The Sound of Swarm. Auditory Description of Swarm Robotic Movements

MARIA MANNONE, Department of Engineering, University of Palermo; DAIS - ECLT, Ca' Foscari University of Venice, Italy
VALERIA SEIDITA, Department of Engineering, University of Palermo, Italy
ANTONIO CHELLA, Department of Engineering, University of Palermo; ICAR-CNR National Research Council, Palermo, Italy

Movements of robots in a swarm can be mapped to sounds, highlighting the group behavior through the coordinated and simultaneous variations of musical parameters across time. The vice versa is also possible: sound parameters can be mapped to robotic motion parameters, giving instructions through sound. In this article, we first develop a theoretical framework to relate musical parameters such as pitch, timbre, loudness, and articulation (for each time) with robotic parameters such as position, identity, motor status, and sensor status. We propose a definition of musical spaces as Hilbert spaces, and musical paths between parameters as elements of bigroupoids, generalizing existing conceptions of musical spaces. The use of Hilbert spaces allows us to build up quantum representations of musical states, inheriting quantum computing resources, already used for robotic swarms. We present the theoretical framework and then some case studies as toy examples. In particular, we discuss a 2D video and matrix simulation with two robo-caterpillars; a 2D simulation of 10 robo-ants with Webots; a 3D simulation of three robo-fish in an underwater search&rescue mission.

CCS Concepts: • **Computer systems organization → Embedded systems**; *Redundancy*; Robotics; • **Networks** → Network reliability.

Additional Key Words and Phrases: sonification, musical spaces, robotic spaces, quantum computing, nature-inspired swarms

1 INTRODUCTION

No matter which language you speak, no matter the color of your eyes or skin, no matter the place you live in, you must have practiced some kind of music. Be it Italian belcanto, Japanese theater, Ghanian songs, Mongolian multiphonic singing, each exemplar of *Homo sapiens sapiens* in their lives had connections with some kind of musical practice.

Music and musical parameters are a word-less, universal, privileged way for humans' communications. Even though different cultures produces different kind of music, each human culture *does have* some form of music. And music always deals with some sonic material, its physical parameters, as events rhythmically distributed across time: pitch, loudness, timbre, and articulation.

Thinking through parameters as the constitutive material of music helps us draw connections with the world of natural music: singing of birds, the rhythm of seasons and heart, the whisper of leaves moved by the wind,

---

Authors' addresses: Maria Mannone, mariacaterina.mannone@unipa.it, Department of Engineering, University of Palermo; DAIS - ECLT, Ca' Foscari University of Venice, Italy; Valeria Seidita, Department of Engineering, University of Palermo, Italy, valeria.seidita@unipa.it; Antonio Chella, Department of Engineering, University of Palermo; ICAR-CNR National Research Council, Palermo, Italy, antonio.chella@unipa.it.

2573-9522/2023/5-ART $15.00
https://doi.org/10.1145/3596203

the roar of thunders. Furthermore, the interest in music-like communication is shared with other non-human animals, who can sing and learn ‘songs,’ such as singing birds [77] and seals [73].

If the variety of auditory phenomena in nature inspires human artistic and scientific research, nature provides us with another source of inspiration. The variety of swarm and self-organization and self-adaptation [94] observed in nature inspires swarm robotic developments, with decentralized autonomous robots [90]. Swarms of robots are often derived from the study of flocking birds [36], schooling fish [20], moving termites [68], and foraging ants [12, 71, 81], to name but a few. Natural and nature-inspired artificial swarms are instances of swarm intelligence [24]. Recent studies on swarm robotics [19, 23, 33, 80] also include micro-robots for medical purposes [22] and to modeling morphogenesis [82]. As an attempt to connect these two worlds, nature-inspired music and nature-inspired robotic swarms, we can wonder if robots can learn music. That is, if we can teach a robotic swarm of birds to sing a human-like music.

Is it possible to teach robots the “language” of music, or at least, use music as an intermediate language between robot-native communication and human language? Let us imagine a future human-robot communication mediated through sound-based interfaces. To make such a connection possible, we have to model basic musical spaces, with points and connections between them, and define the suitable mappings from a world to another one. The behavior of robots in a swarm can also be expressed through parameters, e.g., position, individual identification, motor state, sensors’ activation. And, of course, exchanged signals, which can be thought of as “musical messages” from a robot to another one. To make such a connection possible, we have first to formalize musical spaces and their connections.

The research field of mathematical music theory aims to formalize musical phenomena through the lens of mathematics and computer sciences.

A famous representation of pitches is, for example, provided by Lerdhal [46]. A comprehensive model to investigate pitches and voice leading has been developed by Tymoczko [85, 86].

There are attempts to generalize it through the concept of paths [39]. This study is based on paths in a musical space. Paths, meant as musical gestures, and paths of paths, meants as hypergestures, have been studied [4, 17, 41, 50, 51, 60] in light of category theory [31, 48] and homotopy [35].

Category theory is an abstract branch of mathematics, whose starting point is constituted by the notion of objects (represented as points) and morphisms between them (arrows), verifying associativity and existence of the identity. We can define nested categories, with arrow between them (functors) [48].

We can define spaces of pitches but also of other musical parameters. Xenakis [91] defined timbre spaces to compose new sounds. He also created mappings between physical objects, such as architectures, and musical sequences [92]. Architectures of Sagrada Família [49], shapes of nature, installations [52] can be translated into sound (sonified), once a suitable mapping technique has been chosen. In fact, a visual form can be seen as the path of a drawing hand, becoming in this way a collection of events over time—allowing a mapping to music, which develops itself through time.

The idea of “path” can be applied to several aspects of music; for instance we can define paths in the space of timbres [53, 55]. In a space timbres, the closer the points, the most similar the corresponding instrument timbres [32]. In such a space, timbres are points, and timbre morphings are morphisms. Mathematical details of this approach, with bands of timbres as paths of paths, and mappings from and to a similar space of colors, are described in [55]. The mathematical structure which is exploited is the bigroupoid, where the idea of “group” element is due to the invertibility of some arrows. A bigroupoid is a particular bicategory. In a bicategory, the morphism composition is not associative, but only associative up to an isomorphism [10]. The objects are the 0-cells, the morphisms are the 1-cells, and the morphisms between morphisms are the 2-cells. In a bigroupoid, the 2-cells are strictly invertible, and the 1-cells are invertible up to isomorphism [34]. In a bicategory, we can define a tensor operation [48].

In this article, we propose a generalized musical space based on tensor product of pitch space, timbre space, loudness space, and articulation space. Starting from points in each space, tensor product allows us to obtain 4-uplets of coordinates. Thus, generalized musical paths are arrows across these 4-uplets. While proposing a theoretical model and showing its sound application, we are designing a technique of *sonification* for swarms of robots. An essential framework about data sonification and robot-information sonification is provided in Section 2.

As a third natural source of inspiration, the physics of the *small*, quantum mechanics, suggested Feynman [72] the development of a new computational tool, quantum computing [84]. The quantum paradigm is more and more being applied to robotics [5, 11, 21, 44, 70], to biology [25, 26, 59, 69], and thus we can think of quantum and biological-inspired developments [97]. The recent field of quantum artificial intelligence [88] also involves the swarms: we mention instances of quantum swarm robotics [43, 47, 56] and quantum algorithm optimization [3]. Seeing quantum logic as an instance of fuzzy logic [37, 67], we can include in our discussion fuzzy robotic swarms [78].

Quantum computing is, in fact, a valuable computational tool, which is drawing the attention of researchers in different countries and disciplines. If we draw a connection between musical spaces and Hilbert spaces (used in quantum mechanics), we can use quantum computing to relate human polyphonic music and robotic movements. We consider products of Hilbert spaces to generalize musical and swarm robotic-movement spaces.[1] We also ran a validation test, asking participants to qualitatively judge pleasantness and meaningfulness of the audio samples, regarding clarity of retrieved information. Participants, experts in music, were first asked to listen to the audio files only, and then jointly with the corresponding videos. The obtained feedback is discussed, and it can contributed to the enhancement of our sonification strategy.

Summarizing, the intended contribution is a math-derived framework for a descriptive swarm-robotic sonification. The realized contribution is the proposed formalism based on groupoids and Hilbert spaces, some examples of application to illustrate the proposed ideas, and their validation via a user test.

The article is organized as follows. In Section 2, we summarize relevant literature on data sonification, introducing robot's information sonification and then swarm robotics sonification. A paragraph on approaches of sound studies and quantum mechanics/computing support quantum theory of music. In Section 3, we develop our theoretical approach, and in Section 4, we present our case studies, along with the results of a validation test. We discuss the obtained results and present our conclusions in Section 5.

In the following section, we present some formalism, sketching the proof that our spaces are (real) Hilbert spaces. Before moving on, let us provide more information on robotic swarms and on early attempts of sonifying their information.

2 LITERATURE REVIEW: SONIFICATION AND QUANTUM APPROACHES TO SOUND

In our research, we are interested in a symbolic-based mapping, giving a perceptually-relevant musical outcome. For this reason, we are making heavy reference to studies in mathematical music theory [50]. In the following, we give an overview of data-based sonification approaches, including examples of sonification for robots. Paragraph 2.3 presents the specific framework of our study, with robotic-swarm sonification.

2.1 Data sonification

Here, we summarize selected approaches to data sonification. Two major archives some of the literature is took from are the ICAD archive of auditory-display studies [40] and the Data Sonification Archive of Boston [45].

---

[1] In [2], the delicate problem of human cognition is faced, showing inextricable connections, as an entanglement of ideas, violating a more 'simple' representation of concepts as tensor product of Hilbert spaces representing concepts. However, here we deal with parameter spaces, rather than with concept spaces. Of course, if we model the *thinking of* these spaces, we would get entanglement.

Examples of data sonification range from hypertension [1] to galaxies [7]. On hypertension [1], the authors drew a correspondence between frequency of deaths due to heart attacks and sound beat, with an increased loudness in correspondence of suddenness of events. Concerning galaxies [7], Bardelli et al. focused on a dataset with information on stellar masses, star formation rates, redshifts. The density of galaxies is rendered through a synthetic sound texture, a dense stream of events. Outlier events lead to independent modulations, generating complex sounds. According to [15], "Sonification is the use of non-speech audio to represent data." This is one of the role of sonification, as cited in [38, chapter 2]. Examples of sonification for data exploration purposes are mainly auditory graphs [15, 27, 83] and model-based sonifications [38]. Brown and Brewster [15] sonified line graphs containing two data series, and made tests with different musical instruments. The authors had listeners sketching lines according to what they were hearing. The participants were to draw sketches of sonified graphs of data series, after listening to them. Then, it was evaluated the degree of precision the key elements were identified with. Flowers and Hauer [27] focused on the potential of auditory versus visual data representation, comparing visual and auditory histograms. They found that auditory data representation can be a valid alternative to the more traditional visual one, because the core information was effectively retrieved by listeners. Smith and Walker [83] investigated the benefits of a sonification to data understanding, comparing discrete and continuous stimuli. Finally, model-based sonification [38, 399] is defined as "a sonification technique that takes a particular look at how acoustic responses are generated in response to the user's actions, and offers a framework to govern how these insights can be carried over to data sonification."

2.2 Robots' sonification

Examples of sonification of robotic information range from parameters of motion to robotic expressivity enhancement. For instance, sonification can enhance robotic expressivity [28, 96]. Frid and Bresin [28] compared the effects on listeners of non-specifically designed sounds applied to a NAO's movements, versus a specific, blended sonification. In the second case, the participants were able to more clearly identify an emotional message contained in sounds, distinguishing between "frustration" and "joy." The experiment run by Zhang et al. on robotic motion sonification [96], with real-time synthesis, proved that a robot can be perceived as "happier" and more energetic. According to Zahray and co-authors [93], who focus on real-time music improvisation with a robot, for "human-robot collaboration, it is necessary not only to convey sufficient information about movements, but also to convey that information in a pleasing and even social way to enhance the human-robot relationship." While describing the sonification strategies for the robot Shimon, Zahray et al. consider the following criteria: "enjoyment, ease of use, and conveyance of movement information." In [93], it is considered sonification in relation to emotions. As we will describe later, in our framework the "emotions" could be inspired by the harmonic superpositions and musical structures obtained as output from the robotic motion. For Shimon, high vertical position was mapped to higher pitch, and horizontal movements corresponded to timbre variation, from muted to buzzy sound. For us, timbre will just be used as a label to characterize each robot of the swarm. In an alternative approach, we may associate other properties with timbres. For example, in the case of robots of different sizes, we can have a darker timbre to the larger robots, and a brighter sound for the lighter ones, recalling a physics-inspired cross-modal similarity. Moroni and Monzolli [66] used evolutionary algorithms to create music to control robots. Zhang and co-authors [96] investigated the role of sonification for interactions between robots and children with autism. Children's learning was the topic of another study: Write et al. analyzed to which extend non-verbal feedback, as opposed to verbal feedback, can help children solve an intelligence game [89]. In a completely different scenario, the sonification of robotic movement can enhance the perception of a robot's price [95]. Finally, a codified sound output can be used as a feedback from fleets of robots in farming [42]. In particular, Kamboj and co-authors [42] examine the helpfulness of an auditory feedback from robots' fleets in farming, to quickly inform the human user of a local damage or of any stuck piece. This last example introduces sonification for swarms of robots, which we analyze in more detail in the next paragraph.

