ORIGINAL RESEARCH

Early warning of systemic risk in global banking: eigen-pair R number for financial contagion and market price-based methods

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Abstract

We analyse systemic risk in the core global banking system using a new network-based spectral eigen-pair method, which treats network failure as a dynamical system stability problem. This is compared with market price-based Systemic Risk Indexes, viz. Marginal Expected Shortfall, Delta Conditional Value-at-Risk, and Conditional Capital Shortfall Measure of Systemic Risk in a cross-border setting. Unlike paradoxical market price based risk measures, which underestimate risk during periods of asset price booms, the eigen-pair method based on bilateral balance sheet data gives early-warning of instability in terms of the tipping point that is analogous to the R number in epidemic models. For this regulatory capital thresholds are used. Furthermore, network centrality measures identify systemically important and vulnerable banking systems. Market price-based SRIs are contemporaneous with the crisis and they are found to covary with risk measures like VaR and betas.

Keywords OR in banking · Global financial networks · Systemic risk · Early warning signals · Eigen-pair analysis · Statistical market price-based risk measures · Paradoxical risk measures

1 Introduction

The importance of banking system stability and cross-border propagation of contagion has been underscored by the 2007 Global Financial Crisis (GFC). The role of cross-border exposures in the dynamics of the GFC was most prominent in European banks (Calabrese and Osmetti 2019), which had surprisingly large exposure to US securitized assets (Allen et al. 2011) and resulted in a 50% equity capital loss (valued at \$1.6 trillion). This has led many to conclude that global financial interconnectedness is a major vehicle for macroeconomic

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instability (Soramäki et al. 2007; Degryse et al. 2010; Castrén and Rancan 2014; Simaan et al. 2020) and systemic risk. The latter is associated with the "disruption to the flow of financial services that is caused by the impairment of all or parts of the financial system and has the potential to have serious negative spillovers to the real economy" (IMF-BIS-FSB 2009). This led the Basel Committee to include the cross-jurisdictional activity of banks as a macro-prudential risk indicator (BCBS 2013). This calls for new methods for systemic risk estimation that can capture the growing risk from chains of indebtedness between financial institutions (FIs) and also other economic sectors, both nationally and in a cross-border setting.

The significance of financial network models has come to the forefront post GFC (Haldane 2009), and analogies have been drawn between financial contagion and epidemic models, including the notion of tipping points that has been well known since Furfine (2003). Despite this, remarkably few economists have discussed the necessity to establish the financial contagion systemic risk analog for the reproductive (R) number in epidemiology, which if greater than 1 determines that the spread of infection in the population grows exponentially rather than die out.[1]

We operationalize the model of financial contagion by explicitly adopting the two key causal factors from the epidemic literature relating to transmission rate of infection and cure rates and develop a systemic risk index (SRI) in terms of the spectral stability properties of a dynamical model for the financial contagion (see, May 1972, 1974; Wang et al. 2003; Newman 2010; Markose 2012; Markose et al. 2012; Bardoscia et al. 2017). In a financial system, the 'infection rates' are not homogenous but arise from the exposures that creditors face vis-á-vis counterparties' debt within a structure of balance sheet interconnectedness, which drives the domino effects of debtor defaults. Going forward, note the notion of 'infection rates' in the context of a financial contagion will be called financial exposure rates. In a financial system in which contagion arises from insolvency, regulatory capital provides the buffer against the spread of losses in credit chains and can be seen to be analogous to immune responses that lead to cure rates in epidemic models.

In this paper, a spectral measure of stability (Spectral SRI for short) is developed for the cross-border bilateral net liabilities of national banking systems adjusted for capital buffers of the exposed counterparties. The latter is called *stability matrix* (Markose 2012; Markose et al. 2012). This Spectral SRI is compared and contrasted with the widely popular market price-based SRIs, namely the dynamic conditional correlation (DCC) variants, respectively, of Marginal Expected Shortfall (MES) (Acharya et al. 2012, 2017), and Delta Conditional Value-at-Risk (ΔCoVaR) developed by Adrian and Brunnermeier (2011), along with a hybrid market price and balance sheet based Conditional Capital Shortfall Measure of Systemic Risk (SRISK) by Acharya et al. (2012) and Brownlees and Engle (2016). We gauge the early warning capabilities of the balance sheet network based Spectral SRI and market price based SRIs for the core global banking system. In particular, we see that the Spectral SRI can address the flaws associated with the paradoxical nature of the market price risk measures, in that the latter underestimate risk during boom periods with excessive leverage, are at a local minima directly before market crashes and give no early warning by spiking, at best, contemporaneously with the crisis or after it (Markose 2013; Arsov et al. 2013).

The market price models of systemic risk suffer from the so-called "paradox of volatility", a term coined by Borio and Drehmann (2011) and adopted by others (Brunnermeier and Sannikov 2015; Markose 2013). A well known pointer for paradoxical risk measures, analogous

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[1] See, van den Driessche (2017). The R number is regarded as the "the foremost and most valuable idea that mathematical thinking has brought to epidemic theory" (Heesterbeek and Dietz 1996), which applies dynamical spectral analyses to the modelling of infection transmission for contagious diseases in a population.

to the volatility of asset returns, which is low during boom market conditions, is their negative correlation with the balance sheet variable of leverage. The latter has been identified, especially, in many post GFC studies as a major cause for failure in financial institutions (see Glasserman and Young 2015; Bruno and Shin 2015b; Schularick and Taylor 2012). This problem was presaged by Minsky (1986) who claimed that asset bubbles mask the growing financial system fragility as the enhanced market values for assets and the perception of low risk from paradoxical volatility measures encourage procyclical excessive growth of leverage of banking systems with a commensurate growth of their interbank exposures and liabilities and with also non-bank sectors. Borio et al. (2001) give an extensive discussion on the general problem of procyclicality and underestimation of system failure by market price-based SRIs. Finally, market price based risk measures have been found to be more akin to systematic risk measures that are positively correlated with market returns (Benoit et al. 2013, 2016) rather than systemic risk measures that identity tipping points for system wide cascade failures.

The contribution of this paper is twofold. As the spectral approach relates to the stability of dynamical systems, we first give an appropriate dynamical characterization of the rates of failure of national banking systems when losses from their cross border exposures exceed a given regulatory capital threshold. The Spectral SRI helps to identify the so called tipping point when extant liability and net worth conditions represented within the core global banking network can produce instability. The Spectral SRI is the maximum eigenvalue of the associated generalised stability matrix for the global banking system and yields the maximum expected percentage loss of capital the system as a whole can suffer at each time step. If the SRI is greater than 1, as in the case of the R number, in the absence of external interventions, the exponential rate of losses can lead to system collapse. The main reason that the time path of the R-number based Spectral SRI does not suffer from the flaws of market price SRIs is because the former is the maximum eigenvalue of the stability matrix composed of capital adjusted net liabilities of counterparties. Furthermore, the Spectral SRI is governed by the upper bound given by the maximum row sum (infinity norm) of this matrix representing the network positions of the most leveraged participant of the system. Hence, the size of the Spectral SRI is directly related to instability from growing leverage in the system that can be unfavourably skewed by even one member of the financial network. In the special case, if financial institutions can be assumed to be governed by a single regulatory loss of capital threshold that corresponds to the cure rate, then financial stability requires that the maximum eigenvalue of the stability matrix is less than the capital loss threshold. This follows the simple intuition that the cure rate should be greater than the infection rate for the epidemic to be under control.

Second, we gauge systemic importance and vulnerability of national banking systems in terms of network centrality measures by deriving the associated right and left eigenvectors of the generalised stability matrix of the core global banking system. The network centrality indexes under an appropriate 1-norm is shown to represent the share of the total expected (%) loss of system wide capital either suffered by each banking system in the case of the vulnerability index quantified by the left eigenvector centrality and the contribution to the losses in terms of their systemic importance index given by the right eigenvector centrality. These network centrality measures are prominent in practical monitoring exercises such as in Google PageRank (Page and Brin 1998; Ipsen and Wills 2005) in the world wide web and for identification of 'super-spreaders' in epidemics. They use variants of the von Mises Power Iteration algorithm espoused by us. The necessity of the fixed point algorithm that produces internal consistency of an interconnected system has been recommended by Gauthier et al. (2012) for systemic risk indexes. By tracking these SRI indexes over time for financial networks directly based on contractual balance sheet data for national and global banking

systems, there can be early warning of potential systemic failure and for the identification of sudden changes in the rank order for the centrality of nodes. The latter can help in regulatory interventions as well as the in the design of Pigou type externalities funds that can stabilze the system, Markose et al. (2017).

In epidemic models, the R number depends on the transmission rate of the infection, which is due to some forms of the contacts, either directly or indirectly. In this study, to derive the R number in financial contagion in a cross border setting, we use the BIS Consolidated Banking Statistics (CBS) which yields the direct exposures of national banking systems to one another. For this analysis, we use quarterly data for 18 core reporting banking systems from the BIS CBS on an ultimate risk basis. This takes account of credit risk mitigants, such as collateral, guarantees and credit protection bought, that transfer the banks credit exposure from one counterparty to another. This also takes care of any further adjustments needed while netting gross BIS CBS between reporting banking systems relative to the Tier 1 capital of the BIS reporting banks. What matters in the assessment of the vulnerability of the core cross border banking network is the bilateral net exposures of the 18 BIS banking systems relative to the Tier 1 capital of the internationally active banks in each of the 18 countries. The latter is obtained from Bankscope for the period 2005Q4–2014Q4. As will be discussed, the sectoral break-down is an important advance in the BIS CBS since 2005, which has made the cross border banking network model feasible by reducing the data adjustments that are needed in its absence. For the market price-based SRIs, namely, the DCC-MES, DCC-CoVaR and SRISK, we use the MSCI Financials corresponding with the 18 BIS reporting national banking systems and the MSCI World as proxy for the market index. This paper is amongst the first to give comprehensive applications of these measures in a cross-border setting.

Our findings covering the period 2005Q4–2014Q4 can be summarised as follows. First, we find that the Spectral SRI provides early warning of extreme instability of the core global banking system before the start of the GFC. In contrast to the market data based SRI's, the spectral SRI shows considerable systemic instability through out the period and also in late 2014. We provide a test statistic to determine which of the SRIs also show abnormal deviations from 'normal' values to signal early warning of the impending crisis. At 99% confidence level, only the Spectral SRI showed severe instability of the core global banking network from 2005 Q1 with peaks at 2007 Q1. In contrast, the DCC-ΔCo-VaR and DCC-MES, which both have similar time profiles from 2005 to 2012Q3, momentarily showed two quarters of statistically significant values before the GFC, and they peak contemporaneously with the crisis in 2007 Q3. The SRISK fared the worst and showed no statistically significant early warning with the earliest date for this being 2008 Q2. Secondly, the time series properties of the Spectral SRI are found to correspond closely with the weighted clustering (Fagiolo 2007; Saramäki et al. 2007) for the top 5 highly clustered banking systems of the core global banking network in the pre GFC period. However, the weighted clustering coefficient, in being confined to a second order approximation of interconnectedness, fails to provide early warning signals for the Eurozone crisis that the higher order Spectral SRI gives. Thirdly, we are able to identify the increasing vulnerability of the Dutch and Belgian banking systems prior to the GFC and of the phenomena that the UK banking system replaced the US one as the most systemically important one in 2008 with the collapse of the subprime asset market (Ye et al. 2012). Also noteworthy is the vulnerability of the Portugese banking system in 2012 before the bankruptcy of their major bank Banco Espirito Santo[2] and the marked vulnerability of the Italian banking system, toward the end of 2014. These are results picked up by the

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[2] As will be discussed in Sect. 4, other stress testing methods such as that of the IMF had given a clean bill of health for the Portugese banks for this period.

Spectral SRI and are absent in the market price based SRIs. Fourthly, we find that market price-based SRIs tend to be highly contemporaneous with standard market risk measures, such as the VIX/VSTOXX, making them paradoxical and inversely related to asset price booms and the growth of leverage. That is, market price-based SRIs are found to peak with the crisis or even after it. Finally, we corroborate the Benoit et al. (2013) findings that the market price-based SRIs are determined by systematic risk measures. While the Spectral SRI and SRISK use a capital loss threshold related to the Basel regulatory thresholds to identify tipping points, the lack of similar thresholds in the other market price-based SRIs is found to add to the difficulty of monitoring the onset of potential crises.

The paper is organised as follows: Sect. 2 gives a literature review. Section 3 presents the spectral SRI generalized for a cross-border setting. In Sect. 4 we derive the market price-based SRIs in a cross-border setting. Section 5 describes the data and estimation methodology used in our analysis. In Sect. 6 we provide a network analysis of cross-border exposures in the core global banking network. Section 7 gives the results on cross-border banking network SRI and contrasts with market price-based SRIs. Section 8 analyses rankings of systemic importance and vulnerability over time for different groups of banking systems of countries using both methods. Finally, in Sect. 9 we report conclusions and future work.

2 Literature review

The development of SRIs has taken two main approaches. These are market price oriented statistical SRIs and the financial network approach championed by Haldane (2009). Due to the ease of accessing publicly available market price data on FIs, the majority of SRI analytics have taken this route. The market price based SRIs use a portfolio or weighted average approach to the the Basel II risk measures typically based on stock returns of individual banks, such as Value at Risk and Expected Shortfall. Of particular interest here are the purely market price-based SRIs, namely, the Marginal Expected Shortfall (MES) of Acharya et al. (2012, 2017), the ΔCoVaR of Adrian and Brunnermeier (2011) and a hybrid SRISK developed by Acharya et al. (2012) and Brownlees and Engle (2016). The latter combines balance-sheet data on leverage and equity with market price data.

The lack of bilateral financial exposure data has also led to simulations of financial networks using several assumptions. Examples are entropy maximisation methods (Castrén and Rancan 2014; Mistrulli 2011), correlations or Granger causality methods based on stock price data (see Billio et al. 2012; Diebold and Yılmaz 2014; Hautsch et al. 2015) and sparse network reconstruction models (Torri et al. 2018; Anand et al. 2015; Mahdavi Ardekani et al. 2020). Simaan et al. (2020) proposes a quite novel approach to estimate hidden networks from the interbank market using correlation-based statistical filtering methods. A recent study on cross border systemic risk using copula-based approach is given in Calabrese and Osmetti (2019). They model the joint impact of idiosyncratic and systematic shocks between banks of different countries. Simulation methods for financial networks purported to represent balance sheet interconnectedness introduce model risk and the network topology may have little to do with the contractual financial obligations for cross border banks.

The market price-based SRIs have not been free from criticism. One of the main problems is the absence of any early warning signal and they tend to be "coincident" or "near-coincident" indicators of financial instability spiking contemporaneously with the crisis or after it (Arsov et al. 2013). As indicated, this happens because most mainstream risk management models suffer from the so-called "paradox of volatility". The regime sensitivity of

the statistics for asset returns or GDP growth have been well known, at least since Hamilton (1989). Volatility, correlations and other higher moments of asset returns are known to be low during boom market conditions and become higher only when asset returns are negative during market downturns. Low volatility of returns on stock indexes during boom market conditions encourages procyclical excessive growth of leverage of banking systems without breaching VaR risk limits. Sooner or later, this may result in system collapse while market price based risk measures remain low.[3] In order that regulators are not blindsided by the paradoxically low risk measures that accompany asset price booms and growth of leverage, Brunnermeier and Cheridito (2013) have used a two stage reduced form macro-model that explicitly works to overcome the volatility paradox in their new SRI. Others like Bruno and Shin (2015a, b) have used the reciprocal of volatility in asset returns to proxy leverage and systemic risk. However, in contrast to the work of these authors, the need to avoid paradoxical systemic risk measures is still not widely acknowledged.

