

Research papers

Analysis of the dependence of daily rainfall depth on wet spell duration across Europe

Carmelo Agnese^a, Giorgio Baiamonte^a, Carmelo Cammalleri^{b,*}, Elvira Di Nardo^c, Stefano Ferraris^d, Tommaso Martini^d

^a Department of Agricultural, Food and Forest Sciences, University of Palermo, Viale delle Scienze 13, 90128 Palermo, Italy

^b Department of Civil and Environmental Engineering, Politecnico di Milano, Piazza Leonardo Da Vinci 32, 20133 Milan, Italy

^c Department of Mathematics "G. Peano", Università degli Studi di Torino, Via Carlo Alberto 10, 10123 Turin, Italy

^d Interuniversity Department of Regional and Urban Studies and Planning, Università degli Studi di Torino, Politecnico di Torino, Viale Pier Andrea Mattioli 39, 10125 Turin, Italy

ARTICLE INFO

This manuscript was handled by Amilcare Porporato, Editor-in-Chief

Keywords:

Polylog distribution

Rainfall statistical modelling

Time series

Conditioned rainfall depths

ABSTRACT

The statistical modelling of daily rainfall depth is commonly carried out independently from the rainfall occurrence process (dry/wet alternation). In this study, we investigate the effects of accounting for the dependence between rainfall depth (h) and wet spell duration (ws) using rain gauge data collected on 34 sites across Europe between 1971 and 2016. For each site, a set of conditioned $h|ws$ sub-datasets was generated, and the two-parameter Polylog distribution was fitted on these sub-datasets. The results of the fittings highlight a good capability of the Polylog distribution to capture the statistical behavior of $h|ws$, as well as the presence of statistically significant differences in the various $h|ws$ fittings obtained in some of the sites. This dependence has been quantified as "Weak" in about 13 sites (38%), and "Strong" in 9 sites (26%). The $h|ws$ dependence was directly modelled for each site with a logarithmic function, showing how this approach is able to well reproduce the arithmetic and geometric means of the sub-samples (standard estimate errors of 0.62 and 0.42 mm, respectively). An attempt at explaining the spatial variability of the dependence through a penalized regression highlighted some key factors, such as longitude and distance from the shoreline; however, the selected features only partially explained the variability and the connection to climate factors. Overall, this research introduces a simple procedure to directly account for the $h|ws$ dependence, which can provide a more robust modelling of the rainfall processes, as well as an improved understanding of the rainfall generating processes.

1. Introduction

The statistical modelling of daily rainfall depth plays a fundamental role in hydrology, climatology, and environmental engineering due to its capability to reproduce the stochastic nature of the precipitation process. By quantifying the variability, frequency, and intensity of rainfall events, such models provide essential inputs for flood risk assessment, water resource planning, and the design of hydraulic infrastructures (De Michele et al., 2005; Jehanzaib et al., 2022; Lettenmaier, 2024). Rainfall modelling is also fundamental in agricultural decision-making, where knowledge of precipitation patterns influences crop selection, irrigation scheduling, and drought preparedness (Cavus et al., 2023; Porporato et al., 2015). Furthermore, statistical models enable the detection of shifts in rainfall regimes, offering valuable

insights into the impacts of climate variability and long-term changes.

Intermittency is a distinctive feature of the rainfall process, which manifests in both the fluctuations in rainfall intensity and the alternation of wet and dry periods (Molini et al., 2009; Schmitt et al., 1998). It is well established that rainfall intermittency increases as the observation time scale decreases, with the daily scale being the maximum conventional timescale at which intermittency still manifests (Schleiss and Smith, 2016). Most of the hydrological processes in soil and plants are usually modelled at the daily scale, as the soil acts as a low-pass filter for soil moisture dynamics, and therefore the faster processes of infiltration and evapotranspiration can be embedded in a daily model. Many stochastic models consider rainfall at the daily timescale, with events having a duration of a few hours assumed to be instantaneous (e.g., Porporato and Yin, 2022).

* Corresponding author.

E-mail address: carmelo.cammalleri@polimi.it (C. Cammalleri).

<https://doi.org/10.1016/j.jhydrol.2026.135665>

Received 12 October 2025; Received in revised form 6 May 2026; Accepted 10 May 2026

Available online 14 May 2026

0022-1694/© 2026 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

At the daily scale, the statistical analysis of the sequences of rainy and no-rainy days (named in the following as wet spells and dry spells, respectively) has been widely studied in the past (e.g., De Groen and Savenije, 2006; Foufoula-Georgiou and Lettenmaier, 1986; Manfreda and Rodríguez-Iturbe, 2006; Rodríguez-Iturbe et al., 1987). Agnese et al. (2014) and Baiamonte et al. (2024) demonstrated that some discrete distributions from the Hurwitz–Lerch–Zeta (HLZ) family (Gupta et al., 2008) effectively capture the main statistical features of inter-arrival time series, which represent the time intervals between consecutive rainy days. Furthermore, Agnese et al. (2014) showed that both wet and dry spell distributions can be derived from the inter-arrival time distribution.

In addition to the sequence of wet and dry spells, a complete modelling of the statistical behaviour of rainfall also requires the characterization of rainfall depth. Conventionally, the modelling of rainfall depth and occurrence process (dry/wet) is carried out independently. Continuous probability distributions, such as the exponential (Rodríguez-Iturbe et al., 1987), gamma (Hasan and Dunn, 2010), Weibull (André et al., 2024), and generalized extreme value (GEV), are often used in the scientific literature for the statistical modelling of rainfall depth.

Agnese et al. (2022) showed that the HLZ family can also be successfully used to model rainfall depth frequency when it is treated as a discrete variable. This approximation may offer some advantages, mainly related to the use of a consistent modelling framework for both interarrival times (wet/dry spells) and rainfall depths, without losing generality. This is justified by the way rainfall data are recorded (automated tipping bucket instruments recording 0.1/0.2 mm increments), as well as their general accuracy (e.g., Petroselli and Grimaldi, 2025; Wood et al., 2000). Albeit the same statistical modelling framework is used in that study to analyse daily rainfall depth and interarrival times, they are still treated independently from each other.

Some studies have highlighted a dependence between the rainfall occurrence process and the rainfall depth (e.g., Balistocchi and Bacchi, 2011; Hosseini et al., 2016; Stern and Coe, 1984). For instance, Serinaldi (2009) presented a multisite Markov model based on bivariate copula-based mixed distributions for generating daily rainfall series, preserving both spatial and temporal intermittency as well as hydrologic properties such as wet spell length and daily rainfall amounts. In general, copulas are widely recognized as versatile tools for capturing complex dependence structures, particularly when these relationships are nonlinear or too intricate to be adequately represented by traditional statistical models. Despite this, simpler more direct models of dependence are generally preferable to copula-based approaches when such a direct relationship exists, resulting in computationally lighter and more informative models.

In addition, to model the statistical structure of daily rainfall depth, some studies classified the rainy day with respect to the immediately preceding and following wet or dry status (Buishand, 1978; Chapman, 1998; Suhaila and Jemain, 2009). Chapman (1998) classified rainfall data across 4 different regions (Australia, South Africa, North America and Pacific Islands) according to the number of adjoining wet days. Apart from class 0, which denotes the rainfall depth associated with isolated wet days (i.e., days preceded and followed by dry conditions), Chapman (1998) defined class 1 as rainfalls happening on wet days that are bounded on one side by another wet day and on the opposite side by a dry day. Class 2 refers to rainfalls on wet days flanked on both sides by other wet days, indicating their occurrence within a continuous sequence of rainfall events. Chapman (1998) found significant differences between the rainfall depth probability distributions associated with the classes, and emphasised that neglecting this effect in daily rainfall simulation models leads to significant overestimation of class 0 rainfall and under-estimation for class 2 rainfall.

Similar results were obtained by Suhaila and Jemain (2009) over the Malaysia peninsula. Based on daily rainfall data from 50 rain gauge stations representing four regions, using the same rainfall classification

by Chapman (1998), they concluded that it is crucial in hydrological studies involving rainfall modelling to consider whether rainfall should be modelled according to these temporal classifications.

Although the aforementioned rainfall classes are able to effectively capture the overall structure of rainfall data series, they fail to consider clusters that span multiple rainy days. As a result, they may overlook a potential dependency between wet spell duration and the frequency or rainfall depth. This limitation could be particularly significant in humid regions, which are typically characterized by long wet spells. The importance of this issue was also emphasized by McErlich et al. (2023), who examined ECMWF Reanalysis v5 (ERA5) global precipitation between 1980 and 2019, to determine the existence of a correlation between wet-day frequency and intensity.

To bridge this research gap, the primary aim of this study is to directly investigate the dependence between daily rainfall depth and wet spell duration, building upon the analyses performed on rainfall classes. To capture the complex statistical structure of rainfall, we employ the two-parameter Polylog distribution – a flexible, long-tailed distribution – and the geometric distribution, both belonging to the HLZ family (Baiamonte et al., 2019). These distributions, which were fitted independently on wet spells and rainfall depths in previous studies, are here for the first time conditioned on the size of wet spells. The possibility to capture the dependence of the fittings on wet spells is tested through a simple direct modelling based on site-specific two-parameter logarithmic functions. This direct approach makes possible a more straightforward integration of the modelling of both the intensity and temporal structure of rainfall events, without the need to rely on copulas or more complex approaches.

Rainfall patterns generally exhibit significant spatial and temporal variability, which can lead to misleading conclusions when new methodologies are tested only under specific conditions. For this reason, Europe, where climatic regimes range from oceanic and Mediterranean to continental and boreal, represents an ideal study region, as a wide range of diverse conditions are encountered across a relatively limited spatial area. In this study, we analyse 34 daily rainfall depth time series across Europe using a long-term dataset covering the period from 1971 to 2016, with an average sample length of about 45 years. This dataset builds on the more limited set of sites adopted in previous studies to analyse dry/wet spells and rainfall depth, separately (e.g., Agnese et al., 2022; Baiamonte et al., 2024). The proposed approach will provide a modelling of the dependence of rainfall depth on wet spell duration with a limited number of parameters, making it potentially transferable to a wide range of rainfall regimes. The obtained results can provide new insights into the nature of rainfall variability in Europe and have potential implications for hydrological modelling, infrastructure design, and climate risk assessment.

2. Materials

2.1. Rainfall dataset

This study dataset comprises 34 rain gauge stations distributed across Europe (Fig. 1). The sites were selected to represent different meteo-climatic and rainfall regimes, as well as to ensure a consistent coverage of a common time period of at least 40 years. A preliminary screening of the data identified as the optimal common time window the period 1971–2016, during which all the stations recorded at least 80% of valid daily data in more than 44 years.

