



Research paper

Nonlinear control of gas turbine system under disturbances: An integral backstepping, terminal synergetic and fuzzy logic approach

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ABSTRACT

The deep penetration and inherently uncertain nature of renewable power generation systems make it a challenging task to design reliable control systems for any device interfacing with the power grid. Gas turbines are considered the elective transitional power generation technology. In this article, we propose an advanced robust control system not covered in literature for a power generating Gas Turbine (GT) system utilizing non-linear control theoretical principles. The focused GT system is expressed by a state variables representation of the non-linear mathematical model. GT speed is controlled by determining the fuel demand response of GT under variable load conditions. The novel Integral Backstepping Controller (IBC), Backstepping Controller (BSC), Terminal Synergetic Controller (TSC) and Fuzzy Logic Controller (FLC) are here considered and compared to get the desired GT speed. Overall global asymptotic stability of the GT system has been proved with the help of the Lyapunov theory. Furthermore, stability analysis is formulated to validate the proposed control methods by observing the system's performance subject to noise disturbances. MATLAB/Simulink environment has been used for the implementation of simulations to observe the settling time and state errors. Performance curves for each proposed controller are compared with the classical PID controller. Results obtained from the simulation work are utilized in a comparison analysis based on the graphical and statistical data. Conclusions drawn from the study show that the non-linear controllers have significant improvements as compared to PID controller's performance, with limited increased complexity, while among non-linear controllers, IBC shows the best performance.

1. Introduction

The demand for clean energy is nowadays an imperative due to the higher impacts of CO_2 emissions related to human activities over the global climate. According to the report published by the International Energy Agency (IEA) France, the Net Zero Roadmap 2050 aims to have nearly 90% of the electricity generation by renewable energy sources [1] by 2050. On the other hand, the global climbing trend in purchasing cost of energy requires more significant concerns to develop competitive cost solutions [2]. This steep transition to a net-zero economy calls for transitional technologies that will aid the system to keep the required stability while being ready for further innovations. GT system is a well-known co-generation system that gained popularity in the past few decades [3]. In addition to having extremely efficient fuel combustion and emissions, modern gas turbines may be totally modified to use fuels like green hydrogen, which will significantly reduce carbon emis-

sions. Reduced emissions can be achieved even with modest amounts of hydrogen mixed. Choosing the GT system to be an integral part of the energy systems also relies on its smooth power regulation and faster response in comparison to renewable energy sources in which there is uncertainty associated with power generation.

The fuel selection for GT system plays an important role in this frame. As natural gas is the third largest global energy source with minimum carbon footprint among fossil fuels, it remained a popular choice for GT systems [4], [5]. Several studies cover the technological development of various alternative fuel options for GT systems, like biogas, synthetic gas & hydrogen gas [6], [7]. Hydrogen, in particular, is getting huge attention for the achievement of low carbon future goals [8]. This latter issue encouraged the development of new GT systems with modern combustion hierarchy which enabled the use of hydrogen as input fuel [9], [10]. The modern GT system includes a turbine, a combustion system and a compression system as the basic components. The GT system oper-

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Nomenclature

x_1	Speed of GT system	$\alpha_2, \alpha_3, \alpha_4$	Virtual control for states
x_2	Flow of Gas	e_2, e_3, e_4	Error equations for system's states
x_3	Flow of fuel	z_1, z_2, z_3, z_4	Integral actions of IBC controller
x_4	Valve position	b_1, b_2, b_3, b_4	Tuning gain parameters of IBC
J	Inertia value of load	φ	Macro variable for design of TSC
T_{eo}	Nominal electromagnetic torque	s_0, s_1, s_2	Tuning gain parameters for TSC
D_t	Model disturbance	p, q	Positive bounded integers
T_{CD}	Time constant for discharge	T, T_1	Positive design parameters of TSC
T_a, K_a	Variables of fuel actuation	V_s	Lyapunov candidate function for TSC controller
a, b, c	Positioning values of valve	μ, ω	Positive constants
T_{et}	Electromagnetic torque variation at time t	t_0, t_1, t	Time variables
$u(t)$	Control input for GT system	K_p	Proportional controller gain
k_l	Min. Value of Load	K_i	Integral controller gain
k_f	Max. Value of Fuel	K_d	Derivative controller gain
e_1	Error equation for the speed tracking of GT system	T_i	Integral time constant
x_{1ref}	Reference speed of GT system	T_d	Derivative time constant
V_1, V_2, V_3, V_4	Lyapunov candidate functions for IBC controller		

ation is based on getting air fuel mixture from the compression system, then burning it in the combustion system, thus getting the mechanical power needed to drive an electrical generator for power generation [11]. Above all, a unified control system is the most important part of GT system which is responsible for its safe and reliable operation [12]. Therefore, being the driving element of the GT system, a robust and improved control system is vitally important in smooth power regulation. In the literature, several methodologies have been proposed for designing the control of GT system. Most common and conventional approaches for controlling the industrial systems are based on PI and PID control strategies with linear approximations of the controlled systems [13].

The particle swarm optimization technique has been utilized for the fine tuning of PID controller for GT system control process assuming the ideal system conditions without any external disturbances in [14]. Optimal control theory using model predictive control (MPC) has been reported in [15], [16], the prediction about the future control actions of the GT system made it quite effective and exhibited faster response with smaller offsets to the steady state as compared to the PID controller. In [17], it was reported that a linear quadratic regulator (LQR) can optimize over the whole horizon of the GT operation, thus generating an optimal solution with a fixed control value. On the other hand, for MPC, optimization is made over the receding time horizons thus giving a variable optimal solution. In case the system experiences any external disturbance, the LQR predictions appear inaccurate, conversely, the MPC withstands small external disturbances over finite time horizons. MPC however, for larger external system disturbances with longer time horizon, provides inaccurate predictions of control actions [18]. The Fuzzy-PI controller combination has been tested in [19] for the control of GT system in the presence of errors. PI gains are fuzzified to attain the desired control sequence. The overall performance of this kind of controller decays with larger external disturbances, as it is very hard to define a large number of complex rules to capture a wider scenario of possible perturbations. The GT system structure is very complex and shows a highly non-linear nature, considering several parameters like electric load imbalances, external disturbances, control and speed parameters, fuel injection, etc. [20].

The linear control methods discussed in the literature rely on linear approximations of inherently non-linear system models. To implement such control strategies, it is necessary to define the system's transfer function and a linearized state-space model based on these approximations. However, a critical challenge in this process is determining how accurately and to what extent the essential characteristics of the original system are preserved. In practice, these approximations inevitably differ from the actual system, and therefore cannot fully replicate its

behaviour. Practically, approximations are highly dependent upon the difference between system points of equilibrium and where these linear approximations are made [21]. The main drawbacks of GT system with conventional linear control relate to unstable mechanical output power caused by flaming irregularity of the combustion chamber, mechanical and thermal stress and humming of the machine [22]. Highly non-linear systems, such as GT systems, should desirably have a unified and robust non-linear control as it can handle the system non-linearities much better, as compared to linear control approaches, based on linear approximations, for which the inherent system properties are removed [23–25]. The real-time implementation of non-linear controllers in physical systems such as gas turbine (GT) systems faces several challenges. These include the need for highly precise system modelling, significant modifications to the system architecture, and the use of complex and expensive computational resources to manage control loops [26], [27].