In the context of highly integrated international financial markets, the cross-border exposure of banks has been seen as one of the major catalysts for the GFC (Haldane 2009; Allen et al. 2011; Minoiu and Reyes 2013; Castrén and Rancan 2014; Calabrese et al. 2017). As pointed out by Degryse et al. (2010) in their study of network contagion using the same CBS data, shocks that impact the proper functioning of this densely interconnected network of activities involving cross-border banking may affect not only banks themselves but also the economies they operate in. This is why it is vital to investigate the topological structure of global banking and its vulnerability to liabilities of debtor countries. However, as will be discussed, our analysis differs from Degryse et al. (2010) in that they use former BIS Table 9D data which gave the exposures of reporting banking systems to the debt aggregated across all sectors of countries. Hence, in Degryse et al. (2010) the contagion failure of non-bank sectors from banking sector failure is implicit and relies on an unspecied process they call "spill over effect". This method can only be implemented using ex post simulations of contagion and an ex ante stability metric or an SRI cannot be derived.

Cerutti et al. (2012) go further than Degryse et al. (2010) and give an excellent account of the scenario analysis for the simulation of the (CBS) cross border propagation of shocks that include deleveraging and fire sales (Braouezec and Wagalath 2019; Feinstein 2020). However, the latter are second order effects after a banking system suffers losses on extant credit exposures. While the modeling of second order effects are interesting and can provide evidence for how a crisis can deepen, Cerutti et al. (2012) do not examine whether the BIS data, which they use, could have provided early warning of the GFC with the build up of large exposures of European banking systems to US bank assets prior to 2007 (see Allen et al. 2011).

Econometric techniques based on spatial autoregressive models have also been developed to assess contagion and systemic risk. Calabrese et al. (2017) is an example of BIS CBS data on cross-border bilateral European banks exposures used as input to estimate contagion and systemic risk within an econometric model. Though not directly related to implementing a systemic risk index, Von Peter (2007),Minoiu et al. (2013) and Hattori and Suda (2007) analyse the topological properties of the global banking network using BIS Locational Banking Statistics spanning over 30 years. They study the evolution of connectivity and density of transactions over the period 1978–2010 and find that the global banking system is much more connected during 2000s, when compared to previous decades, and the main trend in global banking network connectivity has a procyclical path with global capital flows.

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[3] The idea of procyclicality of leverage is now acknowledged to be a major source of systemic risk and is well articulated in Adrian and Shin (2010, 2011a).

Another study is that of Castrén and Rancan (2014), who introduced the notion of *Global Macro-Networks*, which combines a network of cross-border exposures of banking systems of countries with the flow of funds between sectors within countries. Similar to Cont et al. (2012) and Battiston et al. (2016), the authors quantify a metric of systemic risk, called the *Loss Multiplier*, based on an average of ex post simulated contagion losses stemming from the failure of each financial participant taken one at a time, in the spirit of Furfine (2003). Despite the seminal nature of their concept of a global macro-net, as contagion losses are greater after the onset of a crisis than before, the Castrén and Rancan (2014) *Loss Multiplier* SRI gives no early warning and peaks well after the GFC. It must be noted that the problem with Furfine (2003) type contagion loss estimates as the basis of SRI is that they are not derived from fixed point algorithms pertaining to the financial network.

Gauthier et al. (2012) have discussed the significance of fixed point algorithms in systemic risk analytics in order that they reflect internal consistency that corresponds to a very high order of interconnections. The use of the fixed point result to financial network analyses is well known from Eisenberg and Noe (2001), who distribute the exogenous loss, which is taken to be the starting point of a contagion in such models, to all participants in the financial network using a fixed point algorithm such that the loss suffered by one participant is internally consistent with that of other participants. However, as the aggregated system losses cannot exceed the initial shock itself, while systemic vulnerability of each of the participants can be easily ascertained from the Eisenberg and Noe (2001) approach, it is hard to gauge, in the absence of simulations, if the financial network has become more or less stable independent of the size of the external shocks.

Battiston et al. (2016) use an approach closest to ours. In their DebtRank metric, although the centrality measure is the result of a fixed point solution, they do not have the concept of the tipping point epidemic R number as an endogenous fixed point solution. The DebtRank centralities directly multiply the exogenously given total capital to obtain the SRI as the total capital loss estimation for the banking system. As will be shown in the next section, as capital measures net worth, it is the capital adjusted interbank net liabilities matrix that should feature in contagion failure through capital inadequacy, rather than the leverage matrix that is used by Battiston et al. (2016).

Our main contribution to the literature is to operationalize the conditions for the stability of an interconnected financial system of assets and liabilities of participants irrespective of the size of the external shocks. This corresponds with Alter et al. (2014), who have found that eigenvector centrality directly extracted from the liabilities based financial network provides the most effective systemic risk estimates for capital losses. In contrast, Karolyi et al. (2017) use the BIS CBS cross border data to assess systemic risk not in a financial network framework but in terms of the Acharya et al. (2012, 2017) MES approach. They find that during periods, as in the run up to the GFC in 2007/2008, of large cross border exposures of banking systems, especially of developed countries, their individual systemic risk MES measures fell. As noted, this is what is expected of paradoxical risk measures that fail to give early warning for the impending crisis.

3 Spectral SRI for cross-border banking networks

In contrast to a portfolio generalisation of risk, as is the case of many market price based SRIs, the spectral method determines the stability of the networked system using the power iteration fixed point algorithm for the matrix representing capital adjusted bilateral exposures,

to characterize the impact of individual node failure on counterparties of counterparties, to a very high order, in the chain of indebtedness.

The main focus here is on the Core Global Banking System Network (CGBSN) (18 BIS reporting countries). Here we will show how the Spectral SRI applies to the instability dynamics that can arise from the cross border balance sheet interconnectedness, which involves only a subset of the balance sheets. We start with a stylized balance sheet of a bank headquartered in a country and has foreign assets and liabilities vis-á-vis banks headquartered in other countries. It is also shown how this relates to the BIS CBS which aggregates cross border exposures of banks headquartered in a country to give the foreign claims at the level of national banking systems.

3.1 Stylized balance sheet for a cross border bank

The balance sheet equation for a bank headquartered in country i denoted as b_i with $A_{b_i 0}$ representing total assets, $L_{b_i 0}$, total liabilities and, $C_{b_i 0}$, equity capital of the bank in country i at some initial date, 0:

$$A_{b_i 0} - L_{b_i 0} = C_{b_i 0}. \tag{1}$$

We distinguish between the assets and liabilities in the balance sheet of bank b_i that are part of the cross-border interbank system and those that are not. This is given in equation (2) with $X_{b_j b_i}$ denoting the liabilities of bank b_j in country j to bank b_i in country i, while $X_{b_i b_j}$ denotes liabilities of bank b_i in country i to bank b_j in country j. The terms $A_{b_i 0}^{\#}$ and $L_{b_i 0}^{\#}$, respectively, denote b_i's non-interbank domestic and foreign assets and liabilities.

$$\sum_{b_j} \left(X_{b_j b_i 0} - X_{b_i b_j 0} \right) + A_{b_i 0}^{\#} - L_{b_i 0}^{\#} = C_{b_i 0}. \tag{2}$$

In order to develop a cross border banking contagion model and the rates of failure of banks, which we will denote by $u_{b_i t} \geq 0$, we track the depletion of equity capital at time $t + 1$ that arises only from cross border interbank defaults from net foreign debtors of b_i. To focus on this, we assume no losses or revaluations from non-interbank assets and liabilities and also no recapitalizations for each bank b_i. The second term in (3), $\sum_{b_j} \left(X_{b_j 0 b_i 0} - X_{b_i 0 b_j 0} \right)^{+} u_{b_j t}^{L}$ yields the total potential losses given as the weighted sum of net exposures of bank b_i vis-á-vis each of its foreign counterparties headquartered in all j countries $j \neq i$ which are pairwise net debtors of b_i. Note, as losses from net debtor banks can result in negative equity in (3), it is not possible to restrict failure rates of banks, $u_{b_i t}$, to be less than 1.

$$C_{b_i t+1} = C_{b_i 0} - \sum_{b_j} \left(X_{b_j 0 b_i 0} - X_{b_i 0 b_j 0} \right)^{+} u_{b_j t}^{L} - (1 - \rho_i) \left(C_{b_i 0} - C_{b_i t} \right). \tag{3}$$

The third term in (3) reflects the role played by the regulatory capital requirement on solvency contagion dynamics for banks. Regulatory capital requirement in country i specifies that cumulative loss of capital at each t, should not exceed a fixed threshold, ρ_i, of initial capital viz $\left(C_{b_i 0} - C_{b_i t} \right) \leq \rho_i C_{b_i 0}$. Likewise, incremental capital losses between t and $t + 1$ should not exceed the same percentage ρ_i of capital $C_{b_i t}$, viz. $\left(C_{b_i t} - C_{b_i t+1} \right) \leq \rho_i C_{b_i t}$.[4] Losses exceeding this threshold signal financial distress, while losses up to the threshold

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[4] Note the respective cumulative and incremental capital loss threshold constraints $\left(C_{b_i 0} - C_{b_i t} \right) \leq \rho \, C_{b_i t} \rightarrow C_{b_i t} \geq (1 - \rho) \, C_{b_i 0}$ and $\left(C_{b_i t} - C_{b_i t+1} \right) \leq \rho \, C_{b_i t} \rightarrow C_{b_i t+1} \geq (1 - \rho) \, C_{b_i t}$. On setting

can be offset by capital buffers. The term $(1 - \rho_i)\left(C_{b_{i0}} - C_{b_{it}}\right)$ in (3) gives the residual cumulative losses till t that are not offset by the permitted initial capital buffer.

3.2 A network epidemic model for solvency contagion across national banking systems

We will now show how the Eq. (3) can be used to track capital depletion and so called solvency contagion at the level of national banking systems which are denoted pairwise simply by (i, j). For this note that $C_{i0} = \sum_{b_i} C_{b_i 0}$ gives the total initial capital of country i's cross border banks and X_{ji} is the cross border liabilities of country j banks to the banking system of country i. Likewise, X_{ij} denotes the cross border liabilities of country i banking system vis-á-vis banking system j. Thus the **Core Global Banking System Network** (CGBSN) (18 BIS reporting countries) is defined by a $N \times N$ directed weighted network represent by the matrix $\mathbf{X}$ such that the sum of an i-th row $\sum_{j=1}^{N} X_{ij}$ represents the total foreign liabilities of the banking system i to all j banking systems represented by the BIS reporting banks of each country j. The sum of the j-th column $\sum_{i=1}^{N} X_{ij}$ represents the total receivable amount in the form of foreign claims of BIS reporting banks in country j. Note that $X_{ii} = 0$.

$$\mathbf{X} = \begin{bmatrix} 0 & X_{12} & \cdots & X_{1j} & \cdots & X_{1N} \\ X_{21} & 0 & \cdots & X_{2j} & \cdots & X_{2N} \\ \vdots & \vdots & 0 & \cdots & \cdots & \cdots \\ X_{i1} & \vdots & \cdots & 0 & \cdots & X_{iN} \\ \vdots & \vdots & \cdots & \cdots & 0 & \vdots \\ X_{N1} & X_{N2} & \cdots & X_{Nj} & \cdots & 0 \end{bmatrix}. \tag{4}$$

To model cross border financial contagion between national banking systems we have

$$C_{it+1} = C_{i0} - \sum_j \left(X_{ji} - X_{ij}\right)^+ u_{jt}^L - (1 - \rho_i)\left(C_{i0} - C_{it}\right). \tag{5}$$

The cumulative rate of capital depletion that is used to assess the rate of failure of the ith national banking system at $t + 1$, denoted by u_{it+1}^L, is given by subtracting the equivalent C_{it+1} from C_{i0} and dividing through by C_{i0} in (5). Hence,

$$u_{it+1}^L = \frac{C_{i0} - C_{it+1}}{C_{i0}} = \frac{\sum_j \left(X_{ji} - X_{ij}\right)^+}{C_{i0}} u_{jt}^L + (1 - \rho_i) u_{it}^L. \tag{6}$$

This yields the form of the dynamics for the failure rates of national banking systems from a loss of capital due to exposures to counterparty national banking systems in a cross border interbank setting. Here, u_{it+1}^L gives the potential failure rate of banking system i at $t + 1$ as arising from two factors given in (6).

(i) The first factor in (6) for the failure of i arises from the balance sheet interconnectedness term $\sum_j \frac{(X_{ji} - X_{ij})^+}{C_{i0}} u_{jt}^L$, which gives the weighted sum of heterogeneous rates of being "infected" at t from the bilateral net exposures of i to those js which are net debtors

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Footnote 4 continued

$C_{b_{it}}^* = (1 - \rho) C_{b_{i0}}$ and $C_{b_{it+1}}^* = (1 - \rho) C_{b_{it}}$, we obtain the term in (3) for the cumulative losses of capital that exceeded the permissible losses that can be offset as $C_{b_{it}}^* - C_{b_{it+1}}^* = (1 - \rho)\left(C_{b_{i0}} - C_{b_{it}}\right)$.

of i, adjusted for i's capital. The weights u_{jt}^L are the individual rates of failure of counterparties j, discussed below.

(ii) Banking systems can be vulnerable due to factors relating to themselves, viz. primarily, the size of initial capital. The term u_{it}^L is the percentage cumulative loss of capital suffered at t by i and is defined as $u_{it}^L = \left(1 - \frac{C_{it}}{C_{i0}}\right)$, $t > 0$, where C_{it} is the remaining capital at t and C_{i0} represents initial capital. In the epidemiology studies, ρ_i is called the "cure rate" and hence $(1 - \rho_i)$ is the rate of not surviving due to the insufficiency of the cure (see Chakrabarti et al. 2008). The factor $(1 - \rho_i) u_{it}^L$ defines the extent to which i cannot mitigate its failure using a percentage ρ_i of its capital as buffer against losses.

(iii) As already noted, u_{it}^L cannot be constrained to be less than 1 as bank losses can exceed 100% of initial capital and C_{it} can be negative at $t > 0$. Hence, u_{it} cannot be modeled as probability of failure. Further, it is known (Bardoscia et al. 2017) that u_{it}^L in the first order dynamic model in (6) apply proportionately rather than follow a (0, 1) indicator function as is the case in Furfine (2003) type contagion stress tests when counterparty losses are propagated only when the counterparty has been deemed to have failed and $u_{it}^L = 1$.

(iv) Finally, as will be shown, it is important that when only losses arising from a subset of the balance sheet is being modelled, a pro rata adjustment to capital should be made to represent this subset of the balance sheet. Or else, the contagion effects will be severely underestimated.

The capital adjustment of the net liabilities of banking system j's counterparties is expressed as $\theta_{ij} = \frac{(X_{ij} - X_{ji})^+}{C_{j0}}$, where the numerator takes only positive net liabilities from i to j and this is equal to zero if $(X_{ij} - X_{ji}) < 0$. The denominator C_{j0} is the initial capital of country j's BIS reporting banks adjusted on a pro rata basis for foreign exposures.[5] This matrix Θ will also be referred to as the stability matrix:

$$\Theta = \begin{bmatrix} 0 & \frac{(X_{12}-X_{21})^+}{C_{20}} & \cdots & 0 & \cdots & \frac{(X_{1N}-X_{N1})^+}{C_{N0}} \\ 0 & 0 & \cdots & \frac{(X_{2j}-X_{j2})^+}{C_{j0}} & \cdots & 0 \\ \vdots & \vdots & 0 & \cdots & \cdots & \cdots \\ \frac{(X_{i1}-X_{1i})^+}{C_{10}} & \vdots & \cdots & 0 & \cdots & \frac{(X_{iN}-X_{Ni})^+}{C_{N0}} \\ \vdots & \vdots & \cdots & \cdots & 0 & \vdots \\ 0 & \frac{(X_{N2}-X_{2N})^+}{C_{20}} & \cdots & \frac{(X_{Nj}-X_{jN})^+}{C_{j0}} & \cdots & 0 \end{bmatrix}. \tag{7}$$

3.3 Eigen-pair method for stability of cross border exposures of national banking systems

The capital loss thresholds ρ_i are important in the determination of the tipping point for the cross border banking system as a whole. We will consider two cases for this. In the homogeneous case, this threshold is assumed to be the same for both banks and the national banking systems, denoted by ρ (without a subscript). A loss of capital in excess of 30% is a

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[5] This adjustment is explained in Sect. 3.1 and quantified in Table B.5 in B.

good benchmark for impending crisis. Alternatively, we will apply the equivalent in terms of absolute capital losses of the Basel II or Basel III criteria of capital adequacy for individual banks of a national banking system. The determination of the ρ_i in the heterogeneous case, the absolute capital loss thresholds for the national banking systems that are equivalent to the regulatory capital requirements for individual banks, is based on the empirical data as will be shown in Sect. 5.2.

