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Abstract. Some thermodynamical properties of solids, such as heat capacity and magnetic susceptibility, have recently been shown to be linked to the amount of entanglement in a solid. However, this entanglement may appear a mere mathematical artefact of the typical symmetrization procedure of many-body wavefunction in solid state physics. Here we show that this entanglement is physical, demonstrating the principles of its extraction from a typical solid-state system by scattering two particles off the system. Moreover, we show how to simulate this process using present day optical lattice technology. This demonstrates not only that entanglement exists in solids but also that it can be used for quantum information processing or as a test of Bell's inequalities.

Entanglement [1] is the property of quantum systems to show correlations that cannot be explained by classical mechanics [2]. In the last two decades entanglement has been recognized as a resource in quantum information processing, such as quantum cryptography [3] and quantum computation [4, 5]. Furthermore, in recent years the role that entanglement plays in the properties of condensed matter systems has received increasing attention. For instance in a typical magnetic solid, the electron orbitals of different atoms overlap giving rise to an exchange interaction

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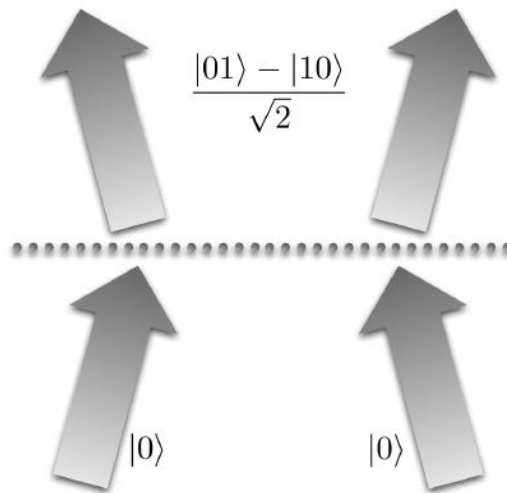$$\frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

$|0\rangle$ $|0\rangle$

Figure 1. Schematic drawing of the entanglement extraction process. Two probe particles initially prepared in a product state are sent toward a solid whose spins are entangled. Each probe interacts with only one spin in the solid. During the collision, due to the exchange interaction between probe and spin, their state is swapped. The two probes come out from the solid in an entangled state.

between the electron spins which can be described by a Heisenberg Hamiltonian, whose thermal states are entangled [6].

It is a common belief that entanglement cannot exist on a macroscopic scale since decoherence effects would destroy all quantum correlations. However, it has been predicted that macroscopic entanglement not only is related to critical phenomena [7] but also that it can exist in solids in the thermodynamic limit [8] even at high temperatures [9]. Indeed there are many works inferring the existence of macroscopic entanglement at various temperatures up to room temperature [10]–[12]. The evidence for this entanglement comes from experiments measuring different thermodynamic properties such as magnetic susceptibility and heat capacity. The main aim of this paper is to show that such entanglement is not an artificial mathematical property but that it can be extracted and therefore used for quantum information processing in the same way as heat can be extracted and used for work in thermodynamics. In particular we will show how such entanglement can be transferred to a pair of particles and subsequently used, in principle, for quantum communication or to test the violation of Bell's inequalities. Furthermore, we propose a scheme to demonstrate entanglement extraction with present day technology using optical lattices. In the next two paragraphs, we outline the basic idea behind our proposal which is then elaborated in the remaining part of the paper.

