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RESEARCH PAPER

Influence of Shallow Foundations on the Response of Steel Wind Towers

Giuseppe Campione[1]

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Abstract

The objective of this study, concerning the soil–structure interaction of shallow reinforced concrete foundations of wind towers with circular cross-sections, was determination in a closed form of the monotonic moment–rotation curve of the soil–foundation complex. This study was based on elastic and plastic analyses of shallow rigid foundations assuming a Winkler soil type including the flexibility of the foundation in the elastic range and the nature of the soil (cohesive and non-cohesive types) through corrective factors of the constant of the Winkler model. The flexibility of the foundations influences the moment–rotation response through the initial rotational stiffness with a coefficient between 1 and 0.7 for a width-to-span ratio between 5 and 2. The nature of the soil is considered through corrective factors of 0.75 and 1.3 of the Winkler constant for cohesive and non-cohesive soil, respectively. Analyses carried out stressed that a possible design valued to be adopted in a steel wind tower with shallow foundations is a diameter of the steel tube 1/15 of the height of the tower, a diameter of foundation 0.75 of the length, and a depth of foundation 1/10 of the diameter and thickness of steel tower ratio diameter equal to 1/10. In this range it was observed that the effects of the soil-to-foundation interaction in the elastic range influences the critical length in the stability of the steel wind tower, with values between 2.5 and 2 (column fixed at the base) in a range of Winkler constant between 0.1 and 1 daN/cm^3. Finally, an experimental validation of the proposed model was carried out with the data available from the literature.

Keywords Wind tower · Shallow foundation · Moment–rotation relationship · Ultimate moment

1 Introduction

Concrete foundation plinths having a circular or polygonal shape in plane or foundations on piles are currently utilized for the construction of wind turbine foundations. We can distinguish shallow, gravity, and deep foundations with piles or shafts. Gravity-based foundations are massive structures in reinforced concrete that have some design peculiarities strictly related to their enormous dimensions: circular or polygonal shapes with diameters that can be greater than 20 m and significant thicknesses greater than 4 m. These structures are generally cast in place and are reinforced with equidistant lower and upper radial bars, i.e., brackets connected to the radial bars (see Fig. 1).

The main variable load in the design of wind towers is the wind, which acts mainly in one direction and induces load cycles with a frequency that must not be close to that of the first mode of vibration of the structure, in order to avoid resonance phenomena. The wind turbine operates according to the strength of the wind; below a certain speed, called cut-in, the machine is unable to start; for start-up to occur, the speed must reach this threshold, which in many cases is of the order of a few m/s. During operation, the "nominal" wind speed is the minimum wind speed that allows the machine to supply the design power; this speed is of the order of about 10 m/s. At high speeds (of the order of 25–35 m/s) the wind turbine is placed out of service for safety reasons (cut-off speed).

The engineering problem is dominated by the soil–structure interaction. In fact, when it is necessary to transfer high bending moments on the ground with a low level of normal stress (this is the case of very high wind towers) and if the foundation soil is weak/soft, risk of

✉ Giuseppe Campione
giuseppe.campione@unipa.it

[1] Department of Engineering, University of Palermo, Viale Delle Scienze, 90128 Palermo, Italy

Fig. 1 Steel reinforcements in shallow foundations for wind tower (photo by the author)

uplifting of foundation with rocking effects arise and nonlinear moment–rotation behavior is observed. The flexibility of the soil–foundation complex strongly influences the stability condition of a steel tower (depending on the effective length involved in the stability problem) and the dynamic characteristics (natural frequencies of the system). The choice of the type of foundation for wind towers depends on the aspects mentioned above and also on fatigue phenomena induced by wind action [1–6]. The performance criteria adopted for the design of wind towers [6–8] must guarantee strength, stability, fatigue, robustness, rotational rigidity, durability, and economy. Improper design of a wind tower due to over-small foundations can determine catastrophic failure, which is shown in Fig. 2.

As indicated in the literature [6] the structural behavior of the tower-foundation-soil system depends on several inputs including the aero-dynamic loads, seismic bedrock motions, tower and foundation dimensions, and soil parameters. According to the literature [7], the permissible gap between the foundation and the soil under different loading conditions is limited. For quasi-permanent load combinations, no gap is permitted, and for unfactored

Fig. 2 Catastrophic failure of a wind tower

extreme loads a maximum gap area of one half of the foundation area is acceptable. An important check on the structure in various loading conditions concerns buckling of the steel tower, vibrations and the strength capacity of the soil–foundation complex, including soil plasticization and RC foundation crises in flexure and/or shear. To examine these effects, it is necessary to determine the moment–rotation diagram of the soil–foundation complex. For analysis of plinths, plates and beams, often considered as rigid elements in the elastic and plastic phases, different approaches, including analytical and numerical ones as well as semi-numerical methods, are available in the literature. With these models it is possible to derive the moment–rotation response under monotonic or cyclic actions. Only a few of the models for monotonic response available in the literature are derived analytically, and they are based on a mechanical approach with a solution in closed form. Very few of them provide equations to be utilized for hand calculation. Most of the models proposed are based on the hypothesis of a Winkler soil type [11]. In the literature [15] it was proposed an analytical equations for the moment–rotation response for a rigid foundation having a square cross-section. Winkler soil models, including uplift-yield, uplift-, and yield-only conditions are considered. In the literature [17] it was proposed an analytical expression derived from a mechanical approach in a closed form moment–rotation relationship for plinths with square and circular cross-sections. Some researchers [16] numerically analyzes the behavior of shallow foundations with a fixed vertical load using PLAXIS 3D. The foundation was considered rigid, while the soil was considered to have nonlinear behavior. Referring to the cyclic and dynamic effects of a wind tower with soil–foundation interaction several studies are present in the literature (see [10, 18]). These studies are not considered in detail here because they are outside the scope of this research. The soil-structure interaction has to be considered for the onshore wind towers with shallow foundations an in this spite te paper gives an original contribution with a simple model abel to inclued in a rigid palstic model the flexibility of foudnatio and the anture of soil though corrective coefficient with a phisical meaning. In addition it is propose a new simple analytical expression for the moment–rotation relationship of shallow foundations with square or circular cross-sections including initial stiffness and ultimate moment. This relationship was deduced in the elastic and plastic range under the hypothesis of a Winkler soil type modificed through corrective factors taking into account the type of soil (cohesive and non-cohesive) and the deformability of the foundation. The proposed model is not to be considered an alternative to the existing sophisticated and applied approaches widely found in the literature, but as an analytical support for a preliminary check on soil-to-foundation problems by hand calculation.

