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Does taking additional Maths classes in high school affect academic outcomes?

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ABSTRACT

Several studies in the mathematical education literature show the effect of students' high school skills in maths on their success at higher levels of education and work. In particular, the importance of maths course taking in US high schools is highlighted to be important for college enrollment and completion. The choice of taking additional maths courses or, as in Italy, of choosing a high-school curriculum with more maths, is not random: it depends on several substantial factors such as gender and socio-economic status. This selection bias implies that the differences in the academic outcomes might be traceable not only to mathematics ability and knowledge. In this paper, the aim is to estimate the treatment effect of attending a relatively new high school curriculum in Italy with more maths, with respect to the traditional track of the scientific "liceo", on two academic outcomes: university enrollment and first-year university performance. After having reduced the selection bias using a caliper multi-level propensity score matching procedure, a multi-state Markov model is used to study the treatment effect on the joint educational outcomes.

0. Introduction

Mathematics is widely regarded as one of the most important school subjects and a central major in high school due to its relevance and application in most fields [1]. According to [2], the reason for the prime importance attributed by the societies to mathematics more than any other science is ascribable to: i) its essential nature in a broad variety of general areas of practice, as the representation of numbers, measurement of time, space, weight, and all sorts of graphical representations and tables; ii) its involvement in specialized practice areas such as optimization, explanation, prediction, decision-making, and problem-solving; iii) its fundamental role in the formulation and foundation as well as in the methods and techniques of many other disciplines. The study of mathematics at university underpins the study of many subjects, not only in science, technology, engineering, and mathematics (STEM) but also in many other fields [3]. In fact, there is at least one mathematics or mathematics-related course in most of the degree programs, and it is often regarded as one of the toughest.

In this paper, our aim is to investigate the effect of studying more mathematics in high school on future educational outcomes. Here, the choice of studying more mathematics in high school depends on many factors related both to the high school system of a country and to students' inclination and characteristics. In the US, students have more flexibility in the choice of attending some courses in high school, and it has been long known that taking additional maths classes is correlated both with college enrollment and success [4,5] and with occupational opportunities [6]. In other countries, such as Italy, the high school system is different. Italian students are not allowed to choose single courses in high school since they choose, at the age of 13, a 5-year high school curriculum among several equal for everybody.

Generally speaking, students who take advanced maths courses, or who choose a high school curriculum with more maths, systematically differ from those who do not in most educational systems [7,8].

Hence, there is a selection bias in the decision to study more mathematics in high school. In the US, selection bias occurs since only the best students decide to study more mathematics in high school [9]. In Italy, the choice of studying more mathematics, that is the choice of attending a specific high school curriculum, is related to socio-cultural factors more than students' skills and preferences.

Several surveys are conducted for investigating mathematical skills in high school at both the international and the national levels. Program for International Student Assessment (PISA) promoted by OECD is the world's largest and most popular educational survey. PISA tests aim to measure teenagers' learning levels in maths, science, and reading, in the international framework. In Italy, there are specific tests managed by INVALSI (*National Evaluation Institute for the School System*). Those tests, similar to PISA ones, are administered every year to pupils of four different levels of education, aiming to measure mathematical and

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Italian language abilities and, since 2018, also English reading and listening abilities.

Investigating students' performance in high school is of paramount importance. It has been shown to be among the most influential factors in university performance [7,10]. However, it is known that students' university performance lies above an incredibly intricate net of multidimensional factors: as the student's high school career, other factors such as the student's socio-economic status [11,12], gender [13–15], and geographical differences [16,17], strongly influence students' academic performance. However, apart from surveys or small-scale analyses based on single institutions, little has been done in Italy to analyze how those factors affect academic outcomes, probably due to the lack of longitudinal micro-data.

In this paper, we reconstruct the longitudinal careers of the students by linking the micro-data of Italian high school students to the micro-data on their first year at university using data coming from different institutional Italian sources. This linkage allows having unique insights into how factors, such as high school performance and socio-economic status, influence students' academic outcomes. Focusing on the Italian case, we aim to evaluate the effect of studying more mathematics in high school on two different and crucial academic outcomes: the decision to enroll at university and the first-year university performance.

Similar to a clinical study, two treatments are considered: the *traditional* track of the scientific "liceo", which is the most popular high school curriculum, and the *applied sciences* track, where more hours are devoted to mathematics-related subjects. Petolicchio [18] compares the in-going and out-going mathematical skills of the students of the two curricula in Italy, underlying the strengths and the weaknesses of both tracks, but as far as the authors' knowledge there are no results about the differences in terms of academic performance between the two scientific tracks. To assess the effect of attending the applied sciences track, we first need to consider the selection bias previously discussed. In randomized studies, differences between treated and control groups can be interpreted as causal effects. On the contrary, when subjects select their treatment, different outcomes may reflect substantial differences in treated and control groups not traceable to treatment effects [19]. In this context, propensity score matching (PSM) has been widely regarded as a useful tool i) to reduce the selection bias by balancing the characteristics of the two groups and ii) to estimate treatment effects in observational studies where the treatment assignment is not random [20]. When dealing with hierarchical data, multi-level propensity score matching (MPSM) is the natural extension of the propensity score [21]. The multi-level or clustered structure adds complexity given that the selection mechanism, as well as the dependency within clusters and the outcomes, may vary considerably across clusters [21]. Educational data represents a perfect example of a hierarchical structure in which individuals (students) are grouped in clusters (schools) and receive a "clustered treatment", which is the high school curriculum. Here, the school "summarizes" the general context in terms of socio-economic level and the neighborhood in which the school is located.