In the ideal scenario, the entanglement between a pair of spins inside a solid can clearly be transferred to another pair of probe particles using only local swap operations. To this end one sends simultaneously a pair of probe particles toward the entangled spin chain in such a way that each probe interacts with a different spin (cf figure 1). We emphasize that the two probes do not interact with each other nor do they experience an interaction mediated by the solid (cf [13]). The entanglement between the probes has been extracted from the spin chain and cannot exist without entanglement in the chain. This is a genuine non-local process between the two
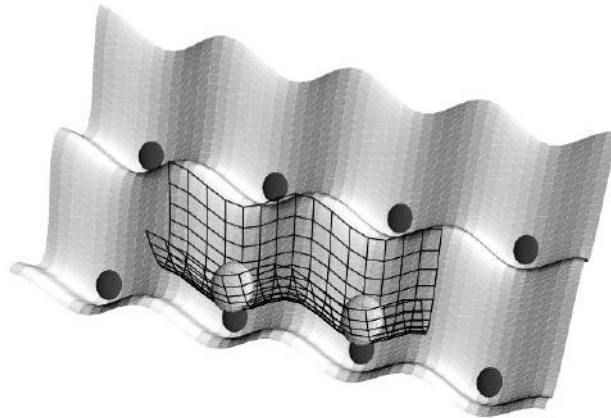
Figure 2. Simulation of the entanglement extraction using optical lattices. Cold neutral atoms (black) are loaded onto an optical lattice generated by counterpropagating lasers in such a way that each potential well is occupied by only one atom. These atoms represent the spin degrees of freedom in a typical solid. To the first potential a second one is superimposed (wireframe) onto which different atoms (white), called markers, are loaded. It is possible to move the markers across the solid by moving adiabatically the second potential. This can be done efficiently by varying the laser parameters of the second lattice. A collision between a marker and a spin can be produced by moving the marker into the same potential well of the spin.

probes like in the case of entanglement swapping [14]. We mention that in the continuum limit, our procedure can be used to extract entanglement from vacuum (some steps in this direction have been taken by [15]). In practice, however, the spin chain is in a mixed entangled state and the scattering interaction between probes and spins in the chain can only partially swap their state. Is entanglement extraction still possible under such realistic conditions? In this paper, we answer this question affirmatively. We will consider only common physical interactions like the Heisenberg or the XY ones. We will show that in general, after one scattering event, entanglement swapping is only partial. In this case, it is interesting to investigate whether by repeating this collision procedure with the same probes, one can achieve a better entanglement extraction.

The most natural way to extract entanglement from entangled electron spins in solids would be to scatter pairs of neutrons off the solid. We will present a proposal for simulation of this process of entanglement extraction with optical lattices [16] that we believe can be implemented at present. Optical lattices are a very useful tool for simulating many quantum effects in solids and have been employed in demonstrating quantum phase transition [16] and quantum gates [17]. For our purposes we need to simulate both the interaction in the solid system as well as the interaction with the external probes. Hamiltonians of entangled spin chains or ladders can be realized using cold neutral atoms trapped in potential wells generated by counterpropagating lasers [18]–[20]. Bosonic atoms (black in figure 2) are loaded on to the lattice in such a way that there is only one atom per lattice site and that the hopping between neighbouring sites is inhibited. Long-lived atomic states are used to simulate the electronic spin in a typical solid. The interaction between atoms can be varied by adjusting the laser parameters and by means of

electric and magnetic fields. In this way, one can produce a variety of common spin Hamiltonians. As probe particles one can use different atoms (white in figure 2), called marker qubits [21]. The main idea is to construct first the optical lattice, loading the atoms in a register that constitutes the simulated solid and then to superimpose a second optical lattice loading auxiliary atoms onto it as shown in figure 2. The auxiliary marker atoms need not be of the same atomic species as the atoms in the solid. The marker atoms can be moved through the solid by varying adiabatically laser parameters of the second optical lattice. These operations do not affect the register atoms which remain confined in their potential wells. In this way, collisions between probes and spins in the solid can be simulated by moving a marker atom close to a register atom letting their orbitals overlap as shown in figure 2. This in principle can be performed with high precision and efficiency as in the case of cold controlled collisions [17]. In this way, one can realize the swap operations needed in our protocol. Such an operation can be repeated several times moving the markers forward to collide with other register atoms.