Moment–rotation relationships for soil-structure interaction.

For rigid footings subjected to moment M and vertical load P, when the bending moment is very high and the vertical load is low with respect to the moment, the contact stresses can become concentrated at the footing edges, as illustrated in Fig. 3. In Fig. 3 two important cases are shown. The case in Fig. 3a) refers to uplifting of a foundation with soil in the elastic range and the one in Fig. 3b) refers to uplifting of a foundation with soil in the plastic range.

According to the literature [19] in the absence of bending moment, the vertical load capacity of a rectangular footing of width B and length L is $P_c = \sigma_{0r} \cdot B \cdot L$, with σ_o the ultimate stress of the soil-to-foundation complex. According to literature [19], the relationship between bending moment M_c and axial load P is

$$M_c = \frac{P \cdot L}{2} \cdot \left(1 - \frac{\sigma}{\sigma_{or}}\right) \tag{1}$$

where

$$\sigma = \frac{P}{B \cdot L} \; q_u = \sigma_{0r} \cdot B \rightarrow \text{for a rectangular cross - section.} \tag{2}$$

For a shallow foundation with a circular cross-section, we consider a circle of equal area with a square cross-section:

$$D = \frac{2B}{\sqrt{\pi}} \cong 1.128B \tag{3}$$

and B = L = D/1.128.

The stress σ0, is the ultimate stress of the structure-foundation complex and it is calculated with the expressions of the literature [20]. In the case of a rectangular cross-section this gives:

$$\sigma_o = \frac{1}{2} \cdot \gamma_{t1} \cdot b \cdot N_\gamma \cdot \left(1 - 0.4 \cdot \frac{b}{L - 3e}\right) + c \cdot N_c \cdot \left(1 + \frac{b}{L - 3e} \cdot \frac{N_q}{N_c}\right) + \gamma_{t2} \cdot d \cdot N_q \cdot \left(1 + \frac{b}{L - 3e} \cdot \tan\phi\right) \tag{4}$$

$$N_q = e^{\pi \tan\phi} \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \quad N_c = \left(N_q - 1\right) \cdot \cot g\phi \quad N_\gamma = 2 \cdot \left(N_q - 1\right) \cdot \tan\varphi \tag{5}$$

Where γt1, γt2 is the volume weight of the soil below the foundation and the backfill soil, *c* is the cohesion of the soil and *d* is the height of the backfill soil.

In this paper the moment–rotation response of a shallow foundation is described with the curve shown in Fig. 4. The model refers to the case of all compressed elastic soil (stage 1); therefore, there follows uplifting of the foundation in the elastic range (point 2 of Fig. 4), the elastic range up to the uplifting of foundation with first soil plasticization (stage 3) and finally a nonlinear branch that approaches infinite rotation when ultimate load, which is *PB/2* for a rigid foundation and *PB/2* for soil having strength σ_o, is reached. The foundation is supposed rigid with respect to the soil; the soil is a Winkler type soil characterized by vertical subgrade stiffness *k*, which is dependent on the soil type, strain level, thickness of the soil medium, load types, size of footing, foundation flexibility, etc.

Many researchers have applied *k* from plate load testing to predict the moment–rotation response of a footing. Some author [18] suggested using unloading subgrade stiffness *k* from a plate loading test which could give good agreement with the experimental moment–rotation curve of a footing. When the Winker model is related to the continuum model, different values of *k* will be calibrated from different responses, such as moment, shear, or deflection of the beam. For example, for the square footing considered in this study, equivalences between the Winkler model and the elastic continuum model can be made to determine different equivalent vertical subgrade stiffnesses (stiffness intensity below the footing) for the footing from the

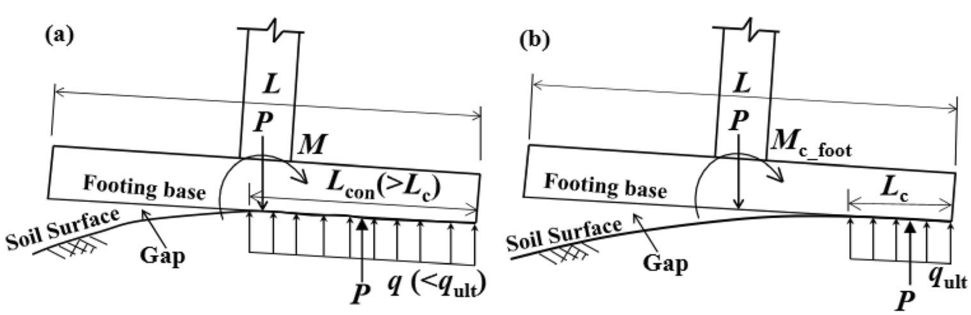

Fig. 3 Footing base rotation with uplifting with soil in: **a** elastic range; **b** plastic range