After balancing the characteristics of the students attending the two tracks using MPSM, a Markov multi-state type model is used to study the treatment effect on the two joint academic outcomes. Those models are typically used in survival analysis to analyze the stage progression of a disease, while they have been rarely used in an educational framework [22]. In this paper, the idea is that stage transitions are represented by students' choices and performances. More specifically, we are interested in estimating the probability of having a specific academic performance, expressed in terms of the number of first-year ECTS, conditioned to the university enrollment in a specific degree program. Those conditional probabilities are estimated for both the unmatched and the matched datasets; the differences between the two treatment groups are studied after the matching procedure for identifying the effect of studying more mathematics on university performance.

The paper is structured as follows: in Section 1 the structure of the Italian educational system is introduced; in Section 2 the data coming from three Italian administrative sources are described; in Section 3 a descriptive analysis is conducted to have a first overview of the existing differences among the two tracks; then, in Section 4 the multi-level propensity score matching procedure and the discrete-time multi-state Markov model are briefly illustrated; finally, the results and conclusions are discussed in Sections 5 and 6.

1. Theoretical framework

Stratification and selectivity describe the structure of the educational system of a country [23,24]. In the Italian high school system, students have apparently freedom to choose the most suitable track. The choice of the type of high school is taken at the age of 13, and it has long-lasting effects on future educational career and labor market perspectives. In 1923, the Gentile reform proposed the division of the high school system into four different paths, intentionally ordered in terms of prestige and selectivity: the humanistic "licei", which allowed access to all the degree programs; the scientific "licei", allowing access to all the degree programs, except law and humanities; the technical schools, allowing the access to few technical degree programs; and finally, the "istituti magistrali", devoted to training the primary school teachers. In 1969, a bill allowed access to university for every high school curriculum, introducing apparently a democratic reform with the intention to reduce the hierarchy among the different curricula.

Nowadays, despite the existence of a large variety of tracks, the Italian high school system could be described as tripartite, with a 5-year academic-oriented generalist education provided by "licei" (with further distinctions in humanities, sciences, languages, pedagogical sciences), 5-year technical schools, and 5-year vocational schools. Each of those tracks is associated with very different outcomes in terms of engagement in further education and labor-market perspectives. The "licei" aim to prepare students to attain successfully a university degree [25,26]. Among all, humanistic and scientific "licei" are the most prestigious and tough tracks. Technical schools have the general objective of providing students with a scientific and technological background in the economic and technological-professional sectors [27], while vocational schools address students to job placement.

The horizontal stratification of the Italian school system implies an inevitable selection process of the students among the different curricula. Theoretically, this selection should be based on students' motivations. However, this selection is actually based on the socio-economic status of the family, which is typically responsible for their children's educational choices [24,28]. On the one hand, upper-class families consider a potential university career for their children, while families from lower classes may prefer technical or vocational schools for their labor market orientation.

After the Second World War, attendance increased at each level of education for all social classes, but socio-economic status is still related to the high school curriculum. Due to the increasing market request for highly qualified people, choosing a scientific or humanistic curriculum is considered a forward-looking choice for the likely achievement of a university degree and a better job [25,29].

Despite the centralization of the Italian high school system, which started in 1870 and has been established by all the other governments till now, it is important to underline the still present divide between northern and southern regions in terms of school and university dropout rates, which are much higher in southern regions.

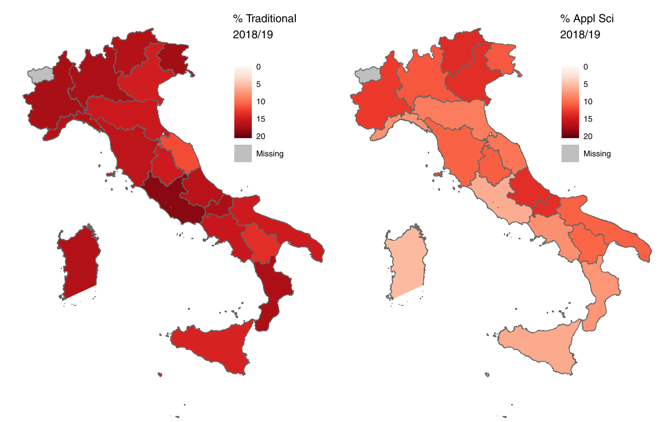

Fig. 1. Percentages of traditional or applied sciences tracks over the total number of tracks offered at the regional level in 2018/19.

Table 1
Number of hours per week devoted to scientific and non-scientific subjects in the two tracks of the scientific “liceo” since the 2010 reform.

