Asymptotically deterministic robust preparation of maximally entangled bosonic states

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We introduce a theoretical scheme to prepare a pure Bell singlet state of two bosonic qubits, in a way that is robust under the action of arbitrary local noise. Focusing on a photonic platform, the proposed procedure employs passive optical devices and a polarization insensitive, nonabsorbing, parity check detector in an iterative process which achieves determinism asymptotically with the number of repetitions. Distributing the photons over two distinct spatial modes, we further show that the elements of the related basis composed of maximally entangled states can be divided in two groups according to an equivalence based on passive optical transformations. We demonstrate that the parity check detector can be used to connect the two sets of states. We thus conclude that the proposed protocol can be employed to prepare any pure state of two bosons which are maximally entangled in either the internal degree of freedom (Bell states) or the spatial mode (NOON states).

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I. INTRODUCTION

Entanglement, the most exotic property of quantum mechanics, is at the heart of the enhancement provided by quantum protocols in many different fields of application [1], ranging from metrology and parameter estimation [2,3], to computation [4], communication, and cryptography [5]. The ability to prepare entangled states with high reliability is thus crucial for the practical development of quantum technologies. Nonetheless, realistic preparations of entangled states are known to be hindered by the ubiquitous interaction with the surrounding environment, whose noisy action is detrimental for the quantum correlations within the system [1,6,7]. For this reason, many different techniques to circumvent the problem have been proposed over time [7–41].

In this work, we first propose a protocol to distil a pure, maximally entangled Bell singlet state of two bosons from a completely depolarized one. We focus on a photonic implementation. The local action of depolarizing channels, which can be efficiently induced by randomized local polarization rotators, transforms any arbitrary state of spatially distinguishable photons into a maximally mixed one. Thus, the proposed procedure can be applied to any arbitrary initial state of two spatially distinguishable photons, regardless of local noises affecting them before the depolarization. Our scheme employs passive optical (PO) devices and a polarization insensitive, nonabsorbing, parity check detector. The latter is a highly nonlinear transformation which performs a quantum nondemolition (QND) measurement capable to discriminate between states with even/odd number of photons. More precisely we only require the detector to distinguish between the cases where in a given location we have a single photon (corresponding to the successful generation of the Bell singlet), and those in which the total number of photons is either zero or equal to two (corresponding to a failure). The nondemolition character of the measurement ensures that in case of failure the whole protocol can be repeated by depolarizing the system once again, resetting it to the maximally mixed state. By doing so, the preparation of the Bell singlet is achieved with a probability scaling to one exponentially with the number of repetitions, thus being asymptotically deterministic.

Different from other entanglement distillation protocols [22–25] allowing only for local operations and classical communication (LOCC), our scheme makes explicit use of the interference effects due to particle indistinguishability when nonlocality is generated by a beam splitter (BS). In this sense, the proposed procedure extends the results obtained in Refs. [42–44], where the authors employed a technique based on spatially localized operations and classical communication (sLOCC) [45–52] to achieve a probabilistic distillation of a Bell singlet state from a singlet subjected to the action of local noisy environments.

Finally, we introduce an equivalence between bosonic bipartite states based on PO transformations. We consider an orthonormal basis of the bipartite Hilbert space composed of only maximally entangled states, and show that it can be divided in two sets of PO equivalent elements. We demonstrate that the two sets can be connected by means of the polarization insensitive, nonabsorbing, parity check detector previously discussed. As the Bell singlet state belongs to one of the two sets, we thus conclude that the proposed procedure allows for the preparation of any arbitrary maximally entangled, pure bipartite state. This comes with a tradeoff in the difficult

