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Bivariate Processes Evolutionary Power Spectral Density Estimation Using Energy Spectrum Equations

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Abstract: In this paper a novel procedure is developed for evolutionary cross power spectra (ECPS) estimation of bivariate nonstationary stochastic processes. Specifically, the ECPS is determined by estimating the statistical moments of energy-like response quantities of lightly damped linear filters excited by nonstationary stochastic processes. In this context, a smoothing procedure is incorporated by using the Savitzky-Golay (S-G) moving average filter to obtain reliable ECPS based even from a limited number of available records. Further, a refinement of the approach is proposed relying on polynomial based functions of the system output. Several numerical examples, including nonstationary processes with known spectra, and historic accelerograms are used to assess the reliability and accuracy of the proposed procedure.

Key Words: Nonstationary Stochastic Processes; Spectral Estimation: Earthquake Inputs

1. Introduction

Many problems in several engineering fields involve multivariate stochastic processes with salient nonstationary features. Seismic ground motions and strong downburst winds, for instance, typically show a slow variation in spectral content with time [1]. In this regard, capturing these features becomes critical in correctly predicting the corresponding structural response [2, 3].

The time-frequency characteristics of these nonstationary random processes can be described in terms of evolutionary auto and cross power spectral density functions [4, 5]. In this regard, several approaches have been developed for appropriately analyzing and estimating the cross power spectra of a nonstationary process. A closed form evolutionary cross power spectra (ECPS) expression has been proposed in [6]. Nevertheless, it has been reported that deriving the closed-form ECPS from nonstationary processes can be cumbersome and lengthy. Further, the wavelets transform (WT) method has been proposed as an alternative tool for estimating the ECPS [7]. Notably, different kinds of wavelet functions can be employed. For instance, Morlet wavelets, Modified Little-Paley wavelets [7], and harmonic wavelets [8] have been used to estimate the ECPS of bivariate nonstationary processes.

In context with the preceding comments, a more physically motivated procedure is perhaps desirable. The procedure is based on a technique first presented in [4, 5, 9]. Specifically, an estimate of the ECPS of bivariate nonstationary random processes can be obtained by determining the first statistical moment

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of appropriately introduced energy-like quantities of two lightly damped single-degree-of-freedom (SDOF) linear systems excited by a bivariate stochastic process [4, 5, 9]. In this regard, the SDOF systems act as bandpass filters, centered around their natural frequencies. Clearly, by appropriately varying the natural frequency of the SDOF systems, a complete description of the ECPS can be achieved. However, some difficulties may be encountered due to the oscillatory trend of the data, which causes the ECPS to be erratic. Note that these issues can be attributed to the limited number of records available, and to the fact that this approach requires the evaluation of the derivatives of the statistical moments of the energy-like quantities.

To address this problem, the original method in [5] is revised and extended, thus enhancing the accuracy of the estimated ECPS. Specifically, a novel approach is presented for appropriately smoothing the first statistical moment of appropriately introduced energy-like quantities, relying on the use of the so-called Savitzky-Golay (S-G) moving average filter [10]. This is done to obtain reliable spectra based even on a relatively small number of available records. Further, a refinement of the approach is investigated by capturing the expected values of these quantities, and their derivatives, utilizing proper polynomial models. Finally, several applications involving both simulated and historic earthquake data are used to assess the accuracy of the proposed approach.

2. Mathematical Background

Consider two independent lightly damped linear SDOF systems (filters) governed by the equation [4, 5]

$$\ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0^2 x = X(t)\,, \tag{1}$$

and

$$\ddot{y} + 2\zeta\omega_0\dot{y} + \omega_0^2 y = Y(t)\,, \tag{2}$$

with $\zeta \ll 1$, representing the damping ratio, and ω_0 representing the natural frequency of the system. Further, $X(t)$ and $Y(t)$ are two broadband nonstationary stochastic processes, which represent the components of the bivariate process $\{X(t), Y(t)\}$. These processes admit an integral representation of the form

$$X(t) = \int_{-\infty}^{\infty} A_x(t,\omega)e^{-i\omega t}dZ_x(\omega)\,, \tag{3}$$

and

$$Y(t) = \int_{-\infty}^{\infty} A_y(t,\omega)e^{-i\omega t}dZ_y(\omega)\,, \tag{4}$$

where $A_x(t,\omega)$ and $A_y(t,\omega)$ are deterministic modulating functions. Further, $Z_x(\omega)$ and $Z_y(\omega)$ are two Gaussian processes with independent increments satisfying the relation

$$< dZ_x(\omega)dZ_y(\omega) >= S_{xy}(\omega)d\omega\,. \tag{5}$$

The cross-covariance function of the two random processes can be expressed as

$$< X(t)Y(t) >= \int_{-\infty}^{\infty} A_x(t,\omega)A_y(t,\omega)S_{xy}(\omega)d\omega = \int_{-\infty}^{\infty} S_{xy}(t,\omega)d\omega\,. \tag{6}$$

Note that the ECPS function $S_{xy}(t,\omega)$ in Eq. (6) lends itself to a physical interpretation similar to that of the evolutionary power spectrum (EPS) function of a bivariate stationary processes.