In the matrix notation, the above dynamics in (6) in the general case of heterogeneous capital loss thresholds is given by:

$$\mathbf{U}_{t+1}^L = \left[\mathbf{\Theta}' + \mathbf{\Gamma} \right] \mathbf{U}_t^L = \mathbf{Q}' \mathbf{U}_t^L. \tag{8}$$

Here $\mathbf{U}_t^L$ is the $1 \times N$ column vector of elements $\left(u_{1t}^L, u_{2t}^L, \ldots, u_{Nt}^L \right)$ defined in (6) for the potential failure rates for each of the banking systems; $\mathbf{\Theta}'$ is the transpose of the $N \times N$ non-negative stability matrix $\mathbf{\Theta}$ given in (7) with each element $\Theta_{ji} = \frac{(X_{ji} - X_{ij})^+}{C_{i0}}$ and $\mathbf{\Gamma}$ is the $N \times N$ identity matrix with the diagonal elements being 1 minus the country specific absolute capital thresholds, ρ_i. Note, matrix $\mathbf{Q}$ or $\mathbf{Q}'$ will be referred to as the generalized stability matrix.

Result 1: System stability with heterogeneous loss threshold ρ_i

The system stability for Eq. (8) is evaluated on the basis of the power iteration of the matrix $\mathbf{Q}' = \left[\mathbf{\Theta}' + \mathbf{\Gamma} \right]$, which yields:

$$\mathbf{U}_{t+1}^L = \left[\mathbf{\Theta}' + \mathbf{\Gamma} \right]^t \mathbf{U}_1^L = \mathbf{Q}'^t \mathbf{U}_1^L \cong g_1 \lambda_{max}^t \left(\mathbf{Q}' \right) \mathbf{v}^{L\#} \left(\mathbf{Q} \right). \tag{9}$$

Following the Perron–Frobenius theorem, a non-negative real square matrix $\mathbf{Q}$ yields non-negative real values for eigenvalues and corresponding eigenvectors with the largest eigenvalue denoted as $\lambda_{max} \left(\mathbf{Q} \right)$ being unique and greater than the next largest eigenvalue. Expressing the eigenvalue equation in matrix notation, the matrix $\mathbf{Q}'$ to the power t in (9) takes the form: $\mathbf{Q}'\mathbf{V} = \mathbf{DV} \rightarrow \mathbf{Q}'^t \mathbf{V}^L = \mathbf{D}^t \mathbf{V}^L$. Here $\mathbf{D}$ is the $N \times N$ identity matrix with eigenvalues of $\mathbf{Q}$ arranged from the highest to the lowest along the diagonal and $\mathbf{V}^L = \left[\mathbf{v}_1^L \mathbf{v}_2^L \ldots \mathbf{v}_N^L \right]$ is the $N \times N$ matrix of the corresponding left eigenvectors of $\mathbf{Q}$. Note the latter are the right eigenvectors of $\mathbf{Q}'$. As the set of eigenvectors of $\mathbf{V}^L$ can span the N dimensional vector space of real numbers $\mathbf{R}^N$, any non-zero, non-negative vector $\mathbf{U}_1$ can be expressed as a linear combination of the vectors in $\mathbf{V}^L$. Hence, we have $\mathbf{Q}'^t \mathbf{U}_1^L = \lambda_1^t \left[g_1 \mathbf{v}_1^L + g_2 \left(\frac{\lambda_2}{\lambda_1} \right)^t \mathbf{v}_2^L + \ldots + g_N \left(\frac{\lambda_N}{\lambda_1} \right)^t \mathbf{v}_N^L \right]$. Note, $g_1, g_2, \ldots, g_n$ are scalars. For some $t > 0$, a speedy convergence to the term with the maximum eigenvalue in (9) is assured as $\lambda_1 > \lambda_2$. Finally, note that $\mathbf{v}^{L\#}$ in (9) is the dominant left eigenvector associated with the maximum eigenvalue $\lambda_{max} \left(\mathbf{Q} \right)$ which is normalized by the 1-norm. This is discussed in Corollary 2b below.

Corollary 1a: Tipping point condition and R number

The stability of the network system in (9) involving the non-negative matrix $\mathbf{Q}$ requires that the following tipping point condition is fulfilled:

$$\lambda_{max} \left(\mathbf{Q} \right) < 1. \tag{10}$$

This is analogous to the R number in epidemiology. The key is to see that only $\lambda_{max}(\mathbf{Q})$ will determine the tipping point for the system in (9) and drives the long term dynamics of the epidemic in terms of containment ($\lambda_{max}(\mathbf{Q}) < 1$) or explosive growth ($\lambda_{max}(\mathbf{Q}) > 1$). [6] The latter case implies that in Eq. (9) as time progresses, without external interventions [7], the system cannot be made stable and there will be full system failure from contagion from any arbitrary size external initial shock with any non-zero $\mathbf{U}_1 \geq 0$. A further property of $\lambda_{max}(\mathbf{Q})$ is that its upper bound is given by the infinity norm $||\mathbf{Q}||_\infty$[8]. This implies that the instability of the system can be exacerbated by a single participant who is typically highly leveraged with its largest net debtor status given by the sum of its positions vis-á-vis counterparties in matrix $\mathbf{Q}$.

Corollary 1b: Speed of system failure

Temporally, the time taken for N banking systems to lose all of their capital, $u_{it} = 1$ implies $||\mathbf{U}_t||_1 = \sum_i u_{it}$ scales with N and $\lambda_{max}(\mathbf{Q}) > 1$. The exogenous arbitrary size shock at time 1 depletes the initial capital of one or more participants and renders the vector $\mathbf{U}_1 \geq 0$ such that at least one element of $\mathbf{U}_1$ is strictly positive. The constant g_1 in (9), $g_1 = \frac{||\mathbf{Q}'\mathbf{U}_1||_1}{\lambda_{max}(\mathbf{Q})}$, is a function of the initial vector of losses as a percentage of capital[9]. Taking logs, the time taken till all N banking systems have lost 100% of capital denoted as t^F:

$$ln(N) = ln(g_1) + t\, ln(\lambda_{max}(\mathbf{Q})) \rightarrow t^F = \frac{ln(N) - ln(g_1)}{ln(\lambda_{max}(\mathbf{Q}))}. \tag{11}$$

Clearly, smaller g_1 and $0 < g_1 < 1$, $ln(g_1) < 0$ and hence time to full system failure, t^F, increases, while higher $\lambda_{max}(\mathbf{Q}) > 0$ and higher g_1, the sooner is the point of full system failure. We hold the view that tipping condition in (10) gives a good *ex ante* indication whether or not system failure can occur. For $\lambda_{max}(\mathbf{Q}) > 1$, systemic failure is not a matter of if but a case of when, on the basis of bilateral data of extant contractual obligations and information in matrix $\mathbf{Q}$. Importantly, it is invariant to the size of the initial shocks, which may be difficult to quantify. Hence, like the epidemic R number, we recommend that regulators as a matter of routine monitor the status of $\lambda_{max}(\mathbf{Q})$ for purposes of systemic risk management.

Result 2: The eigen-pair indexes for systemic risk, systemic importance and vulnerability of banking systems

The eigen-pair result, defined as $\left(\lambda_{max}(\mathbf{Q}); \mathbf{v}^{L\#}(\mathbf{Q}), \mathbf{v}^{R\#}(\mathbf{Q})\right)$, yields the systemic risk index in terms of the maximum eigenvalue $\lambda_{max}(\mathbf{Q})$ of the matrix $\mathbf{Q}$ with $\mathbf{v}^{L\#}(\mathbf{Q})$ being the 1-norm dominant left eigenvector associated with the vulnerability indexes for the banking systems and $\mathbf{v}^{R\#}(\mathbf{Q})$ is the 1-norm dominant right eigenvector for matrix $\mathbf{Q}$, and gives the network centrality of the systemically important banking systems.

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[6] Newman (2010, Chapter 17 on Epidemics on Networks) gives an analogous derivation for the epidemic model to the one in (9).

[7] By external intervention is meant policy drives like bail outs in the context of a financial contagion. In an epidemic, external interventions include social distancing and vaccination.

[8] Note, $||\mathbf{Q}||_\infty = ||\mathbf{Q}'||_\infty$. The infinity norm of a square matrix is the maximum of the absolute row sums of the matrix. In the case of a vector, it is the largest absolute number in the vector.

[9] This result on g_1 is obtained by noting that in (9) for $\mathbf{U}_2$, we have $\mathbf{U}_2 = \mathbf{Q}'\mathbf{U}_1 = g_1 \lambda_{max}(\mathbf{Q}) \mathbf{v}^{L\#}(\mathbf{Q})$. This implies $g_1 = \frac{||\mathbf{Q}'\mathbf{U}_1||_1}{\lambda_{max}(\mathbf{Q})}$ as $||\mathbf{v}^{L\#}(\mathbf{Q})||_1 = 1$.

Corollary 2a: Unique steady state fixed point result incorporated in temporal dynamics for state variables U_{t+1}^L

Inherent to the dynamics of the financial contagion in (9) is the near term time independent fixed point solution:

$$\mathbf{U}^{L\#} = \lambda_{max}\left(\mathbf{Q}\right)\mathbf{v}^{L\#}\left(\mathbf{Q}\right) = \mathbf{Q}'\mathbf{v}^{L\#}\left(\mathbf{Q}\right). \tag{12}$$

Key to the eigen-pair result is the von Mises and Pollaczek-Geiringer (1929) power iteration algorithm where for any real initial non-zero vector $\mathbf{U}_1^L \geq 0$ as in (9), the tth power of the matrix, denoted as $\mathbf{Q}'^t$, can be solved iteratively using an appropriate norm. As we will see, for the economic interpretation of the λ_{max} as a systemic risk index, it is important to use the 1-norm $|| \, . \, ||_1$ [10] to normalize the vector as in the following power iteration algorithm:

$$\mathbf{U}_{t+1}^L = \frac{\mathbf{Q}'\mathbf{U}_t^L}{||\mathbf{Q}'\mathbf{U}_t^L||_1} = \frac{\mathbf{Q}'^t\mathbf{U}_1^L}{||\mathbf{Q}'^t\mathbf{U}_1^L||_1}. \tag{13}$$

The iteration is said to have converged at some t when $\mathbf{U}_{t+1}^L = \mathbf{U}_t^L = \mathbf{v}^{L\#}\left(\mathbf{Q}\right)$ with an epsilon margin of error. The vector $\mathbf{v}^{L\#}\left(\mathbf{Q}\right)$ is the left eigenvector of the matrix $\mathbf{Q}$ and $||\mathbf{Q}'\mathbf{v}^{L\#}\left(\mathbf{Q}\right)||_1 = \lambda_{max}\left(\mathbf{Q}\right)$.

At the point of convergence, multiplying through by $\lambda_{max}\left(\mathbf{Q}\right)$ in the power iteration Eq. (13), we have the eigenvalue Eq. (14) which encapsulates the important fixed point solution of the dynamical system in (12):

$$\mathbf{Q}'\mathbf{v}^{L\#}\left(\mathbf{Q}\right) = \lambda_{max}\left(\mathbf{Q}\right)\mathbf{v}^{L\#}\left(\mathbf{Q}\right) \equiv \mathbf{U}^{L\#}. \tag{14}$$

Corollary 2b: 1-Norm left eigenvector $v^{L\#}$ as vulnerability index for banking systems

From (14), noting that $||\mathbf{v}^{L\#}||_1 = \sum_{i=1}^N v_i^{L\#} = 1$, $\mathbf{v}^{L\#}$ is the 1-norm dominant left eigenvector of $\mathbf{Q}$ such that elements $v_i^{L\#}$ satisfy $0 \leq v_i^{L\#} \leq 1$. Thus $v_i^{L\#}$ can be interpreted to be the probability of failure of the ith banking system or its share of expected systemic losses quantified by λ_{max} as a percentage of total capital. Hence, $v_i^{L\#}$ are characterized as the vulnerability index for the ith banking system.

Corollary 2c: The systemic index λ_{max} measure of maximum expected total (%) loss of capital

Equation (14) and Corollary 2b directly yield the result that $\lambda_{max}\left(\mathbf{Q}\right)$ is the maximum total expected losses as a percentage of capital for the system as whole at each t adjusted for the permissible capital buffers:

$$\lambda_{max}\left(\mathbf{Q}\right) = ||\mathbf{Q}'\mathbf{v}^{L\#}\left(\mathbf{Q}\right)||_1 = \sum_{i=1}^N \left[\sum_{j\neq i} \frac{\left(X_{ji} - X_{ij}\right)^+}{C_{i0}} v_j^{L\#} + (1 - \rho_i)\, v_i^{L\#} \right]. \tag{15}$$

Noting $\mathbf{Q}' = \mathbf{\Theta}' + \mathbf{\Gamma}$ from (8), the first term in the square bracket is the maximum expected losses from net exposures to counterparties as a percentage of capital for the ith

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banking system, while the second term adjusts for the expected percentage of losses not offset by the capital buffer threshold ρ_i. Here we assume that there is a 100% loss of net exposures to counterparties given their probability of failure, $v_j^{L\#}$. Summing over all i, equation (15) justifies the moniker of $\lambda_{max}(\mathbf{Q})$ as the Systemic Risk Index being a system wide measure common to all participants.

There are two further points to mention. From (15), $\lambda_{max}(\mathbf{Q}) = ||\mathbf{\Theta}'\mathbf{v}^{L\#}||_1 + ||\mathbf{Iv}^{L\#}||_1 - ||\mathbf{\Phi v}^{L\#}||_1 = \lambda_{max}(\mathbf{\Theta}) + 1 - ||\mathbf{\Phi v}^{L\#}||_1$. Here $\mathbf{\Phi}$ is a diagonal matrix with values ρ_i. Hence, the 1-norm of $||\mathbf{\Gamma v}^{L\#}||_1 = ||\mathbf{Iv}^{L\#}||_1 - ||\mathbf{\Phi v}^{L\#}||_1 = 1 - \sum_i \rho_i v_i^{L\#}$ where the latter term is the expected value of the (%) capital buffer for the system as a whole. This implies that, in absence of capital buffer, any expected losses from counterparties given by $||\mathbf{\Theta}'\mathbf{v}^{L\#}||_1$ will lead to $\lambda_{max} > 1$. Therefore the stability condition hinges on whether or not the expected total counterparty losses $||\mathbf{\Theta}'\mathbf{v}^{L\#}||_1$ are greater or less than the expected total buffers, both as % of total capital. This leads to the interpretation that if $\lambda_{max} - 1 > 0$, the system will lose capital at this rate at each t. If $\lambda_{max} - 1 < 0$, then as buffers exceed losses, the latter will be reduced at that rate, yielding negative growth of losses as percentage of capital. This is analogous to the interpretation of the R number being greater or less than 1 in terms of the positive/negative growth of the epidemic in the population. To give an example, if $\lambda_{max}(\mathbf{Q}) = 1.09$, then $(1.09 - 1) = 0.09$ is the growth rate of expected total losses as percentage of capital at each time t. For any t, the (%) expected losses is $(1.09^t - 1)$. If $\lambda_{max}(\mathbf{Q}) = 0.83$ then the total expected losses will be offset/reduced by the buffers at the rate of $(0.83 - 1) = -0.17$ at each t.