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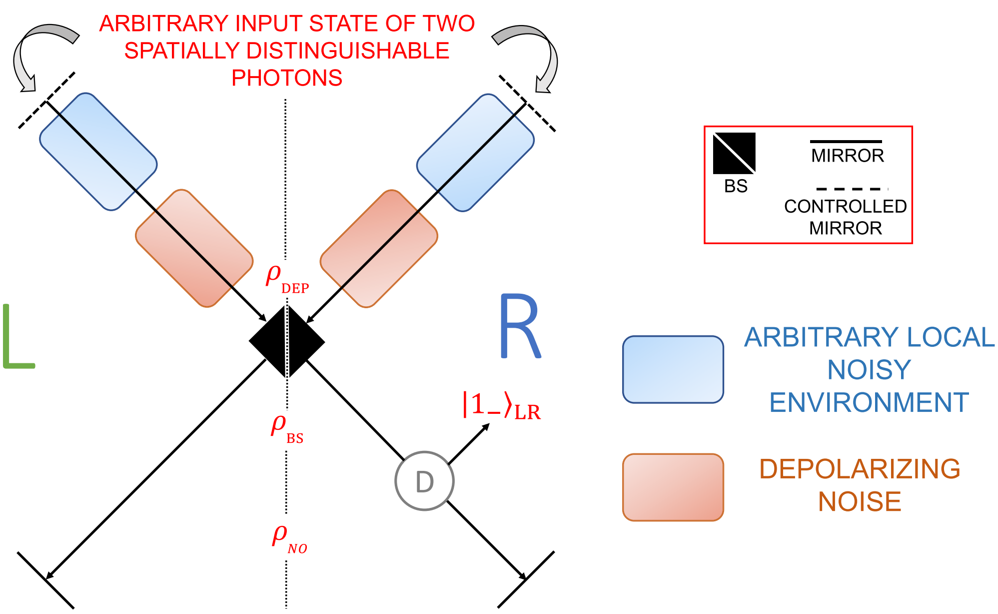

FIG. 1. Schematic representation of the setup. The D element represents a polarization insensitive, nonabsorbing parity check detector. The depolarization noises in the red area (just before the BS) are assumed to be externally activated, while the noise sources in the blue area are environmentally induced. Two actively controlled mirrors and two passive ones are set at the input and output of the interferometer, respectively, to obtain a closed configuration.

realization of the exotic detector required, whose crucial role for quantum information protocols emerges by the relevance of the reported results themselves.

Identical particles are treated via the no-label approach [45,53,54], a mathematical framework which allows to overcome some of the main issues affecting the standard label-based formalism [55,56]. Also, it allows to write multiparticle states without having to explicitly symmetrize/antisymmetrize them as ruled by the symmetrization postulate [45,53,54], thus simplifying the notation.

II. NOTATION

The Hilbert space of two bosonic qubits distributed over two distinct spatial regions, L and R, is ten dimensional. We consider a basis $\mathcal{B} = \mathcal{B}_{\text{LR}} \cup \mathcal{B}_{\text{NO}}$ of maximally entangled states, where

$$\begin{aligned} \mathcal{B}_{\text{LR}} &:= \{|1_\pm\rangle_{\text{LR}}\,,\ |2_\pm\rangle_{\text{LR}}\}, \\ \mathcal{B}_{\text{NO}} &:= \{|1_\pm\rangle_{\text{NO}}\,,\ |U_\pm\rangle_{\text{NO}}\,,\ |D_\pm\rangle_{\text{NO}}\}, \end{aligned} \tag{1}$$

and

$$\begin{aligned} |1_\pm\rangle_{\text{LR}} &:= \frac{1}{\sqrt{2}}(|L\uparrow, R\downarrow\rangle \pm |L\downarrow, R\uparrow\rangle), \\ |2_\pm\rangle_{\text{LR}} &:= \frac{1}{\sqrt{2}}(|L\uparrow, R\uparrow\rangle \pm |L\downarrow, R\downarrow\rangle), \\ |1_\pm\rangle_{\text{NO}} &:= \frac{1}{\sqrt{2}}(|L\uparrow, L\downarrow\rangle \pm |R\uparrow, R\downarrow\rangle), \\ |U_\pm\rangle_{\text{NO}} &:= \frac{1}{2}(|L\uparrow, L\uparrow\rangle \pm |R\uparrow, R\uparrow\rangle), \\ |D_\pm\rangle_{\text{NO}} &:= \frac{1}{2}(|L\downarrow, L\downarrow\rangle \pm |R\downarrow, R\downarrow\rangle). \end{aligned} \tag{2}$$