Clearly, in the case of nonstationary processes, the cross power spectrum function is also time dependent. Further, the stationary cross power spectrum function $S_{XY}(\omega)$ is, in general, a complex function with the property

$$S_{xy}(-\omega) = {S_{xy}}^*(\omega) = S_{yx}(\omega)\,. \tag{7}$$

Therefore, it can be expressed as

$$S_{xy}(\omega) = C_{xy}(\omega) - iQ_{xy}(\omega)\,, \tag{8}$$

where $C_{XY}(\omega)$, referred to as the co-spectrum, is a real-valued even function of ω, while $Q_{XY}(\omega)$, referred to as the quad-spectrum, is a real-valued odd function of ω.

Further, denoting x and y, as x_1 and y_1, respectively, and their derivatives as x_2 and y_2, respectively, it can be proved [5] that the expected value $< x_j(t) y_k(t) >$ is given as

$$\begin{aligned} < x_j(t) y_k(t) >= \frac{\pi}{\omega_0}\Bigg[& \frac{1+(-1)^{j+k}}{2}\omega_0^{j+k-3} C_{xy}(\omega_0) \\ & - i^{j-k+1}\frac{1-(-1)^{j+k}}{2} Q_{xy}(\omega_0)\Bigg] \\ & * \int_0^t A_x(t-\tau,\omega_0) A_y^*(t-\tau,\omega_0) e^{-2\zeta\omega_0\tau} d\tau\,. \end{aligned} \tag{9}$$

Next, introducing an "energy like" quantity [4, 5]

$$\bar{E}_{xy}(t,\omega_0) = \frac{1}{2}\left(\omega_0^2 x(t) y(t) + \dot{x}(t)\dot{y}(t)\right), \tag{10}$$

and taking into account Eq. (9), yields

$$< \bar{E}_{xy}(t,\omega_0) >= \pi Q_{xy}(t,\omega_0) \int_0^t A_x(t-\tau,\omega_0) A_y^*(t-\tau,\omega_0) e^{-2\zeta\omega_0\tau} d\tau\,. \tag{11}$$

The above equation represents a convolution integral associated with the solution of the first order differential equation,

$$\begin{aligned} < \dot{\bar{E}}_{xy}(t,\omega_0) > +2\zeta\omega_0 < \bar{E}_{xy}(t,\omega_0) > &= \pi A_x(t,\omega_0) A_y^*(t,\omega_0) C_{xy}(\omega_0) \\ &= \pi C_{xy}(t,\omega_0). \end{aligned} \tag{12}$$

Further, a similar procedure can be followed for the quad-spectrum. Specifically, defining the quantity

$$\bar{D}_{xy}(t,\omega_0) = \frac{\omega_0}{2}\left(\dot{x}(t) y(t) - x(t)\dot{y}(t)\right), \tag{13}$$

and taking into account Eq. (9), yields

$$< \bar{D}_{xy}(t,\omega_0) >= \pi Q_{xy}(t,\omega_0) \int_0^t A_x(t-\tau,\omega_0) A_y^*(t-\tau,\omega_0) e^{-2\zeta\omega_0\tau} d\tau\,. \tag{14}$$

Therefore, the evolutionary quad-spectrum can be determined by solving the following first-order differential equation

$$\begin{aligned} < \dot{\bar{D}}_{xy}(t,\omega_0) > +2\zeta\omega_0 < \bar{D}_{xy}(t,\omega_0) > &= \pi A_x(t,\omega_0) A_y^*(t,\omega_0) Q_{xy}(\omega_0) \\ &= \pi Q_{xy}(t,\omega_0). \end{aligned} \tag{15}$$