Several works have been reported for the non-linear control of GT system for better and reliable operation. In [28], the non-linear MPC (NMPC) is explained to get some better advancements over the conventional MPC with the linearized stochastic model. However, the overall efficiency is compromised due to computational complexity associated with the definition of constraints and cost functions. The Sliding Mode Controller (SMC), as robust non-linear controller for the GT system, has been proposed in [29]. Although it outperforms the traditional PID controller, the Sliding Mode Controller (SMC) is more sensitive to dynamic variations in model parameters due to its inherent chattering phenomenon, which can make its application more critical. In [30], the Synergetic based non-linear control has been explained. As shown in the paper, the parametric adaptation property eliminates the mentioned chattering problem, like that of SMC, and enhanced the performance. Non-linear Dynamic Inversion (NDI) controller given by [31], with the aid of Kalman filter, achieved better output results of the GT system. The Feedback Linearization controller (FBL), in combination with the state estimations, produced smooth power regulations with GT system model uncertainties [32]. The recursive non-linear control approach of basic backstepping for the GT system was presented in [33], [34]. The method exhibited better performance but, overall, the definition of system error equations for evaluation of control signal made it sensitive to system external disturbances.

The distinct characteristics of various non-linear controllers result in a broad spectrum of performance outcomes. Each control strategy exhibits a unique range of outputs, influenced by specific trade-offs. For instance, Sliding Mode Control (SMC) offers robust handling of system non-linearities, but its inherent chattering effect leads to oscillatory behaviour. Synergetic and backstepping controllers are highly sensitive to model uncertainties due to their fundamental algorithms. Meanwhile,

Nonlinear Dynamic Inversion (NDI) and Feedback Linearization (FBL) show strong dependence on model parameters, which can significantly impact their output performance.

To address all these issues, a unified control system, able to overcome the model uncertainties and disturbances, inherent or external, against a very moderate computational complexity, is highly needed. This research article aims to provide a global asymptotic stable solution of the unified control for a GT system not covered in literature and that is able to maintain a reliable operation. For this study, the contributions are highlighted as follows:

- A novel Integral Backstepping Controller (IBC) is proposed as an enhanced version of the traditional backstepping controller, incorporating integral actions based on system error equations.
- The classical Backstepping Controller (BSC) is also presented and compared with the newly developed IBC.
- An advanced Synergetic-based controller, referred to as the Terminal Synergetic Controller (TSC), is introduced, capable of driving system trajectories to track reference signals within a finite time.
- A Fuzzy Logic Controller (FLC) is proposed to evaluate the dynamic behaviour of the system.
- Lyapunov theory is employed to demonstrate the global asymptotic stability of the proposed non-linear controllers.
- Perturbation analysis is conducted by introducing external disturbances in the form of white noise to assess the stability of the system under non-linear control.
- A comprehensive simulation study is carried out to support the findings and provide detailed insights.

The organization of the article is presented below. Section 2 explains the structure of the GT system along with the non-linear mathematical model and problem formulation, Section 3 covers the proposed work, Section 4 presents all the simulations analysis and finally the article terminates over Section 5 with conclusions and future work.

2. Gas turbine system

2.1. Composition of gas turbine system

The dynamical representation of the GT system comprises three major components; compressor, combustion chamber and the turbine. The graphical illustration of the GT system is given by the Fig. 1. The air inlet allows the natural air to enter the compressor, where it is compressed to high pressure and carried out to the combustion chamber. The fuel valve controls and injects the required fuel quantity into the combustion chamber where inlet guided vane facilitates the air fuel fusion to get burnt for the production of high temperature exhaust gases. The thermal energy of exhaust gases from the combustion chamber is used to drive the turbine. The electrical alternator mounted on the rotary shaft of the turbine generates electrical energy from the rotational mechanical power. The turbine driving speed is governed by the fuel amount so, the control system of GT provides the accuracy of operational range for the fuel valve by monitoring power demand.

2.2. Non-linear mathematical modelling of GT system

William I. Rowen proposed the mathematical representation of the GT system [36]. The advancements in the Rowen's Model resulted in updated non-linear mathematical models as the function of state variables [37], [38] which are presented by the eqs. (1)-(4).

$$\dot{x}_1 = -\frac{1}{2J}x_1 + \frac{1.3}{J}x_2 + \frac{0.201 - T_{eo}}{J} + D_t \quad (1)$$

$$\dot{x}_2 = -\frac{1}{T_{CD}}x_2 + \frac{1}{T_{CD}}x_3 \quad (2)$$

$$\dot{x}_3 = -\frac{1}{T_a}x_3 + \frac{K_a}{T_a}x_4 \quad (3)$$

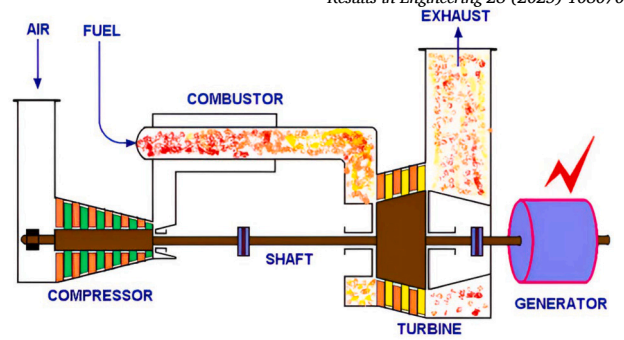


Fig. 1. Simple concept of GT system [35].

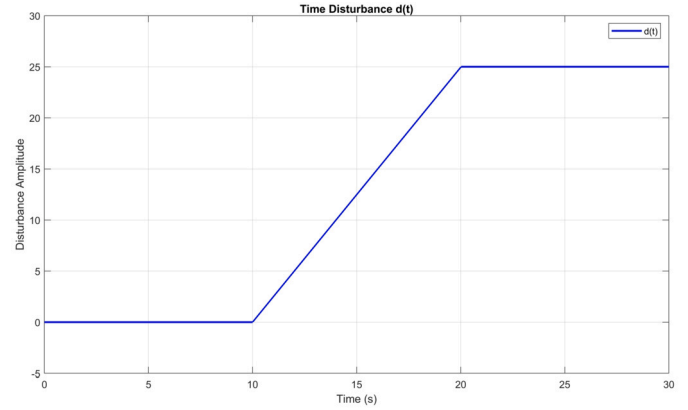


Fig. 2. GT system model disturbance.

$$\dot{x}_4 = -\frac{b}{a}x_4 + \frac{ck_l}{a} + \frac{ck_f}{a}.x_1.u(t) \quad (4)$$

where x_1 is the speed of GT, x_2 represents the gas flow, x_3 is the flow of fuel and x_4 defines the fuel valve position. The term $u(t)$ is the control input signal for the fuel injection to GT. The explanation of supplementary model parameters is listed in nomenclature. The model disturbance D_t due to the variable load conditions, is formulated by the eq. (5) as:

$$D_t = \frac{T_e(t)}{J} + \frac{T_{eo}}{J} \quad (5)$$

where $T_e(t)$ is the electromagnetic torque variation at time t . The variation of the electromagnetic torque $T_e(t)$ is considered as dropping of 10% from its nominal value due to load variations and defined by the bounded conditions as follows:

$$T_e(t) = \begin{cases} 0.2, & 0 \leq t < 3s, \\ 0.2 - 0.01(t-1), & 3s \leq t < 5s, \\ 0.18, & t \geq 5s. \end{cases}$$

The graphical representation of the model disturbance d_t is given by the Fig. 2.