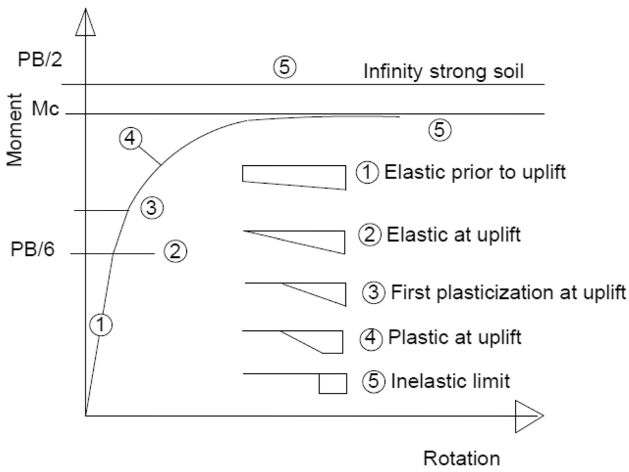

Fig. 4 Proposed moment–rotation model

vertical translational stiffness K_v and rocking stiffness K_θ of the footing on the elastic continuum model. According to [21], for a footing with a square plan on a homogeneous elastic half-space the rocking stiffness of a footing is expressed as

$$K_\theta = \frac{3.6 \cdot G \cdot (B/2)^3}{1 - \upsilon} \quad (6)$$

where μ and G are the Poisson's ratio and the shear modulus of the soil, respectively.

In the case of a rigid surface foundation with a square cross-section of side B placed on a Winkler type soil, the flexural stiffness of the foundation soil complex (which is the slope of the initial linear branch of the moment–rotation relationship) can be obtained by writing the equations of equilibrium (rotation and translation) and obtaining the inverse of the rotation because of a unitary bending moment.

Making the equivalence between the Winkler model and the elastic continuum model gives

$$K_{v\theta} = \frac{K_\theta}{B^4/12} = \frac{5.4}{1 - \upsilon} \cdot \frac{G}{B} \quad (7)$$

In the case of a circular section, considering a circle with an area equal to the square cross-section gives

$$D = \frac{2B}{\sqrt{\pi}} \cong 1.128B \quad (8)$$

if we consider that $G = \frac{E}{2\cdot(1-\upsilon)}$ and $D \cong 1.128B$, substituting θ in Eq. 7 gives

$$k_f \cong \frac{3 \cdot E}{D \cdot (1 - \upsilon^2)} \quad (9)$$

It is interesting to observe from Eq. 9 that the k_f factor is size-dependent.

If we want use k in the expression of the flexural stiffness of square cross-section we have

$$k_f = 0.364 \cdot k \cdot \frac{D^4}{12} \quad (10)$$

The evaluation of k is generally obtained indirectly [21]. By loading the soil with a stiff square or circular steel plate with side a_p = 300 mm with a known weight, it is possible to measure the deflection w. The ratio between the applied pressure and the deflection gives the value of the coefficient of subgrade reaction k_1, correlated with the subgrade coefficient of the soil k at the foundation base b through the following relationships:

$$k = k_1 \frac{a_p}{1.5b} (\text{non-cohensive soil}) \quad (11)$$

$$k = k_1 \left(\frac{b + a_p}{2b}\right)^2 \quad (12)$$

Table 1 gives typical values of mechanical properties of some foundation soils [20]

Adopting Eqs. 11,12 it turns out that for k_1 between 10 and 30 daN/cm^3 and for D = 2000 cm and b_d = 30 cm the

Table 1 Range of mechanical properties of some foundation soils [20]

Soil type	Friction angle φ	Weight density (kN/m^3)	Cohesion (MPa)	E (daN/cm^2)	Poisson coeff v	k Winkler constant (daN/cm^3)
Wet clay	15–25	17–21	–	20.4–153	0.4–0.5	8–10
Compact dry clay	50	18–21	0.0250	153–510	0.1–0.3	10–12
Dry sandy clay	30–45	17–22	0.0020	510–1020	0.4–0.5	8–10
Compact gravel	35–37	18–20	–	1020–2040	0.3–0.4	20–30
Sandy gravel	35–50	18–20	–	0.51–1.53	0.3–0.4	10 to 30
Compact silt	25–30	16–21	0.010	20.4–204	0.3–0.35	–
Compact sand	35–45	18–22	–	510–816	0.3–0.35	8–15
Loose sand	28–34	14–17	–	102–255	0.3–0.35	2–4

corresponding values of k are between 2.5 and 7.7 daN/cm^3 for cohesive soil and between 0.1 and 0.3 daN/cm^3 for non-cohesive soil.

From a geotechnical point of view, assuming that the foundation is rigid with respect to the ground, the distribution of the pressure at failure is non-linear, with a shape depending on the nature of the soil (cohesive or non-cohesive). The different soil pressure profiles shown in [21] for cohesive and non-cohesive soils can be approximated with linear profiles. The relationships between load P and moment M at the fixed section of the steel tube induced by the soil reaction can be calculated with the following expressions:

$$P = \frac{3}{L} \cdot M \quad \text{cohesive soil,} \tag{13}$$

$$P = \frac{6}{L} \cdot M \quad \text{non-cohesive oil,} \tag{14}$$

$$P = \frac{4}{L} \cdot M \quad \text{Winkler soil,} \tag{15}$$

Therefore, calculating the ratio between Eq. 17 and Eq. 15 and between Eq. 17 and Eq. 16, the corrective factors ε of 4/3 and 2/3 equal to non-cohesive and cohesive soil are obtained.