Notice that the elements of basis $\mathcal{B}_{\text{LR}}$ are Bell states entangled in the internal degree of freedom $|\uparrow\rangle\,,\ |\downarrow\rangle$, which, for the photonic implementation considered in the following paragraphs, can be identified with the polarization; instead, the basis $\mathcal{B}_{\text{NO}}$ is composed of NOON states entangled in the spatial degree of freedom.

III. PROCEDURE

The proposed scheme is depicted in Fig. 1. Let us take an arbitrary state of two photons localized in two distinct spatial modes, L and R. If each photon is locally subjected to a depolarizing channel that induces a complete randomization of its polarization degree of freedom, such a state will be mapped into a maximally mixed configuration which can be expressed as a uniform mixture of the elements of the basis $\mathcal{B}_{\text{LR}}$ introduced above, i.e., $\rho_{\text{dep}} := \frac{1}{4}\,\Pi_{\text{LR}}$, where $\Pi_{\text{LR}} := \sum_{|v\rangle \in \mathcal{B}_{\text{LR}}} |v\rangle\langle v|$ is the projector onto the subspace spanned by the elements of the basis $\mathcal{B}_{\text{LR}}$. We now let the two photons impinge on the two input ports of a balanced beam splitter (BS), which mixes the L and R regions inducing, at the level of single particle states, the mappings $|L\rangle \longrightarrow (|L\rangle + |R\rangle)/\sqrt{2}$ and $|R\rangle \longrightarrow (|L\rangle - |R\rangle)/\sqrt{2}$. Applied to the elements of the set $\mathcal{B}_{\text{LR}}$, this achieves the transformations

$$\begin{aligned} |1_-\rangle_{\text{LR}} &\longleftrightarrow -|1_-\rangle_{\text{LR}}\,, \\ |1_+\rangle_{\text{LR}} &\longleftrightarrow |1_-\rangle_{\text{NO}}\,, \\ |2_-\rangle_{\text{LR}} &\longleftrightarrow (|U_-\rangle_{\text{NO}} - |D_-\rangle_{\text{NO}})/\sqrt{2}, \\ |2_+\rangle_{\text{LR}} &\longleftrightarrow (|U_-\rangle_{\text{NO}} + |D_-\rangle_{\text{NO}})/\sqrt{2}. \end{aligned} \tag{3}$$

As a result, the state ρ_{dep} introduced previously is mapped into

$$\rho_{\text{BS}} = \tfrac{1}{4}\,|1_-\rangle_{\text{LR}}\langle 1_-|_{\text{LR}} + \tfrac{3}{4}\rho_{\text{NO}}, \tag{4}$$

where

$$\rho_{\text{NO}} := \tfrac{1}{3}(|1_-\rangle_{\text{NO}}\langle 1_-|_{\text{NO}} + |U_-\rangle_{\text{NO}}\langle U_-|_{\text{NO}} + |D_-\rangle_{\text{NO}}\langle D_-|_{\text{NO}}). \tag{5}$$