2.3. Problem formulation

The intrinsic nature of the GT system in general is highly non-linear and its representation, given by the eqs. (1)-(4), is a very complex non-linear mathematical model. The classical industrial PID controllers are not capable to cater the uncertain behaviour of a non-linear system in case of disturbances. To achieve the global asymptotic stability of a highly non-linear system the subsequent endorsed choice is to design the control using non-linear algorithms under model uncertainties. In this article, we are aiming to propose non-linear controllers for speed control of the GT system subject to varying load and noise disturbances by utilizing the novel non-linear control theory and to ensure minimal

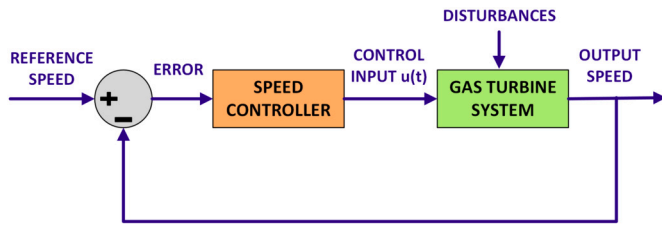


Fig. 3. Closed loop control for GT system.

fuel consumption to minimize costs. The graphical representation of the proposed non-linear control scheme is shown in the Fig. 3.

3. Controller design for gas turbine system

The unified control for the GT system ensures reliable operation which is achieved by the proposed novel non-linear controllers IBC, BC, TSC ζ FLC. In this section, a detailed mathematical formulation has been described for derivation of the proposed non-linear controllers based on non-linear mathematical model of the GT system.

3.1. Integral backstepping controller

The Integral Backstepping technique consists of the update for the classical Backstepping algorithm by introducing the integral action for all system error equations. The proposed IBC obtained from this technique is more intuitive and robust due to the recursiveness summation of the system's errors with the help of integral action [39]. The first error equation for the speed tracking of GT with respect to reference is defined as:

$$e_1 = x_1 - x_{1ref} \quad (6)$$

The integral action for the error e_1 is defined as:

$$z_1 = \int (e_1)dt = \int (x_1 - x_{1ref})dt \quad (7)$$

The first Lyapunov candidate function for the stability of e_1 is given by:

$$V_1 = \frac{1}{2}e_1^2 + \frac{1}{2}z_1^2 \quad (8)$$

By taking the time derivative of the eq. (8), we have:

$$\dot{V}_1 = e_1\dot{e}_1 + z_1\dot{z}_1 \quad (9)$$

Now we introduce the second dynamical error for the system as:

$$e_2 = x_2 - \sigma_2 \quad (10)$$

which can also be written as:

$$x_2 = e_2 + \sigma_2 \quad (11)$$

Now, by taking the time derivative of the error e_1 from the eq. (6), we get:

$$\dot{e}_1 = \dot{x}_1 - \dot{x}_{1ref} \quad (12)$$

By substituting the eq. (1) and eq. (11) in the expression of \dot{e}_1 we can write:

$$\dot{e}_1 = -\frac{1}{2J}x_1 + \frac{1.3}{J}(e_2 + \sigma_2) + \frac{0.201 - Teo}{J} + D_t - \dot{x}_{1ref} \quad (13)$$

Now by substituting the eq. (13) in the eq. (9) and simplifying, we get:

$$\dot{V}_1 = e_1\left(-\frac{1}{2J}x_1 + \frac{1.3}{J}(e_2 + \sigma_2) + \frac{0.201 - Teo}{J} + D_t - \dot{x}_{1ref} + z_1\right) \quad (14)$$

The time derivative of the first Lyapunov candidate function \dot{V}_1 can be proved negative definite if we let:

$$-\frac{1}{2J}x_1 + \frac{1.3}{J}(\sigma_2) + \frac{0.201 - Teo}{J} + D_t - \dot{x}_{1ref} + z_1 = -b_1e_1 \quad (15)$$

where b_1 is the positive gain parameter. The virtual control σ_2 from the eq. (15) can be determined as:

$$\sigma_2 = \frac{J}{1.3}\left(\frac{1}{2J}x_1 - \frac{0.201 - Teo}{J} - D_t + \dot{x}_{1ref} - z_1 - b_1e_1\right) \quad (16)$$

The time derivative of \dot{V}_1 from the eq. (14) can be updated by using the eq. (16), we have:

$$\dot{V}_1 = -b_1e_1^2 + \frac{1.3}{J}e_1e_2 \quad (17)$$

The integral action for the error e_2 is defined as:

$$z_2 = \int (e_2)dt = \int (x_2 - \sigma_2)dt \quad (18)$$

The second Lyapunov candidate function for the stability of e_2 is given by:

$$V_2 = V_1 + \frac{1}{2}e_2^2 + \frac{1}{2}z_2^2 \quad (19)$$

By taking the time derivative of the eq. (19), we have:

$$\dot{V}_2 = \dot{V}_1 + e_2\dot{e}_2 + z_2\dot{z}_2 \quad (20)$$

Now we introduce the third dynamical error for the system as:

$$e_3 = x_3 - \sigma_3 \quad (21)$$

which can also be written as:

$$x_3 = e_3 + \sigma_3 \quad (22)$$

Now, by taking the time derivative of the error e_2 from the eq. (10), we get:

$$\dot{e}_2 = \dot{x}_2 - \dot{\sigma}_2 \quad (23)$$

By substituting the eq. (2) and eq. (22) in the expression of \dot{e}_2 we can write:

$$\dot{e}_2 = -\frac{1}{T_{CD}}x_2 + \frac{1}{T_{CD}}(e_3 + \sigma_3) - \dot{\sigma}_2 \quad (24)$$

Now by substituting the eq. (17) and eq. (24) in the eq. (20) and simplifying, we get:

$$\dot{V}_2 = -b_1e_1^2 + e_2\left(\frac{1.3}{J}e_1 - \frac{1}{T_{CD}}x_2 + \frac{1}{T_{CD}}(e_3 + \sigma_3) - \dot{\sigma}_2 + z_2\right) \quad (25)$$

The time derivative of the second Lyapunov candidate function \dot{V}_2 can be proved negative definite if we let:

$$\frac{1.3}{J}e_1 - \frac{1}{T_{CD}}x_2 + \frac{1}{T_{CD}}(\sigma_3) - \dot{\sigma}_2 + z_2 = -b_2e_2 \quad (26)$$

where b_2 is the positive gain parameter. The virtual control σ_3 from the eq. (26) can be determined as:

$$\sigma_3 = T_{CD}\left(-\frac{1.3}{J}e_1 + \frac{1}{T_{CD}}x_2 + \dot{\sigma}_2 - z_2 - b_2e_2\right) \quad (27)$$

The time derivative of \dot{V}_2 from the eq. (25) can be updated by using the eq. (27), we have:

$$\dot{V}_2 = -b_1e_1^2 - b_2e_2^2 + \frac{1}{T_{CD}}e_2e_3 \quad (28)$$

The integral action for the error e_3 is defined as:

$$z_3 = \int (e_3)dt = \int (x_3 - \sigma_3)dt \quad (29)$$

The third Lyapunov candidate function for the stability of e_3 is given by:

$$V_3 = V_2 + \frac{1}{2}e_3^2 + \frac{1}{2}z_3^2 \quad (30)$$

By taking the time derivative of the eq. (30), we have:

$$\dot{V}_3 = \dot{V}_2 + e_3\dot{e}_3 + z_3\dot{z}_3 \quad (31)$$

Now we introduce the fourth dynamical error for the system as:

$$e_4 = x_4 - \sigma_4 \quad (32)$$

which can also be written as:

$$x_4 = e_4 + \sigma_4 \quad (33)$$

Now by taking the time derivative of the error e_3 from the eq. (21), we get:

$$\dot{e}_3 = \dot{x}_3 - \dot{\sigma}_3 \quad (34)$$

By substituting the eq. (3) and eq. (33) in the expression of \dot{e}_3 we can write:

$$\dot{e}_3 = -\frac{1}{T_a}x_3 + \frac{K_a}{T_a}(e_4 + \sigma_4) - \dot{\sigma}_3 \quad (35)$$