Corollary 2d: Right eigenvector centrality and systemic importance of banking systems

Likewise, applying the power iteration algorithm (13) on $\mathbf{Q}$, from some initial non-zero real valued vector $\mathbf{U}_1^R$, yields the following power iteration equation $\mathbf{U}_{t+1}^R = \frac{\mathbf{QU}_t^R}{||\mathbf{QU}_t^R||_1} = \frac{\mathbf{Q}'\mathbf{U}_1^R}{||\mathbf{Q}'\mathbf{U}_1^R||_1}$, which converges to the right eigenvector $\mathbf{v}^{R\#}(\mathbf{Q})$. The elements of latter represents the systemic importance of each banking system in terms of the percentage loss of capital that it can inflict on the rest of the system in the near term given as a product of $\lambda_{max}(\mathbf{Q})$ and the right eigen vector centrality:

$$\mathbf{U}^{R\#} = \lambda_{max}(\mathbf{Q})\, \mathbf{v}^{R\#}(\mathbf{Q}). \tag{16}$$

From the eigenvalue equation, it can be seen in (17) that the centrality of a node (banking system) is proportional by $\frac{1}{\lambda_{max}(\mathbf{Q})}$ to the weighted sum of the centrality measures of all its neighbours, viz. as a fixed point result, a node is central not only because of the number and size of its weighted links, but also because it is connected to other highly central nodes:

$$v_i^{R\#}(\mathbf{Q}) = \frac{1}{\lambda_{max}(\mathbf{Q})} \sum_{j=1}^{N} \Theta_{ij} v_j^{R\#}(\mathbf{Q}). \tag{17}$$

These eigenvector centrality measures provide valuable analytics on systemic importance and systemic vulnerability of the banking systems in that they provide a robust fixed point *ex ante* quantitative percentage capital loss estimation at the level of each participant banking system and also for the core global banking network as a whole using the SRI $\lambda_{max}(\mathbf{Q})$. Other studies such as Cont et al. (2012), Degryse et al. (2010) and Castrén and Rancan (2014) use *ex post* simulated Furfine (2003) type of stress test calculations for the loss of capital conditional on the failure of a trigger bank.

Result 3: A conservative tipping point condition with heterogeneous capital threshold

As the upper bound of $\lambda_{max}(\mathbf{Q})$ is the infinity norm $||\mathbf{Q}||_\infty$, the more conservative tipping point condition is given by

$$\lambda_{max}(\mathbf{\Theta}) < \rho_i^{min}. \tag{18}$$

The infinity norm of $||\mathbf{Q}||_\infty = ||\mathbf{\Theta}||_\infty + ||\mathbf{\Gamma}||_\infty = ||\mathbf{\Theta}||_\infty + \left(1 - \rho_i^{min}\right)$. As $\lambda_{max}(\mathbf{\Theta})$ has upperbound of $||\mathbf{\Theta}||_\infty$, the stability condition in (9) can now be given as $\lambda_{max}(\mathbf{\Theta}) + \left(1 - \rho_i^{min}\right) < 1$. This gives the result in (18). The condition in (18) highlights the role of the so called weakest link identified by the banking system with the lowest absolute capital loss threshold, ρ_i^{min}, at which that constituent unit falls into distress.[11] Thus, in general, in (9) when many banking systems have low absolute capital loss thresholds, the instability of the system is increased.

Result 4: Global banking system stability with homogeneous loss threshold ρ

In this case, as matrix $\mathbf{Q}$ differs from matrix $\mathbf{\Theta}$ only by a constant multiple of the identity matrix, the so called shift eigenvector theorem gives the result that

$$\lambda_{max}(\mathbf{Q}) = \lambda_{max}(\mathbf{\Theta}) + (1 - \rho). \tag{19}$$

The eigenvector centrality of $\mathbf{Q}$ $(\mathbf{Q}')$ in this case is given by the right (left) dominant eigenvector of the matrix $\mathbf{\Theta}$ (see also Newman 2010, p.663). As discussed in Markose (2012), the stability condition that $\lambda_{max}(\mathbf{Q}) < 1$ in (19), for the case of a homogeneous loss threshold the following tipping point with $\lambda_{max}(\mathbf{\Theta})$ as the Spectral SRI is given by

$$\lambda_{max}(\mathbf{\Theta}) < \rho. \tag{20}$$

Note the condition for system stability in Eq. (20) implies that the expected percentage loss of capital of the system as a whole cannot exceed the homogeneous regulatory capital threshold ρ.

3.4 Operationalizing spectral SRI for BIS consolidated banking statistics (CBS)

We will now use this framework to fit the cross border banking system data given in BIS Consolidated Banking Statistics (CBS). This is done in terms of the new BIS CBS based on the banking sector breakdown.[12] As the CBS are reported to the BIS at an aggregate (banking

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[11] This has been used in Heath et al. (2016).

[12] Note, BIS data on the sectoral break down for foreign claims of the reporting banking systems only started in 2005Q1. At the beginning of our sample period, this sectoral breakdown is given for 9 major BIS reporting banking systems, i.e. Australia, Belgium, France, Ireland, Japan, Portugal, Sweeden, UK, US. From end of 2010, Spain, Italy and Germany started providing the foreign claims of their respective reporting banks on counterparty banks in the destination countries; Austria and Greece from end of 2013; Switzerland only provides partial information whereas India, Turkey and the Netherlands do not report at all. In the absence of the sectoral breakdown, the $N \times N$ network will have foreign debt aggregated across all sectors of countries along the rows of the matrix and banking systems of countries along the columns of the matrix. With the banking sector disaggregation given for the BIS CBS, the matrix representing the network will have the same entities in terms of banking systems, B_i, B_j, respectively, along the rows and columns of the matrix $\mathbf{X}$, which is what is needed for a network representation. Note, the column B_j, by definition, are the BIS reporting lending banks.

system) level rather than individual bank level, the netting is done at the level of the banking systems.

The reporting banks in country i with foreign claims on the banking sector of each country j in the BIS CBS are denote by b_i^R. Those with no foreign claims in a time period are the non-reporting banks, b_i^{NR}. Note that some of these can have liabilities to reporting banks in another country. Thus, banking system of country i in BIS CBS can be defined as $B_i = \sum b_i^R + \sum b_i^{NR}$. In order to show how there is an analogous derivation as in (6), consider the stylized bank balance sheet of a BIS reporting bank, b_i^R, in country i:

$$\sum_j X^F_{B_j b_i^R} + A^{\#DF}_{b_i^R} - \sum_j X^F_{b_i^R B_j} - L^{\#DF}_{b_i^R} = C_{b_i^R}. \tag{21}$$

$X^F_{B_j b_i^R}$: Assets/claims of reporting bank b_i^R of country i on the banking system B_j of country j, with F denoting the foreign status.

$X^F_{b_i^R B_j}$: Liabilities of bank b_i^R of country i owed to the banking system of country j. The latter, by definition, can only be the BIS reporting banks of country j.

$A^{\#DF}_{b_i}$: Domestic interbank and non-interbank domestic and foreign assets of bank b_i^R of country i.

$L^{\#DF}_{b_i^R}$: Domestic interbank and non-interbank domestic and foreign liabilities of bank b_i^R of country i.

$C_{b_i^R}$: Capital of the cross border reporting bank b_i^R in country i.

As in (2), aggregating over all reporting banks in a BIS country i and netting bilaterally with their liabilities to country j's reporting banks, we obtain $\sum_{b^R} C_{b_i^R}$ as in (22), which is the total capital of the BIS reporting banks in country i:

$$\sum_{b^R} \sum_{j \neq i} \left(X^F_{B_j b_i^R} - X^F_{b_i^R B_j} \right) + \sum_{b^R} A^{\#DF}_{b_i^R} - \sum_{b^R} L^{\#DF}_{b_i^R} = \sum_{b^R} C_{b_i^R}. \tag{22}$$

Here, in the first term in brackets we have $\sum_{b^R} X^F_{B_j b_i^R}$, which corresponds to the BIS CBS definition of foreign claims of the reporting banks of country b_i^R on the banking sector B_j of country j. This corresponds to X_{ji} in (6). However, the cross-border liability side of reporting banks in country i, $\sum_{b^R} X^F_{b_i^R B_j}$, is not available. We proxy this by the sum of the foreign claims of banking system j represented by the BIS reporting banks in that country on the banking system of country i, i.e. $\sum_{b^R} X^F_{B_i b_j^R} \equiv X^F_{B_i B_j}$ noting that B_j by definition includes only the BIS reporting banks in country j.[13] Thus, the capital of the reporting banks of country i given in (22), denoted simply as C_i, will be an overestimation of what is needed in terms of their net exposure to the another country j's banking system, which is a net debtor vis-á-vis i. With this proviso, this yields a failure model analogous to (6) for the BIS CBS reporting country banking systems. Finally, the above discussion implies a number of adjustments given in the Online A.

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[13] Note that this proxy includes liabilities of non-reporting banks in country i. This discrepancy is not large as the BIS reporting banks account for the vast majority of total cross-border liabilities (Avdjiev et al. 2015)

4 Market price-based SRIs in a cross-border setting

In this section we provide a brief extension of some popular SRIs based on publicly available market price data in a cross border setting.

4.1 Marginal expected shortfall (MES): cross border case

In our cross-border setting, the MES represents the marginal contribution of the financial system of country i to global systemic risk, measured by the Expected Shortfall (ES) of the global system, proxied by the MSCI World Index. The MES was initially proposed by Acharya et al. (2012, 2017) and later on extended in a dynamic conditional correlation (DCC) version by Brownlees and Engle (2016). The ES at the $\alpha\%$ level is the expected return in the worst $\alpha\%$ of the cases. This can be generalized to the case where returns exceed a given threshold C. Formally, denoting the MSCI World returns at time t by r_{mt} and the country MSCI Financial returns by r_{it}, the conditional ES of the system is defined as:

$$ES_{mt}(C) = E_{t-1}(r_m|r_{mt} < C) = \sum_{i=1}^{N} w_{it} E_{t-1}(r_{it}|r_{it} < C). \tag{23}$$

Here, $w_{i,t}$ are the weights of the country MSCI Financials. In this context, the MES corresponds to the partial derivative of the system ES with respect to the weight of country i in the global financial system:

$$MES_{it}(C) = \frac{\partial ES_{mt}(C)}{\partial w_{it}} = E_{t-1}(r_{it}|r_{it} < C). \tag{24}$$

The higher a country's MES, the higher the individual contribution of the country to the risk of the global system. In order to estimate the MES, we follow the methodologies of Brownlees and Engle (2016) and Benoit et al. (2013) and consider a bivariate GARCH process for the demeaned returns:

$$R_t = H_t^{1/2} V_t. \tag{25}$$

Here $R_t' = (r_{mt}, r_{it})$ denotes a return vector of the world market and the MSCI Financials of country i, respectively. The random vector $V_t' = (\epsilon_{mt}, \xi_{it})$ are the associated standardised innovations and are assumed to be $i.i.d$ with the following first moments: $E(V_t) = 0$ and $E(V_t V_t') = I_2$, a two-dimensional identity matrix. The H_t matrix denotes the conditional variance-covariance matrix:

$$H_t = \begin{pmatrix} \sigma_{mt}^2 & \sigma_{it}\sigma_{mt}\rho_{it} \\ \sigma_{it}\sigma_{mt}\rho_{it} & \sigma_{it}^2 \end{pmatrix}. \tag{26}$$

Here σ_{it} and σ_{mt}, denote the conditional standard deviations and ρ_{it} the conditional correlation between the world market and the country financial returns. Based on Eqs. (25) and (26), Brownlees and Engle (2016) show that the MES can be expressed as:

$$MES_{it}(C) = \sigma_{it}\rho_{it} E_{t-1}\left(\epsilon_{mt}|\epsilon_{mt} < \frac{C}{\sigma_{mt}}\right) + \sigma_{it}\sqrt{1 - \rho_{it}^2} E_{t-1}\left(\xi_{mt}|\epsilon_{mt} < \frac{C}{\sigma_{mt}}\right). \tag{27}$$

Then, the MES can be expressed as a function of the country's return volatility, its correlation with the global index return, and the comovement of the tail distribution. As demonstrated

by Benoit et al. ([2013](#)), the $i.i.d$ assumption on $\xi_{i,t}$ and $\epsilon_{m,t}$ implies that the conditional expectation $E_{t-1}\left(\xi_{mt}|\epsilon_{mt} < \frac{C}{\sigma_{mt}}\right)$ is null. Thus, the MES takes the following form:

$$MES_{it}(\alpha) = \beta_{it}ES_{mt}(\alpha). \tag{28}$$

Under these assumptions, ranking systemically important financial systems is equivalent to sorting financial systems based on their betas (Benoit et al. [2013](#), [2016](#)). Using daily data, this SRI is estimated by the Dynamic Conditional Correlation Model of Engle and Sheppard ([2001](#)), from which the time-varying beta is obtained (Eqs. ([25](#)) and ([26](#))). In order to compare the MES with the network-based SRI based on λ_{max}, we calculate the quarterly MES by averaging the daily MES within each quarter. To obtain our global measure of MES, quarterly MES are averaged across countries, as done by Acharya and Stefen ([2012](#)) to calculate their within country MES.

4.2 SRISK in cross border setting

The SRISK measure was developed by Acharya et al. ([2012](#)) and Brownlees and Engle ([2016](#)). It is an extension of the MES by considering both the liabilities and the size of the financial system. In a cross-country setting, the SRISK corresponds to the expected shortfall of the stock returns of financial firms of a country, conditional on a crisis affecting the global market (e.g., the MSCI World). The country with the largest shortfall is assumed to be the largest contributor to the crisis and can be, therefore, regarded as the most systemically important. Following Acharya et al. ([2012](#)), the SRISK is defined as:

$$SRISK_{i,t} = MAX\left[0; \phi(D_{i,t} + (1 - LRMES_{i,t})W_{i,t}) - (1 - LRMES_{i,t})W_{it}\right]. \tag{29}$$

Here ϕ is the prudential capital ratio, which is assumed to be 8%, D_{it} is the book value of total liabilities of country i's banking system, and $W_{i,t}$ is the market capitalisation or market value of equity of the national banking system. The $LRMES_{i,t}$ is the long-run marginal expected shortfall of country i. If we define leverage as $L_{i,t} = (D_{i,t} + W_{i,t})/W_{i,t}$, SRISK becomes:[14]

$$SRISK_{i,t} = MAX\left[0; (\phi L_{i,t} - 1 + (1 - \phi)LRMES_{i,t})W_{i,t}\right]. \tag{30}$$

Notice that the SRISK is a weighted average of leverage and the LRMES. The $LRMES_{i,t}$ measures the sensitivity of the country's equity return to the downturn of the world market in the case of a financial crash. Acharya et al. ([2012](#)) and Brownlees and Engle ([2016](#)) define a market crash as a hypothetical 40% semiannual world market decline. $LRMES_{i,t}$ is particularly difficult to estimate because it corresponds to an extremely rare event. In addition, the $LRMES_{i,t}$ is typically not available in closed form for this class of dynamic models. We follow the Brownlees and Engle ([2016](#)) simulation procedure to obtain exact LRMES predictions[15] for our 18 reporting countries. The results for this have been reported in the Online D.

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[14] In order to allow comparison with the rest of the SRIs, we scale the SRISK by the country's equity. Hence, throughout the analysis, note we use $\frac{SRISK_{i,t}}{W_{i,t}}$.

[15] This procedure consists of simulating a random sample of the h-period country and market (arithmetic) returns conditional on the information set available on day t. The LRMES for day t is then calculated using the average of the Monte Carlo simulated returns, $LRMES_{i,t} = \frac{\sum_{s=1}^{S} R_{i,t+1:t+h} I\{R_{m,t+1:t+h} < C\}}{\sum_{s=1}^{S} I\{R_{m,t+1:t+h} < C\}}$, where C is the semiannual 40% drawdown and S is the number of runs of simulations, which in this case is $S = 50,000$. $R_{i,t+1:t+h}$ are cumulated returns over a six months period.

In order to compare and contrast the $SRISK$ with λ_{max}, we construct the quarterly $SRISK_{i,t}$ for every country i. To calculate the overall index, we follow Acharya et al. (2012) and take the average of the quarterly $SRISK$ across countries.