In this manner, both the evolutionary co-spectrum and the quad-spectrum can be estimated from Eq. (12) and (15), in which the functions $< \bar{E}_{xy}(t,\omega_0) >$ and $< \bar{D}_{xy}(t,\omega_0) >$, and their derivatives, can be determined from the ensemble average of the outputs of the SDOF filters in Eq. (1) and (2), and by relying on Eq. (10) and (13). Finally, the corresponding ECPS can be obtained as

$$S_{xy}(t,\omega_0) = C_{xy}(t,\omega_0) - iQ_{xy}(t,\omega_0)\,. \tag{16}$$

Note that Eq. (16) provides an estimate of the ECPS of the bivariate process centered at the frequency ω_0, based on the observations of the response to this process of linear oscillators with natural frequency ω_0. It is pointed out, however, that Eq. (12) and (15) require the estimation of both the mean data

$< \bar{E}_{xy}(t,\omega_0) >$ and $< \bar{D}_{xy}(t,\omega_0) >$ and the derivatives $< \dot{\bar{E}}_{xy}(t,\omega_0) >$ and $< \dot{\bar{D}}_{xy}(t,\omega_0) >$, respectively.

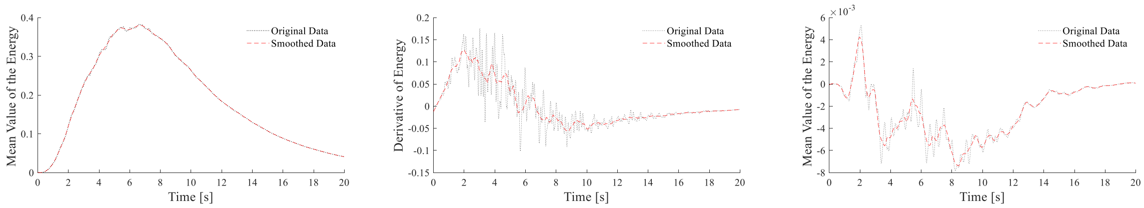
Based on this procedure, reliable estimates can be obtained for the ECPS of nonstationary bivariate processes. However, due to the fact that often only a limited number of records are available, the time derivatives of the energy-like quantities $< \dot{\bar{E}}_{xy}(t,\omega_0) >$ and $< \dot{\bar{D}}_{xy}(t,\omega_0) >$, may present a highly oscillatory trend. This may result, in certain cases, in an unsatisfactory estimate of the corresponding $S_{xy}(t,\omega_0)$.

3. A Novel Procedure for ECPS Estimation

Considering the aforementioned problems related to the accuracy of the procedure, an alternative technique can be adopted, based on a smoothing procedure employing the S-G filter [10]. This is a generalized moving average filter whose coefficients are determined by a non-weighted linear least-squares regression of the data points involving a polynomial model of a specified degree (hereinafter assumed as two).

Notably, a main advantage of the S-G filter is that it can retain the shape of the data, thus capturing their trend with a low computational cost.

In this regard, for each value of the natural frequency of the systems ω_0, the S-G filter can be applied to pertinent numerical data of $< \bar{E}_{xy}(t) >$ and $< \bar{D}_{xy}(t) >$, and the time derivative $< \dot{\bar{E}}_{xy}(t) >$ and $< \dot{\bar{D}}_{xy}(t) >$, leading to less oscillatory approximate representations of the functions. An example of the application of this procedure is shown in Fig. 1(a), where the original numerical data $< \bar{E}_{xy}(t) >$ (black line), vis-à-vis the smoothed data $< E_{xy}(t) >$ by the S-G filter (red line), are reported. Further, analogous results in terms of the time derivative $< \dot{\bar{E}}_{xy}(t) >$ and $< \dot{E}_{xy}(t) >$, and the data $< \bar{D}_{xy}(t) >$ and $< D_{xy}(t) >$ are shown in Fig. 1 (b) and (c), respectively. Specifically, the data from Fig. 1 are produced from 500 bivariate nonstationary stochastic processes generated in conjunction with the Kanai-Tajimi ECPS. Note that a constant value of the linear damping coefficient ($2\zeta\omega_0 = 0.2$) is employed in Eq. (1), (2), (10) and Eq. (13), and the natural frequency value $\omega_0 = 5\ rad/s$ is used.



(a) (b) (c)

Fig. 1. Original and smoothed data. (a) $< \bar{E}_{xy}(t) >$; (b) $< \dot{\bar{E}}_{xy}(t) >$; and (c) $< \bar{D}_{xy}(t) >$.

