proven by a 15th century mathematician from across the globe to suggest that possibility that his work was the original source. (Received September 17, 2019)

1154-VD-2424 **Dushanthi N Herath*** (dherath@maryville.edu), 650, Maryville University Drive, St. Louis, MO 63141. *Incorporating Technology to make Calculus Interactive and Engaging in the Classroom.* Preliminary report.

When it comes to Mathematics, teaching harder concepts with the use of technology helps students to see what is going on clearly and will have them interested in the subject. As faculty, we need to explore exciting ways to keep students motivated, engaged, and be active learners in the class. Maryville University strives towards creating an Active Learning Ecosystem (ALE) in the classroom by providing students and faculty iPads and technology support. Students can use their iPads in class to access their LMS, which is Canvas for the course and many other educational apps. In this presentation, I will be focusing on the GeoGebra app, which I use for teaching calculus for students to be able to visualize and understand challenging concepts. GeoGebra is a Dynamic Mathematics Software (DMS) which is freely available and used for teaching and learning mathematics. I use GeoGebra to explain concepts, and also to let students work individually or in groups to explore concepts learned in the classroom. A few concepts that are taught using GeoGebra are functions, 2D and 3D graphs, vectors, Integrals, limits and differential equations. This presentation will demonstrate several tools in GeoGebra that is used in the calculus course. (Received September 17, 2019)

1154-VD-2517 **Bob Carmichael***, bobc@edfinity.com, and **Sid Grover**, sid@edfinity.com. *A toolkit for breaking expensive textbook/homework bundles: Case studies on using a textbook-agnostic homework system to create affordable online homework to reach every student.*

We present a bouquet of diverse case studies showcasing educators who developed low-cost online homework using Edfinity, an open, WeBWorK-compatible homework system supported by the NSF (Award #1758301). Leveraging Edfinity's community-contributed and peer-reviewed WeBWorK problem repository, educators assembled 'textbook-agnostic' homework mapped to both publisher and OER textbooks, thereby 'unbundling' expensive textbook/homework bundles offered by commercial publishers and pairing quality online homework with any publisher/OER textbook of choice. The case studies also demonstrate collaboration and reuse of assessments within and across institutional boundaries using Edfinity's centrally hosted architecture. These efforts have increased equitable access for students at these institutions, and provided a scalable template for other institutions. (Received September 17, 2019)

1154-VD-2614 **Janine E. Janoski***, janinejanoski@kings.edu. *Axiomatic Systems.*

Euclid stated five axioms for geometry, from which 465 mathematical truths were logically deduced. At King's College, math majors take a Logic and Axiomatics class first semester their freshmen year. As part of the class, each student creates their own axiom system. They create four axioms, find an independence model for each axiom, find a consistency model for their system, and create/prove four statements. In their junior year, students revisit and revamp their axiom system. In this talk, we will discuss these systems and will show examples of such systems. (Received September 17, 2019)

1154-VD-2680 **Timothy Trujillo*** (txt031@shsu.edu). *Infinite paralysis*. Preliminary report.

We describe a mathematical framework to formally express the problem of ‘infinite paralysis’ first pointed out by the philosopher Nick Bostrom [1]. The framework allows the argument to be broken into two main components. First we identify properties of a formal system that lead to a value rule and value order rule that are non-Archimedean. Then we show that instantiations of these formal systems where the codomain is taken to be the standard cardinal arithmetic of set theory suffer from the problem of ‘infinite paralysis’. We conclude with some connections to social welfare theory and infinite dimensional Ramsey theory.

REFERENCES

[1] Nick Bostrom. *Infinite Ethics*. Analysis and Metaphysics, 2013.

(Received September 17, 2019)

1154-VD-2707 **Peyam Ryan Tabrizian*** (ptabrizi@uci.edu), Department of Mathematics, University of California - Irvine, 340 Rowland Hall, Irvine, CA 92697-3875. *Using YouTube videos to promote learning inside and outside the classroom*. Preliminary report.

I will describe my experience of using YouTube videos to promote learning both inside and outside the classroom. I currently have a channel called “Dr Peyam” which has over 30,000 subscribers and over 450 videos, on which I regularly post videos related to my lectures, like the ‘Divergence Theorem’ or ‘Gaussian Elimination,’ as well as others that are accessible to a wider audience, like ‘Half Derivatives’ or ‘The Integral of x^x from 0 to 1.’ Those videos are for students and people around the world who are interested in math, and the advantage is that anyone can watch them online or on their phones, and ask questions and comments. For my students in particular, I upload videos on topics covered in lecture, practice question videos, and videos on further topics. I am also holding virtual office hours by using the live stream option, so that the students can have office hours delivered right to their homes. (Received September 17, 2019)

1154-VD-2770 **Joshua Cole*** (joshua.cole@findlay.edu). *Finitely branching computable trees: a way to think about almost-solvable unsolvable problems*. Preliminary report.

One of the most interesting uses of computability theory is to study the border between the solvable and the unsolvable. A good example is the theory of non-computable computably enumerable sets. When we study sets of real numbers, we can think of a Π_1^0 class with no computable member (which can be characterized as the set of infinite paths through a binary branching tree) as the computationally simplest class of reals lacking a computable member. It is especially nice that the computable tree eventually (in finite time) rules out any finite string that in the end is not the initial segment of any real in the class. We suggest broadening this conception from binary branching trees to finitely branching ones and argue we are still right at the border of the solvable and unsolvable. Thus we suggest it is worthwhile to study together at once the collection of classes of paths through computable finitely branching trees. This line of thinking is based on an idea of Hirschfeldt in his book, *Slicing the Truth*. We discuss examples from computability theory of interesting computable finitely branching trees, note the equivalence with $0'$ -computable trees, and use these ideas to discuss the use of $0'$ oracles in the study of mass problems. (Received September 17, 2019)

Mentoring

1154-VE-70 **John A Kerrigan*** (johnkerr@math.rutgers.edu), 110 Frelinghuysen Road, Piscataway, NJ 08854. *Self-Efficacy in the Flipped Calculus II Classroom*. Preliminary report.

Self-efficacy (Bandura, 1977), an individual’s belief in his or her ability to succeed at a specific task, is a predictor of student performance and persistence in mathematics (Pajares & Miller, 1994; Zeldin & Pajares, 2000). Thus, it is important to understand how students’ self-efficacy changes in different settings. When designed carefully, certain mathematics learning environments are more conducive to students’ development of self-efficacy as they allow for multiple self-efficacy opportunities (Sawtelle, Brewé, & Kramer, 2012). Flipped classrooms (Lage, Platt, & Treglia, 2000) reverse classroom lecture and out-of-class assignments and may increase self-efficacy, as students have opportunities for collaborative work and instructor feedback during class. The purpose of our study was to investigate changes in students’ self-efficacy in a flipped Calculus II course. Quantitative findings included significant increases in students’ self-efficacy in calculus. Qualitative findings revealed that students believe their previous mathematics courses and active learning opportunities impact their mathematics self-efficacy. (Received July 30, 2019)

1154-VE-411 **Samuel Obara*** (so16@txstate.edu), San Marcos TX, 78666. *Role of Instruction on Spatial Skills on Middle School Students.*

The curriculum should give students opportunities to visualize and represent geometric figures with special attention to developing spatial sense. Spatial sense is the mental process used to perceive, store, recall, create, edit, and communicate spatial images. This study was conducted to investigate the effects of instruction on spatial visualization skills of sixth and seventh-grade students. About 40 students from two sites, representing a wide range of socioeconomic status, participated in the study. The spatial visualization unit engaged students with 2-D figures, 2-D pentominoes, 3-D pentominoes, Dime solids and 3-D Dynamic software. The instrument used was the Spatial Visualization Test, with a test-retest reliability of .79, and Cronbach's reliability coefficients for various groups of students ranging from .72 to .86 on the pretest and from .82 to .88 on the posttest. (Received September 03, 2019)

1154-VE-666 **Frederick M Butler*** (fbutler@ycp.edu). *Mentoring Students Through Research Projects.* Preliminary report.

In this paper the author will discuss his experiences conducting research projects with several students. Details will be given about the varying levels and degrees of success of the projects. The author will describe how he used these projects to help in the mentoring of the students who participated. (Received September 09, 2019)

1154-VE-745 **Timmy Ma***, Department of Mathematics, Dartmouth College, HB 6188, Hanover, NH 03755. *Formative versus summative assessment: a way to engage in a math classroom.* Preliminary report.

We discuss the difference between formative and summative assessment and how to engage students in introductory classes such as Calculus. We will also have a demonstration for the audience to get a first-hand look at how Kahoot! can be used in the classroom as a formative assessment and how group quizzes can be used as a summative assessment. (Received September 10, 2019)

1154-VE-1032 **Paul R. Bialek*** (pbialek@tiu.edu), Dept of Mathematics and Computer Science, Trinity International University, 2065 Half Day Rd, Deerfield, IL 60015. *Teaching iGen: Characteristics of Generation Z and implications for the college mathematics classroom.*

Effective teachers need to know their audience. The current generation of traditional college students is from Generation Z (or iGen), which is roughly those born between 1995 and 2012. Gen Z thinks differently from those of us who are Millennials, Generation X, Baby Boomers, etc. We will discuss some of these differences so that you can be a more effective teacher and mentor to your students. (Received September 12, 2019)

1154-VE-1736 **Elizabeth A. Donovan***, 6C Faculty Hall, Murray, KY 42071, and **Lesley W. Wigglesworth.** *Mentoring Cross Institutional Undergraduate Research Projects.*

The positive effects of undergraduate research on students is well documented. Students who participate in undergraduate research experiences report improved communication skills, growth of self-confidence, improved focus on a career path, as well as positive improvements in student learning and attitude. In this talk, we will discuss a year-long research collaboration between undergraduate students at Centre College and Murray State University. Successes and challenges throughout the process the mentoring process, as well as our own reflections, will be presented. (Received September 16, 2019)

1154-VE-1747 **RaKissa D Manzanares*** (rakissa.manzanares@ucdenver.edu), Campus Box 170, P.O. Box 173364, Denver, CO 80217-3364, and **Michael Ferrara, Michael Jacobson** and **Gary Olson.** *Lessons Learned in Designing & Implementing a Peer Mentoring Component to a Comprehensive Training Program for GTAs in the Mathematical Sciences.*

Graduate teaching assistants (GTAs) play a significant role in early undergraduate education, particularly in introductory mathematics and statistics classes, making the pedagogical preparation of this population imperative. This paper focuses on the peer mentoring aspect of a comprehensive pedagogical training program for GTAs developed in a mathematical sciences department at a mid-sized urban-serving university. Our data include classroom observations made by the faculty organizers and bi-weekly reflections submitted by peer mentors and mentees. In addition to providing an overview of our peer mentor training program, we discuss some lessons learned and program modifications that enhanced support for peer mentors that may be of interest to other departments hoping to incorporate peer mentoring into their GTA training. We also share some benefits reported by mentors and mentees in their reflections and briefly outline how the data informed a potential framework for mentor identity. (Received September 16, 2019)

- 1154-VE-1849 **Robert Campbell and Anne Sinko***, 37 S. College Ave, St. Joseph, MN 56374. *In Class Versus Out of Class - A Comparison of Mastery Grading Retakes in an Introductory Statistics Course*. Preliminary report.

Alternative assessment techniques have been a recent avenue of mathematics education research. While there are many different flavors of mastery grading, a common component is that mastery must occur eventually, but the time or pathway an individual student takes to achieve mastery does not matter. In practice, this means that students have built in opportunities for revisions or retakes to show mastery of learning objectives. This study aims to determine what, if any, differences occur with respect to students' successes in demonstrating mastery of ideas as well as differences in student perceptions and anxiety levels when retesting opportunities are conducted outside of standard class time versus inside of standard class time. This talk will present our preliminary results for introductory statistics courses. (Received September 16, 2019)

- 1154-VE-1879 **Suleyman Tek*** (tek@uiwtx.edu), 5432 Nutmeg Trl, San Antonio, TX 78238. *A Support Structure of Mathematics Department at a Hispanic Serving Institution*.

In this presentation, we are going to discuss San Antonio Mathematics Scholars at UIW (SAMS@UIW) program which is an S-STEM project funded by National Science Foundation at the University of the Incarnate Word. The goal of SAMS@UIW is to increase the number of academically talented but financially needy students qualified to enter the STEM workforce or graduate school upon completing a bachelor's degree in Mathematics and Statistics. We discuss how we recruit scholars and discuss the support system that we use for retention and graduation. We also discuss how we use community service opportunities as team bonding activities and also how they can keep it as part of their profession after they graduate. We discuss the successful activities and lessons we learn. (Received September 16, 2019)

- 1154-VE-1939 **Fidele Ngwane***, 807 Hampton Street, Walterboro, SC 29488. *Investigating item parameter distributions*. Preliminary report.

We investigate the effects of item parameters on the amount of information that a test can provide using different Logistic Parametric Models. We run some simulations on various item parameters on short and long exams, and on a different examinee sizes. We also investigate how the distribution of the item parameters affect the test information. (Received September 16, 2019)

- 1154-VE-2216 **Achyuth Syamchandra*** (achyuthc@gmail.com), **Alphonse O'Bannon, Ruchira Sarkar, Nanda Kumar, Roopsa Sen, Ojus Sharma and Aditya Nair**. *An Inquiry into the Relationship Between Participation in Competitive Creative Math Competitions and Measures of Academic Success in High School Students*. Preliminary report.

Creative math competitions are increasingly being used by universities as an indicator of a student's general ability to problem solve.

This study seeks to examine the relationship between participation in a creative math competition and successful academic performance and behavior, using the MAA American Mathematics Competitions. Three years of academic performance data for students who wrote the MAA AMC will be compared to a control group of students who did not take the MAA AMC. The study will also compare non-cognitive measures of academic success, specifically academic self-efficacy, motivation, and engagement. It will also analyze the relationship between demographic factors and competition success, specifically gender, ethnicity, and household income. The authors' hypothesis is that the correlation between participation in creative math competitions and measures of academic success will be high. The results of this study could be used to justify the use of MAA AMC scores as a metric in the college admissions process, and encourage students to participate in creative math competitions.

This talk will present the validity of the authors' hypothesis, the detailed framework of the study, the data that was collected, the analysis and the conclusion. (Received September 17, 2019)

- 1154-VE-2243 **Bernadette Mullins*** (bmullins@bsc.edu), **Catherine Cashio and Allie Ray**. *Inter-rater reliability in departmental assessment*. Preliminary report.

One of the learning outcomes for the mathematics major is for students to be able to use mathematical methods to solve quantitative problems. We assess this problem-solving outcome using a rubric with four dimensions: representation, manipulation, interpretation, and communication. In a small department at a liberal arts college, faculty members collaborated to apply the rubric consistently, and subsequently scored problems independently to assess inter-rater reliability. (Received September 17, 2019)

1154-VE-2553 **Perry Y.C. Lee*** (plee@kutztown.edu), Lytle Hall 267, Kutztown University of Pennsylvania, Kutztown, PA 19530, and **Padraig M. McLoughlin** (mclough@kutztown.edu), Lytle Hall 265, Kutztown University of Pennsylvania, Kutztown, PA 19530. *College Algebra Classrooms - Assessment Study and the Challenges Ahead*. Preliminary report.

I have been using active learning strategies (both Flip and IBL methods) in my 'large' and 'small' College Algebra classrooms since Fall 2014. Also, during this time period, I have been gathering students' pre- and post-assessment data from these classrooms. I will present a summary of the results of the assessment study, and discuss the challenges in conducting such an assessment study in College Algebra. (Received September 18, 2019)

1154-VE-2578 **Elizabeth Schott** (eschott@fsw.edu), **Laurice Garrett** (laurice.garrett@fsw.edu) and **Kelsea Livingstone*** (kelsea.livingstone@fsw.edu). *Creating a Mathematics Community of Best Practices*.

Over the last decade, the faculty members of the math department of Florida SouthWestern State College have explored methods to foster a collaborative community for investigating the best techniques for teaching mathematics. This talk will look at the process initially used to establish a formal community of best practices in our department and trace its progress over the last five years. The discussion will focus on the challenges of maintaining the process over time, the success of our integration into the community of professional development activities within college, and the challenges of maintaining the community to include the selection and presentation of relevant topics, the challenges of incorporating adjuncts, and the challenge of including all faculty across multiple campuses. Finally, we will share the best and worst of our past meetings. (Received September 17, 2019)

1154-VE-2619 **Vira Babenko**, Department of Math and Computer Science, Drake University, Des Moines, IA 50311, and **Maryann Huey*** (maryann.huey@drake.edu), Department of Math and Computer Science, Drake University, Des Moines, IA 50311. *Establishing a Peer-mentoring program for Underrepresented Populations of Students at a Liberal Arts College*.

In this talk, we will describe our vision and associated plans for launching a peer-mentoring program for underrepresented groups in Mathematics, Computer Science, and Data Analytics including females, minorities, first-generation, and LGBTQ students. Mentors were recruited from last-year students and mentees are first-year or transfer students in the Department of Mathematics and Computer Science at Drake University. We will share our successes, challenges, future plans, and recommendations. (Received September 17, 2019)

Modeling and Applications

1154-VF-120 **Raminder Pal Singh*** (raminder.19058@gmail.com). *Modelling and Analysis of Effect of Delay Induced by Toxic Metal on Plant Dry Weight*.

In this paper, a mathematical model is proposed to study the effect of delay induced by toxic metals present in the soil which adversely affect the plant growth by affecting the dry weight of plants. For the analytical study of this adverse effect, the model is divided into root and shoot compartment. The same effect is studied by introducing the delay parameter in the term involving growth rate. The model is governed by system of nonlinear delay differential equations. Three state variables are-Amount of structural dry weight in root, amount of structural dry weight in shoot and concentrations of toxic metal in soil. Hopf bifurcation occurred at a critical value of parameter time delay. To support the analytical results, numerical simulation is done using MATLAB. (Received August 10, 2019)

1154-VF-275 **Mahmoud El-Alem*** (mmelalem@hotmail.com), Department of Math and CS, Faculty of Science, Alexandria University, Alexandria, Egypt, and **Salem Mahdi** (samath2014@yahoo.com), Educational Research and Development Center, Sa'ana, Rep of Yemen. *A Hybrid Evolutionary-Gradient Algorithm for Finding the Global Minimizer of a Nonlinear Optimization Problem*.

A novel design of hybrid evolutionary-gradient method for solving optimization problem will be introduced. The method utilizes in a natural way the simulated-annealing algorithm and the gradient method to enhance the performance of the overall method. The proposed algorithm finds the global minimizer for a nonlinear continuous function of several variables. The gradient method with a line-search globalization ensures convergence to a local minimizer. The gradient method is hybridized with the simulated annealing to escape local minimizers. In

other word, we have combined a globally convergent algorithm "Gradient method" with the global optimization algorithm "Simulated Annealing". The efficiency of the proposed algorithm is tested through extensive numerical experiments on some well-known benchmark test problems. The results are also compared against some existing codes. The results show the competitive performance of the proposed method. (Received August 28, 2019)

1154-VF-852 **Courtney Ann Frazier*** (cfrazier10@my.apsu.edu) and **Ramanjit K. Sahi** (sahir@apsu.edu). *Connections of Molecular Symmetry with mathematics.*

Symmetrical operations in molecules can be observed via group theory and matrices. In our research, we focused on point groups. Specifically, we looked at C_{3v} groups. In here, we saw that different movements of the atoms around the center atom lead to distinct patterns that is easily discernible with mathematics. The modeling of the hydrogen atom's movement around the nitrogen atom was interpreted as an element of the symmetric group of degree 3, i.e. S₃. Moreover, each element in S₃ could also be perceived through the lens of linear transformations. Furthermore, we will study different point groups as well as look at the mathematical aspect of vibrations in different molecules. (Received September 11, 2019)

1154-VF-970 **Jennifer Crodelle*** (crodelle@cims.nyu.edu), 251 Mercer St, New York, NY 10012. *A model for visual circuit development.*

The mammalian primary visual cortex contains neurons that respond preferentially to oriented visual stimuli (e.g., horizontal bars). In the mouse, neurons that prefer similar orientations are scattered randomly throughout the cortex, leading researchers to believe that the mouse visual cortex lacks organization. Experiments have shown, however, that cells sharing an orientation preference are preferentially connected by an electrical connection called a gap junction during the first postnatal week, while chemical synapses have not yet been formed. We construct a comprehensive model of the mouse visual cortex during the first postnatal week of development and analyze the effect of gap-junction coupling on the formation of synaptic connections. Through simulation of the model network, we show that cells containing gap junctions in the first postnatal week learn synaptic connections faster than those that are not connected by a gap junction. Then, we suggest sparse gap-junction coupling as a potential mechanism underlying the random orientation preference map measured in the adult mouse visual cortex. (Received September 12, 2019)

1154-VF-1024 **John LaGrone*** (jlagrone@tulane.edu). *Simulating Bacterial Swimming Dynamics in Stokes Flow with a Stochastic Model of Chemotaxis.*

An important aspect in the study of microorganisms is understanding the mechanisms used to react to environmental stimuli. One such mechanism is the process of chemotaxis, in which the organism detects changes in concentration of a chemical of interest in the fluid environment and utilize these changes to alter their swimming trajectory. Here, a model of swimming bacterial is presented in which the method of regularized Stokeslets is used to describe the fluid interaction. In order to simulate the effect of chemical concentrations, we employ a stochastic decision making process that samples an advecting and diffusing concentration field. (Received September 12, 2019)

1154-VF-1389 **David Deng** and **Michael Xue*** (mxue@vroomlab.com). *Solving parameter estimation problem using Least Square-based Algorithms.*

In today's application of Data Science, a common problem concerns the estimation of unknown parameters in the governing differential equations. This presentation illustrates a machine learning approach to estimate and validate the parameters of two population models, namely, the Malthus model and Verhulst's Logistic model.

The parameter of growth rate in the Malthus model is estimated using a least square-based algorithm applied to a training dataset with historical population data. When validating the resulting model using more contemporary dataset, we discovered inconsistency in the curve fitting of the original model.

To resolve this inconsistency, we turn to Verhulst's Logistic model, whose governing differential equation is nonlinear, with two parameters - growth rate and saturation level. After transforming the model equation with derivatives computed through forward, centered and backward finite difference schemes, we obtain the initial values of the estimation from the least square-based algorithm. These initial values are used to extrapolate the final estimation of parameters through a second iteration of least square-based estimation. The resulting model with the estimated parameters is then ready to make predictions of population in the future. (Received September 15, 2019)

1154-VF-1406 **Remi Drai*** (remi.drai@pepperdine.edu), Pepperdine University, Natural Science Division, 24255 Pacific Coast Highway, Malibu, CA 90263. *Inertial laws in population biology: The role of entropy.*

Among the mathematical approaches developed for population biology, two results have been often compared to Galileo's principle of inertia: Malthus' law of exponential growth and Hardy-Weinberg law in population genetics. In this work we show how these two laws can be derived from the minimization of functionals involving entropy. Our approach for the Malthusian principle uses V. Volterra's "quantity of life". Using calculus of variations and a formalism developed by E. Tonti and G. Strang, one can show that the classical exponential growth model corresponds to the minimization of the following entropic functional: $\int N \ln N + rX dt$ where: $X(t) = \int N(\tau) d\tau$ and r is the Malthusian growth factor.

Interestingly, another form of entropy: $\sum p_i \ln p_i$ (Gibbs-Shannon entropy) plays a role in the other "inertial principle" of population biology: Hardy-Weinberg equilibrium law. Using the Entropy Maximization principle of Jaynes, one can show that the genotype frequencies $X = P(AA)$, $Y = P(Aa) + P(aA)$, $Z = P(aa)$, associated to the minimizing allele frequencies, p_i , satisfy the relation $Y^2 = 4XY$ i.e are in Hardy-Weinberg proportions. (Received September 15, 2019)

1154-VF-1545 **Janelle K. Hammond***, Inria, 2 Rue Simone IFF, 75012 Paris, France, and **Vivien Mallet** and **Ruiwei Chen**. *Meta-modeling and uncertainty quantification of an air quality simulation chain at urban scale.*

With increased pollutant emissions and exposure due to urbanization worldwide, studies on air pollution and health effects have become increasingly common. Air quality simulations at urban scale are a key tool, however models generally depend on parameters such as traffic demand, emissions, and meteorological conditions, often unknown at micro scales. In addition, atmospheric dispersion models make assumptions of simplified or unknown physics, and have high computational costs. Simulations are expensive and subject to high uncertainties from various sources, especially unknown or sparse input data. The quantification of uncertainties using assimilation of observational data can lead to better understanding and use of models. Model Order Reduction methods can render numerous simulations for uncertainty quantification and exposure estimation studies feasible. Our objective is to first construct a reliable reduced-basis type meta-model of an air quality simulation chain at street resolution over the entire metropolitan area of Clermont-Ferrand, France, including models for dynamic traffic assignment, emissions and atmospheric dispersion. Next we study the quantification and propagation of uncertainties by Monte Carlo methods with two years of traffic and pollutant observation data. (Received September 16, 2019)

1154-VF-1667 **Boniface O. Kwach*** (bkwach@kibu.ac.ke), P. O. Box 1699, Bungoma, 50200, Kenya, and **Omolo N. Ongati** and **Olielo Joel Odongo**. *Mathematical Model for Nutrient and Wastes Exchange Across the Placenta.*

This study presents a new mathematical model for nutrient exchange across the placenta which include nutrient exchange from foetus to mother to provide a system of equations in the form, $\vec{Y} = A\vec{Y} + \vec{r}(t)$ and whose solution was analyzed for equilibrium and stability. This model introduces another parameter that takes care of waste elimination from foetus to mother. It was established that the final model is stable compared to the existing models, that is, the eigenvalues of the coefficient matrix are negative real number, λ_1 and complex numbers with negative real parts, λ_2 and λ_3 . This shows that the new model provides one straight line of solutions tending to the origin and a plane of solutions which spiral towards the origin. This gives a more accurate mathematical model for nutrient exchange in the placenta. This model would create a lot of insight into nutrient exchange in the placenta, the elimination of waste from the foetus and open room for further research from the mathematical concept developed..

(Received September 16, 2019)

1154-VF-1929 **Ixtacchuatl Hortencia Obregon*** (ixtacchuatl.obregon@gmail.com). *Minimizing Drug Resistant Cases of Gonorrhoea through Cost-Effective Treatment Plans.*

Gonorrhoea, caused by the bacterium *Neisseria gonorrhoeae*, is the second most prevalent bacterial sexually transmitted infection (STI) with 87 million new cases reported worldwide, according to the World Health Organization (WHO). Gonorrhoea has developed resistance to multiple treatment therapies within the past century due to the introduction and misuse of antibiotics. The decrease of effective treatment plans develops a posing threat of an untreatable gonorrhoea infection and a global health crisis. We aim to study a comprehensive treatment strategy with respect to drug resistance in gonorrhoea and its cost-effectiveness. We develop a mathematical model of gonorrhoea's resistance with respect to two dual treatments recommended by the WHO, ceftriaxone with azithromycin and cefixime with azithromycin. We also perform a cost-benefit analysis which compares the

suggested treatment plans to minimize the emergence of drug resistance. We numerically simulate our model and analyze the reproductive number based on estimated parameters, such that, we conclude the costs of treatments are minimized if more than 50% of individuals are successfully treated. (Received September 17, 2019)

1154-VF-1934 **Tracey G. Oellerich*** (toelleri@gmu.edu), **Maria Emelianenko**, **Robyn Araujo**, **Lance Liotta**, **Alessandra Luchini** and **Abdulaziz Alaraini**. *Adaptability Conditions in Biological Networks*.