2.3 Robotic swarms and their sonification

We find swarms in natural and artificial, nature-inspired environments. Swarm intelligence is an emerging property [14], it is more than the sum of the actions of single components.

A robotic swarm is constituted by a collection of interacting robots, where each robot is performing a simple task, but the swarm as a whole is accomplishing a complex task [14, 23, 33]. The characteristics of a robotic swarm are mainly: redundancy (loosing one robot does not stop the swarm), flexibility (robots are not specialized), and scalability (efficacy of algorithms is size-invariant).

There have been some early attempts to associate sounds with robotic swarms. They include rhythmic pattern generation according to neighboring-robot interactions [13] and identification of robots' individual roles through sound [61]. Other applications involved playing a pre-composed polyphonic music, where robots interpret each role. This is similar to our "inverse" approach, with motion instructions given though input chords. Music has also been an inspiration for motion temporal constraints of robotic motion [16]. In a work by McLurkin [62], harmonic structures are translated into geometries to be displayed by the swarm. Music can thus provide a feedback of multi-robot behavior [79], and, vice versa, robots can aggregate according to the similarity of their emitted timbre [62]. In our research, we first model musical spaces *and* robotic spaces considering structural similarities, and then we propose some sonified examples, where position proximity of a slice of space corresponds to pitch proximity. In the system developed by Santos [79], the user can choose a song, entering the information onto the first robot. It then starts working autonomously, interacting only with their first neighbors. At the beginning, all robots play the same song. Then, the user asks robots to use different timbres. The swarm thus divides itself according to timbre similarity, creating spatial clusters. In this application, there are robot leaders, while in our study all robots are peers, and the user is not supposed to entry at all. Another characterizing element of [79] is the paradigm of ants, with the message exchange inspired by the pheromone gradient system. In that research, there are only local communications from robot to robot. In the next Section, we will present our theoretical approach, that formalizes and generalizes a preliminary study presented during the RO-MAN conference [57].

2.4 Contaminations between sound and quantum theory

In this paragraph, we briefly summarize both relevant and recent studies between quantum theory and music/sound signal processing. Core ideas of quantum mechanics fascinated composers and music researchers. This is the case of Iannis Xenakis [91], who was working with agglomerates and density representations of musical structures. Xenakis also proposed a "synthesis" of music as an additive synthesis from particles, where each particle was constituted by sound units with a specific timbre and duration. His use of transition-probability matrices can be compared with a probability-based approach, close to probability amplitudes of states in quantum mechanics. Curtis Roads, in his works "AI and Music" [74] and "MicroSound" [75], referenced and discussed the "particle synthesis" approach by Xenakis. Another analogy between the world of quantum and of music had been made already in 1947 by the Nobel prize Dennis Gabor, who used wavelets and quantum formalism to describe the basics of acoustics [30]. More recent works are about synthesizers based on quantum measurements [18]; rules in tonal music analyzed in terms of quantum forces [9]; melodies and notes studied in a quantized way and chords as superpositions of states [29]; music cognition studies with quantum formalism [8]; quantum-based systems [64]; criteria to measure the degree of memory of quantum states adapted to musical structures [54], and voice analyzed with quantum measures [76], design and ambiguity in the arts investigated in light of the quantum paradigm [87]. A collection of quantum perspectives on humanities is presented in the book edited by Eduardo R. Miranda [65].

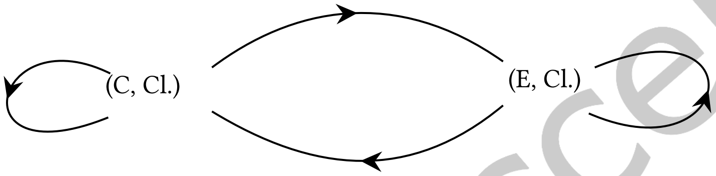
3 THEORETICAL APPROACH

3.1 Tensor product of spaces and Hilbert spaces

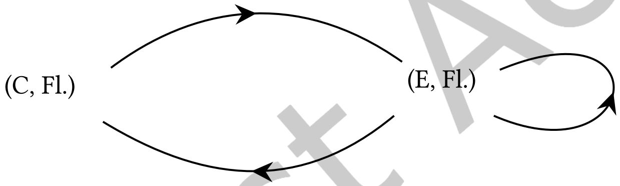
Let us consider two points in the Euclidean space of pitches [46], namely C5 and E5. Similarly, we can choose two points in the Euclidean space of timbres [32], considering for the sake of simplicity two instruments, let's say flute (Fl) and clarinet (CL). We can define paths between points in each space. To define paths between pairs of points in these spaces, we can take the tensor product of the subspaces containing the points:

$$\begin{pmatrix} C & 0 \\ 0 & E \end{pmatrix} \otimes \begin{pmatrix} Fl & 0 \\ 0 & Cl \end{pmatrix} = \begin{pmatrix} (C, Fl.) & 0 & 0 & 0 \\ 0 & (C, Cl.) & 0 & 0 \\ 0 & 0 & (E, Fl.) & 0 \\ 0 & 0 & 0 & (E, Cl.) \end{pmatrix}$$

With the so-obtained pairs of points, we can define paths. Let us imagine two different entities: a clarinet and a flute players switching between C and E. Let us consider an object moving along these lines, as an automaton: we can thus connect this idea with some robotic paths. We can think of robotic paths in the space, and also of their paths in parameters spaces (motor on/off, sensor on/off, single robot activation and recognition...) In our example, the flute can be associated with R_1, and the clarinet with R_2. A possible choice of path(s) for the clarinet is the following:



And, for the flute:



This, with an added time information, can lead to a musical score. The paths we have been defining are invertible. Because the composition of paths is not associative [35], aiming to define some categorical structure we should rather consider classes of paths. A *bicategory* [10] is a structure where morphism composition is not associative, but only associative up to an isomorphism. In a bicategory, the objects are the 0-cells, the morphisms are the 1-cells, and the morphisms between morphisms are the 2-cells. A *bigroupoid* is a bicategory whose "2-cells are strictly invertible, and the 1-cells are invertible up to coherent isomorphism" [34]. Thus, we can focus on bigroupoids, already used to define musical spaces, and in particular, timbre space [55]. For reasons of space, we omit here the precise definitions and proofs of all conditions to be satisfied by a bigroupoid [34, 55]. We consider as bigroupoids the paths connecting points in musical spaces. If we prove that the spaces where these points belong to are Hilbert spaces, then we can relate our musical spaces with the fundamental spaces of quantum mechanics and quantum computing. We can verify that Hilbert axioms are satisfied.

While we do not enter into formal details, we mention recent studies on groupoids on the structure of tensor product of Hilbert spaces [6]. The transition from groupoids to Hilbert spaces is defined as a "degroupoidification," where the structure is lost. The inverse process of path creation is thus a groupoidification. Baez and coauthors

[6] focus on the *span*, which has a monoidal structure, taking the tensor product from groupoids. A span is a triangular structure defined as $X \longrightarrow A$, $X \longrightarrow B$, where $s : X \longrightarrow A$, $t : X \longrightarrow B$ are functors. It represents the set of histories, that is, of paths, leading a system from A to B.

Using $\mathcal{H}$ to denote Hilbert spaces, we can finally define our generalized musical space as:

$$\bigotimes_{i=1}^{4} \mathcal{H}_i^M(t) = \mathcal{H}_P^M \otimes \mathcal{H}_L^M \otimes \mathcal{H}_T^M \otimes \mathcal{H}_A^M, \tag{1}$$

where:

- $\mathcal{H}_P^M$ is the pitch space;
- $\mathcal{H}_L^M$ is the loudness space;
- $\mathcal{H}_T^M$ is the timbre space;
- $\mathcal{H}_A^M$ is the articulation space.

We define the swarm robotic space as:

$$\bigotimes_{j=1}^{4} \mathcal{H}_j^R(t) = \mathcal{H}_C^R \otimes \mathcal{H}_M^R \otimes \mathcal{H}_i^R \otimes \mathcal{H}_S^R, \tag{2}$$

where:

- $\mathcal{H}_C^R$ is the space of positions through coordinates;
- $\mathcal{H}_M^R$ is motor options space;
- $\mathcal{H}_i^R$ is the robot identities space;
- $\mathcal{H}_S^R$ is the sensors space.

Summarizing:

- $\bigotimes_{i=1}^{4} \mathcal{H}_i^M(t)$ is the generalized musical space;
- $\bigotimes_{j=1}^{4} \mathcal{H}_j^R(t)$ is the swarm robotic parameter space, with just a few essential parameters.

Sounds can be obtained as feedback from robotic movements. Conversely, sound can be used to give instructions to robots.

The sonification can be expressed as an arrow:

$$S : \bigotimes_{i=j}^{4} \mathcal{H}_j^R(t) \rightarrow \bigotimes_{i=1}^{4} \mathcal{H}_i^M(t), \tag{3}$$

while sound-based robotic control can be expressed as the arrow:

$$C : \bigotimes_{i=1}^{4} \mathcal{H}_i^M(t) \rightarrow \bigotimes_{i=j}^{4} \mathcal{H}_j^R(t). \tag{4}$$

In particular, the space of positions through coordinates can be mapped to the pitch space: $\mathcal{H}_C^R \rightarrow \mathcal{H}_P^M(t)$, following the projections along axis of the Lerdhal spiral pitch space [46] and the perceptive association of pitch with space displacement; the information on motor (on, off, max power, less power...) can be mapped to loudness, through the instinctive association between energy and loudness: $\mathcal{H}_C^R \rightarrow \mathcal{H}_P^M(t)$. Each robot can be identified through a specific timbre, for example associating each robot with a musical instrument, and thus: $\mathcal{H}_i^R \rightarrow \mathcal{H}_T^M(t)$. Finally, the on/off of sensors, and the choice itself of specific sensors, can be rendered with the activation of non-activation of specific musical articulations: $\mathcal{H}_S^R \rightarrow \mathcal{H}_A^M(t)$. The complete deactivation of a single robot could be rendered with a silence from the corresponding musical instrument, and vice versa.

Inverting all arrows, we can, in fact, use music to give instructions to robots. In category theory, inversion of arrows allows one to create *dual structures*. While, in mathematics, dual structures are not always existing, in our framework we did not make any hypothesis which could impede the inversion of arrows between the tensor product of musical spaces and the tensor product of robotic spaces. Consequently, we can invert arrows, interacting with robots and giving them instructions through sound. In terms of code, this can be translated as a user-given list of parameters, i.e., pitches, and the consequent attribution of physical parameters to robots, i.e., space positions to be reached.

In the following paragraph, we describe a matrix approach to robotic swarms, and we discuss the use of quantum logic gates for a search and rescue mission.

3.2 Matrices

We can relate the local behavior of swarm elements, with their individual movements, 'perception,' and decisions, with the swarm global behavior, which is an emerging effect from local actions and interactions.

In [56, 58], it has been proposed a matrix representation to relate local with global behavior, focusing in particular on pairwise interaction. For a swarm of N robots, we can define a block matrix (eq. 5), whose diagonal blocks R_1, ..., R_N contain information on each single robot, as its 'self-awareness': position, direction, reward (we define the reward in the following paragraph), sensor activation, message sending ON or OFF. The off-diagonal blocks $(R_1 * R_2)$, ..., $(R_N * R_{N-1})$ represent pairwise interactions amongst swarm robots. That is, the i-th robot R_i sends a message to the j-th robot R_j at time t with probability amplitude to stay in a certain position and probability amplitude to be successful. From this information, the R_j robots deduces a possible position to be reached at $t + 1$. Such a decision mechanism can be modeled through a logic gate, as it is described in Section 3.3.

$$S_N(t) = \begin{pmatrix} R_1 & (R_1 * R_2) & \dots & (R_1 * R_N) \\ (R_2 * R_1) & R_2 & \dots & (R_2 * R_N) \\ \dots & \dots & \dots & \dots \\ (R_N * R_1) & (R_N * R_2) & \dots & R_N \end{pmatrix}(t) \tag{5}$$

3.3 Logic gate

Let us summarize the logic gate as it is proposed in [56] for one-dimensional problem, focusing on a 'search' task. The idea of quantum superposition enters characterizes the definition of positions and of reward as an individual measure of target proximity. We define quantized positions along x and y as 'up' (1) and 'down' (0). A generic position is given by the probability amplitude to stay in 0 or 1, respectively. We also quantize the outcome of individual exploration, as 'success' (1) or 'failure' (0). Generic outcome are given by a quantum superposition of success and failure. For the sake of simplicity, in our simulations, we computed the 'reward' as 1 - Euclidean distance from the target: the smaller the distance, the closer the target, the higher the reward.