Let us now introduce the detailed description of the interaction between the probes and the spins in the solid. We stress again that all aspects of the following analysis can be implemented not only with the optical lattices described above but also with any other quantum systems capable of coherent manipulation. Two probes are sent to interact for a time τ each with a single spin of an entangled pair in a solid as shown in figure 1. We suppose that the interaction Hamiltonian is given by an anisotropic XXZ model between probe $i = L, R$ and spin $j = 1, 2$

$$H(\lambda)_{ij} = J \left(\sigma_x^i \sigma_x^j + \sigma_y^i \sigma_y^j + \lambda \sigma_z^i \sigma_z^j \right) \tag{1}$$

where $\sigma_{x,y,z}$ are the Pauli matrices, J is the coupling which we assume to be constant during the collision. In the case of a time-dependent $J(t)$, our analysis still applies by making the substitution: $Jt \to \int_0^t J(t') \, dt'$. The total Hamiltonian is therefore $H_T = H(\lambda)_{1L} + H(\lambda)_{2R}$. This Hamiltonian comes from the same exchange mechanism that gives rise to the interaction between spins in the solid. This Hamiltonian can be easily implemented with optical lattices and marker qubits in a probabilistic fashion. Here we concentrate on two important limits: $\lambda = 1$, which is the Heisenberg (exchange) interaction; $\lambda = 0$, which is the XY interaction. We assume that the spin chain is initially in the ground state, however, our discussion can also be extended to other out-of-equilibrium states. We also assume that during the only dynamics is due to the interaction with the probes. The reduced density matrix of the entangled pair can be written in terms of spin–spin correlation functions by using translational invariance and conservation of angular momentum. The full derivation is presented in [22] and the result is:

$$\rho = \begin{pmatrix} \frac{1}{4} + g_{zz} & 0 & 0 & 0 \\ 0 & \frac{1}{4} - g_{zz} & 2g_{xx} & 0 \\ 0 & 2g_{xx} & \frac{1}{4} - g_{zz} & 0 \\ 0 & 0 & 0 & \frac{1}{4} + g_{zz} \end{pmatrix}, \tag{2}$$

where $g_{zz} = \langle \sigma_z^1 \sigma_z^2 \rangle / 4$ and $g_{xx} = \langle \sigma_x^1 \sigma_x^2 \rangle / 4$ are spin–spin correlation functions and can be numerically estimated for the Heisenberg model. The concurrence of ρ, assuming $|g_{xx}| \geqslant g_{zz}$, is

$$C = \text{Max}[0, -\tfrac{1}{2} + 4 |g_{xx}| - 2g_{zz}]. \tag{3}$$

Note that the results given below apply to a large class of mixed entangled states including, for example, Werner states. Let us first consider what happens after a single collision between
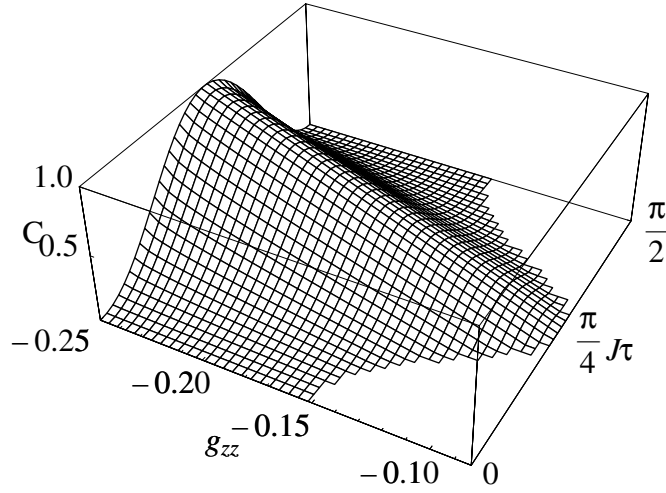
Figure 3. Extracted entanglement for a Heisenberg chain, measured by the concurrence C between the two probes, maximized over the set of initial product states of the probes, after one collision as a function of g_{zz} and τ. Notice how C increases with the correlation g_{zz} and oscillates with the interaction time τ.

the probes in a pure product state e.g. $|00\rangle$ and the spins when $\lambda = 1, 0$. To illustrate the idea let us suppose that the spins are initially in a maximally entangled state $|\psi^-\rangle = 2^{-1/2}(|01\rangle - |10\rangle)$. After the collision the global state of the four particle reads (apart from a global phase factor)

$$|\Psi(t)\rangle = \cos 2J\tau |00\rangle_{LR}|\psi^-\rangle_{12} - \mathrm{i}\sin 2J\tau |\psi^-\rangle_{LR}|00\rangle_{12}. \tag{4}$$