Discipline	Scientific “liceo” track									
	Traditional					Applied Sciences				
	I	II	III	IV	V	I	II	III	IV	V
Mathematics & Computer Science	5	5	4	4	4	7	6	6	6	6
Physics	2	2	3	3	3	2	2	3	3	3
Natural sciences	2	2	3	3	3	3	4	5	5	5
Scientific	**9**	**9**	**10**	**10**	**10**	**12**	**12**	**14**	**14**	**14**
Italian	4	4	4	4	4	4	4	4	4	4
Latin language	3	3	3	3	3	–	–	–	–	–
Foreign language	3	3	3	3	3	3	3	3	3	3
History and geography	3	3	–	–	–	3	3	–	–	–
History	–	–	2	2	2	–	–	2	2	2
Philosophy	–	–	3	3	3	–	–	2	2	2
Technical drawing and arts	2	2	2	2	2	2	2	2	2	2
Gymnastics	2	2	2	2	2	2	2	2	2	2
Religion	1	1	1	1	1	1	1	1	1	1
Non-scientific	**18**	**18**	**20**	**20**	**20**	**15**	**15**	**16**	**16**	**16**
Total amount of hours	**27**	**27**	**30**	**30**	**30**	**27**	**27**	**30**	**30**	**30**

1.1. The scientific “liceo”

Fascism introduced the reform of the Italian high school system, named the Gentile reform, in 1923. Within this reform, the scientific “liceo” was initially a 4-year course compared to the humanistic “liceo”, which was a 5-year course and was considered the elitarian curriculum. Only in 1952, it was transformed into a 5-year course. In Italy, the number of high school students has constantly increased since the Second World War, and the scientific “liceo” is now the most popular high school [30]. Since 1947, several reforms have been proposed within the Italian high school system. However, its previous hierarchical structure introduced by Gentile has been slightly attenuated.

In 2010, the Gelmini reform of the Italian high school introduced a new track within the scientific “liceo”, named the *applied sciences* track. As shown in Table 1, the main difference with the traditional track was the replacement of the Latin language class with more classes devoted to scientific subjects, such as mathematics, computer science, and natural sciences.

2. Data

The dataset used in this paper is built by linking three distinct administrative national sources:

- **ANS-S**: micro-data coming from the National Archive of Schools. It contains information at the track level about the students enrolled in grade 13, i.e. the last year of high school, and other information about the school. The available data covers the period from 2014/15 to 2018/19.
- **INVALSI**: micro-data coming from the National Evaluation Institute for the School System (INV). The statistical unit is the student. INVALSI carries out standardized national assessment tests to evaluate students’ performance in grade 13 both at the school and the student level. Those tests are administered every year – discontinued during the COVID pandemic – to students at four different levels of education, aiming to measure mathematical and Italian language skills and, from 2018, also English reading and listening skills. In addition to the students’ INVALSI test scores, other important variables are the intermediate grades in maths and Italian and the socio-economic status of their family.
- **ANS-U**: micro-level longitudinal data coming from the National Archive of University Students *Anagrafe Nazionale Studenti* (ANS). It is a database comprehensive of all the information about the university careers of all the students enrolled in Italy from 2008 to 2020. This database contains a record for each freshman including information about the high school background and the whole university career until completion or dropout.

Thanks to the ANS-S database, we can compute the school-university transition for every single Italian school in a detailed way that, as far as we know, has never been explored [7,10]. In addition, the linkage of those databases coming from the three different national archives allows investigation of: i) the transition from high school to university at school and individual level; ii) the relationship between student’s performance in high school and university outcomes.

In detail, we only consider the students in the 5th year of high school in 2018/19, which is, at the moment, the only cohort linkable with the university career data. In particular, we have information about the decision to enroll at university and first-year performance only for the 5th-year high school students in 2018/19 who enrolled at university in 2019/20. In addition, we consider only the subset of the students who immediately enrolled at university after high school graduation. Finally, we consider only those schools that include at least one of the two tracks of the scientific “liceo”: the traditional and the applied sciences tracks.

3. Preliminary analysis

This section aims at illustrating a general framework of the differences between the two scientific tracks. Due to the hierarchical structure of our data, the analysis will be conducted on two different levels: at the track level, we will analyze the distribution of the two tracks over the Italian territory; at the student level, we will consider variables such as the Economic, Social and Cultural Status index (ESCS), the INVALSI test scores, and the transition rates to university.

3.1. Track-level analysis

In Fig. 1, we show the percentage of traditional or the applied sciences tracks over the total number of tracks offered at the regional level in 2018/19. On average, the applied sciences tracks make up only the 6.5% of the total number of tracks offered by Italian schools, while the traditional track is the most offered track with the 17%. In detail, the applied sciences track is mainly present in the northern and central Italian regions, while its presence in the southern regions, especially in the Islands, is nearly negligible. On the contrary, the traditional track

Table 2
Students' characteristics of the two tracks of the scientific "liceo". Cohort of the 5th-year high school students in 2018/19.

Variable	Scientific "liceo" track	
	Traditional	Applied sciences
Student's ESCS		
mean	0.53	0.36
sd	0.93	0.92
Gender		
no. of females	25 699	6783
% of females	47.3	32.0
Career regularity		
no. of regular students	51 481	19 150
% of regular students	94.8	90.2
Citizenship		
no. of foreigners	2429	1307
% of foreigners	4.5	6.2
INVALSI score		
Maths		
mean	237.77	240.08
sd	35.40	34.55
Italian		
mean	226.80	222.41
sd	35.61	33.32
English reading		
mean	227.14	223.98
sd	32.61	32.58
English listening		
mean	222.70	221.30
sd	35.54	35.20
Total	**54 327**	**21 237**

is more established in the Italian territory and its distribution is more homogeneous. In this respect, the students attending the traditional track are 1.8 times those attending the applied sciences one in the North of Italy, while the mean ratio is around 3.4 in central and southern regions, and 4.2 in the Islands.