4.3 ΔCoVaR in cross border setting

This measure is based on the concept of Value-at-Risk, denoted as VaR(α), which is the maximum loss within the $\alpha\%$-confidence interval.[16] The CoVaR corresponds to the VaR of the MSCI World Index returns obtained conditionally on some event $C(r_{it})$ observed for country i:

$$Pr\left(r_{mt} \leq CoVaR_t^{m|C(r_{it})}|C(r_{it})\right) = \alpha \tag{31}$$

The ΔCoVaR of country i is then defined as the difference between the VaR of the MSCI World returns conditional on this country's financial system being in distress and the VaR of the MSCI World returns conditional on the financial system of country i being in its median state:

$$\Delta CoVaR_{it}(\alpha) = CoVaR_t^{m|r_{it}=VaR_{it}(\alpha)} - CoVaR_t^{m|r_{it}=Median(r_{it})}. \tag{32}$$

We follow Benoit et al. (2013) and derive the DCC variant of ΔCoVaR as a function of the conditional correlations, volatilities, and VaR:

$$\Delta CoVaR_{it}(\alpha) = \gamma_{it}[VaR_{it}(\alpha) - VaR_{it}(0.5)]. \tag{33}$$

Here $\gamma_{it} = \rho_{it}\sigma_{mt}/\sigma_{it}$ with ρ_{it} denoting the conditional correlation between the returns of the MSCI Financial returns of country i and the MSCI World returns. If we assume that the marginal distribution of the returns is symmetric around zero, ΔCoVaR is strictly proportional to VaR:

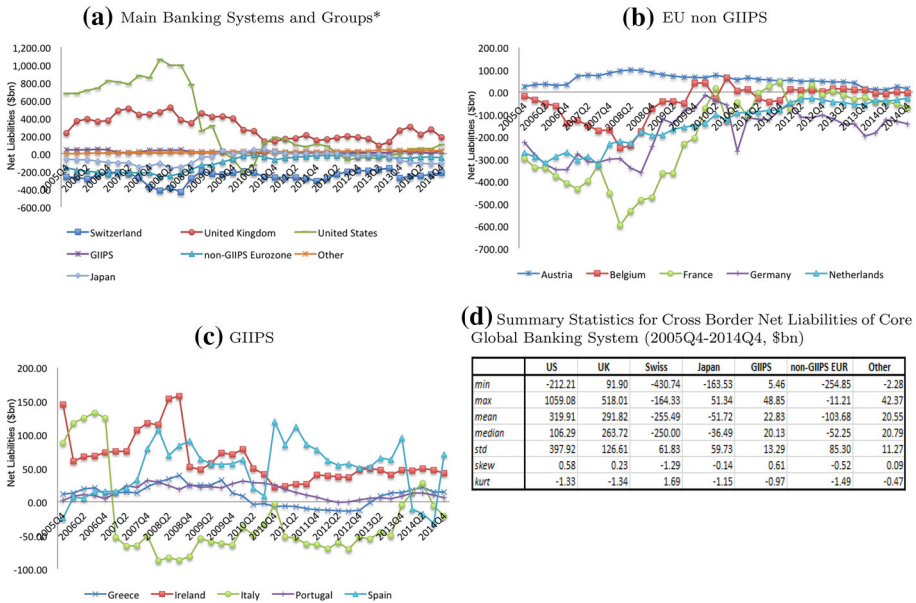$$\Delta CoVaR_{it}(\alpha) = \gamma_{it}VaR_{it}(\alpha). \tag{34}$$

This means that while ΔCoVaR-based rankings and VaR-based rankings of systemic risk may differ in the cross-section (that is, at a given point in time t), in the time-series (for a given financial system i), however, ΔCoVaR is proportional to VaR. As a consequence, as noted by Benoit et al. (2013), forecasting the future evolution of the contribution of a country's financial system i to global systemic risk is equivalent to forecasting its VaR in isolation as given in Eq. (34). We compute the quarterly ΔCoVaR by taking the average within each quarter for each of the countries. In a similar fashion, the overall index is calculated by taking the global average of the quarterly ΔCoVaR across countries.

5 Data

The sample of BIS Consolidated Banking Statistics consists of 18 reporting countries and spans quarterly from 4th quarter of 2005 until 4th quarter 2014.[17] We consider the core counterparts that correspond to the same reporting countries and further, we restrict the analysis to the foreign claims only on the banking sector (see Sect. 3.1).

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[16] A quantile regression model of financial contagion using VaR is presented in Ye et al. (2017).

[17] The countries in the sample are Australia, Austria, Belgium, France, Germany, Greece, India, Ireland, Italy, Japan, Netherlands, Portugal, Spain, Sweden, Switzerland, Turkey, United Kingdom and United States. Of these the so called GIIPS are Greece, Italy, Ireland, Portugal and Spain.

(a) Main Banking Systems and Groups*

(b) EU non GIIPS

(c) GIIPS

(d) Summary Statistics for Cross Border Net Liabilities of Core Global Banking System (2005Q4-2014Q4, $bn)

	US	UK	Swiss	Japan	GIIPS	non-GIIPS EUR	Other
min	-212.21	91.90	-430.74	-163.53	5.46	-254.85	-2.28
max	1059.08	518.01	-164.33	51.34	48.85	-11.21	42.37
mean	319.91	291.82	-255.49	-51.72	22.83	-103.68	20.55
median	106.29	263.72	-250.00	-36.49	20.13	-52.25	20.79
std	397.92	126.61	61.83	59.73	13.29	85.30	11.27
skew	0.58	0.23	-1.29	-0.14	0.61	-0.52	0.09
kurt	-1.33	-1.34	1.69	-1.15	-0.97	-1.49	-0.47

Fig. 1 Cross border net liabilities of the Core Global Banking System ($bn) * GIIPS includes Greece, Ireland, Italy, Portugal and Spain; non-GIIPS Eurozone (EUR) banking systems include Austria, Belgium, France, Germany and Netherlands; Others include Australia, India, Sweden and Turkey . *Source*: BIS consolidated banking statistics with adjustments made for banking system to banking system foreign claims (see Sect. 3.1)

5.1 Foreign claims and capital of core global banking system

The net liabilities trend in Fig. 1 highlights the drastic reversal of the US banking system from being the biggest net borrower to that of net lender after 2008 Q4. In contrast, the UK banking system remains a net borrower throughout the period. In the Eurozone, with the exception of the Netherlands, the core non-GIIPS countries are primarily net lenders with this tapering off in the aftermath of the crisis. The GIIPS are all initially net borrowers and continue to be so, with the exception of the Italian banking system which increasingly expanded its role of net lender post 2007.

The second ingredient needed to construct the Core Global Banking System (CGBS) network is information about the amount of Tier 1 capital, which serves as a buffer for losses incurred by banks. Bankscope data on banks' Tier 1 capital is reported on an annual basis, so data is taken for the end of the year, that is, for the fourth quarter of a given year. Quarters 1 to 3 are then proxied by linear interpolation and discussed in the Online Appendix B.

In order to estimate the DCC-MES, DCC-ΔCoVaR and SRISK SRIs, we use the individual MSCI Financials for the 18 countries and the MSCI World as a proxy for the market return that is faced by the financial system of each country in the sample. Tables of descriptive statistics are provided in Appendix E. The MSCI World Financial Index captures large and mid cap primarily banking firms across 23 Developed Markets.[18] We use daily returns of

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[18] Developed countries include: Australia, Austria, Belgium, Denmark, France, Germany, Hong kong, Ireland, Israel, Italy, Japan, the Netherlands, Norway, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, the United States and United Kingdom.

MSCI Financials for each country during the period 03/10/2005 until 31/12/2014. This data has been retrieved from Bloomberg and it has been adjusted for bank holidays.

5.2 Estimated absolute capital threshold for losses (ρ_i)

In Sect. 3, we argued that the stability of the networked system is also determined by the absolute capital loss threshold, ρ_i for banking system of country i. These thresholds have to be carefully specified in terms of how they relate to the regulatory Tier 1 capital to risk weighted assets ratio (referred to as Tier 1 ratio).[19]

The Basel II criteria for the determination of the on-set of capital inadequacy in a bank from losses of receivables from counterparties, can be defined as follows:

$$\frac{\text{Tier1}_i - \text{LGD}_i}{\text{RWA}_i} < 4\% = T_{\text{RWA}}, \quad \forall\, i. \tag{35}$$

Here LGD is loss given default, RWA is risk weighted assets and Tier1$_i$ is Tier1 capital of i. T_{RWA} is the Tier 1 capital to risk weighted assets ratio. The condition in (35) states that for each i, Tier 1 capital minus losses should not fall below 4% of RWA. However, as the practical aspects of avoiding insolvency requires recapitalization, it is important to see the equivalence of the above Basel rule with permissible LGD as function of absolute Tier 1 capital for each bank i, denoted as ρ_i. We set $\rho_i \cdot \text{Tier1}_i = \text{LGD}_i$. Substituting this into (35) when the condition in (35) is exactly met, we have:

$$\rho_i = MAX\left(0, 1 - T_{\text{RWA}}\frac{\text{RWA}_i}{\text{Tier1}_i}\right). \tag{36}$$

The threshold ρ_i indicates the percentage maximum absolute capital losses bank i can absorb in terms of its Tier 1 capital ratio to RWA and this yields the point of distress.

For instance, when this condition is exactly met viz. $\frac{\text{Tier1}_i}{\text{RWA}_i} = 4\% = T_{\text{RWA}}$, this implies the permissible absolute capital loss is zero, viz. $\rho_i = 0$. In contrast, if a bank has $\frac{Tier1_i}{RWA} = 8\%$, then the absolute capital loss threshold ρ_i is 50%, see footnote 19. As a realistic absolute capital loss threshold, ρ, of 30% is recommended, this implies a Tier 1 ratio of 5.7%. Hence, banks that report very high Tier 1 ratios, should be very closely scrutinized for deflating RWA in the denominator. We calculated ρ_i for the banking system of every BIS reporting country i for all the quarters in order to estimate the Spectral SRI λ_{max} given in Eqs. (9) and (10).

6 Network analysis of the core global banking system (CGBS)

Based on the methodology introduced in Sect. 3, we explore the evolution of the network topology of the core global banking system. For this analysis, we have chosen three specific periods: Fig. 2a represents the pre-crisis period (2006Q1); Fig. 2b depicts the crisis period (2009Q1), while Fig. 2c, shows the network topology after the crisis (2013Q1).

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[19] In general, for financial contagion analysis, we recommend the use of an absolute Tier 1 capital loss threshold that is equivalent to reported Tier 1 risk weighted capital ratio. The reason for this is that it is well known that the risk weighted assets in the denominator of the Tier 1 ratio is not reliable as it can be 'gamed' (Leslé and Avramova 2012). Indeed, as will be shown, seemingly healthy Tier 1 ratios of 8% can imply very large equivalent absolute capital loss thresholds of 50%. As no bank or banking system can survive with such a large absolute percentage loss of capital, we recommend that a 30% absolute capital loss threshold is a more realistic criteria for distress.
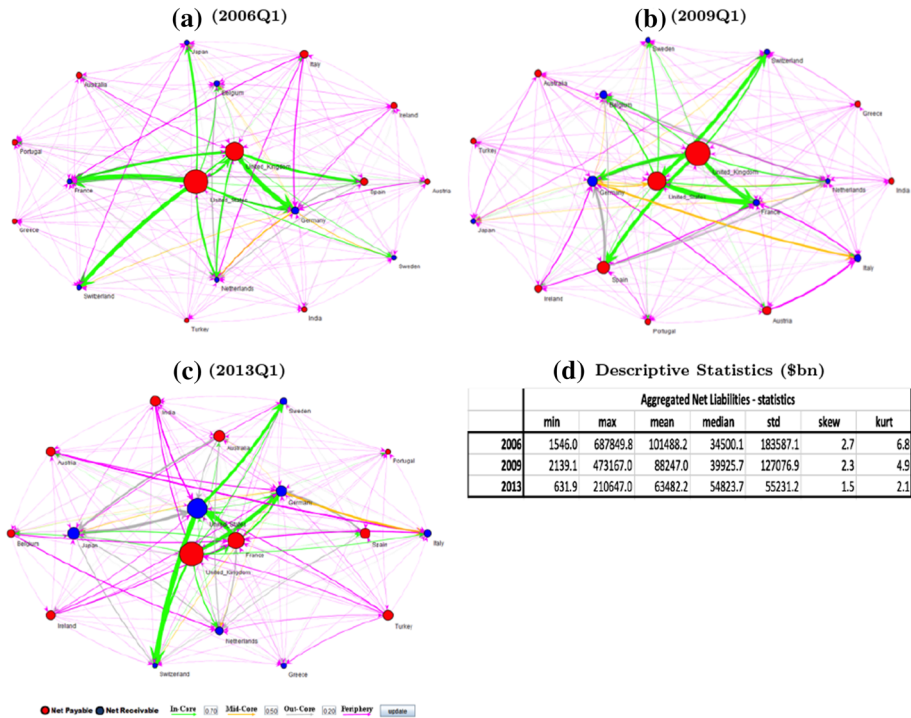
Fig. 2 CGBS network topology. Direction of arrows goes from borrowing banking system to lending banking system of the 18 BIS Reporting Countries. *Note*: Net payables are coded in red. Net receivables are coded in blue. Authors calculations based on the BIS data. The tiered layout in these graphs is constructed according to the net positions whether borrowing or lending. Tiering is based on a ratio of the net position of a country's banking system with that for the one with the highest net position. Those in the top 70 percentile of this ratio belong to the inner core; those in between 70 and 30 percentile are in the mid-core; and the out-core lies between 30 and 10 percentile; those with the lowest ratio belong to the periphery. The links are colour coded based on their tier membership of the originating node (see legend). *Source*: See Fig. 1

These figures represent the network of banking systems of 18 reporting countries, with links representing the net cross-border liabilities. The size of the node is proportional to the total netted position. Blue nodes are net lenders, while red nodes are net borrowers. The links are weighted and directed from the net borrower to the net lender. The thicker the edge, the larger the cross-border net liabilities.

Before the crisis, in Fig. 1a the US is followed by UK in net payables, borrowing heavily from Switzerland, France, Germany and Japan. After the crisis, Fig. 1b, c, US is no longer among the top net borrowers and is overtaken by UK and the GIIPS. As we can see, France started as a net lender and after the Eurozone crisis became the second largest net borrower.

During 2006Q1, the largest net lender is Germany followed by France and the Netherlands. During the crisis France and Switzerland become the most exposed to US and UK debt, while Germany and Belgium occupy the second and third positions, respectively. The Belgian banking system suffered huge losses during 2008 when Fortis group was sold to BNP Paribas and in 2011 Dexia group was dismantled.

