



ALIASING INTERFERENCE AND SIGNAL UNDERSAMPLING IN THERMOELASTIC STRESS ANALYSIS

R. Cappello^{1,2}, G. Pitarresi¹

¹ Department of Engineering, University of Palermo, Italy

² Bristol Composites Institute, University of Bristol, UK

1. Introduction

Thermoelastic Stress Analysis (TSA) relies on evaluating harmonic components of a temperature signal carrying temperature variations induced by cyclic elastic straining. These “thermoelastic” harmonics are typically modulated at frequencies f and $2f$, when f is the main loading frequency applied to the structure. Common algorithms to extract thermoelastic harmonics from a sampled temperature signal employ Discrete Fourier Transform (DFT) or Least Square Fitting based approaches [1]. These require the harmonic to be filtered to have at least four samples per cycle (reduced to two when the harmonic phase is known [1]), i.e. $f < 4f_s$ and $2f < 8f_s$ with f_s being the sampling frequency and f the load frequency. Thus, a thermoelastic “second harmonic” from a test performed with a 15 Hz loading would at least require $f_s = 120$ Hz.

To the authors’ knowledge, there are no works in the literature who have specifically evaluated thermoelastic harmonics by extracting their aliased waves. This possibility is of relevant interest since it would enable the possibility to use slower IR cameras (e.g. low cost microbolometers, [2,3]), and also provide strategies to investigate very high frequency loading scenarios.

This study examines the sampling conditions which enable the filtering of aliased waves in the TSA context. The analysis is performed by analyzing two scenarios: one where aliasing collision frequencies may arise [4], generating aliasing interference which may affect the accuracy in routine oversampled TSA, and the other one where aliased waves of the thermoelastic signals are instead detected by a properly and purposely designed undersampling strategy.

2. Methodology

The relationship that identifies aliased frequencies f_a when sampling a harmonic with frequency f at the sampling rate f_s can be written as [1]:

$$f_a = |f - m \cdot f_s| \quad \text{where } m \text{ is an integer ensuring that } f_a < f_s/2 = f_{ny} \quad (1)$$

The Nyquist frequency, f_{ny} , also known as the folding frequency, reflects aliased frequencies by mirroring frequency f about f_{ny} in a discrete Fourier spectrum.

If the frequencies carrying the thermoelastic harmonics are known, Eq. (1) can then be used to design a processing scheme which uses a subsampling frequency. In particular, Eq. (1) allows to determine the alias frequencies of the thermoelastic harmonics that will appear in the DFT spectrum of the subsampled signal.

A numerical example of a simulated temperature signal is here provided to proof the signal processing concept. Let’s consider a generic signal of the likes:

$$T(t) = T_o + E \sin(2\pi f_E t + \phi_E) + D \sin(2\pi f_D t + \phi_D) + R \sin(2\pi f_R t + \phi_R) + \text{whitenoise} \quad (2)$$

where $T_o = 23\text{ }^\circ\text{C}$, $E = 0.55\text{ }^\circ\text{C}$, $f_E = 15\text{ Hz}$, $D = 0.05\text{ }^\circ\text{C}$, $f_D = 30\text{ Hz}$, $R = 0.01\text{ }^\circ\text{C}$, $f_R = 45\text{ Hz}$. The phases ϕ can have any values and the term *whitenoise* adds uniformly distributed noise having a noise floor of about $0.001\text{ }^\circ\text{C}$. These figures well represent a typical TSA experiment. The E , D and R terms will also be referred to as *first*, *second* and *third harmonics*. In order to extract these from the signal, this is sampled and analysed by the DFT. Two different DFT spectrums are reported in Fig. 1, one obtained from sampling at 180 Hz and one obtained from sampling at 6.25 Hz (i.e. a typical frequency of a microbolometer [3]). In the first case ($f_s = 180\text{ Hz}$), $N = 900$ samples are processed, corresponding to an acquisition time window of 5 sec , while for the subsampling case ($f_s = 6.25\text{ Hz}$), $N = 100$ is chosen, corresponding to a time window of 16 sec .

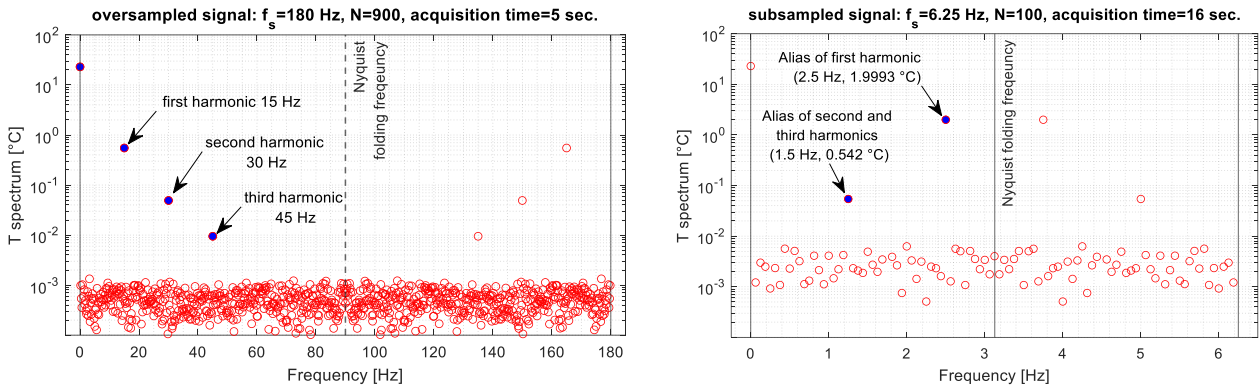


Fig. 1. DFT spectrums of the signal of Eq. (2). Left) oversampling case; Right) subsampling case.

3. Results and conclusions

From Eq. (1) it is found that the subsampled signal produces an alias of the first 15 Hz harmonic at 2.5 Hz , when $m=2$. The DFT in this case has evaluated an amplitude of $E = 0.552\text{ }^\circ\text{C}$ which differs from the nominal one of only 0.36% . It is noticed that the second 30 Hz and third 45 Hz harmonics both have aliases at 1.5 Hz , for respectively $m=5$ and $m=7$. This is then a case where the presence of the third harmonic in the signal is producing an aliasing interference with the second harmonic. As a result, the amplitude of the second harmonic is now less accurate, with an error of 3.4% .

It is also noticed that a careful choice of the number of processed samples (e.g. $N = 100$) in the subsampling case yields coherent sampling, thus avoiding spectral leakage. It is also observed that the noise floor in the subsampling scheme is increased, although this can be reduced by extending further the acquisition time, that is anyway necessarily higher than that of the oversampling case.

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5. References

- [1] O. Breitenstein, W. Warta, and M. Langenkamp (2010). Lock-in thermography: Basics and use for evaluating electronic devices and materials, volume 10. Springer, 2010.
- [2] G. Pitarresi, R. Cappello and G. Catalanotti (2020). Quantitative thermoelastic stress analysis by means of low-cost setups. Optics and Lasers in Engineering, **134**, 106158, 11.
- [3] I. Jiménez-Fortunato, D.J. Bull, O.T. Thomsen, J.M. Dulieu-Barton (2023). Quantitative microbolometer-based thermoelastic stress analysis. Optics and Lasers in Engineering, **160**, 107276.
- [4] J.C. Krapez, G Gardette, and D. Balageas (1995). Lock-In IR Thermography - advantages and problems of some approaches. Int. Workshop on Advanced Infrared Technologies and Applications.