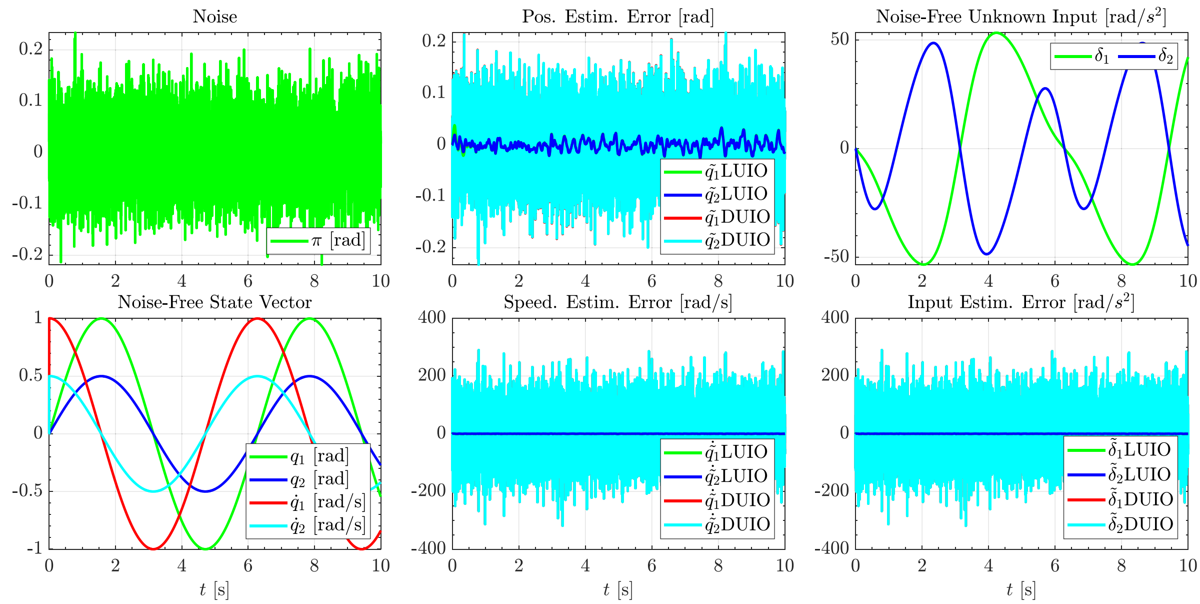

Fig. 4. Simulation results with a two-degree-of-freedom SAR when the system output is perturbed by a white measurement noise with a noise power $p_w = 3.5 \cdot 10^{-6}$. The proposed method, i.e. LUIO, allows a fast and precise estimation of the state vector, with the corresponding unknown inputs, with respect to the DUIO one which is affected by heavy peaking phenomena. The estimation errors of system state and unknown inputs, are with respect to the noise free behavior reported in figure.

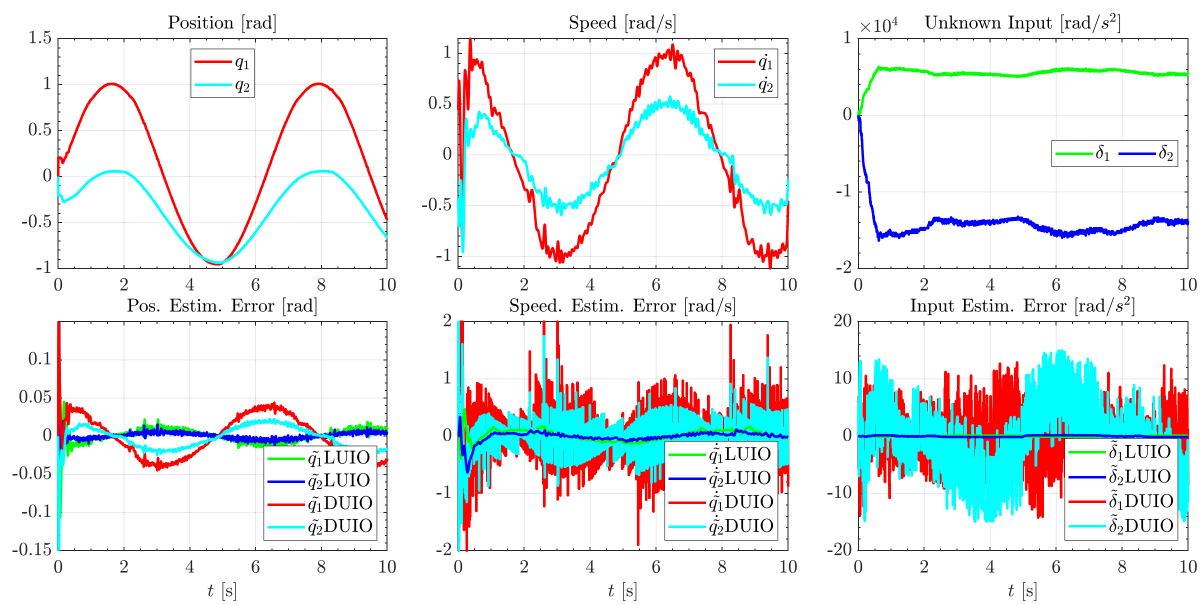

Fig. 5. Experimental results with a SAR. The overall noise affecting the measurement of the robot's joint positions has an average value of 0 radians and a standard deviation of approximately 0.004908 radians. The graphs show how the LUIO outperforms the DUIO in terms of estimation performance and absence of spikes.