Now by substituting the eq. (28) and eq. (35) in the eq. (31) and simplifying, we get:

$$\begin{aligned} \dot{V}_3 = & -b_1e_1^2 - b_2e_2^2 + e_3\left(\frac{1}{T_{CD}}e_2 - \frac{1}{T_a}x_3 + \frac{K_a}{T_a}(e_4 + \sigma_4)\right) \\ & - \dot{\sigma}_3 + z_3 \end{aligned} \quad (36)$$

The time derivative of the third Lyapunov candidate function \dot{V}_3 can be proved negative definite if we let:

$$\frac{1}{T_{CD}}e_2 - \frac{1}{T_a}x_3 + \frac{K_a}{T_a}(\sigma_4) - \dot{\sigma}_3 + z_3 = -b_3e_3 \quad (37)$$

where b_3 is the positive gain parameter. The virtual control σ_4 from the eq. (37) can be determined as:

$$\sigma_4 = \frac{T_a}{K_a}\left(-\frac{1}{T_{CD}}e_2 + \frac{1}{T_a}x_3 - \dot{\sigma}_3 - z_3 - b_3e_3\right) \quad (38)$$

The time derivative of \dot{V}_3 from the eq. (36) can be updated by using the eq. (38), we have:

$$\dot{V}_3 = -b_1e_1^2 - b_2e_2^2 - b_3e_3^2 + \frac{K_a}{T_a}e_3e_4 \quad (39)$$

The integral action for the error e_4 is defined as:

$$z_4 = \int (e_4)dt = \int (x_4 - \sigma_4)dt \quad (40)$$

The final Lyapunov candidate function for the stability of e_4 is given by:

$$V_4 = V_3 + \frac{1}{2}e_4^2 + \frac{1}{2}z_4^2 \quad (41)$$

By taking the time derivative of the eq. (41), we have:

$$\dot{V}_4 = \dot{V}_3 + e_4\dot{e}_4 + z_4\dot{z}_4 \quad (42)$$

Now by taking the time derivative of the error e_4 from the eq. (32), we get:

$$\dot{e}_4 = \dot{x}_4 - \dot{\sigma}_4 \quad (43)$$

By substituting the eq. (4) in the expression of \dot{e}_4 we can write:

$$\dot{e}_4 = -\frac{b}{a}x_4 + \frac{ck_l}{a} + \frac{ck_f}{a}.x_1.u(t) - \dot{\sigma}_4 \quad (44)$$

Now by substituting the eq. (39) and eq. (44) in the eq. (42) and simplifying, we get:

$$\begin{aligned} \dot{V}_4 = & -b_1e_1^2 - b_2e_2^2 - b_3e_3^2 + e_4\left(\frac{K_a}{T_a}e_3 - \frac{b}{a}x_4 + \frac{ck_l}{a}\right. \\ & \left. + \frac{ck_f}{a}.x_1.u(t) - \dot{\sigma}_4 + z_4\right) \end{aligned} \quad (45)$$

The time derivative of the final Lyapunov candidate function \dot{V}_4 can be proved negative definite if we let:

$$\begin{aligned} \frac{K_a}{T_a}e_3 - \frac{b}{a}x_4 + \frac{ck_l}{a} \\ + \frac{ck_f}{a}.x_1.u(t) - \dot{\sigma}_4 + z_4 = & -b_4e_4 \end{aligned} \quad (46)$$

where b_4 is the positive gain parameter. The actual control $u(t)$ from the eq. (46) can be determined as:

$$\begin{aligned} u(t) = & \frac{a}{ck_f.x_1}\left(-\frac{K_a}{T_a}e_3 + \frac{b}{a}x_4 - \frac{ck_l}{a}\right) \\ & + \dot{\sigma}_4 - z_4 - b_4e_4 \end{aligned} \quad (47)$$

The time derivative of \dot{V}_4 from the eq. (45) can be updated by using the eq. (47), we have:

$$\dot{V}_4 = -b_1e_1^2 - b_2e_2^2 - b_3e_3^2 - b_4e_4^2 \quad (48)$$

Hence, the time derivative of the final Lyapunov candidate function \dot{V}_4 given by the eq. (48) is proved negative definite by utilizing the proposed IBC from the eq. (47) and it implies that the asymptotic stability of the system has been achieved [40].

3.2. Backstepping controller

The Backstepping is the recursive method with the absence of integral action for all the error dynamics. The formulation of BC can be done by performing the same steps as carried out for the IBC. The proposed BC for the system given by eqs. (1) to (4) is derived as:

$$\begin{aligned} u(t) = & \frac{a}{ck_f.x_1}\left(-\frac{K_a}{T_a}e_3 + \frac{b}{a}x_4 - \frac{ck_l}{a}\right) \\ & + \dot{\sigma}_4 - b_4e_4 \end{aligned} \quad (49)$$

3.3. Terminal synergetic controller

The Synergetic Control (SC) design approach represents an intuitive method for deriving the control law which is expressed in terms of macro variables. The SC tends to bring the system trajectories from the unknown initial conditions onto the surface of the macro variable located at the origin. Finally the asymptotic stability of the system is achieved by the control law pushing the trajectories to reach the equilibrium points. Additionally, the SC has similar design specifications like that of the sliding Mode Control (SMC) but its chattering free response makes the system's convergence smoother. The proposed TSC is the advancement of the SC, ensuring the system's convergence more swiftly, in a finite time frame [41]. The number of system inputs corresponds to the number of macro variables, $\varphi = \varphi(x, t)$ is then a macro variable for TSC in terms of the error e_1 from eq. (6) and defined as:

$$\varphi = \ddot{e}_1 + s_2\dot{e}_1 + s_1\dot{e}_1 + s_0e_1 \quad (50)$$

where s_2 , s_1 and s_0 are positive gain parameters. The TSC urges the macro variable on the manifold $\varphi = 0$ subject to the following condition as:

$$T\dot{\varphi}^{\frac{p}{q}} + \varphi = 0 \quad (51)$$

where p and q are positive integers bounded by $1 < p/q < 2$ and $T > 0$. From the eq (50) the time derivative of φ is defined as:

$$\dot{\varphi} = \ddot{\ddot{e}}_1 + s_2\ddot{\dot{e}}_1 + s_1\ddot{\dot{e}}_1 + s_0\ddot{e}_1 \quad (52)$$

The fourth time derivative of error e_1 is defined as:

$$\ddot{\ddot{e}}_1 = \ddot{\ddot{x}}_1 - \ddot{\ddot{x}}_{1ref} \quad (53)$$

From the eq. (1), if we take the time derivative of \dot{x}_1 upto the fourth time interval, we have:

$$\ddot{x}_1 = -\frac{1}{2J}\ddot{x}_1 - \frac{1.3}{JT_{CD}T_a}\dot{x}_3 + \frac{1.3K_a}{JT_{CD}T_a} \left[-\frac{b}{a}x_4 + \frac{ck_l}{a} + \frac{ck_f}{a}.x_1.u(t) \right] + \ddot{D}_t \quad (54)$$