3.2. Student-level analysis

The ESCS index represents the socio-economic status of the student. This index, used in both INVALSI and PISA studies, is built with the following variables: the International Socio-Economic Index of Occupational Status (ISEI); the highest level of education of the student's parents, converted into years of schooling; the PISA index of family wealth; the PISA index of home educational resources; and the PISA index of possessions related to "classical" culture in the family home. By construction, the ESCS index has zero mean and standard deviation equal to one [31]. A value of the index higher than zero indicates that students have a socio-economic and cultural level below the Italian average, and vice-versa.

In Table 2, there are the main characteristics of the students attending the two tracks of the scientific "liceo":

i) students from both tracks have a socio-economic status higher than the Italian average, but the applied sciences students have a lower ESCS index than the traditional ones, which confirms that upper-class families prefer the traditional track; ii) the overall percentage of females is 32% in the applied sciences track and 47.3% in the traditional one; iii) the proportions of students with a regular scholastic career and with foreign citizenship are higher in the applied sciences track; iv) on average, students from the applied sciences track perform slightly better in INVALSI maths tests than their peers from the traditional track, and slightly worse in the remaining tests. In Fig. 2, we show the school-university transition rates for both the traditional and applied sciences tracks of the scientific "liceo". Here, the transition rate is computed as the total number of students enrolled at university in 2019/20 over the total number of 5th-year high school students in

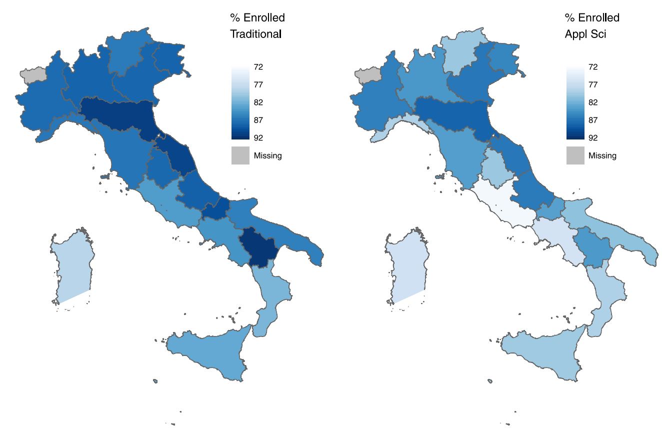

Fig. 2. High school-university transition rates for the two scientific tracks at the regional level, computed as the number of students enrolled at university in 2019/20 over the total number of 5th-year high school students in 2018/19.

2018/19. On average, the transition rates are 85.6% and 82% for the traditional and applied sciences tracks, respectively. However, some regional differences can be observed. The transition rates are higher in northern and central regions for both tracks, but the north-south divide is more evident for the applied sciences track.

The choice of the high school track influences future educational and labor market outcomes. In this respect, it is reasonable that having attended the applied sciences track, where the amount of hours devoted to scientific subjects is higher than at the traditional track, can influence educational choices at university. In Fig. 3, we show how the students of the two scientific tracks are distributed according to the field of study of university enrollment. The distribution of the students among the different degree programs seems to be similar, with the majority of the students of both groups enrolling in engineering and health degree programs. Yet, some differences can be observed: a higher percentage of applied sciences students decide to enroll in computing and engineering programs, while a lower percentage enroll in non-STEM programs, especially in arts and humanities, business and administration, and law.

4. Methods

When interventions are randomly assigned, differences between treated and control groups can be interpreted as causal effects, but when subjects select their treatment, different outcomes may reflect initial differences in treated and control groups rather than treatment effects [19]. In our case, the assignment to the treatment, i.e. the choice of the high school track, is not random since it depends on many factors. Students aged 13 years old do not indeed choose the school track by themselves. The parents are usually responsible for their children, and the choice differs a lot conditionally on socio-economic status [12,25,32,33], the geographical area in which they live [16], the gender of the children [14], and other factors. Therefore, the applied sciences treatment group and the traditional treatment group present substantial differences. To reduce the selection bias inherent to the track choice, we implement a multi-level propensity score matching procedure. As already said, educational data often presents a multi-level structure: students are grouped in classes/tracks that are grouped in schools. Hence, data has a clustered structure in which treatments can be applied to entire clusters (classes/tracks or schools) or individuals (students or teachers), or multiple levels simultaneously (e.g. tracks and students) [34]. Different methods for handling multi-level educational data have been proposed in the literature [35]. Arpino and Cannas [36] made a review of the possible matching procedures in the multi-level context, highlighting the advantages of taking into

