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Research Paper

Analysis of a Hyperbolic Heat Transfer Model in Blood-perfused Biological Tissues with Laser Heating

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Abstract. This paper proposes a hyperbolic heat transport model for a homogeneously perfused biological tissue irradiated by a laser beam. In particular, involving two local energy equations, one for the blood vessel and the other for the tissue, a non-Fourier-like heat equation is introduced and solved analytically using the Laplace transform method. The generalized hyperbolic model obtained is reduced to Pennes' heat transport equation in case the thermal delay time is zero and the solution obtained is in accordance with the numerical and experimental data existing in the literature. In addition, the achieved results also show that the effects of thermal relaxation and blood perfusion on temperature distribution are similar; indeed the highest temperature is expected when the delay time τ_R increases during tissue cooling. Finally, the consequences of the change in the values of the physical parameters characterizing the model are described and the effect of thermal relaxation on the temperature profile in the tissue during and after laser application is investigated.

Keywords: Heat transport, Bioheat, Nonequilibrium thermodynamics, Blood perfusion, Relaxation time, Porosity.

1. Introduction

The study of mathematical models for heat transport in living tissues is an interesting topic for several researchers because heat transfer in biological systems is significant in many diagnostic and therapeutic techniques involving temperature changes. The scrupulous description of the thermal interaction between vasculature and tissues is very important also in relation to constant advances in medical technology.

Currently, modern surgery has a wide range of different surgical techniques available for heating the biological tissue in a well-defined region. All these techniques use specific tools, e.g. devices (cryosurgery), that remove or introduce heat, (laser, radio frequency current, microwave or ultrasound), [1, 2]. Cryosurgery, sometimes referred to as cryotherapy or cryoagulation, is a surgical technique in which freezing is applied to destroy unsuitable tissues. In addition, a different freezing process of cryosurgery, consists in freezing for preservation (cryopreservation), which means that the cells and tissues are stored in a frozen state for transplantation, [3]. Other therapeutic applications include non-invasive thermal therapy that is used for cancer treatment; this method of treatment is chosen according to the location of the tumor, the stage or whether it is resistant to continuous treatment, [4, 5]. Therefore, it is essential to select the extent of the heating of cancer cells in order to avoid damage to the surrounding tissue, thereby achieving an adequate distribution of temperature. The development of effective strategies for the treatment of the cancer has been an important task in the field of medical research.

Mathematical models can play a key role in providing meaningful information for clinicians about possible outcomes and risks which may occur before the onset of thermotherapy treatment to treat cancer. In these medical problems, it is necessary to carry out an accurate analysis of both spatial and temporal transport of heat to biological tissues, [6, 7].

In general, all living systems do not exhibit a uniform temperature in organs and blood. This non uniformity of the temperature induces an energy transfer among organs, tissues and the perfused blood. Heat transport in biological tissues, usually modeled with the bioheat equation, is not simple to analyze since it involves thermal conduction in tissues, convection and perfusion of blood, and metabolic heat generation. In fact, several authors have introduced various mathematical models of bioheat transfer generalizing the Pennes's bioheat equation [8]. This equation describes the thermal behavior of tissue by taking into account several terms influencing the heat transfer at the tissue surface: the heat exchange between the tissue surface and the environment, the conduction through the tissue, the energy transfer due to blood circulation in the tissue, and the heat generation due to local metabolism. Pennes [8], investigating the thermal behavior in forearm skin, proposed the equation:

$$\rho_t c_t \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} + \rho_b c_b w_b (T - T_a) + q_{met}, \tag{1}$$

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where ρ_t, c_t, k, ρ_b and c_b are the density, the specific heat, and the thermal conductivity of skin tissue, the density and specific heat of blood, respectively. Moreover, the quantities w_b, T and T_a are the blood perfusion rate, the skin tissue and arterial blood temperatures, respectively, whereas q_{met} is the metabolic heat generated by the skin tissue. Since temperature variations in biological tissues depend on many phenomena, various generalizations of this equation have been proposed. Among these extensions, a relevant role is played by models of bioheat transfer where the living tissues are assumed to be deformable porous media [9-15]. This approach implies the necessity of introducing two energy equations for the tissue and the blood. In such a framework, important effects such as the vascular geometry and size, the blood flow and direction, the thermal diffusion and the local thermal nonequilibrium between the blood and peripheral tissues are included.

In the modelization of transport phenomena in porous media, it is possible to describe the living structure composed by a fluid phase (the blood) and a solid matrix (the tissues). Because of the metabolism, a volumetric heat generation in the solid part occurs. Recently, Xuan and Roetzel [16, 17], applied this approach to human tissues, where the porous medium models the tissue cells and the interlinked voids where arterial or venous blood flows. By using the principle of local thermal nonequilibrium between the tissue and the blood, the thermal energy exchange between the tissue and the blood in a given volume element is formulated as follows [16]:

$$\begin{aligned} (1-\phi)\rho_t c_t \frac{\partial T_t}{\partial t} &= \nabla\cdot((1-\phi)k_t \nabla T_t) + h_{bt}(T_b - T_t) + (1-\phi)q_{met}, \\ \phi\rho_b c_b \frac{\partial T_b}{\partial t} &= \nabla\cdot(\phi k_b \nabla T_b) - h_{bt}(T_b - T_t) + \phi\rho_b c_b \mathbf{v_b}\cdot\nabla T_b, \end{aligned} \tag{2}$$

$\phi, T_b, T_t, k_t, k_b, \mathbf{v_b}$ and h_{bt} being the porosity of the tissue, the local arterial blood averaged temperature, the local tissue averaged temperature, the tissue thermal conductivity tensor, the blood thermal conductivity tensor, the blood velocity vector and the interstitial convective heat transfer coefficient, respectively. Moreover, the energy equations for both phases are coupled by the interstitial convective heat transfer, representing the heat transfer to the tissue due to blood convection, i.e. the heat exchange rate through the boundary surface between the blood phase and the solid matrix because of local thermal nonequilibrium [14].