Let's have:

$$\delta(t) = -\frac{1}{2J}\ddot{x}_1 - \frac{1.3}{JT_{CD}T_a}\dot{x}_3 + \frac{1.3K_a}{JT_{CD}T_a} \left[-\frac{b}{a}x_4 + \frac{ck_l}{a} \right] + \ddot{D}_t \quad (55)$$

By using expression of $\delta(t)$ from eq. (55) in the eq. (54), we get:

$$\ddot{x}_1 = \delta(t) + \frac{1.3K_a}{JT_{CD}T_a} \left[\frac{ck_f}{a}.x_1.u(t) \right] \quad (56)$$

The fourth time derivative of error e_1 from eq. (53) can be updated by substituting the eq. (56). We thus have:

$$\ddot{e}_1 = \delta(t) + \frac{1.3K_a}{JT_{CD}T_a} \left[\frac{ck_f}{a}.x_1.u(t) \right] - \ddot{x}_{1ref} \quad (57)$$

By substituting the eq. (57) in the eq. (52), the time derivative of $\dot{\varphi}$ can be written as:

$$\dot{\varphi} = \delta(t) + \frac{1.3K_a}{JT_{CD}T_a} \left[\frac{ck_f}{a}.x_1.u(t) \right] - \ddot{x}_{1ref} + s_2\ddot{e}_1 + s_1\ddot{e}_1 + s_0\dot{e}_1 \quad (58)$$

From eq. (51) we can also write $\dot{\varphi}$ as:

$$\dot{\varphi} = -\left(\frac{\varphi}{T}\right)^{\frac{q}{p}} \quad (59)$$

Now by equating the eq. (58) and the eq. (59) and solving for the actual control law $u(t)$ of TSC, we have:

$$u(t) = \frac{JT_{CD}T_a a}{1.3K_a ck_f x_1} \left(-\frac{\varphi}{T} \right)^{\frac{q}{p}} - \beta(t) - s_2\ddot{e}_1 - s_1\ddot{e}_1 - s_0\dot{e}_1 \quad (60)$$

The system stability has been checked by considering an energy like Lyapunov candidate function for the macro-variable φ which is defined as:

$$V_s = \frac{1}{2}\varphi^2 \quad (61)$$

By taking the time derivative of eq. (61), we have:

$$\dot{V}_s = \varphi\dot{\varphi} \quad (62)$$

The time derivative \dot{V}_s can be updated by using the eq. (58) and eq. (60), we get:

$$\dot{V}_s = \varphi \left(-\frac{\varphi}{T} \right)^{\frac{q}{p}} \quad (63)$$

By manipulating the eq. (63), we have:

$$\dot{V}_s = \left(-\frac{1}{T} \right)^{\frac{q}{p}} \varphi^{\frac{(q+p)}{p}} \quad (64)$$

From Lemma 1 the eq. (64) can be written as:

$$\left(-\frac{1}{T} \right)^{\frac{q}{p}} \varphi^{\frac{(q+p)}{p}} \leq \left(-\frac{1}{T} \right)^{\frac{q}{p}} 2^{\frac{(q+p)}{2p}} \left(\frac{1}{2}\varphi^2 \right)^{\frac{(q+p)}{2p}} \leq -T_1 (V_s)^{\frac{(q+p)}{2p}} \quad (65)$$

which proves the \dot{V}_s as negative definite. Hence, the asymptotic stability of the system is ensured.

Lemma 1. [42], [43] A positive definite Lyapunov function is considered to holds the inequality given as:

$$\dot{V}(t) \leq -\mu V^\omega(t), \forall t \geq t_0, V(t_0) \geq 0, \quad (66)$$

where μ and ω are simple constants defined as $\mu > 0$ while $0 < \omega < 1$.

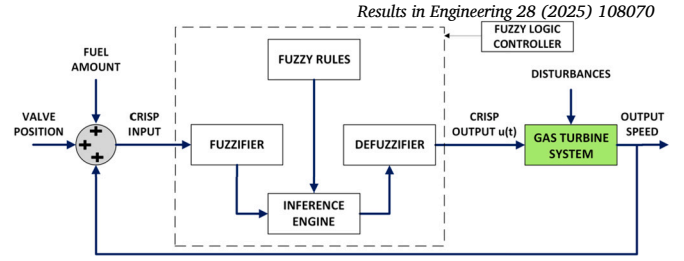


Fig. 4. Fuzzy control for GT system.

For any given initial value for time t_0 , the $V(t)$ holds for the following inequality:

$$V^{1-\omega} \leq V^{1-\omega}(t_0) - \mu(1-\omega)(t-t_0), \quad t_0 < t < t_1 \quad (67)$$

and $V(t) \equiv 0, \forall t \geq t_1$ where t_1 is defined as:

$$t_1 = t_0 + \frac{V^{1-\omega}(t_0)}{\mu(1-\omega)} \quad (68)$$

It can be drawn from the Lemma 1 that the TSC given by the eq. (60) will take finite time t_1 to make the system converge at the equilibrium points.

$$t_1 = \frac{V_s^{\left(\frac{p-q}{2p}\right)}(t_0)}{T_1 \left(\frac{p-q}{2p}\right)} \quad (69)$$

3.4. Fuzzy logic controller

Fuzzy control system is based on the logics that are fuzzy in their representation not having a strict notion of 0's and 1's logic as described by [44]. The fuzzy control relates an event with some degree of its occurrence which lies between the statement of true and false. The model free implementation of fuzzy control systems makes its structure more intuitive and simpler: the fuzzy system based on rules replicates thinking of the human brain into fuzzy logic [45], [46]. Due to these properties, the FLC is capable of capturing the non-linear behaviour of GT system to ensure a reliable operation [47]. The structural representation of a FLC for GT system is depicted in the Fig. 4 and relies on a closed loop control. FLC parts are Membership Functions, Fuzzifier, Fuzzy Rules, Inference Engine and Defuzzifier described as follows.

3.4.1. Membership function

The membership function relates each linguistic variable to input data with its relative degree of membership. The formation of a membership function is defined by the type of shape and its degree which lies between 0 and 1 over the feasible input range. For the proposed FLC, inputs of the GT system are expressed by three input membership functions; gas turbine system's speed (GTSS), fuelling amount (FA) and valve position (VP) given by the Figs. 5-7, where the linguistic variables utilized in the input membership functions are described below:

- l.speed– low speed of GTS
- m.speed– medium speed of GTS
- i.speed– ideal speed of GTS
- h.speed– high speed of GTS
- l.fuel– low fuel input
- m.fuel– medium fuel input
- h.fuel– high fuel input
- l.open– low valve open
- m.open– medium valve open
- h.open– high valve open

The output membership function for the proposed FLC output speed of the GT system is expressed by one defined as controlled signal (CS)

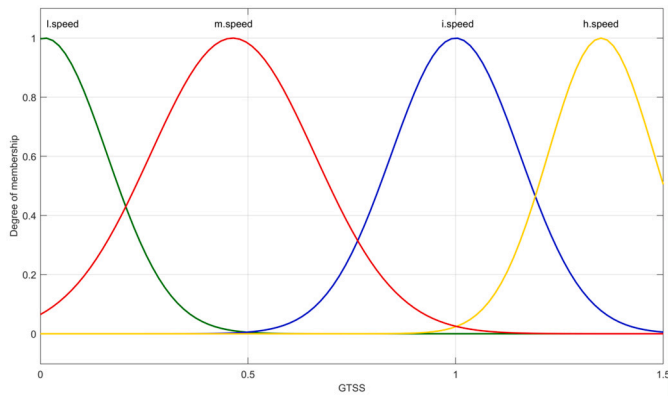


Fig. 5. GTSS membership function.