Rainfall records were collected from well-established open-access rain gauge networks across Europe. Raw data were collected by the network managers, pre-processed according to internal standards, and openly disseminated as daily accumulated values with associated quality flags. Daily data for most stations were obtained from the European Climate Assessment & Dataset (Klein Tank et al., 2002). This dataset includes records from meteorological services in more than 20 European countries, homogenised through a basic quality-control procedure. Data

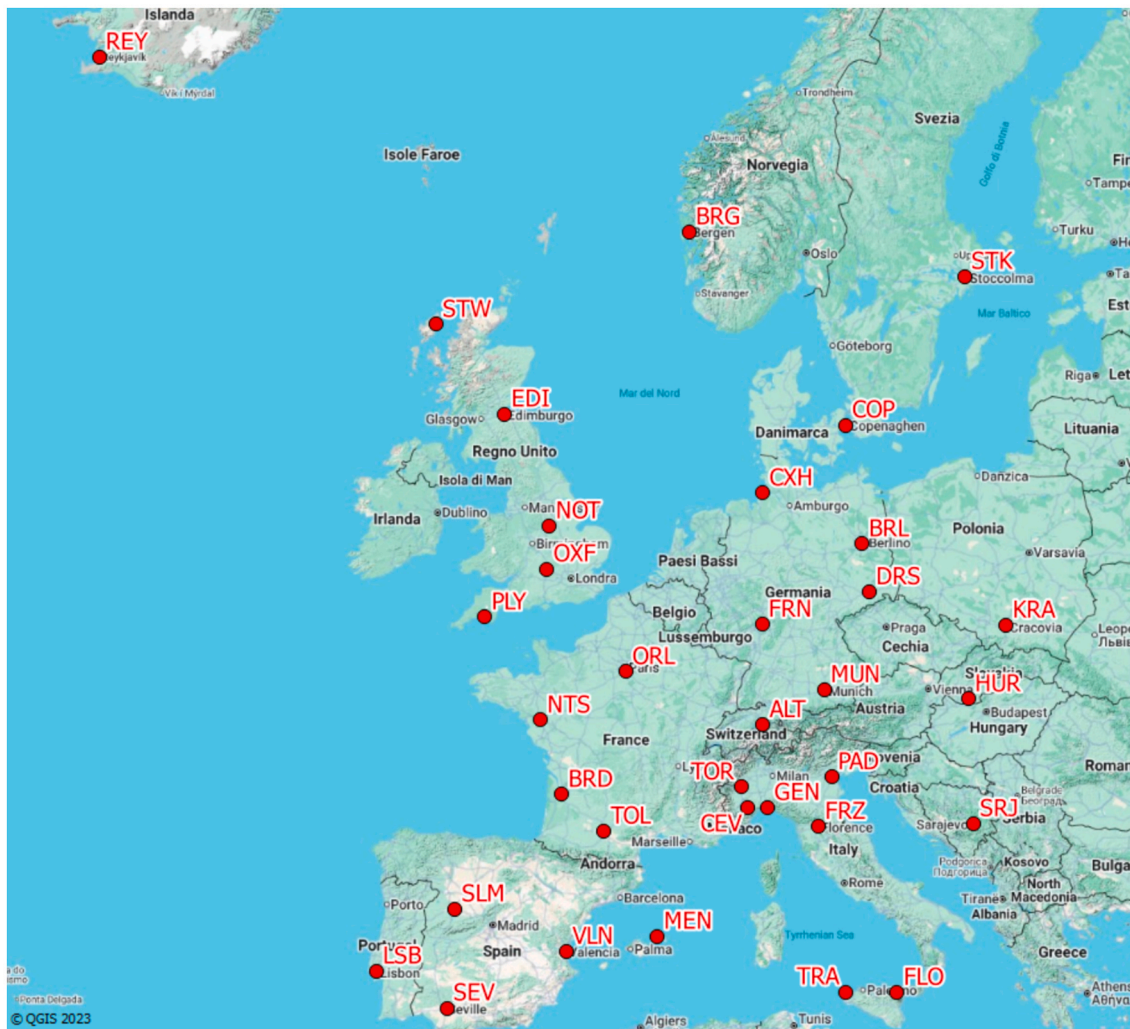


Fig. 1. Location of the rain gauge sites analyzed in this study.

for Norway (Bergen) were retrieved from the Norsk Klima Service Center (<https://seklima.met.no>). For the Italian territory, data were provided by the National Hydrographic and Mareographic Service (SIMN) then decentralized to the Regional Functional Centres, while the records for the PAD (Padua, IT) station are the same as those used in Miniussi and Marani (2020).

Thanks to the extensive temporal coverage for all the stations, missing values were not reconstructed, and the analysis was based solely on actual daily observations.

Rainy days were defined as days with $h \geq 1$ mm, which is the conventional threshold introduced by the World Meteorological Organization (WMO) to distinguish between rainy and dry days. Similarly to Agnese et al. (2022), only the integer part of the observed rainfall depths was used to treat the data as a discrete random variable. Some key statistics of the rainfall records in the selected sites are summarized in Table 1.

2.2. Time series of rainfall depths and wet spells

Let a time series of rainfall data be defined as $\mathbf{h} = \{h_1, h_2, \dots, h_n\}$, where h (mm) represents the rainfall depth recorded at a fixed uniform unit τ of time (e.g., a day). A day i is considered rainy if the rainfall depth $h_i \geq h^*$, where h^* is a fixed rainfall threshold (set to 1 mm in this study, see Section 2.1). Any sequence of m rainy days is a wet spell (ws) of duration m . Given a time series of rainfall data h and the corresponding sequence of wet spells denoted by $\mathbf{ws}$, we constructed for each given site

a bivariate dataset $(\mathbf{ws}, \mathbf{h})$, which is the set of all pairs representing a rainfall depth and its associated cluster. This set is constructed as follows: consider the first wet spell observation ws_1 in $\mathbf{ws}$, which has an arbitrary length $j \in \mathbb{N}^+$, and let $h_1, h_2, \dots, h_j$ be the corresponding rainfall depth observations within the spell. The first j pairs in the dataset $(\mathbf{ws}, \mathbf{h})$ are then defined as $(ws_1, h_1), (ws_1, h_2), \dots, (ws_1, h_j)$. By repeating this process for all observations in $\mathbf{ws}$, the complete bivariate dataset $(\mathbf{ws}, \mathbf{h})$ is constructed. In summary, each rainy day is labelled according to the total duration of the spell to which that day belongs.

We can then define a conditional dataset $h|ws$ as the sub-sample of daily rainfall depths belonging to spells whose total length equals to a given ws . By comparing the empirical distribution of $h|ws$ with that of the full rainfall sample h , we can directly test whether the amount of daily precipitation depends on the duration of the spell in which it occurs.

Due to site-specific rainfall characteristics and limitations in the sampling, the ws for which a reliable analysis can be performed is specifically identified at each site. In other words, the wet spell sample associated to a site (and, consequently, the corresponding $h|ws$ sets) is trimmed according to a combination of a relative and an absolute thresholding criteria: (1) the maximum ws considered for the site is selected as the first value for which the cumulative sample size of $h|ws$ exceeds the 95% of the total h sample (relative criterion); (2) among the conditional samples respecting criteria 1 for the site, only those ws with a minimum h sample size of 75 are considered (absolute criterion). As an example, at a site with 1,000 h observations, only the first consecutive

Table 1

Characteristics of the 34 rainfall records used in the analysis. N is the number of rainy days ($h \geq 1$ mm) $\langle H \rangle$ is the 1971–2016 average total annual rainfall (mm), AM is the arithmetic mean of daily rainfall (mm), GM is the geometric mean of daily rainfall (mm), and $\langle ws \rangle$ is the average length of the wet spells (days). Additionally, lat , lon , z and d characterize the geographic location, whereas CAPE and SI represent the climate conditions. See the main text for the definition and the source of these quantities.

site	N	$\langle H \rangle$ (mm)	AM(mm)	GM(mm)	$\langle ws \rangle$ (mm)	lat (deg)	lon (deg)	z (m a.s.l.)	d (km)	CAPE	SI
ALT	7388	2176	13.5	8.0	2.80	47.04	8.72	1220	290.8	175.4	0.20
BRD	5721	855	6.9	4.2	2.40	44.83	−0.69	47	21.2	135.4	0.16
BRG	8917	2266	11.7	7.1	3.81	60.40	5.31	41	0.1	57.9	0.27
BRL	5018	514	4.7	3.1	1.90	52.46	13.30	51	136.4	97.2	0.19
CEV	3026	820	12.2	6.4	1.80	44.39	8.03	375	33.1	373.1	0.24
COP	5113	566	5.1	3.3	1.94	55.68	12.58	6	0.4	71.4	0.18
CXH	6138	737	5.5	3.6	2.33	53.87	8.71	5	0.0	87.2	0.21
DRS	4714	525	5.0	3.2	1.83	51.09	13.63	106	265.7	115.4	0.20
EDI	5707	604	4.9	3.1	1.97	55.97	−3.21	26	1.5	31.0	0.13
FLO	4643	1139	10.9	5.9	2.28	37.98	14.91	1265	19.6	354.2	0.47
FRN	4945	579	5.2	3.4	1.99	50.14	8.67	49	78.4	228.2	0.13
FRZ	3850	788	9.4	5.4	1.90	43.77	11.26	132	325.0	104.5	0.20
GEN	3573	1198	15.4	7.3	1.85	44.40	8.94	48	0.5	427.5	0.35
HUR	3768	491	6.0	3.7	1.63	47.87	18.18	115	398.2	179.1	0.22
KRA	5004	600	5.5	3.3	1.91	50.08	19.80	237	453.3	153.4	0.34
LSB	3495	719	9.4	5.3	2.23	38.72	−9.15	77	1.5	82.2	0.55
MEN	3031	529	8.0	4.4	1.81	39.85	4.22	91	2.6	578.9	0.42
MUN	5893	871	6.8	4.1	2.14	48.16	11.54	515	299.3	145.9	0.31
NOT	5535	634	5.1	3.3	2.03	53.01	−1.25	117	82.8	42.1	0.11
NTS	5458	731	6.2	3.9	2.24	47.15	−1.61	26	31.3	78.2	0.23
ORL	5021	547	5.0	3.3	1.96	48.72	2.38	89	163.8	84.7	0.08
OXF	5287	587	5.1	3.3	2.02	51.75	−1.26	62	88.6	46.3	0.11
PAD	3805	832	10.0	5.5	1.83	45.41	11.87	0	36.5	428.8	0.13
PLY	6456	912	6.5	4.1	2.49	50.35	−4.12	50	0.2	46.0	0.24
REY	7021	753	4.9	3.3	2.57	64.13	−21.9	52	0.8	28.5	0.20
SEV	2383	506	9.8	5.4	1.89	37.42	−5.88	34	74.9	95.5	0.54
SLM	3027	331	5.0	3.3	1.75	40.96	−5.50	790	280.0	35.2	0.27
SRJ	5181	868	7.7	4.5	2.02	43.87	18.42	630	120.4	174.8	0.10
STK	4724	479	4.7	3.0	1.75	59.35	18.05	44	2.5	75.7	0.26
STW	9340	1107	5.5	3.7	3.55	58.21	−6.38	15	0.3	43.8	0.25
TOL	4496	587	6.0	3.7	1.94	43.62	1.38	151	151.7	179.6	0.13
TOR	3477	862	11.4	6.0	1.93	45.07	7.69	238	104.6	238.3	0.26
TRA	2692	422	7.0	4.0	1.77	38.02	12.54	2	1.3	621.3	0.53
VLN	2022	436	9.9	4.7	1.56	39.49	−0.47	69	12.5	397.1	0.34

ws accounting for a total of at least 950 h data are retained (criterion 1). Among those, only the ws containing more than 75 h observations each are considered (criterion 2).