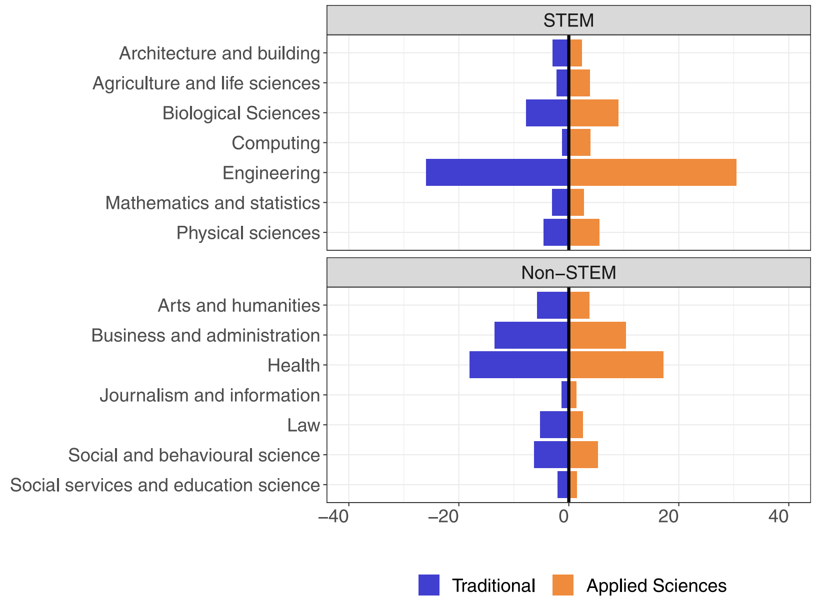

Fig. 3. Percentage distribution of the students of the two tracks of the scientific "liceo" according to the field of study of university enrollment.

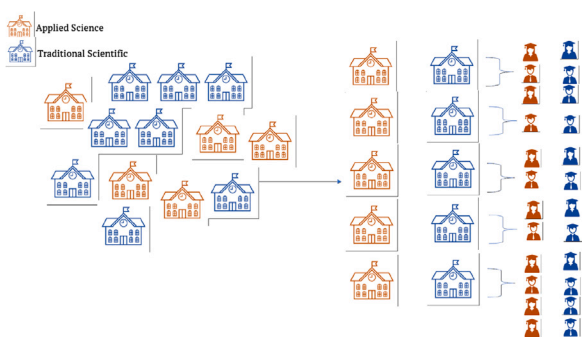

Fig. 4. Illustration of the two-step matching procedure.

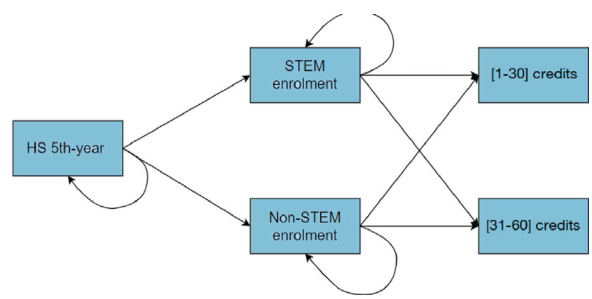

Fig. 5. States and transitions of the multi-state Markov model.

account the hierarchical structure of the data. However, most of the literature concerns cases in which the treatment is administered at the individual level. Inspired by the procedure proposed by Rickles and Seltzer [37], we propose a two-step matching procedure for clustered data in which the treatment is administered at the cluster level. The idea behind this procedure is that we first match the tracks using a single-level logit with the track covariates; then, we match the students using another logit model with random effects given by the paired couples of the two tracks derived in the first step (Fig. 4).

Once a good balance between the two groups has been achieved, we focus on the assessment of the effect of the track on the two educational outcomes: university enrollment and first-year university performance. Given that these two outcomes can occur at two different times, first the enrollment or not enrollment and second the first-year performance, we decided to respect the longitudinal nature of the data and to consider a multi-state type model. In particular, we consider a multi-state model based on the discrete-time Markov chain, also called Markov chain transitional models, popular for the analysis of longitudinal data [38]. As already seen, high school track attendance has a strong influence on university enrollment accordingly to the field of study [39]. At the same time, the field of study makes a difference in the university performance [40]. Hence, we decided to distinguish between the enrollment in a STEM or non-STEM degree program and to evaluate the student's performance in the two fields, in terms of first-year ECTS, accordingly. In this framework, we can imagine that each career step is a state that can occur from the 5th year of high school to the end of the first year of university. Using this representation, it is reasonable to assume a Markov chain where each student has a transition probability from each state to another conditioned to her/his profile. In Fig. 5, the transitions are reported: at $t = 0$, the student is in the 5th year of high school; at $t = 1$, the student can decide to enroll in a STEM or non-STEM degree program, or not enroll at a university, which means that he remains in the initial state; at $t = 2$, there are three states depending on the number of ECTS obtained at the end of the first year: 0, which means that the student remains in the STEM or non-STEM enrollment state, between 1 and 30 ECTS, or between 31 and 60 ECTS, as a rough classification of "bad performing" and "good performing" students.

4.1. Multi-level propensity score matching

Consider a two-level data structure where N units, indexed by i ($i = 1, 2, \ldots, n_j$), are nested in J second-level units (clusters), indexed by j ($j = 1, 2, \ldots, J$). We consider a binary treatment T_{ij} for the first-level unit administered at the cluster level, such that $T_{ij} = 1$ if cluster j is treated and $T_{ij} = 0$ otherwise. Each first-level unit has two potential outcomes Y_{ij}: $Y_{ij}(1)$ under treatment condition, $Y_{ij}(0)$ under control condition. Let $\boldsymbol{Z}$ and $\boldsymbol{X}$ be respectively the matrices of first- and second-level covariates.