In several medical treatments, especially in dermatology, the laser heating of biological tissues is widely used; consequently the details of the heat transfer and of the related thermo-mechanical properties of tissues are essential from a medical viewpoint. Since the thermo-mechanical response of the skin to various therapeutic temperatures during laser irradiation is not completely known, it is important to investigate the thermal behavior of tissues during laser-tissue interaction to avoid thermal damage.

Many mathematical models of heat conduction in biological tissues irradiated with laser are described in the literature [18-22]. In most of these contributions, the temperature distribution in the tissues were obtained by using the heat transfer equation proposed by Pennes (parabolic model). However, although the bioheat equation can be valid is several situations, it has been hypothesized that for heat transfers on small time scales classical model will fail, and a thermal wave theory with finite thermal propagation speed could be more appropriate to describe such phenomena [7].

In fact, a model that considers a finite speed of propagation of thermal energy is crucial in surgical or therapeutic procedures (such as radiofrequency heating, irradiation, ...) where short heating times occur. In such cases, a non-Fourier type model should be considered by introducing a hyperbolic heat transfer equation with a thermal relaxation time τ_R of the tissue. The hyperbolicity guarantees a finite speed of heat propagation, which is inversely proportional to τ_R. Thermal relaxation time represents a parameter for estimating the time required for heat to conduct away from a directly heated tissue region. When continuous wave lasers heat targets for longer than the thermal relaxation time, heat diffuses to adjacent tissues, where it can induce collateral damage.

Likely, the internal structure of biological tissues could have a decisive impact on relaxation time. In addition, it could be modified, for example, by specific nanoparticles carried by the blood and addressed to certain regions of the tissue such as in a neoplastic region. Nanoparticles, as they possess a number of properties such as chemical reactivity, energy absorption, and biological mobility, have an important impact in the treatment of various types of cancer, as evidenced by the numerous nanoparticle-based drug and delivery systems that are in medical treatments [23]. Different nanoparticle-based drug delivery strategies can be utilized to modulate and improve the performance of a drug; also, metal nanoparticles could locally increase the absorption of the laser beam in the tumor region and thus modify the τ_R value dependent on microscopic thermal inertia. So, it is reasonable to expect τ_R play a fundamental role in the thermal destruction of the of cancer cells by targeted absorption of the laser beam.

The current study focuses on the analysis of a non-Fourier bioheat transfer model describing the laser heating of skin tissue; the solutions will be obtained using the Laplace transform method. Therefore, the purpose of this paper is to propose a hyperbolic model of thermal conduction and to carry out an analysis on thermal relaxation of the tissue after laser heating in order to evaluate the time needed for temperature values, caused by the heat absorbed by the laser, decrease until are reached values close to those desired. In this assessment, it is very important to make the appropriate changes to the biological parameters of the model and the intensity of the laser beam so that the risk of tissue damage is reduced.

The structure of the paper is as follows. In Section 2, we introduce the model of heat transport of biological blood perfused tissue, and provide a mathematical formulation in a semi-infinite domain. Then, in Section 3, we present an analytical solution suitable for describing the distribution of the temperature. The results are discussed in Section 4. Finally, Section 5 contains some final comments as well as possible future developments.

2. Mathematical Formulation of Heat Transport Model for Blood Perfused Tissues

In this Section, we consider a semi-infinite fragment of homogenous isotropic biological tissue. Let us suppose that the whole tissue surface is influenced by the laser energy; thus, we will solve the heat transfer equation in a one- dimensional setting, where the unique spatial variable follows the direction of the laser beam. Furthermore, local thermal equilibrium is used to an acceptable approximation for the temperature field in some applications involving blood vessels of small sizes. Therefore, we limit ourselves to the case where the temperatures of the blood and tissues are equal in a certain volume element, i.e., $T_t = T_b = T$, Hence, the two equations (2) reduce to a single equation, say

$$\left[(1-\phi)\rho_t c_t + \phi\rho_b c_b\right]\frac{\partial T}{\partial t} = \nabla\cdot\left\{\left[(1-\phi)k_t + \phi k_b\right]\nabla T\right\} - \phi\rho_b c_b \mathbf{v_b}\cdot\nabla T + (1-\phi)q_{met}. \tag{3}$$

Let us remark that the second term on the right hand side expresses the contribution to heat transfer due to blood perfusion. Thence, in the following, we suppose that the latter corresponds to the perfusion source term that in Penne's equation (1) was taken equal to $\rho_b c_b w_b (T - T_a)$, w_b being the flow rate of blood in the tissue per unit volume. This term is derived under the assumptions that in equilibrium conditions between the capillary tube and the tissue, the venous blood temperature locally is equal to the tissue temperature. Moreover, the arterial temperature is considered uniform throughout the tissue. Our mathematical model turns out to be hyperbolic since we introduce a relaxation time τ_R.