allowing a fast and precise estimation both state vector and unknown input signals also in this case. Furthermore, the estimates provided by the LUIO do not present the peak phenomena that characterize those obtained through the DUIO.

4.3. Experimental validation

The experiment results obtained with a real SAR using the presented approach are finally reported here. The experimental robot is depicted in Fig. 6 and consists of a three-degree-of-freedom kinematic chain, where each link is connected to the previous one by a variable stiffness actuator (VSA) joint, i.e., a *qbmove advanced* (qbrobotics, 2022). Each joint comprises two electric motors, a and b, acting in an agonist-antagonist configuration. More precisely, by varying the angles $\theta_{i,a}$ and $\theta_{i,b}$ of the ith joint, the ith (elastic) torque τ_i is applied to the link according to the experimentally identified formula

$$\tau_i(\theta_{i,a}, \theta_{i,b}) = k_a \sinh(a_a(q_i - \theta_{i,a})) + k_b \sinh(a_b(q_i - \theta_{i,b}))\,,$$

where all the constants involved have nominal values listed in Table 1. In the experiments, the first two joints are active, while the last one is passive, causing the last link to simply act as a load for the previous ones, and are forced to track well-defined desired positions through a suitable internal controller. For the remaining part of the paper, the design and structure of the controller will be seen as a black box. More specifically, the joints are forced to track sinusoidal trajectories, and both LUIO and DUIO, exploiting both joint position and known control input measures, can estimate the entire state vector, i.e. x, and the unknown input vector δ. It should be noted that, in the experiment,

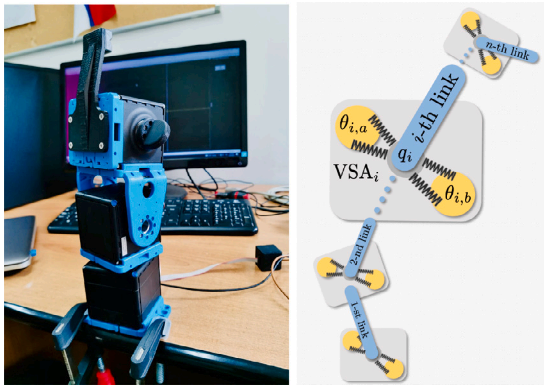

Fig. 6. Picture of the hardware setup used to validate the proposed solution (left) and depiction of the internal functioning of the *soft articulated robot* driven by VSA devices (right).

Table 2
RMSE index comparison between the proposed LUIO and a DUIO approach in experimental results.

Method	RMSE($\dot{\tilde{q}}_1$)	RMSE($\dot{\tilde{q}}_2$)	RMSE($\tilde{\delta}_1$)	RMSE($\tilde{\delta}_2$)
LUIO	$8 \cdot 10^{-2}$	$7 \cdot 10^{-2}$	$9 \cdot 10^{-2}$	$6 \cdot 10^{-2}$
DUIO	$5 \cdot 10^{-1}$	$4 \cdot 10^{-1}$	5	7

the real unknown input can be computed, since the nonlinear model is known, but it is not used in the estimation process. As it can be seen in Fig. 5, the LUIO quickly tracks both state and unknown inputs, respectively, with respect the DUIO. However, the improved performance of the LUIO is numerically quantified in Table 2, by showing a reduction of the Root Mean Square Error (RMSE) index of at least one order of magnitude with respect to the DUIO one in experimental results.

5. Conclusion

This paper presented a Linear Unknown Input-state Observer and its experimental validation on a soft articulated robot. Using the first entries of the Taylor series vector of the system output, the approach showed how to recover the unknown input and system state with better estimation performance and more relaxed unknown input decoupling conditions with respect to existing techniques. Necessary and sufficient conditions, as well as a construction procedure, were described with convergence speed guarantees also in the presence of measurement noise. Future work will be related to closing the loop with a robust control law and its extension to the discrete-time domain.

CRediT authorship contribution statement

Salvatore Pedone: Conceptualization, Formal analysis, Methodology, Software, Validation, Visualization, Writing – original draft, Writing – review & editing. **Adriano Fagiolini:** Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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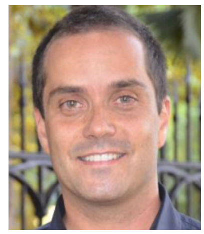

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