

Reversing arrows: *Duality*

Maria Mannone

European Centre for Living Technology, Ca' Foscari University of Venice, Italy
Department of Mathematics and Computer Sciences, University of Palermo, Italy

[This is the preprint. The published article can be found here:

<https://www.tandfonline.com/doi/abs/10.1080/17513472.2021.1979910?journalCode=tmaa20>]

What do you get reversing all arrows? The drawing "Duality" is an homage to mirrors, classical art themes, and abstract mathematics.

I'm looking for beauty in the arts and beauty in science. It's a motivation for research and a motivation for life. I made the drawing *Duality* (Fig. 1) in March 2020, at the beginning of the Covid pandemic. According to Hardy, "When the world is mad, a mathematician may find in mathematics an incomparable anodyne."¹

My research uses the diagrammatic language of category theory, an abstract branch of mathematics, to compare forms and growth processes in nature and to transform them into music and musical variations. Category Theory studies in an abstract way the general aspects of mathematical structures, highlighting their transformations and interactions, and unifying them. From the initial role of "meta-mathematics," categories are more and more used as a formal language to get a general overview of different disciplines, their transformations, and their commonalities. The same mathematics can help create models in precision medicine (I worked on this topic during my postdoc at Ca' Foscari University of Venice).

In *Duality*, I tried to use categories not as a language, but as an inspiration source in itself. A simple, but fundamental, concept as "duality," can be re-read in light of classical artistic themes.

References informing my artwork come from my studies, from Ancient Greek to Theoretical Physics, and, in more recent time, Category Theory. I loved *A True Story* by Lucian of Samosata, where the only true thing was its falsity. I'm still fascinated by the imaginary creatures and landscapes by Hieronymus Bosch, and expressions of the Middle Age in Sicily, with the coexistence of cultures and morphing decorations (Monreale's both figurative and abstract decorations, also mentioned by Hermann Weyl in *Symmetry*).

My principal sources of inspiration are the Baroque, with its *horror vacui*, broken symmetries, and musical counterpoint; the modern Surrealism; Federico Fellini's movies; the visions by Maurits Cornelis Escher with his impossible geometries drew with precision; the mathematical and fantastic view of nature by Antoni Gaudí.

I love free-hand pencil drawing. I started on my own at a very young age, and then I improved my drawing ability during a course of "Scientific Illustration" at the Academy of Fine Arts of Palermo. I also used water-colors, and, moreover, software such as Mathematica to create figurative representations. However, I prefer paper and pencil because they are easy to find and easy to carry along. Pencil technique allows you to create detailed images, as well as quick sketches — I used them also to portrait travelers in Paris' métro.

¹ G. H. Hardy, *A Mathematician's Apology*, page 45. <https://www.math.ualberta.ca/mss/misc/A%20Mathematician%27s%20Apology.pdf>



Fig. 1: *Duality* by M. Mannone

The pencil drawing *Duality* is inspired by the formalism of category theory, exploiting diagrams with points and arrows, and by the fundamental concept of “duality”, described in the following.

A “category” is constituted by objects (points) and morphisms between them (arrows).² The correspondence between the properties of a category C and of the opposed category, C^{op} , with all reversed arrows, is called *duality*. This is the most abstract description of the multifaceted idea of duality, which recurs in different domains of mathematics and physics.³

² There exists the identity morphism, and the composition of morphisms is associative. The morphism mapping a category onto another category is called “functor.” Thus, there can be arrows between arrows, formalizing the idea of transformations between transformations.

³ D. Corfield, Duality as a Category-Theoretic Concept. *Studies in History and Philosophy of Modern Physics*. 59, 55–61. (2017)

The theme of arrows and inverted arrows is present throughout the image. Faces themselves are made out of arrows; there are arrows in the jewels, and the columns of the temple in the background are built with arrows. There are arrows between arrows. Arrows are mainly mirrored and reversed. Mirroring is a theme of particular relevance in physics: we can think of looking-glasses of different shapes, but also of mirroring of physical laws, of properties such as parity, with cases of parity violations (non-invariance with respect to the sign flip of one or all spatial coordinates, e.g., as for beta decay) and other broken symmetries. These topics remind us of quantum field theory, particle physics, and essential properties of nature. In category theory, structures can be transformed into their dual versions by reversing arrows. In this way, a limit (a generalization of the product) becomes a colimit (a generalization of the sum), its dual construction.

In *Duality*, limits and colimits appear in the book, in the (lady's) eyes and earrings (of the lady and her mirror image) and in the hourglasses.

The woman is looking at herself in the mirror, that reverses arrows, facial expressions, hairstyles, and the flow of time within the hourglass. The woman's left-hand gets inside the mirror, and, from the mirror, a skeletal hand gets outside.

Physical mirroring of the lady's face leads to an impossible image, where a terrified expression becomes a malicious smile.

Tragic masks are recursively seen as 2-cells (arrows between arrows); the sad mask is the dual of the happy one, and there are references to braidings, equalizers, and coequalizers, elements which appear in category theory books.

A "braiding" is an isomorphism $x \otimes y \rightarrow y \otimes x$ that can be thought of as a switch in tensor product (for categories having it as product operation).

The equalizer of two functions $f, g : X \rightarrow Y$ is the set of elements such that $f(x) = g(x)$.

The coequalizer is the dual of the equalizer.

The classic concept of mirroring is related to the theme of the Vanitas in Baroque aesthetics. The spelling of the word "Duality" is mirrored. Vertigo of mathematical abstraction leads us toward a dimension where no time exists, and where classical art meets avant-garde, which in turn, meets science.