3. Methods

3.1. Fitting rainfall depth distribution

Agnese et al. (2022) showed that the three parameters HLZ distribution can be successfully used to model the daily rainfall depth. To reduce the complexity of the proposed methodology, we shall restrict ourselves to a special case of the HLZ, namely the two parameter Polylog distribution, whose probability mass function (p.m.f.) is given by:

$$P(h = k) = \frac{\theta^k}{Li_s(\theta)k^s} \quad (1)$$

where θ and s are the parameters of the distribution and

$$Li_s(\theta) = \sum_{k \geq 1} \frac{\theta^k}{k^s} \quad (2)$$

is the polylogarithmic function for $k \in N, \theta \in (0, 1)$. It is worth mentioning that the Polylog distribution reduces to the Geometric distribution (discrete counterpart of the exponential distribution) when $s = 0$.

The Polylog distribution is fitted using the maximum likelihood estimation (MLE) method. As shown, for example, in Gupta et al. (2008), solving the corresponding likelihood equations is equivalent to equating the sample arithmetic mean (AM) and the sample geometric mean (GM) to their theoretical counterparts in the Polylog distribution. The need for

the Polylog distribution over the simpler Geometric distribution is verified with the LLH ratio test (Wilks, 1938). In this test, the statistic D is defined as:

$$D = -2[\ln(L_G) - \ln(L_P)] \quad (3)$$

where $\ln(L_G)$ and $\ln(L_P)$ denote the maximized log-likelihoods under the Geometric and Polylog models, respectively. Assuming the Geometric distribution as the null model (a special case of the Polylog), and the Polylog distribution as the alternative, under the null hypothesis that the additional parameter s is not required (i.e., the Geometric model is sufficient), the statistic D is approximately χ^2 -distributed with one degree of freedom. The null hypothesis is rejected when D exceeds the critical value at the significance level 0.05.

3.2. Effects of wet spell dependence

To verify whether the rainfall depth variable h depends on the length of the wet spell ws , we compare the empirical frequency distribution of the marginal sample of h with the empirical frequency distribution of each conditional sample $h|ws$. The Population Stability Index (PSI) was used to quantify the divergence between the conditional and each marginal distributions, while the independence between h and ws was tested with a permutation-based χ^2 test. The following subsections give the details of each method.

3.2.1. Population stability index

Because the sample $h|ws$ is not independent of the marginal sample h , the independence assumption underlying classical two-sample goodness-of-fit tests (e.g., the two-sample Kolmogorov–Smirnov test) is

violated. Therefore, we decided to employ the Population Stability Index (Yurdakul and Naranjo, 2021), based on information theory, which can be used as a metric to quantify the difference between a probability distribution from a reference one. Thus, given a marginal sample h and a conditional sample $h|ws$, the following index is computed:

$$PSI(ws) = \sum_{k \in h_{pos}} (p_k^h - p_k^{h|ws}) \ln \frac{p_k^h}{p_k^{h|ws}}, \quad (4)$$

where p_k^h is the empirical relative frequency of k in the marginal sample h , $p_k^{h|ws}$ is the corresponding frequency in the conditional sample $h|ws$ and

$$h_{pos} = \{k \leq \max(h) : p_k^h > 0 \text{ and } p_k^{h|ws} > 0\}. \quad (5)$$

That is, the sum is taken over all values observed at least once in both the full sample and the conditional sub-sample.

Rule-of-thumb criteria suggest that a $PSI < 0.1$ indicates small differences, whereas a $PSI > 0.25$ indicates large differences. Instead of relying on fixed values to assess the statistical significance of computed PSI values, Yurdakul and Naranjo (2021) introduced a test based on a normal approximation. Let z_α be the α -th quantile of a standard normal distribution, then the test in our case reads:

$$PSI(ws) > (1/N + 1/M)(B - 1) + z_{0.90}(1/N + 1/M)\sqrt{2(B - 1)}, \quad (6)$$

where N is the size of the marginal sample, M is the size of the conditional sample and B is the number of bins, which, according to Eq. (4), we have defined as the cardinality of h_{pos} . A quantile of 0.9 is used here to discriminate statistically significant differences. For consistency, the goodness of fit of the theoretical distributions (either Polylog or Geometric) to the $h|ws$ samples is also evaluated using the PSI (90-th percentile). In this case, N is set equal to M in Eq. (6).

3.2.2. Chi-squared permutation test

As a more classical test of the dependence between ws and h , to test the null hypothesis $H_0: h$ is independent of ws , we employed a Pearson χ^2 statistic with a permutation-based sampling distribution, thus avoiding the need for large-sample or sparse-cell approximations. After constructing the contingency table of the two variables, we calculated the usual χ^2 statistic of the sample denoted by χ_{obs}^2 . To eliminate any association, we permuted the rainfall depths 5,000 times among the records while keeping the marginal totals fixed, and then computed the resulting χ^2 statistic denoted by χ_{perm}^2 once for each permutation. A bias-corrected p-value was estimated as $\frac{1 + \#\{\chi_{perm}^2 \geq \chi_{obs}^2\}}{5000 + 1}$.

3.3. Logarithmic function fitting

Since the MLE estimation of the Polylog distribution parameters is related to the AM and the GM values, we have analysed how variations in these quantities can be described by ws . First, for each site, normalized conditional AM and GM values, denoted in what follows as y , are computed by dividing $AM|ws$ and $GM|ws$ by their respective full sample AM and GM (reported in Table 1). Successively, the following (discretized) logarithmic growth formula is considered for y ,

$$y = \ln(mws^n), \text{ for } ws \in N, n, \text{ and } m \in R, \quad (7)$$

A single fitting of eq. (7) is performed separately for each site using both AM and GM values, under the assumption that the two normalized datasets $AM|ws$ and $GM|ws$ scale with ws according to the same relationship given by eq. (7). The fit of the two parameters for each site was performed using the *NonlinearModelFit* function in Wolfram Mathematica (Wolfram Research, 2008). This approach enables us to estimate the two means for each given ws and, consequently, to obtain the Polylog (or Geometric) distribution parameters that characterize the distribution of

$h|ws$.

3.4. Exploring the spatial variability of the dependence between h and ws

Differences in the $h|ws$ dependence behaviour are expected across sites. Some of these differences may be attributed to climatic or geographic factors. For this reason, a set of geographic and meteorological variables were used to characterize each site (Table 1), with the aim of interpreting the spatial patterns observed in the results. These characteristics include the standard geographic variables: latitude (lat), longitude (lon), and elevation (z), derived from the stations' metadata; the distance from the shoreline (d), computed with the *Dist2Line* function in the *geosphere* R package (Hijmans, 2026), as well as the rainfall seasonality index (SI), and the Convective Available Potential Energy (CAPE).

The SI is computed according to Walsh and Lawler (1981) as:

$$SI = \frac{1}{R} \sum_{i=1}^{12} \left| x_i - \frac{R}{12} \right| \quad (8)$$

where x_i is the mean precipitation of month i , computed over the entire observation period, and R denotes the mean annual precipitation calculated over the same dataset. The SI theoretically ranges from 0, corresponding to a perfectly uniform distribution of rainfall throughout the year (i.e., identical monthly means), to 1.83, representing the limiting case in which the entire annual precipitation is concentrated in a single month.

The CAPE (J/kg) data are derived from the ERA5 dataset (Hersbach et al., 2020) as the average in the 1970–2016 period of the daily maximum values. CAPE is a common measure of atmospheric instability, and it is here used to quantify the average likelihood to have thunderstorm precipitation at each site.

A penalized multinomial logistic regression method (Friedman et al., 2010) with bootstrap resampling is employed to assess the relevance of these predictors in explaining inter-site differences in $h|ws$ dependence. It is worth noting that, given the limited number of sites, this analysis is intended to provide only some qualitative insight into the observed spatial results. The selection frequency, defined as the proportion of bootstrap samples in which a predictor has a nonzero coefficient, serves as an indicator of variable importance (Meinshausen and Bühlmann, 2010).

4. Results

4.1. Analysis of the $h|ws$ dependence on empirical data

Daily rainfall records for each station are divided in groups based on ws , to firstly analyse the $h|ws$ dependence directly on the empirical data. The plot in Fig. 2 shows the fraction of h values in each ws class, highlighting how the classes $ws = 2$ and 3 are the most numerous in the majority of the sites. This also aligns with $\langle ws \rangle$ in Table 1, as most of the values are between 2 and 3. Following this separation, the χ^2 and PSI tests were performed on the empirical h datasets and the results are summarized in Table 2. While the χ^2 test assesses the dependence between h and ws , the PSI evaluates the similarity between the frequency distribution of marginal h and each conditional $h|ws$, resulting in multiple tests performed for each site. The data in Table 2 show the percentage of tests in which the PSI indicates dissimilarity at the 90-th percentile significance level. Overall, the two tests yield consistent results, detecting statistically significant dependence across most of the same sites. To summarize, the 34 sites are divided into three groups based on the combined results of the two tests: 1) “No” dependence, when less than 10% of the PSI tests indicate dissimilarity and there is no statistically significant relationship according to the χ^2 ; 2) “Strong” dependence, when more than 30% of the PSI tests indicate dissimilarity and the χ^2 test shows a statistically significant relationship; and 3)

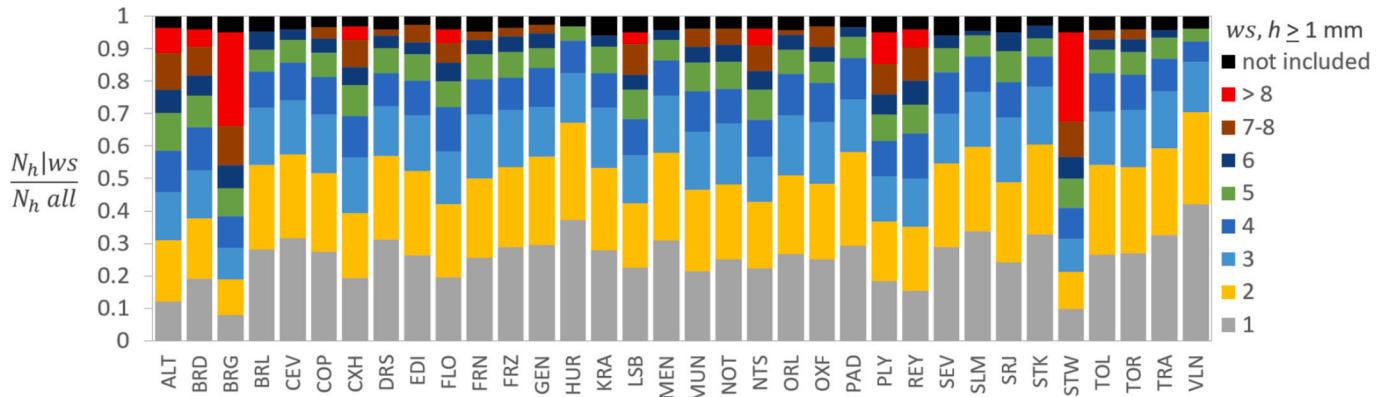


Fig. 2. Numerosity of each $h|ws$ sample relative to the full dataset for each test site. Values excluded due to the trimming procedure (see Section 2.2) are reported as “not included”.

