



XX ANIDIS Conference

Sequential optimization of liquid-column vibration control devices for multi-story structures

Salvatore Dario Di Trapani ^{a,*}, Silvia Vassallo, Chiara Masnata^a, Antonina Pirrotta^a

^a*Department of Engineering, University of Palermo, Viale delle Scienze, Palermo, 90128, Italy*

Abstract

This study presents a novel procedure for optimizing a vibration control system composed of multiple U-shaped liquid containers, collectively referred to as the Multi-Sliding Tuned Liquid Column Damper (M-STLCD), aimed at reducing seismic responses in multi-story buildings. Each liquid container is equipped with a spring-dashpot mechanism that connects the unit to a floor of the main structure and allows controlled horizontal translation. The optimization procedure, structured as a sequence of iterations, installs and tunes one device at a time, while accounting for the effects of previously installed units. This enables the control strategy to progressively adapt to the evolving dynamic characteristics of the structure. For each STLCD unit, the procedure simultaneously determines the optimal installation floor and the structural vibration mode to be targeted. This is achieved by identifying the most effective placement along the building elevation while optimizing the damping ratio and natural frequency of the spring-dashpot mechanism, as well as the head-loss coefficient and the natural frequency associated with the liquid column. To demonstrate the effectiveness of the sequential optimization and placement procedure, the method is applied to different structures subjected to various types of excitation. Beyond validating the optimization framework, the performance of the M-STLCD is comprehensively evaluated in both time and frequency domains, and compared against optimized multi-unit configurations of traditional devices, namely, the Multi-Tuned Mass Damper (the M-TMD) and the Multi-Tuned Liquid Column Damper (the M-TLCD).

© 2025 The Authors. Published by ELSEVIER B.V.

This is an open access article under the CC BY-NC-ND license (<https://creativecommons.org/licenses/by-nc-nd/4.0>)

Peer-review under responsibility of XX ANIDIS Conference organizers

Keywords: sliding tuned liquid column damper; optimal placement; multiple STLCD; vibration control;

* Corresponding author. Tel.: +39 3349327410.

E-mail address: salvatore.dario.ditrapani@unipa.it

1. Introduction

In civil engineering, the dynamic behavior of buildings is significantly affected by actions such as earthquakes and strong winds. Tuned Liquid Column Dampers (TLCDs) have gained notable attention as efficient passive control devices for mitigating such effects, owing to their fluid-based operating mechanism. A TLCD typically consists of a U- or V-shaped container partially filled with liquid, which is usually water, and rigidly connected to the building (Sakai et al. (1989)). In this configuration, energy from external dynamic actions is dissipated through the vertical oscillation of the liquid columns. As the main advantage of the TLCD lies in the inherent damping characteristics of the liquid, this strategy can be adapted to meet a broader range of performance requirements. For instance, TLCDs can be coupled with base-isolation systems to limit excessive base displacements during severe seismic events (Di Matteo et al. (2018)). Moreover, to optimize spatial efficiency and avoid localized overstress, TLCDs can be employed in modular floor configurations, where multiple units are installed on the same story of a building. Modular floor configurations of TLCDs have been investigated in the literature, both in arrangements where several units are tuned to frequencies closely spaced around a central structural frequency of interest (Sadek et al. (1998), Gao et al. (1999)), and in configurations where device parameters are differentiated based on their dynamic contribution to the overall structural response (Mohebbi et al. (2015)). While both Multi-unit TLCD approaches enhance robustness, particularly in scenarios where individual units may not perform optimally, in case of Mohebbi et al. (2015), enabling each TLCD unit to target a distinct structural mode can lead to a more effective overall response mitigation. Nevertheless, it has been demonstrated that conventional TLCDs are generally unsuitable for short-period structures, as their optimal tuning is often constrained by practical geometric limitations. In this context, the Sliding Tuned Liquid Column Damper (STLCD) represents a variant of the conventional TLCD, where the device is connected to the main structure through a spring-dashpot mechanism allowing controlled horizontal translation at the installation floor (Gosh and Basu (2004)). Although the effectiveness of the STLCD has been demonstrated in prior work (Masnata and Pirrotta (2024)), investigations on multi-unit configurations aimed at reducing the global response of multi-story buildings are still lacking. To address this gap, the present work introduces a sequential optimization procedure tailored to the installation of Multi-unit STLCDs (the M-STLCD) within multi-story buildings. In addition to optimizing the dynamic parameters of each STLCD unit, namely the natural frequency and damping ratio of the spring-dashpot mechanism, as well as the liquid column frequency and head-loss coefficient, the proposed procedure also determines the most effective installation floor for each unit composing the M-STLCD, a placement strategy consistent with those previously explored for traditional Tuned Mass Dampers (TMDs) in multi-unit configurations (Chen and Wu (2001)). The effectiveness of this optimization strategy is demonstrated by its application to a short-period, three-story shear-type structure subjected to the recorded seismic event from the 1999 Kocaeli earthquake. After optimization, the performance of the final M-STLCD configuration is numerically assessed in terms of structural response reduction at various floors, compared to the uncontrolled case. Furthermore, for comparative purposes, analogous sequential optimization procedures have been developed for configurations involving Multi-unit TMDs (the M-TMD) and TLCDs (the M-TLCD), with the latter also introducing the novel aspect of optimally placing individual TLCD units along the building elevation.