In equation (3) we have assigned for the heat flux $\mathbf{q}$ a constitutive equation that satisfies the Fourier equation, if we assume for heat flux a Maxwell-Cattaneo-Vernotte like evolution equation

$$\tau_R \frac{\partial \mathbf{q}}{\partial t} + \mathbf{q} = -[(1-\phi)k_t + \phi k_b]\nabla T, \tag{4}$$

we obtain

$$\begin{aligned} \left[(1-\phi)\rho_t c_t + \phi\rho_b c_b\right]\frac{\partial T}{\partial t} &= -\nabla\cdot\mathbf{q} - \phi\rho_b c_b w_b (T - T_a) + (1-\phi)q_{met}, \\ \tau_R \frac{\partial \mathbf{q}}{\partial t} + \mathbf{q} &= -[(1-\phi)k_t + \phi k_b]\nabla T. \end{aligned} \tag{5}$$

and considering an additional term which is responsible for the effect of laser heat source on skin tissue, the resulting hyperbolic bioheat transfer equation (3), in a semi-infinite domain $\Omega = [0, +\infty)$ reads

$$\begin{aligned} &\tau_R\left[(1-\phi)\rho_t c_t + \phi\rho_b c_b\right]\frac{\partial^2 T}{\partial t^2} + \left[(1-\phi)\rho_t c_t + \phi\rho_b c_b + \tau_R\phi\rho_b c_b w_b\right]\frac{\partial T}{\partial t} \\ &= \left[(1-\phi)k_t + \phi k_b\right]\frac{\partial^2 T}{\partial x^2} + \phi\rho_b c_b w_b (T_a - T) + (1-\phi)q_{met} + (1-\phi)\left[q_{laser} + \tau_R \frac{\partial q_{laser}}{\partial t}\right], \end{aligned} \tag{6}$$

Let us model the effect of laser as an internal heat source q_{laser}, and assign the following adiabatic-like boundary and initial conditions:

$$T_x(0,t) = 0, \qquad \lim_{x\to\infty} T_x(x,t) = 0, \quad T(x,0) = T_0, \qquad T_t(x,0) = 0. \tag{7}$$

where T_0 is the initial temperature, i.e., the tissue temperature before heating. Moreover, let us suppose that, for the small values of the absorption coefficient of tissue a ($[a] = m^{-1}$), the laser effect is described by the Beer- Lambert's law [24]

$$q_{laser} = aI_0 \exp(-ax) H(t_{laser} - t), \tag{8}$$

representing the energy absorption of the laser irradiation. Here, I_0 is the constant irradiation intensity at the skin surface ($[q_{laser}] = Wm^{-2}$), $H(t)$ is the Heaviside function, t_{laser} is the instant the laser is removed, and x is the depth of the tissue.

3. The Analytical Solution of the Bioheat Model

In this Section, we determine the analytical solution of the hyperbolic bioheat model (6) by applying the Laplace transform. Before proceeding, it is convenient to write the system choosing the dimensionless variables

$$\eta = At, \qquad \xi = Bx, \qquad \theta(\xi,\eta) = C(T - T_a), \tag{9}$$

wherein, A, B, C are suitably chosen. Hence, the dimensionless thermal wave-like bioheat transfer equation during laser irradiation becomes

$$\begin{aligned} &\frac{\tau_R^2 A^2(1+\tilde{\beta})}{T_0 C}\frac{\partial^2\theta}{\partial\eta^2} + \frac{\tau_R A(1+\tilde{\beta}+\Lambda)}{T_0 C}\frac{\partial\theta}{\partial\eta} \\ &= \frac{\tau_R B^2(\alpha_t+\alpha_b)}{T_0 C}\frac{\partial^2\theta}{\partial\xi^2} - \frac{\tau_R\tilde{\beta}w_b}{t_0 C}\theta + \frac{\tau_R}{T_0\rho_t c_t}q_{met} + \frac{\tau_R a I_0}{T_0\rho_t c_t}\exp\left(-a\frac{\xi}{B}\right)\left[H\left(\frac{\eta_{laser}}{A} - \frac{\eta}{A}\right) + \tau_R A\frac{\partial}{\partial\eta}\left(H\left(\frac{\eta_{laser}}{A} - \frac{\eta}{A}\right)\right)\right], \end{aligned} \tag{10}$$

where

$$\tilde{\beta} = \frac{\phi\rho_b c_b}{(1-\phi)\rho_t c_t}, \quad \Lambda = \tau_R w_b\tilde{\beta}, \quad \alpha_t = \frac{k_t}{\rho_t c_t}, \quad \alpha_b = \frac{k_b}{(1-\phi)\rho_t c_t}, \quad \eta_{laser} = At_{laser} \tag{11}$$

with $[\alpha_t] = [\alpha_b] = m^2 s^{-1}$. Moreover, without loss of generality, we can assume that the coefficients of the derivatives of the temperature are equal to 1, i.e.,

$$\frac{\tau_R^2 A^2(1+\tilde{\beta})}{T_0 C} = 1, \qquad \frac{\tau_R A(1+\tilde{\beta}+\Lambda)}{T_0 C} = 1, \qquad \frac{\tau_R B^2(\alpha_t+\alpha_b)}{T_0 C} = 1, \tag{12}$$

along with the following expressions of the coefficients A, B, C:

$$A = \frac{1+\tilde{\beta}+\Lambda}{\tau_R(1+\tilde{\beta})}, \quad B = \frac{1+\tilde{\beta}+\Lambda}{\sqrt{\tau_R(\alpha_t+\alpha_b)(1+\tilde{\beta})}}, \quad C = \frac{(1+\tilde{\beta}+\Lambda)^2}{T_0(1+\tilde{\beta})}. \tag{13}$$