In this talk we extend adaptability conditions for biological networks to include singular systems with non-hyperbolic equilibria. The proposed theoretical extension is compatible with the notions of homeostasis and robust perfect adaptation (RPA) and clarifies the relationship between the two. The new condition is derived using the notion of Moore-Penrose pseudoinverse and is implemented using a numerically efficient algorithm. The proposed approach is tested on several synthetic systems that are shown to exhibit homeostatic behavior yet lie outside of the scope of earlier work. (Received September 16, 2019)

1154-VF-1971 **Timothy D. Comar*** (tcomar@ben.edu) and **Stefano Chiaradonna**. *Age-Structured Models for HPV Vaccination: A Student Research Experience*.

This presentation will explore a long-term student research project that developed and analyzed age-structured vaccination models for Human Papilloma Virus (HPV). One of the models used impulsive differential equations, and the used an agent-based model. These models are used to investigate the consequences of a sufficient fraction of the population not completing the vaccination regimen. Specifically, we will describe and compare these models and the conclusions one can draw from these models. We will also discuss the challenges and benefits for student researchers of approaching this complex problem via these two modeling paradigms. (Received September 16, 2019)

1154-VF-2011 **Hadi Safari Katesari*** (hadi.safari@siu.edu) and **S. Yaser Samadi** (ysamadi@siu.edu). *Modeling volatility dependence for count time series data via copulas*.

Multivariate count data can be found in many areas such as biology, epidemiology, psychology, actuarial science, economy, etc. In this paper, we propose a novel copula-based model for analyzing the dependencies in count time series data. Discrete marginal copulas are used to estimate and obtain the volatility dependencies among the count correlated data. Particularly, our proposed copula-based volatility model is very effective and efficient for modeling high-dimensional discrete asset returns data. Moreover, the proposed copula-based volatility model can capture both the linear and nonlinear dependencies between asset returns variables efficiently. Parameters of the proposed model are estimated by the method of inference function for margins. Simulation study is applied to highlight the validity of our theoretical results. (Received September 17, 2019)

1154-VF-2060 **Nawa Raj Pokhrel***, 1 Drexel Dr., New Orleans, LA 70125, and **Chris P. Tsokos** and **Keshav Pokhrel**. *A Stochastic Predictive Model to Determine the Network Vulnerability Risk*.

Precise quantification of vulnerability and level of security for the computer network is the challenging issue from the long time. Security experts and administrators in the subject area used to act based on common security metrics, their proficiencies and experiences lacking formal statistical model. This paper propose a stochastic model to quantify the risk associated with the computer network in conjunction with the Common Vulnerability Scoring System (CVSS) framework. The model we developed uses the attack graph to represent the network environment where network probability risk is calculated based on the attack path. The cumulative probability of the given attack path helps the system administrator to implement the appropriate security measure to protect the network and adopt proactive security measures against potential attacks. Gaining in depth understanding of risk associated with the computer network helps individual to implement decisions like deployment of security products and even design of network topologies. (Received September 17, 2019)

1154-VF-2075 **Omondi J. Ouno*** (ouno@mmarau.ac.ke), P.O.Box 861, Narok, Kenya, Narok, Kenya, and **Dr. Boniface o. Kwach** (bkwach@kibu.ac.ke), P. O. Box 1699, Bungoma, Kenya. *Demand Dynamics for Capital Shares in Nairobi Security Exchange*.

In this paper we present the notion that the quantity of demand for capital shares follows a Geometric Brownian motion with mean reversion. This is because when the quantity of demand is taken over time and the values observed are transformed by taking the natural logarithms for each quantity of demand; the transformed quantities of demand carries properties similar to those of price. Properties like lognormal distribution, random walk process, mean reversion etc will be obeyed. A number of scholars have proved this fact. They include Egan (2007) who observes that normal and lognormal distributions are the two mostly used distributions in the analysis of financial returns and prices and that the latter gives accurate distribution of prices. Another scholar

is Liang (2003) who showed that demand for a commodity is a random walk process and can be modeled as a Geometric Brownian motion. As can be realized, demand and price have similar properties and that is the reason this paper discusses another property obeyed by quantity of demand which is considered over time. This is the property of mean reversion.

(Received September 17, 2019)

1154-VF-2352 **Yonathan Admassu** and **Celes Woodruff*** (woodruca@jmu.edu). *Automated Identification of Sinkholes*. Preliminary report.

High resolution digital elevation models (DEMs) have proven useful in mapping geomorphic features indicative of past geologic hazards, such as sinkholes. Location mapping is necessary for studying factors controlling the development of sinkholes and also for city planning. Although sinkholes are easily discernible on high resolution DEMs by their rounded outlines, their large number has led to the investigation of automated mapping techniques to identify them. We use curvature, sphericity, and circularity to create a method to automatically identify sinkholes given DEM data from locations in Virginia. We will discuss our methods and results, as well as limitations and future work. (Received September 17, 2019)

1154-VF-2513 **Leif Zinn-Brooks***, zinn@math.ucla.edu, and **Marcus Roper**. *Multi-compartment modeling of Neurospora circadian rhythm*. Preliminary report.

Neurospora crassa has been an excellent model organism for studying circadian rhythms for more than half a century. In *Neurospora* cells, circadian timekeeping is regulated by interlocking positive and negative feedback loops, which lead to oscillating levels of White Collar Complex (WCC) and Frequency (FRQ) protein over time. Previous differential equation models (Dovzhenok et. al., 2015; Bellman et. al., 2018) have replicated these oscillations in a single nucleus. However, *Neurospora* cells are multinucleic; FRQ and WCC activity needs to be coordinated between nuclei for circadian rhythm to be maintained. In this work, we adapt the previous models to track FRQ and WCC levels in multiple nuclear compartments. We show that diffusion of FRQ and WCC protein between compartments is sufficient to coordinate mRNA and protein activity across the entire cell. We also study how number of frq mRNAs scales with compartment size. (Received September 17, 2019)

1154-VF-2652 **Sean M Laverty*** (slaverty@uco.edu), 100 N University Dr, Box 129, Edmond, OK 73034, and **Brittany E Bannish** (bbannish@uco.edu), 100 N University Dr, Box 129, Edmond, OK 73034. *Mathematical model of annual growth in trees: a response to perturbation by fire*. Preliminary report.

We present an overview of the structure and dynamics of annual wood growth in trees. First we discuss typical, natural growth under ideal circumstances and follow this by discussing growth in the aftermath of major wood injury. Motivated by preliminary data from recently burned trees, we present a spatio-temporal mathematical model of individual tree growth and preliminary model results. Ultimately we are interested in the response of trees to repeated fires, and the interaction between fire and the healing process which shape the long-term health of forests. (Received September 17, 2019)

1154-VF-2704 **Amanda Metzner** and **Dan Zwillinger***, 600 District Ave, Burlington, MA 01803. *Cheating and its detection in games such as poker and paintball*.

Kohn poker is a well-studied three card version of two-person poker with known optimal strategy. Current implementations of Kohn poker assume both players engage fairly, but this may not always be the case. We analyze a new aspect of Kohn poker, when one or both players may cheat. The analysis covers naive play (when one player cheats and believes the other player is honest) and adaptive play (both players know that each is cheating). Our analysis extends to cheating detection; like an "inspection game" a player can only determine if the other player is cheating some percentage of the time. We determine the change in payoff and the ability to determine if cheating is occurring. This type of analysis is applied to more complex team games such as paintball. Fair play for paintball means that the teams have equal capabilities. Cheating play for paintball means: having a more powerful paintball gun, using a drone to see further, and/or moving outside the geographically defined game area. In these cases, both the advantage of cheating and the ability to detect cheating is determined.

Approved for public release; unlimited distribution. Not export controlled per ES-FL-091619-0204 (Received September 17, 2019)

1154-VF-2824 **Ivan Ramirez Zuniga*** (ivr3@pitt.edu), Pittsburgh, PA 15206, and **Jonathan E Rubin** (jonrubin@pitt.edu), **David Swigon** (swigon@pitt.edu) and **Gilles Clermont**. *A Data-driven Mathematical Study of the Role of Energy in Sepsis*. Preliminary report.

When pathogens enter the body, a whole cascade of pro and anti-inflammatory reactions occur in which the immune system activates and attempts to eliminate the invaders and prevent an infection. Under normal circumstances, if the initial pathogen load is sufficiently limited, the immune response would satisfactorily eliminate the intruder and the host will become healthy again. However, in some cases, a systemic overreaction of the immune may occur, leading to persistent inflammation, nitric oxide accumulation, tissue damage, organ dysfunction, and possibly death. Such systemic overreaction is called sepsis and it has been found to be associated to lower levels of energy in the form of adenosine triphosphate (ATP) as well as over production of lactate and nitric oxide. In this work we present an extended mathematical model of ordinary differential equations that includes the dynamics of the acute immune response along with the energy requirements to fight an infection as well as typically measured variables in the ICU. We tuned the model with available animal data from a study done on baboons of the species *Papio ursinus* that were infused with *Escherichia coli* (*E. coli*). We compare the outcomes of survivors versus non-survivors and explore the role of energetics in both groups. (Received September 18, 2019)

Outreach

1154-VG-212 **Jeanette Shakalli***, jshakall@gmail.com. *Program on Math Outreach in Panama*.

Talented mathematicians all around the world inspire kids and adults of all ages to rejoice in the beauty of science and math through various methods, like magic, juggling and music. The Program on Math Outreach of the National Secretariat of Science, Technology and Innovation (SENACYT) of the Republic of Panama was born in 2016 with the purpose of inviting these exceptional mathematicians to Panama so that they could share their passion for mathematics with the general audience. The goal of the Program is to convince Panamanians that math is not only fun but it also has many interesting applications. We also want to inspire Panamanian youth to study math. In this presentation, we will briefly discuss some of the logistics, the marketing strategy and the publicity that are necessary to get people to attend, and in general, what has worked and what has not. This Program is an example of an extracurricular activity that others might be interested in replicating in their own communities. (Received August 23, 2019)

1154-VG-354 **Luis Caceres*** (luis.caceres1@upr.edu), Mathematical Sciences Department, UPRM, Mayaguez, PR 00680, and **Omar Colon**. *OMPR Summer Camp*.

Very few efforts are made in Puerto Rico for meeting the needs of academically talented students in mathematics. OMPR Summer Camp is a five days' camp whose objective is to offer activities that introduces important concepts and develops abilities of those who may become leaders in the next generation. On the other hand, students are faced with challenging problems and academic staff work as advisors for them, providing tips but encouraging students to work on their own. The topics of the camp include geometry, number theory, combinatorics, algebra and game theory. OMPR Summer Camp caters exclusively to Puerto Rico Middle and High School students, in both public and private schools, and is exclusively merit based, with a very rigorous selection process. The selection process starts with more than 3000 students and finish with 20 students. OMPR Summer Camp is free of charge, thus no student is ever unable to participate due to financial need and the target population is made up almost exclusively of members of underrepresented minorities. The OMPR Camp is the only opportunity for most of the very talented students in PR. The 2019 Camp was supported by the MAA and some other donors. (Received September 02, 2019)

1154-VG-455 **Luis Caceres** (luis.caceres1@upr.edu), Mathematical Sciences Department, UPR-Mayaguez, PO Box 9000, Mayaguez, PR 00681, **Meike Akveld*** (akveld@math.ethz.ch), HG J 55, Department of Mathematics, Raemistrasse 101, 8092 Zurich, Switzerland, and **Rafael Sanchez** (lamonedar@gmail.com) and **Jose Heber Nieto** (jhnieto@gmail.com). *Mathematical kangaroo: The world's largest international math competition*.

The Mathematical Kangaroo competition, organized by AKSF, has been held for more than 25 years and has as its main objective the popularization of mathematics. This is achieved through a competition that includes striking mathematical problems, with an approach that is not necessarily taught regularly in the classroom. Currently over 80 countries participate and all contribute to the database from which the final questions are selected, in 6 different levels for students from elementary school to High School. In this Talk details of the

Mathematical Kangaroo competition are presented, including selected problems of several years and levels. This activity is undoubtedly an important teaching resource to help foster mathematics and help the understanding of mathematical concepts, ideas and applications. The problems of this competition stimulate the search for patterns, development of algorithms and the use of basic notions in non-typical situations, helping to develop creativity. On the other hand, this competition has served as a mechanism for the collaboration of math professors in different educational projects. (Received September 05, 2019)

1154-VG-876 **Hansun To*** (hto1@worchester.edu), 486 Chandler Street, Worcester, MA 01602, and **Maria Fung** (mfung@worchester.edu), 486 Chandler Street, Worcester, MA 01602.
Worcester State Math Academy Circle Project 2019-2020. Preliminary report.

This is a project under the Dolciani Mathematics Enrichment Grants (DMEG) of the MAA program. We have a Saturday Math Circle (Worcester State Math Academy Circle) for talented high school students from Worcester, MASS. We meet on Saturday mornings from November to February for a total of 10 sessions. We have recruited a group of 20 students from Worcester Public High Schools who are interested in improving their problem-solving skills, while working on topics that are not part of the standard secondary mathematics curriculum. We prepare the participants and administer the AMC 10 and AMC 12 competitions in February 2020. Once the competitions are done, we concentrate on problem solving sessions as an introduction to mathematical research. The project targets communication of mathematics skills and proficiency in problem-solving. This project supports participants to strengthen their mathematical abilities and eventually contribute to the mathematics college community. We will be discussing the progress of this program during this talk. (Received September 11, 2019)

1154-VG-946 **Brian Drake*** (drakebr@gvsu.edu). *Incorporating Python in a Linear Programming Course.*

Our undergraduate math majors and minors are offered an introductory course in linear programming. It builds on their linear algebra skills to develop some theory of standard algorithms, while also using coding to investigate optimization applications with thousands of variables. In the past we have used Maple as a unified platform for linear algebra and programming; in this presentation we describe using Python as an alternative. We share some example activities and discuss the student experience in one semester of the course. (Received September 12, 2019)

1154-VG-1119 **Sarah L Sparks*** (sarah.sparks@unco.edu), **Alees Lee**, **Gulden Karakok** and **Katie Morrison**. *Problems from Math Teachers' Circle Outreach Program.* Preliminary report.

The Northern Colorado Math Circles program is one of the outreach efforts of the School of Mathematical Sciences which aims to reach both middle school students and in-service teachers. The goal of this outreach program is to enrich teachers' mathematics content knowledge through engaging in rich mathematical problems, and to increase student and teacher enthusiasm for mathematics. Both student and teacher participants work on math problems that also include mathematics content that they do not necessarily experience in their school curriculum. Furthermore, we asked our teacher participants to create challenging math problems for their students to apply in their classrooms. In this session, we will share a sample of math problems that we facilitated with middle school in-service teachers and discuss their experiences with these problems. We will share teacher participant created math problems and our observations of connections between their experiences solving math problems and creating math problems. (Received September 13, 2019)

1154-VG-1134 **Andrew Penland*** (adpenland@email.wcu.edu), Stillwell 443, Western Carolina University, Cullowhee, NC 28723. *Human Cellular Automaton Activities for Fun and Mathematical Insight.*

Cellular Automata are discrete mathematical objects that offer relatively descriptions, but can have deep and fascinating behaviors. We will describe some activities that make the subject more accessible to students and non-mathematicians. Audience members will have the opportunity to actually become part of a Cellular Automaton. (Received September 13, 2019)

1154-VG-1311 **Jessica Ohrlein*** (jeo2136@columbia.edu). *Maps, Bridges, Networks, and Art Galleries: Introducing Secondary Students to Graph Theory through Classic Problems.*

Over the past few years, I have taught several iterations of two different one-hour courses in graph theory for middle and high school students. Both courses are based around classic and accessible problems in graph theory. One course briefly explores the Bridges of Königsberg problem, map coloring, and ways of describing centrality in networks. The other course goes in-depth into the art gallery problem. I will give a survey of the courses

and show student feedback, how the classes have changed over time, and what I plan to change in the future. (Received September 14, 2019)

1154-VG-1779 **Stephanie L Hurtt*** (stephanie.hurtt@unco.edu), **Gulden Karakok**, **Katherine Morrison**, **Cathleen Craviotto** and **Elizabeth Sasse**. *Math Circles Program to Facilitate Challenging Tasks*.

The Northern Colorado Math Circles program aims to enrich mathematics for middle school students by providing a three-day camp where students work on challenging, often open-ended mathematics tasks, which they would not typically see in their own mathematics classrooms. The goal of this outreach program is to improve students' ability to problem-solve, to reason mathematically, and to promote enthusiasm for mathematics. At the end of the camp, to evaluate our work with students, we asked students to complete an anonymous survey. The survey allowed us to collect data about student beliefs and attitudes surrounding mathematics, problem-solving, and the camp in general. In this presentation, we will share how we structured summer camps with support from various sources and the MAA Dolciani Mathematics Enrichment Grants and problems we used with students. In addition, we will share themes across the three years of the program regarding students' perspectives on problem-solving and mathematical growth mindset. We will also discuss the implications of the program as well as suggestions for improvement and implementation of the program in other locations. (Received September 16, 2019)

1154-VG-2161 **Lisa Schneider*** (lmschneider@salisbury.edu). *Exploring the Use of Presentations and Oral Examinations in Abstract Algebra Courses*. Preliminary report.

Teaching abstract algebra for the first time, I sought to explore different ways of measuring student understanding. In this talk, I will discuss the incorporation of student presentations of problems and oral examinations in a sequence of abstract algebra courses (spanning two semesters) with the advantages and disadvantages. I will share the adjustments I made during the semester to student presentation days as well as a look to changes for future courses based on student feedback and learning outcomes. (Received September 17, 2019)

1154-VG-2266 **Shannon R Lockard*** (slockard@bridgew.edu). *A Student Exploration of Spherical Geometry*.

Foundations of Geometry is an upper level mathematics course at Bridgewater State University. Taken primarily by future elementary and secondary educators, the course covers a variety of advanced topics in geometry. While course topics have centered around Euclidean geometry in my own sections of the course, I have been looking for ways to incorporate ideas of other types of geometries. In this presentation, I will discuss a student project that focuses on spherical geometry. I will present the objectives of the project and share student work that culminated in a three dimensional visual display of several important ideas in spherical geometry. (Received September 17, 2019)

1154-VG-2282 **Patrick X Rault*** (prault@unomaha.edu), 6001 Dodge Street, Mathematics Department, Omaha, NE 68182, and **Janice Rech** (jrech@unomaha.edu), 6001 Dodge Street, Mathematics Department, Omaha, NE 68182. *A tale of two circles: math with urban middle school teachers and rural secondary school teachers*. Preliminary report.

We will compare and contrast two math teacher circles. The first circle serves the Omaha, Nebraska metropolitan area. As a partnership with the Omaha STEM Ecosystem and the Henry Doorly Zoo, topics in this circle are usually in the form of "Math with a Musician," "Cooking with Math," or "Math and Gerrymandering." Though the target constituency is middle school mathematics teachers, these topics draw music teachers, culinary arts instructors, and in general other teachers from fields related to the topic of the day. The second circle is international, serving the borderlands region of Southern Arizona with some participants coming across the border from Sonora Mexico. This circle brings together Secondary School mathematics teachers from a wide region, connecting them with mathematics professors who introduce them to alternative pedagogies and new mathematical theories. We will discuss the idiosyncrasies of both types of Math Teacher Circle, and share ideas for topics that may be of use in your math teacher circle. (Received September 17, 2019)

1154-VG-2368 **Edwin Herman*** (eherman@uwsp.edu), Department of Mathematical Sciences, University of Wisconsin-Stevens Point, 2001 Fourth Avenue, Stevens Point, WI 54481. *Critical Thinking at the University of Wisconsin-Stevens Point*. Preliminary report.

Active learning is useful, but it is better when it is combined with dispositional changes in student thinking. To this end, the University of Wisconsin-Stevens Point is in the second year of a pilot program designed to help students "master key critical thinking skills and dispositions" as well as "foster the development of a supportive community of practice" (Critical Thinking Center, UWSP). Many participants – myself included – meet the

program expectations by using active learning pedagogies. The program includes more than forty faculty from a diverse range of departments (including mathematics). This talk will discuss the pilot program and how it is being integrated into the university curriculum as a whole, as well as my practices for including critical thinking and active learning techniques into the mathematics classroom. (Received September 17, 2019)

Teaching and Learning Introductory Mathematics

1154-VI-28 **Rabia Shahbaz*** (rshahbaz@ggc.edu), 1000 University Center Lane, Building W, Suite 3207, Lawrenceville, GA 30097, and **Janice Alves** (jalves@ggc.edu), 1000 University Center Lane, Lawrenceville, GA 30043. *Revamping of College Algebra*.

College algebra is a required math course for many undergraduate majors. Due to the higher failure rate in this course, it is considered as a gatekeeper to post-secondary success. In fact, a 2017 Washington Post and several other articles stated the difficulty of passing college algebra to the point where it is ranked as the number 1 most failed course in college.

The reasons behind students' lack of success in college algebra are varied; nonetheless, their struggles are exacerbated by the lack of helpful class resources. To address this issue, a mini Affordable Learning Grant was written, which supported the development of comprehensive and student-friendly guided notes. To further alleviate students' cost burden in these courses, a free online platform for Homework was piloted.

The purpose of this presentation is to delineate our experience of developing and incorporating guided notes and using the free online HW platform. Additionally, course data and student feedback regarding these resources will be shared. Lastly, this presentation will allow audience to explore an expanding repository of open educational resources that are available for higher education. (Received June 28, 2019)

1154-VI-129 **Jay A Malmstrom*** (jmalmstrom@occc.edu), 7777 S May Ave, Oklahoma City, OK 73159. *Using an Activity to Illustrate Conditional Probability and Baye's Formula*.

In the abstract, conditional probability can be a difficult concept for students in a liberal arts math class to grasp. A simple activity using colored disks in tins can be used to illustrate the concept as well as to justify Baye's Formula. (Received August 12, 2019)

1154-VI-176 **Rasitha R. Jayasekare*** (rjayasek@butler.edu), Dept. of Math, Statistics & Acutarial Sc., Butler University, 4600, Sunset Avenue, Indianapolis, IN 46208. *Data Science for All: The first course in Data Science*.

Butler University introduced a Minor in Data Science to provide the opportunity, to all majors, to add data mining skills to their existing major. "Introduction to Data Science" is the first data science course of the minor. The course requires basic statistics and computer science as prerequisites. It gives a first hands-on experience in data science, provides an experiential overview of current issues and ethics of big data, and introduces data mining algorithms and their applications. Software packages R and SAS are taught in the course. In this talk I will present the development, curriculum, and techniques of this first course in data science. (Received August 19, 2019)

1154-VI-450 **Monica Deni Morales Hernandez*** (mmoraleshernandez@adelphi.edu) and **Josh Hiller** (johiller@adelphi.edu). *A preliminary report: In and in front of a Math for ESL classroom*. Preliminary report.

It is no secret that ESL students face additional barriers to success in the math classroom; I should know because I was one. To help our students transition to their new educational environment Adelphi started a joint venture termed Adelphi University International (AUI). The main objective of AUI is to recruit and help international students with limited English skills by providing intensive non credit English language courses and smaller dedicated courses with a full time dedicated faculty line. In this talk I will provide my perspective as both a former ESL student and the current Math faculty for AUI. In particular I will explore challenges and strategies that may be implemented in other classrooms. (Received September 04, 2019)

1154-VI-662 **Melanie Butler*** (mbutler@msmary.edu). *Preparation is key to reading mathematics*.

This talk is for anyone who teaches or plans to teach mathematics. We will discuss why we should include reading instruction in mathematics courses. Once we are motivated, we will talk about how we can help our students learn to read mathematics by preparing them before the assignment. This discussion will include a pre reading heuristic which students can use to prepare and begin to read mathematics. Practical advice and techniques from education research will be shared. Faculty can use the strategies immediately in their classrooms. (Received September 09, 2019)

1154-VI-1009 **Jenna P. Carpenter*** (carpenter@campbell.edu), Po Box 115, Buies Creek, NC 27506.
Using Coded Messages to Teach One-to-One and Inverse Functions.

Students in college algebra and precalculus often struggle to grasp the concepts of one-to-one and inverse functions. Introducing these concepts in the context of designing a scheme to code and de-code messages taps into an intuitive framework in which both of these concepts are clear. Used as a running theme throughout the lesson, with a coding assignment for homework, greatly improves both student mastery of and interest in these topics. (Received September 12, 2019)

1154-VI-1012 **Kaitlyn A Perry*** (k.perry@wingate.edu). *Integrating Energy Use into Curriculum.*

Wingate University received an Energy Conservation Grant from the Jessie Ball DuPont Educational Fund to help transform the campus into a more sustainable campus. Instead of only changing the buildings, we decided to bring the ideas of sustainability into the curriculum to engage the students and increase their energy awareness. In this talk, we will go over the ways to use data from energy savings to introduce sustainability in a variety of introductory math courses. (Received September 12, 2019)

1154-VI-1334 **Kevin Bombardier*** (bombardierk@uwplatt.edu). *Student Perceptions on Adding Mathematical Journals into a Precalculus Course.*

Incorporating mathematical journals into the classroom can be a useful tool for both the students and the instructor. A mathematical journal consists of assignments with open-ended questions. Students may be asked to reflect on concepts, write thoughts about a mathematical topic, or discuss their thoughts on the course. We will discuss how this technique was implemented in a precalculus course. The students' feedback will also be discussed. (Received September 14, 2019)

1154-VI-1410 **Reva Kasman*** (rkasman@salemstate.edu), Salem State University, Salem, MA.
Designing a Calculus Course for Middle School Teachers. Preliminary report.

As part of a Master of Arts in Teaching Middle School Mathematics, our graduate students may take a course introducing them to Calculus. This is not a topic that these in-service and pre-service teachers will likely bring directly into their own classrooms, and for many it is the first exposure to such an advanced area of mathematics. Yet there are rich and accessible opportunities for students to explore the main themes and "big questions" which motivate calculus. Using inquiry-based activities that start with familiar middle school ideas, students in the class are able to build deep conceptual understanding as well as key calculational tools in calculus. (Received September 15, 2019)

1154-VI-1588 **Rebecca Metcalf*** (rmetcalf@bridgew.edu), Mathematics Department, 355 DMF Science Building, Bridgewater, MA 02325. *Massachusetts Department of Higher Education Mathematics Pathways Project & Bridgewater State University.* Preliminary report.