2. Mathematical Model of the M-STLCD-Controlled Multi-Story Structure

Consider the system depicted in Figure 1, representing an n -story, shear-type planar frame equipped with the M-STLCD, subjected to a base acceleration $\ddot{x}_g(t)$. The M-STLCD control device consists of n -STLCD units, one per floor, each comprising a U-shaped container filled with liquid, assumed to have the density ρ of water, whose oscillatory motion is influenced by gravity g . Each container is connected to the main structure through a spring-dashpot mechanism that allows horizontal translation at the installation floor. The n -story structure, discretized into n lumped masses M_j ($j = 1, \dots, n$), has n dynamic degrees-of-freedom (DOF), represented by the ground-relative displacements collected in the $n \times 1$ vector $\mathbf{x}_s(t) = [x_1(t); x_2(t); \dots; x_n(t)]$. The M-STLCD introduces $2n$ additional dynamic DOF, n associated with the horizontal motion of the sliding containers relative to the respective installation floors, and n associated with the vertical motion of the liquid columns relative to their equilibrium position at rest.

These additional DOF are collected in the $n \times 1$ vectors $\mathbf{y}(t) = [y_1(t); y_2(t); \dots; y_n(t)]$ and $\mathbf{u}(t) = [u_1(t); u_2(t); \dots; u_n(t)]$, respectively. In the close-up of the generic j^{th} STLCD unit (with $j=1, \dots, n$) shown in Figure 1, installed at the j^{th} floor, the total liquid column length is denoted as $L_j = 2L_{v,j} + L_{h,j}$, where $L_{v,j}$ and $L_{h,j}$ represent the vertical and horizontal liquid segments, respectively. Based on these lengths, the horizontal-to-total length ratio is defined as $\alpha_j = L_{h,j} / L_j$. The horizontal and vertical cross-sectional areas of the j^{th} container are denoted as $A_{h,j}$ and $A_{v,j}$, respectively, with their area ratio denoted as $r_j = A_{h,j} / A_{v,j}$. The j^{th} unit has a total mass $M_{t,j}$, constituted of the container mass, $m_{c,j}$, and the liquid mass, $m_{l,j} = 2\rho A_{h,j} L_{v,j} + \rho A_{v,j} L_{h,j}$. Dissipative effects due to the liquid motion are modeled through the head-loss coefficient ξ_j , while the natural frequency of the liquid column is defined as $\omega_{l,j} = \sqrt{2g / L_j}$. Regarding the j^{th} spring-dashpot unit, its damping and stiffness properties are denoted by $c_{c,j}$ and $k_{c,j}$, respectively, and can be expressed in canonical form as $\zeta_{c,j}$ and $\omega_{c,j}$. The governing equations for the M-STLCD-controlled multi-story structure are defined as follows:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = -\tilde{\mathbf{M}}\ddot{\mathbf{x}}_g(t) \tag{1}$$

where $\mathbf{x}(t) = [\mathbf{x}_s(t)_{[nx1]}; \mathbf{y}(t)_{[nx1]}; \mathbf{u}(t)_{[nx1]}]$ is the $3n \times 1$ vector collecting $\mathbf{x}_s(t)$, $\mathbf{y}(t)$ and $\mathbf{u}(t)$.