An interesting observation is that the GIIPS countries did not increase their overall borrowing from 2006Q1 to 2009Q1, but instead they changed the composition of their lenders.
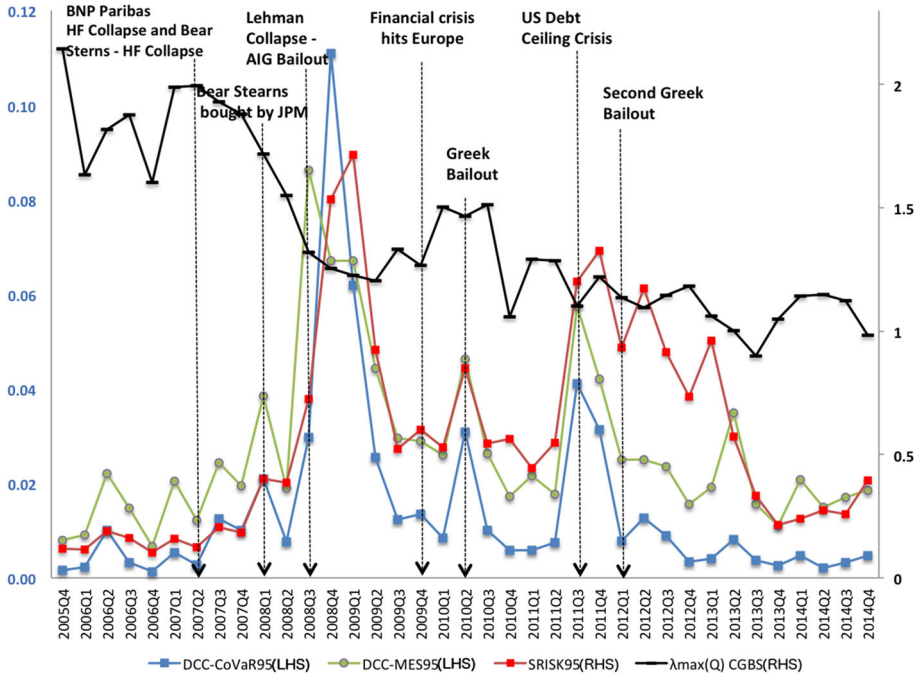
Fig. 3 Systemic Risk Indexes (SRI): Spectral SRI Maximum Eigenvalue $\lambda_{max}(Q)$ (RHS axis) of the Core Global Banking System Network versus DCC-ΔCoVaR (LHS axis), DCC-MES (LHS axis) and SRISK (RHS axis) during the period 2005Q4–2014Q4

In fact, during 2009Q1, GIIPS countries borrowed heavily from Non-GIIPS Eurozone countries, as in the case of the strong borrowing by Italy from France and by Germany and Spain from UK. This effect vanishes as we approach 2013Q1, where the non-GIIPS Eurozone countries decreased their overall lending and their exposure to GIIPS countries.

The only banking systems that belong to the inner-core tier in Fig. 2 in all three periods are the US and UK. We note that France joins US and UK in the in-core as second largest net borrower by 2013.

The mid-core tier in Fig. 2 is populated initially only by Germany, joined by France during the crisis in 2009. The outer-core banking systems are mostly Belgium, Spain, and the Netherlands. From the GIIPS, we observe only Spain in the out-core as net borrower, whereas the remaining GIIPS country banking systems occupy the periphery.

We also report the weighted clustering coefficient in the Online C as a key network statistic for the core global banking network which has been identified in Sect. 1 as giving early warning for high counterparty risk.

7 Spectral and market price-based SRIs

The level of instability of the Core Global Banking System Network (CGBSN), reflected by the maximum eigenvalue (λ_{max}) of matrix **Q** given in (9) and (10) is depicted in Fig. 3 as a dark line (right axis). As we can see, $\lambda_{max}(Q)$ is already high (at about 2.3) before the crisis started at the end of 2005, and never falling below 1.5 till 2008Q1 and showing a peak at 1.8

at the starting point of the 2007 crisis. The point here is that Spectral SRI estimated from a higher order power iteration of matrix **Q** in (9) shows a unstable level of interconnectedness through out the sample period based on the large net cross border exposures of banking systems relative to the adjusted Tier 1 capital and heterogeneous capital loss thresholds given in (36). The extremely high value of $\lambda_{max}(Q)$, estimating capital losses of over 150% for the core global banking network before 2006, provides reasonable early warning for the crisis that was about to come, with a peak corresponding to the collapse of two Bear Stearns hedge funds followed by that of two BNP Paribas hedge funds in 2007Q3, which were highly exposed to subprime mortgage backed securities. After the Subprime Crisis unraveled, λ_{max} starts decreasing throughout 2008 to 1.3 reflecting the drastic fall in cross border liabilities of the US. The λ_{max} starts picking up again from 2009Q3, when the crisis had transmuted into the Eurozone sovereign debt crisis and remained high until 2010Q3. Since $\lambda_{max}(Q)$, which signals system instability, includes both counterparty interconnectedness and also weakness from own capital inadequacy vis-à-vis net exposures to debtor banking systems, the high $\lambda_{max}(Q)$ in 2010 reflects the case of Greek banks being close to insolvency. This is partially addressed with the first Greek bailout in 2010 Q3 (€110bn in May 2010). The situation deteriorates again leading to a second Greek bailout in February 2012, Arghyrou and Kontonikas (2012).

We note that although we observe fluctuations of λ_{max}, the CGBSN has always been unstable except briefly in 2013 Q3 and approaching 2014 Q4, as only at these points has it been slightly below the tipping point threshold of 1 given in Eq. (10). The two cases in 2014, in particular, where our Eigen-pair R number based analyses showed up the systemic risk when other methods to failed to do so are that of Portugal and Italy (see Sect. 8.2). In the Online B Table B.4 we give an approximation of the percentage of unadjusted total Tier 1 capital of the cross border banks in the BIS core global banking systems that can potentially be lost from contagion risk. The extreme λ_{max} in Fig. 3 signalling losses of over 200% of the adjusted Tier 1 capital associated with the cross border banking system exposures, is calculated to be between 50% to around 60% of total (unadjusted) Tier 1 capital of the CGBSN in the period 2005Q4 to 2008Q1. Such losses were experienced in the wake of the GFC. In contrast, the other statistical SRIs were just contemporaneous with the crisis and provided no early warning signals for the financial losses that occurred.

In Fig. 3, we can see that DCC-MES spikes with the Lehman collapse, while the same happens for DCC-ΔCoVaR and for SRISK after this event. By 2009 Q4, the three market price based SRIs subside to pre GFC levels, when in contrast, the spectral SRI picks up in anticipation of the Greek crisis. The market based SRIs peak with the Greek bailout showing no sign of early warning. Further, note the added information in the post 2008 period in the higher order interconnectedness being analyzed in the Spectral SRI which is over and above the second order (neighbour's neighbours) based clustering coefficient for the CGBSN in Figure C.10.

The market based SRIs also seem to coincide with subsequent key events in financial markets, such as the US Debt Ceiling Crisis (2011Q3) and the second Greek bailout. The marked feature of the spectral SRI is its countercyclicality with market volatility and shows high systemic risk with growing bank liabilities and leverage. SRISK in combining leverage data with that of market prices, at least in the period of 2011 Q3 to 2013 Q2, shows similar high systemic risk as the spectral SRI, unlike the pure market based SRIs.

In Table 1, we provide specially constructed T-Statistics to determine the statistical significance for the early warning capabilities of the four SRIs shown in Fig. 3 for the period 2005–2014. We use the notion of 'abnormal' deviations of the SRIs from a normal non-crisis reference value similar to the determination of abnormal returns in event studies (see Elad and

Table 1 T-statistics of the Systemic Risk Indexes. Positive values at 1% significance are reported in bold

Test period	T- statistics			
	$\lambda_{max}(Q)CGBS$	DCC-ΔCoVaR	SRISK	DCC-MES
2005Q4	**22.36**	−16.28	−0.39	−6.02
2006Q1	**12.52**	−6.50	−0.38	−3.51
2006Q2	**16.26**	**6.35**	0.80	**4.93**
2006Q3	**17.54**	−1.89	0.37	2.09
2006Q4	**11.68**	−22.44	−0.67	−11.60
2007Q1	**19.58**	**4.42**	0.35	**7.02**
2007Q2	**19.94**	−5.39	−0.23	0.18
2007Q3	**18.56**	**9.27**	1.08	**6.18**
2007Q4	**18.75**	**7.13**	0.72	**5.14**
2008Q1	**14.79**	**20.07**	**3.28**	**14.61**
2008Q2	**11.30**	**6.03**	**3.14**	**5.80**
2008Q3	**6.35**	**8.76**	**4.81**	**9.67**
2008Q4	**5.02**	**11.90**	**4.89**	**9.92**
2009Q1	**4.74**	**8.30**	**3.72**	**6.48**
2009Q2	**4.23**	**7.10**	**4.47**	**5.97**
2009Q3	**7.04**	**5.56**	**3.70**	**4.86**
2009Q4	**5.72**	**5.92**	**3.24**	**5.13**
2010Q1	**11.91**	**4.95**	**3.19**	**4.61**
2010Q2	**11.37**	**10.36**	**5.26**	**7.83**
2010Q3	**13.22**	**5.10**	**3.92**	**4.98**
2010Q4	1.33	**2.91**	**3.62**	**2.93**
2011Q1	**6.79**	**4.31**	**2.76**	**5.85**
2011Q2	**6.97**	**4.59**	**3.66**	**3.99**
2011Q3	2.36	**10.28**	**4.76**	**9.64**
2011Q4	**5.56**	**7.43**	**3.44**	**6.62**
2012Q1	**3.50**	**4.34**	**2.72**	**4.99**
2012Q2	2.30	**5.71**	**3.10**	**4.18**
2012Q3	**3.78**	**4.35**	**2.91**	**4.14**
2012Q4	**4.94**	−1.20	1.95	2.07
2013Q1	1.37	0.17	1.88	**3.17**
2013Q2	0.05	2.55	**4.23**	**5.07**
2013Q3	−2.67	−0.82	**2.74**	**2.94**
2013Q4	1.32	−5.62	1.47	−1.08
2014Q1	**3.34**	1.32	1.70	**3.56**
2014Q2	**3.57**	−4.85	1.77	1.11
2014Q3	2.76	−0.88	1.43	1.43
2014Q4	−0.36	1.33	2.32	**2.79**

Reference values for the non-crisis period are the average of the first 5 quarters 2005Q4–2006Q4 for the DCC-ΔCoVaR, SRISK and DCC-MES while we use 1 as reference value for $\lambda_{max}(Q)$ as derived in Eq. (10). Standard deviations used in the t-statistics for the DCC-ΔCoVaR, SRISK and DCC-MES refer to the cross sectional standard deviation of countries SRIs in each quarter, while the standard deviation of the right eigenvector of Eq. (16) is used for $\lambda_{max}(Q)$. Note, the right eigenvector centrality for each of the 18 national banking systems is the measure of their systemic importance

Bongbee 2017; Mackinlay 1997).[20] The T-statistic measures the likelihood that the actual value of the index is different than a reference non-crisis value[21]. The larger the absolute deviations of the SRI over and above the reference value for SRI at each quarter adjusted for the relevant cross-sectional standard deviation, the higher the probability that the SRI can signal statistically significant values for instability in the system (Elad and Bongbee 2017; Mackinlay 1997). Note that with 18 cross-sectional observations per quarter and a degree of freedom of 17, the critical value at 99% significance is 2.57. The statistically significant values for the T-Statistics for the abnormal values of the SRIs at 99% are marked in red in Table 1. Test results for spectral SRI, $\lambda_{max}(Q)$, are positive and above the critical value since 2005Q4. In contrast, the DCC-Δ Co-VaR and DCC-MES, which both have similar time profiles from 2005 to 2012Q3, momentarily signaled two quarters of statistically significant 'abnormal' values before the GFC, but thereafter peaking contemporaneously with the crisis in 2007 Q3. SRISK shows the worst early warning capability of all indexes as its t-statistics are above the critical value only from 2008Q1.

An explanation for this is given by Benoit et al. (2013), who argue that the SRISK tends to depend on leverage during boom market conditions, while during crisis periods tends to depend on the beta of the country, as correlations increase while equity prices drop and the LRMES increases. However, for the sample analysed, the SRISK mainly behaves as a purely market price-based SRI except for the Eurozone crisis period. For further analysis, see the Online D of this paper.

We explore the degree of cyclicality of these market based SRIs, which we have argued signals their paradoxical nature that vitiates their capacity to give early warning for financial crisis. All SRIs are analyzed for their comovements with market volatility by computing the R^2's from a regression of the VIX index against the systemic risk measures at different time lags to gauge their explanatory power.[22] As documented by Adrian and Shin (2011b), standard risk management risk measures, such as the VaR of financial equity returns, fluctuate procyclically with measures of risk such as the implied volatility of traded options on banks' shares. This means, that financial intermediaries are deleveraging and withdrawing credit precisely when the financial system is under stress, serving to amplify the downturn. In Table 2, we report contemporaneous and lagged regression R^2 coefficients (for up to 2 lags) between the different SRIs and the VIX (Column 1). In addition, taking into account that most countries in our sample are European, we also report the R^2's with the VSTOXX (Column 2).[23] Finally, in Column 3, we report the R^2's of a regression of the TED spread on the systemic risk measures. The TED spread is the difference between the 3-month T-bill and the rate at which banks lend to each other on a 3-month period (measured by the Libor) and it is often used as a good proxy for interbank credit risk and perceived health of the banking system.

We calculate the R^2's based on 50,000 bootstrapped samples. We report the standard errors in parentheses. In Table 2 we can see that the market price-based SRIs are contemporaneous

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[20] An alternative approach to early warning signals can be found in Savona (2014).

[21] For all market price-based SRIs we use the first five quarters average 2005Q4–2006Q4 to calculate the reference value. The $\lambda_{max}(Q)$ reference value is 1 as derived in Eq. (10). The standard deviation used in the t-statistics for the market price-based SRIs refers to the cross sectional standard deviation of countries SRIs in each quarter (Elad and Bongbee 2017), while our spectral SRI uses the standard deviation of the values for the right eigenvector of Eq. (16).

[22] The VIX is the acronym for the Chicago Board Options Exchange Volatility Index. This is often used as a good proxy for international risk aversion. A high value of the VIX corresponds to a more volatile market.

[23] This is the European counterpart of the VIX, based on the EURO STOXX 50 realtime options prices and is designed to reflect the market expectations of near-term up to long-term volatility.

Table 2 Bootstrapped R^2's from a regression of the VIX, VSTOXX and TED spread (taken as the last day of every quarter) on the SRIs

Systemic risk measures	VIX (1)	VSTOXX (2)	TED Spread (3)
$\lambda_t^{max} CGBS$	0.119	**0.211**∗∗	0.104
	(0.080)	(0.103)	(0.108)
$\lambda_{t-1}^{max} CGBS$	0.055	0.113	0.200
	(0.055)	(0.086)	(0.124)
$\lambda_{t-2}^{max} CGBS$	0.035	0.079	**0.242**∗∗
	(0.041)	(0.072)	(0.134)
$DCC - MES_t$	**0.820**∗∗∗	0.828∗∗∗	0.273∗
	(0.060)	(0.056)	(0.184)
$DCC - MES_{t-1}$	0.370∗∗∗	0.333∗	0.036
	(0.194)	(0.193)	(0.060)
$DCC - MES_{t-2}$	0.147∗	0.147∗	0.033
	(0.136)	(0.129)	(0.049)
$DCC - \Delta CoVaR_t$	**0.784**∗∗∗	0.720∗∗∗	0.152∗
	(0.063)	(0.078)	(0.091)
$DCC - \Delta CoVaR_{t-1}$	0.294∗∗	0.265∗∗	0.026
	(0.162)	(0.147)	(0.044)
$DCC - \Delta CoVaR_{t-2}$	0.058	0.074	0.025
	(0.060)	(0.067)	(0.030)
$SRISK_t$	**0.510**∗∗∗	**0.646**∗∗∗	0.025
	(0.146)	(0.100)	(0.040)
$SRISK_{t-1}$	0.133	0.194	0.087
	(0.103)	(0.120)	(0.064)
$SRISK_{t-2}$	0.018	0.031	**0.227**∗∗∗
	(0.028)	(0.043)	(0.081)

DCC-MES, DCC-ΔCoVaR and SRISK are calculated at a 5% tail. Standard errors in parentheses. *Significance at 10%; **Significance at 5%; ***Significance at 1%. Own calculations

with the VIX/VSTOXX, with R^2's of 80% in the case of DCC-MES and over 70% for DCC-ΔCoVar at time t and this falls off greatly as the number of lags increases. The λ_{max} SRI exhibits the lowest contemporaneous explanatory power with the VSTOXX (21%). Market price-based SRIs lose their explanatory power with time lags and in some cases, they even lose significance after the contemporaneous period, as the case of the DCC-ΔCoVaR. Another interesting finding can be seen in Column 3, where λ_{max} and SRISK exhibits a low but significant explanatory power with respect to the TED spread at time lag 2, thus giving some advance signals for the potential tightening of credit conditions in the interbank market. The rest of the market price-based SRIs are not able to reflect this phenomenon, not even contemporaneously.