To consider the flexibility of the foundation, it was introduced [4] a reduction factor of stiffness defined as F_c. for a rectangular cross-section.

To consider the flexibility of a rectangular cross-section to correct the rotation flexural stiffness of a rigid foundation [17] introduces the F_c. corrective function. This function is expressed as:

$$F_c = \frac{4 \cdot \cos\left(\frac{\beta_W \cdot B}{2}\right) + \cosh\left(\frac{\beta_W \cdot B}{2}\right)}{2 + \cos(\beta_W \cdot B) + \cosh(\beta_W \cdot B)} \tag{16}$$

where β_w is

$$\beta_w = \sqrt[4]{\frac{k \cdot B}{4 \cdot E \cdot I}} \tag{17}$$

with E the elastic modulus of concrete and I the inertia of the rectangular cross-section of the plinth having base B and height s.

From Eq. 15 if the flexural stiffness of the foundation increases, the hypothesis of a rigid foundation is verified (F_c approaches 1). Increasing the stiffness of the soil (k increases) a greater height of the foundation is required to respect the hypothesis of a rigid foundation.

For a circular cross-section the F_c function is calculated referring to a circular plate of radius R, with constant thickness s (in the case of variable thickness an average value is assumed), loaded by a uniform load and supported in the center by a steel tube of diameter r_0. The ratio between the deflection along the circle of radius R and the uniform settlement obtained by supposing infinity rigidity of the foundation gives the flexibility function F_c. A solution of the deflection for a circular plate can be found in textbooks [22].

The elastic surface of a circular plate under distributed pressure has the following expression:

$$w = \frac{\sigma \cdot (R - r)^4}{64 \cdot D} \cdot \left[-\beta^4 + 2 \cdot \left(\beta^2 - k_{10} - 2\right) \cdot \left(\beta^2 - \rho^2\right) + \rho^4 - 4 \cdot k_{10} \cdot \beta^2 \cdot \lg\frac{\rho}{\beta} - 8 \cdot \rho^2 \cdot \lg\frac{\rho}{\beta}\right] \tag{18}$$

with $\beta = r_o/R$, μ arbitrary, $\rho = r/R$, and $D = \frac{E \cdot s^3}{12 \cdot (1 - \upsilon^2)}$.

and

$$k_{10} = \frac{1 - \mu + (1 + \mu) \cdot \left(\beta^2 - 4 \cdot \lg\beta\right)}{1 + \mu + (1 - \mu) \cdot \beta^2} \tag{19}$$

The uniform settlement calculated for unit load $P = 1$ assuming a rigid foundation on a Winkler type soil gives

$$w_o = \frac{4 \cdot (P = 1)}{\pi \cdot D^2} \tag{20}$$

If the ratio is calculated between the settlements at the tip of the plate ($r = R$) and the settlement at the section $r = r_o$, it is possible to obtain the F_c coefficient, which gives the flexibility of the foundation in relation to the soil characteristics (soil assumed to be Winkler type). Calculating F_c through Eq. (21) for $r = R$ ($r = 1$, with $m = 0$) Eq. 22 gives

$$F_{c_o} = 1 - \frac{w(r = R) - w_0(r = r_o)}{w_o} \tag{21}$$

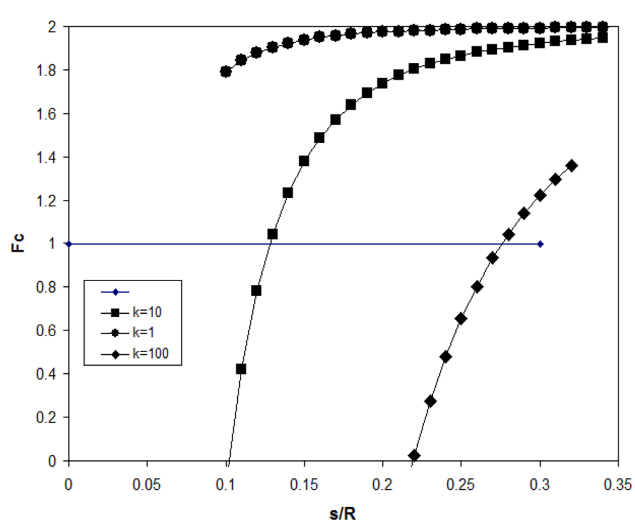

Fig. 5 Variation of Fc function with thickness of slabs for k = 0.5, 10, 100 daN/cm3 and ro/R = 0.2

The case examined here is that shown in Fig. 5. It refers to a shallow foundation having a circular cross-section of radius R and diameter D, while r is the radius of the steel tube of the wind tower measured at the base. From the graph we see that if s increases the flexural stiffness of the foundation increases, and the hypothesis of a rigid foundation is verified (F_c approaches 1). Increasing the stiffness of the soil (k increases) a greater height of the foundation is required to respect the hypothesis of a rigid foundation. For rigid foundation s/R (R = D/2) should be higher than 0.1 for k = 10 daN/cm^3 and higher than 0.25 for k = 100 daN/cm^3.