Students' lack of mathematical preparedness can be a major obstacle for timely completion of an academic degree. Both national and Massachusetts data support the contention that students who enroll in developmental coursework are less likely to graduate. Since March 2012, the Massachusetts Department of Higher Education (MA DHE) has been transforming developmental mathematics education to systematically improve the percentage of students who succeed in gateway mathematics courses. Their recommendations include using high school GPA for mathematics placement, forward placing students into gateway mathematics courses with co-requisite remediation and establishing pathways to these gateway courses. This session will provide more background on the MA DHE recommendations and describe how the Mathematics Department at Bridgewater State University has responded to them. (Received September 16, 2019)

1154-VI-1724 **Jordan Nikkel*** (jnikkel@cbu.edu) and **Cathy Grilli** (cgrilli@cbu.edu). *Using Peer Facilitators in Calculus Labs.* Preliminary report.

Christian Brothers University includes an 80-minute lab with its Calculus I course. Historically, it was an opportunity for students to investigate and discover concepts in groups. It was also an opportunity for students to work on interesting questions or reinforce classroom material. These were originally tied to specific sections of Calculus I. Scheduling difficulties led to the labs which may cross sections. With growing enrollment, faculty were needed to teach other courses, so the department ran a pilot to see how academically talented upperclassmen could run the labs. Faculty anecdotally noticed that students became more engaged when working with peer facilitators. However, the lab sessions evolved into problem sessions with the student facilitators. Recently, in order to return to the original intention of the labs, the department has initiated a training session where student facilitators experience a lab before directing a session. This paper will share our experiences with using peer

facilitators including what has worked, what has not worked, and what we would still like to try. (Received September 16, 2019)

1154-VI-1904 **L Franklin Kemp*** (lfkmp@yahoo.com), 3004 Pataula Lane, Plano, TX 75074-8765.
Observations from (Co)Sine Clock beyond the Unit Circle Definition.

(Co)Sine Clock is a unit circle clock that tells time and trigonometry. The clock shows cosine and sine are even and odd functions, respectively. Study of the dynamics of cosine and sine and their derivatives show they are a self-contained family. Simple interpolations lead to their power series which appear to defy periodicity. (Received September 16, 2019)

1154-VI-1986 **Bori Mazzag*** (borim@humboldt.edu), Department of Mathematics, Humboldt State University, Arcata, CA 95521. *Co-requisite courses and the STEM pathway: a report from Humboldt State University.*

The California State University (CSU) system mandated a curricular reform and abolished all developmental courses in the system starting in the 2018-19 academic year. The talk will describe the new Mathematics curriculum developed and implemented at Humboldt State University by a team of faculty. The talk will focus on curriculum for students in the STEM-pathway. It will describe the interventions and support system for students, including co-requisite courses, supplemental instruction and a recent pilot with embedded tutors. The talk will present data on course-level student outcomes as well as some results from learning outcomes assessment. The presentation will also briefly touch on an in-house faculty learning community for instructors of the new co-requisite courses. (Received September 17, 2019)

1154-VI-1997 **Jila Niknejad*** (jila@ku.edu), 405 Snow Hall, 1460 Jayhawk Blvd., Lawrence, KS 66049, and **Joseph Phillip Brennan** (brennanj@ku.edu), 405 Snow Hall, 1460 Jayhawk Blvd, Lawrence, KS 66049. *Coordinating Calculus with Limited Resources.*

Coordinating calculus at large state universities is done in a variety of styles. For state schools lacking an abundance of resources and personnel, best practices appear unattainable and reality forces coordinators to optimize their strategies under heavy constraints. At the University of Kansas, we follow a traditional lecture-discussion model and improve student experiences wherever possible. In this talk we discuss the challenges of coordination and how technology and data collection can improve the experience for students and their graduate teaching assistants. (Received September 17, 2019)

1154-VI-2019 **Ganga Acharya*** (gcharya@cnm.edu), 525 Buena Vista SE, Albuquerque, NM 87106. *An Approach to Improve Students' Participation and Performance in Introductory Math Classes.* Preliminary report.

Class-participation, office-hour attendance, and learning and understanding of expected mathematical concepts is decreasing for the students who use online homework, though they get better grade in online Homework. As an effort to increase students' participation in classroom, and improve test performance, parallel sections of introductory math sections are taught with certain treatment in some sections and outcomes are compared. The complete research process and outcomes will be presented in the talk. (Received September 17, 2019)

1154-VI-2190 **Brian E. Moore*** (brian.moore@ucf.edu), FL, and **Erin Saitta, Michele Gill, Jacquelyn J. Chini, Melissa Dagle** and **Xin Li.** *Active Learning in College Calculus: Supporting Instructors and Targeting Cultural Change.*

An interdisciplinary team of faculty from education, mathematics, chemistry, and physics, at a large public research university, devised and implemented a program to support mathematics faculty as they adapted unfamiliar active learning techniques to their calculus classrooms. The program supported two cohorts of three faculty and five graduate teaching assistants as they selected, designed, implemented, examined, and revised active learning strategies over the course of a year. Altogether, the autonomy that was given to the faculty who participated, along with the expertise of faculty from other departments and the overall plan for course and faculty development, provided an excellent environment for promoting faculty growth. Data collected through surveys, interviews, and observations reveal some persistent and sustainable changes in calculus instruction, especially among the faculty who participated in the program, but the broader culture of the department appears more resistant to change. (Received September 17, 2019)

1154-VI-2213 **Tyson H Walsh*** (tyson.walsh@westpoint.edu), D/MATH Thayer Hall, West Point, NY 10996, **Matt Webb** (matthew.webb@westpoint.edu), D/MATH Thayer Hall, West Point, NY 10996, **Lee Evans** (lee.evans@westpoint.edu), D/Math Thayer Hall, West Point, NY 10996, and **Kayla Blyman** (kayla.blyman@westpoint.edu), D/Math Thayer Hall, West Point, NY 10996. *Achieving a Baseline for Preparation in the Mathematics Classroom.*

One of the biggest challenges instructors face in the classroom is students coming to class unprepared and in complete “receive” mode. The concept of reading ahead for the next lecture, having questions prepared on that lecture’s material, and, as a result, actively participating during the lecture, is a cornerstone in the ideal student-teacher paradigm. Multiple assessment techniques, reading quizzes and peer assessments, require minimal time to grade, help achieve a baseline level of preparation, and allow for rich classroom discussions. Reading quizzes, short assessments covering the text chapter’s key words and concepts, at the start of the lecture covering the current day’s topic motivates the student to come to class having read the lecture material ahead of class. Offset from days incurring a reading quiz, a teacher may instead hold a peer assessment, wherein students grade one another’s notes, homework, or other lecture preparation material for completeness and neatness. The results of these two ideas shows immediate improvement in student participation in the classroom after implementation. (Received September 17, 2019)

1154-VI-2223 **Bernadette Boyle*** (boyleb7@sacredheart.edu). *Finding the missing values: An activity for introducing limits in a Calculus course.*

When first introducing students to finding the limit of a function from its graph, students often struggle with the concept. A common mistake the students make it to simply think of a limit as the value of the function for that x-value. This presentation will share an activity used in my Calculus classes to introduce the concept of a limit before actually defining what a limit is. We will discuss the activity’s effectiveness and the students’ reaction to it. (Received September 17, 2019)

1154-VI-2294 **Yun Lu*** (lu@kutztown.edu). *Application Projects in Calculus II.*

In this presentation, I will talk about my experience of implementing application projects to promote students’ interests and their learning performance during Calculus II classes. I will share some of my approaches as well as students’ feedback and performance if time allows. (Received September 17, 2019)

1154-VI-2462 **Jennifer Zakotnik-Gutierrez*** (jennifer.zakotnik@unco.edu). *Instructors’ and Administrators’ Experiences: Perspectives from the Other Side of Developmental Mathematics Reform.*

In response to the failures of developmental mathematics programs, many colleges have implemented program reforms aimed at increasing student success in college courses and educational programs. Research regarding program reform is largely quantitative and focused on measures of student success outcomes such as developmental course completion, college-level course enrollment and completion, and graduation rates. Instructors and administrators are notably absent in the literature, although both are integral to program design and student success. This study provides a qualitative view of one urban community college’s efforts to replace its developmental math program with a five-week intensive developmental course attached to a 10-week college algebra course with corequisite support. An activity theory lens was used to describe and analyze the perspectives of instructors and administrators, identifying tensions and contradictions which arose from the reform process and act as drivers of change. I will discuss the results of the study, including the implications of these tensions and contradictions. (Received September 17, 2019)

1154-VI-2562 **Chamila K. Ranaweera*** (chamila.ranaweera@colorado.edu), Boulder, CO 80302. *Improving students’ knowledge retention through daily quizzes in pre-calculus class: An experimental approach.* Preliminary report.

Students who are in the pre-calculus class are struggling to manage their time for the continuous study process. Often, they wait until the last minute for exam preparation. This leads to the outcome of taking classes only for the grade not to master the skills that they need to move forward into other calculus classes. Daily quizzes are designed to help the students to self-evaluate their study habits mathematical skills and improve their knowledge retention by studying, reviewing their work more regular manner. First five minutes of the class is spent taking short quizzes from the previous day’s materials. If they realize the quiz questions are very hard and need more explanation on the material more time will be spent on either working as groups to answer the quiz or instructor clarification. This process helps to develop a more active inclusive classroom environment, peer learning opportunities while improving self -motivation and self-awareness learning process. (Received September 17, 2019)

- 1154-VI-2568 **Torina Lewis***, 223 James P. Brawley SW, Atlanta, GA 30314. *A Propensity Score Matching Analysis of the Effects of Assessment and Learning in Knowledge Spaces (ALEKS) on Student Achievement at a Historically Black University.*

We applied recent advances in propensity-score matching (PSM) to quantify the effectiveness of Assessment and Learning in Knowledge Spaces (ALEKS) delivered as a corequisite model in Differential Calculus at a Historically Black College and University. Specifically, we examined whether students enrolled in a redesigned Differential Calculus course achieved higher final exam grades, or recorded a higher course average than closely matched students with selected covariates after using ALEKS. In order to avoid the selectivity problem, the “MatchIt” package in R was used to analyze student data from fall 2016 – spring 2019. Results indicate that implementing ALEKS as a corequisite approach in Differential Calculus showed an enhancement of student mathematical skills and a moderate positive relationship between the final course grade earned and the number of topics mastered. Moreover, the corequisite approach yielded a moderate effect on spring 2018 final exam scores, but the effect on final course grades was minimal. (Received September 17, 2019)

- 1154-VI-2611 **Timothy B Flowers*** (flowers@iup.edu). *Teaching a Redesigned Mathematics Course for Business Majors.* Preliminary report.

For many years our department offered a *Business Calculus* course, similar to courses offered at many other institutions. This course met the liberal studies mathematics requirement for all students with a major housed in our College of Business. However, since it was a calculus course, most students needed one or two courses of algebra prerequisites before reaching the class (based on their placement test scores). With this in mind, the College of Business requested we revamp the course to include more applications and ideas of calculus (e.g., marginal cost) while excluding some of the typical calculus machinery (e.g., the quotient rule). They also requested it be an introductory course that required no more than one algebra prerequisite. In this talk, I will summarize the work of several of my colleagues to develop our new *Business Mathematics* course to these specifications. I will also report on my experiences teaching the course in the first semester in which it was ever offered, including what went well and what did not for me and for the students. (Received September 17, 2019)

- 1154-VI-2618 **Melanie Pivarski*** (mpivarski@roosevelt.edu). *Using an Ellipse to Compute the Elapse of an Eclipse in Multivariate Calculus.*

On our university’s convocation day in 2017, there was a total solar eclipse, visible from parts of the United States. This eclipse motivated our multivariate calculus class to model its length, using ellipses, Kepler’s laws, areas, eccentricity, and error estimates. The problem illuminated the need to think carefully about geometric details, such as the difference between a sphere and a point particle, and how assumptions can dramatically affect the end result. (Received September 17, 2019)

- 1154-VI-2801 **Amy Been Bennett*** (amybeen@email.arizona.edu). *What’s the Norm? Teaching Introductory Mathematics via Active Learning in Collaborative Learning Spaces.* Preliminary report.

As undergraduate mathematics instructors continue to implement active learning strategies and practices, researchers investigate the factors that contribute to classroom environments that are conducive to this approach. In this study, four “novice” instructors share their reflections, challenges, dilemmas, and personal growth from teaching introductory mathematics courses via an active learning approach. Instructors navigated institutional demands and innovative, “flexible” learning spaces to make reasonable pedagogical decisions. This study uses a multi-pronged analysis to explore the alignment of introductory mathematics content and norms, an active learning approach to teaching, and the physical learning space. By examining the practicality of their decisions with respect to teaching norms and obligations, this study emphasizes the many resources and supports that instructors utilize to improve their teaching. (Received September 18, 2019)

- 1154-VI-2810 **Silva Chang*** (silva.chang@colorado.edu) and **Anne Dougherty**. *Mastery-Based Grading in a Large Calculus Course.*

Standards-based grading is a grading scheme which is based on a student’s mastery of course learning outcomes. In fall 2018 we implemented mastery-based grading in a large calculus course with over 800 students. Core topics were identified and multiple opportunities to demonstrate mastery were offered. After each midterm exam, students with low scores were given several chances to raise their scores by taking proctored online tests. We present data on student participation rates, the effect on exam averages, and student satisfaction with the grading scheme. (Received September 18, 2019)

1154-VI-2820 **Brendan Kelly*** (kelly@math.harvard.edu), 1 Oxford Street, SC 422, Cambridge, MA 02138, and **Emina Alibegovic**, **Rebecca Noonan-Heale**, **Anna Schoening** and **Amanda Cangelosi**. *Shortening Pathways to Completing Calculus*. Preliminary report.

Across the country, calculus requirements act as a gatekeeper to many STEM disciplines. STEM persistence is a multifaceted complex issue that interfaces with critical issues of social justice and national economic competitiveness, requiring our immediate attention. A recent strategy in higher education is to attend to the basic observation: the more college math courses a student has to take before enrolling in their first calculus class, the less likely the student is to complete their calculus requirements. This observation leads us to the following question: How can we change this paradigm and build a sequence of entry-level courses that promotes productive mathematical practices, encourages ambitious interactive instruction, highlights mathematics as a sense-making tool that explains the world in which we live, better serves students, and shortens our students' pathways to completing calculus? Today we are excited to share an open source curriculum that makes progress on this difficult question. This session will focus on the ways in which the new curriculum builds a mathematical story, satisfies students' intellectual needs, and supports ambitious instruction. (Received September 18, 2019)

Algebra

1154-VL-481 **Joseph Kirtland*** (joe.kirtland@marist.edu), Department of Mathematics, Marist College, 3399 North Road, Poughkeepsie, NY 12601. *A Note on a Class of Generalized Nilpotent Groups Introduced by Bechtell and Doerk*. Preliminary report.

All groups are finite with $\Phi(G)$ denoting the Frattini subgroup of a group G . If G is nilpotent with subgroups H and K where $H \leq K$, it is well known that $\Phi(H) \leq \Phi(G)$ and that $\Phi(H) \leq \Phi(K)$. However, as demonstrated by the symmetric group S_3 , there are non-nilpotent groups that also satisfy these properties. In 1965 H. Bechtell introduced a class of groups which satisfy the property that $\Phi(H) \leq \Phi(G)$ for all subgroups H of a group G . About 30 years later K. Doerk introduced a class of solvable groups which satisfy the property $\Phi(H) \leq \Phi(K)$ when $H \leq K \leq G$ for a group G . These two classes are identical when restricted to solvable groups. In this short talk, the author will extend the work done by Bechtell and Doerk by presenting some additional properties and structural results concern this class of solvable groups. (Received September 05, 2019)

1154-VL-622 **Michael Evan Niemeier*** (mniemeie@math.fsu.edu). *A Correspondence Between Central Extensions and Homotopy Classes of Maps*. Preliminary report.

It is well known that isomorphism classes of central extensions of groups, $0 \rightarrow A \rightarrow E \rightarrow G \rightarrow 1$, correspond to homotopy classes of maps between Eilenberg-MacLane spaces, $[K(G, 1), K(A, 2)]$. We will show a similar result, which is weakly equivalent path components of central extensions of simplicial groups, simplicial groupoids, presheaves of simplicial groups, and presheaves of simplicial groupoids all are in correspondence with their respective homotopy classes of maps $[\overline{W}G, \overline{W}^2A]$. We will show this by rephrasing the central extension definition in terms of an action and weakening E/A to only be weakly equivalent to G . These generalized central extensions are interesting as they may not have a cross section unlike the usual central extensions of groups. (Received September 08, 2019)

1154-VL-644 **John A Miller*** (john_miller5@baylor.edu), Sid Richardson Science Building, One Bear Place #97328, Waco, TX 76798, and **Markus Hunziker** and **Mark Sepanski**. *Explicit Pieri Inclusions*. Preliminary report.

Pieri inclusions, that is, embeddings of summands of Weyl Modules that arise via the Pieri rule, were first studied by Weyman in his thesis and first given explicitly by Olver in 1982. More recently, these maps have appeared in the work of Eisenbud, Fløstad, and Weyman and of Sam and Weyman to compute pure free resolutions for classical groups. In this talk we give a more efficient algorithm for computing these maps and describe the map in a general closed form. (Received September 09, 2019)

1154-VL-907 **Casey M. Pinckney*** (pinckney@math.colostate.edu). *Independence Complexes of Finite Groups*.

Certain problems involving groups are made easier by appropriate choices of generators. Thus understanding generating sets is of interest in group theory. Our goal is to describe minimal generating sets for certain finite groups in a visual way. We will explore some interesting combinatorics that arises from this view. More specifically, let G be a finite group. We define an independent set of G to be a collection of group elements that is a minimal generating set for some subgroup of G . These independent sets form a simplicial complex (called the Independence Complex) whose vertices are elements of G and whose faces of size k correspond to

independent sets with k generators. We describe the structure and combinatorics of the resulting simplicial complex. (Received September 11, 2019)

1154-VL-1088 **Karlee Westrem*** (westr106@du.edu), 1408 Maple Grove Road #105, Duluth, MN 56721. *Groups of Units Modulo $f(x)$.*

For a prime p and an irreducible polynomial $f(x)$ over \mathbb{Z}_p , the ring of polynomials over \mathbb{Z}_p modulo $f(x)$ denoted $\frac{\mathbb{Z}_p[x]}{\langle f(x) \rangle}$ is a finite field and the set of nonzero elements is a cyclic group under multiplication. We investigate the structure of the groups of units of $\frac{\mathbb{Z}_p[x]}{\langle f(x) \rangle}$ when $f(x)$ is reducible over \mathbb{Z}_p . (Received September 13, 2019)

1154-VL-1159 **Cheryl Lynn Ferguson*** (fergersoncl@jay.washjeff.edu), 50 S. Lincoln St., Washington, PA 15301, and **Roman Wong.** *Check digit schemes and the Dihedral group.*

We explore the different uses of identification numbers, the common errors made when transcribing them, and the check schemes associated with them. We investigate the mathematical strengths and flaws of different check digit schemes and how they work. We show that the general mod 10 schemes do not catch all transposition errors. We also investigate the Verhoeff Scheme and how the Dihedral group D_{10} is used to form check digits to catch the most common entry errors. (Received September 13, 2019)

1154-VL-1535 **Muhammad Inam*** (muhammad.inam@amu.edu). *The word problem for one relation Adian inverse semigroups.*

We show that if the Schützenberger graph of every positive word, that contains an R -word only once as it's subword, is finite over an Adian presentation $\langle X|u = v \rangle$, then the Schützenberger graph of every positive word is finite over the presentation $\langle X|u = v \rangle$. This enable us to solve the word problem for some classes of one relation Adian inverse semigroups. (Received September 16, 2019)

1154-VL-1566 **O'Neill Kingston*** (oneillk@iastate.edu). *Jeux de taquin.*

A Young tableau is a collection of boxes filled with letters from an alphabet. It sounds simple enough, but these unassuming creatures have numerous applications in areas ranging from representation theory to algebraic geometry. This leads to a natural question: what happens if you poke a hole in one of them? The answer, which we'll investigate in this talk, may surprise you! (Received September 16, 2019)

1154-VL-1877 **Alison E Becker*** (aebecker@uwm.edu). *A New Method to Find Relations Between Invariants of Representations.*

Invariant theory looks at the polynomial functions that remain unchanged as a result of actions of linear groups. The Second Fundamental Theorem of Invariant Theory (SFT) states what the relations are between the generating set of invariants. In many cases, these relations are unknown. This talk provides an overview of a Monte Carlo method of finding the relations between invariants given by the SFT. We will discuss new relations discovered using this method on the orthogonal group action on the set of polynomial functions of matrices. (Received September 16, 2019)

1154-VL-2288 **Brittany Riggs*** (bariggs@ncsu.edu) and **Hoon Hong.** *Optimality of an Improved Bound on Pólya's Positivity Theorem (Univariate Case).* Preliminary report.

A polynomial with non-negative coefficients can have no positive real root. One way to determine that a polynomial f has no positive real root is to produce a product gf that has non-negative coefficients. Pólya provided such a polynomial g of sufficiently large degree. I will provide an improved bound on the degree of g and discuss its optimality. (Received September 17, 2019)

1154-VL-2485 **Haydee M Lindo*** (haydee.m.lindo@williams.edu). *Endomorphism invariant modules and ring classifications.* Preliminary report.

I will speak on modules which are invariant under endomorphisms of their envelopes. This will include connections to the general theory of trace modules with some preliminary applications to ring classifications and conjectures involving modules with no self-extensions. (Received September 17, 2019)

1154-VL-2557 **Naomi Krawzik*** (naomikrawzik@my.unt.edu). *Graded Hecke algebras for the symmetric group acting in positive characteristic.*

We examine deformations of a skew group algebra arising from the action of the symmetric group on polynomial rings in positive characteristic. Over fields of characteristic zero, analogues of Lusztig's graded affine Hecke algebras are all isomorphic to analogues of symplectic reflection algebras. But when the characteristic of the underlying field divides the order of the group, new deformations arise. These noncommutative algebras exhibit relations deforming both the commutation relation in the polynomial algebra and also the action of the group

on the vector space. We classify the resulting deformations using Poincaré-Birkhoff-Witt conditions. (Received September 17, 2019)

1154-VL-2567 **Ann E. Rogers***, arogers1936@gmail.com, IL. *On the Weyl Algebras (I)*, by Jacques Dixmier Translated by Ann E. Rogers. Preliminary report.

In 1968, distinguished French mathematician Jacques Dixmier published one of the most renowned articles in the history of abstract algebra; "Sur l'algèbres de Weyl." It was the first comprehensive treatment of the non-commutative ring which is now known as the Weyl Algebra $A_n(k)$. In this paper, Dixmier presents fundamental structural and dimensional properties of the first Weyl Algebra $A_1(k)$, including a complete characterization of the generators of its automorphism group. In addition to being a canonical example of both a ring of differential operators as well as of a simple Noetherian domain, $A_n(k)$ has appeared in a surprisingly diverse number of contexts, including algebraic D -module theory and Lie representation theory. Most remarkably, this beautiful noncommutative algebra has proved to be the fundamental mathematical structure upon which quantum mechanical theory rests. We present the important highlights of Dixmier's foundational paper, taken from the first known complete translation into English. (Received September 17, 2019)

1154-VL-2573 **Matthew D. Jaynes*** (matthew_jaynes@baylor.edu), One Bear Place Room 97328, Waco, TX 76798-7328, and **Markus Hunziker**. *Hilbert Series of Modules of Covariants and Branching*. Preliminary report.

As demonstrated in the work of Enright & Hunziker, there is a correspondence between certain highest weight $\mathfrak{sp}(n, \mathbb{C})$ - and $\mathfrak{so}(2m, \mathbb{C})$ -modules which gives rise to a "transfer" theorem that relates the Hilbert series of an infinite-dimensional representation of the former to that of a finite-dimensional representation of the latter. To the end of an explicit description of such polynomials, we develop in special cases a new (diagrammatic) approach to the restriction of irreducible $\mathfrak{so}(2m, \mathbb{C})$ representations to $\mathfrak{gl}(m, \mathbb{C})$. (Received September 17, 2019)

1154-VL-2610 **Tyler D Anway*** (tyler.anway@mavs.uta.edu). *Approximations of Totally Acyclic Complexes*. Preliminary report.

For Noetherian local rings Q and R we can approximate totally acyclic complexes over R by more simple, often periodic, totally acyclic complexes over Q . We will discuss how, when Q is a ring of finite CM type, we can begin to classify the R -complexes by their approximations. (Received September 17, 2019)

1154-VL-2658 **Rebekah J. Aduddell*** (rebekah.aduddell@mavs.uta.edu) and **David A. Jorgensen** (djorgens@uta.edu). *The Natural Middle of a Complete Resolution*. Preliminary report.

Given a complex over a noetherian, local ring R , we present two new invariants called the critical and co-critical degrees. This allows us to define the natural middle of a complex as the portion containing the simplest syzygy modules. Additionally, we have a few preliminary results. (Received September 17, 2019)

Analysis

1154-VM-34 **Ritu Dhankar*** (p20180024@goa.bits-pilani.ac.in), Department of Mathematics, BITS Pilani K K Birla Goa Campus, Zuarinagar, 403726, India, and **Prasanna Kumar**, Department of Mathematics, BITS Pilani K K Birla Goa Campus, Zuarinagar, 403726, India. *Sendov conjecture for polar derivative of polynomials*.

Sendov conjecture states that, for a polynomial with each zero in the closed unit disk in the complex plane, every closed disk of radius 1 centered at a zero will contain a critical point of it. In this paper, we present the analogue of it for 'polar derivative' of polynomials, which includes the relative location of zeros of polynomials with that of their polar derivatives. We also prove some special cases, which might eventually answer the same for Sendov conjecture.

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(Received September 13, 2019)

1154-VM-193 **Mary Vaughan***, maryo@iastate.edu, and **Pablo Raúl Stinga**, stinga@iastate.edu.
Fractional derivatives in one-sided weighted Sobolev spaces.