With this idea in mind, we build a logic gate. We suppose a broadcast communication for our robotic swarm: each robot sends messages with its position and reward in terms of probability amplitudes for up/down along x and y, and yes/no regarding the reward. Similarly, each robot receives messages from all the other robots.

The proposed logic gate (Table 1) involves pairs of robots: the i-th robot (R_i) communicates position and reward, and the j-th robot (R_j), according to the probability amplitude of 'success' of R_i, decides to reach it or to explore other regions of the experimental arena.

The inputs are the xy-position and reward of the i-th robot R_i, and the outputs include the suggested xy-position of the j-th robot R_j, but not its reward, which is calculated once R_j gets to the suggested new position [58]. The gate also returns the reward of R_i by just copying it. Restricting our analysis to cases with reward = $|1\rangle$, the gate is reversible. The truth table 1 contains the following elements:

- $|q_0\rangle$ is the x-position of R_i at t_1;

- $|q_1\rangle$ is the y-position of R_i at t_1;
- $|q_2\rangle$ is the reward of R_i at t_1;
- $|q_3\rangle$ is the suggested x-position of R_j at t_2;
- $|q_4\rangle$ is the suggested y-position of R_j at t_2.

Table 1. Truth tables for two robots R_i, R_j on the xy-plane

q_0	q_1	q_2	q_4	q_3	q_2
x-pos	y-pos	reward	y-pos	x-pos	reward
R_i	R_i	R_i	R_j	R_j	R_i
0	0	0	0/1	0/1	0
0	0	1	0	0	1
0	1	0	0/1	0/1	0
0	1	1	1	0	1
1	1	1	1	1	1
1	0	0	0/1	0/1	0
1	1	0	0/1	0/1	0
1	0	1	0	1	1

If we have N robots, we should in principle compute $N!$-times the logic gate. However, we compute it just once by letting only the most successful robot enter the circuit.

To test our logic gate, we imagined a 2D scenario and we evaluated pros and cons. We run some tests using only the logic gate in a loop.

We considered an ant-inspired search&rescue scenario, with ten robots starting from close positions at t_0, as ants sorting out from a nest, looking for food, here represented by a target at a certain distance. The robots can acknowledge the distance from the target through smell or sound, according to the nature of the considered scenario.

The minimal algorithm we choose for our search and rescue 2D example is the pseudocode 1, with a quantum circuit and a classical step of initial reshuffling, needing to ensure convergence. More discussion on this topic is given in Section 5.

Algorithm 1 quantum search and rescue

```
Set initial positions of robots as state superpositions
if All robots have a too-low reward then
    Random reshuffling of positions
end if
Finding the most successful robot, which enters the logic gate
Quantum circuit implementing the logic gate (called here 'gate' for short)
```

In an optional step, all robots can be asked to reach the most successful robot. However, in 3 our of 4 trials, this was not needed, so we can safely omit it. Overall, we can distinguish between a *local* dimension of the algorithm, and a *global* one; see Figure 1.

While the logic circuit models pairwise interactions only, the modeling of local-global interactions can be obtained from the matrices described in [56]. Of these matrices, only the off-diagonal blocks contain the logic-gate information. Hidden in the overall block-matrix representation, there is the local-global behavior and a connection with the environment. For instance, a change of the target position determines a change of the

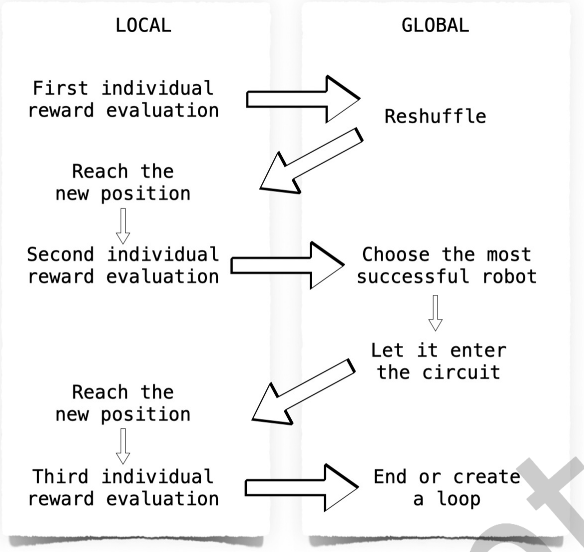


Fig. 1. Information exchange, in our algorithm, between local and global levels of a robotic swarm.

probability amplitude of R_1 to be successful. R_1, thus, in its broadcast message, will include its position (unchanged) and its reward (changed). Similarly, all other robots will communicate their changed reward, and thus all the N diagonal blocks $R_1, ..., R_N$ will have a different element describing their reward. Because each robot communicates to the other robots its position and reward, also the $N!$ non-diagonal blocks $R_i * R_j$ will change. In fact, the content of the message will be different, as well as the suggested new position for the following time point. Because a change of the target position, or a more precise information on it, can be triggered by external factors, we can connect environmental information and issues with the local robotic behavior, which in turn, determines the overall swarm behavior. As environmental factors, we can think of the wind shifting a food source or bringing the smell directly to the ants' nest, or a falling rock blocking the visual toward the goal, or a sudden noise masking the sound coming from the target.

We can also envisage the following circular behavior, triggered by, let's say, an environmental effect on the food position at t:

→ changes of the single robot acting on the environment (e.g., different pheromone production according to an environmental food change)
→ change of behavior for all robots, modifying individual sub-matrices
→ change of pairwise interactions, modifying off-diagonal sub-matrices
→ change of individual robot positions at $t + 1$
→ change of the pheromone-information released.

This representations reminds to the studies of [63] concerning feedback in the domain of social sciences.

4 EXAMPLES OF IMPLEMENTATIONS

To illustrate how the proposed sonification scheme works, we present here four examples of "regular" application, i.e., robots→sounds, and one "inverse" strategy, i.e., sounds→robots. The sonification examples are presented

Table 2. Characteristics of the selected participants to the experiment.

index	geography	profession
$P1$	Croatia-UK	computer scientist and musician/improviser, founder and leader of the Female Laptop Orchestra, and former professor at the University of Greenwich
$P2$	Italy	orchestral conductor and former professor at the Conservatory of Palermo
$P3$	UK	professor at the University of Plymouth, computer scientist, composer, and performer
$P4$	Italy-USA	professor at the Berkeley University, mathematician and composer
$P5$	Nigeria	mathematician and musician, working at the Irish Business School
$P6$	Italy	composer, professor at the Conservatory of Palermo, and supervisor of Teatro Massimo
$P7$	Canada-USA	professor of opera theater at the University of Minnesota

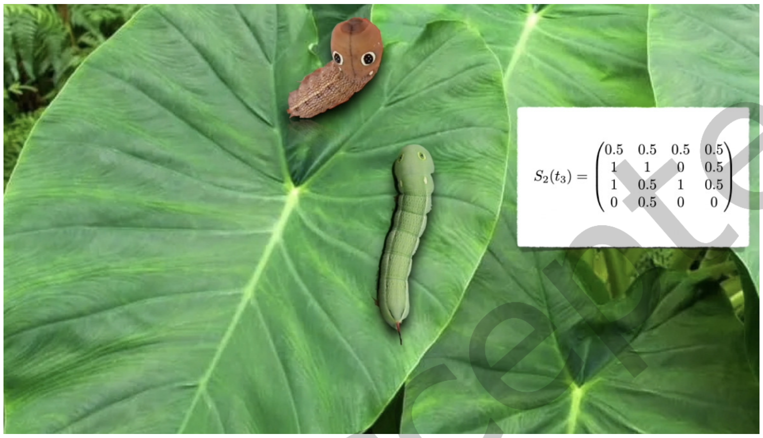
according to the their increasing complexity. We first focus on time, pitch, and timbre, adding each time one dimension to the motion: 1D caterpillar paths, 2D ant-inspired paths, 3D robotic-fish paths. Then, we take into account all parameters, that is, time, pitch, timbre, loudness, and articulation, proposing a simulation with approximately 2D water-surface robots. In addition, motion in the second and third examples is modeled according to the logic gate presented in paragraph 3.3, for 2D and 3D motion, respectively. In the first example, we propose a different individual motion scheme, where the two robots try to explore the leaf avoiding each other. In the fourth example, we have two robots following parallel paths, and another robot moving in the opposite way, representing an approximate exchange of motion and position, highlighted via sound parameters. In the fifth example, we present an application of the inverse code, where notes are given as input and robotic positions are obtained as the output. This fifth example exemplifies how sound can be used as a controller for a swarm-robotic motion, and not merely as an output of it. This last example opens new scenarios with human-friendly robot-human interaction systems based on chords and chord sequences.

To qualitatively assess the degree of pleasantness of the sonified samples and meaningfulness in terms of clarity of retrieved information, we run a first validation test. We implemented a Google form, which can be accessed from this link. The results are available at this folder. We asked participants to listen to our four examples of sonification, first without and then with the corresponding videos, and rate pleasantness, understandability, and match between music and robotic motion. Before seeing the videos, participants rated the degree of pleasantness and understandability of musical sequences, and they expressed eventual mind images they visualized. After seeing the corresponding videos, they rated the match between music and motion, and again the understandability of musical sequences, and its pleasantness. We used a scale from 1 to 5, with 5 associated to the best result. The participants did not receive any previous instruction regarding content and context of the experiment and the simulations, expect the overall, generic title "this is a test on robotic-motion sonification."

In particular, we focused on seven selected participants with professional expertise in the musical and interdisciplinary domains, from the USA, Europe, and Africa. We are grateful to them for the time and attention they dedicated to our test. For privacy reasons, we are indicating the names of the participants as $P1$–$P7$. Geographic provenience and profession of each participant are listed in Table **??**

4.1 1D: A pair of caterpillars

Let us present a first example of sonification of robotic movements. We consider an approximation of one-dimensional movement of two robotic versions of *Hippotion celerio* at the caterpillar state. Figure 2 shows a screenshot from the proposed video, with the explicit matrices. The law of swarm motion in this case is the following: the caterpillars have to explore the line-space, but they should preferably avoid ending up in the same point. Thus, their reward is 1 when they are separated. Their motion is a balance between the need to explore and the need to stay clear from the other individual. In this example, we do not consider a quantum logic gate, for the sake of simplicity.



$$S_2(t_3) = \begin{pmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 1 & 1 & 0 & 0.5 \\ 1 & 0.5 & 1 & 0.5 \\ 0 & 0.5 & 0 & 0 \end{pmatrix}$$

Fig. 2. A screenshot from the video of our 1D simulation, with pictures of *Hippotion celerio.*