(a) SYSTEMIC IMPORTANCE (Right Eigenvector Centrality, $V^{R\#}$)

(b) DCC-MES

(c) DCC-ΔCoVaR
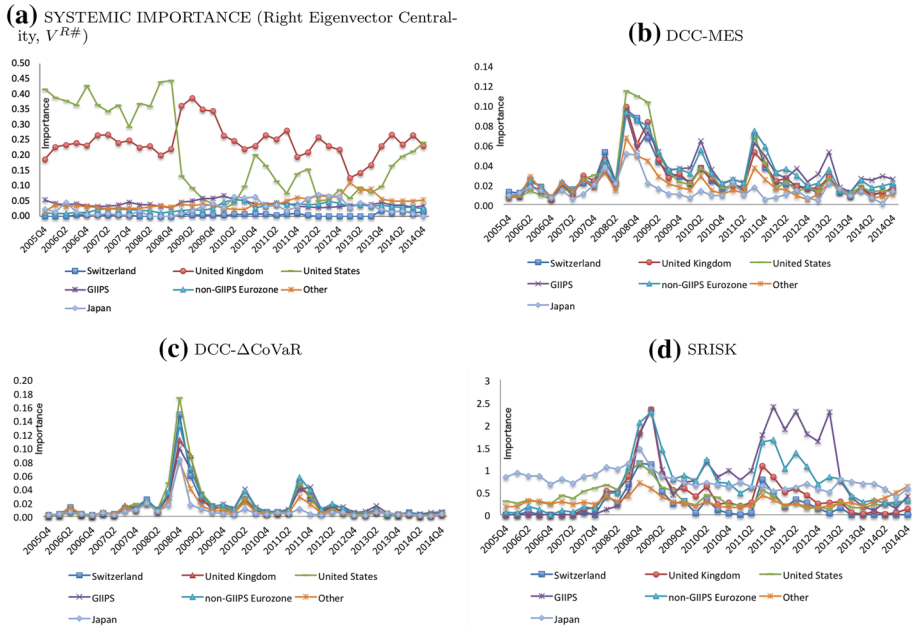
(d) SRISK

Fig. 4 Rankings of Systemic Importance for Core Europe, US, UK, Japan, Switzerland and average for other countries (2005Q4–2014Q4). *Note*: Estimations based on Right Eigenvector Centrality ($V^{R\#}$), DCC-MES, DCC-CoVaR and SRISK. GIIPS: Portugal, Italy, Ireland, Greece, Spain; Eurozone non-GIIPS: Germany, France, the Netherlands, Belgium, Austria; Other: Australia, India, Sweden, Turkey

8 Systemic risk importance and vulnerability in the core global banking system

As derived in Sect. 3 above, the eigen-pair method not only delivers an index for systemic stability, based on the maximum eigenvalue (λ_{max}) of the generalised stability matrix (Q), but also delivers simultaneously its associated right ($V^{R\#}$) and left ($V^{L\#}$) eigenvectors. The right eigenvector centrality gives a cardinal ranking, see Eq. (17), of the systemically important banking systems according to their capacity to cause loss of capital by their potential failure. The left eigenvector can give the rank order of banking systems' vulnerability to contagion and the percentage loss of capital they can suffer as implied by Eq. (12).

The *Right Eigenvector Centrality* ($V^{R\#}$) for the major economies and blocs in our sample is depicted in Fig. 4a. Here we observe the large systemic threat posed by the US in the run up to the GFC. The drastic reversal of the US in terms of being a net borrower in the CGBS is clear from Fig. 4a as the UK banking system assumes top systemic importance from 2008 Q4. In 2014 Q4 the right network centrality of the US begins to overtake that of the UK. Compared to the high centrality measures for the US and UK, other banking systems do not pose much risks to the system.

In the case of DCC-MES and DCC-ΔCoVaR, Fig. 4a, b, there is no significant difference between the banking systems for their systemic importance and the time profiles of the latter simply mimic the respective SRIs in Fig. 3. This is because these SRIs are an average of individual banking systems quantile risk measures. As the SRISK SRI includes leverage, there are considerable differences in the time profiles of systemic importance across banking
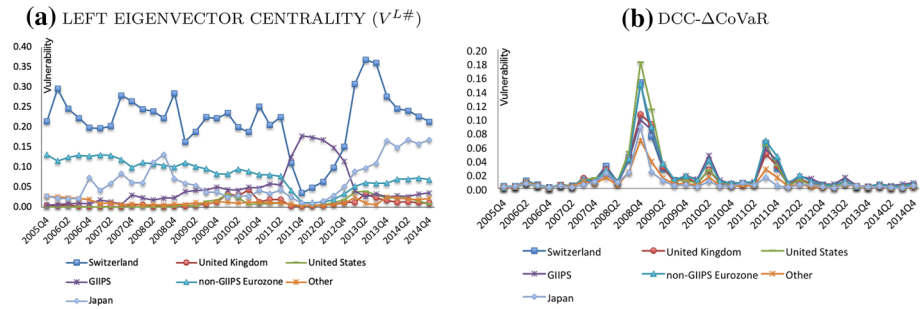
Fig. 5 Rankings of Systemic Vulnerability for US, UK, Japan, Switzerland and averages for GIIPS, non-GIIPS Eurozone and other countries (2005Q4–2014Q4)

systems. Interestingly, as SRISK includes stock market prices in Fig. 4d, it ends up placing the US only a poor second in terms of systemic importance in the run up to the GFC. Indeed, SRISK ranks Japan as the most systemically important in the pre GFC period. During the Lehman collapse, the UK and Eurozone banking systems become systemically important. During the Eurozone crisis, the GIIPS dominate in terms of their SRISK in Fig. 4d.

While the market based SRIs can give some ranking of systemic importance of banking systems, only ΔCoVaR can give us a ranking for those that are susceptible to these potential losses. In Fig. 5 we plot the *Left Eigenvector Centrality* ($V^{L\#}$, Panel (a)), while in Panel (b), we plot the vulnerability rankings given by the DCC-ΔCoVaR.[24] In Fig. 5a, we can see a different story to the one shown in Fig. 4a. Switzerland is the most systemically vulnerable banking system in the CGBSN almost throughout the whole sample period with the exception of the Eurozone crisis. The non-GIIPS Eurozone countries rank second until 2011Q2, when the vulnerability of GIIPS becomes prominent. Japan is ranked the third most vulnerable banking system, after non-GIIPS Eurozone bloc except during the Eurozone crisis and after this Japan ranks second after Switzerland.

In Fig. 5b the DCC-ΔCoVaR shows very little difference in the vulnerability ranking of banking systems with the exception of the period of the Lehman collapse and to a lesser extent during the two Greek bailout events. During the Lehman collapse, DCC-ΔCoVaR shows the US to be the most vulnerable, followed by Switzerland, Eurozone non-GIIPS and UK.

8.1 Systemic importance and vulnerability of core Eurozone countries

In this section, we look more closely at the Eurozone crisis. We start with a subset of Fig. 4 that covers the core Eurozone banking systems. For this subset, the first main result for the *Right Eigenvector Centrality* ($V^{R\#}$) measure for systemic importance in Fig. 6a shows that the potential contributions to capital loss is dominated by French banks and followed to a lesser extent by German banks. This problem is post GFC and relates to the Eurozone crisis. In fact, the potential for the French banks to trigger contagion losses increased sharply in the midst of the Eurozone Crisis.

The *Left Eigenvector Centrality* ($V^{L\#}$), which measures the vulnerability of core Eurozone banking systems, see Fig. 7a, shows that Dutch and Belgian banks are the most vulnerable

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[24] Vulnerability, in a ΔCoVaR context, is defined as the VaR of the country, conditional on the VaR of the market (Adrian and Brunnermeier 2011).
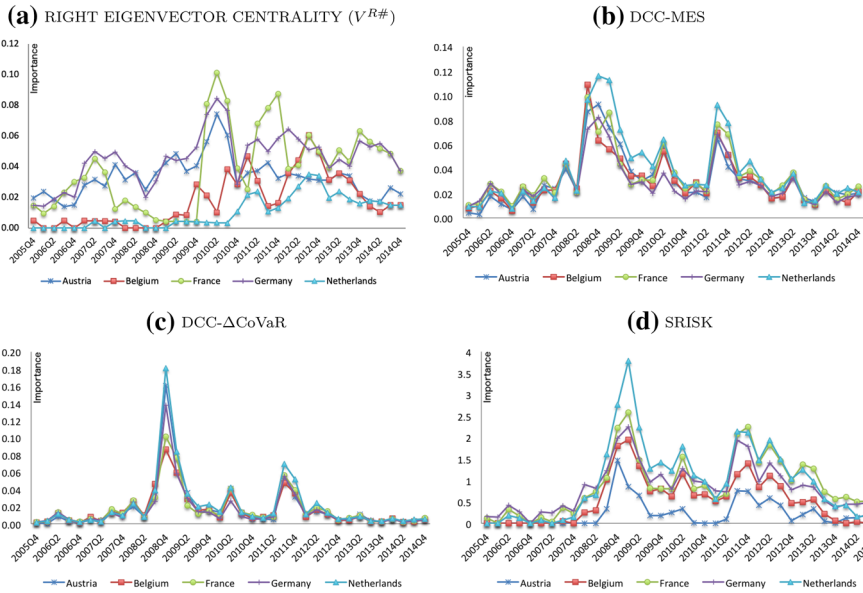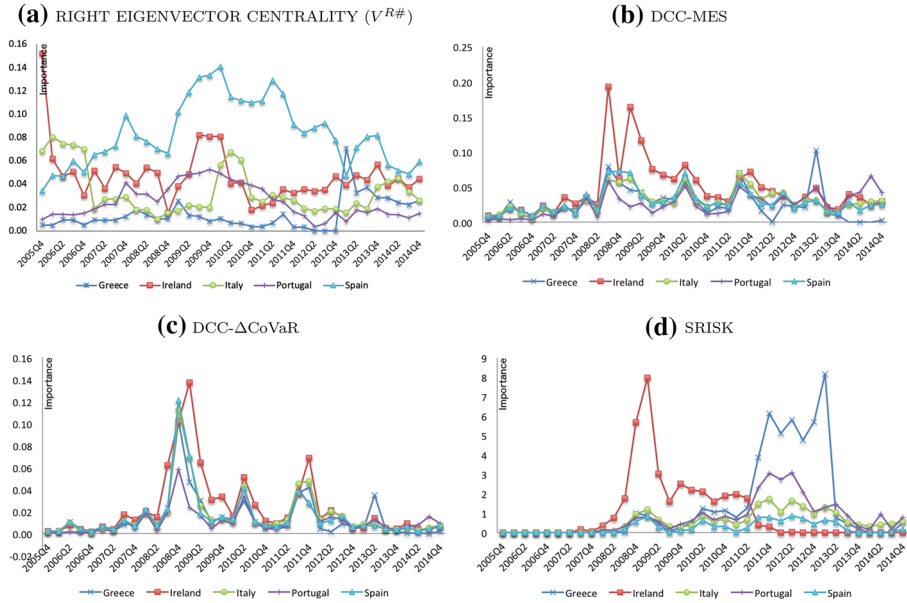
Fig. 6 Rankings of systemic importance for core Eurozone countries (2005Q4–2014Q4) . *Source*: Own calculations
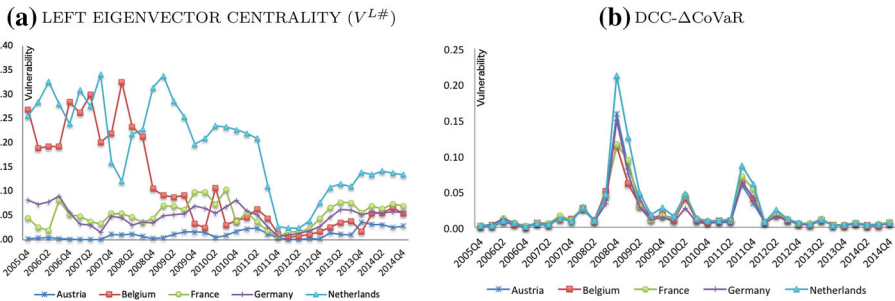
Fig. 7 Rankings of systemic vulnerability for core Eurozone countries (2005Q4–2014Q4) . *Source*: Own calculations

and their problems strongly predate the GFC. During this period, two financial companies, Fortis NV and ABN AMRO, were experiencing great difficulties due to the US subprime exposure. Prior to its collapse in 2008, Fortis was the largest financial services company in Belgium, named as Fortis SA/NV, and in the Netherlands, named as Fortis NV. On the 13th of October 2008, the Belgian government designed and approved a mechanism to compensate certain categories of Fortis shareholders. In the same month, the Dutch government became involved and bailed-out the bank. This event is reflected in Fig. 7a, where the Left Eigenvector Centrality of Belgian banking system almost halves (from 0.70 to 0.50 from 2007Q4–2008Q4). The Dutch left centrality index almost doubles (from 0.5 to 0.84 in 2009 Q2) due to repercussions from the UK bailout of RBS which had acquired the Dutch bank ABN AMRO. The subsequent recapitalization of core Eurozone banks reduced their vulnerability indexes. We finally note that after 2011 Q4, the vulnerability of Dutch banking system increases from a low point.
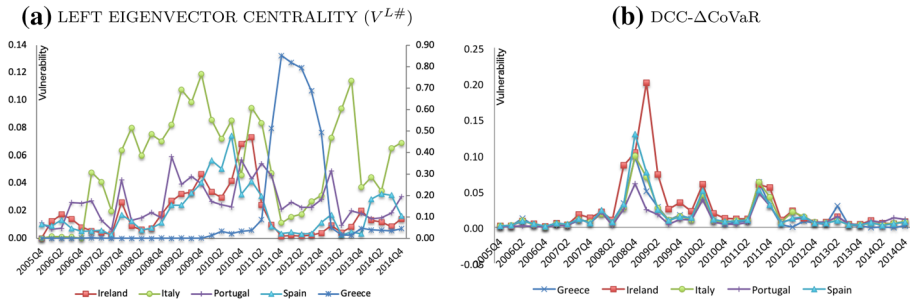
(a) RIGHT EIGENVECTOR CENTRALITY ($V^{R\#}$)

(b) DCC-MES

(c) DCC-ΔCoVaR

(d) SRISK

Fig. 8 Rankings of systemic importance for Eurozone periphery countries (2005Q4–2014Q4) . *Source*: Own calculations

The DCC-MES and DCC-ΔCoVaR based systemic importance rankings for this subset of banking systems in Fig. 6b, c remain relatively low and homogeneous in the run up to the GFC. In Fig. 6a, b, they start picking up during 2008Q2, reaching their maximum during 2008Q4, with the exception of Belgium, which peaks in 2008Q3. After that, they only react in the face of critical market events, such as the Greek bailout during 2010Q2, the growing uncertainty over Greece staying in the Eurozone (2011Q4) and the second Greek bailout (2012Q1). The market based systemic importance or vulnerability measures of individual banking systems are entirely contemporaneous with the three main market shocks, viz. the Lehman collapse and the Greek bailouts. In particular in Fig. 7b, DCC-ΔCoVaR, unlike the left eigenvector centrality ranking, gives no early warning of the weakness in the case of Netherlands and Belgium in the pre GFC period.

8.2 Systemic importance and vulnerability of Eurozone periphery countries

Figure 8 illustrates the contributions to the SRIs by the individual GIIPS. In Fig. 8a we plot the *Right Eigenvector Centrality* ($V^{R\#}$), which shows that, on average, the systemic importance of the GIIPS was higher than that of the non-GIIPS Eurozone banking systems that was shown in Fig. 6a. When compared with the right centrality indexes of France and Germany, Spain can potentially inflict a higher percentage of capital loss in the system by its failure. In 2009 Q4 and 2010 Q4, Ireland and Italy respectively, pose some threats, but to a much lesser extent than Spain. Only in 2013 Q1 does Greece dominate in terms of systemic importance among the GIIPS.