This state is a superposition of the initial state and of the swapped state. By tracing out the two spins one obtains the reduced density matrix ρ_{LR} of the two probes. We use the concurrence [23] to measure the entanglement extracted. For a density matrix of two qubits ρ let us define $\tilde{\rho} \doteq \sigma_y \otimes \sigma_y \rho^* \sigma_y \otimes \sigma_y$ and $R = \rho\tilde{\rho}$. The concurrence is defined as $C = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$ where λ_i are the square roots of the eigenvalues of R labelled in decreasing order. The concurrence for the two probes is simply $C = \sin^2 2J\tau$ and is equal to 1 when $J\tau = \pi/4$. Thus no matter how small τ is, it is still possible to extract some entanglement from the singlet state.

The extracted entanglement (i.e. the concurrence between the probes), maximized over the set of initial product states of the probes, is plotted in figure 3, for $\lambda = 1$, as a function of $g_{xx} = g_{zz}$, including therefore the case of mixed states of spins in the solid. Note that ρ is entangled for $-1/4 \leqslant g_{zz} < -1/12$. As is shown in the plot the entanglement between probes decays when g_{zz} decreases. This confirms the intuitive expectation that more the spins in the solid are entangled, the more such entanglement can be transferred to the probes.

We want to stress that for a translational invariant system the correlation functions can never reach the maximum value $g_{xx} = g_{zz} = -1/4$. Pure maximal entangled states cannot exist in a homogeneous spin chain. Nevertheless in figure 3, we show the concurrence for all possible values of g_{zz}.

The state of the spins and that of the probes are swapped when $J\tau = \pi/4$ and the entanglement extracted is maximum. In an optical lattice implementation, this can be achieved in a time comparable to the trapping frequency of the atoms ($10\,\mu\mathrm{s}$ for ^{87}Rb [21]). In the case of solid J is of the order of 1 K giving approximately $\tau \sim 10^{-11}\,$s [11]. We also note that entanglement

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oscillates with τ, a feature which reflects the fidelity of the swapping operation. The probes are always entangled for $g_{zz} \lesssim -0.16$ no matter how small τ is. However, as shown in figure 3, for $-0.16 < g_{zz} < -1/12$, τ must be chosen appropriately in order to extract some entanglement. Notice that obviously no entanglement can be extracted for $g_{zz} \geqslant -1/12$ since the two spins are not entangled. The important point is that as long as some entanglement is shared between two spins in the solid (i.e. for $g_{zz} < -1/12$), our procedure is capable of transferring part of it to the probes. Similar results applies also to the XY model $\lambda = 0$.

Once τ is fixed for a specific experimental setup, the amount of entanglement extracted is less than the entanglement in the pair of spins as long as $J\tau < \pi/4$. Is it possible to extract *more* entanglement by repeating the collision process *several* times? This can be done in optical lattices by moving the marker qubits so they interact with different identically prepared chains (cf figure 2). Although this approach has some conceptual analogies with the so-called homogenization process [24], the two schemes are not equivalent since the transformation induced by the interaction Hamiltonian (1) is not a global partial SWAP transformation. Indeed with a local interaction generated by the spin Hamiltonian ($\lambda = 1$), the full transformation of the four spins is

$$U_{12LR} = \text{PSW}_{1L} \otimes \text{PSW}_{2R}, \quad \text{where} \tag{5}$$

$$\text{PSW} = e^{iJt} \left(\cos 2Jt \, \mathbb{1} - i \sin 2Jt \, \text{SWAP} \right), \quad \text{and} \tag{6}$$

$$\text{SWAP} = |00\rangle\langle 00| + |01\rangle\langle 10| + |10\rangle\langle 01| + |11\rangle\langle 11|, \tag{7}$$

swaps two qubits.