We highlight that ρ_{BS} in Eq. (4) is a classical mixture of the Bell singlet state $|1_-\rangle_{\text{LR}}$ and of NOON states. Crucially, the former is characterized by an odd number of photons in each spatial mode, while an even number (zero or two) characterizes the latter. This fact can be exploited to distil the singlet as follows. We employ a polarization insensitive, nonabsorbing, parity check detector D. By monitoring one of the two spatial modes, such a detector is capable to distinguish whether it contains an odd or an even number of photons. In the first case, ρ_{BS} is projected onto the subspace spanned by the Bell states composing $\mathcal{B}_{\text{LR}}$ via the projection operator Π_{LR} previously introduced, giving the desired singlet $|1_-\rangle_{\text{LR}}$. In this case, occurring with probability $p_{\text{LR}} = \text{Tr}[\Pi_{\text{LR}}\rho_{\text{BS}}] = 1/4$, we collect the state and conclude the process. If D registers an even number of photons, instead, ρ_{BS} is projected onto the subspace spanned by the NOON states in basis $\mathcal{B}_{\text{NO}}$ via the projection operator $\Pi_{\text{NO}} := \sum_{|k\rangle\in\mathcal{B}_{\text{NO}}} |k\rangle\langle k|$. This scenario, which occurs with probability $p_{\text{NO}} = \text{Tr}[\Pi_{\text{NO}}\rho_{\text{BS}}] = 3/4$, leaves the system in the state ρ_{NO} of Eq. (5). In this case, we act on the system with another beam splitting operation, getting the state

$$\xi_{\text{LR}} := \tfrac{1}{3}(|1_+\rangle_{\text{LR}}\langle 1_+|_{\text{LR}} + |2_+\rangle_{\text{LR}}\langle 2_+|_{\text{LR}} + |2_-\rangle_{\text{LR}}\langle 2_-|_{\text{LR}}). \tag{6}$$

The two photons are now subjected to local depolarizing channels once again, resetting the system to the completely depolarized state ρ_{dep} we started with. The process can thus be repeated a second time without having to inject new photons in the setup, leading to the generation of a Bell singlet state with total probability $p^{(2)}_{\text{LR}} = 1/4 + (3/4)(1/4)$. Proceeding this way, the jth iteration returns $|1_-\rangle_{\text{LR}}$ with probability $p^{(j)}_{\text{LR}} = \sum_{n=1}^{j}(1/4)(3/4)^{n-1}$, which converges exponentially to one for $j \to \infty$. We emphasize that such an iterated implementation can be achieved with the closed configuration depicted in Fig. 1. Here, two actively controlled mirrors close the input arms of the interferometer after the photons have been injected in the setup, while two other mirrors are set on the output modes after the detector. In this way, the two particles are reflected back into the same BS and noisy channels, allowing for the process to be repeated without requiring further resources.

IV. AMPLITUDE DAMPING-BASED IMPLEMENTATION

In this section we discuss an alternative implementation of our scheme which adopts two local amplitude damping channels instead of the depolarizing ones.

In this case, the noisy environments map two spatially separated qubits into the pure ground state $|L\downarrow, R\downarrow\rangle$. Placing a polarization rotator (PR) (see below) on the spatial mode L (to fix a framework), we get $|L\uparrow, R\downarrow\rangle = (|1_+\rangle_{\text{LR}} + |1_-\rangle_{\text{LR}})/\sqrt{2}$. From Eq. (3), we notice that the BS transforms this state into $(|1_-\rangle_{\text{NO}} - |1_-\rangle_{\text{LR}})/\sqrt{2}$. The detector D can now be employed to distill a Bell singlet state with probability $p_{\text{LR}} = 1/2$. When the system is found in state $|1_-\rangle_{\text{NO}}$, instead, the process is repeated analogously to the case where depolarizing channels are employed. At the jth iteration, the singlet is distilled with probability $p^{(j)}_{\text{LR}} = \sum_{n=1}^{j} 1/2^n$, which again converges to one exponentially when $j \to \infty$.

V. PASSIVE OPTICAL EQUIVALENCE

We introduce PO operations as the set of transformations which can be obtained by a proper sequence of BSs, polarization BSs (PBSs), polarization-dependent or -independent phase shifters (PDPSs/PIPSs), and local polarization rotators (PRs). We further define two states to be PO equivalent if they can be obtained one from the other by means of PO operations.