Table 2

Summary of the outcomes of the PSI and χ^2 test of independence. The PSI (%) column reports the percentage of cases in which the frequency distribution of h and $h|ws$ are significantly different at the 90-th percentile. The χ^2 column reports the outcome of the χ^2 test of independence at the 0.01 level of significance. An asterisk indicates that the null hypothesis of independence is not rejected. The “dependence group” column provides the classification of the dependence at each site according to the two tests (see main text for the definition of each class), the n and $\log(m)$ columns reports the parameters of the logarithmic growth functions, whereas the SEE AM/GM columns report the Standard Error of the Estimates for the two means in each site.

site	PSI(%)	χ^2	dependencegroup	n	$\log(m)$	SEE AM(mm)	SEE GM(mm)
ALT	8.3	*	No	0.1321	0.8192	1.17	0.67
BRD	45.5		Strong	0.3409	0.6360	0.70	0.49
BRG	42.9		Strong	0.3193	0.4856	0.82	1.03
BRL	0	*	No	0.0844	0.9238	0.32	0.14
CEV	33.3		Strong	0.3916	0.7162	0.80	0.48
COP	14.3	*	Weak	0.0656	0.9465	0.37	0.13
CXH	30		Strong	0.2329	0.7554	0.31	0.22
DRS	14.3		Weak	0.0697	0.9377	0.36	0.16
EDI	9.5		Weak	0.1570	0.8605	0.35	0.15
FLO	20		Weak	0.2119	0.7843	0.62	0.29
FRN	0	*	No	0.0785	0.9243	0.28	0.12
FRZ	0		Weak	0.0964	0.9218	0.58	0.51
GEN	42.9		Strong	0.3720	0.7168	1.26	0.77
HUR	20		Weak	0.2136	0.8694	0.48	0.28
KRA	14.3		Weak	0.1465	0.8811	0.52	0.31
LSB	20		Weak	0.2523	0.7404	0.62	0.32
MEN	16.7		Weak	0.2778	0.8031	0.78	0.21
MUN	0	*	No	0.0113	0.9728	0.99	0.44
NOT	0	*	No	0.0673	0.9355	0.28	0.17
NTS	27.3		Weak	0.2168	0.7731	0.44	0.31
ORL	0	*	No	0.0635	0.9407	0.27	0.18
OXF	8.75	*	No	0.1554	0.8668	0.30	0.23
PAD	0	*	No	0.1119	0.9086	0.69	0.40
PLY	25	*	Weak	0.1663	0.8007	0.42	0.22
REY	36.4		Strong	0.1295	0.8371	0.35	0.24
SEV	32.6		Strong	0.4214	0.6673	0.89	0.56
SLM	0	*	No	0.1179	0.9138	0.22	0.13
SRJ	0		Weak	0.1602	0.8615	0.59	0.30
STK	0	*	No	0.0800	0.9445	0.26	0.20
STW	47.6		Strong	0.2350	0.6255	0.42	0.45
TOL	0	*	No	0.0592	0.9448	0.42	0.18
TOR	14.3		Weak	0.3452	0.7277	0.67	0.29
TRA	0	*	No	0.1104	0.9309	0.32	0.12
VLN	60		Strong	0.6314	0.6808	1.93	0.44

“Weak” dependence, all other cases. Overall, the sites are fairly evenly distributed across the three groups, with 12 sites classified as “No” dependence, 13 as “Weak” dependence, and 9 as “Strong” dependence. The PSI percentage thresholds used for the classification were selected following a sensitivity analysis reported in the [Supplementary Materials \(Fig. S1\)](#), and it represents the combination that maximises the stability of the distribution of the stations in the three classes.

Since the PSI test is performed independently for each $h|ws$, we can analyse these results in greater detail as a function of ws . The bar plots in [Fig. 3](#) display the results of the PSI test separately for the three

dependence groups. From these plots, it can be observed that significant differences between h and $h|ws$ (PSI > 90-th percentile) occur more frequently in the “Weak” and “Strong” dependence groups compared to the “No” dependence group. Additionally, statistically significant differences between h and $h|ws$ are more often observed for $ws = 1$ and $ws > 4$.

4.2. Fitting of conditional $h|ws$ with the Polylog distribution

As described in [Section 3.1](#), the Polylog distribution is assumed to be

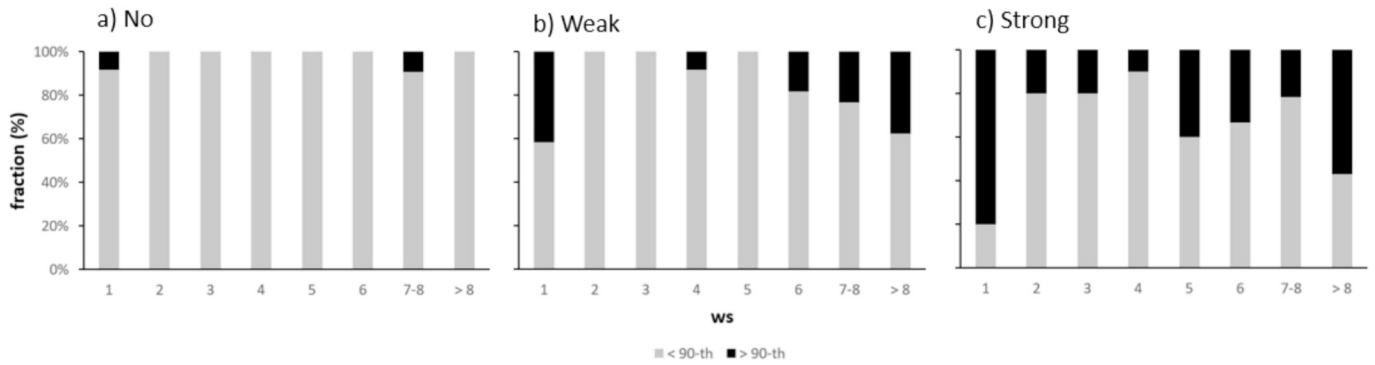


Fig. 3. Analysis of the similarity between the full dataset h and the conditional subsets $h|ws$ based on the PSI test. Cases when $PSI > 90$ -th percentile are considered statistically different.

a suitable model for fitting both the marginal h and the conditional $h|ws$. The goodness of fit of the MLE-based estimations is assessed using the PSI test. At the 10% significance level (90% confidence), none of the 34 marginal h fittings, nor any of the 281 conditional $h|ws$ fittings, exhibit statistically significant deviations from the observed samples.

The plots in Fig. 4 show the fitted distributions for three representative examples, each corresponding to one of the dependence groups defined in Section 4.1 (Table 1). In addition to the good visual fitting of the Polylog distribution to the empirical data, these plots also highlight how all the conditional $h|ws$ fittings ($ws = 1$ –6) closely resemble the marginal h (All data) in BRL (Fig. 4a; “No” dependence), whereas both the fitted and empirical distributions are more dispersed for the other two sites (Fig. 4b and 4c, “Weak” and “Strong” dependence, respectively).

Fig. 5 presents the range of variability of the parameters estimated for the Polylog distribution across all 34 sites as a function of ws . In general, it is possible to observe how, on average, the parameters estimated for the marginal sample (All) closely resemble those obtained for $ws = 2$ or 3, as these latter are often the most numerous sub-samples (see Fig. 2). The parameter θ increases from $ws = 1$ to $ws = 3$ and then decreases approximately linearly, whereas the parameter s tends to decrease with ws , approaching values close to 0 (i.e., as for the Geometric distribution) for $ws > 7$.

Given the observed tendency of the Polylog distribution parameters to approach those of a Geometric distribution as ws increases, the latter distribution was also fitted and tested on both the marginal h and conditional $h|ws$ datasets. The plots in Fig. 6 summarize the results of the PSI and LLH tests for the Geometric distribution. As shown in Fig. 6a, the Geometric distribution is generally unsuitable for the marginal datasets (“All”) according to the PSI at the 90-th percentile. However, the number of cases where the Geometric distribution is a suitable option increases with ws and reaches 100% for $ws > 5$. Similarly, the results of

the LLH test (Fig. 6b) show that the Geometric distribution becomes a suitable alternative to the Polylog distribution in many cases for $ws \geq 5$.

Fig. 7 shows the variability of the parameter of the Geometric distribution as a function of ws . As for the Polylog distribution, the parameter values for the marginal samples (“All”) closely resemble those for $ws = 2$ and 3. However, an overall increasing trend is observed here, which is opposite to that of the corresponding parameter in the Polylog distribution (Fig. 6a). This behaviour is likely because the single parameter in the Geometric distribution must compensate for the non-zero s values present at small ws or in the marginal samples (“All” on the x-axis).

4.3. Modelling of the $h|ws$ dependence with the logarithmic function

The normalized moments introduced in Section 3.3 (and denoted as y) are fitted for each site with Eq. (7) by applying the latter to both AM and GM. This implies that the values of $AM|ws$ and $GM|ws$ can be obtained by multiplying the same value of y in Eq. (7) for the corresponding mean value (either AM or GM) of the marginal dataset h .

Fig. 8 plots three examples of fitting, each representing one of the groups defined in Section 4.1. Overall, these plots show a good capability of the proposed logarithmic model to capture the dependence of y on ws in all three cases. Note that for each ws two points are shown: one for AM and one for GM. Additionally, Fig. 8 shows that the shape of the logarithmic function is flatter in the “No” dependence site (Fig. 8a) compared to the other two, and that its values keep increasing within the range of observed ws values in the case of the “Strong” dependence site (Fig. 8c).

