

# BOOK OF ABSTRACTS

## NEW TRENDS IN MANY-VALUED LOGIC AND CATEGORICAL ALGEBRA

An overview of  
the recent trends in  
many-valued logic  
and categorical algebra  
in the context of the  
SDF project scopes

### **Some properties of implicative semilattices**

Corentin Vienne

### **On actions and split extensions in varieties of hoops**

Manuel Mancini

### **Towards a notion of coherent and ideal actions in ideally exact contexts**

Federica Piazza

### **A generalization of the Brauer-Fowler theorem for p-elements**

Alessandro Dioguardi Burgio

### **Properties of continued fractions in the field of p-adic numbers**

Giuliano Romeo

– *Coffee Break* –

### **Fuzzy multi-criteria decision making and conditional random quantities**

Lydia Castronovo

### **Some Questions on Fuzziness and Nilpotency in Lie Algebras**

Gianmarco La Rosa

### **Nonlocal Carrier's Problems**

Giuseppe Failla

### **Polymorphic Word Representation: A mathematical framework for “Real World” LUT-driven string-pattern matchings and its applications to XAI**

Giuseppe Giacopelli

### **Fuzzy Weighting Scheme for Artificial Experts**

Giuseppe Filippone

**wednesday,  
march 18th, 2026**

**14.00 hr**

DMI, Aula 5  
via Archirafi, 34  
Palermo

#### **Organising Committee**

Lydia Castronovo  
Giuseppe Failla  
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Manuel Mancini  
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Università degli Studi di Palermo  
Dipartimento di Matematica e Informatica

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AND CATEGORICAL ALGEBRA**

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Palermo  
March 18th, 2026

# On actions and split extensions in varieties of hoops

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BL-algebras provide the algebraic semantics of *Basic Logic*, the logic of continuous  $t$ -norms, capturing the common fragment of *Lukasiewicz*, *Gödel*, and *Product* logics. It is well known that, up to isomorphism, every continuous  $t$ -norm behaves locally as one of the three fundamental ones: the Lukasiewicz  $t$ -norm  $x \cdot_L y = \max\{x + y - 1, 0\}$ , the Gödel  $t$ -norm  $x \cdot_G y = \min\{x, y\}$ , or the product  $t$ -norm  $x \cdot_P y = xy$ .

Each  $t$ -norm induces a residuation defined by  $x \rightarrow y = \sup\{z \in [0, 1] \mid z \cdot x \leq y\}$ . The residuations associated with the three fundamental continuous  $t$ -norms were studied by P. Hájek, who provided axiomatizations of the corresponding varieties of algebras: the variety  $\mathbf{MValg}$  of *MV-algebras* forms the algebraic semantics of Lukasiewicz Logic; the variety  $\mathbf{GAlg}$  of *Gödel algebras* forms the algebraic semantics of Gödel Logic; and the variety  $\mathbf{PAlg}$  of *product algebras* forms the algebraic semantics of Product Logic. Finally, P. Hájek introduced the variety  $\mathbf{BLAlg}$  of *BL-algebras*, which provides the algebraic semantics of Basic Logic. From a categorical point of view, the variety  $\mathbf{BLAlg}$  is an ideally exact category. Moreover, if  $\mathbf{2}$  denotes the two-element Boolean algebra, then the semi-abelian categories  $(\mathbf{BLAlg} \downarrow \mathbf{2})$ ,  $(\mathbf{MValg} \downarrow \mathbf{2})$ ,  $(\mathbf{GAlg} \downarrow \mathbf{2})$ ,  $(\mathbf{PAlg} \downarrow \mathbf{2})$  are equivalent, respectively, to the varieties of basic, Wajsberg, Gödel, and product hoops.

The aim of this talk is to study internal actions and split extensions in the variety of hoops, with particular attention to split extensions with *strong section*. Such extensions are described in terms of *strong external actions*, i.e., a pair of maps satisfying a set of identities related to the axioms satisfied by the hoop [1, 2]. We prove that for any hoop  $X$  there is a natural isomorphism  $\mathbf{EAct}_{\text{ss}}(-, X) \cong \mathbf{SplExt}_{\text{ss}}(-, X)$  between the functor of strong external actions on  $X$  and the functor of isomorphism classes of split extensions with strong section with kernel  $X$ . We also show that this notion trivializes for MV-algebras, while in the variety of Gödel hoops strong external actions coincide with those of basic hoops.

This is joint work with Giuseppe Metere, Federica Piazza and Marco Elio Tabacchi.

## References

- [1] M. Mancini, G. Metere, F. Piazza and M. E. Tabacchi, *On split extensions of product hoops*, 2025 IEEE International Conference on Fuzzy Systems (FUZZ), Reims, France, 2025, 1–5.
- [2] M. Mancini, G. Metere, and F. Piazza, *On actions and split extensions in varieties of hoops: the case of strong section*, *Studia Logica*, to appear (2026). Preprint available at [arXiv:2510.06886](https://arxiv.org/abs/2510.06886).