Considering the nature of the soil and the flexibility of the foundation the rotational stiffness of the foundation becomes:

$$k_f = k \cdot \frac{(0.886 \cdot D)^4}{12} \cdot F_c \cdot \varepsilon \tag{22}$$

In the linear-elastic branch (branch 1) the relation between the moment and the rotation is expressed as

$$M = k_f \cdot \theta \tag{23}$$

If a finite depth soil model is used instead of a Winkler model, more accurate results are expected, especially in term of settlements. This kind of approach could be useful for the analyses of geotechnical aspects of problems; for structural purposes, in the opinion of the author, because the Winkler model adopted was modified to consider cohesive and non-cohesive soils and the flexibility of the foundation, it seems sufficiently accurate to analyze the structural problem too.

To determine the moment–rotation relationship, we refer to the cases of Fig. 6. For $e = D/6$ and soil in the elastic range the bending moment and rotation are expressed as

$$M = P \cdot \frac{D}{6} \quad \theta = 2.97 \cdot P / \left(D^3 \cdot k\right) \tag{24}$$

For $e > D/6$ with first plasticization of the soil the rotation and the moment for the first plasticization of the ground and uplifting (see Fig. 6a) are

$$M_y = \frac{1}{36} \cdot D^3 \cdot \sigma_0 \cdot \cos^3(\alpha/2) \tag{25}$$

$$\theta_y = \frac{\sigma_o}{k} \cdot \frac{2}{3 \cdot D \cdot (1 - \cos(\alpha/2))} \tag{26}$$

Once the elastic phase has been overcome, the moment rotation curve is nonlinear, and the case examined is that of Fig. 6b).

Determining the non-linear response of the moment-to-soil system combining the equilibrium translation and rotation conditions gives

$$P = \sigma_o \cdot \frac{D^2}{4} \cdot (\alpha - \sin\alpha) \tag{27}$$

$$P \cdot \frac{D}{2} + M - \frac{D^2}{4} \cdot (\alpha - \sin\alpha) \cdot \sigma_o \cdot \frac{D}{2} \cdot \cos\left(\frac{\alpha}{2}\right) = 0 \tag{28}$$

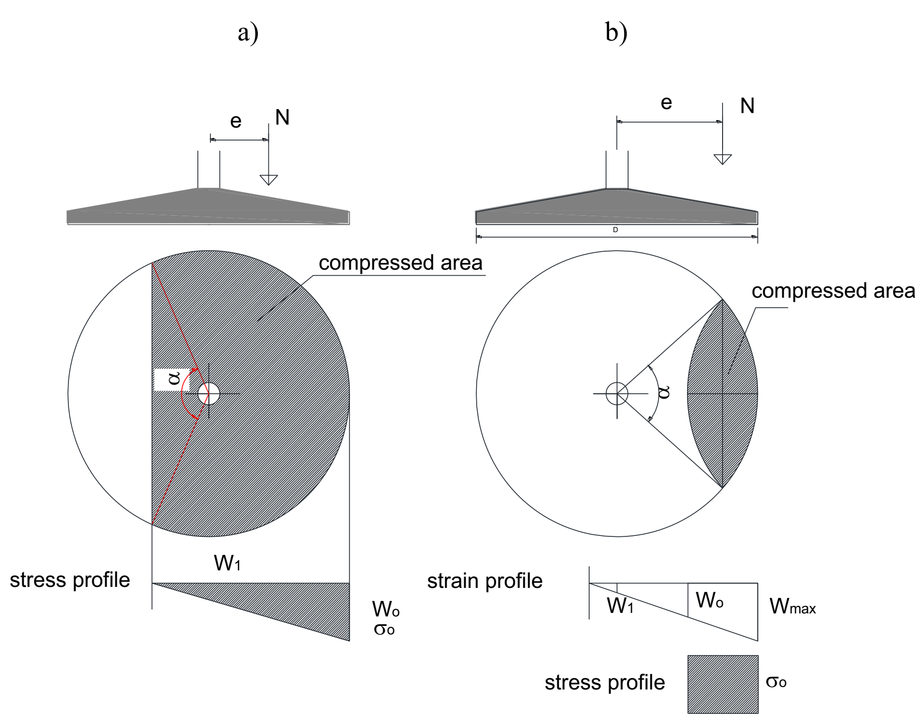

Fig. 6 Proposed model: **a** elastic phase; **b** plastic phase

with α the center angle defining the position of the compressed area.

In addition, by geometrical considerations in the hypothesis of a plane section, it is possible to derive the relationship between the rotation of the section for each value of the angle α and the settlements of the foundation in the following form:

$$\theta = \left(\sigma_{or} - \frac{4 \cdot P}{\pi \cdot D^2}\right) \cdot \frac{1}{k} \cdot \frac{1}{D/2 \cdot \cos(\alpha/2)} \tag{29}$$

Deriving α from Eq. 32 gives

$$\alpha = 2 \cdot ar\cos\left\{\left[\left(\sigma_{or} - \frac{4 \cdot P}{\pi \cdot D^2}\right) \cdot \frac{2}{k \cdot D}\right] \cdot \frac{1}{\theta}\right\} \tag{30}$$

When the rotation approaches infinity, Eq. 30 gives α=π.

$$\lim_{\theta \to \infty}\left(2 \cdot ar\cos\left\{\left[\left(\sigma_{or} - \frac{4 \cdot P}{\pi \cdot D^2}\right) \cdot \frac{2}{k \cdot D}\right] \cdot \frac{1}{\theta}\right\}\right) = \pi \tag{31}$$