Beside the qualitative good agreement between the empirical normalized means and the logarithmic functions, a quantitative evaluation of the goodness of the model was performed by directly comparing the means ($AM|ws$ and $GM|ws$) modelled according to the logarithmic

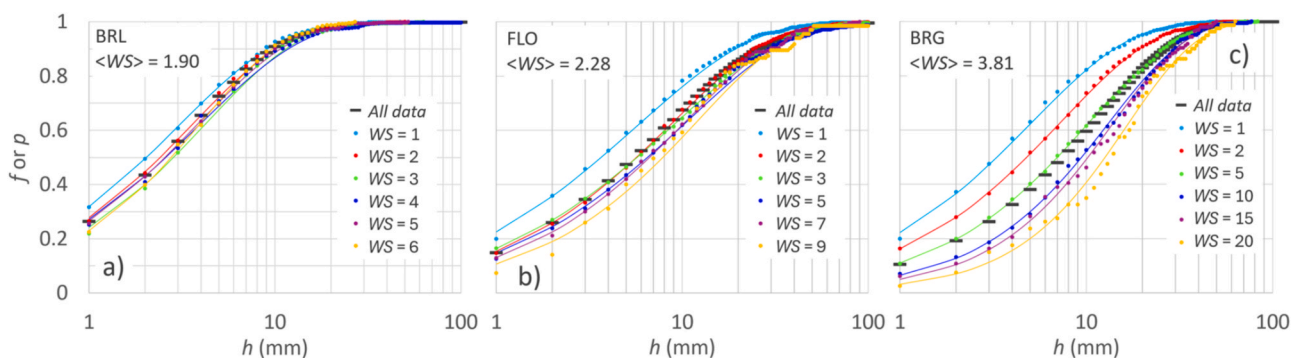


Fig. 4. Examples of fitting of the Polylog distribution for three study sites: (a) BRL (“No” dependence), (b) FLO (“Weak” dependence), and (c) BRG (“Strong” dependence). The values in parentheses $\langle WS \rangle$ are the average ws value for each site.

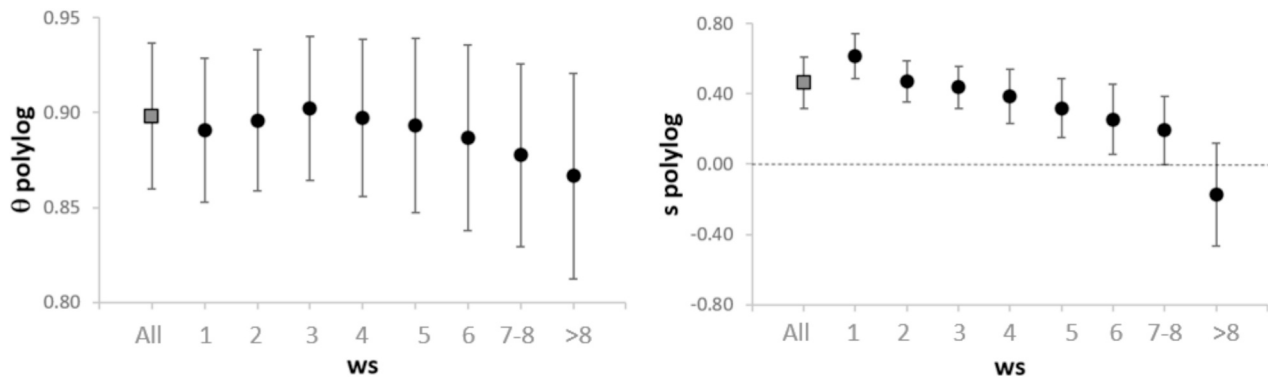


Fig. 5. Variability of the Polylog distribution parameters – θ in panel (a) and s in panel (b) – as a function of ws for the 34 sites. Points represent the average values while the whiskers indicate ± 1 standard deviation. The values labeled “All” correspond to the marginal h sample, whereas the remaining data correspond to the conditional sub-samples $h|ws$.

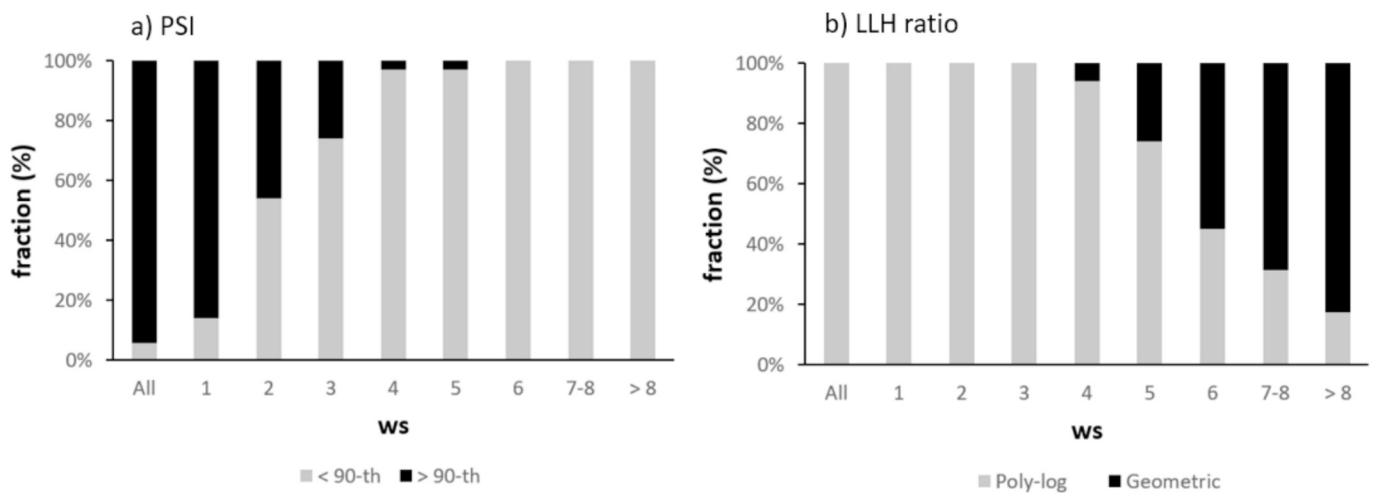


Fig. 6. Summary of the results of the PSI (a) and LLH ratio (b) tests for the Geometric distribution. In panel a), the black bars represent the percentage of cases where the Geometric distribution is not a good fit, whereas in panel b) the black bars represent the percentage of cases where the Geometric distribution should be preferred over the Polylog distribution. The column “All” refers to the marginal h samples, whereas the other columns correspond to each $h|ws$ conditional sub-samples.

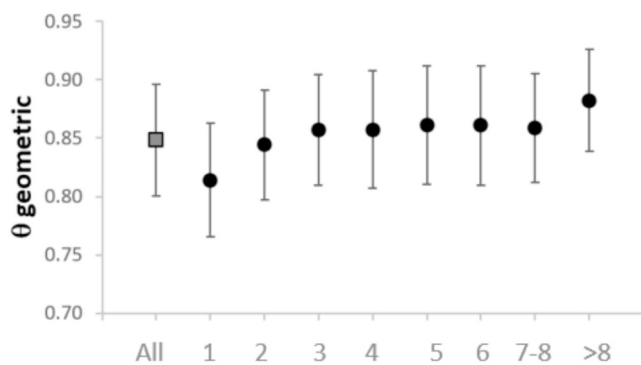


Fig. 7. Variability of the Geometric distribution parameter with ws for the 34 sites. Points represent the average values while the whiskers depict ± 1 standard deviation. The values labeled “All” correspond to the marginal h samples, whereas the other data correspond to the conditional sub-samples $h|ws$.

function with the empirical values. The results, shown in Fig. 9, demonstrate general consistency. On average, standard error of the estimates (SEE) values of 0.62 and 0.42 mm are obtained for AM and GM, respectively, with negligible systematic biases (mean error, ME, of 0.07 and -0.09 mm, respectively). The relative error (RE, SEE/mean) values are approximately 7.6% for AM and 8.4% for GM. SEE values for each

site are reported in Table 2, showing good performances across the board, with SEE lower than 1 mm in the majority of the cases (64 out of 68 values).

The values obtained for the two parameters of the logarithmic function have a clear distinct range when separated in the three dependence groups (see Table 2), as further exemplified in Fig. 10. The parameter m is clearly lower for the “Strong” dependence class compared to the other two groups (Fig. 10a), whereas n is much higher in the “Strong” dependence group, on average. In terms of inter-site variability, the n parameter is more variable for the “Strong” dependence group, whereas the parameter m is more variable for the “Weak” group. Since the n parameter represents the growth rate, its greater values for the “Strong” and “Weak” groups compared to the “No” dependence group is expected, as suggested by the flat shape of the logarithmic function for the sites in this latter group (see Fig. 8a as an example). Since the obtained results are a function of the threshold used to define a wet day (1 mm), a sensitivity analysis of the effect of this choice on the fitting of the logarithmic function is reported in the Supplementary Materials (Fig. S2), showing how most of the above considerations remain valid also for a smaller or a larger threshold.

4.4. Spatial analysis of the dependence

The proposed model can reproduce the $h|ws$ dependence in each given site based on four quantities: the two AM and GM values computed

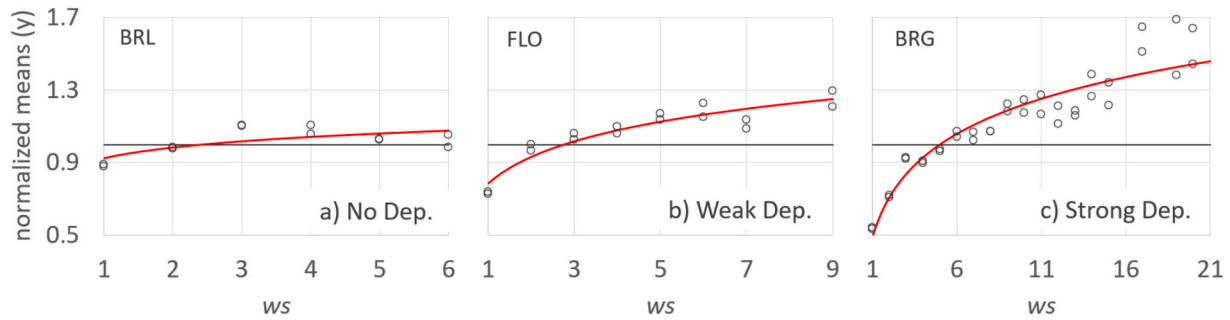


Fig. 8. Examples of the logarithmic function fitted to the normalized means (y) for three sites, each representing a different dependence group: a) “No” dependence, BRL; b) “Weak” dependence, FLO, and c) “Strong” dependence, BRG.