In this example, we identify R_1 (green caterpillar) with the timbre of flute, and R_2 (red caterpillar) with the timbre of oboe. We consider position $|0\rangle$ as the pitch G4, and $|1\rangle$ as B4. Because of the simplicity of this example, we stressed the movements steps through musical glissando from one pitch to the other one. Glissando represents thus a state superposition of positions. We considered loudness $|1\rangle$ ($|0\rangle$) when the motors are ON (OFF). We do not propose changes in articulations, because we consider robotic sensor as ON for the entire simulation. With this information, we can build up paths in the Hilbert spaces across the points (flute, G), (flute, B) for the first robot, and (oboe, G), (oboe, B) for the second one. Diagonal sub-matrices are, in this example, of the form:

$$R_1(t) = \begin{pmatrix} x_1(t) & reward_1(t) \\ motor_1(t) & \dot{x}_1(t) \end{pmatrix}, \tag{6}$$

where the information is mainly: 'where I am' ($x(t)$)—each robot has no incertitude on its own position and reward—'where I go' ($\dot{x}(t)$), 'what I found' ($reward(t)$). Similarly for $R_2(t)$. The interaction sub-matrices are the following ones:

$$(R_1 * R_2)(t) = \begin{pmatrix} \beta_1^x(t) & \delta_1(t) \\ 0 & \beta_2^x(t+1) \end{pmatrix}, \tag{7}$$

and

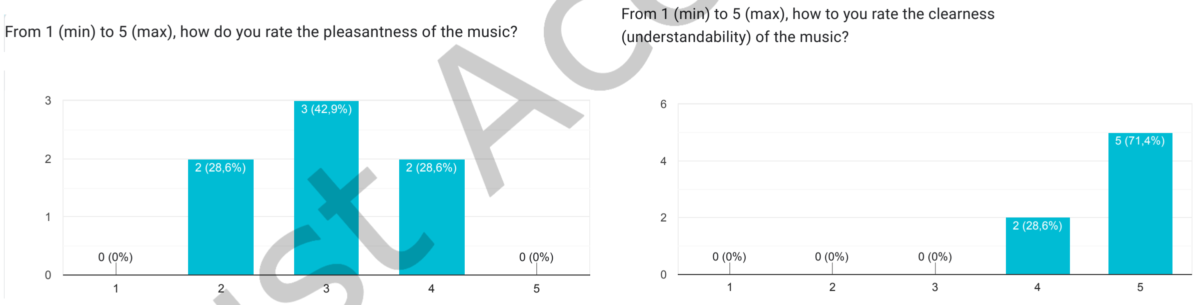
$$(R_2 * R_1)(t) = \begin{pmatrix} \beta_2^x(t) & \delta_2(t) \\ 0 & \beta_1^x(t+1) \end{pmatrix}. \tag{8}$$

The complete matrix at time t is thus:

$$S_2(t) = \begin{pmatrix} x_1(t) & reward_1(t) & \beta_1^x(t) & \delta_1(t) \\ motor_1(t) & \dot{x}_1(t) & 0 & \beta_2^x(t+1) \\ \beta_2^x(t) & \delta_2(t) & x_2(t) & reward_2(t) \\ 0 & \beta_1^x(t+1) & motor_2(t) & \dot{x}_2(t) \end{pmatrix} \quad (9)$$

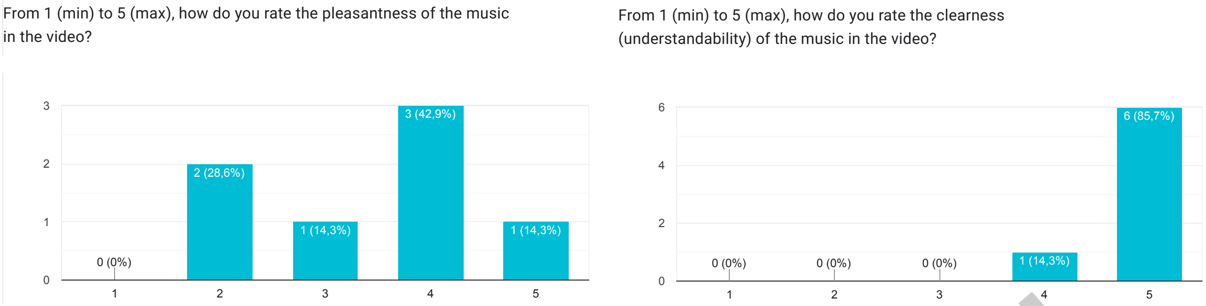
In this way, we obtain the matrices shown in the video simulation. We created a musical *crescendo* in correspondence of the motor transitions, from OFF (0) to ON (1), as $0 \longrightarrow 0.1 \longrightarrow 1$, and *diminuendo* for the inverse movement. We also implemented a discrete-pitch *glissando* between G and B, the endpoints of the imaginary segment along which caterpillars are moving. In the next examples, we did not include any glissando for reasons of auditory clarity.

The first video was characterized by an alternation of movements along a line, with rising and lowering chromatic musical scales. The qualitative answers of participants confirm that they mostly grasped the idea: *Colour green, stairs, ascending higher*; *Uccellini che si inseguono* (Italian for *Little birds chasing each other*); *Things going up and down*, *Canon*, *Mountains* (probably for the idea of rising and lowering paths on them). One of the participants had not a clear visualization, and another one indicated *Something beautiful but possibly dangerous - animal or insects*, showing a clear insight on a small-sized motion of insect-like robots. The majority of participants rated 3/5 the pleasantness of music (Figure 3a), and five participants gave it the maximum rate of understandability (Figure 3b). Once the video has been shown, the results were spread, with rates of pleasantness ranging from 2 to 5/5, and the majority assessed on 4/5 (Figure 4a). Clearness remained stable, with six participants assessed on 5/5, and just one on 4/5 (Figure 4b). The match between music and image was rated well: four participants gave it the maximum value, and the others were spread between 3/5 (one person) and 4/5 (two persons; see Figure 5). Table 3 breaks down the result into the individual participants' replies.



(a) (b)

Fig. 3. First sonification, ratings before watching the video.



(a) (b)

Fig. 4. First sonification, ratings after having watched the video.

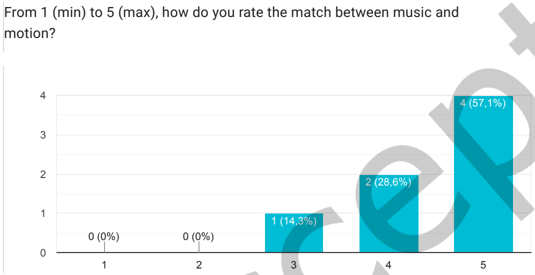


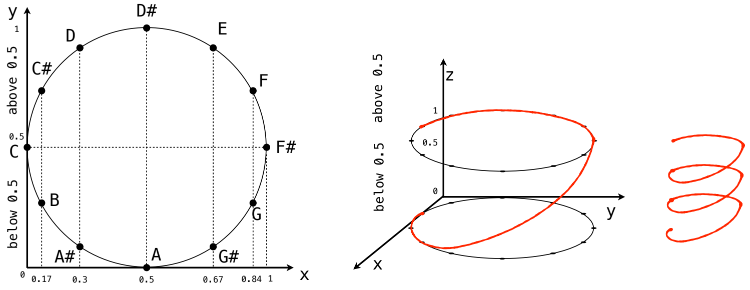
Fig. 5. First sonification, rating of music-motion match.

Table 3. Replies of participants $P1$-$P7$ to the questions on the first test

	#1	#2	#3	#4	#5	open
$P1$	4	5	4	5	5	Colour green, stairs , ascending higher
$P2$	4	5	5	5	5	Uccellini che si inseguono
$P3$	3	5	4	5	5	Things going up and down, at different speeds.
$P4$	2	5	2	5	3	no clear images appeared to me while listening; I tried, unsuccesfully, several times
$P5$	3	4	4	5	5	Canon... Overlay
$P6$	2	4	2	4	4	sea
$P7$	3	5	3	5	4	something beautiful but possibly dangerous - animal or insect

4.2 2D: Ten little ants

In this second example, we simulate a foraging ant-inspired 2D simulation, with 10 robots, in form of e-pucks. For the sake of simplicity, we used the same timbre for all robots. Pitch choice has been performed according to the structure of Figure 6. Our second example includes only movements on the plane, while our third example (section 4.3) also involves movements across the space. Spiral-like octave distribution is suggested by Lerdhal and other music theorists [46]. For the sake of simplicity, we divided the vertical axis into two parts. Positions belonging to the lower side have been rendered with one octave-lower pitches.



(a) (b) (c)

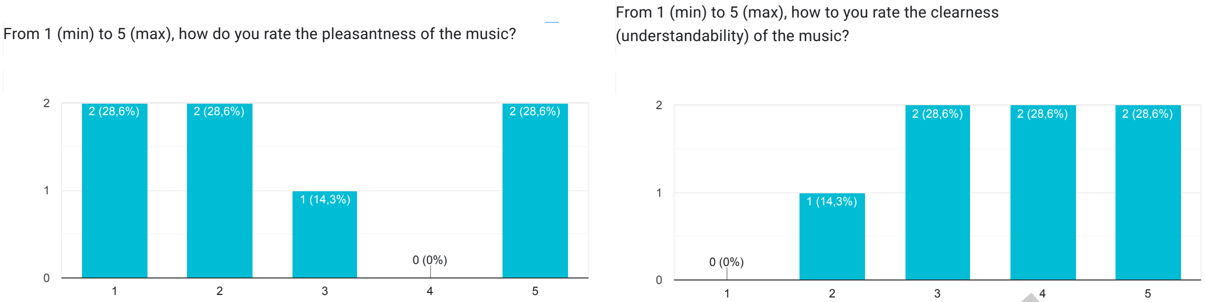
Fig. 6. Pitch choice structure

Table 4. Replies of participants $P1$-$P7$ to the questions on the second test

	#1	#2	#3	#4	#5	open
*P*1	2	5	1	5	4	Trumpet players
*P*2	5	4	5	5	5	Tensione e contrasto
*P*3	5	3	4	4	5	A number of conflicting things.
*P*4	2	5	2	5	3	a castle
*P*5	3	3	4	4	4	Trio... Sonata
*P*6	1	2	1	4	4	bottom of the dark sea
*P*7	1	4	1	5	5	a violent death or threat

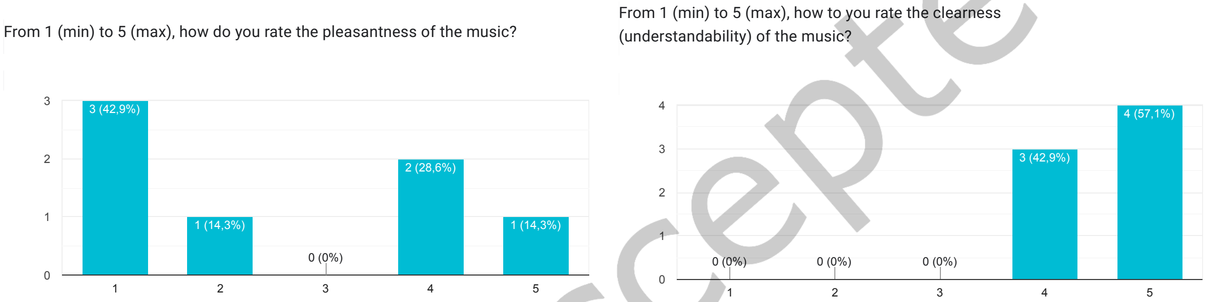
Our video shows the movements of pitches in the pitch-class circle, and the movements of robots in an underwater scenario, obtained with software Webots. For this example, we implemented the Pseudocode 1. Figures 10a, 10b, 10c show the starting, middle, and final step of the simulation, respectively.

The audio sample of the second video contains ten trumpet lines, first playing a cluster, then spreading, and finally converging to the same pitch. Timbre has been recognized (*Trumpet players*), as well as the feeling of tension given by dissonances: *Tensione e contrasto*, Italian for *Tension and contrast*, *A number of conflicting things*. One of the listeners isolated three main lines: *Trio*; another one referred to the semantic field associated with multiple trumpet players, suggesting *A castle*. The remaining two participants considered a 'negative' connotation associated with the harmonic tension and timbral violence of brass, writing *Bottom of the dark sea* and *A violent death or threat*, respectively. In next experiments, we may try to use more neutral tones, to privilege the feeling of tension and contrast given by the resulting and temporary dissonant harmonies, rather than having the attention focused on the timbre itself as an element of tension. The rating of pleasantness (Figure 7a) is aligned with the qualitative replies: two participants selected 1/5; other two of them, 2/5; one 3/5, and only the remaining two, 5/5. Clearness (Figure 7b) was slightly more spread, with only one participant indicating 2/5, and the remaining equally distributed between 3/5, 4/5, and 5/5. After having watched the corresponding video, pleasantness ratings continued to be rather spread (Figure 8a). However, the perception of understandability improved: four participants rated it as 5/5, and the remaining, 4/5 (Figure 8b). Correspondingly, the match between music and motion was rated well, with the majority of replies of 4/5 and 5/5 (Figure 9). Table 4 breaks down the result into the individual participants' replies.



(a) (b)

Fig. 7. Second sonification, ratings before watching the video.



(a) (b)

Fig. 8. Second sonification, ratings after having watched the video.