Using the previous coefficients in the relations (11), leads us to the further positions:

$$\gamma = \frac{\tau_R w_b \tilde{\beta}(1+\tilde{\beta})}{(1+\tilde{\beta}+\Lambda)^2}, \quad \Gamma = \frac{\tau_R}{T_0 \rho_t c_t}, \quad \lambda = \frac{\tau_R a I_0}{T_0 \rho_t c_t}, \quad g(\eta) = F(\eta) + A\tau_R \frac{\partial F(\eta)}{\partial \eta}, \tag{14}$$

with

$$F(\eta) = H\left(\frac{\eta_{laser} - \eta}{A}\right). \tag{15}$$

Then equation (10) can be rewritten in terms of the dimensionless variables as follows:

$$\frac{\partial^2 \theta}{\partial \eta^2} + \frac{\partial \theta}{\partial \eta} = \frac{\partial^2 \theta}{\partial \xi^2} - \gamma\theta + \Gamma q_{met} + \lambda \exp\left(-a\frac{\xi}{B}\right) g(\eta), \tag{16}$$

whereas the dimensionless initial and boundary conditions are expressed by:

$$\begin{aligned} &\theta(\xi,0) = \theta_0, \qquad \frac{\partial \theta}{\partial \eta}(\xi,0) = 0, \\ &\frac{\partial \theta}{\partial \xi}(0,\eta) = 0, \qquad \lim_{\xi\to\infty} \frac{\partial \theta}{\partial \xi}(\xi,\eta) = 0, \end{aligned} \tag{17}$$

with

$$\theta_0 = \frac{(1+\tilde{\beta}+\Lambda)^2}{T_0(1+\tilde{\beta})}(T_0 - T_a). \tag{18}$$

By taking Laplace transform with respect to t [25], and defining

$$\hat{\theta}(\xi,s) := \mathcal{L}[\theta(\xi,\eta)], \qquad \hat{g}(s) := \mathcal{L}[g(\eta)], \tag{19}$$

after multiplying the equation (16) by $\exp(-s\eta)$ and integrating in $[0,+\infty)$, one gets:

$$\frac{\partial^2 \hat{\theta}}{\partial \xi^2}(\xi,s) - (s^2+s+\gamma)\hat{\theta}(\xi,s) = -(1+s)\theta_0 - s\Gamma q_{met} - \lambda \exp\left(-a\frac{\xi}{B}\right)\hat{g}(s); \tag{20}$$

Moreover, the Laplace transform of the boundary conditions yields:

$$\frac{\partial \hat{\theta}}{\partial \xi}(0,s) = 0, \qquad \lim_{\xi\to\infty} \hat{\theta}(\xi,s) = 0. \tag{21}$$

Solving the homogeneous equation associated with equation (20), and testing with a particular solution $\hat{\theta}_{part}$, one gets the general solution:

$$\hat{\theta}(\xi,s) = \hat{\theta}_{hom}(\xi,s) + \hat{\theta}_{part}(\xi,s), \tag{22}$$

having the following explicit form:

$$\hat{\theta}(\xi,s) = c_1 \exp\left(\sqrt{s^2+s+\gamma}\,\xi\right) + c_2 \exp\left(-\sqrt{s^2+s+\gamma}\,\xi\right) + \frac{(1+s)\theta_0 + s\Gamma q_{met}}{s^2+s+\gamma} - \frac{\lambda \hat{g}(s) B^2}{a^2 - B^2(s^2+s+\gamma)} \exp\left(-\frac{a}{B}\xi\right). \tag{23}$$

Substituting the boundary conditions (21) in the derivative respect to ξ of the general solution, the expression of the coefficients c_1, c_2 are determined:

$$c_1 = 0, \quad c_2 = \frac{\lambda a \hat{g}(s) B}{[a^2 - B^2(s^2+s+\gamma)]\sqrt{s^2+s+\gamma}}. \tag{24}$$

Moreover, by substituting the relations (24) into equation (23), the function $\hat{\theta}(\xi,s)$ on the Laplace domain becomes:

$$\hat{\theta}(\xi,s) = \frac{\lambda a \hat{g}(s) B}{[a^2 - B^2(s^2+s+\gamma)]\sqrt{s^2+s+\gamma}} + \exp\left(-\sqrt{s^2+s+\gamma}\,\xi\right) + \frac{(1+s)\theta_0 + s\Gamma q_{met}}{s^2+s+\gamma} - \frac{\lambda \hat{g}(s) B^2}{a^2 - B^2(s^2+s+\gamma)} \exp\left(-\frac{a}{B}\xi\right). \tag{25}$$

In order to get the wanted temperature, it is necessary to compute the inverse of the Laplace transform

$$\theta(\xi,\eta) = \mathcal{L}^{-1}\left\{\hat{\theta}(\xi,s)\right\}; \tag{26}$$

this computation is not easy and involves different steps. For this reason, we split the calculation of each addend, and use the inverse Laplace tables [25]. The inverse Laplace transform of the first term of equation (25) is obtained by applying the convolution theorem of Laplace transform,

$$\mathcal{L}^{-1}\left\{F(s)G(s)\right\} = \int_0^\eta \mathcal{L}^{-1}\left\{F(s)\right\}_{s\to u} \mathcal{L}^{-1}\left\{G(s)\right\}_{s\to\eta-u} du = \int_0^\eta F(u)G(\eta-u)du, \tag{27}$$

leading to:

$$\mathcal{L}^{-1}\left\{\frac{\lambda a\hat{g}(s)B}{\left[a^2-B^2\left(s^2+s+\gamma\right)\right]\sqrt{s^2+s+\gamma}}\exp\left(-\sqrt{s^2+s+\gamma}\,\xi\right)\right\}=\lambda aB\int_0^{\eta}\exp\left(-\frac{\eta-v}{2}\right)I_0\left(\frac{1}{2}i\sqrt{4\gamma-1}\times\sqrt{(\eta-v)^2-\xi^2}\right)H(\eta-v-\xi)$$
$$\times\int_0^{v}F(u)\exp\left(-\frac{v-u}{2}\right)\left[-\frac{A\tau_R}{B^2}\cosh\left(\frac{\sqrt{-4B^2\gamma+4a^2+B^2}}{2B}(v-u)\right)+\frac{A\tau_R-2}{\sqrt{-4B^2\gamma+4a^2+B^2}}\sinh\left(\frac{\sqrt{-4B^2\gamma+4a^2+B^2}}{2B}(v-u)\right)\right]du\,dv \tag{28}$$
$$-\lambda aB\int_0^{\eta}\exp\left(-\frac{\eta-v}{2}\right)I_0\left(\frac{1}{2}i\sqrt{4\gamma-1}\times\sqrt{(\eta-v)^2-\xi^2}\right)H(\eta-v-\xi)\times\left[\frac{2A\tau_RF(0)\exp\left(-\frac{v}{2}\right)}{B\sqrt{-4B^2\gamma+4a^2+B^2}}\exp\left(-\frac{\eta}{2}\right)\sinh\left(\frac{v}{2}\frac{\sqrt{-4B^2\gamma+4a^2+B^2}}{B}\right)\right]dv,$$

with

$$\hat{g}(s)=(1+A\tau_Rs)\hat{F}(s)-A\tau_RH\left(\frac{\eta_{laser}}{A}\right). \tag{29}$$

The inverse Laplace transform of the second term of Eq. (25)

$$\mathcal{L}^{-1}\left\{\frac{1+s}{s^2+s+\gamma}\theta_0\right\}=\theta_0\exp\left(\frac{\eta}{2}\right)\left[\cos\left(\frac{\eta}{2}\sqrt{4\gamma+1}\right)+\frac{1}{\sqrt{4\gamma+1}}\sin\left(\frac{\eta}{2}\sqrt{4\gamma+1}\right)\right] \tag{30}$$

The inverse Laplace transform of the third term of Eq. (25) is given by

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+s+\gamma}\Gamma q_{met}\right\}=\Gamma q_{met}\exp\left(-\frac{\eta}{2}\right)\left[\cos\left(\frac{\eta}{2}\sqrt{4\gamma+1}\right)-\frac{1}{\sqrt{4\gamma+1}}\sin\left(\frac{\eta}{2}\sqrt{4\gamma+1}\right)\right]. \tag{31}$$

Moreover, the inverse Laplace transform of the fourth term of Eq. (25) is:

$$\mathcal{L}^{-1}\left\{\frac{-\lambda\hat{g}(s)B^2}{a^2-B^2(s^2+s+\gamma)}\exp\left(-\frac{a}{B}\xi\right)\right\}=-\lambda B\exp\left(-\frac{a}{B}\xi\right)\int_0^{\eta}F(u)\exp\left(-\frac{\eta-u}{2}\right)$$
$$\times\left[-\frac{A\tau_R}{B}\cosh\left(\frac{\sqrt{-4B^2\gamma+4a^2+B^2}}{2B}(\eta-u)\right)+\frac{A\tau_R-2}{\sqrt{-4B^2\gamma+4a^2+B^2}}\sinh\left(\frac{\sqrt{-4B^2\gamma+4a^2+B^2}}{2B}(\eta-u)\right)\right]du \tag{32}$$
$$+\frac{2\lambda B\exp\left(-\frac{a}{B}\xi\right)A\tau_RF(0)}{\sqrt{-4B^2\gamma+4a^2+B^2}}\exp\left(-\frac{\eta}{2}\right)\sinh\left(\frac{\eta}{2}\frac{\sqrt{-4B^2\gamma+4a^2+B^2}}{B}\right).$$

Finally, by introducing the inverse Laplace transform of all the addends in relation (25), the temperature $\theta(\xi,\eta)$ writes as

$$\theta(\xi,\eta)=\mathcal{L}^{-1}\left\{\frac{\lambda a\hat{g}(s)B}{[a^2-B^2(s^2+s+\gamma)]\sqrt{s^2+s+\gamma}}\exp\left(-\sqrt{s^2+s+\gamma}\,\xi\right)\right\}+\mathcal{L}^{-1}\left\{\frac{1+s}{s^2+s+\gamma}\theta_0\right\}+\mathcal{L}^{-1}\left\{\frac{s}{s^2+s+\gamma}\Gamma q_{met}\right\}$$
$$+\mathcal{L}^{-1}\left\{\frac{-\lambda\hat{g}(s)B^2}{a^2-B^2(s^2+s+\gamma)}\exp\left(-\frac{a}{B}\xi\right)\right\}. \tag{33}$$

In conclusion, coming back to the original physical variables, the following solution for $\theta(x,y)$ is obtained:

$$T(x,t)=\frac{T_0(1+\tilde{\beta})}{(1+\tilde{\beta}+\Lambda)^2}\cdot\theta(Bx,At)+T_a. \tag{34}$$

Table 1. Thermal and laser properties of the model.