It can be seen from Fig. 1 that $< \dot{E}_{xy}(t) >$ becomes significantly smoother by using the S-G filter. However, although the above-described procedure improves the accuracy of the numerical evaluation of $< \dot{\bar{E}}_{xy}(t) >$, in some circumstances the corresponding ECPS estimated using Eq. (10), (13) and (16) may still present an oscillatory trend.

In this regard, an additional step can be introduced to obtain a more accurate representation of the ECPS from the available records. An appropriately chosen model of the mean energy-like quantities can be defined to fit the corresponding data, for each value of the natural frequency ω_0. Specifically, the model can be constructed by employing the polynomial functions in [11]. In this context, define two approximate functions $\tilde{E}_{xy}(t)$ and $\tilde{D}_{xy}(t)$ in the interval $[0,\ t_f]$, where t_f is the final time instant, with boundary conditions given as

$$\tilde{E}_{xy}(0) = \dot{\tilde{E}}_{xy}(0) = \tilde{D}_{xy}(0) = \dot{\tilde{D}}_{xy}(0) = 0\,,$$
and
$$\tilde{E}_{xy}(t_f) = E_{xy_f};\ \dot{\tilde{E}}_{xy}(t_f) = \dot{E}_{xy_f};\ \tilde{D}_{xy}(t_f) = D_{xy_f};\ \dot{\tilde{D}}_{xy}(t_f) = \dot{D}_{xy_f}\,, \tag{17}$$

where the initial conditions have been already assumed in dealing with the SDOF systems in Eq. (1) and (2). Further, $(E_f, \dot{E}_f)$ and $(D_f, \dot{D}_f)$ are given by the original mean data $< \bar{E}_{xy}(t_f) >$, $< \bar{D}_{xy}(t_f) >$ and the derivatives $< \dot{\bar{E}}_{xy}(t_f) >$ and $< \dot{\bar{D}}_{xy}(t_f) >$ by numerical simulations of the SDOF systems for each value of ω_0.

Next, an approximate model of $\tilde{E}_{xy}(t)$ and $\tilde{D}_{xy}(t)$ for each value of ω_0 can be expressed as a combination of shifted Legendre polynomials $P_p(t)$ of order p, and Hermite interpolating polynomials $H(t)$. In particular, the shifted Legendre polynomial of order $p+1$ can be evaluated by the recursive formula

$$P_{p+1}(t) = \frac{2p+1}{p+1}\left(\frac{2t - t_f}{t_f}\right)P_p(t) - \frac{p}{p+1}P_{p-1}(t), \qquad p = 2, 3, \dots, \tag{18}$$

where $P_0(t) = 1$ and $P_1(t) = (2t - t_f)/t_f$. Further, the Hermite interpolating polynomials $H(t)$ are defined as

$$H(t) = \sum_{k=0}^{3} a_k t^k\,, \tag{19}$$

where the coefficients a_k can be determined by satisfying the conditions in Eq. (17). Specifically, denote $H_1(t)$ the Hermite interpolating polynomials of $< E_{xy}(t) >$. That is,

$$H_1(0) = \dot{H}_1(0) = 0\,,$$
and
$$H_1(t_f) = E_{xy_f};\ \dot{H}_1(t_f) = \dot{E}_{xy_f}\,, \tag{20}$$

and $H_2(t)$ the Hermite interpolating of $< D_{xy}(t) >$, that is

$$H_2(0) = \dot{H}_2(0) = 0\,,$$
and
$$H_2(t_f) = D_{xy_f};\ \dot{H}_2(t_f) = \dot{D}_{xy_f}\,. \tag{21}$$

In this manner, an approximate polynomial of $< E_{xy}(t) >$ can be expressed as

$$\tilde{E}_{xy}(t) = \sum_{p=0}^{N-1} \alpha_p t^2 (t - t_f)^2 P_p(t) + H_1(t)\,, \tag{22}$$

and an approximate model of $< D_{xy}(t) >$ can be expressed as

$$\tilde{D}_{xy}(t) = \sum_{p=0}^{N-1} \beta_p t^2 (t - t_f)^2 P_p(t) + H_2(t)\,, \tag{23}$$

where N is an appropriate number of shifted Legendre polynomials. Note that α_p and β_p are coefficients that can be evaluated via a nonlinear least-squares approach, minimizing the error between $< E_{xy}(t) >$ and $\tilde{E}_{xy}(t)$, and between $< D_{xy}(t) >$ and $\tilde{D}_{xy}(t)$.

On this base, a simple scheme for the estimation of the ECPS $S_{xy}(t, \omega_0)$ can be built. Specifically, the following procedure can be used for each value of ω_0 from $[\omega_{0,min}, \omega_{0,max}]$.