The $3n \times 3n$ mass, damping, and stiffness matrices, along with the $3n \times 1$ array accounting for inertial contributions due to the ground excitation $\ddot{\mathbf{x}}_g(t)$, are respectively organized into submatrices that separately group the components associated with the main structure and those related to the M-STLCD, as follows:

$$\mathbf{M}_{[3nx3n]} = \begin{bmatrix} \mathbf{M}_{s[nxn]} & \mathbf{0}_{[nxn]} & \mathbf{0}_{[nxn]} \\ \mathbf{M}_{c[nxn]} & \mathbf{M}_{c[nxn]} & \mathbf{M}_{c,l[nxn]} \\ \mathbf{M}_{c,l[nxn]} & \mathbf{M}_{c,l[nxn]} & \mathbf{M}_{l[nxn]} \end{bmatrix}; \mathbf{C}_{[3nx3n]} = \begin{bmatrix} \mathbf{C}_{s[nxn]} & -\mathbf{C}_{c[nxn]} & \mathbf{0}_{[nxn]} \\ \mathbf{0}_{[nxn]} & \mathbf{C}_{c[nxn]} & \mathbf{0}_{[nxn]} \\ \mathbf{0}_{[nxn]} & \mathbf{0}_{[nxn]} & \mathbf{C}_{l[nxn]} \end{bmatrix}; \tag{2 a-d}$$

$$\mathbf{K}_{[3nx3n]} = \begin{bmatrix} \mathbf{K}_{s[nxn]} & -\mathbf{K}_{c[nxn]} & \mathbf{0}_{[nxn]} \\ \mathbf{0}_{[nxn]} & \mathbf{K}_{c[nxn]} & \mathbf{0}_{[nxn]} \\ \mathbf{0}_{[nxn]} & \mathbf{0}_{[nxn]} & \mathbf{K}_{l[nxn]} \end{bmatrix}; \tilde{\mathbf{M}}_{[3nx1]} = \begin{bmatrix} \mathbf{M}_{s[nxn]} \boldsymbol{\tau}_{[nx1]} \\ \mathbf{M}_{c[nxn]} \boldsymbol{\tau}_{[nx1]} \\ \mathbf{M}_{c,l[nxn]} \boldsymbol{\tau}_{[nx1]} \end{bmatrix}$$

With $\boldsymbol{\tau} = [1; \dots; 1]$ being a $n \times 1$ vector of ones. The other submatrices in (2) are diagonal submatrices defined as:

$$\mathbf{M}_{c,l[nxn]} = \lceil \rho A_{h,j} L_{h,j} \rceil; \mathbf{M}_{c[nxn]} = \lceil \rho A_{h,j} (2L_{v,j} + \frac{1}{r_j} L_{h,j}) + m_{c,j} \rceil;$$

$$\mathbf{M}_{l[nxn]} = \lceil \rho A_{h,j} (2L_{v,j} + r_j L_{h,j}) \rceil;$$

$$\mathbf{C}_{c[nxn]} = \lceil c_{c,j} \rceil; \mathbf{C}_{l[nxn]} = \lceil \frac{1}{2} \rho A_{h,j} r_j \xi_j |\dot{u}_j(t)| \rceil$$

$$\mathbf{K}_{c[nxn]} = \lceil k_{c,j} \rceil; \mathbf{K}_{l[nxn]} = \lceil 2\rho A_{h,j} g \rceil$$
(3 a-g)

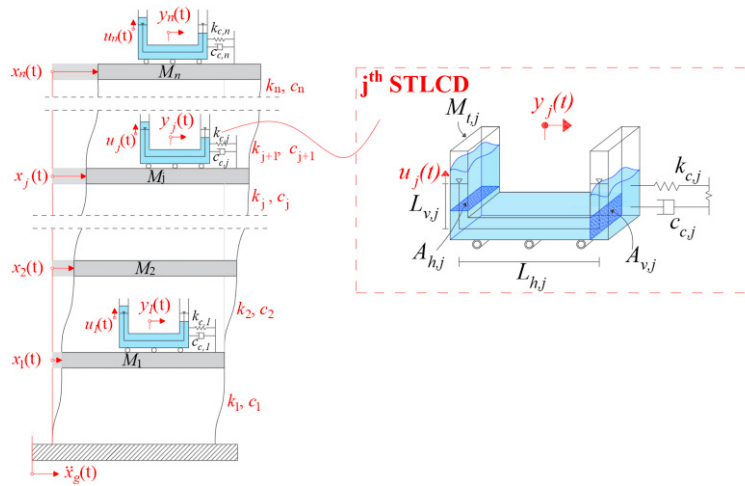


Figure 1. M-STLCD-controlled n -story structure.