Fractional derivatives can be defined in the most general way in the distributional sense. We start from this point and then show that the Marchaud pointwise formulas for these operators indeed hold for functions in more general classes. The classes we consider are weighted Sobolev spaces with one-sided Sawyer weights. These classes capture the one-sided nature of fractional derivatives. The pointwise and norm limits as the orders of the derivatives converge to an integer are also analyzed. (Received August 21, 2019)

1154-VM-244 **Kimberly D Ayers***, kayers@carroll.edu. *Chain Recurrence in Graph Determined Hybrid Systems.*

In this talk, we'll consider a family of dynamical systems on the same compact metric space. We then consider the dynamics given when the given flow shifts between these different flows at regular time intervals. We further require that shifts be allowed by a given directed graph. We then define a type of set, called a chain set, that exhibits many similar properties to chain transitive sets of flows. We will see that many analogous results from control theory need to be adapted and refined for this particular series. This work is included as part of a recent publication in the Journal of Dynamical and Control Systems. (Received August 26, 2019)

1154-VM-288 **Thomas Wunderli*** (twunderli@aus.edu), PO Box 26666, The American University of Sharjah, Sharjah, Sharjah, United Arab Emirates. *Lower semicontinuity and Γ -convergence of a class of linear growth functionals with L^1 Data.*

We prove some new results for linear growth functionals $f_\Omega \phi(x, Du)$, $u \in BV(\Omega)$, where $\phi(x, p) =: g(x, \|p\|)$ if $\|p\| \leq \beta\psi(x)\|p\| + k(x)$ if $\|p\| > \beta$, $\psi \in C(\Omega) \cap L^\infty(\Omega)$, $\psi \geq 0$ on Ω , and ϕ is convex in p . In particular, we give conditions on ϕ for which $f_\Omega \phi(x, Du)$ is lower semicontinuous in $L^1(\Omega)$ with $k \in L^1(\Omega)$ and $g(\cdot, \|p\|) \in L^1(\Omega)$ for each $p \in \mathbb{R}^N$. Notably, we make no continuity or lower semicontinuity assumptions for ϕ in (x, p) and no differentiability assumption for ϕ in p , as is done in earlier work. We also consider more general linear growth functionals $f_\Omega g(x, \|Du\|)$ with $g(x, \|p\|)$ convex in $\|p\|$ and prove Γ -convergence of functionals of the form $f_\Omega \phi(x, Du)$ to $f_\Omega g(x, \|Du\|)$. Finally, functionals with specified trace values for u are also considered. (Received August 29, 2019)

1154-VM-390 **Yauheniya Shviadok*** (yauheniya.shviadok@cameron.edu), 2800 W Gore Blvd, Lawton, OK 73505, and **Ioannis K. Argyros** and **Samundra Regmi**. *Developments in the semi-local convergence of Newton's method.*

Newton's method is a very effective tool for generating a sequence approximating a locally unique solution of a nonlinear equation involving Banach space valued operators under some conditions on the initial data. We present the developments of the semi-local convergence of Newton's method starting from the famous for its simplicity and clarity Kantorovich criterion for solving nonlinear equations. It turns out that this criterion can be weakened leading to a wider convergence region, tighter error bounds on the distances involved and a more precise location of the solution. These advantages are obtained under the same Lipschitz conditions as before. Hence, the applicability of Newton's method is extended in cases not covered before. (Received September 04, 2019)

1154-VM-421 **Min Ranabhat*** (ranabhat@ksu.edu) and **Diego Maldonado**. *Recent updates on Poincaré and Sobolev inequalities in the Monge-Ampère quasi-metric structure.*

The developments of Poincaré and Sobolev inequalities in the Monge-Ampère quasi-metric structure, at their early stages, were based on the decay rate for the distribution function of Green's function associated to the linearized Monge-Ampère operator. In this talk, we will see a completely different approach to establish Poincaré and Sobolev inequalities as well as some improvements of these inequalities under various assumptions on the Monge-Ampère measure. (Received September 03, 2019)

1154-VM-449 **Lorenzo Riva*** (lri07661@creighton.edu), 2500 California Plaza, Creighton University, Davis Hall, Omaha, NE 68178. *Low Regularity Non- $L^2(\mathbb{R}^n)$ Local Solutions to the gMHD- α system.*

The Magneto-Hydrodynamic (MHD) system of equations governs viscous fluids subject to a magnetic field and is derived via a coupling of the Navier-Stokes equations and Maxwell's equations. It has recently become common to study generalizations of fluids-based differential equations. Here we consider the generalized Magneto-Hydrodynamic alpha (gMHD- α) system, which differs from the original MHD system by the presence of additional non-linear terms (indexed by the choice of α) and replacing the Laplace operators in the equations by more general Fourier multipliers with symbols of the form $-|\xi|^\gamma/g(|\xi|)$. In a paper by Pennington the author considered the problem with initial data in Sobolev spaces of the form $H^{s,2}(\mathbb{R}^n)$ with $n \geq 3$. Here we consider

the problem with initial data in $H^{s,p}(\mathbb{R}^n)$ with $n \geq 3$ and $p > 2$, with the goal of minimizing the regularity required to obtain unique existence results. (Received September 04, 2019)

1154-VM-625 **Jeramiah A. Hocutt*** (jahocutt@ua.edu). *Entire Functions, Entirely Discontinuous*. Preliminary report.

Paul Halmos defined a notion of Cesàro continuity and classified the real functions that satisfy this property. In a note in the MAA Monthly, we examined the analogous notion of Cesàro differentiability and the real functions that satisfied this condition. This led to a surprising conclusion - that everywhere differentiable functions are nowhere continuous. We now turn our attention to Cesàro differentiability in the complex plane. Utilizing a well-known functional equation, we arrive at a similarly contrived, yet no longer surprising, conclusion - that the functions satisfying this notion are entire functions, which are entirely discontinuous. (Received September 08, 2019)

1154-VM-682 **Jonathan Stanfill*** (jonathan_stanfill@baylor.edu), Department of Mathematics, Baylor University, One Bear Place #97328, Waco, TX 76798-7328, and **Guglielmo Fucci**, **Fritz Gesztesy** and **Klaus Kirsten**. *Determining Values of Spectral ζ -Functions for Sturm-Liouville Operators*. Preliminary report.

Under appropriate hypotheses on regular as well as singular Sturm-Liouville operators associated with differential expressions of the type $\tau = r^{-1}[-(d/dx)p(d/dx) + q]$, the associated spectral ζ -function can be found through complex contour integration techniques to equal the residue of explicit functions, F (essentially, Fredholm determinants), involving a canonical system of fundamental solutions of $\tau y = zy$. The asymptotic behavior of F allows for a deformation of contours to find explicit expressions for the spectral ζ -function for $n \in \mathbb{N}$. We end by providing various concrete examples.

This is based on joint work with Guglielmo Fucci, Fritz Gesztesy, and Klaus Kirsten. (Received September 09, 2019)

1154-VM-859 **Ryan Shifler***, Salisbury University, 1101 Camden Ave., Salisbury, MD 21801. *Quantum Bruhat Graphs, Asymptotics, and Undergraduate research*.

Let $\text{Gr}(k, n)$ be the Grassmannian. The quantum multiplication by the first Chern class $c_1(\text{Gr}(k, n))$ induces an endomorphism \hat{c}_1 of the finite-dimensional vector space $\text{QH}^*(\text{Gr}(k, n))|_{q=1}$ specialized at $q = 1$. Our main result is a case that a conjecture by Galkin holds. It states that the largest real eigenvalue of \hat{c}_1 is greater than or equal to $\dim \text{Gr}(k, n) + 1$ with equality if and only if $\text{Gr}(k, n) = \mathbb{P}^{n-1}$. The result can be interpreted with graph theory and regular polygons. This is the outcome of an undergraduate research project. (Received September 11, 2019)

1154-VM-959 **Beth Schaubroek*** (beth.schaubroek@usafa.edu). *An alternative definition of exponential functions in time scales using basis functions*. Preliminary report.

In time scales, the exponential function is defined using the cylinder transformation. Here we establish an alternative, but equivalent, definition of the exponential function using an infinite series. This series uses basis functions in time scales that are analogous to power functions. We also provide series definitions of sine and cosine. (Received September 12, 2019)

1154-VM-1004 **Kirsten R Messer*** (kirstenmesser@yahoo.com). *Basis functions in time scales*. Preliminary report.

The theory of time scales unifies discrete difference equations and continuous differential equations. In this talk, we begin with a review of foundational time scales definitions. We then describe basis functions in a time scale that are analogous to the power functions in traditional calculus. We close the talk by discussing some of the key properties of these basis functions. (Received September 12, 2019)

1154-VM-1097 **Javad Namazi*** (namazi@fdu.edu). *Shape through sound*.

We look into new developments about hearing the shape of a geometrical object through its sound. This goes back to the questions raised in Mark Kac's celebrated 1966 article. (Received September 13, 2019)

1154-VM-1112 **Jessica N Gibson*** (gibsonjn@jay.washjeff.edu), 60 South Lincoln Street, Washington, PA 15301, and **Roman Wong** (rwong@washjeff.edu), 60 South Lincoln Street, Washington, PA 15301. *A sequence of means and generalization*.

We investigate a sequence defined recursively by the seeds a_1 and a_2 , and for $n \geq 3$, $a_n = \frac{1}{2}(a_{n-1} + a_{n-2})$, the arithmetic mean of the previous two terms. The sequence is non-monotonic and the net distance between consecutive terms decreases by half. Thus the sequence is Cauchy and so is convergent. In a way similar to the

derivation of Binet’s Formula, we use diagonalization method to find an explicit formula of a_n and determine the limit of the sequence. A Java program is used to test the accuracy of the term prediction. We also investigate some generalization of this sequence. (Received September 13, 2019)

1154-VM-2176 **Grant C. Wagner*** (grantwagner1729@gmail.com). *Two Competitive Lotka-Volterra Systems with Nonlinear Competition.*

We examine two modifications of the classical Lotka-Volterra equations to account for nonlinear interspecific and intraspecific interactions by performing a complete analysis of the critical points of both systems. In the first modification, we show that it exhibits similar properties of the classical model; in the second modification, we show that it exhibits new properties not seen in the classical model. (Received September 17, 2019)

1154-VM-2387 **Danqi Yin*** (dyin@coe.edu), 1220 1st Ave NE, Cedar Rapids, IA 52402, and **Brittney Miller.** *Classifying solutions for an important functional equation in Complex Analysis.*

Let $\varphi : \mathbb{D} \rightarrow \mathbb{D}$ be a non-constant rational map in the Hardy space, and let C_φ act on act on the Hardy space.

Set $\sigma(z) = \frac{1}{\varphi^{-1}(\sqrt{z})}$, $\psi(z) = \frac{z\sigma t(z)}{\sigma(z)}$, and $\varphi(\infty) = \lim_{|z| \rightarrow \infty} \varphi(z)$. Then,

$$(C_\varphi^* f)(z) = \frac{f(0)}{1 - \varphi(\infty)z} + \sum \psi(z)f(\sigma(z))$$

where the sum is taken over the branches of σ . The kernel of C_φ^* is a subset of H^2 such that if $f \in \ker(C_\varphi^*)$, then $C_\varphi^*(f) = 0$. It is very important but also turns out to be very hard to classify the kernel of C_φ^* . According to the previous research, the following functional equation point out a possible way to classify the kernel of C_φ^* .

$$U(z)f(z) + zU'(z)f(U(z)) = 0$$

U is always represented by one of following 5 forms: $-z$, $\frac{m}{z}$, $m - z$, $\frac{-z}{1 + mz}$, $\frac{-m + z}{1 + tz}$, where $m, t \in \mathbb{R}$ and $m \neq 0, t \neq 0$. To classify the kernel of C_φ^* , the research was conducted to investigating the solutions of each case of u for the functional equation. (Received September 17, 2019)

1154-VM-2684 **Thomas J Osler*** (osler@rowan.edu). *A heuristic derivation of the Boole summation formula.*

In the paper [1], the Gregory-Leibniz series for Pi/2 was shown to exhibit a remarkable feature. The sum to 500,000 terms exhibits an error as early as the sixth decimal place as expected by the alternating series test. However, the next ten decimal places are correct! And there is more of the same astonishing behavior. In the paper [1], every digit for Pi/2 is correct except those underlined. Below these underlined digits is the amount that must be added to make the number correct. (These numbers, 1, -1, 5, -61, . . . , are a special sequence known as Euler numbers.) It is the purpose of this short note to give a heuristic derivation of the Boole summation formula which uses Euler numbers and explains the above remarkable feature. [1] Borwein, J. M., Borwein, P. B., Dilcher, K., Pi, Euler Numbers, and Asymptotic Expansions, The American Mathematical Monthly, 96 (1989), pp. 681-687. (Received September 17, 2019)

1154-VM-2690 **David Herron, Abigail Richard*** (richaab@mail.uc.edu) and **Marie Snipes.** *Chordal Hausdorff Convergence and Quasihyperbolic Distance.*

We present relationships between multiple types of convergence. In particular, we examine types of convergence of sets including Hausdorff convergence, Gromov-Hausdorff convergence, etc. In studying these relationships, we gain a better understanding of the necessary conditions for the quasihyperbolic metric to pointed Gromov-Hausdorff converge. (Received September 17, 2019)

Applied Mathematics

1154-VN-85 **Sunil Giri*** (sgiri2012@fau.edu) and **Necibe Tuncer.** *Backward Bifurcation in vector-borne model with direct transmission.*

This paper deals with the study of time since infection structured vector borne model with the direct transmission. The model is analyzed to investigate the dynamical behavior of the system. Analysis of the existence and stability of equilibria reveals the existence of backward bifurcation i.e. where the disease-free equilibrium(DFE) coexists with the endemic equilibrium(EE) when the reproduction number \mathcal{R}_0 is less than unity. This aspect shows that in order to control vector borne disease, it is not sufficient to have reproduction number less than unity although necessary. Thus, the infection can persist in the population even if the reproduction number is less than unity. Numerical simulation is presented to see the bifurcation behaviour in the model. By taking the reproduction number as the bifurcation parameter, we find the system undergoes backward bifurcation at $\mathcal{R}_0 = 1$. Thus, the

model has backward bifurcation and may have one or two positive endemic equilibrium when $\mathcal{R}_0 < 1$ and unique positive endemic equilibrium whenever $\mathcal{R}_0 > 1$. (Received August 22, 2019)

1154-VN-150 **Parth Mukeshbhai Shah*** (parthshah2908@gmail.com) and **G. C. Samanta**. *Stability of cosmological models in modified gravity.*

Modified gravity theories have received increased attention lately to understand the late time acceleration of the universe. Among numerous extensions to Einstein's theory of gravity, $f(R)$ theories have received several acknowledgments. In our current work we try to understand the acceleration of the universe in modified geometric space using dynamical system analysis. This technique also allows understanding the behavior of the universe and its stability analysis which could then be compared with observational data. (Received August 16, 2019)

1154-VN-164 **Ugur Abdulla, Jonathan Goldfarb and Ali Hagverdiyev*** (ahaqverdiyev2011@my.fit.edu), 300 Cornell Ave, Melbourne, FL 32901. *Optimal Control of Coefficients in Parabolic Free Boundary Problems Modeling Laser Ablation.*

In this talk we consider new variational formulation of the inverse Stefan type free boundary problem introduced in U.G. Abdulla, *Inverse Problems and Imaging*, 7, 2(2013), 307-340; 10, 4(2016), 869-898. Optimal control framework is employed where coefficient of the PDE and free boundary are components of the control vector. Gradient descent algorithm based on Frechet differentiability in Hilbert-Besov spaces complemented with preconditioning or increase of regularity of the Frechet gradient through implementation of the Riesz representation theorem is obtained. Computational analysis of a Model Example through implementation of preconditioning, calibration of preconditioning, identification of control parameters separately and simultaneously as well as sensitivity analysis with respect to different initial guesses is pursued. Motivation for this research arises in biomedical engineering problem on the laser ablation of biological tissues. The presentation is based on recently submitted work in Journal of Computational and Applied Mathematics. (Received August 16, 2019)

1154-VN-287 **aubain H Nzokem*** (aubain14@mathstat.yorku.ca) and **Neal Madras** (madras@mathstat.yorku.ca). *Epidemic Dynamics and adaptive vaccination strategy :scalar-renewal equation approach.*

We use analytical and numerical methods to investigate the continuous vaccination strategy effects on the infectious disease dynamics in the closed population and the demographically opened population. The methodology and key assumptions are based on Breda et al (2012). We show that the cumulative force of infection for the closed population and the endemic force of infection in the demographically opened population can be reduced significantly by combining two factors: the vaccine effectiveness and the vaccination rate. The impact of these factors on the force of infection can transform an endemic steady state into to a disease free state. (Received August 29, 2019)

1154-VN-373 **Katherine Thai*** (katherine.thai@rutgers.edu), **Jacob Chang** (jchang4@nd.edu), **Rachel Duquette** (duquetra@bc.edu) and **Tongyu Zhou** (tz5@williams.edu). *Combining Genetic Algorithms and Machine Learning (CgALM) for Modeling Complex Systems.*

Essential for services such as communication, navigation, and weather prediction, satellite constellations must minimize loss of coverage while subject to constraints on space traffic and the number of satellites available. Genetic algorithms (GAs) offer a versatile method of optimization with demonstrated success in applied problems. In the case of a computationally expensive problem, the necessity of repeated fitness evaluations prevents convergence of a GA in a feasible time frame. Implementing a more efficient surrogate model to estimate the expensive objective function poses a potential solution to this dilemma. Recent advances in machine learning methods, in particular neural networks, make them a compelling candidate as a surrogate function. A GA incorporating an ensemble of neural networks as a surrogate function is evaluated on a set of canonical test problems, including those with discrete inputs and multimodal objective functions, and applied to the problem of constellation design. This modified GA performs 4.5 times faster than the basic GA. This work was done as part of Research in Industrial Projects for Students at the Institute for Pure and Applied Mathematics. (Received September 02, 2019)

1154-VN-407 **Miguel Fuentes*** (mmfuentes@udel.edu), **Shreya Gupta**, **Sarika Aggarwal** and **Aviva Prins**. *Risk Assessments and Measurements of Privacy Leaks within Google's Ads Data Hub.*

In Google's Ads Data Hub (ADH), advertisers can analyze ad campaign data by utilizing a combination of internally collected data and Google's event-level ad data. ADH employs its own privacy measures that filter SQL queries and output so that advertisers only obtain aggregate results, thereby protecting end-user privacy.

Even with these existing protections, targeted and untargeted privacy leaks can occur. The goal of this project was to develop methods and algorithms that measure the risk of privacy leaks in Google's ADH. Throughout this project, we had to consider preserving the utility of ADH for advertisers.

To address our goal, we modified the Special Unique Detection Algorithm (SUDA) commonly utilized in Statistical Disclosure Control. Our new score, PIRATE, the Probabilistic Identification Risk and Attacker Threat Estimate, uses probability to assess the likelihood of uniquely identifying a record in a dynamic dataset. We applied PIRATE to simulated ADH data and successfully calculated the risk for each row. This work was done at the Institute for Pure and Applied Mathematics (IPAM) as a part of the Research in Industrial Projects for Students (RIPS) program. (Received September 03, 2019)

1154-VN-452 **Bhuvanewari Buna Sambandham*** (buna.sambandham@dixie.edu), **Aghalaya S Vatsala** and **Vinodh K Chellamuthu**. *Numerical Results for Linear Sequential Caputo Fractional Boundary Value Problems with Mixed Nonhomogeneous Boundary Conditions*. Preliminary report.

In our recent work, we developed the numerical results for linear sequential Caputo fractional boundary value problems with mixed homogeneous boundary conditions. We used Green's function representation to compute the solutions for linear sequential Caputo fractional boundary value problems. In this work, we plan to develop the numerical code for the boundary value problems with general nonhomogeneous terms with mixed nonhomogeneous boundary conditions. This will be useful in solving the nonlinear Caputo fractional boundary value problem with Caputo mixed boundary conditions by monotone method. (Received September 04, 2019)

1154-VN-461 **Dimplekumar Navinchandra Chalishajar*** (dipu17370@gmail.com), 435 Mallory Hall, Virginia Military Institute, Lexington, VA 24450. *Exact Controllability of Third order Dispersion Equation*.

We have investigated the controllability of third order dispersion equation/KDV equation using monotone operator theory. Then we study the weaker notion of the Lipschitz continuity called "Method of Integral Contractor" and study the controllability of the third order dispersion equation. Examples are given to illustrate the theory. (Received September 04, 2019)

1154-VN-580 **Imad A Jaradat*** (iajaradat@just.edu.jo), Department of Mathematics & Statistics, Jordan University of Science and Technology, P.O.Box 3030, Irbid, 22110, Jordan, and **Marwan T Alquran** (marwan04@just.edu.jo), Department of Mathematics & Statistics, Jordan University of Science and Technology, P.O.Box 3030, Irbid, 22110. *Taylor series in higher dimensional fractal space*.

The primary goal of our project is to provide an analytical handling of partial differential equations exhibited entirely in a fractional derivative sense (FPDE). In this regard, the bivariate Taylor series expansion has been modified and incorporated with the celebrated differential transform idea. In light of this, the FPDE is transformed into a difference equation which can be resolved by dint of a successive iteration pattern. The integer projection of the obtained series solutions of such hybrid equations match with their well-known corresponding solutions of the classical versions. This displays the PDEs in a more generalized form. Finally, we provide a graphical visualization to illustrate the role of the fractional derivative parameters which they are acting as homotopy parameters in the usual meaning. (Received September 07, 2019)

1154-VN-771 **Joseph Allen Morgan***, 2800 W. Gore Blvd, Lawton, OK 73505, and **Narayan Thapa**. *Parameter Identification in a Parabolic Partial Differential Equation from the Overspecified Data*.

We consider a time-dependent linear diffusion equation with initial and boundary data. The key aim of this identification problem is to determine time dependent diffusion coefficient from the Overspecified data. Several finite-difference schemes are used to determine the solution and the time dependent coefficient. The accuracy and stability of the methods are discussed and compared. Numerical examples are presented to demonstrate relevant features of the developed computational schemes. (Received September 10, 2019)

1154-VN-1045 **Netra P Khanal***, 401 W. Kennedy Blvd., Tampa, FL 33606. *Differential Equation Model for Carbon Dioxide Emission and the Impact of Major Contributors*. Preliminary report.

Solid, liquid, and gaseous fuels are considered to be major contributors to the emission of carbon dioxide in the atmosphere- the one that causes Global Warming. In this study, we develop a differential equation to model carbon dioxide emission while considering the impact of major contributors as an input function. The input function is estimated using a differential operator as data a smoother and penalized least square criteria is implemented to select a model using functional data analysis techniques. The proposed model is applied to the

historical data of carbon dioxide emission and its most significant constituents in the continental United States from 1882 to 2014. The data is obtained from the Carbon Dioxide Information Analysis Center (CDIAC), the primary climate change data and information analysis center of the United States Department of Energy. The proposed model is expected to be useful to understand the dynamics of major attributing variables and their impacts on the rate of change of carbon dioxide emission. (Received September 12, 2019)

1154-VN-1092 **Taufik Meklachi***, 320 Hoya Ave, Harrisburg, PA, and **Shari Moskow** and **John Schotland**. *Asymptotic Analysis of Resonances of Small Volume High Contrast Linear and Nonlinear Scatterers*.

Abstract. We consider the asymptotic analysis of the resonances of high contrast small volume scatterers for linear and nonlinear scalar waves. The resonance problem is formulated as a nonlinear eigenvalue problem. We derive asymptotic formulae for linear scatterers and nonlinear scatterers of Kerr type. Our results are illustrated with numerical examples. (Received September 13, 2019)

1154-VN-1144 **Haley Colgate*** (h_colgate@coloradocollege.edu). *Measuring Gerrymandering in Colorado*. Preliminary report.

After the 2020 census, every state in the U.S. will redraw their congressional and legislative voting districts. When plans are proposed, they will be examined for compliance to state laws, and in Colorado, Amendments Y and Z, passed in 2018, will require that plans be free of partisan bias. This summer we gathered data and ran analysis on Colorado to contextualize measures of gerrymandering when applied to the current voting districts. We used statistical sampling methods with real population and voting data to model election outcomes and partisan advantage under thousands of possible district plans. Future work will examine communities of interest and fairness criteria specific to the state of Colorado. With the upcoming redistricting and new census data, our work will serve as the backbone of a tool that may be used to analyze proposed plans, both by politicians and by the public. (Received September 13, 2019)

1154-VN-1150 **Ramanjit K. Sahi*** (sahir@apsu.edu), **Samuel N. Jator** (jators@apsu.edu) and **Mary I. Akinyemi** (makinyemi@unilag.edu.ng). *Solving Black-Scholes Equation using Exponentially-Fitted Backward Differentiation Formula*. Preliminary report.

A family of Exponentially-Fitted Block Backward Differentiation Formula (EFBBDF) is formulated based on linear combination of polynomial and exponential functions. The EFBBDF is applied to solve the Black-Scholes partial differential equation (PDE) after reducing the PDE into a system of ordinary differential equations resulting from the semi-discretization of the PDE via the method of lines. The stability and convergence of the EFBBDF are discussed. Numerical experiments are performed to validate the performance of the method. (Received September 13, 2019)

1154-VN-1166 **Jeb Collins*** (jbcoll12@gmail.com) and **Jehanzeb Chaudhry**. *A Posteriori Error Estimation for the Spectral Deferred Correction Method*.

The spectral deferred correction method is a relatively new method for solving systems of ordinary differential equations. This method utilizes basic finite difference methods and iterates on them to obtain higher accuracy than the original method can provide. This talk will discuss methods to estimate the error in this method a posteriori using adjoint-based methods. This necessitates finding a nodally equivalent finite element method, which essentially fills in the holes between the nodes of the finite difference approximation. A standard adjoint-based error representation formula is then used to find the error in a particular quantity of interest. (Received September 13, 2019)

1154-VN-1251 **Yung-Sze Choi** and **Jieun Lee*** (jieun.lee@uconn.edu). *Existence of standing pulse solutions to a skew-gradient system*. Preliminary report.

From vegetation patterns in an ecological system to propagating waves in a nerve conduction system, patterns emerge everywhere in nature. Reaction-diffusion systems have been used to explain the mechanism of pattern formation from a mathematical perspective. There are certain reaction-diffusion systems with skew-gradient structure that exhibit localized patterns such as fronts and pulses. While there are many literatures on the existence of pulse solutions to activator-inhibitor type skew-gradient systems, the study of nonlinear inhibitor equation is still limited. In this study, we investigate standing pulse solutions in a 2-component skew-gradient system in which cubic nonlinearities are involved with both activator and inhibitor. Using a variational approach, we establish the existence of standing pulse solutions with a sign change under appropriate parameter constraints. In addition, we explore some qualitative properties of the standing pulse solutions. (Received September 14, 2019)

1154-VN-1269 **Vinodh Kumar Chellamuthu**, 225 S University Avenue, Saint George, UT 84770, and **Jisun Otterson** and **Ammon Taylor***. *Assessing the Role of Temperature in Dengue Fever Outbreak Dynamics with Wolbachia Transinfection Control Methods*. Preliminary report.

Dengue is a mosquito-borne viral infection that is usually found in tropical and subtropical regions around the world. The cycle of dengue transmission can be broken by infecting mosquitoes with Wolbachia bacterium, which reduce the level of dengue virus in the mosquito and shorten the host mosquito's lifespan. Several studies have shown that infecting mosquitos with Wolbachia and releasing them (a process called Transinfection) can spread this bacterium to the local mosquito population and mitigate the impact of the disease. We developed a mathematical model to investigate the efficacy of Wolbachia in blocking dengue virus transmission. Our model also incorporates the local temperature data, which can affect the procreation and growth of mosquitos. Furthermore, our simulation results provide new insights into the effectiveness of Wolbachia in reducing dengue at a population level. (Received September 14, 2019)

1154-VN-1360 **Ram Sharan Adhikari*** (radhikari@rsu.edu), 1701 W Will Rogers Blvd, Claremore, OK 74017. *Numerical Stability Analysis Of a Weak Rectangular Method For a Class of Stochastic Differential Equations*.