A summary measure of the covariates is the propensity score defined as:

$$\pi_{ij} = \pi_{ij}(\boldsymbol{X}, \boldsymbol{Z}) = P(T_{ij} = 1 | \boldsymbol{X}, \boldsymbol{Z}).$$

Usually, in observational studies, the propensity score must be estimated from the data. To this end, propensity score estimates are typically obtained with either logistic fixed- or random-effects models. Here, we propose a procedure to deal with the multi-level structure of our data which works as follows:

1. A single-level logit model is fitted to estimate the propensity score at the cluster level:

   $$g(\pi_j) = g(P(T_j = 1|\boldsymbol{Z})) = \boldsymbol{\gamma_0} + \boldsymbol{\gamma}_Z^T \boldsymbol{Z},$$

   where $\boldsymbol{\gamma_0}$ is the vector of intercept, $\boldsymbol{\gamma}_Z$ is the vector of the cluster covariates effects and g denotes the logit link function. The most common PSM methods are greedy and optimal matching [20]. Despite the superior performance of the optimal matching in terms of balance, this method has been regarded as computationally expensive and troublesome. Greedy matching, instead, can sometimes be a too simplistic approach, leading to not well-balanced groups. The introduction of a caliper in the greedy matching approach is a good compromise between the classical greedy matching and the optimal matching approach. We apply a caliper propensity score matching that allows matching among units within a specific threshold of the propensity score [21]. Formally, let I_1 and I_0 denote the set of treated and control units, respectively, and let indicate with A_r the set of control clusters matched to the treated cluster $r \in I_1$:

   $$A_r = \{k \in I_0 : \hat{\pi}_k = \min_{k \in I_0} |\hat{\pi}_r - \hat{\pi}_k| < c_1\}, \quad (1)$$

   where c_1 is the caliper imposed on the cluster-level covariates. After (1) has been constructed for all clusters in the treatment group, the matched dataset M is built:

   $$M = \{r : A_r \neq \emptyset\} \cup \left\{ \bigcup_r A_r \right\}.$$

2. A random effect logit model is fitted to estimate the propensity score at the individual level, considering the couples obtained in the previous step and belonging to the matched dataset M as random effects:

   $$g(\pi_{ir}) = g(P(T_{ir} = 1|\boldsymbol{X})) = \boldsymbol{\alpha} + \boldsymbol{\beta}_X^T \boldsymbol{X}, \boldsymbol{\alpha} \sim N(\alpha_0, \sigma_\alpha^2) \quad (2)$$

   where $\boldsymbol{\alpha}$ is the vector of the random effect, $\boldsymbol{\beta}_X$ is the vector of individual-level covariates effects. As in the first step we use a one-to-one caliper matching at the individual level based on the estimated propensity scores (2).

In this procedure, both the first- and second-level calipers are set to 0.05, as usually implemented [41]. To assess the goodness of the resulting balance, we use the difference in means, the most commonly chosen method for exact matching [42].

It is important to underline that we decided to use this specific matching procedure because it was the most rational choice with respect to the aim of our work. However, other matching procedures were carried out, finding no relevant differences with the selected procedure.

4.2. The multi-state Markov model

Generally, given S = 1, …,R a discrete set of states, a stochastic process $\{M_n\}$ is called a Markov chain if for all times $n \geq 0$ and all states $i_0, \dots, i, j \in S$,

$$P(M_{n+1} = j|M_n = i, M_{n-1} = i_{n-1}, \dots, M_0 = i_0) =$$
$$= P(M_{n+1} = j|M_n = i) = P_{ij}.$$

P_{ij}, often referred to as *one-step transition probability*, denotes the probability that the chain, whenever in state i, moves to state j one unit of time later. Loosely speaking, $\{M_n\}$ is characterized by the so-called Markov property that states that the probability of being in a given state at a certain time depends only on the state occupied in the immediately previous time and not on the whole process history. P_{ij} are then entries of the so-called *transition probability matrix*, **P**, that in our case can be represented as:

$$\mathbf{P} = \begin{array}{c} \text{Not enr.} \quad \text{STEM} \quad \text{Non-STEM} \quad \text{[1-30] ECTS} \quad \text{[31-60] ECTS} \\ \begin{bmatrix} P_{11} & P_{12} & P_{13} & 0 & 0 \\ 0 & P_{22} & 0 & P_{24} & P_{25} \\ 0 & 0 & P_{33} & P_{34} & P_{35} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{l} \text{Not enr.} \\ \text{STEM} \\ \text{Non-STEM} \\ \text{[1-30] ECTS} \\ \text{[31-60] ECTS} \end{array} \quad (3)$$

The zero entries of matrix (3) identify the non-allowed transitions. The rows of **P** satisfy the condition $\sum_{j=1}^{5} = 1$. We are assuming *time homogeneity* of the Markov chain, hence, the transition probabilities do not depend on the time n. In a discrete-time multi-state Markov model, covariates are usually linked to the transition probabilities through a set of multinomial logistic regressions. More specifically, for any rows of the transition probability matrix a multinomial logistic regression is fitted:

$$log\left(\frac{P_{ij}}{P_{ii}}\right) = b_{ij,0} + b_{ij}^T X_{TREATMENT}, \ i \neq j$$

$X_{TREATMENT}$ is the only covariate considered in the model and it identifies the two treatments: traditional and applied sciences tracks.