PO equivalence allows to divide basis $\mathcal{B}$ in Eq. (1) in two sets of equivalent states:

$$\mathcal{S}_1 := \{|1_\pm\rangle_{\text{LR}}, |2_\pm\rangle_{\text{LR}}, |1_\pm\rangle_{\text{NO}}\}, \quad \mathcal{S}_2 := \{|U_\pm\rangle_{\text{NO}}, |D_\pm\rangle_{\text{NO}}\}. \tag{7}$$

Focusing on $\mathcal{S}_1$, mappings $|1_-\rangle_{\text{LR}} \leftrightarrow |1_+\rangle_{\text{LR}}$ and $|2_-\rangle_{\text{LR}} \leftrightarrow |2_+\rangle_{\text{LR}}$ can be obtained by locally applying a π-PIPS to one of the two spatial modes, while a PIPS of $\pi/2$ achieves $|1_-\rangle_{\text{NO}} \leftrightarrow |1_+\rangle_{\text{NO}}$. Connections $|1_-\rangle_{\text{LR}} \leftrightarrow |2_-\rangle_{\text{LR}}$ and $|1_+\rangle_{\text{LR}} \leftrightarrow |2_+\rangle_{\text{LR}}$ can be obtained by means of a local PR performing the operation $|\uparrow\rangle \leftrightarrow |\downarrow\rangle$ on one mode. This set of local transformations relating Bell states were firstly introduced in Ref. [22]. We now extend them by noticing from Eq. (3) that the nonlocality generated by a BS can be employed to achieve the transformation $|1_+\rangle_{\text{LR}} \leftrightarrow |1_-\rangle_{\text{NO}}$. Considering $\mathcal{S}_2$, instead, mappings $|U_-\rangle_{\text{NO}} \leftrightarrow |U_+\rangle_{\text{NO}}$ and $|D_-\rangle_{\text{NO}} \leftrightarrow |D_+\rangle_{\text{NO}}$ can be realized by a local $\pi/2$-PIPS on one spatial mode, while $|U_-\rangle_{\text{NO}} \leftrightarrow |D_-\rangle_{\text{NO}}$ and $|U_+\rangle_{\text{NO}} \leftrightarrow |D_+\rangle_{\text{NO}}$ can be obtained by applying a PR to both modes. Sets $\mathcal{S}_1$ and $\mathcal{S}_2$ and the related intraset PO relations are depicted in Fig 2.

We now show that a link between the two sets can be established by employing the (non-PO) detector D described above. To move from $\mathcal{S}_1$ to $\mathcal{S}_2$, we start from state $|2_+\rangle_{\text{LR}} \in \mathcal{S}_1$. We apply a PBS on one arbitrary spatial mode, placing D at the output of one of its ports before recombining the outputs in another PBS. Notice that such a Mach-Zehnder-like setup behaves as a polarization sensitive, nonabsorbing, parity check detector. This allows to discriminate the component $|L\uparrow, R\uparrow\rangle$ of state $|2_+\rangle_{\text{LR}}$ from the one $|L\downarrow, R\downarrow\rangle$. Combining the two spatial modes in a BS now leads to either state $|U_-\rangle_{\text{NO}} \in \mathcal{S}_2$ or $|D_-\rangle_{\text{NO}} \in \mathcal{S}_2$, respectively, as can be computed using Eq. (3), and recalling that $|L\uparrow, R\uparrow\rangle = |2_+\rangle_{\text{LR}} + |2_-\rangle_{\text{LR}}$, $|L\downarrow, R\downarrow\rangle = |2_+\rangle_{\text{LR}} - |2_-\rangle_{\text{LR}}$. To move from $\mathcal{S}_2$ to $\mathcal{S}_1$, instead, let us begin with $|U_-\rangle_{\text{NO}} \in \mathcal{S}_2$. Acting on it with a beam splitting operation, we obtain $(|2_+\rangle_{\text{LR}} + |2_-\rangle_{\text{LR}})/\sqrt{2}$. A PR set on the R spatial mode gives $(|1_+\rangle_{\text{LR}} + |1_-\rangle_{\text{LR}})/\sqrt{2}$, which is transformed by a second BS into $(|1_-\rangle_{\text{NO}} - |1_-\rangle_{\text{LR}})/\sqrt{2}$. The detector D can now be employed to discriminate the odd component ($|1_-\rangle_{\text{LR}}$) from the even one ($|1_-\rangle_{\text{NO}}$), both belonging to $\mathcal{S}_1$. Given the intraset connections discussed above, we have thus found a link