Assuming α=π, Eq. (31) gives

$$M_c = P \cdot \frac{D}{2} \cdot \left(1 - \frac{4 \cdot P}{\pi \cdot D^2 \cdot \sigma_o}\right) \tag{32}$$

To represent the nonlinear response (branch 4 of Fig. 4) the following equation was utilized:

$$M = \frac{k_f \cdot \theta}{1 + k_f \cdot \theta / M_c} \tag{33}$$

For strength verification of an RC foundation in flexure and in shear it has to be verified that the moment M_f must be higher than M_c. The moment M_f is calculated as the product of the resultant of the contact pressures of the compressed area of soil for the distance from the external perimeter of the steel tube, giving

$$M_f = R^3 \cdot \left(\frac{\pi \cdot \alpha}{180} - \sin\alpha\right) \cdot \sigma_o \cdot \left(\sin\frac{\alpha}{2} - r\right) \tag{34}$$

The shear force is the resultant of the compressed areas and it is expressed as

$$V_f = R^2 \cdot \left(\frac{\pi \cdot \alpha}{180} - \sin\alpha\right) \cdot \sigma_o \tag{35}$$

The ultimate moment of the cross-section involved in the flexure mechanism is

$$M_{uf} = 0.9 \cdot d \cdot A_s \cdot f_y \tag{36}$$

A_s being the area of radial bars enclosed in the angle α.

Therefore, for flexural strength verification we must have $M_f < M_{uf}$ and for shear $V_f < V_{us}$ with V_{us}, calculated with the expressions available in the literature for members without shear reinforcements.

In addition, the maximum design bending moment $M_{sd} = N_{sd}$ e must be lower than M_c, N_{sd} being the axial load on the steel wind tower and e the total eccentricity due to wind cabin, rotor, and blades.

2 Experimental Validation

For experimental validation of the proposed model, a comparison was made with several experimental investigations available in the literature [23–25].

The first one considered is the experimental research of [23] concerning both cyclic and monotonic loading, simulating extreme wind conditions on 1:15 scaled models of wind turbine steel towers connected by stud bolt adapters to reinforced concrete shallow foundations embedded in a sandy soil. Figure 7 shows the moment-axial force domain of the soil foundation complex. In the same figures the design values are also given.

Figure 8 shows the moment–rotation diagrams deduced analytically with the proposed model and the one deduced experimentally, showing good agreement. The data assumed were σ_{0r} = 2 MPa and k = 4 daN/cm^3.

The real structures had wind turbine towers 85 m tall supporting 3.5 MW aerogenerators founded on truncated-conical shallow footings with a diameter of D = 19 m, heights H = 3 m h = 0.85 m, and diameter of steel tubes d = 5.55 m. The load P was 100 kN. The loading system was designed based on a predefined wind-induced Ultimate Limit State (ULS) Design Load Condition (DLC), selected among the most severe of a typical set given a maximum bending moment of 81,450 kNm with axial load P = 120 kN.

Figure 8 shows the moment–rotation diagrams deduced analytically with the proposed model and the one deduced experimentally, showing good agreement. Figure 8 gives the curve obtained with the proposed model with geotechnical parameters given in Table 1 for compact and loose sands.

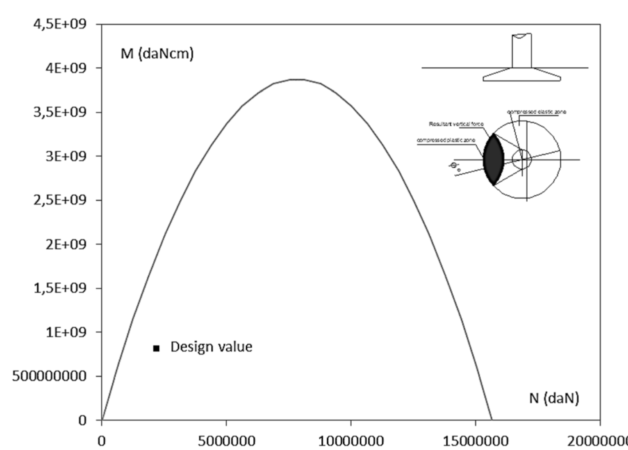

Fig. 7 Moment-axial force domain for soil foundation complex

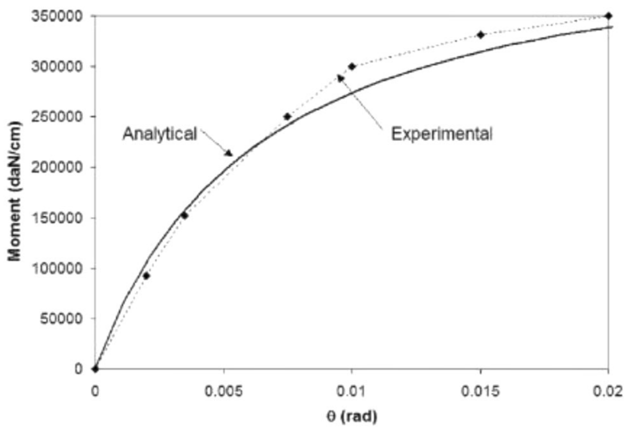

Fig. 8 Moment–rotation response of specimen tested in [23]

The upper curve was obtained for $\sigma_{0r} = 2$ MPa and $k = 4$ daN/cm^3, both possible values for the kind of sand utilized which fits the experimental response very well.