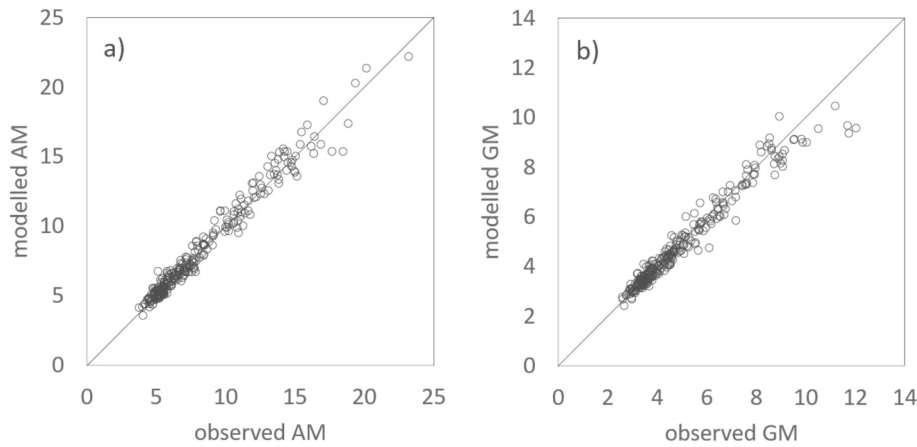


Fig. 9. Relationship between empirical and modelled (with the logarithmic functions) mean values: a) arithmetic mean, AM, and b) geometric mean, GM.

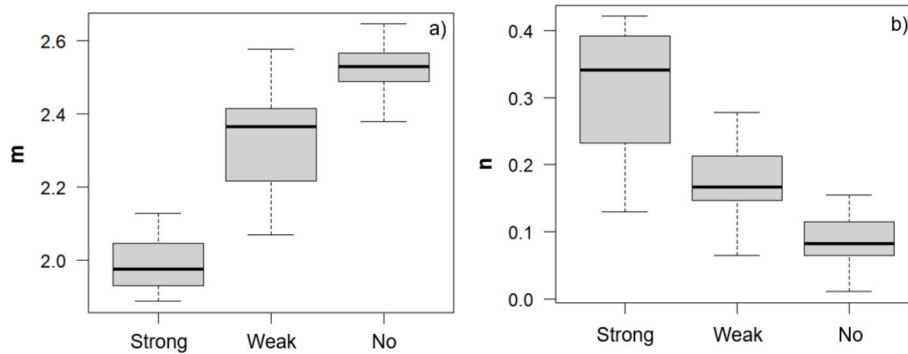


Fig. 10. Boxplots of the variability of the logarithmic function parameters (Eq. (2)) when separated in the three dependence groups (see Table 2).

on the marginal h sample (needed to rescale y) and the two parameters of the logarithmic growth function. In the studied datasets, the two mean values show some clear joint spatial patterns, depicted in Fig. 11. In this map, the ratio GM/AM is reported, showing a clear south-north increasing trend, with values of the ratio around 1/2 in the south and up to 2/3 in the north.

Given the strong relationship between the two parameters of the logarithmic function and the three dependence groups, the joint behaviour of these parameters can be concisely investigated by analysing the spatial distribution of the dependence groups. The map in Fig. 12 shows the spatial distribution of the sites according to the above-mentioned classification of the dependence. At first glance, no clear spatial patterns can be discerned from this map, beside a slightly higher concentration of “Strong” dependence sites in the west compared to the east.

The spatial distribution of the class labels (No, Weak, Strong) in Fig. 12, is formally analysed with respect to the meteo-climatic and geographical variables using the method described in Section 3.4. Overall, using this approach, the quantities most often associated with the dependence classes are distance from the shoreline (d), longitude and SI, with a selection frequency of 0.939, 0.835 and 0.796, respectively. However, it is worth highlighting how the accuracy of this model is rather limited (0.53, 95% CI: 0.35–0.70, under leave-one-out cross-validation), particularly for the “Weak” dependence class.

A similar approach, based on a penalized linear regression, was employed to explore which of the aforementioned variables are most associated with the continuous ratio GM/AM. In this case, the most frequently selected predictors were CAPE, latitude, and SI, with selection frequencies of 0.999, 0.908, and 0.843, respectively. Using only these three predictors, the reduced model achieved a leave-one-out

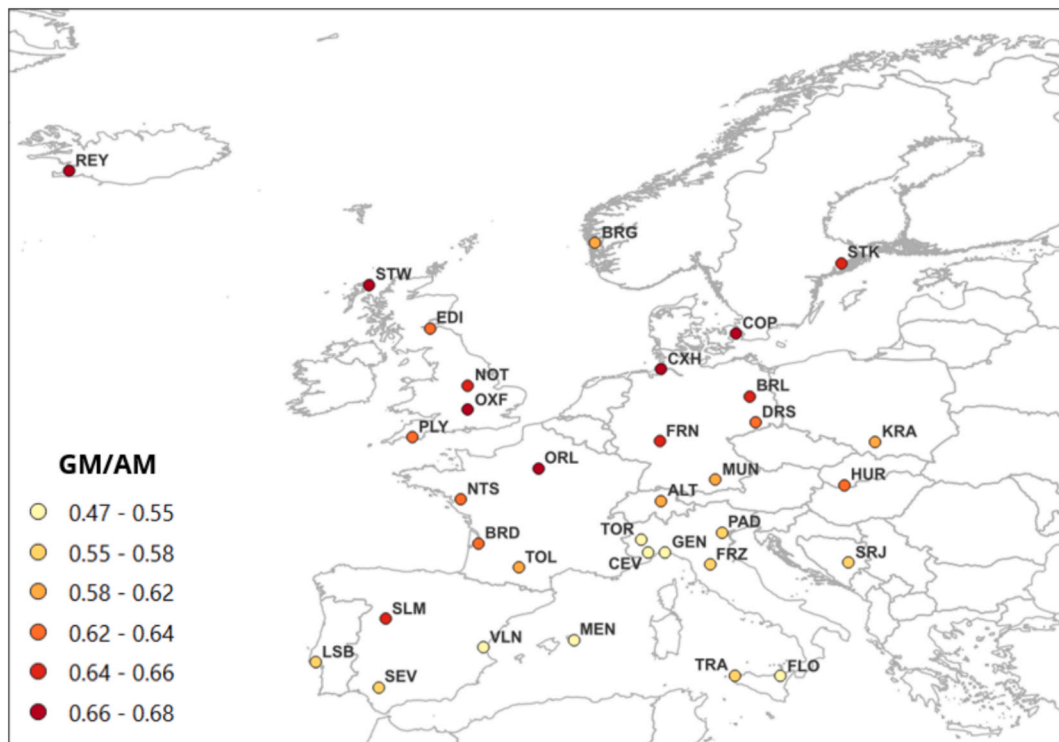


Fig. 11. Spatial distribution of the ratio GM/AM for the 34 test sites. The two mean values are computed on the full h marginal dataset of each site.

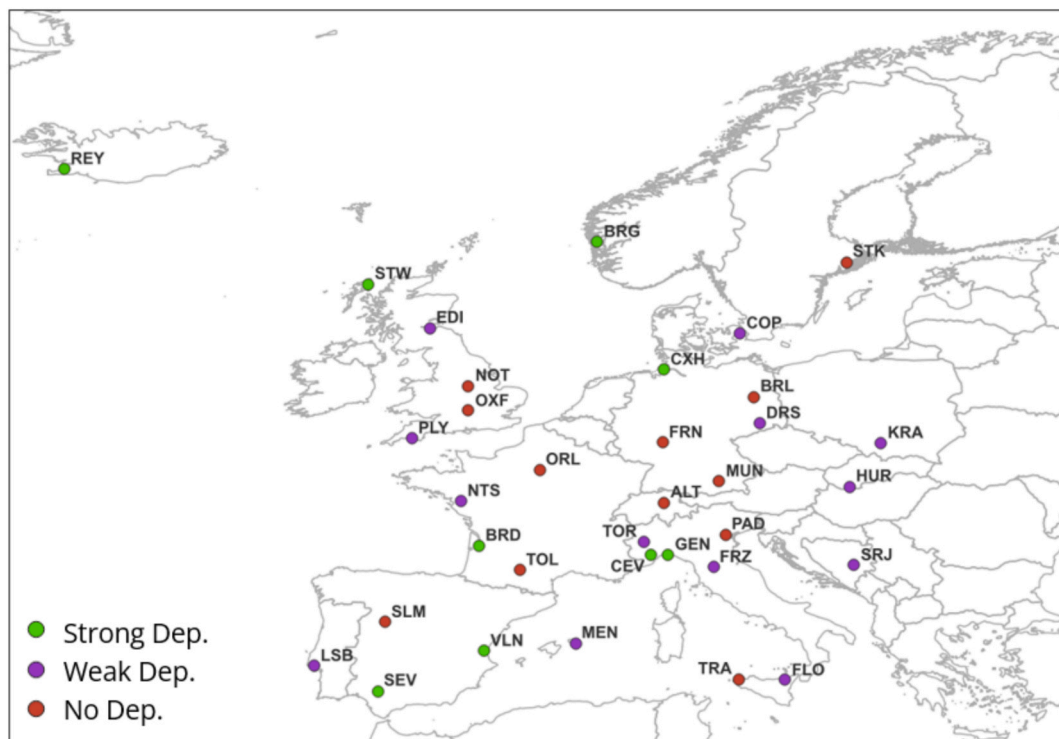


Fig. 12. Spatial distribution of the study sites according to the dependence grouping (see Table 2).

cross-validated RMSE of 0.038 and an R^2 of 0.482. Interestingly, while the SI emerges as a relevant predictor in both the classification and regression, suggesting a key role of precipitation climatology, the relative importance of the other covariates differs. To compare the predictive skill of the two reduced models, we computed a Normalized Skill Score (NSS) for each, defined in general as the relative gain in predictive

power compared to a naive baseline. For the classification, the NSS was defined as the improvement over the no-information rate (i.e., the proportion of the most frequent class), whereas for the regression, we simply considered the R^2 computed above, which can be interpreted as the relative gain in predictive performance compared to a naive model that predicts the mean of the target variable for all observations. Under

these definitions, the classification model achieved a NSS of 0.19, while the regression model reached a higher value of 0.48, indicating stronger relative performance in capturing the variability of the continuous outcome. These results should be interpreted as exploratory indications rather than confirmatory evidence, as they merely suggest that both dependence groupings and the GM/AM ratio may reflect underlying spatial and seasonal structures that warrant further investigation using a larger number of rain gauge sites.