The proposed weak rectangular method has the potential to overcome some of the numerical instabilities that are often experienced when using explicit Euler-Maruyama method. This work also aims to determine the mean-square stability region of the weak rectangular method for linear stochastic differential equations with multiplicative noises. In this work, a mean-square stability region of the weak rectangular scheme is identified, and step-sizes for the numerical scheme where errors propagation are under control in well-defined sense are given. The main results are illustrated with numerical examples. (Received September 15, 2019)

1154-VN-1378 **Joon Hyuk Kang*** (kang@andrews.edu). *Uniqueness pattern of survival of two species of animals in a domain*.

We concentrate on the uniqueness of the positive solution for the general elliptic system with homogeneous boundary conditions. This system is the general model for the steady state of a competitive interacting system. The techniques used in this paper are upper-lower solutions, maximum principles and spectrum estimates. The arguments also rely on some detailed properties for the solution of logistic equations. (Received September 15, 2019)

1154-VN-1507 **Amanda K Gute*** (agute@unc.c.edu), 9201 University City Blvd, Fret 350B, Charlotte, NC 28223, and **Xingjie Helen Li** (xli47@unc.c.edu), 9201 University City Blvd, Fret 350C, Charlotte, NC 28223. *Consistent Finite Difference Scheme for 1D Nonlocal to Local Diffusion Problem: Analysis and Simulation*.

In a recent paper a quasi-nonlocal coupling method was introduced to couple a nonlocal diffusion model directly with the classical local diffusion in one dimensional space. This new coupling framework removes interfacial inconsistency, ensures the flux balance, and satisfies the maximum principle. However, the numerical scheme proposed in that paper does not maintain all of these properties on a discretization level. In this talk we resolve this issue with a new first order scheme that provides first order consistency, stability, and the maximum principle throughout the entire domain including the transitional region. These scheme properties are rigorously proved and numerically confirmed via a series of benchmark examples. (Received September 16, 2019)

1154-VN-1508 **Uyen-Minh Le***, 100 N University Dr, Box 129, Edmond, OK 7303, **Mehmet Aktas** (maktas@uco.edu), 100 N University Dr, Box 129, Edmond, OK 73034, and **Emily Hendryx** (ehendryx@uco.edu), 100 N University Dr, Box 129, Edmond, OK. *The diffusion Frechet function and its application in classifying ECG waveforms*. Preliminary report.

The diffusion Frechet function (DFF) provides a means of comparing graphical networks. With nodes obtaining lower DFF values playing more essential roles in the whole network, the DFF is known to exhibit stability in capturing network features. We apply this technique to compare and classify electrocardiogram (ECG) waveforms. In effort to distinguish data from patients that experience a type of cardiac event from those who do not, we first transform each ECG signal into a network. We then use the DFF method in conjunction with the random forest classification algorithm to capture the network's distinguishing attributes and predict whether a patient is susceptible to a cardiac event. This talk presents a brief overview of the DFF, our ECG network construction, as well as the results of our ECG classification experiments. (Received September 16, 2019)

- 1154-VN-1540 **Alrazi M Abdeljabbar*** (aabdeljabbar@pi.ac.ae), Khalifa University, Abu Dhabi, United Arab Emirates. *A bilinear Bäcklund transformation is presented for a (3+1)-dimensional generalized KP equation with variable coefficients.*

A bilinear Bäcklund transformation is presented for a (3+1)-dimensional generalized KP equation with variable coefficients. Two classes of exponential and rational traveling wave solutions with arbitrary wave numbers are computed, based on the proposed bilinear Bäcklund transformation. (Received September 16, 2019)

- 1154-VN-1541 **Thomas Andrew Cook*** (thomas.cook@pfeiffer.edu). *On the spectral theory of 1-D Schrödinger operator with random sparse potentials.*

The goal of this talk is to develop a spectral theory for the Schrödinger operator with sparse random potential. To do this, we will first reformulate theories for sparse deterministic potentials through the development of a unitary operator known as the monodromy operator. We will construct an example to show that for sparse potentials the Bargmann estimate is too rough of an estimate for the number of negative eigenvalues. Lastly, we will show that there is a spectral transition from singular continuous to pure point spectrum of certain Schrödinger operators with random sparse potentials. (Received September 16, 2019)

- 1154-VN-1582 **Francesca Bernardi*** (fbernardi@fsu.edu), **Nicholas G. Cogan** and **M. Nicholas J. Moore**. *Hollow-Fiber Microfiltration Systems: Stokes Flow Solution in a Semi-Infinite Channel with Permeable Walls.*

Most wastewater management facilities aimed at water purification in the United States utilize hollow-fiber microfiltration. In these systems, pipes are split into thousands of micrometer-scale capped tubes with permeable walls. As wastewater flows through the filter, foulants are captured by the membraned walls allowing clean water to exit. Understanding the fluid dynamics is a fundamental step towards controlling the fouling process and enhancing the efficiency of microfiltration. We investigate the flow of wastewater through a single hollow-fiber tube. Starting from an infinite channel with permeable walls, we solve the Stokes flow problem in the channel interior for all permeability regimes. Then, we generalize the result to a semi-infinite channel with permeable walls capped at one end to mimic a single hollow-fiber system. Comparison with experiments and future directions will be discussed. (Received September 16, 2019)

- 1154-VN-1614 **Caleb Logemann*** (logemann@iastate.edu) and **James Rossmannith**. *High Order Local Discontinuous Galerkin Method for Solving Nonlinear Thin Film Equations.*

Thin Film Equations are useful in many applications including ice buildup on aircraft. We introduce a Local Discontinuous Galerkin method for numerically solving this type of equation. Time stepping is implemented with a Runge-Kutta IMEX scheme to maximize efficiency. The nonlinear convection term is able to be handled explicitly, while the nonlinear diffusion term is handled implicitly. This allows for much larger time steps to be taken. Our method efficiently solves the implicit nonlinear fourth order diffusion equation by linearizing the operator and doing a Picard iteration. We are able to achieve high order convergence with a minimal number of Picard iterations. (Received September 16, 2019)

- 1154-VN-1665 **Goodness Agboola***, 2800 W Gore Blv., Department of Mathematical Sciences, Cameron University, Lawton, OK 73505, and **Parshuram Budhathoki**. *Polynomial multiplication using Karatsuba and Toom-Cook algorithms and their applications.* Preliminary report.

In this presentation, we will discuss multiplication algorithms that can be applied to solve problems in applied fields such as Cryptography. The Schoolbook method to multiply two n terms polynomials takes n^2 steps, which is quadratic in input size. We will talk about faster methods like Karatsuba's algorithm [$n^{\lceil \log_3(2) \rceil}$ steps] and Toom-3 algorithm [$n^{\lceil \log_3(5) \rceil}$ steps] and their applications. (Received September 16, 2019)

- 1154-VN-1717 **Malcolm Alexander Gehlbach*** (mgehlbach8@gmail.com), Cameron University, 2800 W Gore Blvd, Attn: Math Department (Dr. Joshi), Lawton, OK 73505, and **Janak R Joshi**. *Reaction-Advection-Dispersion Equation.*

We study the transportation of a chemical or biological tracer carried by water through a uniform, one-dimensional, saturated porous medium and derive the simple mathematical model based on Mass balance that incorporate advection, dispersion, and diffusion. Mass balance states that the rate of change of the total mass in arbitrary section of the medium must equal the net rate that the mass flows into the section through its boundaries, plus the rate that the mass is created or destroyed within the section. (Received September 16, 2019)

1154-VN-1967 **Adebowale J. Sijuwade*** (adebowale.sijuwade@wsu.edu), Department of Mathematics and Statistics, Washington State University, Pullman WA, 99164, Pullman, WA 99163, **Mostafa Rezapour** (mostafa.rezapour@wsu.edu), Department of Mathematics and Statistics, Washington State University, Pullman WA, 99164, Pullman, WA 99163, and **Thomas J. Asaki** (tasaki@wsu.edu), Department of Mathematics and Statistics, Washington State University, Pullman WA, 99164, Pullman, WA. *Solving PDEs with Neural Networks.*

Recent developments have been made to learn partial differential equations using neural networks. We investigate the effectiveness of this approach in the case of Poisson and diffusion systems. Classical numerical approaches face the problem of discretization on irregular domains, while small neural networks can effectively provide approximations to solutions of certain systems on complex domains. We explore the influence of random weight initialization on the neural network by appealing to an averaging approach. Each system can be studied in terms of its penalty, which can be assessed via correlation. We investigate the suitability of different loss functions and conclude with remarks on the advantages and disadvantages of this approach compared to classical numerical approaches. (Received September 18, 2019)

1154-VN-1985 **Oliver Nina, Jamison M Moody*** (jmm1995@byu.edu) and **Clarissa Milligan.** *Augmentations with Random Triplet Mining: An Approach for Unsupervised Clustering and Manifold Learning.*

Unsupervised clustering is a very relevant, open area of research in machine learning with many applications in the real world. Learning the manifold in which images lie and measuring the proximity distance of the sample points to the clusters in their latent space is non-trivial. Recent deep learning methods have proposed the use of autoencoders for manifold learning and dimensionality reduction in an effort to better cluster image samples. However, offline training of autoencoders is cumbersome and rather tedious to update. We introduce a novel method that uses a triplet network architecture in order to replace autoencoders, thus avoiding the need to pre-train autoencoders offline. Because our framework can be trained online, we can train our network with data augmented pairs which allows us to build a more robust encoder and improve accuracy. Our method remains competitive compared with other current methods while we obtain state of the art results on Fashion-MNIST dataset. (Received September 17, 2019)

1154-VN-2026 **Robert Erra*** (robert.erra@epita.fr), 14-16 rue Voltaire, Le Kremlin-Bicetre, 94270, and **Alexandre Letois** and **Mark Angoustures.** *A new algorithm for fast approximation of the diameter of a sparse very large graph.* Preliminary report.

We consider the computation of the diameter of (real-world) sparse very large graphs, with hundred of millions or billions of nodes. At this scale, we can only compute an approximation. Most of the known algorithms (like iFUB) use the same strategy and tactics: we select a small subset S of the nodes and, with a BFS for nodes in S , we update a lower and an upper bound for the diameter. One of the problems is to find a *good* and if possible small set S , *i.e.* good candidates to start the BFS. We propose here a new algorithm to compute S , let G a graph:

- We compute communities of G : we use a fast (but approximate) algorithm like Louvain.
- We then create a quotient graph G/C : each point of this quotient graph represents a community.
- We can then compute the exact diameter of the quotient graph G/C .
- We then have some diametral points of G/C , which are communities of G .
- We can now define S as the smallest community of this list.

Picking nodes from those communities are good candidates to start a BFS instead of a random point of the graph. We will present some examples (and some counter examples). (Received September 17, 2019)

1154-VN-2131 **Lauren J Sauer*** (lauren.jo.sauer@gmail.com). *Stress-strength Inference for the Multicomponent System Based on Progressively type-II censored samples from Pareto Distributions.* Preliminary report.

A system of k components, where the strengths of all k components are independent and have identical distribution and each component is subject to a common random stress, is investigated. This system is alive only if at least s ($\leq k$) component strengths exceed the stress. This is also called a multicomponent stress-strength problem. In this talk, the maximum likelihood estimate of the multicomponent system reliability and the related confident intervals of the system reliability are presented based on progressively type-II censored samples from Pareto distributions. An intensive Monte Carlo simulation study is conducted to compare the impact from difference progressive censoring schemes. (Received September 17, 2019)

1154-VN-2160 **Yu Cao*** (cao20@iu.edu). *Rayleigh-Bénard Problem (2D): A Determining Form and Algebraic Bounds on the Global Attractor.*

We construct a determining form for the 2D Rayleigh-Bénard (RB) problem in a strip with solid horizontal boundaries, in the cases of no-slip and stress-free boundary conditions. The determining form is an ODE in a Banach space of trajectories whose steady states comprise the long-time dynamics of the RB system. In fact, solutions on the global attractor of the RB system can be further identified through the zeros of a scalar equation to which the ODE reduces for each initial trajectory. The twist in this work is that the trajectories are for the velocity field only, which in turn determines the corresponding trajectories of the temperature. This is a joint work with Michael Jolly and Edriss Titi.

In the case of stress-free boundary conditions, we show bounds for various Sobolev norms of solutions in the global attractor. All these bounds are algebraic in the viscosity and thermal diffusivity, a significant improvement over previously established estimates. The sharpness of the bounds are tested with numerical simulations. This is a joint work with Michael Jolly, Edriss Titi and Jared Whitehead. (Received September 17, 2019)

1154-VN-2168 **Adnan Sljoka*** (adnan.sljoka@riken.jp). *Rigidity of graphs and applications to protein signalling and experimental design.*

To understand how proteins function one must examine their internal flexibility and dynamics. One critical way proteins regulate their function is through a phenomena known as ‘allostery’ which has been coined as ‘second secret of life’. Allostery is a universal phenomena which involves regulation and/or signal transduction induced by a perturbation at one site on the protein which is a topographically distinct site from an active site. The molecular mechanisms that give rise to allostery are still poorly understood. In this talk, we treat a protein as a molecular multigraph where vertices represent atoms and edges various molecular bonding and non-bonding constraints. Molecular theorem in rigidity theory prescribes a combinatorial characterization of rigidity and flexibility which allows us to rapidly decompose a protein graph into rigid and flexible connection. Extending this theory, we have developed mechanistic models of allostery as transmissions of rigidity across molecular graphs. Recently, rigidity transmission allostery theory and algorithms have been used to decipher allostery in enzymes, membrane proteins and to guide biochemical NMR experiments in protein design (Sljoka, Science 2017, Nature Communication 2018, Journal of American Chemical Society 2019). (Received September 17, 2019)

1154-VN-2179 **Emma J Moore*** (emma.moore379@topper.wku.edu), 1906 College Heights Hill Blvd, Department of Mathematics, Western Kentucky University, Bowling Green, KY 42101, and **Ahmet Ozkan Ozer** (ozkan.ozer@wku.edu), 1906 College Heights Hill Blvd, Department of Mathematics, Western Kentucky University, Bowling Green, KY 42101. *Finite difference approximations for controlling hyperbolic-type partial differential equations of a piezoelectric system.* Preliminary report.

Semi-discretized Finite Difference approximations for a system of wave-type partial differential equations (PDE), modeling vibrations on a piezoelectric beam, are considered. It is shown that these approximations do not mimic the observability and/or stabilizability features of the PDE. This is mainly due to the loss of the uniform gap among the eigenvalues of the approximated finite dimensional model. To obtain a uniform gap, and therefore an exponential stability result for the closed-loop system with a feedback controller, we consider an indirect filtering technique which involves adding viscosity terms to the PDE. After filtering, as the mesh parameter goes to zero, the approximated solution space covers the whole infinite-dimensional solution space, and a uniform gap is achieved.

The results will be simulated by a Wolfram Demonstration Project. (Received September 17, 2019)

1154-VN-2281 **Victor Churchill*** (victor.a.churchill.gr@dartmouth.edu). *Identifying damage in sea ice from sparse laser strain measurements.*

We discuss several methods for identifying damage in sea ice when given laser strain or displacement measurements at only a few sparse locations in the domain of interest. We begin by modifying the equations of linear elasticity in order to account for damage in the displacement field. We then present a standard method for solving an inverse problem of this type which minimizes a data misfit cost function that is constrained by the aforementioned partial differential equations. We consider several regularization schemes for this method. Finally, we propose a method which minimizes an unconstrained cost function with respect to two variables via alternating minimization. Our results using both simulated and real data suggest that this method, which allows for variation away from both the given data as well as the model, is promising. (Received September 17, 2019)

- 1154-VN-2334 **Lu Duc Vy*** (lu.vy@ucdenver.edu), 2435 W. Cornell Ave, Denver, CO 80236, and **Yaning Liu**. *Variance Reduction Methods Based on Multilevel Monte Carlo for Option Pricing*. Preliminary report.

When modeling the value of an asset, the model complexity often exceeds the grasp of an analytic solution. When this is the case, simulation becomes the best alternative. What began in the Los Alamos Laboratories as Monte Carlo estimation evolved over the next 70 years to become something ubiquitous in financial mathematics. Today, Monte Carlo computational methods are so heavily used that pseudo-random numbers alone hardly suffice. Predicting the modern market requires efficiency, and to this end, a number of variance reduction techniques emerged. In this paper, we juxtapose two: these are a quasi Monte Carlo method utilizing random start digitally scrambled Halton sequences and the multi-level Monte Carlo estimator. We apply their combined advantages to various mathematical models in finance, including option pricing models, and compare the results with traditional Monte Carlo estimation. (Received September 17, 2019)

- 1154-VN-2351 **Hayley Guy*** (hguy@ncsu.edu), **Alen Alexanderian** and **Meilin Yu**. *A distributed active subspace method for scalable surrogate modeling of function valued outputs*. Preliminary report.

I will discuss a distributed active subspace method for training surrogate models of complex physical processes with high-dimensional inputs and function valued outputs. The proposed method combines benefits of active subspace methods for input dimension reduction and KL expansions used for spectral representation of the output field. I will demonstrate the performance of the approach using an example from biotransport. (Received September 17, 2019)

- 1154-VN-2390 **Kathryn A Boddie*** (kathryn.boddie@louisiana.edu), Mathematics Department, University of Louisiana at Lafayette, P.O. Box 43568, Lafayette, LA 70504. *A Minimal Time Solution to the Firing Squad Synchronization Problem with Von Neumann Neighborhood of Extent 2*.

Cellular automata provide a simple environment in which to study global behaviors. One example of a problem that utilizes cellular automata is the Firing Squad Synchronization Problem (FSSP), first proposed in 1957. A new extension to the standard FSSP to a different neighborhood definition - a Von Neumann Neighborhood of extent 2 - will be presented. An 8 state 651 rule minimal time solution to the extended problem will be described and presented along with an outline of the proof of the correctness of the solution. (Received September 17, 2019)

- 1154-VN-2391 **Michael Lamar*** (michael.lamar@centre.edu), **Jordan Turley** and **Zeyang Huang**. *A Statistical Learning Model for Embedding Rating Scale Data*. Preliminary report.

Embedding models produce low-dimensional representations of data and typically require pairwise distances as inputs although some work has been done to extend the embedding approach to allow for co-occurrence inputs. In this work, we further extend the idea to sparse, rating-scale input data. The learned embedding has the property that individuals with similar preferences are embedded near one another and objects that are similarly preferred are embedded near one another. This similarity based embedding then provides a representation of the data that can be efficiently clustered using any number of traditional algorithms (e.g k-Means, Mixture of Gaussians, etc.) in order to classify the individuals and objects into similarity groups based on their preferences in an unsupervised way. We demonstrate the utility of this embedding model on multiple data sets. We evaluate the quality of the learned embedding by comparing the ratings they predict with the actual ratings in test cases reserved from the data set before learning. We show that the resulting embedding algorithm is fast, simple, and intuitive, and its predictions match those of other more complex approaches. (Received September 17, 2019)

- 1154-VN-2392 **Abhinandan Chowdhury***, chowdhury@savannahstate.edu, Savannah, GA 31404. *Stochastic Functional Expansion for Identifying the Effective Heat Conductivity Coefficient of Polydisperse Suspension*.

We consider a random two-phase medium which represents a matrix containing an array of non-overlapping spherical inclusions with random radii. A statistical theory of transport phenomena in the medium is constructed by means of the functional (Volterra-Wiener) series approach for identifying the effective heat conductivity of a polydisperse spherical suspension. An approximate model based on power-series expansion of the kernels with respect to the volume fraction is developed. The functional series for the temperature is rendered virial in the sense that its truncation after the p -tuple term asymptotically correct to the order γ^p where γ is the mean number of spheres per unit volume - also proportional to the volume fraction. The case $p = 2$ is considered in detail and the needed kernels of the functional series are found to the order γ^2 . The truncated Volterra-Wiener

expansion is applied consistently to derive the equations for the kernels and their contributions to the overall (effective) modulus are identified. In this way, not only the effective conductivity, but also all needed correlation functions can be expressed in closed form, correct to the said order. (Received September 17, 2019)

1154-VN-2490 **Zigen Song, Melinda Baxter, Mingwu Jin, Jian-Xiong Wang, Ren-Cang Li, Talon Johnson*** (talon.johnson@mavs.uta.edu) and **Jianzhong Su** (su@uta.edu). *Sparse Sampling and Fully-3D Fast Total Variation Based Imaging Reconstruction for Chemical Shift Imaging in Magnetic Resonance Spectroscopy.*

We propose a 3-dimensional sparse sampling reconstruction method, aiming for chemical shift imaging in magnetic resonance spectroscopy. The method is a Compressed Sensing (CS) method based on the interior point optimization technique that can substantially reduce the number of sampling points required, and the method has been tested successfully in hyperpolarized ^{13}C experimental data using two different sampling strategies. (Received September 17, 2019)

1154-VN-2500 **Radoslav G Vuchkov*** (rvuchkov@ucmerced.edu), 5200 N Lake Rd, Merced, CA 95343, **Cosmin Gheorghita Petra** (petra1@llnl.gov), 7000 East Ave, Livermore, CA 94550, and **Noemi Petra** (npetra@ucmerced.edu), 5200 N. Lake Road, Merced, CA 95343. *Quasi-Newton Methods for Infinite-Dimensional Inverse Problems Governed by PDEs.*

Many engineering applications are modeled as optimization problems governed by differential equations. Newton's method is usually the method of choice to solve these problems due to its superior convergence properties compared to gradient-based or derivative-free optimization algorithms. However, deriving and computing second-order derivatives needed by Newton's method often is not trivial and sometimes not possible. In such cases quasi-Newton algorithms are a great alternative since they use only gradient information to build approximations to the second derivative. In this talk, we provide a new derivation of the Broyden-Fletcher-Goldfarb-Shanno (BFGS) quasi-Newton method update in an infinite-dimensional Hilbert space setting. The key component of this derivation is the formulation of variational problems related to the quasi-Newton methods within the space of self-adjoint Hilbert-Schmidt operators. Similarly by changing the constraints of the variational problem we obtain updates for other quasi-Newton methods such as Davidon-Fletcher-Powell, Symmetric Rank One, and Powell-Symmetric-Broyden. We show numerical results for an inverse problem governed by a partial differential equation and demonstrate the desired mesh-independent property of the quasi-Newton methods. (Received September 17, 2019)

1154-VN-2552 **Wei Zhu*** (min.shu@uconn.edu), 1 University Pl, Stamford, CT 06901, and **Wei Zhu** (wei.zhu@stonybrook.edu). *Diagnosis and Prediction of the 2015 Chinese Stock Market Bubble.*

We perform a novel analysis of the 2015 financial bubble in the Chinese stock market by calibrating the Log Periodic Power Law Singularity (LPPLS) model to two important Chinese stock indices, SSEC and SZSC, from early 2014 to June 2015. The back tests of the 2015 Chinese stock market bubbles indicate that the LPPLS model can readily detect the bubble behavior of the faster-than-exponential increase corrected by the accelerating logarithm-periodic oscillations in the 2015 Chinese Stock market. The existence of log-periodicity is detected by applying the Lomb spectral analysis on the detrended residuals. The Ornstein-Uhlenbeck property and the stationarity of the LPPLS fitting residuals are confirmed by the two Unit-root tests (Phillips-Perron test and Dickery-Fuller test). According to our analysis, the actual critical day can be well predicted by the LPPLS model as far back as two months before the actual bubble crash. Compared to the traditional optimization method used in the LPPLS model, we find the covariance matrix adaptation evolution strategy (CMA-ES) to have a significantly lower computation cost, and thus recommend this as a better alternative algorithm for LPPLS model fit. (Received September 17, 2019)

1154-VN-2615 **Min Shu*** (min.shu@uconn.edu), 1 University Pl, Stamford, CT 06901, and **Wei Zhu** (wei.zhu@stonybrook.edu). *Real-time prediction of cryptocurrency Bubble Crashes.*

In the past decade, cryptocurrency as an emerging asset class has gained widespread public attention because of their extraordinary returns in phases of extreme price growth and their unpredictable massive crashes. We proposed an advance bubble detection methodology based on the log-periodic power law singularity (LPPLS) model for identifying bubbles of the cryptocurrencies including Bitcoin, Bitcoin Cash, Litecoin, Ethereum, and Ethereum Classic. We found that the LPPLS confidence indicator based on this new method is an outstanding instrument to effectively detect the bubbles and accurately forecast the bubble crashes, even if a bubble exists in a short time. We method can provide real-time detection of bubbles and advanced forecast of crashes to warn of the imminent risk of cryptocurrency bubbles. (Received September 17, 2019)

- 1154-VN-2828 **Mohammad Tabanjeh*** (mt173@nyu.edu), PO Box 129188, Saadiyat Island, Abu Dhabi, United Arab Emirates. *Preconditioning Techniques for solving ill-conditioned Large Linear Systems of equations.*

Matrix preconditioning is a popular tool for the acceleration of the solution of an ill-conditioned systems of linear equations. Our goal is to use the multiplicative and additive preconditioning techniques to accelerate the convergence of the iterative solution algorithms of an ill-conditioned system of linear equations and to achieve more accurate numerical solution in less time. We will propose a rank-one modification as alternative technique to the multiplicative preconditioning since the later method cannot be applied to structured or sparse matrices. Thus to achieve our goal, we will add a properly scaled random rank-one modification matrices or small-rank modification matrices to the input matrix A until we arrive at a well-conditioned matrix and a simplified solution. (Received September 18, 2019)

Geometry

- 1154-VO-159 **Anh Tran, Kyle Andrie and J Mealy*** (jmealy@austincollege.edu). *Staircase metric tori.*

After a brief introduction to the category, staircase metric (SCM) geometry and its accompanying methodology, we discuss compact examples of (positive definite signature) SCM manifolds; specifically, we investigate examples of compact SCM surfaces of higher genus. Constructions are given of new SCM tori, and variously featuring: almost everywhere (a.e.) negative curvature; a.e. zero curvature; and a.e. positive curvature. The asymptotic behaviors of geodesics on these tori and other SCM manifolds are analyzed. (Received August 16, 2019)

- 1154-VO-170 **Patricia Baggett*** (pbaggett@nmsu.edu), Dept of Mathematical Sciences, MSC 3MB PO Box 30001, New Mexico State University, Las Cruces, NM 88003-8001. *An Approach to Accommodating a Congenitally Blind Student in an Inclusive University Geometry Course for Preservice K-8 Teachers.*

In this talk we will describe the tools and other supports, and the selection of curricular units that we chose, in attempting to offer a successful university geometry course for preservice K-8 teachers in which one blind student was enrolled. The course, taught in spring semester 2019, was not a proof course, but a hands-on course in which students used rulers, protractors, compasses, scissors, and calculators to design, build, measure, and study geometric objects. We found that high-quality 2-D tactile drawings on “swell” paper, made by a PIAF (Picture in a Flash) machine, and units in which 3-D objects were constructed, worked well for the blind student. We will describe the technology that was used in the class, such as a talking calculator, a braille note-taker and a printer that yielded printed braille, called an embosser. And we will describe particular curricular units that worked well for both the sighted students and the blind student. Students’ anonymous evaluations of the course at the end of the semester will also be discussed. (Received August 17, 2019)

- 1154-VO-1068 **Rishi Raj Subedi*** (rsubedi@gsu.edu), 2909 Cedar Brook Dr., Decatur, GA 30033. *Six-dimensional Lie-Einstein spaces with diagonal metric form.* Preliminary report.