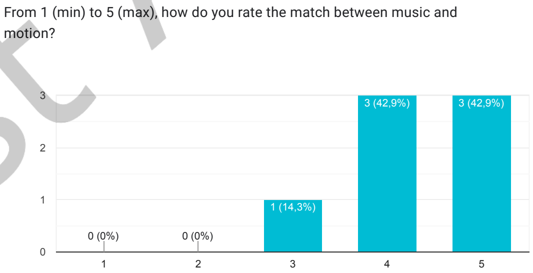
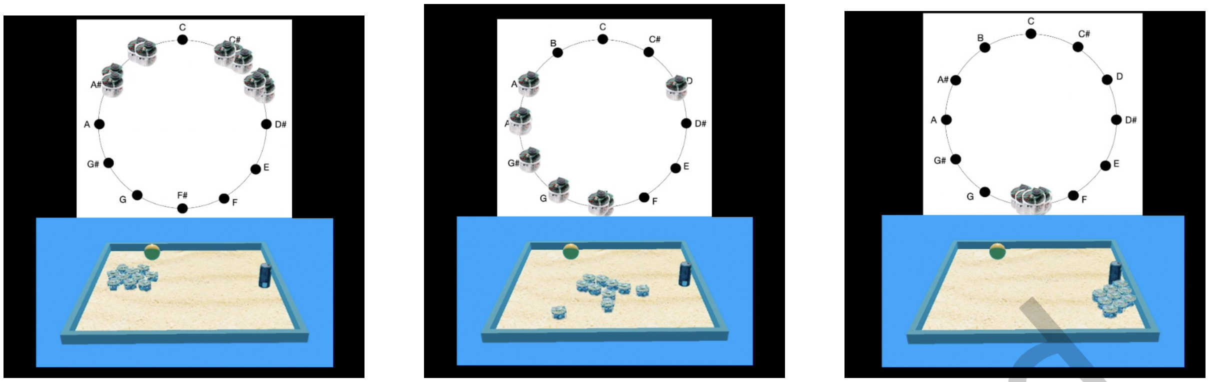


Fig. 9. Second sonification, rating of music-motion match.

4.3 3D: Three robo-fish in the ocean

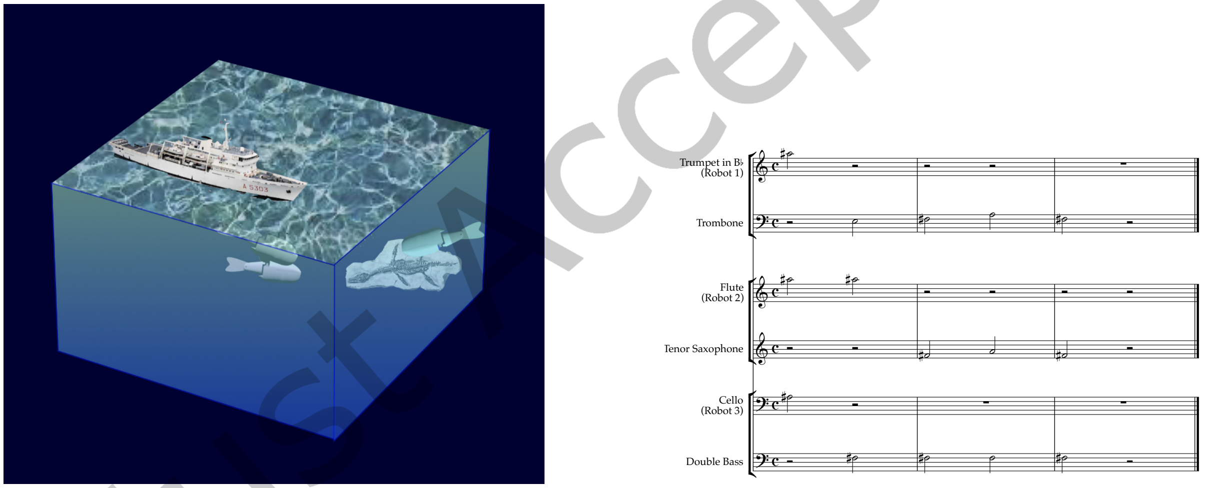
In our third example, we simulated a swarm of three swimming robots, implementing the Pseudocode 1. In fact, as the swarm motion law, the robots have to converge to a target as a search and rescue mission. We simulated a 3-dimensional underwater motion. Figure 11a shows a screenshot from the video simulation. Here we considered Flute, Trumpet, and Cello to distinguish the three robots in the upper pitch register, and Tenor Saxophone,



(a) (b) (c)

Fig. 10. Initial (a), middle (b), and final (c) steps of our 2D-simulation video.

Trombone, and Double Bass for the lower register. Figure 11b presents the notes obtained through the example sonification.



(a) (b)

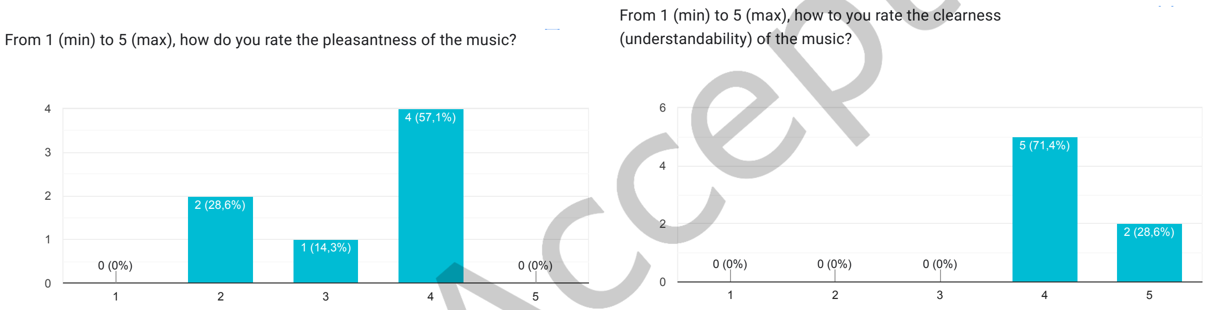
Fig. 11. A screenshot from our our 3D underwater swarm simulation video (a) and its sonification (b).

The third sample was the only sonification of a 3-dimensional motion, where the pitch range was correspondingly expanded. Participants seemed to perceive correctly the idea. Their qualitative answers included *Potenza e corposità* (Italian for *Powerfulness and thickness*), and *Mountains coming out of the sea*. Other listeners indicated *Something of mysterious* and *Epic*. Thus, the feeling of a different size and space of motion seems to be pretty clear. A participants wrote: *...two entities moving in a space... one smaller... the other bigger or heavier.* This is in correspondence with the alternation between higher and lower-range pitches, associated with bodies of different size (because of crossmodal correspondences having a physical foundation). Four participants rated

Table 5. Replies of participants *P*1-*P*7 to the questions on the third test

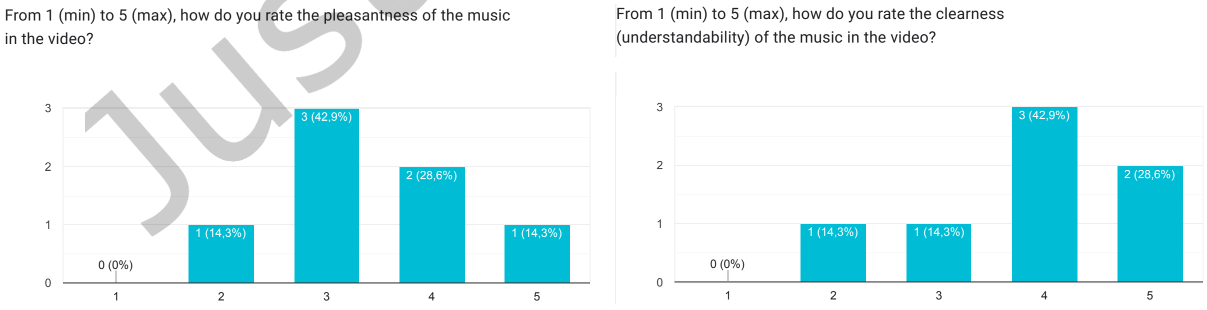
	#1	#2	#3	#4	#5	open
*P*1	4	4	3	4	2	Coming together
*P*2	5	4	5	5	5	Potenza e corposità
*P*3	4	4	4	2	3	I visualise two entities moving in a space, one smaller or lighter and another bigger or heavier.
*P*4	2	5	2	5	3	no images came to my mind while listening, after several times
*P*5	4	4	3	3	3	Epic
*P*6	2	4	3	4	4	mountains coming out of the sea
*P*7	3	5	4	4	4	antonioni film l'avventura - something mysterious and ominous but not necessarily physically dangerous

the pleasantness (Figure 12a) as 4/5, and the remaining split between 2/5 and 3/5. The majority of replies rated understandability as pretty high (five listeners chose 4/5, and the remaining 5/5; see Figure 12b). After having seen the associated video simulation, one participant rated pleasantness as 5/5, while the majority was assessed on 3/5 (Figure 13a). However, understandability then became more spread between 2 and 5/5, with a peak on 4/5 (Figure 13b). A spread outcome was also obtained by the match between music and motion, with the majority (three listeners) that assessed on a medium value, 3/5 (Figure 14). Table 5 breaks down the result into the individual participants' replies.



(a) (b)

Fig. 12. Third sonification, ratings before watching the video.



(a) (b)

Fig. 13. Third sonification, ratings after having watched the video.

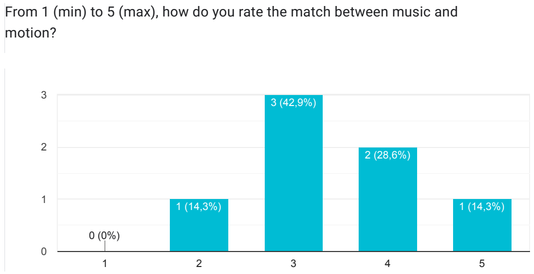


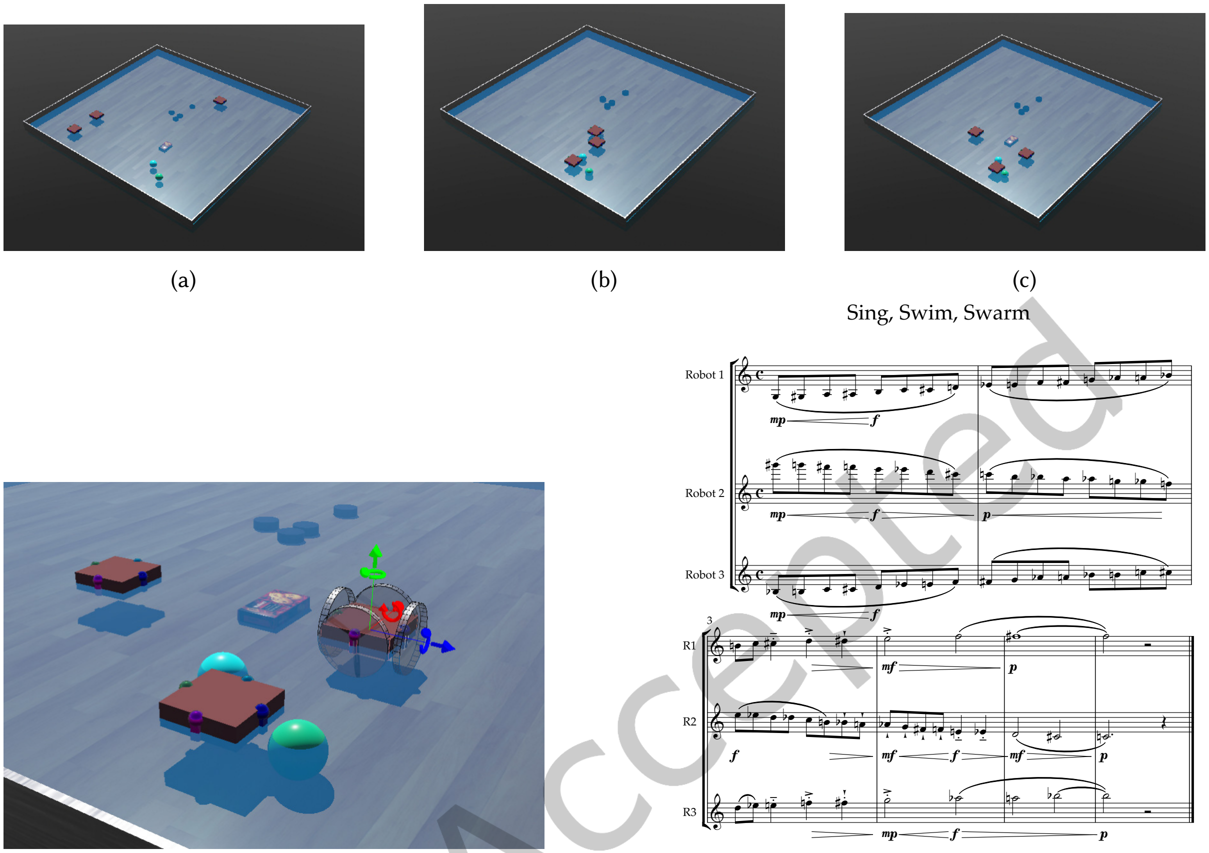
Fig. 14. Third sonification, rating of music-motion match.