3. Optimization Procedure for the Dynamic Parameters and Placement of the n -STLCD Units

The sequential M-STLCD optimization procedure has been developed to achieve a global reduction of the dynamic response of multi-story structures. As is common practice in structural control applications, the optimization process identifies the optimal set of significant device parameters. The procedure consists of several iterations, equal in number to the structural DOF. Within the generic i^{th} iteration, the optimal damping ratio and natural frequency of the spring-dashpot mechanism (respectively $\zeta_{c,j}$ and $\omega_{c,j}$), along with the optimal head-loss coefficient and natural frequency of the liquid column (respectively ξ_j and $\omega_{l,j}$) are determined together with the optimal installation floor. This floor is subsequently excluded from future iterations, resulting in a final M-STLCD configuration with exactly one device per floor. Each parameter is selected from a predefined discrete search space, defined by appropriate lower and upper bounds and explored with fixed incremental steps. Within the generic i^{th} iteration, the optimal parameters and floor placement are determined by minimizing a specific objective function, denoted as Of_i . Consistent with previous studies, this objective function specifically targets the reduction of absolute accelerations, for which optimization aimed at higher structural modes is beneficial for multi-story buildings (Chen and Wu (2001)). The expression of Of_i is as follows:

$$Of_i = \sqrt{\sum_{j=1}^n \eta_{\ddot{x}_a i, j}} = \sqrt{\sum_{j=1}^n \frac{\sigma_{\ddot{x}_a i, j}^2}{\sigma_{\ddot{x}_a (i-1), j}^2}} \quad \text{with } i=1, \dots, n \quad (4)$$

Where $\sigma_{\ddot{x}_a i, j}^2$ is the variance of the absolute acceleration at the j^{th} floor during the i^{th} iteration, $\sigma_{\ddot{x}_a (i-1), j}^2$ is the corresponding variance at the previous iteration, and their ratio $\eta_{\ddot{x}_a i, j}$ represents a normalized measure of response. Thus, using the objective function expressed in Equation (4), normalization is performed relative to the previous iteration, resulting in a parameter that simultaneously accounts for the response at all floors as well as the previous system configurations. Clearly, at the first iteration ($i = 1$), the reference configuration corresponds to the uncontrolled structure denoted as state 0 (subscript 0), with $\sigma_{\ddot{x}_a 0, j}^2$ representing the acceleration variance at the j^{th} floor in the uncontrolled state. The procedure is fully automated and compatible with any computational environment (e.g., MatLab), requiring only structural data, base excitation input, parameter bounds, and objective function definition. The procedure is structured as follows:

1. **Initialization:** The algorithm generates the structural base acceleration input, defines the response of the uncontrolled structure in terms of absolute acceleration variances ($\sigma_{x_{0,j}}^2$), and initializes the full-sized system matrices ($3n \times 3n$ for \mathbf{M} , \mathbf{C} , and \mathbf{K} , and $3n \times 1$ for $\tilde{\mathbf{M}}$), with zeros for all STLCD-related terms. These matrices serve as the restore-point configuration.
2. **1st iteration (placement of the 1st STLCD):** The algorithm evaluates every combination of STLCD parameters (natural frequencies, damping ratios, head-loss coefficients) across all possible floors, temporarily reducing matrices by removing zero rows and columns corresponding to inactive DOF. The configuration that minimizes Of_i (Equation 4) is identified, determining optimal parameters and installation floor. This optimal floor is excluded from subsequent iterations, and matrices and variances are updated accordingly, establishing a new baseline.
3. **Subsequent iterations (placement of successive STLCD units):** The procedure repeats iteratively, identifying the optimal parameters and placement of each successive STLCD unit, until one optimally tuned device is installed on every floor.