I. Use Eq. (1) – (2) to estimate the response $x(t)$ and $y(t)$ and the derivative $\dot{x}(t)$ and $\dot{y}(t)$ of the linear lightly damped SDOF systems for each record of the nonstationary process $\{X(t), Y(t)\}$.

II. Use Eq. (10) and (13) to determine the energy-like quantities for each record, and then estimate $< \bar{E}_{xy}(t) >$, $< \bar{D}_{xy}(t) >$ by ensemble average, and evaluate their derivatives $< \dot{\bar{E}}_{xy}(t) >$, $< \dot{\bar{D}}_{xy}(t) >$.

III. Use the S-G filter to smooth $< \bar{E}_{xy}(t) >$, $< \dot{\bar{E}}_{xy}(t) >$, and $< \bar{D}_{xy}(t) >$, $< \dot{\bar{D}}_{xy}(t) >$, obtaining smoothed data $< E_{xy}(t) >$, $< D_{xy}(t) >$, $< \dot{E}_{xy}(t) >$ and $< \dot{D}_{xy}(t) >$.

IV. Determine the Hermite interpolating polynomials $H(t)$ in Eq. (19), considering the boundary conditions in Eq. (20) – (21).

V. Choose a suitable number of N shifted Legendre polynomials, for the approximating polynomials in Eq. (22) and (23), and find the coefficients α_p and β_p by applying a nonlinear least-squares procedure using the model in Eq. (22) and (23) to fit the smoothed data $< E_{xy}(t) >$ and $< D_{xy}(t) >$, and their derivatives $< \dot{E}_{xy}(t) >$ and $< \dot{D}_{xy}(t) >$.

VI. Use the coefficients α_p and β_p in the shifted Legendre polynomials and Hermite interpolating polynomial to generate the time dependent approximate functions $\tilde{E}_{xy}(t)$ and $\tilde{D}_{xy}(t)$, and their derivatives $\dot{\tilde{E}}_{xy}(t)$ and $\dot{\tilde{D}}_{xy}(t)$

VII. Compute the evolutionary co-spectrum $C_{xy}(t, \omega_0)$, as in Eq. (12). That is,

$$\dot{\tilde{E}}_{xy}(t, \omega_0) + 2\zeta\omega_0 \tilde{E}_{xy}(t, \omega_0) = \pi C_{xy}(t, \omega_0)\,, \tag{24}$$

and evolutionary quad-spectrum $Q_{xy}(t, \omega_0)$, as in Eq. (15). That is,

$$\dot{\tilde{D}}_{xy}(t, \omega_0) + 2\zeta\omega_0 \tilde{D}_{xy}(t, \omega_0) = \pi Q_{xy}(t, \omega_0)\,. \tag{25}$$

VIII. Finally, derive the ECPS $S_{xy}(t, \omega_0)$ from $C_{xy}(t, \omega_0)$ and $Q_{xy}(t, \omega_0)$, as in Eq. (16). That is

$$S_{xy}(t, \omega_0) = C_{xy}(t, \omega_0) - iQ_{xy}(t, \omega_0)\,. \tag{26}$$

4. Numerical Applications

To assess the reliability of the proposed procedure, in this section several applications are presented. In each case, 500 samples of the bivariate processes $\{X(t), Y(t)\}$ have been synthetized from pertinent ECPS using the concept of spectral representation of stochastic processes. In this regard, the original mean data $< \bar{E}_{xy}(t) >$ and $< \bar{D}_{xy}(t) >$ are computed by an ensemble average employing Eq. (10) and (13), respectively. Then the quantities are smoothed by the S-G filter. Further, the approximate polynomials $\tilde{E}_{xy}(t)$ and $\tilde{D}_{xy}(t)$ are determined from Eq. (22) and (23), respectively. Note that in the ensuing numerical applications, a constant value of the linear damping coefficient ($2\zeta\omega_0 = 0.2$) is employed in the proposed procedure. Thus, the critical damping ratio is given as ($\zeta = 0.1/\omega_0$), with ω_0 varying in the range $[\omega_{0,min}, \omega_{0,max}]$, and the initial natural frequency $\omega_{0,min} = 1\, rad/s$, the highest value of the damping ratio is $\zeta = 0.1$. The rationale behind this choice is related to the fact that, in this manner, the computed approximate functions $\tilde{E}_{xy}(t)$ and $\tilde{D}_{xy}(t)$ both present a similar trend, with a characteristic smooth decay as time elapses (see Fig. 1) for all the values of ω_0. This represents a beneficial feature that facilitates the procedure related to the application of Eq. (24), (25) and (26).