4.4 Five sound parameters, 2D-motion of aquatic robots

Our fourth example is a sonification where all the mentioned sound parameters, that is, time, pitch, timbre, articulation, and loudness are exploited. We present a simulation of floating toy-swarm of three robots moving between given coordinates, sensing obstacles, and capturing images of the ground. The physics of the system is included in the simulation. The robots are simple tablets of wood, with GPS and distance sensors on each side. The sensors emerge from the water surface. On the bottom face of each robot, there is a camera, capturing pictures of the ground. The robots are in fact to recognize objects according to their color. On each side, there are also propellers, which are visible by clicking on them on the simulator software. This is a simple, original robot we called *RoboWood.*

The controllers are in C and the chosen simulation environment is Webots. The motion algorithm is also original. Each robot knows the GPS coordinates of the middle points of each one of its sides. Thus, it can calculate its barycenter and its orientation with respect to a given target. According to the coordinate difference between each robot's side and the target, the robot autonomously chooses which propeller or couple of propellers to activate. The output information we print in a file are the barycenter position at each discrete time-point, the distance sensors outcomes, the detection of objects on the ground. The sonification we present here is composed, to give an idea of how a complete auditory output should sound like. With a suitable code, we could also obtain a formal map from the simulation output parameters. We considered higher pitches at the north-east corner of the scenario, and lower pitches at the south-west, approximately. During one step of the simulations, two of the two robots get closer, and thus also their pitches are closer. We choose to describe the robotic proximity, signaled by distance sensors (even though the robot are not equipped yet with lateral camera devices, to recognize each other), by an articulation variation. The closer the robots, the more staccato the notes. This can be compared with distance sensors and proximity alarms used in cars. The fourth parameter is loudness. In our sonification, it follows the activation level of motors. To distinguish the robots, we considered B♭-clarinet sound for the first and third robot, and flute sound for the second robot. The main steps of the simulation are represented in Figures 15 (a-c). Figure 15(d) shows the propellers of one of the robots. Finally, in Figure 15 (e), we present the resulting micro-piece, titled *Sing, Swim, Swarm.*

The fourth simulation featured three robots, with two of them moving along parallel paths, which were associated with parallel melodic lines. In severe counterpoint of Western music, parallel intervals are either prohibited (fifths and octaves), or restricted (not more than three consecutive intervals), to increment diversity and make the different voices recognizable. Thus, we are not surprised that listeners mostly distinguished only two lines of motion, identifying and thus merging the parallel lines into only one line. One listener indicated *Two parallel things going up and up, one reaching the end before the other.* This is correctly perceived, because the two parallely-moving robots stop before the third one. Other participants also got the idea of different elements moving around: *People moving around, Night, walking tiptoes.* A participant thought of a single human agent,



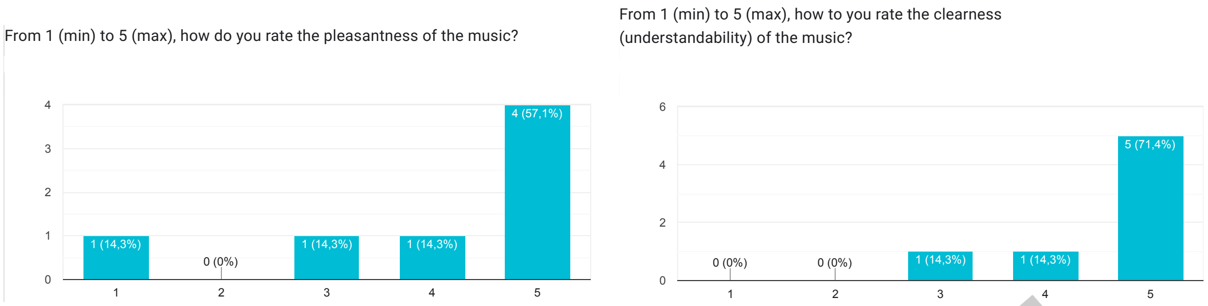
(a) (b) (c)

(d) (e)

Fig. 15. The three main steps of the simulation in an aquatic scenario (a-c); the detail of robots, visibile by clicking on them (d) and the sonification score (e). Robots 1 and 3 start from the south-west, while robot 2 starts from north. We used here the newly-coded *RoboWood*. The complete simulation can be accessed from this link.

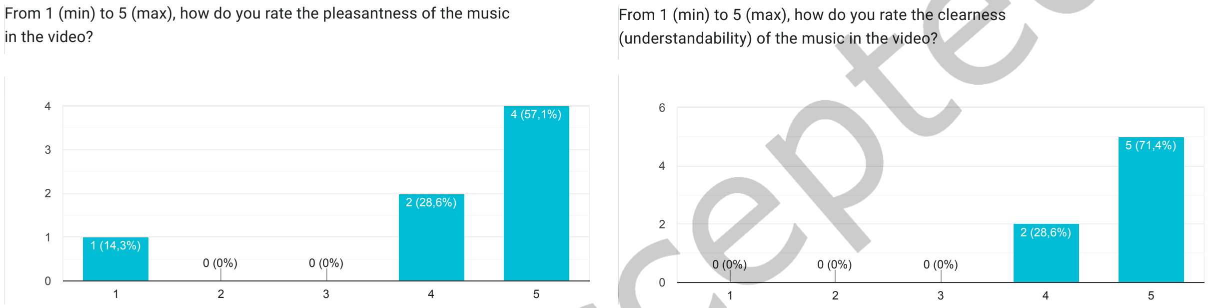
interacting with a moving living being: *Child playing some kind of game – alone – chasing a butterfly*. This last comment caught the idea of a less-moving element, the child, chasing a smaller, lighter, randomly-moving element, the butterfly. This corresponds with the image of an element moving less or at least stopping its motion before the other one. The reply *Animation* gives the idea of cartoon-style soundtrack, with a precise correspondence between movement and sound. The comment *Leaves of trees in the wind* reminds of a non-organized movement of multiple, light, and small elements. Four out of seven participants rated pleasantness as 5/5 (Figure 16a); clearness was mostly rated as the top value (Figure 16b). These results were almost confirmed after having seen the video simulation, with a rating of 5/5 on pleasantness by four participants (Figure 17a), and 5/5 on understandability by five participants (Figure 17b). Regarding the correspondence between music and motion, three listeners indicated 5/5, other three of them indicated 4/5, and only one rated it as 2/5 (Figure 18). Table 6 breaks down the result into the individual participants' replies.

We concluded the test with an optional section of overall, qualitative comments. According to *P*3, With the movies the music works very well for all examples. However, *P*4 stated: *The acoustic material didn't really engage*



(a) (b)

Fig. 16. Fourth sonification, ratings before watching the video.



(a) (b)

Fig. 17. Fourth sonification, ratings after having watched the video.

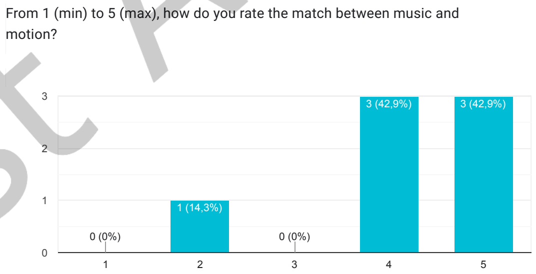


Fig. 18. Fourth sonification, rating of music-motion match.

with me; it was difficult for me to match the sounds with the images. Interestingly, this reply came from a composer used to work with timbral masses of sound, rather than thinking in terms of lines and counterpoint. *P6* stated: *I think a little bit more of precision is needed.* These two comments can help us refine the sonification techniques for our further research. For instance, we can shape timbral masses as a field of sound, to be modified by the movement of robots. We can also exploit more fine-grained correspondences between motion and sound, to increase the perception of precision. From the user test, we can evaluate some take-home findings: the expected correspondence of perceived motion was overall confirmed. It can be improved via a more refined position-sound

Table 6. Replies of participants $P1$-$P7$ to the questions on the fourth test

	#1	#2	#3	#4	#5	open
$P1$	5	5	5	5	5	People moving around
$P2$	5	5	5	5	5	Movimento
$P3$	5	3	5	4	5	Two parallel things going up and up, one reaching the end before the other.
$P4$	1	5	1	5	2	night, walking tiptoes
$P5$	4	5	4	5	4	Animation
$P6$	3	4	4	4	4	leaves of trees in the wind
$P7$	5	5	5	5	4	child playing some kind of game - alone - chasing a butterfly?

mapping, maybe using consistently continuous-glissandos. We could generalize the work with a variable number of robots and timbres. Next, we could also keep unchanged some parameters such as pitch, loudness, and timbre, and check if the articulation variations are clearly perceived by all participants.

4.5 The inverse code

The presented four examples involve sonification as a feedback from robotic movements. It is possible to do the inverse, indicating input chords as positions of robots at the beginning or for each time point. The skeleton of an inverse code has, as input, chords, and as output, robots' positions. The additional materials contain the inverse code in Jupyter, where user-given input notes are used as instructions. A snippet of the code for a 10-robot swarm is shown in Figure 19.

In [4]:

```
# User input of 10 notes!!! (at each time point...)

for i in RoboSound._registry:
    i.note = input("Enter a note:")

for i in RoboSound._registry:
    print(i.name, i.note)
```

```
Enter a note:C
Enter a note:C#
Enter a note:D
Enter a note:B
Enter a note:C
Enter a note:D
Enter a note:E
Enter a note:A
Enter a note:B
Enter a note:C
R1 C
R2 C#
R3 D
R4 B
R5 C
R6 D
R7 E
R8 A
R9 B
R10 C
```

(a)

In [5]:

```
# setting up positions and create the sound file


audio = []

for x in range(11): # it should be between 1 and 11 in this case
    valuez = AudioSegment.from_file("notes_/tC.mp3") # just to initialize
    audio.append(valuez)

for i in RoboSound._registry:
    if (i.note == 'C'):
        i.betax = 0
        i.betay = 0.5
        valuez = AudioSegment.from_file("notes_/tC.mp3")
        audio.append(valuez)
    if (i.note == 'B'):
        i.betax = round(np.random.uniform(0,0.17), 3)
        i.betay = round(np.random.uniform(0,0.5), 3)
        valuez = AudioSegment.from_file("notes_/tB.mp3")
        audio.append(valuez)
```

(b)

In [6]:

```
# graph

for i in  RoboSound._registry:
    plt.scatter(i.betax, i.betay, s = 400, marker = 'o', color = 'black')

plt.axis([0, 1, 0, 1])

plt.show()
```

(c)

Fig. 19. User-given pitches (a), a snippet of the inner code (b), and positions of the robots obtained as output (c).

5 DISCUSSION AND CONCLUSIONS

In this article, we proposed an approach to build musical spaces, robotic spaces, and their connections. Sound can be provided as feedback from robotic movements, but, within an inverse code, it can be given by the (human) user to robots to convey instructions. In this way, sound, and music in particular, becomes a privileged way of interaction between humans and robots, bringing robotic information closer to a human-understandable language. We focuses on swarms of robots for our examples. Then, we propose four examples of implementation, where simulated robotic movements are mapped to sound. We also present an example of inverse code, where user-input chords are mapped to robotic positions. We also ran a user test to validate the proposed technique. We asked music professionals to listen to the music samples first, and then jointly with their corresponding videos. We asked the professionals to rate enjoyability and understandability of samples, qualitatively assessing the amount of retrieved (and retrievable) information from sonification.

Our study may advance the literature, joining concepts and formalism from mathematical music theory, suggestions from cross-modal correspondences, and characteristics of generic robotic swarms. Three out of four simulation examples are inspired by different naturalistic scenarios, and the fourth example is based on a new robot type. Such a difference of scenarios and simulated devices shows the generality of the proposed approach and its flexibility. Other works in the literature present specific choices of sound parameters to represent elements of robotic motion [42], often considering a symbolic use of pre-defined sound sequences [89], or compare different sound features [93]. Conversely, we describe a general flexible framework based on sound spaces, which leaves room for specific choices, and yet can in principle be widely understood, being based on human perception and complexity of music. In fact, specific choices lie inside equivalence classes which ensure a degree of understandability of motion characteristics. Our scheme can be further generalized by including more parameters and make them correspond to other swarm-robotic features, or enriching the existing ones. For example, we used timbre to label each robot of the swarm. This parameter can be made flexible through variations of the spectral centroid according to specific motion: a robot approaching the correct target can have a slightly brighter timbre, while a robot approaching the wrong object can get a slightly darker timbre.