4. M-STLCD Control Strategy: Results of the Optimized Configuration

To numerically evaluate the effectiveness and versatility of the M-STLCD strategy, the sequential optimization procedure has been applied to two benchmark structures, a three-story and a six-story building, under distinct base excitations. First, both buildings have been subjected to a real accelerogram recorded during the 1999 Kocaeli earthquake in Turkey. Subsequently, each building has been driven by a harmonic base acceleration in resonance with its fundamental mode. The three-story building model is characterized, from bottom to top, by lumped masses of $M_1 = 6.0 \times 10^4$ kg, $M_2 = 4.5 \times 10^4$ kg and $M_3 = 3.0 \times 10^4$ kg (being $M_{tot} = M_1 + M_2 + M_3$ the total building mass) and interstory stiffness values of $k_1 = 4.0 \times 10^7$ N/m, $k_2 = 2.1 \times 10^7$ N/m and $k_3 = 2.0 \times 10^6$ N/m. The modal damping ratios are set to 5%, 2%, and 3% for the 1st, 2nd, and 3rd modes, respectively, while the natural frequencies $\omega_1 = 7.50$ rad/sec, $\omega_2 = 17.23$ rad/sec, and $\omega_3 = 35.23$ rad/sec indicate a relatively stiff system. The six-story building model adopts structural parameters previously used in the literature, and its fundamental frequencies also indicate a stiff system (Chen and Wu (2001)). For the M-STLCD strategy applied to the three-story building, each unit has been assigned a mass ratio of 4%, equally divided between the sliding container (2%) and the liquid (2%), resulting in a cumulative mass ratio of 12% relative to M_{tot} . To achieve the same cumulative mass ratio of 12% for the six-story building, each M-STLCD unit has been assigned a mass ratio of 2%. The horizontal-to-total length ratio α_j has been set to 0.85. Based on practical considerations, the optimal parameters of each unit were searched within the following ranges: $\omega_{c,j} \in [0.64\omega_{min}, 1.36\omega_{max}]$, with ω_{min} and ω_{max} being respectively the fundamental frequency and the highest natural structural frequency; $\zeta_{c,j} \in [0.005, 0.05]$, $\zeta_j \in [1, 80]$ and $L_j \in [3, 50]$ m. For comparison, the M-STLCD has been evaluated against optimized configurations of the Multi-Tuned Mass Damper (M-TMD) and the Multi-Tuned Liquid Column Damper (M-TLCD), with each unit assigned the same mass ratio as in the M-STLCD strategy, i.e., 4% for the three-story building and 2% for the six-story building.

Table 1 reports peak values of absolute accelerations and interstory drifts for the three-story building equipped with the final M-STLCD configuration, compared with analogous final configurations for the M-TMD and the M-TLCD. As can be observed, the M-STLCD strategy significantly reduces structural responses at all floors, decreasing accelerations at the 2nd floor from 8.51 m/s² to 5.22 m/s² (39% reduction) and interstory drift between the 2nd and 1st floors from 0.0191 m to 0.0141 m (26% reduction), thus remaining competitive with the M-TMD. Peak values clearly indicate that the response reduction achievable with the M-TLCD is limited, slightly exceeding the uncontrolled structure drift at the 3rd-to-2nd interstory (0.0840 m). A similar trend can be observed in terms of absolute acceleration response reductions at the top floors of both the three-story and six-story buildings, where the M-STLCD and M-TMD consistently achieve meaningful response attenuation throughout the entire time history, whereas the reduction obtained by the M-TLCD remains less pronounced.

Furthermore, under resonant excitation at the 1st structural mode frequency, the M-STLCD again demonstrates

response reductions comparable to those achieved with the M-TMD for both buildings (Figures 3(A) and (B)).

Table 1. Peak floor accelerations and interstory drifts in the three-story structure: M-STLCD vs M-TMD and M-TLCD control strategies under Kocaeli earthquake.