$$\mathcal{S}_1 \xleftrightarrow{PO+D} \mathcal{S}_2 \tag{8}$$

which allows to transform any two arbitrary maximally entangled states in $\mathcal{B}$ into the other. As these include the Bell singlet state, the proposed scheme can be employed to prepare any maximally entangled state of two photonic qubits.

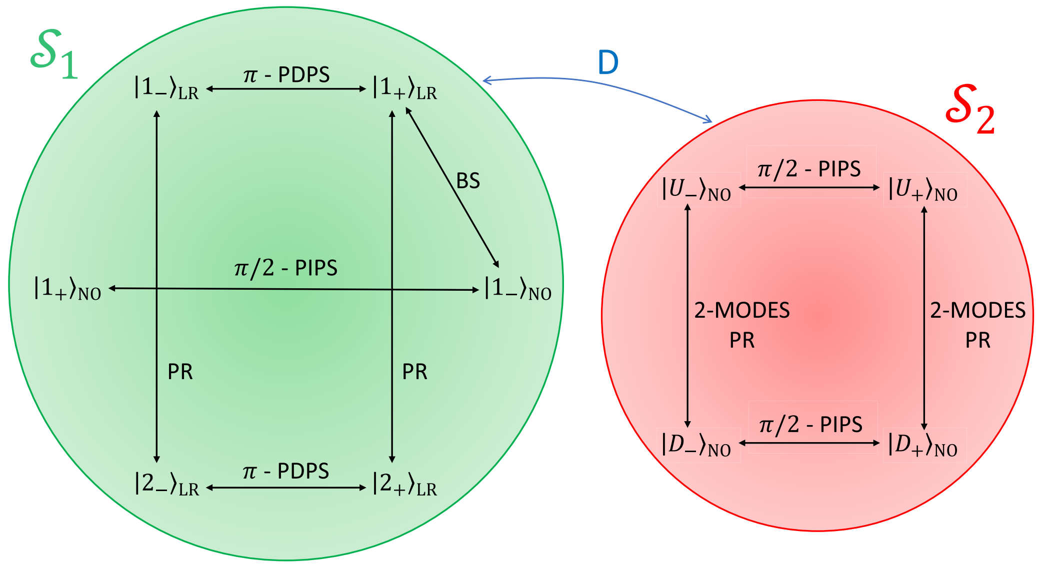

FIG. 2. Structure of passive optical equivalent maximally entangled states of two photons. The figure shows two sets of PO equivalent maximally entangled states of two bosonic qubits distributed over two spatial modes. Examples of PO transformations connecting them are reported for each set. All the depicted PO transformations are assumed to occur on a single arbitrary spatial mode, except when two-modes is stated. θ-PDPS/PIPS are polarization dependent/independent phase shifters inducing a phase θ on the spatial mode they are set on; PRs are 90° polarization rotators, and BSs are beam splitters. The two sets are linked by a polarization insensitive, nonabsorbing, parity check detector D (see main text).

VI. FAULTY PARITY CHECK DETECTOR

We conclude our analysis by accounting for possible errors occurring during the parity check detection.