5. Discussion

The analyses reported in this study highlights a clear and statistically significant dependence between daily rainfall depth (h) and wet spell duration (ws) across most of the analysed European sites. The combination of permutation-based χ^2 tests and the Population Stability Index (PSI) reveals that this dependence is not merely due to isolated cases, but it represents a structural feature of rainfall dynamics across many climates. Using the χ^2 permutation test, the null hypothesis of independence between daily h and ws was rejected at the majority of the study sites (20 out of 34). Albeit simple, this test suggests that further investigations into the dependence between these two variables is warranted to improve our understanding of rainfall's statistical properties. The additional analyses based on the PSI test provides further insights into the structure of this dependence: the observation that, compared to the intermediate durations, the conditional empirical frequency distributions $h|ws$ differ more from the marginal distribution at $ws = 1$ (isolated rainy days) and $ws > 4$ (long rainy periods) supports the idea that rainfall intensity is not uniformly distributed across spell lengths. This dependence seems to follow a non-monotonic pattern influenced by both atmospheric dynamics and statistical processes. Short wet spells often result from brief, scattered local storms, whereas long wet spells are associated with large-scale weather systems that bring long lasting rain over a wide area (Brunner and Dougherty, 2022). These results extend and complement the analysis of Chapman (1998), where only the isolated rainy days (class 0) were investigated separately, whereas all class 1 and class 2 days were pooled regardless of the duration of the wet spells. Both Chapman (1998) and Suhaila and Jemain (2009) reported reduced rainfall depth on isolated rainy days, a finding confirmed by our analysis. Given that short wet spells ($ws = 2-3$ days) typically consist of an equal mix of class 1 and class 2 days, while class 2 events are predominant in long-duration spells, it is possible that the differences among these three classes noted by Chapman (1998) may partially explain the dependence observed in our study. However, our results suggest a more structured dependence, as the relationship between h and ws keep increasing in some cases (e.g., site BRG) extending well beyond the expected influence of the initial and ending day ($ws > 15$ days).

An important implication of our findings is that marginal rainfall depth distributions (h) might often represent a superposition of several sub-populations ($h|ws$), each behaving differently. In particular, the similarity between the marginal and some conditional frequencies is also likely influenced by the relative size of the sub-samples (see Fig. 2). Indeed, although cases of $ws = 1$ are the most frequent, they span only a single day, whereas events with $ws > 4$ are less common but last longer. In the majority of the our sites, the sub-sample size of $h|ws$ for $ws = 2$ tends to be either the largest or the second largest, which helps explain the similarity between this conditional and the marginal frequency distributions.

A global-scale dependence between wet-day intensity and frequency was recently reported by McErlich et al. (2023) using ERA5 data. While our results confirm their findings, we also show how this dependence is not always significant across all study sites. This variability in dependence may explain why fitting h has proven challenging in some cases in the past, often requiring more sophisticated distributions for a good fitting (Agnese et al., 2022; Verma et al., 2011). By conditioning on ws , we are able to disentangle part of this hidden heterogeneity.

Overall, according to the PSI, the fitting of the Polylog distribution is notable, particularly in its ability to model both marginal and conditional datasets. Although the Geometric distribution rarely provides a good fit for the marginals, it becomes a suitable and simpler alternative for many conditional distributions, particularly for large ws values. It is important to highlight that this behaviour may be partly explained by the limited size of the $h|ws$ samples for $ws > 4$, which may reduce the robustness of the PSI test and compromise the reliability of the results. Indeed, in about 70% of the sites the h values for $ws > 4$ represents less than 20% of the total sample. However, the tendency of the parameters of the Polylog distribution to approach those of the Geometric ($s = 0$) for $ws > 4$ suggests a possible physical explanation to this behaviour and not only a relationship with sample size. More detailed analyses of the effects of sample size on the fittings may be needed to further investigate this aspect.

Although the Polylog distribution demonstrated a good capability in reproducing the observed frequency in all the considered conditional samples $h|ws$, its practical application may be limited as a ws -specific sampling and fitting approach would be required for its use, involving the estimation of a large number of parameters ($2 \times$ number of unique ws , in the case of the Polylog distribution). Additionally, this approach would preclude reproducing the behaviour for ws with limited sample size, for which a direct fitting is not feasible. The use of a logarithmic function overcomes this drawback, as the AM| ws and GM| ws values can be simulated from the marginal AM and GM using only two growth parameters, allowing the Polylog parameters to be estimated from these theoretically inferred mean values.

While the logarithmic growth function performs well in all the test sites, clear differences were observed in the obtained parameters. It is well known that different types of rain (frontal, convective, orographic, etc.) can exhibit different statistical characteristics. Dong et al. (2019) identified a relationship between rainfall intensity in China and both the precipitable water and the CAPE index, whereas McErlich et al. (2023) observed that global spatial patterns in the relationship between wet-day frequency and intensity are related to similarity in vertical velocity and convective available potential energy distributions. Our attempt at explaining the differences among the sites using a penalized regression method with selected features did not yield to conclusive evidence, besides a possible primary influence of rainfall seasonality and geographic location. Both quantities may highlight a relationship between the observed results and the climatic regimes across Europe, where climate in Europe generally exhibits both south-north and east-west patterns, as well as a clear influence of the distance from the sea. The limited explanatory value of this analysis may be partly due to the limited number of sites included and would require further investigation using a more extensive dataset or a gridded precipitation product. A clearer spatial pattern was instead observed for the ratio GM/AM, with a tendency toward lower values at the stations located in southern Europe. This ratio equals 1 when all the data of the sample are the same, and it decreases as variability increases. The observed pattern is consistent with climatological rainfall characteristics across Europe, where precipitation is generally more variable in the southern Mediterranean regions where most of the lowest GM/AM values are found.

It is well-known that even rain gauge-based rainfall observations may be affected by relatively large uncertainty (e.g., Schnepfer et al., 2023), which in turn, influence conclusions drawn from such observations. In this study, the main objective is to compare conditional distributions, $h|ws$, with the marginal distribution of h . Since both distributions are derived from the same dataset, with the conditional distributions representing subsets of the marginal one, the effects of measurement errors are expected to be evenly distributed and therefore minimal. Measurement errors may, however, influence the observed spatial differences among sites, as the magnitude and characteristics of such errors can vary from one site to another. Errors due to snow effects and low temperature are expected to affect the rain gauges in the north and north-east Europe more strongly, where cold climates without a dry

season are more common. Extending the analysis to a larger set of stations or a gridded dataset may help mitigate this issue in future studies and provide more robust evidence of spatial differences.

Seasonality is another source of variability in the statistical behaviour of rainfall. Our results indicate that the Polylog is able to satisfactorily reproduce the empirical frequency even when the entire yearly dataset is used. However, this does not preclude the possibility that seasonal sub-sampling can further disclose additional characteristics of the rainfall process that are hidden in the yearly datasets, particularly for sites with a strong seasonality in rainfall patterns (e.g., Mediterranean area). Indeed, [Sirangelo et al. \(2011; 2015\)](#) highlighted strong seasonality in rainfall occurrence and dry spells in southern Italy using a non-homogeneous Poisson process model, and [Baiaomonte et al. \(2024\)](#) showed significant differences in the fitting of both dry and wet spells between April–September and October–March. Different behaviour between humid and arid environment were also observed in [Ratan and Venugopal \(2013\)](#). However, despite the established role of seasonality, issues related to limited sample sizes would need to be addressed for such analyses, which is why specific analyses on seasonality were not considered in this study. Finally, the results of this study are based on the assumption that the amount of rain in a day is affected only by the duration of the wet spell and not by the position of the rainfall within the sequence chain. A natural extension of this analysis would be to investigate the effect of rainy-day position within the wet spell, for instance, by introducing the simple three class classification of [Chapman \(1998\)](#) and [Suhaila and Jemain \(2009\)](#) into our dependence approach. This would enable a more detailed analysis of the temporal behaviour of rainfall with the rainy event.

6. Summary and conclusions

The results of this study highlight the existence of a dependence between daily rainfall depth (h) and the duration of the rainfall event (ws). Overall, greater daily rainfall depths are associated with longer durations, with both the arithmetic mean (AM) and geometric mean (GM) exhibiting a logarithmic growth pattern as ws increases. A two-parameter Polylog distribution appears suitable for capturing the statistical behaviour of the $h|ws$ samples in all 34 test sites, with a common tendency toward the Geometric distribution for longer wet spells ($ws > 4$). Although the proposed model seems able to reproduce the $h|ws$ dependence at each site with a relatively limited number of parameters (4, AM and GM plus two parameters of the logarithmic function), significant variations in the strength of this dependence were observed among sites. Further analysis would be needed to identify the underlying factors controlling this dependence and the relationship with climate regimes.

CRedit authorship contribution statement

Carmelo Agnese: Writing – review & editing, Validation, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Giorgio Baiaomonte:** Writing – review & editing, Validation, Investigation, Formal analysis, Data curation, Conceptualization. **Carmelo Cammalleri:** Writing – review & editing, Writing – original draft, Validation, Methodology, Investigation, Formal analysis, Conceptualization. **Elvira Di Nardo:** Writing – review & editing, Validation, Investigation, Formal analysis, Conceptualization. **Stefano Ferraris:** Writing – review & editing, Data curation, Conceptualization. **Tommaso Martini:** Writing – review & editing, Writing – original draft, Validation, Methodology, Investigation, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

We are very thankful to the rainfall data providers, who made it possible to perform this analysis. The authors would like to acknowledge the NODES project, funded under MUR–M4C2 1.5 of the PNRR with resources from the European Union – NextGenerationEU (Grant agreement no ECS00000036), as well as the MUR PRIN project SUNSET 202295PFKP-003 and Fondazione CRT fundings 2023 and 2025. E. Di Nardo is member of the INDAM – GNAMPA group.

Appendix A. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.jhydrol.2026.135665>.

Data availability

Data used in this analysis and outputs can be shared upon request to the corresponding author.