In our study, we also considered global/local features of a robotic swarm. In fact, sonification constitutes a method to represent the swarm behavior at a micro and marco level. In our approach, each individual behavior is represented by a sound at each time point—changing pitch according to robot's position—and a polyphony to the swarm. Thus, musical features such as voice leading are here expression of the emerging swarm behavior.

We made the hypothesis of a component-wise connection, and we reached the objective to build up a theoretical model and an algorithm for practical applications. In two out of three examples, we used a quantum circuit implementing a logic gate. The pros of using a quantum approach are the conceptual simplicity of the overall machinery and the rapidity of convergence to the target with a mixture of quantum and non-quantum approach. The non-quantum approach converges quite rapidly but not as fast as the mixing of quantum and non-quantum. The main cons is the inadequacy of this formula for a quantum-only approach: in fact, while initial positions of robots are improved, in the sense that they get closer to the target, they are then stuck in the middle (approximately) of the arena. Thus, the step of reshuffling to scramble initial positions seems necessary. Concerning the number of robots, we considered 2 for the 1D example, 10 for the 2D example, and 10 for the 3D example. In the fourth example, we considered an approximately planar motion, but we took into account five sound parameters. With particular regard to the second and third example, where we exploited the quantum circuit, we do expect a scalability: that is, by adding more robots, we do expect similar to the obtained ones. We anyway notice that, the more are the robots, the more likely we are that one robot makes it closer to the target. With few robots, an eventual position 'stuck' in the middle of the arena can be solved by looping the reshuffle & circuit structure, where the reshuffle is however limited to the region of space with higher reward.

To reduce the $N!$ logic gate computations for each pair of robots to just 1 computation, we let only the first successful robot (the one closer to the target) enter the gate. In our model, each robot sends information broadcast with position and reward (target proximity in our computational examples), with some incertitude described as quantum superposition of states. The behavior of each single robot is autonomous but, at the same time, it is connected with the decisions of other robots of the swarm. All robotic movements contribute to the emerging swarm behavior. A change in the environment is reflected by a change of target proximity perception for at least one robot. In our model, all robots notice the change. A change of reward even for just one robot is reflected into the message sent broadcast. Thus, this information influences those pairwise terms involving that robot, potentially affecting decisions on new positions to be reached at the following time points. When the robots reach their new positions, they have a different perception of target proximity (that we called *reward* in our analysis), and thus the messages they send each other are different. New messages imply new inputs for quantum circuits, that imply new positions reached. We can describe this effect through a system with feedback, as the social model of "chance" and "necessity" described for social sciences [63]. A change in the environment is reflected into a change of rewards, enacting the described cycle. The cycle is interrupted when all robots have reached the target. These effects can be recovered from matrices introduced in [56] and summarized here, mainly residing in pairwise interaction terms (off-diagonal elements of the overall block-matrix). A more detailed description of feedback effect, maybe also using diffusion differential equations to describe swarm's overall movements, can be the topic of further research.

This study is meant to be a small contribution toward a deeper understanding and modeling of robotic swarm phenomena, as a benchmark of swarm-understanding theories, with music as a unique and human-friendly investigation tool.

6 DATA AVAILABILITY

The links to our videos are provided in the captions of Figures 2, 10, and 11. Our codes for the 2D example are available at the GitHub folder https://github.com/medusamedusa/10_little_ants. This folder contains another copy of the videos as well. The Jupyter codes for the 2D case (direct and inverse) are original. As the output, the direct Jupyter code provides sound files with a chord and the positions for each robot at each time point. Through an opportune change of coordinates, these positions are then taken by the Webots code as steps to be reached by robots. The Webots code also includes an obstacle-avoidance script. The Webots code has been adapted from sections 2 and 3 of the public repository https://github.com/albertbrucelee/webots-e-puck_robot-tutorial by A. Alfrianta. Data of the 3D example can be retrieved at https://github.com/medusamedusa/quantum_robo_sound. The Jupyter code for the 3D case is original. These are public repositories. The codes for *RoboWood* can be freely accessed at https://github.com/medusamedusa/RoboWood. The robotic structure, its motion code, and their implementations in C and Webots are original.

7 ACKNOWLEDGMENTS

The research leading to these results takes place within the framework of the project "ARES, Autonomous Robotics for the Extended Ship," funded by the Italian Ministry of University and Research under grant agreement ARS01_00682. The *Comitato di Bioetica* (BioEthics Committee) of the University of Palermo approved our experimental study during the meeting no. 138/2023 on March 30, 2023, and registered it on April 3, under protocol number UNPA-CLE - Prot. 51537-03/04/2023.

REFERENCES

[1] Team 31. 2022. Sonification of Hypertension & Heart Disease 1 Design. https://www.afyako.com/data-visuals/data-sonification.

[2] D. Aerts, L. Beltran, S. Geriente, and S. Sozzo. 2021. Quantum-Theoretic Modeling in Computer Science. *International Journal of Theoretical Physics* 60 (2021), 710–726.

[3] M. Alvarez-Alvarado, F. Alban-Chacón, E. Lamilla-Rubio, C. Rodríguez-Gallegos, and W. Velásquez. 2021. Three novel quantum-inspired swarm optimization algorithms using different bounded potential fields. *Nature Scientific Reports* 11 (2021), 11655. https://www.nature.com/articles/s41598-021-90847-7

[4] J. S. Arias-Valero and E. Lluis-Puebla. 2020. Some remarks on hypergestural homology of spaces and its relation to classical homology. *Journal of Mathematics and Music* 14, 3 (2020), 245–265.

[5] P. Atchade-Adelomou, P. Alonso-Linaje, J. Albo-Canals, and D. Casado-Fauli. 2021. qRobot: A Quantum Computing Approach in Mobile Robot Order Picking and Batching Problem Solver Optimization. *Algorithms* 14, 194 (2021). https://www.mdpi.com/1999-4893/14/7/194

[6] J. Baez, E. Hoffnung, and C. Walker. 2008. Groupoidification Made Easy. ArXiV, https://arxiv.org/pdf/0812.4864.pdf.

[7] S. Bardelli, C. Ferretti, L. A. Ludovico, G. Presti, M. Rinaldi, and M. Rauterberg. 2021. A Sonification of the zCOSMOS Galaxy Dataset. In *Culture and computing, interactive cultural heritage and arts: 9th international conference, C&C 2021, held as part of the 23rd HCI international conference, HCII 2021, virtual event*, Vol. 12794. 171–188. https://air.unimi.it/retrieve/handle/2434/856154/1836679/2523_final.pdf

[8] P. beim Graben and R. Blutner. 2019. Quantum approaches to music cognition. *Journal of Mathematical Psychology* 91 (2019), 38–50.

[9] P. beim Graben and R. Blutner. 2020. Gauge models of musical forces. *Journal of Mathematics and Music* 15, 1 (2020), 17–36.

[10] J. Bénabou. 1967. Introduction to bicategories, part I. In *Reports of the Midwest Category Seminar.* Lecture Notes in Mathematics, Vol. 47. Springer, 1–77.

[11] P. Benioff. 1998. Quantum robots and environments. *Physical Review A* 58 (1998), 893.

[12] S. Berman, Q. Lindsey, M. S. Sakar, V. Kumar, and S. C. Pratt. 2011. Experimental Study and Modeling of Group Retrieval in Ants as an Approach to Collective Transport in Swarm Robotic Systems. *Proc. IEEE* 99, 9 (2011), 1470–1481.

[13] T. Blackwell. 2003. Swarm music: improvised music with multi-swarms. In *Proc. AISBSymposium in Artificial Intelligence and Creativity Arts Science.* 41–49.

[14] E. Bonabeau, G. Theraulaz, and M. Dorigo. 1977. *Swarm Intelligence: From Natural to Artificial Systems.* Oxford University Press, New York.

[15] L. M. Brown and S. A. Brewster. 2003. Drawing by Ear: Interpreting Sonified Line Graphs. In *Proceedings of the 2003 International Conference on Auditory Display.* Boston, ICAD 03–152–156.

[16] C. Chopra and M. Egerstedt. 2014. Heterogeneous multi-robot routing. In *American Control Conference.* 5390–5395.

[17] T. L. Clark. 2021. On the topological characterization of gestures in a convenient category of spaces. *Journal of Mathematics and Music* 15, 1 (2021), 37–61.

[18] O. Costa Hamido, G. A. Cirillo, and E. Giusto. 2020. Quantum synth: a quantum-computing-based synthesizer. In *AM '20: Proceedings of the 15th International Audio Mostly Conference.* 265–268.

[19] E. Şahin. 2004. Swarm robotics: From sources of inspiration to domains of application. In *International Workshop on Swarm Robotics (Lecture Notes in Computer Science, Vol. 3342)*, E. Şahin and W. M. Spears (Eds.). Springer, Berlin/Heidelberg.

[20] J. Delcourt, N.W. Bode, and M. Denöel. 2016. Collective Vortex Behaviors: Diversity, Proximate, and Ultimate Causes of Circular Animal Group Movements. *The Quaterly Review of Biology* 91, 1 (2016), 1–24.

[21] D. Dong, C. Chen, C. Zhang, and C. Chen. 2006. Quantum robot: structure, algorithms and applications. *Robotica* 4 (2006), 513–521.

[22] X. Dong and M. Sitti. 2020. Controlling two-dimensional collective formation and cooperative behavior of magnetic microrobot swarms. *The International Journal of Robotic Research* 39, 5 (2020), eabe4385. https://journals.sagepub.com/doi/full/10.1177/0278364920903107

[23] M. Dorigo, G. Theraulaz, and V. Trianni. 2020. Reflections on the future of swarm robotics. *Science Robotics* 5, 49 (2020), eabe4385.

[24] R. Eberhart, Y. Shi, and J. Kennedy. 2001. *Swarm Intelligence.* Morgan Kaufman, Burlington, MA, USA.

[25] P. S. Emani, J. Warrell, A. Anticevic, and et al. 2021. Quantum computing at the frontiers of biological sciences. *Nature Methods* 18 (2021), 701–709. https://www.nature.com/articles/s41592-020-01004-3

[26] A. K. Fedorov and M. S. Gelfand. 2021. Towards practical applications in quantum computational biology. *Nature Computational Science* 1 (2021), 114–119. https://www.nature.com/articles/s43588-021-00024-z

[27] O. H. Flowers and T. A. Hauer. 1992. The ear's versus the eye's potential to assess characteristics of numeric data: Are we too visuocentric? *Behavior Research Methods, Instruments, & Computers* Z4, 2 (1992), 258–264.

[28] E. Frid and R. Bresin. 2021. Perceptual Evaluation of Blended Sonification of Mechanical Robot Sounds Produced by Emotionally Expressive Gestures: Augmenting Consequential Sounds to Improve Non-verbal Robot Communication. *International Journal of Social Robotics* 14 (2021), 357–372.

[29] B. Fugiel. 2022. Quantum-like melody perception. *Journal of Mathematics and Music* (2022). https://doi.org/10.1080/17459737.2022.2049383

[30] D. Gabor. 1947. Acoustical quanta and the theory of hearing. *Nature* 159, 4044 (1947), 591.

[31] M. Grandis. 2020. *Higher Category Theory.* World Scientific, Singapore.

[32] J. Grey. 1977. Multidimensional Perceptual Scaling of Musical Timbres. *The Journal of the Acoustical Society of America* 61 (1977), 1270–1277.

[33] H. Hamann. 2018. *Swarm Robotics: A Formal Approach.* Springer, Cham.

[34] K. A. Hardie and R. W. Kieboom. 2001. A Homotopy Bigroupoid of a Topological Space. *Applied Categorical Structures* 9 (2001), 311–327.

[35] A. Hatcher. 2002. *Algebraic Topology.* Cornell University, USA.

[36] C. H. Hemelrijk and H. Hildenbrandt. 2012. Schools of fish and flocks of birds: Their shape and internal structure by self-organization. *Interface Focus* 2 (2012), 726–737.

[37] M. Hemmo and O. Shenker. 2020. Quantum Mechanics as a Theory of Probability. In *Quantum, Probability, Logic*, M. Hemmo and O. Shenker (Eds.). Springer, Cham, 337–351.

[38] T. Hermann, A. Hunt, and Neuhoff J. G. (Eds.). 2011. *The Sonification Handbook.* Logos Publishing House, Berlin.

[39] J. R. Hughes. 2022. Generalizing the Orbifold Model for Voice Leading. *Mathematics* 10, 939 (2022), 1–26. https://www.mdpi.com/2227-7390/10/6/939

[40] ICAD. 2022. ICAD archive. https://icad.org/.

[41] F. Jedrzejewski. 2019. *Hétérotopies musicales: Modèles mathématiques de la musique.* Hermann, Paris.