Parameters	Value [2]	Parameters	Value [3]
$\rho_t\left[Kg/mm^3\right]$	1200 x 10^{-9}	$\rho_b\left[Kg/mm^3\right]$	1080 x 10^{-9}
$c_t\left[J\cdot Kg^{-1}\cdot K^{-1}\right]$	3222	$c_b\left[J/Kg\cdot K\right]$	3300
$k_t\left[W/mm\cdot K\right]$	0.42 x 10^{-3}	$k_b\left[W/mm\cdot K\right]$	0.508 x 10^{-3}
$q_{met}\left[W/mm^3\right]$	368 x 10^{-9}	$w_b\left[s^{-1}\right]$	8.3 x 10^{-3}
$\tau_R[s]$	10	ϕ	0.3
$T_a\left[°C\right]$	36	$T_0\left[°C\right]$	35.1
$I_0\left[W/mm^2\right]$	122 x 10^{-3}	$t_{laser}\left[s\right]$	15
$a\left[mm^{-1}\right]$	70 x 10^{-3}		

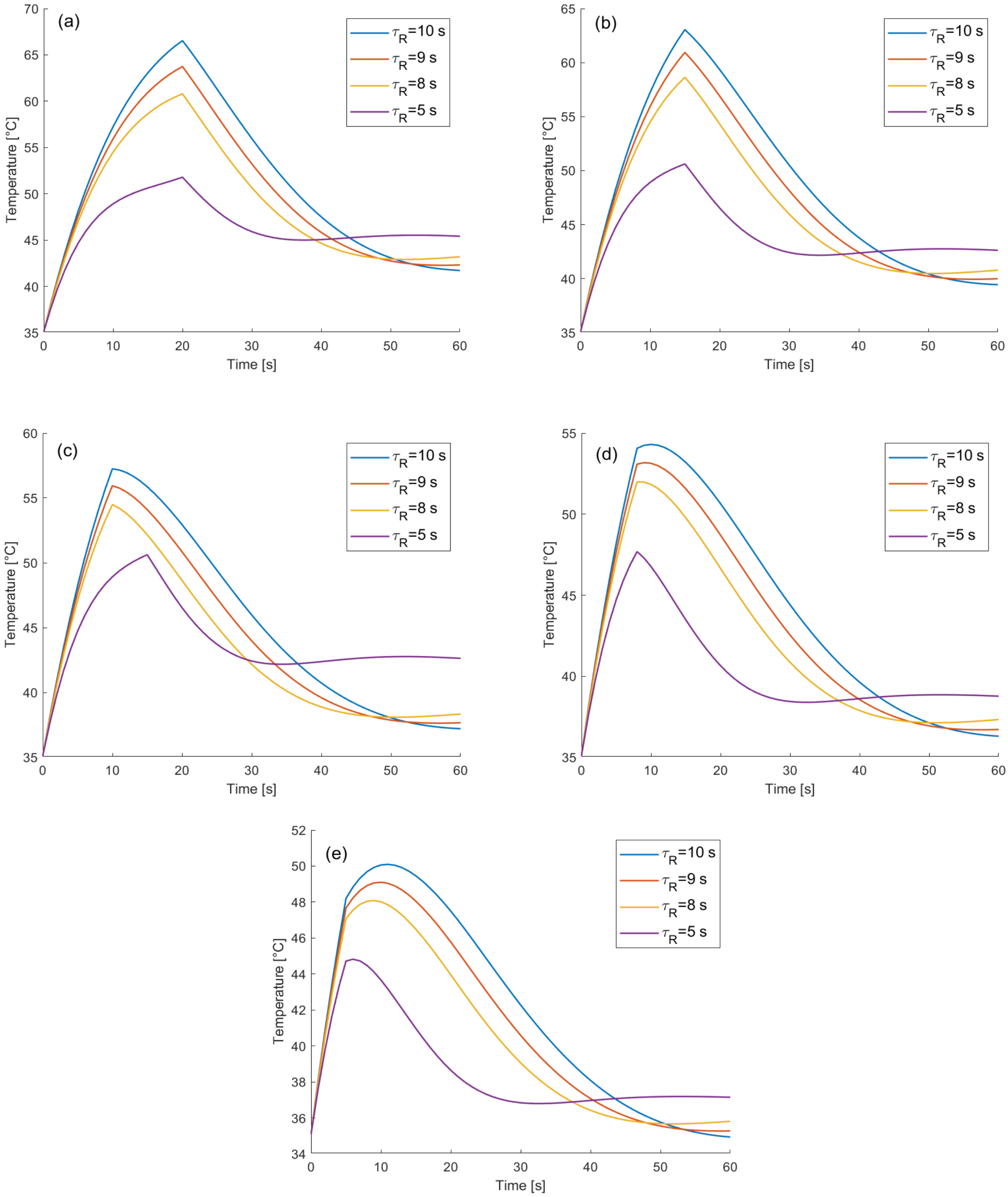

Fig. 1. The skin surface temperature profile over time vs relaxation times τ_R, (a) t_{laser} =20s (b) t_{laser} =15s (c) t_{laser} =10s (d) t_{laser} =8s (e) t_{laser} =5s (f) t_{laser} =2s.

4. Results and Discussion

In the present section, we attempt to explain the influence of important parameters of the model on temperature profiles. A summary of all relevant parameters used is given in Table 1. The solution obtained in the previous section is here analyzed in order to discuss the distribution of the temperature in a biological tissue (808 nm laser, 122 kWm^{-2}) as a function of the thermal relaxation time τ_R also considering different values for blood perfusion ω_b [26].

Thus, the effects of laser irradiation time, laser intensity on the temperature distributions in the layered skin during the laser beam can be described. During the action of the laser it is necessary to control the growth of the temperature. In fact, too high temperatures could cause undesirable and often irreversible damages to the skin and surrounding tissues (full-thickness epidermal necrosis including hair follicles, dermal necrosis, dermal edema and hemorrhage, [26]).