In this talk, we address a few basics on Lie Algebra and Differential Geometry, then focus on Einstein spaces in dimension six. Next, we present results pertaining to the existence of such “Lie-Einstein” spaces. The case where the nilradical is of dimension four and five is solved completely assuming the diagonal metric form. Finally, we show the cases where the Einstein spaces are known to exist in dimension six. (Received September 13, 2019)

- 1154-VO-1170 **Cecilia C Fitzgerald*** (cecfitzgerald@gmail.com), 50 S Lincoln St, Washington, PA 15301, and **Roman Wong.** *The path from Fermat point to Steiner points.*

In this paper we will explore Steiner tree problems, which deal with minimizing the total distance of a network connecting a set group of n points, with $n=3$ and $n=4$. The relationship between the Fermat points of triangles and the Steiner points of quadrilaterals will be explored. We will also investigate the number of Steiner points needed to minimize the total distance of the network for quadrilaterals. (Received September 13, 2019)

- 1154-VO-1434 **Sergiy Koshkin*** (koshkins@uhd.edu) and **Ivan Rocha** (rocha.ivan36@gmail.com). *Caustics of light rays and Euler’s angles of inclination.* Preliminary report.

Euler used intrinsic equations expressing the radius of curvature as a function of the angle of inclination to find curves similar to their evolutes. We interpret the evolute of a plane curve as the caustic (envelope) of light rays normal to it, and study the Euler’s problem for general caustics. The resulting curves are characterized explicitly in two special cases: when the rays are at a constant angle to the curve (skew evolutes) and when they are reflections of parallel rays by it (caustics by reflection). Aside from the analogs of classical solutions

known to Euler (logarithmic spirals, cycloids), we encounter infinite-dimensional families of self-similar curves that solve delay differential equations. (Received September 15, 2019)

1154-VO-1903 **Alan R Parry*** (alan.parry@uvu.edu). *Normal Vectors and Space Curves.*

It is well known that the curvature and torsion of a curve in \mathbb{R}^3 determines the curve up to a rigid motion. We also know that the binormal vector function determines the curvature and torsion. Moreover, the unit tangent vector function determines the curve up to translation. However, an exercise in do Carmo's classical text on Curves and Surfaces claims that knowing the unit normal vector function at every point also determines the curvature and the torsion. We provide counter examples to do Carmo's exercise and show what condition must be added to obtain the correct result and its proof. (Received September 16, 2019)

1154-VO-2066 **Yuxuan Zheng*** (wendy.zheng@prismsus.org), 19 Lambert Drive, Princeton, NJ 08540, Princeton, NJ 08540. *Improved Upper Bound for Three Dimensional Universal Covering Problem.*

In 1914, Lebesgue proposed the problem about the smallest area of a universal covering in two dimensions, where universal covering is defined as a point set that can contain any subset with diameter 1. P. Gibbs made the latest progress in 2018 and he reduced the area of the universal covering to 0.8441. My research focuses on finding the smallest volume of a solid that is a universal covering in three dimensions. I have proved that a regular octagon with side length $\frac{\sqrt{6}}{2}$ is a universal covering, and its volume can be reduced to $\frac{5}{2} - \sqrt{3}$, or 0.7679 by cutting three corners off from the octahedron. Furthermore, I develop the method proposed by J. Pal in 1920 to a double-rotation method, I proved that a decahedron with opposite plane distance 1 is a universal covering. Cutting off two corners from that decahedron gives a better universal covering with volume 0.7586. I also raise a conjecture that a regular dodecahedron with side length $\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5+3}}$ and volume 0.6938 is a universal covering. (Received September 17, 2019)

1154-VO-2286 **Sherry Sarkar*** (sherry.sarkar@outlook.com), **Alex Xue** and **Pablo Soberón**. *Quantitative combinatorial geometry for concave functions.*

Helly's theorem is a fundamental result about the intersection properties of convex sets. It states that given n convex sets in \mathbb{R}^d , if the intersection of every $d + 1$ of the sets is non-empty, then the intersection of all the convex sets is non-empty. In 1982, Bárány, Katchalski, and Pach proved a volumetric extension of Helly's theorem stating that if the intersection of every $2d$ of the convex sets has volume at least one, then the volume of the intersection of all the sets is at least d^{-2d^2} . In fact, one can show that a loss factor in the volume of the intersection is necessary. Our results characterize conditions that are sufficient for the intersection of a family of convex sets to contain a "witness set" which is large under some concave or log-concave measure. The possible witness sets include ellipsoids, zonotopes, and H -convex sets. Our results also bound the complexity of finding the best approximation of a family of convex sets by a single zonotope or by a single H -convex set. We obtain colorful and fractional variants of all our Helly-type theorems and also exact quantitative versions of Tverberg's theorems. (Received September 17, 2019)

1154-VO-2662 **Daniel Martin*** (daniel.martin@trincoll.edu), **Lan-Hsuan Huang** and **Hyun Chul Jang**. *Rigidity of Hyperbolic space.*

The goal of this talk is to give a characterization of hyperbolic space using a geometric invariant defined for a class of manifolds that are asymptotic to hyperbolic space. This result is inspired by a characterization of Euclidean space called the rigidity of the Riemannian positive mass theorem. In this talk we discuss the history and context of our result, as well as some of the main ideas of its proof. (Received September 17, 2019)

Graph Theory

1154-VP-202 **Zhongyuan Che*** (zxc10@psu.edu), Penn State University, Beaver Campus, 100 University Drive, Monaca, PA 15061. *Peripheral convex expansions of resonance graphs.* Preliminary report.

The resonance graph of a plane elementary bipartite graph G , denoted by $Z(G)$, is a graph whose vertex set is the set of all perfect matchings of G and two vertices are adjacent in $Z(G)$ if their symmetric difference is the boundary of a finite face of G . In this talk, we will show that the resonance graph of a plane elementary bipartite graph G can be obtained from an edge by a sequence of peripheral convex expansions with respect to a reducible face decomposition of G if and only if the infinite face of G is forcing, that is, the induced subgraph obtained by removing all vertices on the boundary of G is either empty or has a unique perfect matching. This further

generalizes the corresponding result published in *Discrete Applied Mathematics* 2019 for the peripheral convex expansion structure of the resonance graph of a 2-connected outerplane bipartite graph. (Received August 22, 2019)

1154-VP-293 **Gary Chartrand** and **James Hallas*** (james.r.hallas@wmich.edu), 1903 W Michigan Ave, Kalamazoo, MI 49008-5248, and **Ebrahim Salehi** and **Ping Zhang**. *Irregularity Strength and Beyond*.

As far back as 1880, in an attempt to solve the Four Color Problem, there have been numerous examples of certain types of graph colorings that have generated other graph colorings of interest. These types of colorings only gained momentum a century later, however, when in the 1980s, edge colorings were studied that led to vertex colorings of various types, led by the introduction of the irregularity strength of a graph. In this talk, we take another look at the concept of irregularity strength and describe a related concept. Results and conjectures are presented on this topic. This is joint work with Gary Chartrand, Ebrahim Salehi, and Ping Zhang. (Received August 29, 2019)

1154-VP-713 **Jian-Bing Liu*** (j10068@mix.wvu.edu), Department of Mathematics, West Virginia University, Morgantown, WV 26506, and **Hong-Jian Lai**, **Ping Li**, **Jiao Li** and **Miaomiao Han**. *On weighted modulo orientation of graphs*.

In 2018, Esperet, De Verclos, Le and Thomass introduced the problem that for an odd prime p , whether there exists an orientation D of a graph G for any mapping $f : E(G) \rightarrow \mathbb{Z}_p^*$ and any \mathbb{Z}_p -boundary b of G , such that under D , at every vertex, the net out f -flow is the same as $b(v)$ in \mathbb{Z}_p . Such an orientation D is called an $(f, b; p)$ -orientation of G . Esperet et al indicated that this problem is closely related to modulo p -orientations of graphs, including Tutte's nowhere zero 3-flow conjecture. Utilizing properties of additive bases and contractible configurations, we showed that every $(4p - 6)$ -edge-connected planar graph G admits an $(f, b; p)$ -orientation and that every $(12p^2 - 28p + 15)$ -edge-connected signed graphs admit $(f, b; p)$ -orientations. We also reduced the Esperet et al's edge-connectivity lower bound for certain graphs families including complete graphs, chordal graphs and bipartite graphs. (Received September 09, 2019)

1154-VP-773 **N A Newman*** (newman@troy.edu), Troy University Dothan, 400 Adams Hall, 501 University Drive, Dothan, AL 36303, and **Matt Noble**. *A Few Questions on Color-Critical Subgraphs*.

In our work, we define a k -tuple of positive integers (x_1, \dots, x_k) to be a χ -sequence if there exists a k -chromatic graph G such that for each $i \in \{1, \dots, k\}$, the order of a minimum i -chromatic subgraph of G is equal to x_i . Denote by \mathcal{X}_k the set of all χ -sequences of length k . A very difficult question is to determine, for a given $(x_1, \dots, x_k) \in \mathcal{X}_k$, the set of all integers y such that $(x_1, \dots, x_k, y) \in \mathcal{X}_{k+1}$. We propose a few variants of this question and elaborate upon a number of partial results along the way. (Received September 10, 2019)

1154-VP-881 **Paul Horn** and **Lauren M. Nelsen***, [nelson@uindy.edu](mailto:nelsen@uindy.edu). *Diffusion of PageRank on graphs*.

Personalized PageRank has found many uses in not only the ranking of webpages, but also algorithmic design, due to its ability to capture certain geometric properties of networks. In this talk we will discuss graph curvature and how we can use curvature lower bounds to understand the diffusion of PageRank on graphs. (Received September 11, 2019)

1154-VP-1310 **Julianna Doo Ree Kim*** (julianna.kim1478@gmail.com), 2930 Francis Avenue APT 305, Los Angeles, CA 90005. *Characterize Sandpiles on Two and Three Vertices Graphs*.

A sandpile is a function that maps the vertices of a graph to the nonnegative integers. A vertex is stable if its image is smaller than the number of lines connected to it. In toppling, for every unstable vertex, one sand is sent down every line that starts at and leaves the vertex. We map graphs with varying lines and sinks and topple the vertices until all vertices are stable. There have been characterizations of sandpiles on $\mathbb{Z} \times \mathbb{Z}$ and graphs with finite vertices with only one line between each neighboring vertices. In this talk, we determine necessary and sufficient conditions to stabilize sandpiles on two and three vertices graphs with varying number of lines connecting them. (Received September 14, 2019)

1154-VP-1398 **Zachary Tyler King*** (zking@uco.edu), 6616 Fawn Canyon Drive, Oklahoma City, OK 73162. *Iterated Line Graphs of Graphs with Regular and Bi-Regular Partitions*.

Graph theory has many important applications to discrete mathematics and mathematical modeling. One tool that has been used to understand the underlying structure of graphs is the line graph. In 1965, van Rooij and

Wilf characterized iterated line graphs by the growth of their vertex count. In 2017, Balch, Milligan, and Lane-Harvard detailed the properties of the iterated line graphs of regular graphs, bi-regular graphs, and stars. This presentation will detail new research being done to extend those results to larger classes of graphs, particularly graphs with regular and bi-regular sub-graphs or vertex partitions. (Received September 15, 2019)

1154-VP-1452 **Stephen G. Hartke** and **Luke L. Nelsen*** (nelsen11@uindy.edu). *Subcubic Planar Graphs Are 7-Square-Choosable.*

A subcubic planar graph is a graph such that every vertex has degree at most three and which can be embedded in the plane without any edge crossings. The square of a graph G is the graph obtained from G by adding edges between pairs of vertices which have distance 2 in G . Wegner (1977) conjectured that the square of any subcubic planar graph has chromatic number at most 7, and Cranston and Kim (2008) asked if this conjecture could be extended to list coloring. We prove this stronger result, that the square of any subcubic planar graph has list-chromatic number at most 7. (In other words, all subcubic planar graphs are 7-square-choosable.) Our proof uses the discharging method and analyzes the choosability of many small subgraphs for our list of reducible configurations. However, graph choosability is a Π_2^P -complete problem (which is higher than NP-complete in the polynomial complexity hierarchy). To successfully verify the list of reducible configurations, we develop and apply new computational methods. (Received September 15, 2019)

1154-VP-1570 **Lucian Mazza*** (lmazza@mix.wvu.edu). *Some Recent Advances in Group Coloring of Graphs.* Preliminary report.

Group Coloring is a generalization of vertex coloring. The vertices of a graph are colored with the elements of a group in such a way that the colors of two adjacent vertices do not differ by a group value assigned to the edge between them. Some basic properties and definitions will be presented, followed by more recent results. These recent (not yet published) results will include advances in group coloring of multigraphs as well as a previously unknown relationship between subgroup structure and group coloring. (Received September 16, 2019)

1154-VP-1809 **Alex Kodess, Brian Kronenthal, Diego Manzano-Ruiz** and **Ethan Noe*** (enoe483@live.kutztown.edu). *Classification of three-dimensional real algebraically defined monomial graphs by girth.*

In this talk we will discuss bipartite graphs with algebraic equations determining adjacency. With copies of \mathbb{R}^3 for each partite set, we will present a classification of infinite families of these graphs by girth. We will also explain the motivation behind studying these graphs and talk about some of the tools we used to obtain our results. (Received September 16, 2019)

1154-VP-1815 **Tyler Russell*** (tyler.russell11@aol.com), **Isaac Brown** and **Brendan Miller.** *Inflection Points and Comparability of Reliability Polynomials.* Preliminary report.

The reliability of a network is broadly defined to be the probability that the system remains operational after the failure of some of its components. This reliability is expressed as a polynomial in p , the probability an individual component remains operational. Graves and Milan have shown that reliability polynomials may have arbitrarily many inflection points. These results, however, have only been proven for one variation of reliability. In this paper, we extend these results for other variations of reliability. Many of these results are extended by finding a comparability relationship between two kinds of reliability. (Received September 16, 2019)

1154-VP-1829 **Farid Bouya*** (fbouya1@lsu.edu) and **Bogdan Oporowski** (bogdan@math.lsu.edu). *Seymour's Second Neighborhood Conjecture from a Different Perspective.*

Seymour's Second Neighborhood Conjecture states that every orientation of every simple graph has at least one vertex v such that the number of vertices of out-distance 2 from v is at least as large as the number of vertices of out-distance 1 from it. We present an alternative statement of the conjecture in the language of linear algebra. (Received September 16, 2019)

1154-VP-2143 **Alex Brandt, Nathan Tenpas** and **Carl Yerger*** (cayerger@davidson.edu). *Planar Graphs with Girth 20 are Additively 3-choosable.*

The *additive choice number* of a graph G , denoted $ch_{\Sigma}(G)$, is the minimum positive integer k such that whenever each vertex is given a list of at least k positive integers, vertex labels can be chosen from respective lists in such a way that adjacent vertices have distinct sums of labels on their neighbors. Recently, bounds on the additive choice number have been obtained for planar graphs under certain girth assumptions. In this talk, we give a brief history of these bounds and present a proof that for a planar graph G with girth at least 20, $ch_{\Sigma}(G) \leq 3$. Our approach applies the Combinatorial Nullstellensatz to streamline arguments within a proof that uses the discharging method. (Received September 17, 2019)

1154-VP-2212 **Zechariah Hazel** and **Josh Laison***, Mathematics Department, Willamette University, Salem, OR 97301, and **Allison Kerkhoff**. *Stamping numbers of graphs*. Preliminary report.

We present research in graph theory done as part of an interdisciplinary collaboration with students of mathematics and art. For a graph G , we say a *stamp* of G is a minimal cycle in G . We distinguish some of the stamps of G as labeled and the rest as unlabeled, and ask for the smallest number of labeled stamps needed to remove the symmetries of G , or equivalently, to identify every vertex of G unambiguously in terms of the labeled stamps. This smallest number is the *stamping number* of G . We find stamping numbers of complete graphs and some circulant graphs. We'll also show some of the work from our group art exhibit. (Received September 17, 2019)

1154-VP-2291 **Arvind Ayyer**, **Daniel Hathcock*** (dhathcock3@gatech.edu) and **Prasad Tetali**. *Enumerating Acyclic Orientations of Complete Multipartite Graphs*.

An *acyclic orientation* (AO) of an undirected graph is an assignment of direction to each of its edges without introducing a directed cycle. We study the enumeration of AOs for complete multipartite graphs. Our results include: an explicit formula for the the number of AOs of complete multipartite graphs, which answers a question raised by Cameron et. al. (2014). We also provide a bijection between AOs of complete bipartite graphs with a fixed unique sink vertex and permutations with a prescribed *exceedance set*, relating two combinatorial objects not previously known to be connected.

Our techniques involve counting AOs on partially unlabeled complete multipartite graphs (i.e. counting AOs up to certain isomorphisms), then recovering the total count through a relabeling process.

Enumerating AOs is of interest for the connections they share with graph coloring. Indeed, the number of AOs is given by the chromatic polynomial evaluation $|\chi_G(-1)|$. This enumeration problem is also studied in computer science as a $\#P$ -complete Tutte polynomial evaluation with unknown approximability. Finally, it appears in biology as the enumeration of *branched polymers*, and statistical physics as the *Ursell function*. (Received September 17, 2019)

1154-VP-2427 **Kevin Iga*** (kiga@pepperdine.edu), Pepperdine University, Natural Science Division, 24255 Pacific Coast Hwy., Malibu, CA 90263. *Adinkras: Pictures of supersymmetry and its friends*.

The study of the supersymmetry algebra in physics has led to certain decorated graphs called Adinkras, which are a visual and accessible way to understand important representations of this algebra. The classification of Adinkras has, in turn, connected to a wide range of areas of math, like error correcting codes, cohomology, spin structures, and partially ordered sets. These graphs are naturally embedded in Riemann surfaces, where features of the Adinkra become geometric features of the surfaces. Adinkras are furthermore an avenue for making many of these topics accessible to undergraduates. (Received September 17, 2019)

1154-VP-2503 **Jozsef Balogh**, **Felix Clemen**, **Emily Heath*** (eheath3@illinois.edu) and **Mikhail Lavrov**. *On the ordered size Ramsey number of paths*.

An ordered graph is a simple graph with an ordering on its vertices. We are interested in the ordered path P_n with n edges whose vertices appear in increasing order. The ordered size Ramsey number of P_r versus P_s is the minimum number of edges in an ordered graph H such that every red-blue coloring of the edges of H contains either a red copy of P_r or a blue copy of P_s . In this talk, we will present upper and lower bounds on this number which are tight up to a polylogarithmic factor and discuss connections to other Ramsey numbers for paths. (Received September 17, 2019)

1154-VP-2551 **Jackson Autry***, jautry@math.tamu.edu, and **Christopher O'Neill**. *Sequentially Embeddable Graphs*.

We call a (not necessarily planar) embedding of a graph G in the plane *sequential* if its vertices lie in \mathbb{Z}^2 and the line segments between adjacent vertices contain no interior integer points. We prove (i) a graph G has a sequential embedding if and only if G is 4-colorable, and (ii) if G is planar, then G has a sequential planar embedding. (Received September 17, 2019)

1154-VP-2570 **Manuel A Davila*** (mdavila9@calstatela.edu), 2440 Continental Ave., El Monte, CA 91733. *Inequalities of Kazhdan Constants on Isomorphic Graph Pairs*. Preliminary report.

Let G be a finite group, and let Γ be a subset of G . The Kazhdan constant of the pair (G, Γ) is defined to be the maximum distance we can guarantee that an arbitrary unit vector in an arbitrary nontrivial irreducible unitary representation space of G can be moved by some element of Γ . Different pairs (G_1, Γ_1) and (G_2, Γ_2) may give rise to isomorphic Cayley graphs. We investigate the question, to what extent is the Kazhdan constant a graph invariant? In other words, if the pairs yield isomorphic Cayley graphs, must the corresponding Kazhdan

constants be equal? There exist some instance where given isomorphic Cayley graphs our Kazhdan constant will be equal. The Kazhdan constant of (G_1, Γ_1) relates to various invariants of the corresponding Cayley graph. For that reason, if two pairs (G_1, Γ_1) and (G_2, Γ_2) produce isomorphic Cayley graphs, we can bound one Kazhdan constant in terms of the other, by relating both to graph invariants. (Received September 17, 2019)

1154-VP-2666 **Hong-Jian Lai, Mingquan Zhan and Taoye Zhang*** (tuz3@psu.edu), 120 Ridge View Dr, Dunmore, PA 18512, and **Ju Zhou.** *Hamiltonicity of 3-connected Line Graphs with Diameter Three.*

Saito conjectured that every 3-connected line graph of diameter at most 3 is hamiltonian unless it is the line graph obtained from the Petersen graph by adding at least one pendant edge to each of its vertices. This conjecture is proved. (Received September 17, 2019)

1154-VP-2724 **Jay Cummings*** (jay.cummings@csus.edu), 6000 J Street, Shasta Hall 253, Sacramento, CA 95819. *Hunters and Rabbits on Graphs.*

An invisible, omniscient rabbit is hiding behind some bushes. At each time-step, a collection of hunters shoot at some of the bushes, and if the rabbit is behind a bush that they fire at, the rabbit is killed. Otherwise, the rabbit hops to a neighboring bush and they try again. In this talk we investigate how many hunters are needed to guarantee a kill in finite time. We will give the answer for blowups or paths, the infinite ray and path, and the box product of a path with a cycle. (Received September 17, 2019)

1154-VP-2779 **Grant Fickes, Dylan Green, Jonelle Hook, Karen McCreedy*** (karenmccreedy@kings.edu), **Kathleen Ryan, Jill Stifano and Nathaniel Sauerberg.** *Properly connected graphs and going the distance.*

An edge-colored path is properly colored if adjacent edges receive distinct colors. An edge-colored graph is properly connected if each pair of vertices in the graph is connected by at least one properly colored path. In such a graph we extend the idea of diameter to that of proper diameter, a function of both the graph and its coloring, which is defined to be the maximum length of a shortest properly colored path between any two vertices in the graph. In this talk we will investigate properties of proper diameter for bipartite graphs. (Received September 17, 2019)

Number Theory

1154-VS-44 **Elizabeth M Reid*** (elizabeth.reid@marist.edu), Department of Mathematics, Marist College, 3399 North Road, Poughkeepsie, NY 12601. *Using Inclusion-Exclusion to find Bent and Balanced Monomial Rotation Symmetric Functions.*

There are many cryptographic applications of Boolean functions. Recently, research has been done on monomial rotation symmetric (MRS) functions which have useful cryptographic properties. Here we use the inclusion-exclusion principle to develop a formula for the weight of degree d short monomial rotation symmetric functions in n variables. We then expand on this method to construct a formula for the weight of d -functions. From these results we classify bent and balanced functions of these forms. (Received July 16, 2019)

1154-VS-835 **Darleen S Perez-Lavin***, 715 Patterson Office Tower, Lexington, KY 40506. *Plus-Minus Davenport Constant - a zero-subsum problem on Finite Abelian Groups.*

Let G be a finite abelian group, written additively. The Davenport constant $D(G)$ is the smallest positive number s such that for any set $\{g_1, g_2, \dots, g_s\}$ of s elements in G , with repetition allowed, there exists a subset $\{g_{i_1}, g_{i_2}, \dots, g_{i_t}\}$ such that $g_{i_1} + g_{i_2} + \dots + g_{i_t} = 0$. The plus-minus Davenport constant, $D_{\pm}(G)$, is defined similarly but instead we only require that $g_{i_1} \pm g_{i_2} \pm \dots \pm g_{i_t} = 0$. In this talk, we study the best known estimates for $D_{\pm}(G)$ when $G = C_2 \oplus C_3^n$. (Received September 11, 2019)

1154-VS-890 **Gabriel Khan, Mizan R. Khan, Joydip Saha and Peng Zhao*** (peng.zhao@indstate.edu), Department of Mathematics, Indiana State University, Terre Haute, IN 47809. *A Conjectural Inequality for Visible Points in Lattice Parallelograms.*

Let $a, n \in \mathbb{Z}^+$, with $a < n$ and $\gcd(a, n) = 1$. Let $P_{a,n}$ denote the lattice parallelogram spanned by $(1, 0)$ and (a, n) , that is,

$$P_{a,n} = \{t_1(1, 0) + t_2(a, n) : 0 \leq t_1, t_2 \leq 1\},$$

and let

$$V(a, n) = \# \text{ of visible lattice points in the interior of } P_{a,n}.$$

In this talk, we present some interesting results for $V(a, n)$. The numerics and graphs suggest the conjecture that for $a \neq 1, n - 1$, $V(a, n)/n$ satisfies the inequality

$$0.5 < V(a, n)/n < 0.75.$$

(Received September 11, 2019)

1154-VS-928 **Byungchul Cha, Adam Claman, Joshua Harrington, Ziyu Liu, Barbara Maldonado, Alexander Miller, Ann Palma and Wing Hong Tony Wong***, 15200 Kutztown Road, Kutztown, PA 19530, and **Hongkwon Yi**. *Factorization of prime powers with the same sum.*

Integers with different partitions that share the same product is a topic with long term integers in the field of number theory. In this talk, we will present the necessary and sufficient condition for a prime power to have different factorizations that share the same sum. (Received September 12, 2019)

1154-VS-978 **Caleb Ji**, calebj@wustl.edu, **Joshua Kazdan***, jkazdan@stanford.edu, and **Vaughan McDonald**, vmcdonald@college.harvard.edu. *Patterns of Primes in the Intersection of Beatty and Chebotarev Sets.*

We study the prime numbers that lie in Beatty sequences of the form $\lfloor \alpha n + \beta \rfloor$ and have prescribed algebraic splitting conditions. We prove that the density of primes in both a fixed Beatty sequence with α of finite type and a Chebotarev class of some Galois extension is precisely the product of the densities $\alpha^{-1} \cdot \frac{|C|}{|G|}$. Moreover, we show that the primes in the intersection of these sets satisfy a Bombieri-Vinogradov type theorem. This allows us to prove the existence of bounded gaps for such primes. As a final application, we prove a common generalization of the aforementioned bounded gaps result and the Green-Tao theorem. (Received September 12, 2019)

1154-VS-1265 **Freda Li*** (fli@wesleyan.edu). *Regularity Conditions on Hermitian Lattices.* Preliminary report.