It is easy to demonstrate that, for $\lambda = 0$ (XY model), the state $|\psi^+\rangle_{LR}|\psi^-\rangle_{12}$ is an eigenstate of H_T. It is thus a fixed point of the evolution transformation U_{12LR} and the state of the probes approaches the state $|\psi^+\rangle$ as the number of collisions goes to infinity, independently of the input state and of how small τ is. This is a counterintuitive result: the state of the probes do not converge to the state of the bath but to an orthogonal state. The reason is that XY interaction does not generate a partial SWAP operation but something similar to a partial *i*SWAP operation

$$i\text{SWAP} = |00\rangle\langle 00| + i|01\rangle\langle 10| + i|10\rangle\langle 01| + |11\rangle\langle 11|, \tag{8}$$

that swaps the state of the two qubits but with a i factor. The composition of two of these transformations produces a minus sign that transform $|\psi^+\rangle$ in $|\psi^-\rangle$. In figure 4, we show how the concurrence grows increasing the number of collision with the spins.

For $\lambda = 1$ (Heisenberg model), there is no eigenstate of the total Hamiltonian which is the tensor product of a maximally entangled state of the probes and a maximally entangled state of the two spins.

When the state of the two spins in the solid is mixed, the transformation has still a unique fixed point [25, 26]. This follows form the Banach–Cacciopoli theorem for contractive map. The transformation we are considering is the tensor product of two partial swap transformation which are contractive maps and thus it is itself a contractive map. The entanglement of the fixed point depends on g_{zz}, g_{xx} and τ. For example, for a Heisenberg interaction, when $g_{zz} = -0.2$ and $J\tau = 0.6$, the concurrence of the fixed point is $C \simeq 0.28$. However, when the two spins in
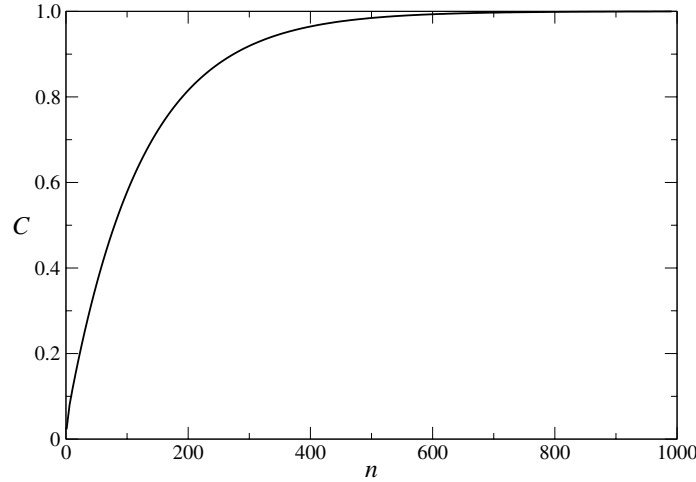
Figure 4. Concurrence between the probes as a function of the number of collisions. We supposed the initial state (2) with $\eta = 0$ and $J\tau = 0.2$. The data are well described by the fit function $C(n) = 1 - \exp(-\kappa n)$ where $\kappa = -8.3 \times 10^{-3}$.

the chain are in a mixed state, the entanglement extracted by the probes after repeated collisions is always less than that after only one collision for $\lambda = 1$ (Heisenberg interaction) and 'nearly' always, i.e. except for mixed states which are very close to a pure state, for $\lambda = 0$ (XY interaction). Therefore repeated collisions do not increase the extracted entanglement.

Under a different hypothesis, it is possible by repeated collisions to homogenize the state of the probes to the state of the spins in the solid. By modifying the interaction between the four spins 1, 2, L and R we can build a Hamiltonian that generates a global SWAP transformation

$$\mathrm{PSW}_{1L,2R} = \mathrm{e}^{\mathrm{i}Jt} \left(\cos 2Jt \mathbb{1}_{1L} \otimes \mathbb{1}_{2R} - \mathrm{i} \sin 2Jt \mathrm{SWAP}_{1L} \otimes \mathrm{SWAP}_{2R} \right). \tag{9}$$

This can be found by inspection and reads in the computational basis of $12LR$

$$\begin{aligned}
H_{12LR} = &\; |0000\rangle\langle 0000| + |0101\rangle\langle 0101| + |1010\rangle\langle 1010| + |1111\rangle\langle 1111| \\
&+ (|0001\rangle\langle 0100| + |0010\rangle\langle 1000| + |1011\rangle\langle 1110| + |1101\rangle\langle 0111| \\
&+ |0011\rangle\langle 1100| + |1001\rangle\langle 0110| + \mathrm{h.c.}).
\end{aligned} \tag{10}$$

Notice that the elements of the first two rows can be generated by pairwise interactions as in (1). The elements of the third row instead are four spin processes. With this interaction, it is proven by the homogenization process [24] that the probes state will approach the state of the spins no matter the initial state.