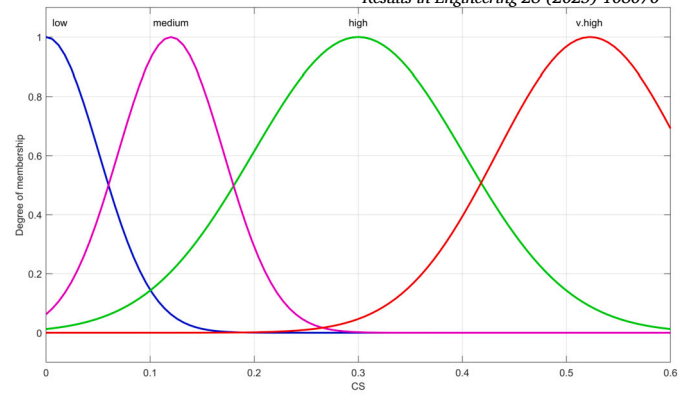


Fig. 8. CS membership function.

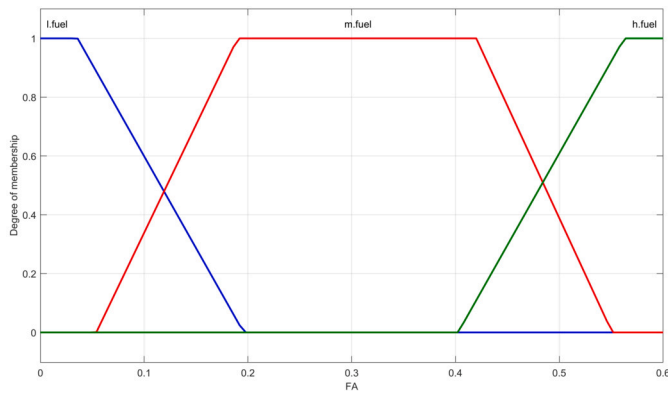


Fig. 6. FA membership function.

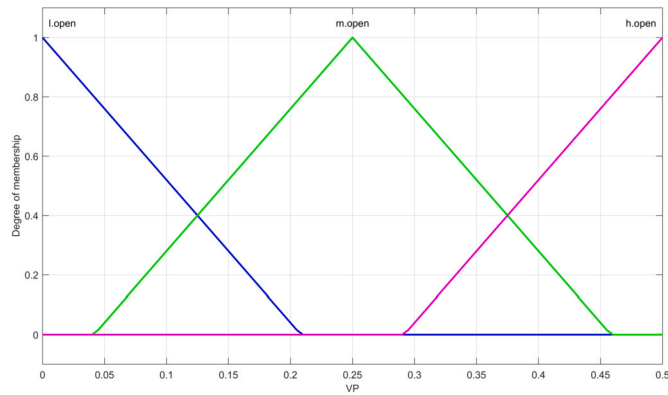


Fig. 7. VP membership function.

Table 1
Fuzzy Rules for Inference System.

GTSS	FA	VP	CS
<i>i.speed</i>	<i>m.fuel</i>	<i>l.open</i>	<i>low</i>
<i>i.speed</i>	–	<i>l.open</i>	<i>medium</i>
<i>i.speed</i>	<i>h.fuel</i>	<i>l.open</i>	<i>v.high</i>
<i>m.speed</i>	–	<i>l.open</i>	<i>high</i>
<i>l.speed</i>	<i>m.fuel</i>	<i>l.open</i>	<i>high</i>
<i>l.speed</i>	–	<i>l.open</i>	<i>v.high</i>
<i>h.speed</i>	<i>m.fuel</i>	<i>l.open</i>	<i>low</i>
<i>m.speed</i>	<i>m.fuel</i>	<i>l.open</i>	<i>medium</i>
<i>h.speed</i>	–	<i>l.open</i>	<i>low</i>
<i>i.speed</i>	<i>h.fuel</i>	<i>l.open</i>	<i>low</i>
<i>h.speed</i>	<i>h.fuel</i>	<i>l.open</i>	<i>low</i>
<i>m.speed</i>	<i>h.fuel</i>	<i>l.open</i>	<i>high</i>
<i>l.speed</i>	<i>l.fuel</i>	<i>m.open</i>	<i>medium</i>
<i>i.speed</i>	<i>l.fuel</i>	<i>m.open</i>	<i>medium</i>
<i>h.speed</i>	<i>l.fuel</i>	<i>m.open</i>	<i>low</i>
<i>m.speed</i>	<i>l.fuel</i>	<i>m.open</i>	<i>high</i>
<i>l.speed</i>	<i>m.fuel</i>	<i>m.open</i>	<i>v.high</i>
<i>i.speed</i>	<i>m.fuel</i>	<i>m.open</i>	<i>v.high</i>
<i>i.speed</i>	<i>m.fuel</i>	<i>m.open</i>	<i>v.high</i>
<i>m.speed</i>	<i>m.fuel</i>	<i>m.open</i>	<i>high</i>
<i>l.speed</i>	<i>m.fuel</i>	<i>m.open</i>	<i>v.high</i>
<i>i.speed</i>	<i>h.fuel</i>	<i>m.open</i>	<i>low</i>
<i>h.speed</i>	<i>h.fuel</i>	<i>m.open</i>	<i>medium</i>
<i>m.speed</i>	<i>h.fuel</i>	<i>m.open</i>	<i>medium</i>
<i>l.speed</i>	<i>l.fuel</i>	<i>h.open</i>	<i>high</i>
<i>i.speed</i>	<i>l.fuel</i>	<i>h.open</i>	<i>low</i>
<i>h.speed</i>	<i>l.fuel</i>	<i>h.open</i>	<i>low</i>
<i>m.speed</i>	<i>l.fuel</i>	<i>h.open</i>	<i>low</i>
<i>l.speed</i>	<i>m.fuel</i>	<i>h.open</i>	<i>medium</i>
<i>l.speed</i>	<i>m.fuel</i>	<i>h.open</i>	<i>medium</i>
<i>i.speed</i>	<i>m.fuel</i>	<i>h.open</i>	<i>low</i>
<i>h.speed</i>	<i>m.fuel</i>	<i>h.open</i>	<i>low</i>
<i>m.speed</i>	<i>m.fuel</i>	<i>h.open</i>	<i>low</i>
<i>l.speed</i>	<i>h.fuel</i>	<i>h.open</i>	<i>medium</i>
<i>i.speed</i>	<i>h.fuel</i>	<i>h.open</i>	<i>low</i>
<i>h.speed</i>	<i>h.fuel</i>	<i>h.open</i>	<i>low</i>
<i>m.speed</i>	<i>h.fuel</i>	<i>h.open</i>	<i>low</i>
<i>l.speed</i>	<i>l.fuel</i>	<i>l.open</i>	<i>low</i>
<i>h.speed</i>	<i>l.fuel</i>	<i>l.open</i>	<i>low</i>

given by the Fig. 8, where the linguistic variables utilized in the output membership function are described below:

- low– low speed signal
- medium– medium speed signal
- high– high speed signal
- v.high– very high speed signal

3.4.2. Fuzzifier

The first step in a fuzzy system is the fuzzification process, where the fuzzifier converts crisp input data from the system’s key variables into degrees of membership corresponding to predefined input membership functions.

3.4.3. Fuzzy rules

The fuzzy rules are the actual logical statements which intuitively relate the scenarios from input membership function with their linguistic variable data for generation of the desired output data for the output membership functions. Rules phrases are constructed by using **If-then** statements, where the input members are conditional **If** statement and output members are the resultant **then** statement linked via **and-or** logical operators. The fuzzy rules definition for the proposed FLC of GT system are listed in the Table 1.

