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**Asymptotic expansion of correlation functions for  $\mathbb{Z}^d$  covers of hyperbolic flows.** (English. French summary) [Zbl 1510.37039](#)

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The authors give asymptotic expansions for  $t \rightarrow \infty$  of the correlation functions  $C_t(f, g)$  of sufficiently regular observables  $f, g$ , for  $\mathbb{Z}^d$  extensions of a suitable class of hyperbolic flows, in an abstract setup. Such asymptotic expansions are of the form

$$C_t(f, g) = \sum_{p=0}^P C_p(f, g) t^{-\frac{d}{2}-p} + o(t^{-\frac{d}{2}-P}), \quad P \in \mathbb{N},$$

and the class under study includes the finite horizon periodic Lorentz gas in dimension 2 (i.e., a Sinai billiard) and geodesic flows on abelian covers of compact Riemannian manifolds with negative curvature.

Reviewer: [Valeria Ricci \(Roma\)](#)

**MSC:**

- [37C30](#) Functional analytic techniques in dynamical systems; zeta functions, (Ruelle-Frobenius) transfer operators, etc. Cited in **2** Documents
- [37A50](#) Dynamical systems and their relations with probability theory and stochastic processes
- [37A60](#) Dynamical aspects of statistical mechanics
- [37D40](#) Dynamical systems of geometric origin and hyperbolicity (geodesic and horocycle flows, etc.)
- [37A25](#) Ergodicity, mixing, rates of mixing

**Keywords:**

dynamical systems; hyperbolic flows; Sinai billiard; geodesic flow

**Full Text:** [DOI](#) [arXiv](#)

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