The second research examined for comparison is that of [24], which refers to monotonic compressive tests on a rigid foundation of side 305 mm and height 55 mm placed on compact gravel. Different eccentricities were adopted. For the calculations, $\varphi = 36°$, $\gamma = 19$ kN/m^3, and $k = 20$ daN/cm^3 were assumed, in agreement with the values suggested in Table 1 for compact gravel. The graph in Fig. 9 shows the strength domain and the experimental results obtained by the load–settlement curves obtained experimentally by [24], showing good agreement.

The third case examined is that of [25], which refers to monotonic compressive tests on a rigid foundation of side $B = 2000$ mm and height 400 mm placed on compact wet clay. A fixed load of 130 kN was assumed, with an eccentricity equal to $e/L = 1/6$. For the calculation were assumed $k = 8$ and 10 daN/cm^3, in agreement with the values suggested in Table 1 for compact wet clay. The graph in Fig. 10 shows the moment–rotation curves

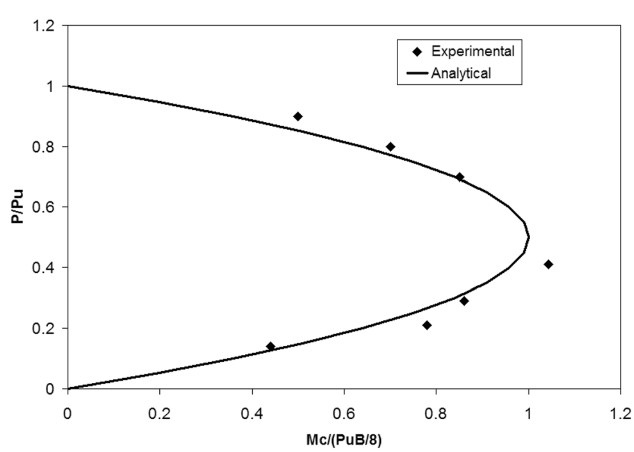

Fig. 9 Strength domain (Experimental data of [24])

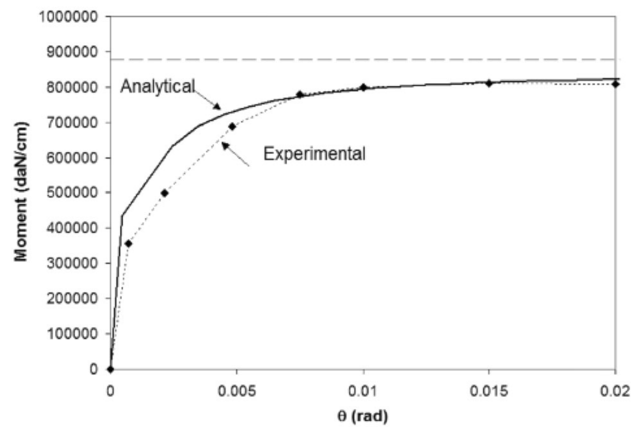

Fig. 10 Moment–rotation curves (Data of [25])

obtained experimentally by Algie [30] and the analytical solution obtained with the model proposed here. In this case the results obtained show that the proposed model is in good agreement with the experimental results, reproducing the whole nonlinear response only when the lower limits of the geotechnical parameters given in Table 1 for this kind of soil are assumed.

3 Effect of Foundation on the Buckling Load of the Wind Pole

For a preliminary buckling check on the stability of a steel tower in the presence of a shallow foundation, the vertical design load N_{sd} is lower than the critical load expressed as:

$$P_{cr} = \frac{\pi^2 \cdot E \cdot J}{\beta \cdot L} \tag{37}$$

with

$$\beta = 2 + \frac{E \cdot J}{L \cdot k_f} \text{ with } k_f = k \cdot \frac{(0.886 \cdot D)^4}{12} \cdot F_c \cdot \varepsilon \tag{38}$$

In Eq. (38) J is the inertia of the steel pipes, L the length of the pile, and k_f the elastic stiffness of the shallow foundation.

The study case refers to a wind pole with $d = 1/25$ L, $t = 1/100$ d, and $D = 1/5$ L.

Equation 39 was derived with a simple linear equation that for $k_f = \infty$ gives $\beta = 2$, and for $k_f = 0$ gives $\beta = \infty$.

Figure 11 shows the variation of β with the height-to-diameter ratio of the pole. The graph in Fig. 11 shows that for soil with a high value of k (1, 10,100) the variation of β is between 2 and 4 for low L/d ratios, but for soil with very low stiffness ($k = 0.1$ daN/cm^3) the value of β drops to 14 and a major risk of buckling due to flexibility of the soil-to-foundation complex arises.

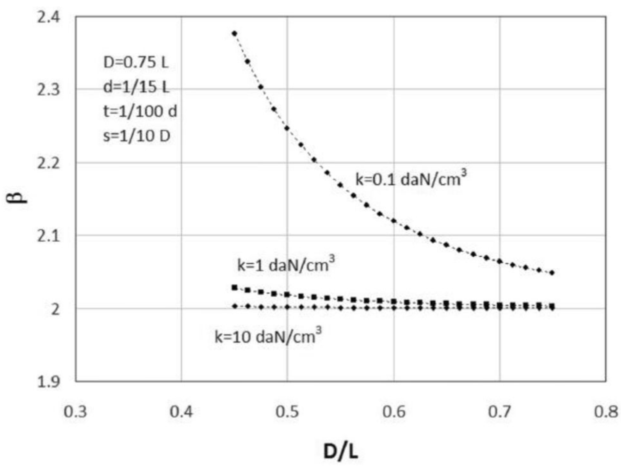

Fig. 11 Variation of β with L/d for given k

The variation in the critical length βL also influences the strength check on the wind tower. In the case of a steel wind tower of diameter d ($d = 2r$) and thickness t, local buckling and overall buckling effects must be checked for, calculating the second-order moment, which should be lower than the ultimate moment depending on the class section.