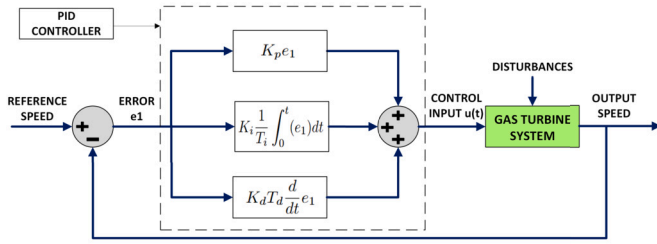


Fig. 9. PID control for GT system.

3.4.4. Inference engine

The inference engine is the main element of the fuzzy system which works as a replica of human brain for establishing the connection between logical statements. Fuzzy inference engine urges to link the linguistic data of the inputs to generate a degree of membership for the output linguistic variable by translating the fuzzy rules. In the proposed FLC, the Mamdani Inference is utilized for non-linear mapping as the suitable choice because Sugeno Inference generates constant membership degree or the linear output membership functions [48].

3.4.5. Defuzzifier

Defuzzification is the terminating step of fuzzy system operation. The depicted output membership functions from the inference engine with the corresponding relative membership degree of each linguistic variable is mapped over the output data in the same crisp form as initially fed to the fuzzy system. The output of the proposed FLC is the controlled response which is used to stabilize the GT system under study.

3.5. PID controller

PID control technique is the conventional feedback loop control for the system stabilization for various process control and automation industrial applications. The PID control structure facilitates its application over the energy generating systems by sequential logics and selective functional blocks [49]. The property of the PID control with fine tuning makes its implementation simple as it keeps updating the system errors through the feedback loop. The PID controller for the GT system can be proposed using the system error equation e_1 from the eq. (6) and it is given by:

$$u(t) = K_p e_1 + K_i \frac{1}{T_i} \int_0^t (e_1) dt + K_d T_d \frac{d}{dt} e_1 \quad (70)$$

where K_p is the proportional gain, K_i is the integral gain and K_d is the derivative gain. The gains tuning of PID enables the system stabilization to ensure optimal performance. The structure for closed loop PID control for the GT system under consideration has been shown in the Fig. 9.

4. Simulations analysis

The performance analysis of the proposed study has been made by doing numerical simulations with the help of MATLAB/Simulink version R2024b. The coding language/environment of Simulink are built-in blocks and graphical programming approach to develop Plant and Controller models. While the mathematical equations/expressions are written in MATLAB C/C++ syntax. The speed reference x_{1ref} for the GT system is selected to be 1 p.u. , given that the GT system operates under maximum load without any torque load. The performance of the proposed controllers IBC, BSC, TSC and FLC formulated by the eqs. (47), (49), (60) and Section 3.4 respectively obtained using the non-linear model of GT system, has been illustrated and compared with PID controller by the simulation work. The parametric values for the mathematical model of the GT system being used in the simulations are given in the Table 2 as [38].

Table 2
Numerical Values of Parameters for GT System.

Parameters	Numerical Values
J	$8 \times 10^{-4} \text{ kg.m}^2$
T_{eo}	0.2 N.m
T_{CD}	0.2 s
T_a	0.4
K_a	1
a	0.5
b	1
c	1
k_i	0.2
k_f	0.7

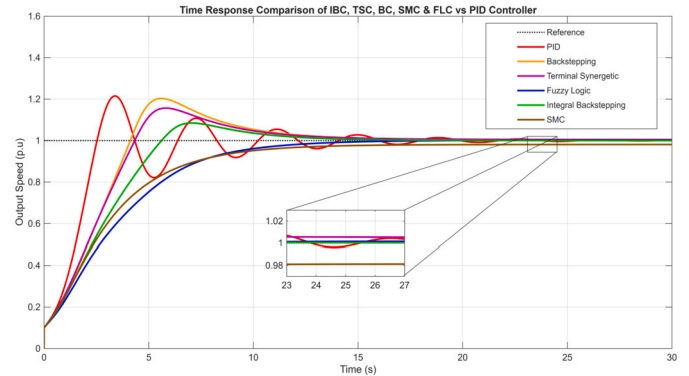


Fig. 10. GT system speed tracking for IBC, TSC, BSC, SMC & FLC vs PID.

The output speed time response curves of the proposed controllers IBC, TSC, BSC and FLC for the speed x_1 of the GT system given by eq. (1) have been plotted against the reference value as shown in Fig. 10. For the comparative analysis, the performance curves of the conventional PID controller and well known SMC from literature also has been given along with the proposed controllers. From the Fig. 10, it has been observed that all the proposed controllers are tracking the reference in a quiet smooth fashion while that of the PID controller goes through several oscillations. In terms of the converging time, the BSC has approx. 15 sec, both FLC and SMC have approx. 14 sec which is improved by using the TSC which shows the converging time of approx. 12 sec, while the IBC gained the converging time of approx. 10 sec and above all the PID controller has larger converging time of approx. 20 sec. The PID controller also has large steady state errors in comparison to BSC and FLC which have very minimal steady state errors and also for SMC larger steady state has been observed due to its inherent chattering effect. Moreover, the novel IBC outperformed overall while tracking the reference by depicting almost zero steady state and faster converging time which validates the upgradation of IBC from BSC.

In GT system, the fuel demand response program is of much importance due to its criticality associated with the cost factors. To understand the fuel consumption of GT system operating under the proposed controllers, a graphical representation of the fuel amount x_2 has been given by the Fig. 11. It has been observed that the PID controller shows an oscillatory behaviour in terms of fuel amount and also presents large peaks, while BSC has comparably a much lower peak value which is further improved by the TSC for the fuel amount. The FLC shows a smooth trend but the overall fuel amount is still higher than the IBC. Similarly, the SMC fuel amount is also higher than IBC but comparably low than FLC. All the proposed controller's behaviour reflected the gradual improvement in terms of the fuel amount consumption but cumulatively the IBC depicted a favourable choice as the cost effective solution which ensured its better performance overall.

For the state error e_1 given by the eq. (6), the time response curves of the proposed controllers have been plotted for comparison with PID

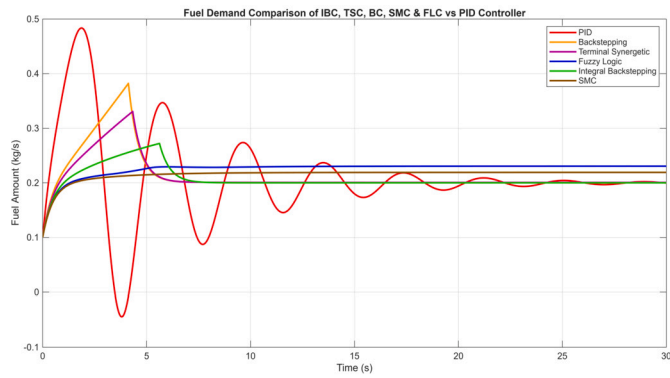


Fig. 11. GT system fuel amount for IBC, TSC, BSC, SMC & FLC vs PID.