5. Results

In this section, the results of the MPSM procedure and of the multi-state Markov models are shown.

First, the results of the MPSM procedure are shown in Fig. 6. This procedure provides a good balance of the characteristics of the two groups. At the track level, the imbalance in the overall percentage of females in favor of the traditional track has been almost totally removed, while a good balance has been reached concerning the macro-regional location of the schools. The school's ESCS is the only variable for which the groups are still a bit unbalanced. At the student level, the student's ESCS is now almost entirely balanced, as well as gender, career regularity, and citizenship. A good balance is also obtained for the INVALSI score in mathematics, originally higher for applied sciences students. Instead, the whole balancing procedure determines a slight worsening of the other INVALSI subjects' scores.

Then, the results of the multi-state Markov model are reported in Table 3. Overall, it can be observed the differences between the two tracks in terms of estimated transition probabilities shrunk after the balancing procedure. Yet, some differences persist.

The results of the model fitted on the matched data highlight that students from the applied sciences track have a lower probability to enroll at university than those from the traditional track. On the one hand, students who attended the applied sciences track have a higher probability of enrolling in a STEM degree program than their traditional scientific peers; on the other hand, the probability of enrolling in non-STEM degree programs is reversed. As for the academic performance, before the matching procedure, the probability to remain in the state STEM, i.e. the probability of not obtaining any ECTS after having enrolled in STEM, was significantly higher for the applied sciences track, but this difference is no longer significant after the matching procedure. As well as, before the matching, the probabilities of being enrolled in a STEM program and moving to the non-zero ECTS states were significantly higher for the students from the traditional track, while those differences are no more significant after the matching. After the matching, there are no more differences between the two tracks in terms of transition probabilities from non-STEM to any state except for the probability of transitioning to more than 30 ECTS, which is significantly lower for the students of the applied sciences track.

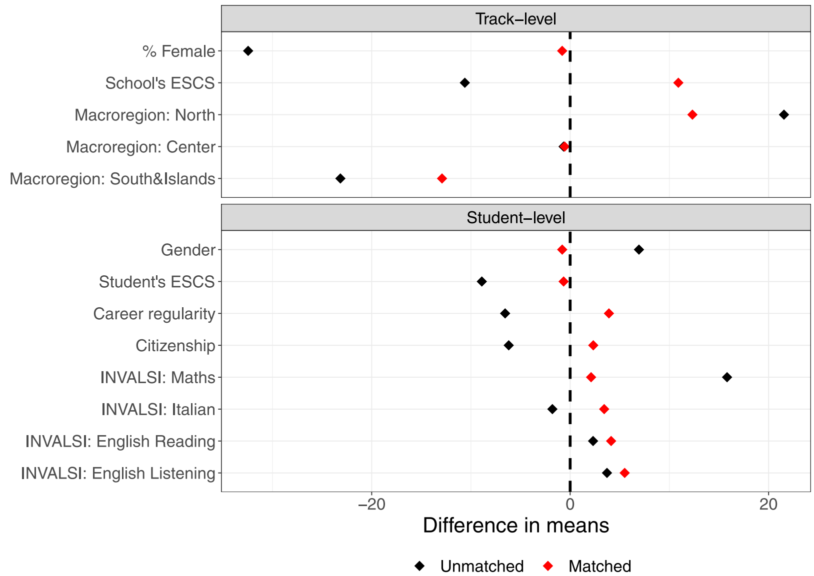

Fig. 6. Difference in means between the two tracks of the scientific "liceo" of the track- and student-level covariates before (red) and after (black) the MPSM procedure. Positive values indicate an imbalance in favor of the applied sciences track and negative values in favor of the traditional track.

Table 3
Estimated transition probabilities for both tracks of the scientific "liceo" before and after the MPSM procedure. CIs are in brackets.

Transition	Applied Sciences	Traditional Scientific
	Unmatched	
HS 5th-year → HS 5th-year	0.180 (0.175, 0.185)	0.142 (0.138, 0.145)
HS 5th-year → STEM	0.487 (0.481, 0.494)	0.425 (0.421, 0.430)
HS 5th-year → Non-STEM	0.332 (0.326, 0.340)	0.434 (0.429, 0.438)
STEM → STEM	0.075 (0.070, 0.080)	0.060 (0.057, 0.064)
STEM → [1–30] ECTS	0.280 (0.271, 0.289)	0.257 (0.251, 0.262)
STEM → [31–60] ECTS	0.644 (0.636, 0.654)	0.683 (0.677, 0.689)
Non-STEM → Non-STEM	0.079 (0.073, 0.086)	0.055 (0.052, 0.058)
Non-STEM → [1–30] ECTS	0.211 (0.202, 0.221)	0.189 (0.184, 0.194)
Non-STEM → [31–60] ECTS	0.709 (0.698, 0.720)	0.756 (0.750, 0.762)
	Matched	
HS 5th-year → HS 5th-year	0.181 (0.174, 0.188)	0.159 (0.153, 0.166)
HS 5th-year → STEM	0.477 (0.467, 0.486)	0.432 (0.424, 0.441)
HS 5th-year → Non-STEM	0.342 (0.334, 0.351)	0.408 (0.399, 0.418)
STEM → STEM	0.077 (0.070.0.084)	0.070 (0.064, 0.077)
STEM → [1–30] ECTS	0.278 (0.267, 0.291)	0.271 (0.259, 0.284)
STEM → [31–60] ECTS	0.644 (0.631, 0.657)	0.658 (0.645, 0.671)
Non-STEM → Non-STEM	0.081 (0.072, 0.089)	0.071 (0.064, 0.078)
Non-STEM → [1–30] ECTS	0.215 (0.202, 0.229)	0.194 (0.183, 0.206)
Non-STEM → [31–60] ECTS	0.704 (0.689, 0.718)	0.735 (0.723, 0.748)