A rise in temperature during the laser session of course depends on the irradiation time, laser intensity and type of the exposed tissue. In Fig. 1, it is shown the non-Fourier-type temperature evolution equation as a function of relaxation time τ_R and for different time exposure duration of the laser. Let us remark that τ_R plays an important role in the temperature evolution, as expected, whereas the thermal delay time has a major influence on the temperature distribution, i.e., as τ_R increases the tissue temperature decreases more slowly. In particular, increasing the value of the relaxation time, the action of laser provides higher values for the temperature, while once the laser action is stopped (for example after 20 seconds in Fig. 1(a)), it is observed a decrease of temperature fluctuations. The temperature profile, from a natural state around $T_0 = 35.1$ °C, during the process of irradiation increases until a maximum acceptable value is attained.

Figure 2(a) clearly shows the influence of porosity on the evolution of the skin surface temperature. Higher porosity values provide smaller variations in temperatures.

In Fig. 2(b), the effects of the rate of blood perfusion ω_b under bioheat model with a fixed relaxation time on the temperature variation is depicted. Higher values of the rate of blood perfusion have the effect of increasing the convective heat loss due to faster blood flow; this allows the skin to exhibit lower values of surface temperature.

Finally, in Fig. 3, the trend of the temperature is represented varying the intensity of the laser. Comparison between our theoretical and the experimental results get by Zhou et al. [28] demonstrate that the behavior of the temperature results agreed with the experimental data. Some discrepancies are due to the presence of additional material parameters in the two models, such as porosity of the tissue and evaporation on skin surface.

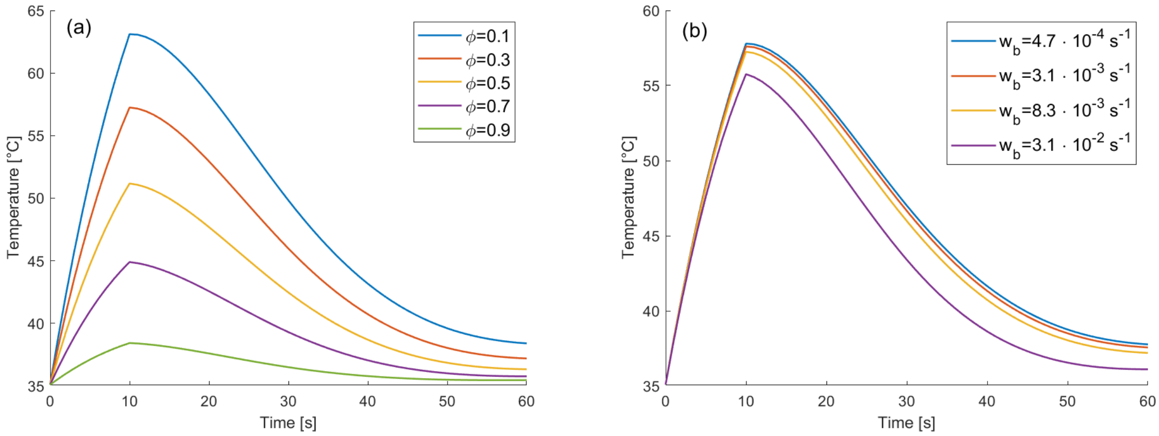

Fig. 2. Temperature profiles for different values of (a) ϕ, (b) w_b, with I_0=2.0 W mm^{-2}, τ_R=10s, t_{laser}=10s.