4.1. Correlated and Modulated Bivariate Processes with separable spectra

In this section, the case of a correlated and modulated bivariate process with spectra of the Kanai-Tajimi and Clough-Penzien family is considered [1, 12]. First, the stationary bivariate process $\{A(t), B(t)\}$ is simulated by an AR algorithm [13, 14]. Then, the process $\{A(t), B(t)\}$ can then be correlated as follows

$$\begin{Bmatrix} \bar{X}(t) \\ \bar{Y}(t) \end{Bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{Bmatrix} A(t) \\ B(t) \end{Bmatrix}, \tag{27}$$

where $\{\bar{X}(t), \bar{Y}(t)\}$ is the correlated stationary process. Then, correlated bivariate nonstationary process $X(t)$ and $Y(t)$ can be generated respectively by multiplying the modulating envelope functions

$$\begin{Bmatrix} X(t) \\ Y(t) \end{Bmatrix} = \begin{Bmatrix} g_1(t)\bar{X}(t) \\ g_2(t)\bar{Y}(t) \end{Bmatrix}. \quad (28)$$

In this manner, the analytical expressions of the evolutionary co-spectrum and quad-spectrum can be given as [4, 5]

$$C_{xy}(t, \omega_0) = g_1(t)g_2(t)(cos\theta sin\theta S_1(\omega_0) + sin\theta cos\theta S_2(\omega_0)), \quad (29)$$

with

$$Q_{xy}(t, \omega_0) = 0, \quad (30)$$

where $S_1(\omega_0)$ is a two-sided power spectrum corresponding to the stationary process $A(t)$, $S_2(\omega_0)$ is a two-sided power spectrum corresponding to the stationary process $B(t)$, and $g_1(t)$ and $g_2(t)$ are modulating envelope functions. Then, Eq. (26) can be used to determine the ECPS $S_{xy}(t, \omega_0)$. In this example, both $S_1(\omega_0)$ and $S_2(\omega_0)$ are represented by the classical Kanai-Tajimi spectrum [1]

$$S_1(\omega_0) = S_{01} \frac{1 + 4\zeta_s{}^2 \left(\frac{\omega_0}{\omega_{g1}}\right)^2}{\left[\left(1 - \left(\frac{\omega_0}{\omega_{g1}}\right)^2\right)^2 + \left(2\zeta_s \frac{\omega_0}{\omega_{g1}}\right)^2\right]}, \quad (31)$$

and

$$S_2(\omega_0) = S_{02} \frac{1 + 4\zeta_s{}^2 \left(\frac{\omega_0}{\omega_{g2}}\right)^2}{\left[\left(1 - \left(\frac{\omega_0}{\omega_{g2}}\right)^2\right)^2 + \left(2\zeta_s \frac{\omega_0}{\omega_{g2}}\right)^2\right]}, \quad (32)$$

where $S_{01} = 0.04\, cm^2 s^{-3} rad^{-1}$, $\zeta_s = 0.24$, $\omega_{g1} = 10\, rad/s$, $S_{02} = 0.025\, cm^2 s^{-3} rad^{-1}$, and $\omega_{g2} = 20\, rad/s$. Further, the modulating envelope functions of the Shinozuka-Sato type [15] are given as

$$g_1(t) = \frac{e^{-0.25t} - e^{-0.5t}}{0.25}, \quad (33)$$

and

$$g_2(t) = \frac{e^{-0.4t} - e^{-0.8t}}{0.4}. \quad (34)$$

In the following, the estimated ECPS by the proposed procedure is compared to the target one in Eq. (29). Specifically, Fig. 2 shows the contour plot of the target and estimated spectrum, while in Fig. 3 comparisons in both the frequency and time domain are reported. As it can be seen, a satisfactory agreement is achieved between the target ECPS and the proposed procedure-based ECPS.