References

- Agnese, C., Baiaomonte, G., Cammalleri, C., 2014. Modelling the occurrence of rainy days under a typical Mediterranean climate. *Adv. Water Resour.* 64, 62–76. <https://doi.org/10.1016/j.advwatres.2013.12.005>.
- Agnese, C., Baiaomonte, G., Di Nardo, E., Ferraris, S., Martini, T., 2022. Modelling the frequency of interarrival times and rainfall depths with the Poisson Hurwitz–Lerch zeta distribution. *Fractal Fract.* 6, 509. <https://doi.org/10.3390/fractalfract6090509>.
- André, J., D'Andrea, F., Drobinski, P., Muller, C., 2024. Regimes of precipitation change over Europe and the Mediterranean. *JGR Atmos.* 129, e2023JD040413. <https://doi.org/10.1029/2023JD040413>.
- Baiaomonte, G., Mercalli, L., Berro, D.C., Agnese, C., Ferraris, S., 2019. Modelling the frequency distribution of inter-arrival times from daily precipitation time-series in North-West Italy. *Hydrol. Res.* 50, 339–357. <https://doi.org/10.2166/nh.2018.042>.
- Baiaomonte, G., Agnese, C., Cammalleri, C., Di Nardo, E., Ferraris, S., Martini, T., 2024. Applying different methods to model dry and wet spells at daily scale in a large range of rainfall regimes across Europe. *Adv. Stat. Clim. Meteorol. Oceanogr.* 10, 51–67. <https://doi.org/10.5194/ascmo-10-51-2024>.
- Balistrocchi, M., Bacchi, B., 2011. Modelling the statistical dependence of rainfall event variables through copula functions. *Hydrol. Earth Syst. Sci.* 15, 1959–1977. <https://doi.org/10.5194/hess-15-1959-2011>.
- Brunner, M.I., Dougherty, E.M., 2022. Varying importance of storm types and antecedent conditions for local and regional floods. *Water Resour. Res.* 58, e2022WR033249. <https://doi.org/10.1029/2022WR033249>.
- Buishand, T.A., 1978. Some remarks on the use of daily rainfall models. *J. Hydrol.* 36, 295–308. [https://doi.org/10.1016/0022-1694\(78\)90150-6](https://doi.org/10.1016/0022-1694(78)90150-6).
- Cavus, Y., Stahl, K., Aksoy, H., 2023. Drought intensity–duration–frequency curves based on deficit in precipitation and streamflow for water resources management. *Hydrol. Earth Syst. Sci.* 27, 3427–3445. <https://doi.org/10.5194/hess-27-3427-2023>.
- Chapman, T., 1998. Stochastic modelling of daily rainfall: the impact of adjoining wet days on the distribution of rainfall amounts. *Environ. Model. Software* 13, 317–324. [https://doi.org/10.1016/S1364-8152\(98\)00036-X](https://doi.org/10.1016/S1364-8152(98)00036-X).
- De Groen, M.M., Savenije, H.H.G., 2006. A monthly interception equation based on the statistical characteristics of daily rainfall. *Water Resour. Res.* 42, 2006WR005013. <https://doi.org/10.1029/2006WR005013>.
- De Michele, C., Salvadori, G., Canossi, M., Petaccia, A., Rosso, R., 2005. Bivariate statistical approach to check adequacy of dam spillway. *J. Hydrol. Eng.* 10, 50–57. [https://doi.org/10.1061/\(ASCE\)1084-0699\(2005\)10:1\(50\)](https://doi.org/10.1061/(ASCE)1084-0699(2005)10:1(50)).
- Dong, W., Lin, Y., Wright, J.S., Xie, Y., Yin, X., Guo, J., 2019. Precipitable water and CAPE dependence of rainfall intensities in China. *Clim. Dyn.* 52, 3357–3368. <https://doi.org/10.1007/s00382-018-4327-8>.
- Foufoula-Georgiou, E., Lettenmaier, D.P., 1986. Continuous-time versus discrete-time point process models for rainfall occurrence series. *Water Resour. Res.* 22, 531–542. <https://doi.org/10.1029/WR022i004p00531>.
- Friedman, J., Hastie, T., Tibshirani, R., 2010. Regularization paths for generalized linear models via coordinate descent. *J. Stat. Softw.* 33. <https://doi.org/10.18637/jss.v033.i01>.
- Gupta, P.L., Gupta, R.C., Ong, S.-H., Srivastava, H.M., 2008. A class of Hurwitz–Lerch Zeta distributions and their applications in reliability. *Appl. Math. Comput.* 196, 521–531. <https://doi.org/10.1016/j.amc.2007.06.012>.
- Hasan, M.M., Dunn, P.K., 2010. A simple Poisson–gamma model for modelling rainfall occurrence and amount simultaneously. *Agric. For. Meteorol.* 150, 1319–1330. <https://doi.org/10.1016/j.agrformet.2010.06.002>.
- Hersbach, H., Bell, B., Berrisford, P., et al., 2020. The ERA5 global reanalysis. *Q. J. R. Meteorol. Soc.* 146, 1999–2049. <https://doi.org/10.1002/qj.3803>.
- Hijmans, R., 2026. Geosphere: Spherical Trigonometry. R package version 1.6-7, <https://github.com/rspatial/geosphere>.

- Hosseini, A., Hosseini, R., Zare-Mehrjerdi, Y., Abooi, M.H., 2016. Capturing the time-dependence in the precipitation process for weather risk assessment. *Stoch. Environ. Res. Risk Assess.* 31, 609–627. <https://doi.org/10.1007/s00477-016-1285-8>.
- Jehanzaib, M., Ajmal, M., Achite, M., Kim, T.-W., 2022. Comprehensive review: advancements in rainfall-runoff modelling for flood mitigation. *Climate* 10, 147. <https://doi.org/10.3390/cli10100147>.
- Klein Tank, A.M.G., Wijngaard, J.B., Können, G.P., et al., 2002. Daily dataset of 20th-century surface air temperature and precipitation series for the European climate assessment. *Int. J. Climatol.* 22, 1441–1453. <https://doi.org/10.1002/joc.773>.
- Lettenmaier, D., 2024. Role of Rainfall in hydrological modeling and water resource planning. *J. Geogr. Nat. Disasters* 14, 319. <https://doi.org/10.35841/2167-0587.24.14.319>.
- Manfreda, S., Rodríguez-Iturbe, I., 2006. On the spatial and temporal sampling of soil moisture fields. *Water Resour. Res.* 42, 2005WR004548. <https://doi.org/10.1029/2005WR004548>.
- McErlich, C., McDonald, A., Schuddeboom, A., Vishwanathan, G., Renwick, J., Rana, S., 2023. Positive correlation between wet-day frequency and intensity linked to universal precipitation drivers. *Nat. Geosci.* 16, 410–415. <https://doi.org/10.1038/s41561-023-01177-4>.
- Meinshausen, N., Bühlmann, P., 2010. Stability selection. *J. R. Stat. Soc. Ser. B Stat. Methodol.* 72, 417–473. <https://doi.org/10.1111/j.1467-9868.2010.00740.x>.
- Miniussi, A., Marani, M., 2020. Estimation of daily rainfall extremes through the metastatistical extreme value distribution: uncertainty minimization and implications for trend detection. *Water Resour. Res.* 56, e2019WR026535. <https://doi.org/10.1029/2019WR026535>.
- Molini, A., Katul, G.G., Porporato, A., 2009. Revisiting rainfall clustering and intermittency across different climatic regimes. *Water Resour. Res.* 45, 2008WR007352. <https://doi.org/10.1029/2008WR007352>.
- Petroselli, A., Grimaldi, S., 2025. Evaluating rain gauge precipitation measurements using a giant rain gauge as a benchmark. *Theor. Appl. Climatol.* 156, 324. <https://doi.org/10.1007/s00704-025-05540-w>.
- Porporato, A., Feng, X., Manzoni, S., Mau, Y., Parolari, A.J., Vico, G., 2015. Ecohydrological modeling in agroecosystems: examples and challenges. *Water Resour. Res.* 51, 5081–5099. <https://doi.org/10.1002/2015WR017289>.
- Porporato, A., Yin, J., 2022. In: *Ecohydrology: Dynamics of Life and Water in the Critical Zone*. Cambridge University Press. <https://doi.org/10.1017/9781108886321>.
- Ratan, R., Venugopal, V., 2013. Wet and dry spell characteristics of global tropical rainfall. *Water Resour. Res.* 49, 3830–3841. <https://doi.org/10.1002/wrcr.20275>.
- Rodríguez-Iturbe, I., Cox, D.R., Isham, V., 1987. Some models for rainfall based on stochastic point processes. *Proc. R. Soc. Lond. Ser. A* 410, 269–288. <https://doi.org/10.1098/rspa.1987.0039>.
- Schleiss, M., Smith, J.A., 2016. Two simple metrics for quantifying rainfall intermittency: the burstiness and memory of interamount times. *J. Hydrometeorol.* 17, 421–436. <https://doi.org/10.1175/JHM-D-15-0078.1>.
- Schmitt, F., Vannitsem, S., Barbosa, A., 1998. Modeling of rainfall time series using two-state renewal processes and multifractals. *J. Geophys. Res.* 103, 23181–23193. <https://doi.org/10.1029/98JD02071>.
- Schnepper, T., Groh, J., Gerke, H.H., Reichert, B., Pütz, T., 2023. Evaluation of precipitation measurement methods using data from a precision lysimeter network. *Hydrol. Earth Syst. Sci.* 27, 3265–3292. <https://doi.org/10.5194/hess-27-3265-2023>.
- Serinaldi, F., 2009. A multisite daily rainfall generator driven by bivariate copula-based mixed distributions. *J. Geophys. Res.* 114, 2008JD011258. <https://doi.org/10.1029/2008JD011258>.
- Sirangelo, B., Ferrari, E., De Luca, D.L., 2011. Occurrence analysis of daily rainfalls through non-homogeneous Poissonian processes. *Nat. Haz. Earth Syst. Sci.* 11, 1657–1668. <https://doi.org/10.5194/nhess-11-1657-2011>.
- Sirangelo, B., Caloiro, T., Coscarelli, R., Ferrari, E., 2015. A stochastic model for the analysis of the temporal change of dry spells. *Stoch. Environ. Res. Risk Assess.* 29, 143–155. <https://doi.org/10.1007/s00477-014-0904-5>.
- Stern, R.D., Coe, R., 1984. A model fitting analysis of daily rainfall data. *J. R. Stat. Soc. Ser. A* 147, 1. <https://doi.org/10.2307/2981736>.
- Suhaila, J., Jemain, A.A., 2009. Investigating the impacts of adjoining wet days on the distribution of daily rainfall amounts in Peninsular Malaysia. *J. Hydrol.* 368, 17–25. <https://doi.org/10.1016/j.jhydrol.2009.01.022>.
- Verma, P., Yeates, J., Daly, E., 2011. A stochastic model describing the impact of daily rainfall depth distribution on the soil water balance. *Adv. Water Resour.* 34, 1039–1048. <https://doi.org/10.1016/j.advwatres.2011.05.013>.
- Walsh, R., Lawler, D., 1981. Rainfall seasonality: description, spatial patterns and change through time (British Isles, Africa). *Weather* 36 (7), 201–208. <https://doi.org/10.1002/j.1477-8696.1981.tb05400.x>.
- Wilks, S.S., 1938. The large-sample distribution of the likelihood ratio for testing composite hypotheses. *Ann. Math. Stat.* 9, 60–62. <https://doi.org/10.1214/aoms/1177732360>.
- Wolfram Research, 2008. NonlinearModelFit, Wolfram Language function, <https://reference.wolfram.com/language/ref/NonlinearModelFit.html> (updated 2025).
- Wood, S.J., Jones, D.A., Moore, R.J., 2000. Accuracy of rainfall measurement for scales of hydrological interest. *Hydrol. Earth Syst. Sci.* 4 (4), 531–543. <https://doi.org/10.5194/hess-4-531-2000>.
- Yurdakul, B., Naranjo, J., 2021. Statistical properties of the population stability index. *J. Risk Model Validation* 14 (4), 89–100. <https://doi.org/10.21314/JRMV.2020.227>.