In the case of a wind pole subjected to a first order moment M^I and having a deflection at the top induced by wind action equal to δ_o, the increase in flexural moment due to second-order effects proves to be:

$$\frac{M^{II}}{M^I} = 1 + \frac{1}{M^I} + \frac{N \cdot \delta_o}{1 - N/N_{cr}} \tag{39}$$

For the strength check

$$M = M^I + M^{II} \leq M_{rd} \tag{40}$$

with M_{rd} the ultimate moment of the cross-section for a fixed axial force and obtained from the strength domain expressed as:

$$\frac{M}{M_{rd}} = 1 - \left(\frac{N}{N_{rd}}\right)^{1.7} \tag{41}$$

The flexural strength is obtained multiplying the strength modulus by the yielding stress of the steel and by a reductive coefficient that considers the slenderness of the cross-section.

According to European and international standards, the reductive coefficient depends on the slenderness of the cross-section and the type of steel. The cross-section of a tubular member is compact ($d/t < \lambda_p$), non-compact ($\lambda_p < d/t < \lambda_r$), or slender ($d/t > \lambda_r$), where d/t is the diameter-to-thickness ratio, $\lambda_p = 0.07E/f_y$ and $\lambda_r = 0.31E/f_y$. Eurocode 3 [26] gives the following expression for the c_1 reductive coefficient:

$$c_1 = 1 - 0.155 \cdot \left(1 - \frac{t}{d} \cdot \frac{E}{10 \cdot f_y}\right)^2 0.1 \leq \frac{d}{t} \cdot \frac{f_y}{E} \leq 0.357 \tag{42}$$

For verification of the whole pole a strength domain (moment-axial force diagram) is constructed and it is penalized to take into account of instability through the χ factor defined in [31].

For a circular cross-section the ultimate axial force is

$$N_{sd} \leq \chi \cdot N_{rd} = \frac{1}{\phi + \sqrt{\phi^2 + A \cdot f_y / P_{cr}}} \cdot A \cdot f_{yd} \; \phi = 0.5\left[1 + 0.21 \cdot \left(\sqrt{\frac{A \cdot f_y}{P_{cr}}} - 0.2\right) + \sqrt{\frac{A \cdot f_y}{P_{cr}}}\right] \tag{43}$$

The strength domain is that shown in Fig. 12. The graph shows the strength domain for the gross and reduced cross-section considering the slenderness of the cross-section. The reduced compressive strength χN_{rd} and the design calculus including second-order effects are also considered. It must be stressed that load and global second-order effects significantly reduced the load-carrying capacity of a member.

4 Conclusions

In this study the moment–rotation capacity curve of shallow foundation systems of wind towers with rectangular, square, and circular cross-sections was determined in a closed form. The model includes the initial rotational stiffness of the foundation and the ultimate moment related to a crisis of the soil and of the structures. The Winkler soil model was adopted for both the elastic state and plastic model to capture the non-linear behavior of the soil. The main parameters governing the response were Winkler

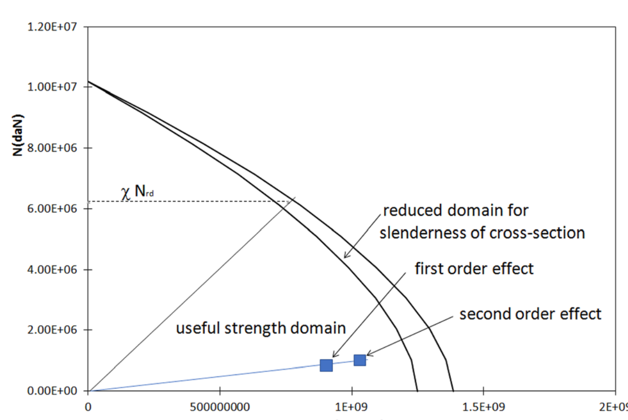

Fig. 12 Strength domain for steel pole with hollow cross-section

modulus, ultimate stress of soil-to-complex foundation and level of axial force. It was observed that the flexibility of foundations influences the moment–rotation response though the initial rotational stiffness with a coefficient between 1 and 0.7 for a width-to-span ratio between 5 and 2. The nature of the soil is considered though corrective factors of 0.75 and 1.3 of the Winkler constant for cohesive and non-cohesive soils, respectively. Analyses carried out stressed that a possible design valued to be adopted in a steel wind tower with shallow foundations is a diameter of the steel tube 1/15 of the height of the tower, a diameter of foundation 0.75 of the length and a depth of foundation 1/10 of the diameter and thickness of steel tower ratio diameter equal to 1/10. In this range is was observed that the effects of the soil-to-foundation interaction in the elastic range influence the critical length in the stability of the steel wind tower with values between 2.5 and 2 (column fixed at the base) in a Winkler constant range between 0.1 and 1 daN/cm^3. Finally, an experimental validation of the proposed model was carried out with the data available from the literature. It can potentially be useful for preliminary design, although no actual time history of loads deriving from coupled aeroelastic simulations in the time domain are considered. The proposed model is not to be considered an alternative to the existing sophisticated and applied approaches widely found in the literature for soil-to-foundation problems but as an analytical support for a preliminary check to be utilized for hand calculation.

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Data availability No data, models, or code were generated or used during this study.

Declarations

Conflict of interest The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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