Errors may occur when the system state $|1_-\rangle_{LR}\langle 1_-|_{LR}$ is mistakenly detected as an even-parity state, and (or) when the system state ρ_{NO} in Eq. (5) is wrongly detected as an odd-parity state. Accounting for these events amounts to substituting the previously defined projectors Π_{LR} and Π_{NO} with $\Pi'_{LR} := (1-\epsilon)\,\Pi_{LR} + \epsilon'\,\Pi_{NO}$ and $\Pi'_{NO} := (1-\epsilon')\,\Pi_{NO} + \epsilon\,\Pi_{LR}$, respectively, where error probabilities ϵ, ϵ' are considered. Correspondingly, the system is projected into the states

$$\begin{aligned} \rho'_{LR} &= \tfrac{1}{4}[(1-\epsilon)\,|1_-\rangle_{LR}\langle 1_-|_{LR} + 3\,\epsilon'\,\rho_{NO}]/p'_{LR}, \\ \rho'_{NO} &= \tfrac{1}{4}[3\,(1-\epsilon')\,\rho_{NO} + \epsilon\,|1_-\rangle_{LR}\langle 1_-|_{LR}]/p'_{NO} \end{aligned} \tag{9}$$

with related probabilities

$$\begin{aligned} p'_{LR} &= (1-\epsilon)/4 + 3\epsilon'/4, \\ p'_{NO} &= 3(1-\epsilon')/4 + \epsilon/4 = 1 - p'_{LR}. \end{aligned} \tag{10}$$

We now quantify the amount of quantum correlations present in the faulty state ρ'_{LR} we collect. Since when no errors occur we expect to get the singlet state $|1_-\rangle_{LR}$, we focus on the entanglement in polarization. To do so, we calculate the concurrence [57], obtaining

$$C(\rho'_{LR}) = \frac{1-\epsilon}{1-\epsilon+3\,\epsilon'}. \tag{11}$$

Notice that the amount of entanglement in ρ'_{LR} depends on both the error probabilities ϵ and ϵ', ranging from $C(\rho'_{LR}) = 0$ (separable state) when $\epsilon = 1$, to $C(\rho'_{LR}) = 1/(1+3\epsilon')$ when $\epsilon = 0$. Figure 3 reports the concurrence $C(\rho'_{LR})$ as a function of ϵ and ϵ'. We remark that, however, not all the scenarios are relevant. When $\epsilon' = 1$, for example, it is enough to collect the photons when the detector signals an even parity state to achieve a state with nonzero entanglement (unless $\epsilon = 0$, too).

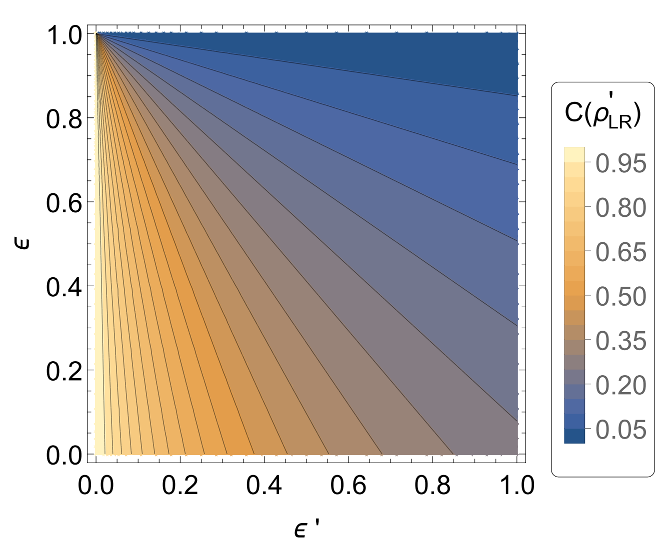

FIG. 3. Concurrence of the prepared state ρ'_{LR}, as a function of the error probabilities ϵ and ϵ' characterizing a faulty parity check detection.