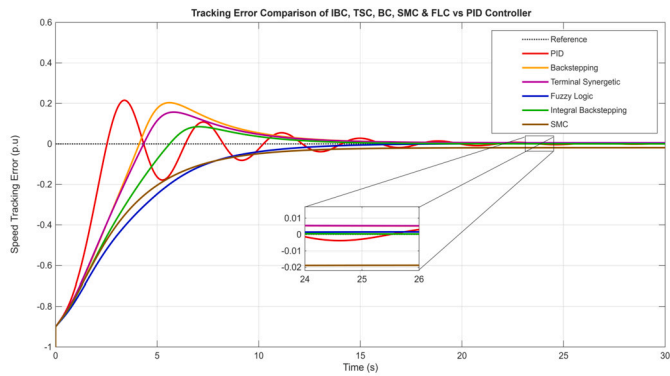


Fig. 12. GT system speed error for IBC, TSC, BSC, SMC & FLC vs PID.

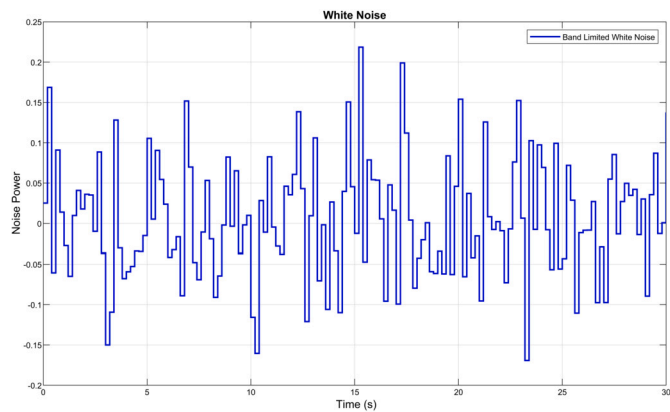


Fig. 13. Band Limited White Noise.

controller and literature SMC which is demonstrated by the Fig. 12. It can be observed that the BSC shows some steady state error, while TSC and FLC show very small steady state errors and negligible as compared to both SMC and PID controllers. The IBC exhibits almost zero steady state error in comparison to all the other algorithms, showing the best performance while tracking the given reference value.

To check the stability and robustness of the proposed controllers, the GT system has been further subjected to disturbance along with the inherent disturbance D_i given by the eq. (6). To achieve this, some perturbations have been introduced in the GT system in the form of white noise as given by the Fig. 13.

After the introduction of disturbances in the GT system, the time response curves for the proposed controllers IBC, TSC, BSC and FLC for the speed x_1 of the GT system given by eq. (1) have been again plotted against the reference value as shown by the Fig. 14. It has been observed that the BSC has shown an additional peak in its trajectory with small

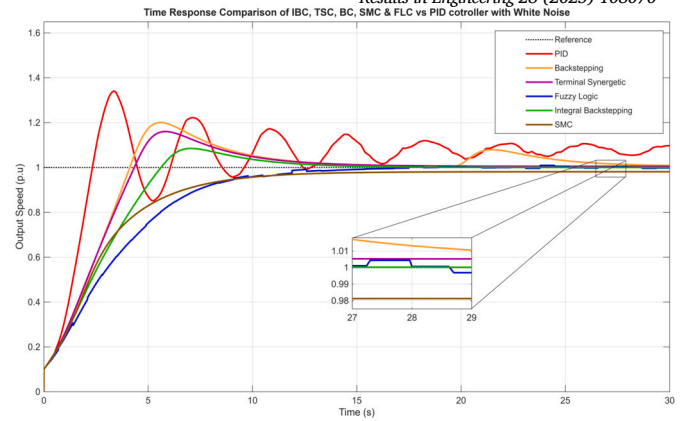


Fig. 14. GT system speed tracking for IBC, TSC, BSC, SMC & FLC vs PID under White Noise.

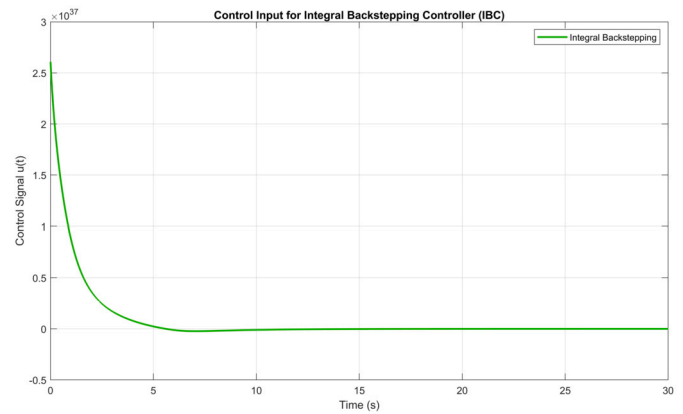


Fig. 15. Input control signal of IBC.

steady state error, the TSC has shown a small steady state error but with smooth trajectory while the FLC showed some deformation in its trajectory with negligible error. The SMC response also has a smooth trend to noise but the steady state error still persists in the trajectory. The PID controller response has been very affected by the noise as it underwent too much distortion and also got deflected from the reference value. The response of IBC remained unchanged due to noise and smoothly tracked reference without any steady state error. The IBC has shown better performance with system inherent disturbance as well as with additional noise disturbances as compared to all other controllers in terms of the converging time, state error and oscillations. Overall, the response of the IBC reflected better stability and the robustness due to its integral property of errors which made the proposed IBC outstanding and validates the upgradation of the non-linear backstepping.

The input control signal $u(t)$ of GT system for the proposed IBC controller given by the eq. (47) has been illustrated by the Fig. 15. It can be observed that the IBC calculated the input values in a smooth trend, which gradually decreased as the GT system started tracking the reference value and ultimately went to zero as the GT system got stable at the desired speed. A brief numerical comparison against the performances of all the proposed controllers has been formulated in the Table 3 by representing the respective converging time and status for steady state.

5. Conclusion

The inherent non-linear nature of the GT generation system is quite critical for the reliable and safe operation in getting the desired regulated output power. This research article intended to develop a unified control system for the stable working of GT system by achieving its controlled speed. The mathematical non-linear model of the GT sys-

Table 3
Comparison Analysis of Proposed Controllers.

Controller	Converging Time (s)	Steady State Error
IBC	10 or less	No
TSC	12 or less	No
BSC	15 almost	Yes
FLC	14 almost	Yes
SMC	14 almost	Yes
PID	20 or greater	Yes

tem has been used to propose the novel backstepping non-linear controller IBC along with other non-linear controllers BSC, TSC and FLC. In the presence of GT system inherent disturbances, the Lyapunov theory has been utilized to prove mathematically with global asymptotic stability of the system. The simulation analysis has been carried out in MATLAB/Simulink environment for all the proposed controllers. Performance curves of the proposed controllers have been illustrated in graphical figures and compared with the conventional PID controller. Based on simulation results a comparison study has been established reflecting the better performance of the proposed non-linear controllers in contrast to conventional PID controller. Numerical values obtained showed that the IBC outperformed overall others in view of steady state errors, converging time and oscillations. The robustness and stability of the proposed non-linear controllers have been tested via additional white noise introduction in the GT system. Simulations highlighted the better performance of the proposed non-linear controller under such additional system disturbances and the IBC performance curves validated the addition of integral actions for the upgrade of the backstepping algorithm. The future work of this study will be based on the implementation of an experimental platform for verification and deployment on actual systems.

CRedit authorship contribution statement

Sheraz Ahmad Babar: Writing – original draft, Software, Methodology, Investigation, Conceptualization. **Eleonora Riva Sanseverino:** Writing – review & editing, Validation, Supervision, Resources, Project administration. **Rossano Musca:** Writing – review & editing, Validation, Supervision, Conceptualization. **Marco Ferraro:** Validation, Supervision, Resources, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

No data was used for the research described in the article.

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