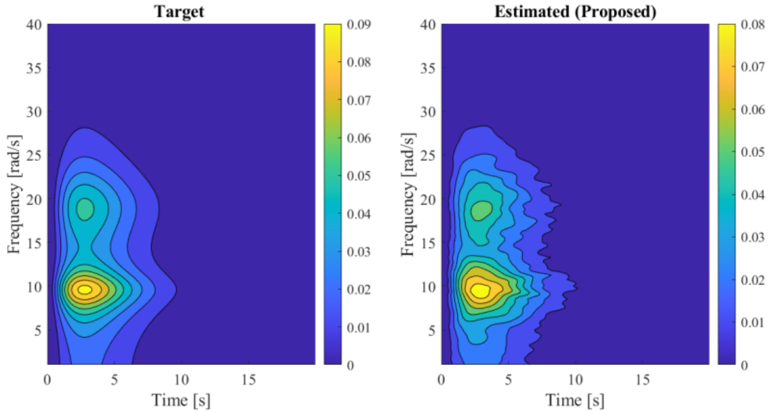
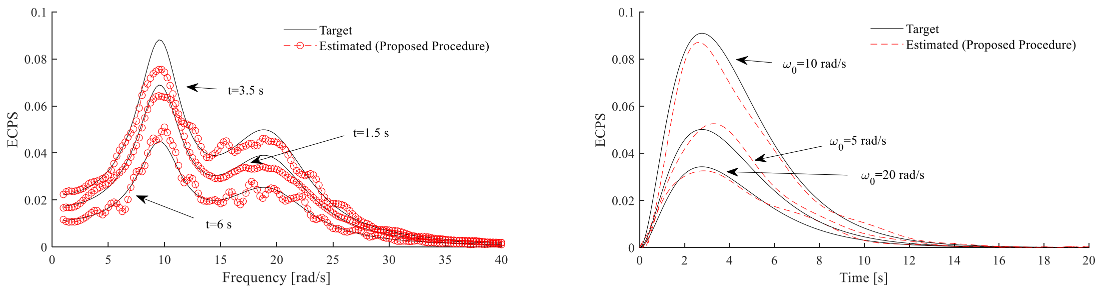


Fig. 2. Contour plot of the Kanai-Tajimi ECPS.



(a) (b)

Fig. 3. Kanai-Tajimi ECPS. (a) at different time instants; and (b) at different frequency values.

4.2. Correlated Nonstationary Processes with Non-separable Spectra

Further, to assess the reliability of the procedure for different bivariate stochastic processes, a nonstationary process compatible with the non-separable ECPS is used. In this regard, 500 uncorrelated nonstationary samples $\{\bar{X}(t), \bar{Y}(t)\}$ are generated by the spectral representation methods [3, 16], and the non-separable spectra compatible with $\{\bar{X}(t), \bar{Y}(t)\}$ are given as [17-19]

$$S_{\bar{X}}(t, \omega_0) = S_1 \left(\frac{\omega_0}{5\pi}\right)^2 e^{-0.15t} t^2 e^{-\left(\frac{\omega_0}{5\pi}\right)^2 t}, \tag{35}$$

where $S_1 = 0.4\, cm^2 s^{-3}$, and

$$S_{\bar{Y}}(t, \omega_0) = S_2 \left(\frac{\omega_0}{5\pi}\right)^2 e^{-0.15t} t^2 e^{-\left(\frac{\omega_0}{5\pi}\right)^2 t}, \tag{36}$$

where $S_2 = 0.8\, cm^2 s^{-3}$. Note that these two non-separable spectra comprise some of the main characteristics of the seismic vibrations, such as the decline of the dominant frequency over time. Further, the processes are correlated by the linear transformation in Eq. (27). In this regard, the proposed procedure is implemented to estimate the ECPS of the nonstationary processes. The estimated ECPS by the proposed procedure is compared to the target ECPS, which is given as

$$S_{xy}(t, \omega_0) = cos\theta sin\theta S_{\bar{X}}(t, \omega_0) + sin\theta cos\theta S_{\bar{Y}}(t, \omega_0)\,. \tag{37}$$

In this regard, Fig. 4 and 5 show the target ECPS in Eq. (37) and its estimates based on the proposed procedure. As it can be seen from Fig. 4 and 5, again a satisfactory agreement is achieved between the proposed procedure-based EPS and the target one.