Structural Response	Units	Uncontrolled Structure	M-TMD $\mu_d = 4\%$	M-TLCD $\mu_l = 4\%$	M-STLCD $\mu_s = 4\%$
Absolute accelerations					
$\ddot{x}_{a,3}(t)$	[m/sec ²]	5.69	3.61	5.00	4.19
$\ddot{x}_{a,2}(t)$	[m/sec ²]	8.51	4.10	8.45	5.22
$\ddot{x}_{a,1}(t)$	[m/sec ²]	5.31	3.07	5.22	3.61
Interstory drifts					
$x_3(t) - x_2(t)$	[m]	0.0834	0.0656	0.0840	0.0718
$x_2(t) - x_1(t)$	[m]	0.0191	0.0125	0.0173	0.0141

.....Uncontrolled Structure - - - M - TLCD - - - M - STLCD - - - M - TMD

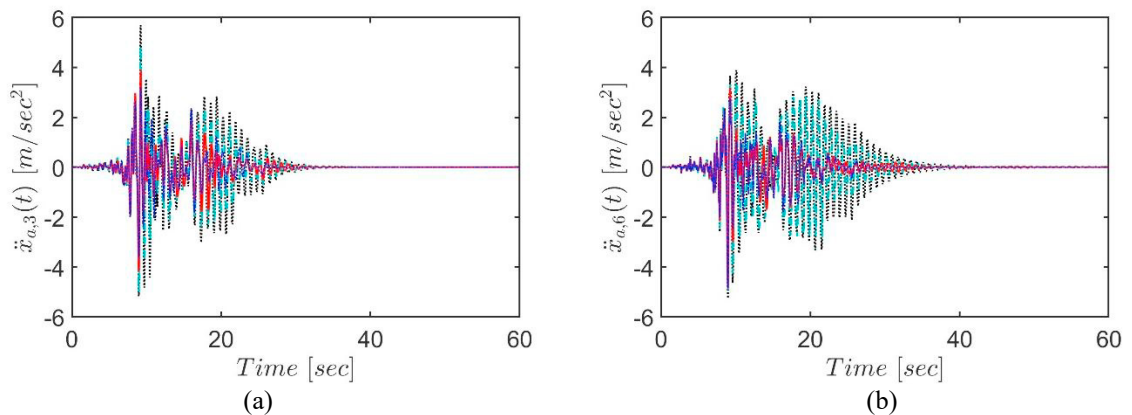


Figure 2. Top floor absolute accelerations under the 1999 Kocaeli earthquake for: (a) three-story building (4% unit mass ratios); (b) six-story building (2% unit mass ratios), comparing the M-STLCD, M-TMD, and M-TLCD.

.....Uncontrolled Structure ---M – TLCD —M – STLCD —M – TMD

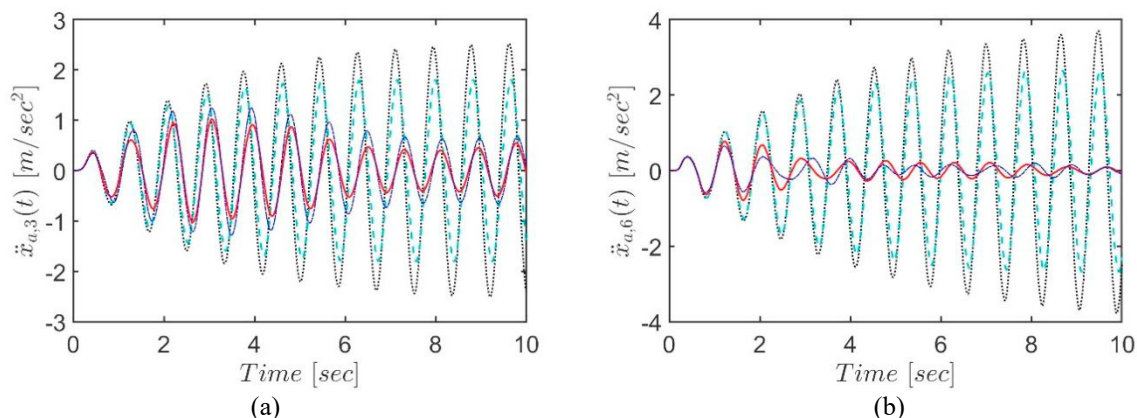


Figure 3. Top floor absolute accelerations under harmonic base excitation at the 1st structural mode frequency for: (a) three-story building (4% unit mass ratios); (b) six-story building (2% unit mass ratios), comparing the M-STLCD, M-TMD, and M-TLCD.