One of the main questions in the study of quadratic forms is the representation problem, which asks: given a quadratic form f , for which integers a does there exist a solution to $f(x) = a$? This has led to the study of different types of quadratic forms, which include regular and strictly regular quadratic forms. I will discuss my work on how properties that hold for such quadratic forms have natural analogues in the study of Hermitian forms. (Received September 14, 2019)

1154-VS-1668 **Benjamin Kibort Breen***, Department of Mathematics, Dartmouth College, 6188 Kemeny Hall, 27 North Main Street, Hanover, NH 03755. *Class groups and narrow class groups of S_n -fields of even degree.*

We describe heuristics in the style of Cohen-Lenstra for class groups and narrow class groups of S_n -fields of even degree. Our work follows from a general model centered on the 2-Selmer group of a number field. In support of our conjectures, we report on experimental evidence for S_4 -fields. (Received September 16, 2019)

1154-VS-1702 **Mohammad Javaheri*** (mjavaheri@siena.edu), 515 Loudon Road, Siena College, School of Science, Loudonville, NY 12211. *A Note on Polynomial Sequences Modulo Integers.*

We study the uniform distribution of the polynomial sequence $\lambda(P) = (\lfloor P(k) \rfloor)_{k \geq 1}$ modulo integers, where $P(x)$ is a polynomial with real coefficients. In the nonlinear case, we show that $\lambda(P)$ is uniformly distributed in \mathbb{Z} if and only if $P(x)$ has at least one irrational coefficient other than the constant term. In the case of even degree, we prove a stronger result: $\lambda(P)$ intersects every congruence class modulo every integer if and only if $P(x)$ has at least one irrational coefficient other than the constant term. (Received September 16, 2019)

1154-VS-1886 **Benjamin Hutz, Grayson Jorgensen, Brandon Gontmacher and Srinjoy Srimani*** (srinjoy_srimani@brown.edu), 69 Brown Street, Box #7125, Providence, RI 02912, and **Shuofeng Xu**. *Characteristic 0 Automorphism loci of the moduli spaces of dynamical systems \mathcal{M}_3 and \mathcal{M}_4 and Applications.*

Let $f : \mathbb{P}^1 \rightarrow \mathbb{P}^1$ be a rational function, where \mathbb{P}^1 is the projective line. We know that conjugating f by an element of PGL_2 will produce another rational function with similar dynamical behavior. Define Rat_d as the set of degree d rational maps on \mathbb{P}^1 and the moduli space of degree d rational maps, \mathcal{M}_d , as the quotient space $Rat_d/PGL_2(\bar{\mathbb{Q}})$.

The automorphism loci of the moduli space corresponds to the set of rational functions with non-trivial automorphism groups and can be further broken down into loci where the automorphism group contains a particular finite subgroup of PGL_2 . We are able to describe the automorphism loci in \mathcal{M}_3 and \mathcal{M}_4 in terms of some normal forms, i.e., n -parameter families in Rat_3 and Rat_4 such that n equals to the dimension of the

appropriate automorphism loci in the moduli space. Using these normal forms, we are also able to obtain some uniform boundedness results on the rational preperiodic structures of several one-parameter families with non-trivial automorphism groups. Additionally, we look at applications of the automorphisms to statistics over finite fields. (Received September 16, 2019)

1154-VS-1947 **Rachel Louise Petrik*** (rachel.petrik@uky.edu), 719 Patterson Office Tower, Lexington, KY 40506. *Improvements of the Chevalley-Warning Theorems*. Preliminary report.

The Chevalley-Warning theorems are a collection of results that give lower bounds for the number of solutions to systems of equations over finite fields. In particular, for a system of equations over a finite field, \mathbb{F}_q where $\text{char}(\mathbb{F}_q)=p$, if the number of variables is strictly greater than the sum of the degrees of the equations, then the number of solutions is congruent to 0 mod p . These bounds are best possible, as shown by simple cases. In 2011, D. R. Heath-Brown gave improvements to these results by excluding these simple cases. I have improved some of Heath-Brown’s results in this area. Some of these improvements have resulted in best possible bounds. In this talk, I will briefly introduce the classical Chevalley-Warning theorems and then discuss my results. (Received September 16, 2019)

1154-VS-2017 **Niraek Jain-Sharma*** (niraekjs@berkeley.edu), **Tanmay Khale** (tnkhale@gmail.com) and **August Liu** (m1868@cam.ac.uk). *Explicit Burgess Bounds for Composite Moduli*.

Let χ be any Dirichlet character with modulus q . Sums of the form $S(M, N) := \sum_{n=M+1}^{M+N} \chi(n)$, known as *character sums*, arise naturally in many number-theoretic contexts, such as in the study of quadratic non-residues, primitive roots, and Dirichlet L -functions. Bounds on character sums can be directly translated to useful bounds in each of these situations. In his 1962 paper “On Character Sums and Primitive Roots”, D.A. Burgess proved an *epsilon bound* on short character sums for any character χ with a prime modulus p ; shortly thereafter, in two other papers, Burgess extended this result to composite moduli as well. However, the composite moduli bounds were never made explicit. In this paper, we extend Burgess’ result and provide *explicit* bounds for any modulus $q > 2$. (Received September 17, 2019)

1154-VS-2088 **Alan Talmage*** (abt5217@psu.edu). *Simultaneous Cubic and Quadratic Diagonal Equations Over the Primes*.

The system of equations

$$\begin{aligned} u_1 p_1^2 + \dots + u_s p_s^2 &= 0 \\ v_1 p_1^3 + \dots + v_s p_s^3 &= 0 \end{aligned}$$

has prime solutions (p_1, \dots, p_s) for $s \geq 13$, assuming that the system has solutions modulo each prime p . This is proved via the Hardy-Littlewood circle method, with the main ingredients in the proof being Wooley’s work on the corresponding system over the integers [?] and results on Vinogradov’s mean value theorem. Additionally, a set of sufficient conditions for the local solvability is given: If both equations are solvable modulo 2, the quadratic equation is solvable modulo 3, and at least 7 of each of u_i, v_i are not zero modulo p for each prime p , then the system has solutions modulo each prime p . (Received September 17, 2019)

1154-VS-2094 **Chad Awtrey*** (cawtrey@elon.edu) and **Briana Brady**. *Automorphisms of 2-adic fields of degree twice an odd number*.

When we begin studying p -adic number fields, one of the first results we see is a formula for the number of nonisomorphic quadratic extensions. For example, we learn there is a unique unramified quadratic extension of the 2-adic numbers (defined by a quadratic polynomial that is irreducible mod 2) and six totally ramified extensions (these can be defined by Eisenstein polynomials), up to isomorphism. As with all quadratic extensions, each of these has exactly two automorphisms. In this talk, we generalize this situation. In particular, we show that if $n > 0$ is odd and K is any totally ramified extension of the 2-adic numbers of degree $2n$, then K has exactly two automorphisms. This result allows us to count isomorphism classes of totally ramified 2-adic fields of degree $2n$ by discriminant, which was not known previously. This in turn allows us to show easily that there are $2^{n+2} - 2$ such classes, which, while probably not well known, can be deduced from recent work of M. Monge (2011). Moreover, our result also allows us to develop canonical Eisenstein polynomials defining each class, extending previous work which has produced defining polynomials for cases $n \leq 11$. (Received September 17, 2019)

1154-VS-2106 **Patrick Daniels*** (pdaniel1@umd.edu), 4176 Campus Drive, College Park, MD 20742. *A Tannakian framework for displays and Rapoport-Zink spaces*.

We develop a Tannakian framework for group-theoretic analogs of displays, originally introduced by Bueltel and Pappas, and further studied by Lau. We use this framework to generalize the purely group-theoretic definition of

Rapoport-Zink spaces given by Bueltel and Pappas, and to show that this definition coincides with the classical one in the case of unramified EL-type local Shimura data. (Received September 17, 2019)

1154-VS-2175 **Brian Hopkins** and **Aram Tangboonduangjit*** (aram.tan@mahidol.edu). *Rational Polynomials Based on the Fibonacci Numbers.*

We investigate interpolating polynomials determined by certain Fibonacci numbers and find expressions for their values on all integers beyond the defining sets. This leads to several new identities involving the Fibonacci numbers and the harmonic numbers. (Received September 17, 2019)

1154-VS-2193 **Ralph P Grimaldi*** (grimaldi@rose-hulman.edu). *Ternary Pell Strings - The Palindromes.*

For $n \geq 1$ let a_n count the number of ternary strings $s_1 s_2 s_3 \dots s_n$ where (i) $s_1 = 0$; (ii) $s_i \in \{0, 1, 2\}$, for $2 \leq i \leq n$; and, (iii) $|s_i - s_{i-1}| \leq 1$, for $2 \leq i \leq n$. Then $a_1 = 1$, $a_2 = 2$, $a_3 = 5$, $a_4 = 12$, and $a_5 = 29$. In general, for $n \geq 3$, $a_n = 2a_{n-1} + a_{n-2}$, and a_n equals P_n , the n th Pell number.

For these P_n strings of length n , now let pal_n count the number of palindromes of length n that appear among the P_n strings. We find that $pal_n = P_{\frac{n}{2}}$ for n even, while $pal_n = P_{\frac{n+1}{2}}$ for n odd.

Then, for the pal_n palindromic strings of length n , we determine (i) the number of occurrences of each of the symbols 0, 1, 2; (ii) the sum of all the entries in the pal_n palindromes; (iii) the number of levels, rises and descents that occur within the strings; (iv) the number of runs that occur within the strings; (v) the number of inversions and coinversions for the strings; and, (vi) the sum of all the strings considered as base 3 integers. (Received September 17, 2019)

1154-VS-2363 **Eugene Fiorini** (eugenefiorini@muhlenberg.edu) and **Froylan Maldonado*** (froylan.mal@gmail.com), 4972 Bunnell Street, San Diego, CA 92113, and **Sabrina Traver, Peterson Lenard** and **Tony W. H. Wong** (wong@kutztown.edu). *On Some Properties of Latin Square Determinants.*

A Latin square is an $n \times n$ matrix in which the symbols $\{1, 2, \dots, n\}$ appear in each row and column of the matrix without repetition. An isotopic Latin square is a Latin square whose first row and first column are comprised of the symbols $\{1, 2, \dots, n\}$ in order. Two Latin squares are in the same isotopic class if one can be transformed into the other by a series of row, symbol, and column permutations. In this talk, we discuss the determinants of various $n \times n$ Latin squares, the correlation between isotopic classes and those determinants, and several integer sequences associated with those isotopic classes. (Received September 17, 2019)

1154-VS-2518 **Chatchawan Panraksa*** (chatchawan.pan@mahidol.edu), 999 Phutthamonthon 4 Rd., Phutthamonthon, Pathom 73170, Thailand. *On Uniform Boundedness of $x^d + c$.* Preliminary report.

The finiteness of torsion points of rational elliptic curves can be viewed as the uniform boundedness of rational periodic points of their associated Latté's maps. For a polynomial in the form

$$f(x) = x^d + c,$$

it is still unsolved for the existence of an upper bound of its rational periodic points. This is a special case of Morton and Silverman's uniform boundedness conjecture. Under the assumption of the abc-conjecture, we prove the uniform boundedness for

$$x^d + c.$$

(Received September 17, 2019)

1154-VS-2597 **John Lien*** (jlilien4@lsu.edu), 2150 Duncan Drive, Baton Rouge, LA 70802. *Higher Reciprocity Laws, Modular Forms of Weight One and Their Galois Representations.*

Due to a result by Serre and Deligne, for all Hecke eigenforms of weight one, there is an associated linear two dimensional complex representation which factors uniquely through a finite Galois extension K/\mathbb{Q} . If K is the splitting field of $f(x)$, then the p th coefficient of the modular form gives information about how $f(x)$ factors mod p . In this paper, we will discuss various pieces of information which can be used to explicitly identify $f(x)$ including the level of the modular form, its character, and the parity of its coefficients. (Received September 17, 2019)

1154-VS-2612 **Matthew Krauel*** (krauel@csus.edu). *Some properties of functions behaving like Jacobi forms.* Preliminary report.

In this talk we will examine functions that behave similar to, but are not, Jacobi forms. We then explain how these functions can be recharacterized and studied in a different manner. (Received September 17, 2019)

1154-VS-2659 **Owen Gary-Dennis Ekblad*** (oekblad@umich.edu), **Abigail G Loe**, **Alvaro Jose Cornejo**, **Marietta Elizabeth Geist** and **Kayla Iman Harrison.** *Minimal Discriminants of Rational Elliptic Curves with Specified Isogeny.*

By a rational elliptic curve, we mean a projective variety of genus 1 that admits a Weierstrass model of the form $y^2 = x^2 + Ax + B$ where A and B are integers. For a rational elliptic curve E , there is a unique quantity known as the minimal discriminant which has the property that it is the smallest integer (in absolute value) occurring in the \mathbb{Q} -isomorphism class of E . In 1975, Hellegouarch showed that the elliptic curve $y^2 = x(x+a)(x-b)$ for relatively prime integers a and b comes equipped with an easily computable minimal discriminant. Recently, Barrios extended this result to all rational elliptic curves with non-trivial torsion subgroup. This project gives a classification of minimal discriminant for rational elliptic curves that admit an isogeny of degree $N \geq 2$. This work is part of PRiME (Pomona Research in Mathematics Experience, NSF-1560394). (Received September 17, 2019)

1154-VS-2683 **Marietta Elizabeth Geist*** (geistm@carleton.edu), **Alvaro Jose Cornejo**, **Kayla Iman Harrison**, **Abigail G Loe** and **Owen Gary-Dennis Ekblad.** *The Modified Szpiro Conjecture and Elliptic Curves with Specified Isogeny.*

Given three positive, relatively prime integers a, b, c such that $a+b = c$, it is rare to have the product of the primes dividing them to be smaller than each of the three. In 1985, David Masser and Joseph Oesterlé made this precise through their celebrated “*ABC Conjecture*.” In 1988, Oesterlé showed that the *ABC* conjecture is equivalent to the modified Szpiro conjecture which states that for each $\epsilon > 0$ there are finitely many rational elliptic curves $N_E^{6+\epsilon} < \max\{c_4^3, c_6^2\}$ where N_E is the conductor of E and c_4 and c_6 are the invariants associated to a minimal model of E . Recently, Barrios showed that for a rational elliptic curve E , there is an explicit lower bound which depends only on the torsion subgroup of E . Our project seeks to create databases of rational elliptic curves in order to study the relationship between the modified Szpiro conjecture and elliptic curves with specified isogeny degree. This work is part of PRiME (Pomona Research in Mathematics Experience, NSF-1560394). (Received September 17, 2019)

1154-VS-2715 **Julia Cai*** (julia.cai@yale.edu), **Benjamin Hutz**, **Leopold Mayer** and **Max Weinreich.** *Automorphisms of Rational Functions over Fields of Characteristic $p > 0$.*

Given an algebraically closed field K , the group $\mathrm{PGL}_2(K)$ acts on the the set of all rational maps $\phi : \mathbb{P}^1(K) \rightarrow \mathbb{P}^1(K)$ by conjugation. Given a map ϕ , we can compute its automorphism group $\mathrm{Aut}(\phi)$, which is the stabilizer of ϕ in $\mathrm{PGL}_2(K)$ under this group action. It is known that $\mathrm{Aut}(\phi)$ is a finite group. Restricting our attention to fields of positive characteristic, we use the classification of finite subgroups of $\mathrm{PGL}_2(K)$ to show that every finite subgroup is isomorphic to $\mathrm{Aut}(\phi)$ for some ϕ .

The action of conjugation creates a natural equivalence relation on Rat_d , the space of degree- d rational maps. We can then consider the quotient space $\mathcal{M}_d(K)$. Under this relation, equivalent maps have isomorphic automorphism groups, so the set of all maps with a non-trivial automorphism group is well defined in $\mathcal{M}_d(K)$. We call this the automorphism locus. The automorphism locus of \mathcal{M}_2 has been studied over fields of characteristic 0; we describe the automorphism locus of $\mathcal{M}_2(\overline{\mathbb{F}_p})$, for all primes p , by following techniques from the proof of the former. When $p = 2$, it turns out that the automorphism locus is not Zariski-closed. (Received September 17, 2019)

1154-VS-2814 **Sky Pelletier Waterpeace***, Mathematics Department, Rowan University, 201 Mullica Hill Rd, Glassboro, NJ 08028. *A Novel Generating Function for a Parametrized Family of Möbius-like Arithmetic Functions Related to the Riemann ζ Function.*

We introduce a generating function for a novel generalized Möbius μ function defined in terms of partial sums of geometric series in $\frac{1}{p^s}$ and $-\frac{1}{p^s}$, and others, for primes p , and for complex $s = \sigma + it$, where $\sigma, t \in \mathbb{R}$, as usual. The standard Möbius $\mu(n)$ and Louiville $\lambda(n)$ functions are seen to be special cases of the general function. (Received September 18, 2019)

Probability and Statistics

1154-VT-250 **John C. Wierman*** (jwierma1@gmail.com), Dept. of Applied Mathematics & Statistics, 100 Whitehead Hall, Johns Hopkins University, Baltimore, MD 21218. *Progress on bounding site percolation thresholds of Archimedean lattices*. Preliminary report.

Percolation models are infinite random graph models which have applications to phase transitions and critical phenomena. In the site percolation model, each vertex in an infinite graph G is retained independently with probability p and deleted otherwise, while an edge is retained only if both of its endpoints are retained. Similarly, in bond percolation models, the edges are retained independently. The percolation threshold is the critical probability $p_c(G)$ such that if $p > p_c(G)$ there is positive probability that the random subgraph of retained elements has an infinite connected component, while the probability that all of its components are finite is one if $p < p_c(G)$. There are a few lattice graphs for which the site percolation threshold is exactly known, but rigorous bounds for unsolved lattices have been very inaccurate. The substitution method, which has been successful for computing relatively accurate bounds for bond percolation thresholds, is being adapted to site percolation models. Recent progress in improving rigorous bounds for site percolation on the Archimedean lattices, which are vertex-transitive tilings of the plane by regular polygons, will be surveyed. (Received August 27, 2019)

1154-VT-482 **Josh Hiller*** (johiller@adelphi.edu), One South Ave, Garden City, NY 11530, and **Monica Morales-Hernandez**. *Combining statistical literacy with real world data-skills: What should a first course in data science for computer science majors cover?* Preliminary report.

What is the best way to present data science to students with minimal mathematical maturity but computer programming experience? This is a question that the department of Mathematics and Computer Science at Adelphi has struggled with for some time. To help answer it, a new required course was introduced in the computer science major. This course, Statistics and Data Analytics, has dual objectives of teaching statistical literacy and advanced data science techniques. However, the authors (who are responsible for teaching the course) have had difficulty finding the correct balance of topics to extract the course's full potential. In this talk we will go over different iterations of the curriculum and student feedback. We would appreciate comments from the audience that might help us improve our course going forward. (Received September 05, 2019)

1154-VT-612 **Majid Noroozi*** (mnoroozi@knights.ucf.edu), Department of Mathematics at UCF, 4393 Andromeda Loop N, Orlando, FL 32816, and **Marianna Pensky** (marianna.pensky@ucf.edu), Department of Mathematics at UCF, 4393 Andromeda Loop N, Orlando, FL 32816. *Clustering in Sparse Popularity Adjusted Stochastic Block Model*.

In the present talk, we introduce the Sparse Popularity Adjusted Stochastic Block Model (SPABM) which is a special case of the Popularity Adjusted Stochastic Block Model (PABM). Since the real-life networks are usually sparse, studying a model that is designed to deal with such networks is very useful. One of the shortcomings of well-studied block models such as the SBM and the DCBM is that they do not allow to efficiently model sparsity in networks, while the SPABM can be used to effectively model the sparsity by allowing a node to be inactive in some communities, yet active in the others. Clustering is one of the fundamental problems in network analysis. To detect the communities in the networks that fit the SPABM, we propose to use a subspace clustering method. We also estimate the true number of communities in such networks using the estimated connection probabilities. Experiments on synthetic and real data sets demonstrate the effectiveness of the clustering and estimation approaches. (Received September 08, 2019)

1154-VT-1005 **Bhikhari Tharu***, Spelman Lane SW Box 953, Atlanta, GA 30314, and **Nirajan Dhakal**. *On the use of Bayesian quantile regression method to explore the historical trends in extreme precipitation and their connections with large-scale climate patterns over the contiguous USA*.

Probability distribution function (PDF) of precipitation is expected to change under the warmer climate. In this study, the Bayesian quantile regression method was employed to analyze trends of Annual Daily and Monthly Maximum precipitation indices at different quantile levels and their teleconnections with large-scale climate patterns over the Contiguous USA. Historic precipitation time series for 65 years (1950 -2014) for 1108 sites were used for the analysis. Two significant oscillations: the ENSO and the NAO were used as covariates in the Bayesian quantile regression model. A tremendous spatial variability on the spatial distribution of significant trends was observed with ENSO or NAO. However, for many regions of the USA, a stronger teleconnection between climate indices and precipitation indices for upper quantile level was observed as compared to lower

quantile levels. The trends detected using the Bayesian quantile regression method were undetected or overlooked by other previously used approaches for trend detection. (Received September 12, 2019)

1154-VT-1046 **Cong Wang*** (cong960@nmsu.edu), **Tonghui Wang** (twang@nmsu.edu), **David Trafimow** (dtrafimo@nmsu.edu) and **Hunter A. Myuz** (hamz@nmsu.edu). *Necessary sample sizes for specified closeness and confidence of matched data under the skew normal setting.*

Previous researchers have shown how to compute a prior confidence intervals (as opposed to sample-based confidence intervals) for means, assuming normal distribution (Tramow, 2017; Tra mow and MacDonald, 2017); or for locations, assuming skew normal distributions (Tramow et al. 2018). The present work extends a prior thinking to an important case not addressed previously, where the researcher is interested in the difference between means or the difference in locations across two matched samples. The proposed procedure can be used under the assumption that both samples come from normal distributions or skew-normal distributions. Computer simulations support the equations presented, along with an example with real data. (Received September 12, 2019)

1154-VT-1066 **Md Sazib Hasan*** (sazib25@gmail.com), 368 S Mall Drive, Unit I 206, St George, UT 84790, and **Kalimuthu Krishnamoorthy**. *Confidence intervals for the mean and a percentile based on zero-inflated lognormal data.*

The problems of estimating the mean and an upper percentile of a lognormal population with nonnegative values are considered. For estimating the mean of a such population based on data that include zeros, a simple confidence interval (CI) that is obtained by modifying Tian's [Inferences on the mean of zero-inflated lognormal data: the generalized variable approach. Stat Med. 2005;24:3223—3232] generalized CI, is proposed. A fiducial upper confidence limit (UCL) and a closed-form approximate UCL for an upper percentile are developed. Our simulation studies indicate that the proposed methods are very satisfactory in terms of coverage probability and precision, and better than existing methods for maintaining balanced tail error rates. The proposed CI and the UCL are simple and easy to calculate. All the methods considered are illustrated using samples of data involving airborne chlorine concentrations and data on diagnostic test costs. (Received September 13, 2019)

1154-VT-1402 **Melissa Innerst*** (melissainnerst@gmail.com) and **Jack D Tubbs**. *Lehmann ROC Regression for Longitudinal Data.*

The Lehmann family of Receiver Operating Characteristic (ROC) curves has the flexibility to accommodate the comparison of multiple markers, covariates, and longitudinal data. This work focuses on the application of the Lehmann ROC regression method to longitudinal data. The performance of the Lehmann method is compared to that of the beta ROC regression method for the analysis of data from a simulated single dose-response study. This is done twice, with data generated from the Weibull and normal distributions. (Received September 15, 2019)

1154-VT-1501 **Grant Lee Innerst***, 441 E King St, Shippensburg, PA 17257, and **David J Kahle**. *Solving Statistical Estimation Problems with Algebraic Geometric Tools.*

This work arises from the realization that statistical estimation problems often display a strong algebraic structure. This structure can be leveraged to use powerful software from the algebraic community to find solutions that were previously unattainable. Although we touch on multiple different estimation problems and algebraic techniques, our main focus is on using tools from numerical algebraic geometry to solve estimation problems that fall into the class of minimum chi-square estimators. (Received September 15, 2019)

1154-VT-1755 **Durga H Kutal***, kutald@uww.edu, and **Khyam N Paneru**. *E-Bayesian Estimation on Cure Model for Right Censored Data.* Preliminary report.

This study considers a cure model for right censored data. We consider E-Bayesian (the expectation of Bayesian estimate) method, Bayesian estimation method and the maximum likelihood method to estimate model parameters in a cure model with right censored data. The Bayesian and E-Bayesian approaches are studied using the Markov Chain Monte Carlo Method. We compare the performance of E-Bayesian, Bayesian and maximum likelihood methods. Furthermore, we apply to a real data set for illustration. (Received September 16, 2019)

1154-VT-1869 **Mark Landry*** (landrym5@msu.edu), **Paige Percy** and **Cheuk-Yin Lee**. *Multiple Points of Fractional Brownian Motion in \mathbb{R}^d .*

In 1987, Nils Tongring showed that if X is a Brownian motion in \mathbb{R}^2 , E is a fixed, nonempty compact set in the plane with positive logarithmic capacity of order k , where k is a positive integer, then E contains k -tuple points for almost all paths. We define x to be a k -tuple point for the path ω if there are times $t_1 < t_2 < \dots < t_k$ such that $x = X(t_1, \omega) = X(t_2, \omega) = \dots = X(t_k, \omega)$. In this paper, we extend these findings to fractional Brownian

motion where the assumption of independent increments is relaxed. Using the property of local nondeterminism, we show that if B is a fractional Brownian motion in \mathbb{R}^d with Hurst index H such that $Hd = 1$, and E is a fixed, nonempty compact set in \mathbb{R}^d with positive logarithmic capacity of order k , then E contains k -tuple points with positive probability. We also show that under the same conditions, but with Hurst index H such that $Hd > 1$, then if E is a fixed, nonempty compact set in \mathbb{R}^d with positive capacity with respect to the function $\phi(s) = \frac{1}{s^{k(d-1/H)}}$, then E contains k -tuple points with positive probability. (Received September 16, 2019)

1154-VT-2090 **Ziwei Ma*** (ziweima@nmsu.edu), 1290 Frenger Mall MSC 3MB / Science Hall 236, Las Cruces, NM 88003-8006, and **Tonghui Wang** (twang@nmsu.edu), 1290 Frenger Mall MSC 3MB / Science Hall 236, Las Cruces, NM 88003. *Inference on the difference of location parameters under Multivariate skew-normal setting*. Preliminary report.

In our previous work, we proposed three methods to construct confidence regions for location parameter under the multivariate skew-normal setting. However, to inference on the difference of location parameters between two population attracts more attentions for many practitioners who concern more on answering if there is a significant change of location parameter based on two samples. Under this scenario, the related sampling distributions are studied, and there are two methods, based on pivotal quantity and confidence distribution, proposed to construct the confidence regions of the difference of location parameter under the multivariate skew-normal setting. A case study, analyzing the traffic accident data from the National Highway Traffic Safety Administration, is presented as well. (Received September 17, 2019)

1154-VT-2117 **Jessica Murphy*** (jessica.murphy@ucdenver.edu), **Pitshou Nzazi Duki, Nicholas Weaver** and **Audrey Hendricks**. *Accessible Analysis of Longitudinal Data with Linear Mixed Effects Models: There's an App for That*.