6. Conclusions

Studying the effect of mathematics on students' academic outcomes is not trivial. In fact, academic outcomes, such as university enrollment and first-year university performance, depend on an incredibly intricate net of multi-dimensional factors related to students' characteristics and school outcomes, such as socio-economic status, gender, and the type of high school track. As a proxy of the role of mathematics on students' careers, we considered the high school track attended. In particular, we focused on the two tracks of the scientific "liceo": the traditional and the applied sciences tracks, the latter offering more hours devoted to mathematics and mathematics-related subjects.

This study represents the first attempt to explore the relationship between two tracks, differing in the number of hours devoted to mathematics classes in high school, with respect to university performance in Italy. While research on mathematics has primarily focused on the differences in educational outcomes at the high school level [14], as far as the authors' knowledge this is the first study that investigates the impact of mathematics on academic outcomes in Italy. In fact, the absence of longitudinal microdata has hindered the study of the high school-to-university transition. In this paper, we disentangle the relationship between high school and university performance using a multi-level propensity score approach. In this respect, the use of the multi-level propensity score approach has proven to be essential in addressing the unbalance between the two tracks observed at both the track and the student levels regarding gender composition, socio-economic status, and high school performance.

Then, we used a discrete-time multi-state Markov model to study the effect of studying more mathematics on university enrollment and first-year performance in terms of ECTS. Our results highlight that, after having balanced, the gap between the two tracks of the scientific "liceo": i) has been reduced in terms of the transition from high school to university; ii) has almost disappeared in terms of university performance. In detail, the estimated transition probabilities show that students from the applied sciences track tend to enroll less at university than their traditional scientific peers. Studying more mathematics is a push factor for university enrollment in more scientific programs, as students from the applied sciences track tend to enroll more in STEM programs than those from the traditional track. As for the university performance, the only remark is that traditional students in non-STEM programs have a higher probability of obtaining more than 30 ECTS, probably because of the more humanistic nature of the traditional track.

Yet, beyond this evidence, we are aware that our results suggest it is hard to disentangle entirely the hierarchical structure of the Italian high school system, in which the scientific traditional track occupies a better rank compared to the applied sciences one. This is due to several unobservable variables, likely given by the expectations of the students and the teachers, and the general educational atmosphere that cannot be measured. Another sort of limitation is given by the cancellation of many traditional scientific schools during the propensity score matching procedure. Moreover, the procedure did not produce a good balance of the macro-regional allocation of the two tracks. As a more general consideration, it seems that in Italy, the traditional scientific track, though has fewer hours devoted to maths-related classes than the applied sciences track, has a higher consideration in the hierarchical Italian high school system. This result reflects indeed the still alive conjecture among upper- and middle-class families that humanistic subjects give a wider-ranging education compared to scientific ones [24]. The recent growth in enrollment in the applied sciences track, accounting for approximately 35% of students in scientific "licei" during the 2020/21 academic year, denotes a notable shift in educational preferences. This trend reflects an expanded acknowledgment of the significance of practical skills and their pertinence in today's competitive job market. Nevertheless, it is important to acknowledge that disparities exist in the growth of this trend in Italy, both geographically and by gender. Geographically, there are noticeable differences between Northern and Southern Italy, with the former showing a higher diffusion of applied sciences tracks, as shown in Fig. 1. Gender disparities are another significant concern. As introduced in Table 2, females continue to be underrepresented in applied sciences (32%) and traditional scientific tracks (47%), limiting their participation in STEM-related fields.

In conclusion, while this study focuses solely on one type of high school (the scientific high school), it provides valuable insights into the broader Italian school system. The findings underscore the urgent need for interventions at the high school level to mitigate the impact of educational paths and of the diverse high school environments on students' subsequent educational choices. Further research can be carried out to investigate the effect of attending "licei" against the broad class of technical schools, whose university enrollment rates are increasing over the last decades in Italy.

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CRediT authorship contribution statement

Andrea Priulla: Conceptualization, Data curation, Formal analysis, Methodology, Software, Validation, Writing – original draft. **Martina Vittorietti:** Conceptualization, Data curation, Formal analysis, Methodology, Software, Validation, Writing – original draft. **Massimo Attanasio:** Funding acquisition, Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The authors do not have permission to share data.

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