5. Conclusions

This study presented a novel sequential optimization procedure for the Multi-unit STLCD (M-STLCD). As demonstrated through peak response and time-history analyses, the procedure effectively tuned each M-STLCD unit and identified its optimal installation floor by carefully matching structural and input characteristics. Despite the objective function specifically targeting acceleration reductions, the M-STLCD strategy also significantly reduced interstory displacements. Conversely, the Multi-unit TLCD (M-TLCD) strategy did not achieve satisfactory results, highlighting the intrinsic difficulty in identifying optimal parameters within feasible ranges for stiff structures. Consequently, the proposed M-STLCD extends the applicability of liquid-based control devices to multi-story, short-period buildings, effectively reducing their overall dynamic response. With performance comparable to the Multi-unit TMD (M-TMD), the M-STLCD represents a viable alternative whose primary strengths lie in modularity and the cost-effective utilization of water.

Acknowledgements

The authors gratefully acknowledge financial support from the Italian Ministry of Education, University and Research (MIUR) under the P.R.I.N. 2022 National Grant “Innovative metamaterial components and absorbers for vibration mitigation (METAVIBRA)” (Project Code 2022LA43E2_001; University of Palermo Research Unit). A. Pirrotta gratefully acknowledges the support received from the project SiciliAn MicronanOTech Research And Innovation Center “SAMOTHRACE” (MUR, PNRR-M4C2, ECS_0000022), spoke 3-Universita’ degli Studi di Palermo “S2-COMMs-Micro and Nanotechnologies for Smart & Sustainable Communities.

References

- Sakai, F., Takeda, S., Tamaki, T., 1989. Tuned liquid column damper—new type device for suppression of building vibrations, Proceedings of the International Conference on Highrise Buildings. Nanjing, China, pp. 926–931.
- Di Matteo, A., Furtmüller, T., Adam, C., Pirrotta, A., 2018. Optimal design of tuned liquid column dampers for seismic response control of base-isolated structures. Acta Mechanica 229, pp. 437–454. doi: [10.1016/j.proeng.2017.09.060](https://doi.org/10.1016/j.proeng.2017.09.060)
- Sadek, F., Mohraz, B., Lew, H. S., 1998. Single - and multiple-tuned liquid column dampers for seismic applications. Earthquake Engineering & Structural Dynamics 27, pp. 439–463. doi: [10.1002/\(SICI\)1096-9845\(199805\)27:5<439::AID-EQE730>3.0.CO;2-8](https://doi.org/10.1002/(SICI)1096-9845(199805)27:5<439::AID-EQE730>3.0.CO;2-8)
- Gao, H., Kwok, K. S. C., Samali, B., 1999. Characteristics of multiple tuned liquid column dampers in suppressing structural vibration. Engineering Structures 21, pp. 316–331. doi: [10.1016/S0141-0296\(97\)00183-1](https://doi.org/10.1016/S0141-0296(97)00183-1)
- Mohebbi, M., Dabbagh, H. R., Shakeri, K., 2015. Optimal Design of Multiple Tuned Liquid Column Dampers for Seismic Vibration Control of MDOF Structures. Periodica Polytechnica Civil Engineering 59, pp. 543–558. doi: [10.3311/PPci.7645](https://doi.org/10.3311/PPci.7645)
- Ghosh, A., Basu, B., 2004. Seismic vibration control of short period structures using the liquid column damper. Engineering Structures 26, pp. 1905–1913. doi: [10.1016/j.engstruct.2004.07.001](https://doi.org/10.1016/j.engstruct.2004.07.001)

- Masnata, C., Pirrotta, A., 2024. Optimal control of base-isolated systems with sliding TLCD under stochastic process. *Engineering Structures* 318, 118754. doi: [10.1016/j.engstruct.2024.118754](https://doi.org/10.1016/j.engstruct.2024.118754)
- Chen, G., Wu, J., 2001. Optimal Placement of Multiple Tune Mass Dampers for Seismic Structures. *Journal of Structural Engineering* 127, pp. 1054–1062. doi: [10.1061/\(ASCE\)0733-9445\(2001\)127:9\(1054\)](https://doi.org/10.1061/(ASCE)0733-9445(2001)127:9(1054))