Longitudinal mouse models are commonly used to study possible causal factors associated with human health and disease. However, the statistical models applied in these studies are often incorrect. If correlated observations in longitudinal data are not modeled correctly, they can lead to biased and imprecise results. Therefore, we provide an interactive Shiny App to enable appropriate analysis of correlated data using linear mixed effects (LME) models. Using the app, we re-analyze a dataset published by Blanton et al (Science 2016) that modeled mice growth trajectories after microbiome implantation from nourished or malnourished children. We then compare the fit and stability of LME models with different parameterizations. While the model with the best fit and zero convergence warnings differed substantially from the two-way ANOVA model chosen by Blanton et al, both models found significantly different growth trajectories for microbiota from nourished vs. malnourished children. We also show through simulation that the results from the two-way ANOVA and LME models will not always be consistent, supporting the need to model correlated data correctly. Hence, our app provides easy implementation of LME models for accessible and appropriate analysis of studies with longitudinal data. (Received September 17, 2019)

1154-VT-2337 **Ruth Burkhalter*** (ruth.burkhalter@usd.edu). *Bootstrap Control Chart for Pareto Percentiles*. Preliminary report.

Lifetime percentile is an important indicator of product reliability. However, the sampling distribution of a percentile estimator for any lifetime distribution is not a bell shaped one. Therefore, the well-known Shewhart-type control chart cannot be applied to monitor the product lifetime percentiles. In this talk, Bootstrap control charts based on maximum likelihood estimator (MLE) and moment method are proposed for monitoring Pareto percentiles. An intensive simulation study is conducted to compare the performance among the proposed MLE Bootstrap control chart, moment method Bootstrap control chart and Shewhart-type control chart. Finally, an application is presented. (Received September 17, 2019)

Topology

1154-VU-769 **Zeinab Bandpey*** (zeinab.bandpey@morgan.edu), Morgan State University, E cold Spring Lane, Baltimore, MD 21251. *Compact and extremally disconnected spaces via generalized continuous functions*.

In [1], the class of compact and extremally disconnected spaces were studied using several investigative tools such as filters, graphs, functions, multifunctions and subsets of the space. These different approaches of investigation produced significant characterizations and properties of this important class of spaces. In [2] we introduced three forms of generalized continuous functions by studying the class of u -continuous functions of Joseph, Kwack and Nayyar [3] using the concepts of $\alpha\alpha$ -set of Njastad [4]. The generalized continuous forms introduced there are:

α -continuous, semi- α -continuous and strongly u -continuous functions. In the present study we investigate the class of compact and extremally disconnected spaces using these generalized continuous functions. (Received September 10, 2019)

1154-VU-1295 **Mark Hughes and Thomas Liddle*** (thomasliddle34@gmail.com), thomasliddle34@gmail.com, and **Jamison Moody and Spencer Reschke.** *Using reinforcement learning to find the slice genus.*

One interesting problem in topology involves studying the types of surfaces which can be bounded by a given knot. If the surfaces are orientable and contained in 3-dimensional space then we call them Seifert surfaces. If we allow our surfaces instead to sit inside 4-dimensional space, then we call them slice surfaces. The smallest genus of any slice surface for a knot K is called the slice genus of K . One way to find the slice genus of a knot is by reducing a braid word that represents the knot, though there is no clear procedure for doing this optimally. In this talk, I will discuss an approach to using reinforcement learning for accomplishing this task. Reinforcement learning is a form of artificial intelligence that learns from repetition, with good decisions being reinforced in the learning to facilitate quick improvement. In this talk, we will focus on different algorithms in reinforcement learning that could be used to solve this problem and why some could be more effective than others. (Received September 16, 2019)

1154-VU-1532 **Vincent Longo*** (vincent.longo@huskers.unl.edu), NE. *Tunnel Number and Unknotting Numbers of Twist Spun Knots.* Preliminary report.

In 2004, Satoh proved that $b - 1$ is an upper bound for the unknotting number of the twist spin of a classical knot K , where b is the bridge number of K . This bound, however, does not seem to be sharp for many knots. For meridionally primitive knots K , we provide a strict improvement to this upper bound by showing that the tunnel number $t(K)$ is an upper bound for the unknotting number of the twist spin of K . We additionally show that $t(K) + 1$ is an upper bound for any K . (Received September 16, 2019)

1154-VU-1624 **Ryan J. Jensen***, jensenrj@sfasu.edu. *The Datasaurus and Persistent Homology.* Preliminary report.

The datasaurus dozen is a generalization of Anscombe's quartet. It is a collection of 12 data-sets in R^2 , all of which have essentially the same summary statistics (X -mean, Y -mean, X -standard deviation and Y -standard deviation), but which are visually very different. For example the Datasaurus data-set is in the shape of a Tyrannosaurus rex, another data-set is in the shape of an oval and another in the shape of an "X." The datasaurus dozen was designed to urge people to "never trust summary statistics alone; always visualize your data" (from <https://www.autodeskresearch.com/publications/samestats>). In this talk, we will give a brief introduction to persistent homology, a tool from algebraic topology which is used to describe the shape of topological spaces. We will then show how persistent homology can differentiate between the some of the data-sets in the datasaurus dozen. (Received September 16, 2019)

1154-VU-2283 **Eric Anderson, Jonah Amundsen, Christopher Davis and Daniel Guyer*** (guyerdm7106@uwec.edu). *The C-complex clasp number of links.*

In knot theory, a link is a disjoint union of circles, (i.e. components), in 3-dimensional space, and a goal of knot theory is to measure the interaction between the various components of a link. Recently, the surfaces bounded by these components, together referred to as a C -Complex, have been used as one such measure. We ask the question, "Given a link, what is the least number of clasps amongst all C -complexes bounded by that link?" For two-component links, we have found a precise formula for the minimal number of clasps. While in the case of links consisting of three components, we prove a bound in terms of a generalization of the classical linking number called the triple linking number, while relating this problem to minimal perimeter polyominoes. (Received September 17, 2019)

1154-VU-2339 **Ik Jae Lee***, Department of Mathematics, Rowan University, 201 Mullica Hill Rd., Glassboro, NJ 08028, and **David N Yetter.** *Stratified spaces, Directed Algebraic Topology, and State-Sum TQFTs.* Preliminary report.

In this talk, we apply the theory of directed topology developed by Grandis to the study of stratified spaces by describing several ways in which a stratification or a stratification with orientations on the strata can be used to produce a related directed space structure. This provides a setting for the constructions of state-sum TQFTs with defects, similar construction of a Dijkgraaf- Witten type TQFT. (Received September 17, 2019)

1154-VU-2369 **Max Folkers***, 701 North C St, Indianola, IA 50125, and **Eli Marriott and Miguel Sanchez**. *Random grid diagrams*.

In this talk we will look at the distribution of the number of components in the population of knots and links represented by grid diagrams. We write programs in the R programming language to generate random samples of grid diagrams. We analyze the number of components and splittability of the links in these samples using statistical methods to gain a better understanding of the population of grid diagrams and the knots and links they represent. (Received September 17, 2019)

1154-VU-2442 **Leona Sparaco*** (lsparaco@holycross.edu). *Character Varieties of a Family of 2-Bridge Knots*. Preliminary report.

Let M be a hyperbolic 3-manifold of finite volume. The $SL_2(\mathbb{C})$ character variety of M is essentially the set of all representations $\rho : \pi_1 \rightarrow SL_2(\mathbb{C})$ up to trace equivalence. This is an algebraic set which encodes many geometric properties of M . In this paper we will compute the $SL_2(\mathbb{C})$ character variety for an infinite family of 2-bridge knots. (Received September 17, 2019)

1154-VU-2595 **Marion Campisi and Luis Torres*** (luis.torres@sjsu.edu). *The disk complex and topologically minimal surfaces in the 3-sphere*. Preliminary report.

David Bachman introduced topologically minimal surfaces as generalizations of incompressible and strongly irreducible surfaces. These surfaces have been useful in problems that deal with stabilization, amalgamation, and isotopy of Heegaard splittings and bridge spheres for knots. In this talk, we discuss joint work with Marion Campisi which shows that the disk complex of a compact, orientable surface in the 3-sphere is homotopy equivalent to a wedge of spheres, all of the same dimension. This leads to a proof that genus $g > 1$ Heegaard surfaces of the 3-sphere are topologically minimal with index $2g - 1$. (Received September 17, 2019)

1154-VU-2674 **Josh Thompson*** (joshthom@nmu.edu) and **Davin Hemmila** (dahemmil@nmu.edu). *Surgery on Penrose Tilings*. Preliminary report.

The Cartwheel is a well-known Penrose Tiling of the plane. Near the center of this tiling one finds a group of tiles forming a decagon. This decagon, a so-called Cartwheel, forces the remaining tiling. While the tiling is aperiodic, the edges of this decagon bound periodic regions called "Conway worms". We define two surgeries on this tiling that exploit topological properties of these worms and prove results about the resulting objects. The first map collapses the worms and glues the remaining portions together in such a way that the image is quasi-isometric to the original. The second map is a quotient map that identifies each worm and identifies similar edges of the Cartwheel. The image of this map contains a Klein bottle. We discuss this procedure as well as extensions to other geometric designs. (Received September 17, 2019)

1154-VU-2835 **Wako T Bungula*** (wbungula@uwlax.edu), 1725 State Street, La Crosse, WI 54601. *Clustering Algorithms: Stability of TDA Mapper Graphs*.

When a point cloud data is fed into a TDA Mapper algorithm, clustering algorithm is applied to create the nodes of the graph. Tamal Dey et. al. showed that the filtration of mapper graphs exist if a cover satisfies certain condition and if the data is topological space. If the data is point cloud, Single linkage and DBSCAN give filtration of mapper graphs as the parameter, cover size, increases, where as KMeans, average linkage and complete linkage do not. In this talk, we I will present the reasons the parameter cover size is important, Single linkage and DBSCAN give filtration of mapper graphs, and why KMeans, average linkage, and complete linkage do not give filtration mapper graphs. Finally, I will present that if a data is perturbed by a delta, then the bi-filtrations of the mapper graphs of respective data are at most 2δ -interleaved. (Received September 18, 2019)

Other Topics

1154-VV-299 **Pan Shun Lau*** (plau@unr.edu). *Convexity and star-shapedness of joint matricial range*. Let $\mathbf{A} = (A_1, \dots, A_m)$ be an m -tuple of bounded linear operators acting on a Hilbert space \mathcal{H} . Their joint (p, q) -matricial range $\Lambda_{p,q}(\mathbf{A})$ is the collection of $(B_1, \dots, B_m) \in \mathbf{M}_q^n$, where $I_p \otimes B_j$ is a compression of A_j on a pq -dimensional subspace. This definition covers various kinds of generalized numerical ranges for different values of p, q, m . In this talk, we will show that $\Lambda_{p,q}(\mathbf{A})$ is star-shaped if the dimension of \mathcal{H} is sufficiently large. If $\dim \mathcal{H}$ is infinite, we consider the joint essential (p, q) -matricial range

$$\Lambda_{p,q}^{ess}(\mathbf{A}) = \bigcap \{ \text{cl}(\Lambda_{p,q}(A_1 + F_1, \dots, A_m + F_m)) : F_1, \dots, F_m \text{ are compact} \},$$

ans show that it is always non-empty, compact and convex. This is the joint work with Chi-Kwong Li, Yiu-Tung Poon, Nung-Sing Sze. (Received August 29, 2019)

1154-VV-693 **C. Bryan Dawson*** (bdawson@uu.edu). *The Lake Wobegon Paradox.*

A recurring sketch in the radio show *A Prairie Home Companion* is “Lake Wobegon,” where “all the children are above average.” In this talk we present a simple example of a real-valued function on an interval for which it seems that all the functions’ values are above average. The paradox is then resolved with the help of the hyperreal numbers. (Received September 09, 2019)

1154-VV-868 **Tin-Yau Tam*** (ttam@unr.edu). *Weak log-majorization of unital trace-preserving completely positive maps.*

Let $\Phi : M_n \rightarrow M_n$ be a unital trace preserving completely positive map and $A \in M_n$ be a positive definite matrix. Weak log-majorization and weak majorization between $\Phi(A)$ and A are studied. Determinantal inequalities between $\Phi(A)$ and A are obtained as a consequence. By considering special classes of unital trace preserving completely positive map, some known matrix inequalities such as Fischer’s inequality and Matic’s inequality are rediscovered. An affirmative answer to a question of Tam and Zhang in 2019 is given.

This is a joint work with Pan Shun Lau. (Received September 11, 2019)

1154-VV-1047 **Treena Basu***, 1600 Campus Road, Los Angeles, CA 90041, and **Ron Buckmire, Kanadpriya Basu and Nishu Lal.** *“Will They or Won’t They?”: Predictive Models of Student College Commitment Decisions Using Machine Learning.*

Every year, academic institutions invest considerable effort and substantial resources to influence, predict and understand the decision-making choices of applicants who have been offered admission. In this study, we applied several supervised machine learning techniques to four years of data on 11,001 students, each with 35 associated features, admitted to a small liberal arts college in California to predict student college commitment decisions. By treating the question of whether a student offered admission will accept it as a binary classification problem, we implemented a number of different classifiers and then evaluated the performance of these algorithms using the metrics of F -measure and area under the receiver operator curve. The results from this study indicate that the logistic regression classifier performed best in modeling the student college commitment decision problem, i.e., predicting whether a student will accept an admission offer, with an AUC score of 79.6%. The significance of this research is that it demonstrates that many institutions could use machine learning algorithms to improve the accuracy of their estimates of entering class sizes, thus allowing more optimal allocation of resources and better control over net tuition revenue. (Received September 12, 2019)

1154-VV-1110 **Chris Ahrendt*** (ahrendcr@uwec.edu). *Bifurcations of a discrete analog of the Bernoulli differential equation.*

Using the framework of the time scale calculus, we focus on a discrete analog of the classic Bernoulli differential equation. We briefly describe the derivation of the discrete equation. We then explore the bifurcations that occur as the parameters vary; six distinct bifurcation diagrams will be discussed which give a complete characterization of solution behavior in the non-regressive case.

The time scale calculus generalizes and unifies differential and difference equations, but also is a source of many interesting results that do not have a direct analog to these classic cases. An introduction to the key results of the time scale calculus that are used in this work will be provided. (Received September 13, 2019)

1154-VV-1131 **Naiomi T Cameron and Everett N Sullivan*** (everettsu@gmail.com). *Peakless Motzkin Paths with Marked Level Steps at Fixed Height.*

A Motzkin path is a path starting and ending on the x -axis which uses up, down and level steps and never goes below the x -axis. We consider Motzkin paths where any number of level steps on the x -axis are allowed to be marked. It is known that the number of such paths corresponds to a matrix which turns out to be a pseudo-involution in the Riordan group. We provide a combinatorial proof of this result by means of a sign-reversing involution on pairs of signed marked Motzkin paths. We also extend this result to the class of matrices that count Motzkin paths where level steps at a fixed height are allowed to be marked. (Received September 13, 2019)

1154-VV-1593 **Joseph J. Rushanan*** (jjr@mitre.org), 202 Burlington Rd, Bedford, MA 01730, and **Andres Molina-Markham.** *Positioning, Navigation, and Timing Trust Inference Engine.*

Critical infrastructure and military users increasingly need to infer the assurance of position, navigation, and timing (PNT) information. Such inference considers multiple sources of information, such as GPS and other

sensor inputs, awareness information, and auxiliary sources, e.g., network data. The challenge is to fuse trust assumptions and assessments of these sources into useful assurance metrics that can be scrutinized and refined. We present PNTTING, a PNT Trust Inference Engine that facilitates this trust fusion according to probabilistic models with rigorous semantics. PNTTING's probabilistic models describe relations between inputs and outputs and how inputs are transformed and combined. Input transformations and other relations are encoded via probabilistic models developed by assurance model designers and PNT engineers. This talk focuses on the inference and the mathematical and computational frameworks based on emerging work in probabilistic programming languages. In addition, our talk also highlights two complementary challenges to the appropriate computational framework: the need for realistic test vectors and the problem of creating trust models for the inputs and their dependencies. (Received September 17, 2019)

1154-VV-1674 **A Sophie Aiken*** (saiken@coloradocollege.edu). *Maximally sized upward and downward closed P -free families.*

Let $[n]$ denote the set $\{1, 2, \dots, n\}$. Let $\mathcal{B}(n)$ denote the Boolean lattice, which is comprised of the power set of $[n]$ partially ordered by set inclusion. We will look at the maximum size of upward and downward closed families of sets chosen from $\mathcal{B}(n)$ which forbid the presence of a weak subposet P , and in many cases we will give a description of the elements in these families. We say P is a weak subposet of a family \mathcal{F} if there is an injection from P to \mathcal{F} which preserves the set relations in P and may allow for additional instances of set inclusion. We will give results for common posets including chains, forks, diamonds, harps, Js, Ns, and butterflies, and we will make conjectures about batons, brooms, complete bipartite graphs, and crowns. (Received September 16, 2019)

1154-VV-1855 **Victor Ginsburg*** (vvg5123@psu.edu), **Ricardo Velasquez** (rivezu97@utexas.edu) and **Hayden Julius** (hjulius@kent.edu). *On maps preserving Lie products equal to a rank-one nilpotent.*

Linear preserver problems are one of the most active areas of research in matrix theory. Linear maps that preserve zero Lie products have been well-studied. We characterize bijective linear maps on complex $n \times n$ matrices that preserve Lie products equal to a rank-one nilpotent. Surprisingly, these maps have a different description from maps preserving zero Lie products. (Received September 16, 2019)

1154-VV-1863 **Mohamed Allali*** (allali@chapman.edu). *Color Space Mathematical Modeling.*

Images are the most effective medium of human communication and when processed under the control of students and teachers, they put mathematical ideas in an exciting new light. In this talk, I will show how to represent colors mathematically and how to deal with a color space as a vector space. This concept can be incorporated as a practical and interesting problem in mathematics courses. (Received September 16, 2019)

1154-VV-1921 **Denis A. Aliyev*** (aliyevda@vmi.edu) and **Craig Lee Zirbel** (zirbel1@bgsu.edu). *Tree-penalized TSP: A hybrid objective function for linear data ordering.*

Understanding the structural relationship in the given data can greatly facilitate data analysis and decision making. Many tools, like multidimensional scaling and hierarchical clustering have been developed and used for this purpose. Seriation is another method. Given a sample of n objects and the corresponding dissimilarity matrix, seriation aims to produce a linear ordering of the objects. One uses the ordering to produce a heat map visualization of the reordered dissimilarity matrix and thus understand the structure of the data. Good orderings should reflect the underlying data structure and result in heat maps that allow for clear interpretation of the data structure. There have been developed a substantial number of seriation methods. Which methods produce good orderings? Some seriation methods consistently produce orderings that are more helpful for understanding and visualization of the structure of data than other methods. In this paper we introduce a new seriation method, called tree-penalized TSP (tpTSP), which compares favorably with other considered methods. Hybrid in nature, the method benefits from the strengths of two popular types of seriation methods, TSP and Optimal Leaf Order, but avoids their key pitfalls. (Received September 16, 2019)

1154-VV-1923 **Benjamin R Prather*** (bprather@math.fsu.edu). *Steiner Algebras.* Preliminary report. Steiner triple systems are used to generalize Cayley-Dickson algebras. These algebras provide introductory motivation to pursue research in combinatorics, abstract algebra and analysis. (Received September 16, 2019)

- 1154-VV-2273 **Kenneth Horwitz*** (ken.horwitz@njit.edu), **Angela R Garretson** (angela@njit.edu) and **Bruce Bukiet** (bukiet@njit.edu). *Utilizing Inquiry Based Learning to Prepare Urban High School Students for Success in College Mathematics*. Preliminary report.

In this presentation, we discuss an innovative year-long project that combines extensive professional development for teachers and inquiry-based math instruction and skills for success in life for rising high school seniors from an urban district in a college setting. This partnership brings together the leadership of the City of Newark, NJ, (Mayor's Office) the Newark Board of Education (Superintendent's Office) and New Jersey Institute of Technology (Office of the President) for the purpose of changing the paradigm from less to more underrepresented minorities pursuing STEM degrees. The gateway to success in STEM majors is a student's readiness for Calculus 1. Thus, the project works to build the skills of the students through an inquiry based math enrichment program over the summer and a Precalculus or Calculus course on the university campus during the academic year. (Received September 17, 2019)

- 1154-VV-2309 **Adam J Hammett** and **James Ford McElroy*** (jfmcelroy@cedarville.edu), 251 N. Main St. No. 5091, Cedarville, OH 45314. *How often do two permutations meet in the minimal element?*

Let S_n denote the collection of length n permutations, where we think of a given $\pi \in S_n$ as a *word*, $\pi = \pi(1)\pi(2)\cdots\pi(n)$. Two permutations of S_n are said to be comparable in the weak order lattice if one is closer, in a natural way, to the identity permutation $12\cdots n$ than the other. We find a necessary and sufficient condition for r permutations of S_n to have trivial meet $12\cdots n$ in the weak order lattice. Using this condition and the principle of inclusion-exclusion, we compute the probability generating function for the probability that r independent and uniformly random permutations of S_n have trivial meet. (Received September 17, 2019)

- 1154-VV-2382 **Eugene Fiorini** (eugenefiorini@muhlenberg.edu) and **Bryan Walker*** (walkebw0@sewanee.edu), 193 Edgewood Lane, Winchester, TN 37398, and **Sabrina Traver, Robert Argus, Cristian Hernandez and Benjamin Nagle**. *The Topological Structure of a Family of Excellent Morse Functions*.

A smooth function f of n variables is called an excellent Morse function, if every critical point is distinct and non-degenerate. For a Morse function of two variables, all critical points are local maxima, minima, or saddle points. The topological structure of a two-variable Morse function on a compact domain has finitely many critical points and can be associated with a tree, called an A -tree, whose vertices are the connected components of the level surfaces of the function. Each vertex is of degree 1 (local maximum or minimum) or 3 (saddle point). Counting the number of excellent Morse functions associated with each tree is equivalent to playing a game of "plates and olives," introduced by Liviu I. Nicolaescu, in which a plate or olive is added, combined, or removed depending on whether the associated vertex is a local maximum, local minimum, or saddle point. In this talk we discuss results which involve encoding the topological structure of excellent Morse functions on the 2-sphere in an A -tree and attempt to enumerate the number of successful games. That is, we consider the minimum number of ways in which a game of plates and olives can be resolved to an empty plate for a specific family of A -trees called A_{IN} -trees. (Received September 17, 2019)

- 1154-VV-2489 **Areeba Ikram*** (aikram@mines.edu). *Lyapunov Inequalities for Discrete Fractional Nonlinear Boundary Value Problems*. Preliminary report.

We will begin by presenting background for discrete fractional calculus. Then, we will explore properties of solutions to certain nonlinear third-order boundary value problems. In particular, we will discuss a technique for obtaining Lyapunov inequalities for these BVPS which uses the integral form of a solution to a related second-order boundary value problem. Finally, we will mention a generalization of this technique for obtaining Lyapunov inequalities to higher-order BVPs. (Received September 17, 2019)

- 1154-VV-2643 **Joshua Harrington, Kedar Karhadkar, Madeline Kohutka, Tessa Stevens*** (txs473@case.edu) and **Tony W. H. Wong**. *Two dependent probabilistic chip-collecting games*.

Alice and Bob take turns to collect chips in the following manner. In each turn, Alice tosses a fair coin, which decides whether she collects a or b chips, where a and b are positive integers. If Alice collects a chips, then Bob collects b chips, and vice versa. We consider two variants of game play that have different rules in determining the winner. Namely, the winner of Game 1 is the first player to collect at least n chips, while the winner of Game 2 is the first player to collect a positive number of chips congruent to 0 modulo n . We fully determine the formula for the winning probabilities of each player in Game 1, and determine the best and worst case scenarios in terms of winning probabilities in Game 2. (Received September 17, 2019)

1154-VV-2651 **Segun Victor Ofe*** (svo003@shsu.edu). *Positional games and the probabilistic method.* Preliminary report.

We apply the probabilistic method to sequences of positional games to generate lower bounds for generalizations of Ramsey numbers. We will provide examples of sequences that give rise to classical Ramsey numbers and other well-studied Ramsey numbers. (Received September 17, 2019)

1154-VV-2736 **Liljana Babinkostova** (liljanababinkostova@boisestate.edu), **Robert J. Erbes** (robert.erbes@inl.gov), **William R. Myers** (riley.myers@student.nmt.edu) and **William Unger*** (williamunger@u.boisestate.edu). *Security of generalized lightweight substitution permutation ciphers.* Preliminary report.

The study of the security of substitution permutation network (SPN) ciphers has been rigorous because of the need for encryption standards for everyday computing. There exist metrics on how to quantify the security of such cryptosystems such as differential and linear branch numbers for the S-Box parameter. However, most of the literature focuses on the binary case, but such metrics can be generalized over any field of prime power. We construct metrics to measure the security of parameters in the non-binary setting by generalizing known metrics. We implement a version of the GIFT cryptosystem in base 3 opposed to base 2 as a proof of concept of a SPN that is not base 2. Finally, we examine attacks on the binary constructed ciphers in order to generalize them for use in cryptosystems with a generic base. (Received September 17, 2019)

1154-VV-2789 **Hassan Almusawa*** (almusawah@vcu.edu), Virginia Commonwealth University, Department of Mathematics and Applied Mathema, Richmond, VA 23284, and **Ryad Ghanam** and **Gerard Thompson**. *Symmetries of the Canonical Geodesic Equations of Five-Dimensional Nilpotent Lie Algebras.*

Symmetries of the canonical geodesic equations of indecomposable nilpotent Lie groups of dimension five are constructed. For each case, the associated system of geodesics is provided. In addition, a basis for the associated Lie algebra of symmetries as well as the corresponding non-zero Lie brackets are listed and classified. (Received September 18, 2019)

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- 08 General algebraic systems
- 11 Number theory
- 12 Field theory and polynomials
- 13 Commutative rings and algebras
- 14 Algebraic geometry
- 15 Linear and multilinear algebra; matrix theory
- 16 Associative rings and algebras
- 17 Nonassociative rings and algebras
- 18 Category theory; homological algebra
- 19 K -theory
- 20 Group theory and generalizations
- 22 Topological groups, Lie groups
- 26 Real functions
- 28 Measure and integration
- 30 Functions of a complex variable
- 31 Potential theory
- 32 Several complex variables and analytic spaces
- 33 Special functions
- 34 Ordinary differential equations
- 35 Partial differential equations
- 37 Dynamical systems and ergodic theory
- 39 Difference and functional equations
- 40 Sequences, series, summability
- 41 Approximations and expansions
- 42 Fourier analysis
- 43 Abstract harmonic analysis
- 44 Integral transforms, operational calculus
- 45 Integral equations
- 46 Functional analysis
- 47 Operator theory
- 49 Calculus of variations and optimal control; optimization
- 51 Geometry
- 52 Convex and discrete geometry
- 53 Differential geometry
- 54 General topology
- 55 Algebraic topology
- 57 Manifolds and cell complexes
- 58 Global analysis, analysis on manifolds
- 60 Probability theory and stochastic processes
- 62 Statistics
- 65 Numerical analysis
- 68 Computer science
- 70 Mechanics of particles and systems
- 74 Mechanics of deformable solids
- 76 Fluid mechanics
- 78 Optics, electromagnetic theory
- 80 Classical thermodynamics, heat transfer
- 81 Quantum theory
- 82 Statistical mechanics, structure of matter
- 83 Relativity and gravitational theory
- 85 Astronomy and astrophysics
- 86 Geophysics
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- 92 Biology and other natural sciences
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- 94 Information and communication, circuits
- 97 Mathematics education