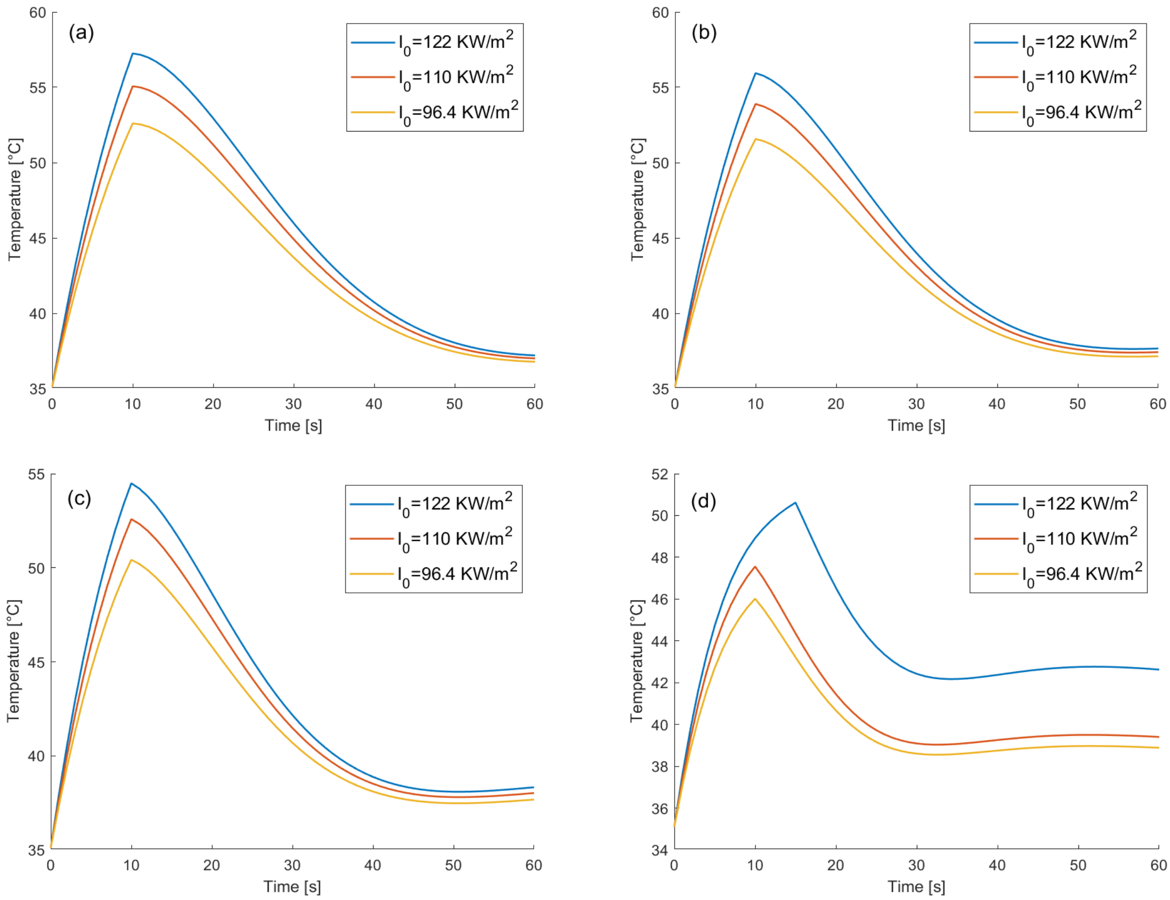

Fig. 3. Temperature profile over time vs laser intensity I_0, with (a) τ_R=10s, (b) τ_R=9s, (c) τ_R=8s, (d) τ_R=5s.

5. Conclusions

The health and the treatment efficiency during laser surgeries and laser medical applications are very important. Thus, predicting tissue temperature during laser irradiation is one of the most significant topic in medical field. An attempt to get a more accurate description of heat transfer in nonhomogeneous materials leads to the application of the Maxwell-Cattaneo-Vernotte equation.

Progress in research on the phenomena of heat transfer to a finite propagation speeds have led to increased interest in solutions of the hyperbolic equation, [27]. The bioheat transfer equation, has also been used in a wide range of applications to describe heat transport in blood perfused tissues, [17] by a variety of generalized non-Fourier type models.

Similar models can be used in many therapeutic treatments involving temperature changes. For example, in the development of cryosurgical techniques for the administration of maximum cell destruction within the tumor while preserving healthy tissue; in the treatment of hyperthermia, useful for tumors that are sensitive to heat, the body tissue is exposed to high temperatures. It is usually used with other forms of cancer therapy, such as radiation therapy and chemotherapy, because hyperthermia may make some cancer cells more sensible to radiation or hurt other cancer cells that radiation cannot damage, (Yuan, [17]). Furthermore, there is also a great interest in the thermal properties of the skin, in order to understand the conditions that lead to thermal damage that can occur when the skin is in contact with high temperature heat sources. Zhou et al., [28] have numerically studied the thermal damage to biological tissues caused by laser irradiation through a dual phase-lag bioheat transfer model.

Due to long-thermal relaxation time of tissue, the non-Fourier model is more reliable for depicting the propagation thermal phenomenon and evaluating the temperature distribution than the classical Fourier one [29].

In the present paper, under suitable approximations, we have solved analytically in one-dimensional setting hyperbolic heat equation including a source term for blood perfusion and laser irradiation on the surface tissue considered as internal heat source.

The exact solution found here can be used to study the evolution of temperature within tissues during thermal therapy. In particular, the effects of thermal relaxation time τ_R on blood perfusion on tissue w_b and temperature T are discussed.

It has been observed that increased perfusion induces a drop in local temperature. In addition, increased energy is concentrated in the skin surface due to large thermal relaxation time leading to a finite speed of heat conduction. Theoretical curve analysis is crucial to identify tissue parameters. The range of variability of these parameters can be investigated in order to determine the appropriate amount of irradiation to which the tissue should be subjected for therapy and such that it does not cause damage. This topic will be a subject of future studies. The results show that due to laser-induced heat, the hyperbolicity of the model provides temperatures higher than those obtained with a parabolic model. The thermal effect of the laser beam depends heavily on the absorption and scattering coefficients, the intensity and the wavelength of the laser. In a future work we will consider a more realistic situation, in order to develop a nonlinear theoretical model that includes in the generalized bioheat equation (6) a temperature-dependent relaxation time and blood perfusion.

Author Contributions

C.F. Munafò and P. Rogolino planned the scheme, initiated the project, and suggested the model; C.F. Munafò and P. Rogolino computed the analytical solution and analyzed the numerical results; C.F. Munafò and P. Rogolino developed the mathematical modeling and examined the theory validation. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed, and approved the final version of the manuscript.

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Conflict of Interest

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Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

Nomenclature

T_t	Local tissue averaged temperature [K]	T_b	Local blood averaged temperature [K]
ρ_t	Mass density of tissue [Kg/m^3]	ρ_b	Mass density of blood [Kg/m^3]
c_t	Specific caloric of tissue [J/Kg K]	c_b	Specific caloric of blood [J/Kg K]
k_t	Thermal conductivity of tissue [W/mm K]	k_b	Thermal conductivity of blood [W/mm K]
q_{met}	Metabolic source [W/mm^3]	w_b	Blood perfusion [1/s]
T_a	Arteries temperature [K]	τ_R	Relaxation time [s]
T_0	Initial temperature [K]	ϕ	Porosity
I_0	Irradiance [W/mm^2]	a	Absorption coefficient [1/mm]
$\mathbf{v}_b$	Blood velocity vector [mm/s]	h_{bt}	Convective heat transfer coefficient [W/mm K]

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