VII. CONCLUSIONS

In this work, we have presented a procedure to robustly prepare maximally entangled states of two photonic qubits undergoing arbitrary local noise. The protocol employs PO transformations and a polarization insensitive, nonabsorbing, parity check detector to distil a Bell singlet state from a completely depolarized one. As the local depolarization of spatially distinguishable photons leads to the maximally mixed state regardless of the previous dynamics, the proposed scheme transforms any arbitrary initial state into the Bell singlet. In this way, the preparation is robust to the action of any local noise affecting the photons before their state is reset by the depolarization. We highlight that, in case a photon is lost during a noisy interaction, a new depolarized photon can be injected to recover the process. Via a QND measurement, the protocol is iterative and prepares the desired state with a probability which scales exponentially with the number of repetitions, thus being asymptotically deterministic.

We have introduced a formal equivalence based on PO transformations, showing that it allows to divide maximally entangled states of two qubits distributed over two distinct spatial modes in two sets of PO equivalent states. A link between the two sets has been established through the polarization insensitive, nonabsorbing, parity check detector. Since the Bell singlet state belongs to one of the two sets, we conclude that the scheme enables a robust generation of any arbitrary maximally entangled state of two photonic qubits.

We emphasize that, to achieve the correct interference patterns, the PO transformations realized by BSs or PBSs require the two photons to be indistinguishable in all the degrees of freedom but the spatial one and, at most, the polarization. In light of this, the PO equivalence defined in this work can be ultimately interpreted as a connection between two synchronized sources of single photons satisfying the above requirement and the set of maximally entangled bipartite states. Our work provides clear insights on the role played by indistinguishability as a tool to achieve a generation of entanglement which is robust to environmental noise, whatever the noise. Moreover, the externally induced noise acts as an ally toward this goal, in contrast to standard protection techniques where noise constitutes a detrimental trait to be avoided. Notice that the low-dimensional basic scheme proposed here is strategic, since it allows to focus on the main underlying physical mechanisms and their interpretation.

In a real-world implementation of our setup, the required PO transformations, including the realization of the depolarizing channels, can be reliably produced with commercially available devices such as mirrors, beam splitters, and optical fibers. Given the resetting function of the depolarizing channels, possible errors introduced by the mirrors do not affect the performances of the setup as long as the photons are not lost. Moreover, we have not considered photon number-preserving errors introduced by the beam splitter, as very highly efficient beam splitters are currently employed in the labs. Therefore, the realization of the polarization insensitive, nonabsorbing, parity check detector constitutes the main obstacle to be tackled and motivates experimental developments in different platforms. We have analyzed the case when faulty detections are involved, quantifying the entanglement between the two resulting photons as a function of the errors due to the parity check detector. To this regard, we remark that the proposed scheme can still be used substituting such a detector with commercially available single photon detectors performing a coincidence measurement on the two output modes, achieving the preparation of the desired maximally entangled state with probability $p_{\text{LR}} = 1/4$. In the latter case, where the photon is absorbed by the detector, deferred measurements can be employed after running the quantum protocol which exploits the desired resource state conditionally [47,49,51].

It is interesting to compare our scheme with the standard entanglement distillation protocol [1,22,58]. While the latter requires entangled states as input, the resetting action of the depolarization of both qubits in our procedure admits initially unentangled states. Also, the proposed method transforms a pair of qubits into a pure maximally entangled state, either Bell-like or NOON-like, with asymptotic certainty, while the standard distillation protocol requires n copies of a bipartite mixed state to probabilistically extract $k < n$ copies of Bell singlet states. On the other hand, our technique is well-suited for the preparation of entangled particles but not for their distribution to remote parties, as the extraction of the desired states occurs after the BS and no noise is assumed to act afterwards. We thus envision a combined implementation of the two schemes: entangled qubits prepared with our scheme propagate through noisy channels toward distant parties, providing the initial entangled states required for the application of the standard distillation protocol.

We finally highlight that the reported results hold for any type of bosonic system, thus not being limited to photons. We foresee an extension of our procedure to fermions, clarifying the role of particle statistics in the preparation of entangled states [59]. Moreover, we aim at widening the analysis of PO transformations to systems of $N > 2$ particles, looking for a suitable generalization of the protocol presented in this work to prepare multipartite entangled states.

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