

# Reply to Comment by P.P. Abrantes et al. on: Dispersion Interaction between Two Hydrogen Atoms in a Static Electric Field by G. Fiscelli et al.

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In their Comment on our Letter *Dispersion Interaction between Two Hydrogen Atoms in a Static Electric Field* [1], P.P. Abrantes et al. address one of the main points discussed in our Letter, that is the possibility to manipulate interatomic interactions through an external static electric field. In our paper we have shown that the interaction between two ground-state atoms can be significantly modified exploiting an external static electric field, and even turned from attractive to repulsive, depending on the strength of the external field and the geometrical configuration. In their Comment, Abrantes et al. point out that it is the electrostatic contribution between the electric dipoles induced in both atoms by the external field that it is dominant and can become repulsive, overcoming the usual attractive dispersion force. They write the interatomic force as the sum of a classical electrostatic dipole-dipole interaction and a dispersion interaction modified by the external field, and point out that it is the total force that changes its sign. As we discuss below in more detail, we partially agree with their interpretation of this result. Essential points, in our opinion, are the exact definition of the dispersion interaction and how it is separated from the (classical) electrostatic contribution when the atoms are polarized by the static electric field, clarifying which quantity is evaluated at any step.

In our paper [1] we evaluate the total interaction energy between the two ground-state hydrogen atoms with a fourth-order perturbative approach, including only contributions from the mixing of the (perturbed) ground state with the atomic excited states with principal quantum number  $n = 2$ . It is this interaction energy that we have named *dispersion energy*, and it is given by Eqs. (6) and (9) of our paper; the classical electrostatic interaction between the permanent induced dipoles is not included, and it should be added to get the complete potential energy. It seems that this classical dipole-dipole interaction is what Abrantes et al. have considered in the force term  $F_z^{\text{el}}$ , together with the dispersive contribution they indicate with  $F^{\text{CP}}$ . Because this classical dipole-dipole force can be attractive or repulsive according to the direction of the external field relative to the atomic distance and can overcome the usual dispersion

force, we agree with the considerations of Abrantes et al. on this point. This contribution takes into account only the static atomic dipoles induced by the external field, and not their fluctuations, that can be modified by the external field. Abrantes et al. then add a Casimir-Polder contribution  $F^{\text{CP}}$  (far-zone only), in order to obtain their total force when the external electric field is present. We believe that their evaluation of this latter contribution is not complete, due to the presence of other relevant terms not included in their calculation. They first consider the change of the static atomic polarizability due to the external field, that appears in the far-zone dispersion energy (two-photon exchange), and they find that this effect is negligible for typical values of the external field. This is totally consistent with our results. In fact, this effect is a higher-order one in the electric charge (see the expression of  $\delta\alpha_0$  in the Comment), that we have neglected because we have included only contributions up to the fourth-order in the electric charge. Also, it is due to a two-photon exchange. However, this is not the only change to the dispersion interaction energy due to the external field. As we have shown in our paper, where the change of the dispersion interaction energy due to the external field is obtained from first principles by a direct fourth-order perturbative calculations, there are other terms coming from a one-photon exchange. They yield the  $r^{-3}$  (near zone) and  $r^{-4}$  (far zone) new contributions given in Eqs. (15) and (17) of our paper, respectively for a perpendicular or parallel orientation of the electric field with respect to the atomic distance. These new contributions we have found in [1], originating from a one-photon exchange, can also yield a change of the sign of the dispersion force (similarly to the classical electrostatic dipole-dipole interaction mentioned above). As discussed in detail in our paper, they can be interpreted in terms of correlations of atomic fluctuating (induced) dipoles, and have not been included in the estimate of the total potential by Abrantes et al. These new terms have a strong analogy with the atom-plate Casimir-Polder interaction, as shown in [1]. It is worth to compare these dispersive contributions with the electrostatic dipole-dipole interaction considered in the Comment by Abrantes et al. It is easy to see that they are of the same order in the near-zone ( $k_0 r \ll 1$ ) and in the intermediate zone ( $k_0 r \sim 1$ ), while in the far zone,  $k_0 r \gg 1$ , our  $r^{-4}$  becomes smaller than the electrostatic term (their ratio scales as  $(k_0 r)^{-1}$ ).

In conclusion, we agree with Abrantes et al. that it is the total interaction that can be turned from attractive to repulsive exploiting the static external field. However,

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this change can originate from both the classical electrostatic dipole-dipole force and a new “mixed” term originating from a one-photon exchange (see Eqs. (9) and (10) of [1]). This mixed term is not considered in the evaluation of Abrantes et al., where only the effect of the change of the static polarizability is included in the dispersive term. In other words, we believe that from one side it is correct that the classical electrostatic dipole-dipole interaction between the induced dipoles can change the sign of the total force, as pointed out by Abrantes et al.; from the other side, however, also other dispersion contributions (from correlated fluctuating dipoles, and arising from one-photon exchange) can, in addition, turn the to-

tal force from attractive to repulsive, as Eqs. (15-18) of our paper [1] show. We wish to point out that the content of the present Comment of Abrantes et al. is in any case very valuable, also because it has allowed us to further clarify and point out the different possible effects of external static electric fields on interatomic interactions.

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[1] G. Fiscelli, L. Rizzuto, and R. Passante, *Dispersion Interaction between Two Hydrogen Atoms in a Static Electric*

*Field*, Phys. Rev. Lett. **124**, 013604 (2020).