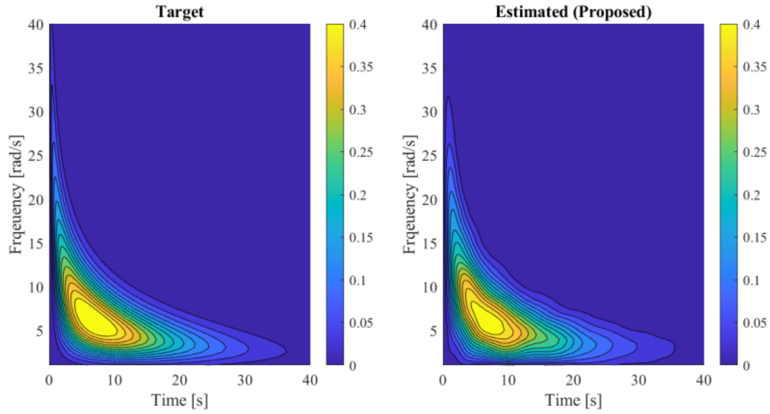
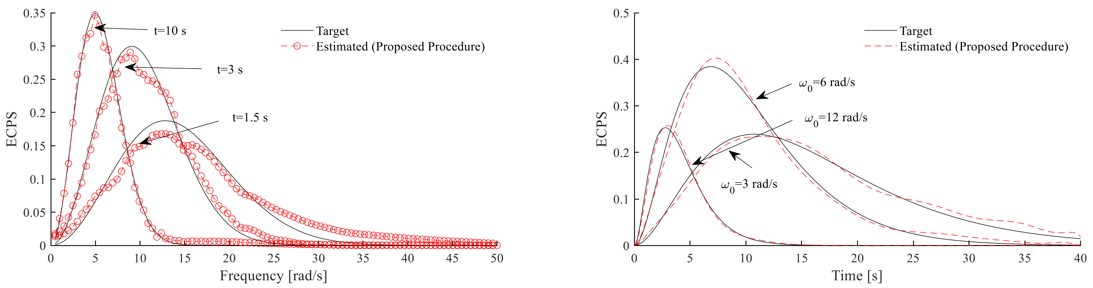


Fig. 4. Contour plot of the Non-separable ECPS.



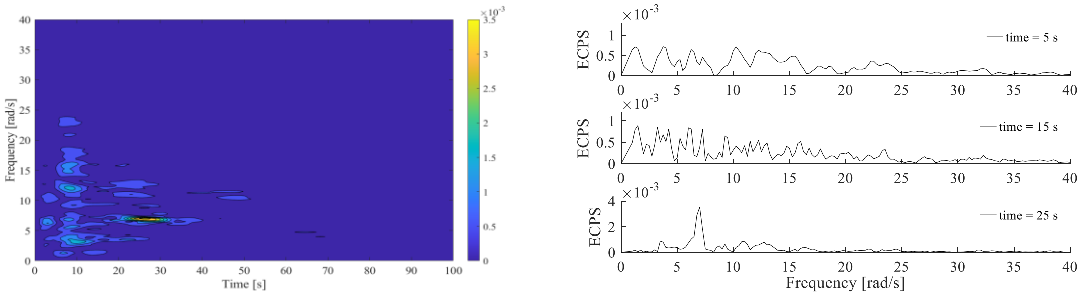
(a) (b)

Fig. 5. Non-separable ECPS (a) at different time instants; and (b) at different frequency values.

4.3. *Application to Historic Earthquake Data*

In this section, earthquake events data are used to assess if the proposed procedure is reliable for real time histories. Consider the Kocaeli earthquake on 08/17/1999, exhibiting a moment magnitude of 7.6. This strong motion caused extensive damage in Turkey. In this regard, 30 records of 0° and 30 records of 90° Kocaeli earthquake ground motions are used to estimate the ECPS by the proposed procedure.

Further, the ECPS estimated by the proposed procedure is shown in Fig. 6 and 7. As it can be seen, the proposed procedure yields result that conform with the physical attributes of the earthquake.



(a) (b)

Fig. 6. ECPS of the Kocaeli earthquake by the proposed procedure. (a) Contour plot; and (b) at different time instants.

5. Concluding Remarks

In this paper the problem of estimating evolutionary cross power spectra (ECPS) of bivariate nonstationary stochastic processes, with emphasis on earthquake records, has been addressed. For this purpose, a technique has been developed based on the determination of the expected value of appropriately introduced energy-like quantities of lightly damped SDOF linear systems, which act as filters, subject to stochastic processes. Next, the so-called and thus Savitzky-Golay (S-G) smoothing filter has been used to smooth these quantities and their derivatives to better capture the trend of the ECPS from even a limited number of records. Further, approximate polynomial models of these functions have been introduced for a more accurate estimation of the ECPS. In this manner, appropriately varying the natural frequency of the systems, the time-dependent spectral content of the bivariate nonstationary process is determined along the frequency dimension. Comparison of the results obtained using the proposed procedure with two examples, with known exact solutions, has confirmed the reliability and the accuracy of the technique. Further, records of the Kocaeli earthquake have been used to demonstrate the applicability of the proposed procedure for recorded seismic events.

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