Until now we concentrated on extracting bipartite entanglement, however, our scheme can also be adapted to extract genuine multipartite entanglement. Let us consider n spins in a chain, each scattering a spin probe. Let the initial state of the chain be a W state:

$$|W_n\rangle = \frac{1}{\sqrt{n}} (|100\cdots 0\rangle + |010\cdots 0\rangle + \cdots + |000\cdots 1\rangle), \tag{11}$$

and the probes be in state $|\underline{0}\rangle = |00\cdots 0\rangle$. W states can be ground states of some Hubbard related Hamiltonians [27], critical spin chains [7, 28] and fermionic lattice models of high T_c superconductivity [9]. The state of the system at time τ is the analogue for n spins of equation (4) with $|W_n\rangle$ instead of $|\psi^-\rangle$. Thus the reduced density matrix for the n probes reads

$$\rho = \cos^2 2J\tau |\underline{0}\rangle\langle\underline{0}| + \sin^2 2J\tau |W_n\rangle\langle W_n|. \tag{12}$$

Notice that when $J\tau = \pi/4$, the state of the chain is fully swapped onto that of the probes as in the case with $n = 2$. For any value of $J\tau \neq k\pi/2$, where k is an integer, it can be shown that the above state contains genuine multipartite entanglement [28] (i.e. it cannot be written as a mixture of biseparable states).

So far we have assumed that each probe interacts with a single spin in the chain. This assumption is, in some cases, unrealistic. For instance it is difficult, with present day technology, to address a single spin in a solid. For example, in neutron scattering experiments, the spin of the incoming neutron interacts with the total angular momentum of a bunch of spins in the solid [29]. It is thus more realistic to analyse a model in which two probes interact with several spins at once. Let us consider a chain of L spins and let us assume that each neutron, being a wavepacket of a certain width, interacts with a different subset of N spins. We will assume that the spin of the probe is coupled to the total angular momentum of the N spins, which is equivalent to assuming that each probe is equally coupled to each of the N spins in the chain. This model is equivalent to the ones considered in other contexts [30], and for $\lambda = 0, 1$ its eigenstates are known. Let us consider a W_L state as the initial state of the chain. Though this is not the most general entangled state of a spin chain, this model can be solved analytically. The concurrence for the two probes for $\lambda = 0$ is $C = (2N/L)\sin^2 2J\sqrt{N}\tau$ and reaches the maximum value $2N/L$ for $J\tau = \pi/(4\sqrt{N})$ (the entanglement for the Heisenberg model $\lambda = 1$ is greater than zero but is always less than that of the XY model). Notice that $2N/L$ is just the fraction of spins interacting with the probes and thus it is the maximum bipartite entanglement that can be extracted. We note that these results have been obtained for a W state of the chain. This state is symmetric under permutations of any number of spins which means that the entanglement is equally distributed among all spins. However, for Heisenberg and XY chain models, long-range correlations are very small and the entanglement extracted should decrease with N and with the distance between the two beams of neutrons. From the above analysis, it emerges that a necessary condition to extract entanglement within this realistic model is that segments of the spin chains should be entangled. This analysis can be extended to the case of non-equilibrium states and it would be interesting to see if the entanglement extracted could be larger than in the case of the W state.

We have described a scheme to extract entanglement from spins in a solid by scattering probe particles. In this way, macroscopic thermal entanglement of solids can be converted into a useful resource for quantum information processing. To illustrate the scheme, we have proposed an optical lattice implementation where all aspects of our procedure can be realized with present day technology. In this context, optical lattices offer only a simulation of entanglement extraction. This is why we have discussed how neutron scattering can be used to achieve entanglement extraction from a real solid as well as the physical limitations of this process. We hope that our ideas in the long run will lead to an entangling procedure for neutrons in the same way as parametric down conversion is for creating entangled photons.

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