[42] A. Kamboj, T. Ji, and K. Driggs-Campbell. 2022. Examining Audio Communication Mechanisms for Supervising Fleets of Agricultural Robots. In *2022 31st IEEE International Conference on Robot and Human Interactive Communication (RO-MAN).* 293–300. https://doi.org/10.1109/RO-MAN53752.2022.9900859

[43] A. Koukam, A. Abbas-Turki, V. Hilaire, , and Y. Ruichek. 2021. Towards a Quantum Modeling Approach to Reactive Agents, In 2021 IEEE International Conference on Quantum Computing and Engineering (QCE). *International Journal of Computers Communications and Control*.

[44] L. Lamata, M. Quadrelli, C. de Silva, P. Kumar, G. Kanter, M. Ghazinejad, and F. Khoshnoud. 2021. Quantum Mechatronics. *Electronics* 10 (2021), 2483. https://doi.org/10.3390/electronics10202483

[45] S. Lenzi, P. Ciuccarelli, Y. Hua, and H. Liu. 2022. Data Sonification. https://sonification.design/.

[46] F. Lerdahl and R. Jackendoff. 1983. *A Generative Theory of Tonal Music.* MIT Press, Cambridge, Massachussets.

[47] Z. Li, W. Liu, G. Li-E, and L. Li. 2019. Path Planning Method for AUV Docking Based on Adaptive Quantum-Behaved Particle Swarm Optimization. *IEEE Access Multidisciplinary* 7 (2019), 78665–78674.

[48] S. Mac Lane. 1978. *Categories for the Working Mathematician.* Springer, New York.

[49] M. Mannone. 2017. Networks of Music and Images. *Gli Spazi della Musica* 6, 2 (2017), 39–52. https://www.ojs.unito.it/index.php/spazidellamusica/article/view/2528

[50] M. Mannone. 2018. Introduction to Gestural Similarity in Music. An Application of Category Theory to the Orchestra. *Journal of Mathematics and Music* 12, 2 (2018), 63–87. https://arxiv.org/abs/1904.10340

[51] M. Mannone. 2018. Knots, Music and DNA. *Journal of Creative Music Systems* 2, 2 (2018), 1–22. https://www.jcms.org.uk/article/id/523/

[52] M. Mannone. 2022. A musical reading of a contemporary installation and back: mathematical investigations of patterns in Qwalala. *Journal of Mathematics and Music* 16, 1 (2022), 80–96.

[53] M. Mannone and J. S. Arias-Valero. 2022. Some mathematical and computational relations between timbre and color. In *Mathematics and Computation in Music 2022 (Lecture Notes in Artificial Intelligence)*. 127–139.

[54] M. Mannone and G. Compagno. 2022. Characterization of the degree of Musical non-Markovianity. https://doi.org/10.5920/jcms.975

[55] M. Mannone, G. Santini, E. Adedoyin, and C. E. Cella. 2022. Color and Timbre Gestures: An Approach with Bicategories and Bigroupoids. *Mathematics* 10, 4 (2022), 663. https://www.mdpi.com/2227-7390/10/4/663

[56] M. Mannone, V. Seidita, and A. Chella. 2022. Categories, Quantum Computing, and Swarm Robotics: A Case Study. *Mathematics* 10, 3 (2022), 372. https://doi.org/10.3390/math10030372

[57] M. Mannone, V. Seidita, and A. Chella. 2022. Quantum RoboSound: Auditory Feedback of a Quantum-Driven Robotic Swarm. In *2022 31st IEEE International Conference on Robot and Human Interactive Communication (RO-MAN)*. 287–292. https://doi.org/10.1109/RO-MAN53752.2022.9900578

[58] M. Mannone, V. Seidita, and A. Chella. 2023. Modeling and Designing a Robotic Swarm: a Quantum Computing Approach. *Swarm and Evolutionary Computation* 79 (2023), 101297.

[59] V. Marx. 2021. Biology begins to tangle with quantum computing. *Nature Methods* 18 (2021), 715–719. https://doi.org/10.1038/s41592-021-01199-z

[60] G. Mazzola and M. Andreatta. 2007. Diagrams, gestures and formulae in music. *Journal of Mathematics and Music* 1, 1 (2007), 23–46. https://hal.archives-ouvertes.fr/hal-01161060/document

[61] J. McLurkin. 2004. Stupid robot tricks: A behavior-based distributed algorithm library for programming swarms of robots (Master's Thesis).

[62] J. McLurkin. 2022. The Swarm Orchestra: Temporal Synchronization and Spatial Division of Labor for Large Swarms of Autonomous Robots. https://groups.csail.mit.edu/mac/projects/amorphous/6.978/final-papers/jamesm-final.pdf. , 12 pages.

[63] P. Mella. 2008. Observing Collectivities: The Combinatory Systems Approach in Social Sciences. *The International Journal of Social Sciences* 3, 1 (2008), 213–223.

[64] E. R. Miranda. 2021. Quantum Computer: Hello, Music! In *Handbook of Artificial Intelligence for Music: Foundations, Advanced Approaches, and Developments for Creativity*, E. R. Miranda (Ed.).

[65] E. R. (ed.) Miranda. 2022. *Quantum Computing in the Arts and Humanities.* Springer, Cham.

[66] A. Moroni and J. Manzolli. 2010. From evolutionary composition to robotic sonification. In *European Conference on the Applications of Evolutionary Computation.* Springer, 401–410.

[67] S. Nădăban. 2021. From classical logic to fuzzy logic and quantum logic: a general view. *International Journal of Computers Communications and Control* 16, 1 (2021), 4125. https://doi.org/10.15837/ijccc.2021.1.4125

[68] C. Noirot and J. P. E. C. Darlington. 2000. Termite Nests: Architecture, Regulation and Defence. In *Termites: Evolution, Sociality, Symbioses, Ecology*, Takuya Abe, David Edward Bignell, and Masahiko Higashi (Eds.). Springer Netherlands, Dordrecht, 121–139. https://doi.org/10.1007/978-94-017-3223-9_6

[69] C. Outeiral, M. Strahm, J. Shi, G. M. Morris, S. C. Benjamin, and C. M. Deane. 2021. The prospects of quantum computing in computational molecular biology. *Wires Computational Molecular Science* 11, 1 (2021), e1481. https://wires.onlinelibrary.wiley.com/doi/10.1002/wcms.1481

[70] C. Petschnigg, M. Brandstötter, and H. Pichler. 2019. Quantum Computation in Robotic Science and Applications. In *IEEE International Conference on Robotics and Automation (ICRA).*

[71] L. Pitonakova, R. Crowder, and S. Bullock. 2020. Task Allocation in Foraging Robot Swarms: The Role of Information Sharing . *Autonomous Robots* 17 (2020), 93–105. https://direct.mit.edu/isal/proceedings/alif2016/28/306/99491

[72] J. Preskill. 2021. Quantum computing 40 years later. In *Feynman Lectures on Computation, 2nd ed.*, A.J.G. Hey (Ed.). Taylor & Francis Group, Abingdon, UK.

[73] A. Ravignani and M. Garcia. 2022. A cross-species framework to identify vocal learning abilities in mammals. *Philosophical Transactions B* 377 (2022), 202003946. https://pure.mpg.de/rest/items/item_3342335_5/component/file_3350714/content

[74] C. Roads. 1980. Artificial Intelligence and Music. *Computer Music Journal* 4, 2 (1980), 13–25.

[75] C. Roads. 2004. *Microsound*. MIT Press, Cambridge, Massachusetts, USA.

[76] D. Rocchesso and M. Mannone. 2020. A Quantum Vocal Theory of Sound. *Quantum Information Processing* 19, 292 (2020), 1–28. https://link.springer.com/article/10.1007/s11128-020-02772-9.

[77] E. M. Rose, N. H. Prior, and G. F. Ball. 2022. The singing question: re-conceptualizing birdsong. *Biological Reviews* 97 (2022), 326–342. https://onlinelibrary.wiley.com/doi/epdf/10.1111/brv.12800

[78] A. Sabra and W. Fung. 2020. A Fuzzy Cooperative Localisation Framework for Underwater Robotic Swarms. *Sensors* 20 (2020), 5496. https://doi.org/10.3390/s20195496

[79] F. Santos. 2016. Musical abstractions for multi-robot coordination. (2016). https://webcache.googleusercontent.com/search?q=cache:p7wV_7QkEScJ:https://smartech.gatech.edu/bitstream/handle/1853/55049/SANTOSFERNANDEZ-THESIS-2016.pdf+&cd=2&hl=it&ct=clnk&gl=it

[80] M. Schranz, M. Umlauft, M. Sende, and W. Elmenreich. 2020. Swarm Robotic Behaviors and Current Applications. *Frontiers in Robotics and AI* 7 (2020).

[81] W.-M. Shen, P. Will, A. Galstyan, and C.-M. Chuong. 2020. Hormone-Inspired Self-Organization and Distributed Control of Robotic Swarms . *Autonomous Robots* 17 (2020), 93–105. http://www.agentgroup.unimo.it/Zambonelli/didattica/cas/L14/Shen_RobotSwarms.pdf

[82] I. Slakov, C. Carrillo-Zapata, D. Jansson, H. Kaandorps, and J. Sharpe. 2018. Morphogenesis in robot swarms. *Science Robotics* 3, 25 (2018).

[83] D. R. Smith and B. N. Walker. 2005. Effects of Auditory Context Cues and Training on Performance of a Point Estimation Sonification Task. *Applied Cognitive Psychology* 19, 8 (2005), 1065–1087.

[84] J. Stolze and D. Suter. 2004. *Quantum Computing: A Short Course from Theory to Experiment*. Wiley, Weinheim, Germany.

[85] D. Tymoczko. 2006. The Geometry of Musical Chords. *Science* 313, 5783 (2006).

[86] D. Tymoczko. 2016. In Quest of Musical Vectors. In *Mathemusical Conversations*. Institute for Mathematical Sciences, National University of Singapore, 256–282.

[87] B. Verstegen, E. Özcan, and S. Delle Monache. 2022. Quantum Mechanics, Ambiguity and Design: Towards a Framework. In *C&C '22: Creativity and Cognition*. 575–582.

[88] A. Wichert. 2020. *Principles of Quantum Artificial Intelligence*. World Scientific, Singapore.

[89] L. L. Wright, A. Kothiyal, Kai O. Arras, and B. Bruno. 2022. How a Social Robot's Vocalization Affects Children's Speech, Learning, and Interaction. In *2022 31st IEEE International Conference on Robot and Human Interactive Communication (RO-MAN)*. 279–286. https://doi.org/10.1109/RO-MAN53752.2022.9900811

[90] A. S. Wu, R. P. Wiegand, and R. R Pradhan. 2020. Response probability enhances robustness in decentralized threshold-based robotic swarms . *Swarm Intelligence* 14 (2020), 233–258. https://par.nsf.gov/servlets/purl/10182080

[91] I. Xenakis. 1971. *Formalized Music*. Pendragon, New York.

[92] I. Xenakis. 1976. Le pavilon Philips: à l'aube d'une architecture [The Philips Pavilion: At the Dawn of Architecture]. In *Musique Architecture*. Casterman, Paris, 129.

[93] L. Zahray, R. Savery, L. Syrkett, and G. Weinberg. 2020. Robot Gesture Sonification to Enhance Awareness of Robot Status and Enjoyment of Interaction. In *2020 29th IEEE International Conference on Robot and Human Interactive Communication (RO-MAN)*. 978–985. https://doi.org/10.1109/RO-MAN47096.2020.9223452

[94] F. Zambonelli, N. Bicocchi, G. Cabri, L. Leonardi, and M. Puviani. 2011. On Self-adaptation, Self-expression, and Self-awareness in Autonomic Service Component Ensembles. In *2011 Fifth IEEE Conference on Self-Adaptive and Self-Organizing Systems Workshops*.

[95] B. J. Zhang, C. A. Sanchez, and N. T. Fitter. 2022. Using the Price Sensitivity Meter to Measure the Value of Transformative Robot Sound. In *2022 31st IEEE International Conference on Robot and Human Interactive Communication (RO-MAN)*. 301–307. https://doi.org/10.1109/RO-MAN53752.2022.9900678

[96] B. J. Zhang, N. Sigafoos, R. Moffit, I. Syed, L. S. Adams, J. Fick, and N. T. Fitter. 2022. SonifyIt: Towards Transformative Sound for All Robots. *IEEE Robotics and Automation Letters* 7, 4 (2022), 10566–10572.

[97] K. Zhu and M. Jiang. 2010. Quantum Artificial Fish Swarm Algorithm. In *Proceedings of the 8th World Congress on Intelligent Control and Automation*.