With more than 400% growth of cross-border banking liabilities of countries such as Spain and Ireland between 2003 and 2008 (Bruno and Shin 2014), it is not surprising that

(a) LEFT EIGENVECTOR CENTRALITY ($V^{L\#}$)   **(b)** DCC-ΔCoVaR

Fig. 9 Rankings of systemic vulnerability for Eurozone periphery countries (2005Q4–2014Q4). *Note*: Greece's left eigenvector centrality is on the right axis . *Source*: Own calculations

they become systemically important as seen from their *Right Eigenvector Centrality* measure in Fig. 8a.

Now, focusing on the *Left Eigenvector Centrality* ($V^{L\#}$) in Fig. 9a, we note that the vulnerability of Italy stands out as the greatest, both in the period around GFC and in the recent years of 2013/14. However, in the case of Greece during the period 2011Q3–2013Q1, its vulnerability index goes to one (right axis), indicating the dire insolvency of its banking system. Something that is worth noting is that following the IMF/EU bailout package for the Portuguese banking system during 2011 (Fig. 9a), there is a substantial reduction in its left eigenvector centrality. However, soon after, we see in Fig. 9a that its vulnerability increases again during 2013Q3 in the run up to the collapse of the major Portuguese Banco Espirito Santo in August 2014, generating fears of a second round of contagion in the Eurozone.[25] Note that, in general, the vulnerability index increases for banking systems when their bank equity capital is eroded by non-performing loans. In a similar vein, Italy's Left Eigenvector Centrality in Fig. 9a is able to capture the growing instability of its banking system in recent years. Three large-medium size banks failed at the end of 2015 (Banca Etruria, Cassa di Risparmio di Ferrara and Bancha delle Marche), along with many small banks and others in serious distress (i.e. Monte Dei Paschi di Siena). The spectral method shows up the great vulnerability of the Italian banking systems in recent years among the GIIPS, an event which is not anticipated by any of the other SRIs presented here.

In keeping with what has been previously noted, the market based DCC-MES and DCC-ΔCoVar SRIs for the individual GIIPS in Fig. 8b, c show substantial jumps only during the main shocks events. These SRIs rank Ireland as being the most systemically important with the exception of Greece during the Greek crisis and Portugal in the 2014 period. The SRISK in Fig. 8d reflects the leverage data and provides risk profiles for Greece and Portugal which are similar to the left centrality/vulnerability index of the spectral approach given in Fig. 9a.

8.3 What explains market price-based sris?

In this section we ask what are the possible reasons for the divergence in rankings of systemically important banking systems as given by the different SRIs. To answer this question, we follow Benoit et al. (2013) and calculate the R^2's from a time-series and cross-sectional

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[25] On the 10 August 2014, the Financial Times queried how the IMF Progress Report of January 2014 had given the Portugese banking system a clean bill of health just before Banco Espirito Santo collapsed in Portugal in August 2014, see Financial Times 10 August 2014: *http://www.ft.com/cms/s/0/2965c812-1e29-11e4-bb68-00144feabdc0.html*

regression of the SRIs presented above and some of the ingredients used to construct them, namely, the beta of the country's banking system, total book value of liabilities, value of equity and Value at Risk.[26] As pointed out by Benoit et al. (2013), under certain conditions, the DCC-MES and beta tend to identify the same systemically important financial institutions. This begs the question of why not rank institutions/countries by the betas in the first place? This relates to the confusion between *systemic* and *systematic* risk with the latter encapslated in the beta. Indeed, this can be seen in Table 3, where the R^2's shows an almost perfect cross-sectional correlation between the DCC-MES and beta and there is 85% R^2 for correlation between DCC-ΔCoVar and beta. Hence, if a ranking is done for systemic importance of country's financial systems using DCC-MES, this will be equivalent to ranking according to their betas.

Although the SRISK is a function of beta, it does not seem to be very sensitive to beta, in both the cross-section and time-series, with an R^2 of 0.189 and 0.249, respectively. The SRISK seems to be explained by leverage (LVG), with a cross-sectional R^2 of 0.812. Cross-sectional correlations are very important in this case, as these are the ones that provide the rankings of systemic importance at a given point in time. However, the relative impact of the beta and leverage factors on SRISK is strongly influenced by market conditions. Thus, as seen in Fig. 3 and also in Figs. 4, 5, 6, 7, 8 and 9, we note that there is a lack of early warning in the run up of the GFC. This is caused by the beta factor being greater than the one for leverage during conditions of market boom. Only in the post GFC period does the SRISK behave like the spectral SRI.[27]

Finally, in Table 3 we find that the DCC-ΔCoVaR is explained with a 80% R^2 in the time-series by its VaR. A high correlation in the time-series domain means that while rankings may differ, for a given country, DCC-ΔCoVaR is proportional to its VaR. This means that forecasting the future evolution of the contribution of a country to systemic risk is equivalent to forecasting its VaR in isolation. This provides evidence that most SRIs can be summarised by using standard risk measures such us VaR and beta. While we have made use of a very limited sample, our findings are in line with those studies using larger samples (Benoit et al. 2013).

9 Conclusions

One of the clear shortcomings of macro-prudential analyses has been the lack of systemic risk models that can, even after the event, identify the threats from large exposures (see Allen et al. 2011) that the global banking system had to US banking sector liabilities in the run up to the GFC. It is increasingly being understood, and as underscored in this study, that paradoxical systemic risk measures based on market price data will fail to give early warning of banking system crises from growing interbank liabilities. For this, network models based on balance sheet linkages are needed. However, despite a number of such network models that have been based on cross border BIS banking flows data involving global banking systems (Degryse et al. 2010; Castrén and Rancan 2014; Cerutti et al. 2012), these have had to rely on simulation methods to assess the average size of losses that the global banking network can suffer having triggered the failure of each of the participants. This approach has failed

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[26] When we estimate cross-sectional regressions, we regress cross-country SRIs against cross-country betas at each t. In contrast, in the time-series domain, we regress individual country SRIs against individual country beta *for that particular country*. In both cases, the R^2s are retrieved and subsequently averaged.

[27] This is seen in Engle et al. (2015) who find systemic risk given by SRISK is high even in the 2012/13 period for European financial institutions.

Table 3 R^2 statistics (average, minimum, maximum and standard deviation) obtained from a regression of a systemic risk measure (MES, ΔCoVaR and SRISK, on firm characteristics, namely, equity capital (EQ), liabilities (LTQ), leverage (LVG), beta and VaR

MES	Time-series					Cross-section				
	EQ	LTQ + EQ	LVG	BETA	VaR	EQ	LTQ + EQ	LVG	BETA	VaR
Avg	0.065	0.126	0.327	0.631	0.508	0.042	0.041	0.163	**1.000**	0.387
SD	0.071	0.110	0.230	0.350	0.274	0.046	0.044	0.185	**0.000**	0.266
Max	0.295	0.376	0.678	1.000	0.874	0.183	0.185	0.603	**1.000**	0.852
Min	0.000	0.002	0.000	0.107	0.035	0.000	0.000	0.000	**1.000**	0.000
ΔCoVaR	EQ	LTQ + EQ	LVG	BETA	VAR	EQ	LTQ + EQ	LVG	BETA	VaR
Avg	0.033	0.076	0.324	0.064	**0.802**	0.040	0.035	0.149	0.859	0.341
SD	0.029	0.103	0.252	0.088	**0.247**	0.053	0.041	0.171	0.077	0.228
Max	0.088	0.426	0.736	0.319	**0.985**	0.266	0.194	0.578	0.966	0.869
Min	0.000	0.000	0.001	0.000	**0.057**	0.000	0.000	0.000	0.597	0.011
SRISK	EQ	LTQ+EQ	LVG	BETA	VAR	EQ	LTQ + EQ	LVG	BETA	VaR
Avg	0.132	0.230	**0.814**	0.248	0.656	0.063	0.072	**0.812**	0.189	0.356
SD	0.154	0.167	**0.209**	0.167	0.179	0.079	0.094	**0.172**	0.188	0.320
Max	0.546	0.482	**0.992**	0.614	0.866	0.326	0.344	**0.996**	0.654	0.925
Min	0.000	0.000	**0.284**	0.004	0.159	0.000	0.000	**0.444**	0.000	0.000

We report two types of regressions: time series regressions for each of the 18 countries and cross-sectional regressions for the 37 data points available (2005Q4–2014Q4). Bold figures indicate the explanatory variable with the highest R^2's. Authors calculations

to give a reliable metric for gauging if a data based network of financial obligations has become more or less prone to tipping into a state of contagion from one period to the next. The advances being made to the BIS Consolidated Banking Statistics since 2005 that gives banking sector liabilities of a country to the BIS reporting cross border banks of another country on an ultimate risk basis, makes it possible to develop a network model of foreign claims of a country's banking system on that of another.

The spectral and fixed point eigen-pair approach, widely used in the epidemic and cascade models, has been absent from many economic models of such phenomena. We have shown how a financial contagion model can produce an index analogous to the famous R number for epidemic models. We give a step by step guide, in a more general setting than in Markose (2012), on how the framework of Newman (2010, section 17) and Wang et al. (2003) can be adapted for tipping points for networks and then implemented for cross border exposures of national banking systems using publicly available BIS CBS.

Firstly, to apply the analytic fixed point eigen-pair results in Eqs. (9), (18), (19) and (20), we have discussed in Sect. 3.1 how the BIS CBS has to be adjusted in the cases when the banking sector breakdown of foreign bank liabilities of a country to the BIS reporting banks of another has not been given. As all contagion models have to specify thresholds for failure, much care was exercised in determining the heterogeneous absolute capital loss thresholds implied by the extant Basel regulatory capital requirement that signals distress. When assessing the potential for contagion failures from capital inadequacy for what is effectively a subset of activities of banks, it is recommended that pro rata adjustments for capital are done.

The Spectral SRI based on the maximum eigenvalue (λ_{max} ($\mathbf{Q}$)) given in Eqs. (9) and (15) for the core global banking system network (CGBSN) can be interpreted as the maximum expected percentage loss of capital exceeding the permissible capital buffer for the CGBSN from a contagion that may arise from arbitrary sized shocks. In Fig. 3 we find that λ_{max} ($\mathbf{Q}$) manifests extreme contagion losses of over 50% of the total adjusted capital in the CGBSN in the run up to the 2007 GFC and there are also similar uptrends before the Eurozone crisis. The right eigenvector centrality has been shown to give the near term proportionate (to the λ_{max} ($\mathbf{Q}$)) capital losses that each banking system can inflict on the CGBSN, giving its systemic importance. Likewise, the product of the λ_{max} ($\mathbf{Q}$) and the left eigenvector centrality gives the share of the total systemic losses (as percentage) of CGBSN adjusted capital that each of them can lose signaling their vulnerability. We have noted that sudden changes in the rank order of these network centrality indexes with respect to the top rank should be taken seriously by regulators as it can identify new 'super spreaders' and those that are vulnerable to failure. The difference in the time profiles of the weighted clustering coefficient and the Spectral SRI in Figure C.10 and Fig. 3 respectively, shows the richer information especially in the higher order approximation of interconnectedness in the latter as opposed to the former.

Another objective of the paper is to compare the results of the Spectral SRI with those for the standard market price-based SRIs, namely, the DCC-MES of Acharya et al. (2017), the SRISK of Acharya et al. (2012) and the DCC-ΔCoVaR of Adrian and Brunnermeier (2011), in a cross border setting. Our main finding is that the market price based measures exhibit an extremely high contemporaneous correlation with market sentiment measures such as the VIX/VSTOXX, which results in little early warning capabilities. Likewise, most of them tend to spike perfectly with market events such as the Lehman collapse and two Greek bailouts and only during those times are they able to identify systemically important financial systems. The overwhelming systemic importance of the US banking system pre GFC and the subsequent systemic importance of the UK banking system found in the Spectral SRI analysis in Fig. 4a do not feature in the market based SRI results. Indeed, only DCC-ΔCoVaR makes

a distinction between systemic important and vulnerable banking systems. Within subsets of banking systems, while the Spectral SRI identifies the vulnerability of the Dutch and Belgian banking systems to predate the 2007 GFC and to highlight the continued weakness of the Italian banking system especially in 2014, the market based SRIs are silent in this respect. Finally, it is surprising that SRISK, which combines leverage with market price-based data is not able to capture the increasing instability of the core global banking system pre GFC. An explanation for this, is that while leverage is increasing in the run up to the crisis, the reliance on the market price determined long run marginal expected shortfall (LRMES) term nullifies this effect and prevents SRISK from giving any early warning.

The present study demonstrates the importance of the informational content of cross-border balance sheet BIS CBS data aggregated at the level of the national banking system in a network setting for assessing global banking stability. It is not only the severe threats from the US banking system that was signalled pre GFC, but also less obvious vulnerability of the Belgian cross border banks in this period and that of the Portuguese banking system in 2012/2013. Future research should widen the scope of this methodology by including financial exposures of sectors within countries and between sectors from different countries, as in the Castrén and Rancan (2014) framework. This will require additional modeling of how failure propagates in non-banking sectors.

References

Acharya, V., Engle, R., & Richardson, M. (2012). Capital shortfall: A new approach to ranking and regulating systemic risks. *American Economic Review, 102*(3), 59–64.

Acharya, V., Pedersen, L., Philippon, T., & Richardson, M. (2017). Measuring systemic risk. *Review of Financial Studies, 1,* 2–47.

Acharya, V. V., & Stefen, S. (2012). *Analyzing systemic risk of the European banking sector. Handbook on systemic risk*. Cambridge: Cambridge University Press.

Adrian, T., & Brunnermeier, M. (2011). CoVaR. Working Paper, Princeton University and Federal Reserve Bank of New York.

Adrian, T., & Shin, H. (2010). Liquidity and leverage. *Journal of Financial Intermediation, 19*(3), 418–37.

Adrian, T., & Shin, H. (2011a). Financial intermediary balance sheet management. *Annual Reviews in Financial Economics, 3,* 289–307.

Adrian, T., & Shin, H. S. (2011b). Procyclical leverage and value-at-risk. FRB of New York Staff Report (338).

Allen, F., Beck, T., Carletti, E., Lane, P., Schoenmaker, D., & Wolf, W. (2011). Cross-border banking in Europe: Implications for financial stability and macroeconomic policies. Centre for Economic Policy Research.

Alter, A., Craig, B., & Raupach, P. (2014). Centrality-based capital allocations and bailout. *IMF Working Paper, 14*(237), 1–40.

Anand, K., Craig, B., & von Peter, G. (2015). Filling in the blanks: Network structure and interbank contagion. *Quantitative Finance, 15*(4), 625–636.

Arghyrou, M., & Kontonikas, A. (2012). The EMU sovereign-debt crisis: Fundamentals, expectations and contagion. *Journal of International Financial Markets, Institutions and Money, 22*(4), 658–677.

Arsov, I., Canetti, E., Kodres, E., & Mitra, S. (2013). "Near-Coincident" Indicators of Systemic Stress. IMF Working Paper WP/12/115.

Avdjiev, S., McGuire, P., & Wooldridge, P. (2015). Enhanced data to analyse international banking. *BIS Quarterly Review*.

Bardoscia, M., Battiston, S., Caccioli, F., & Caldarelli, G. (2017). Pathways towards instability in financial networks. *Nature Communications, 8,* 1–7. https://doi.org/10.1038/ncomms14416.

Battiston, S., Caldarelli, G., D'Errico, M., & Gurciullo, S. (2016). Leveraging the network: A stress-test framework based on debtrank. Available at SSRN 2571218.

BCBS. (2013). Global systemically important banks: Updated assessment methodology and the higher loss. Bank for International Settlements.

Benoit, S., Colletaz, G., Hurlin, C., & Perignon, C. (2013). A theoretical and empirical comparison of systemic risk measures. *HEC Paris Research Paper No. FIN-2014-1030*.

Benoit, S., Colliard, J.-E., Hurlin, C., & Christophe, P. (2016). Where the risks lie: A survey on systemic risk. *Review of Finance, 20,* 1–59.

Billio, M., Getmansky, M., Lo, A. W., & Pelizzon, L. (2012). Econometric measures of connectedness and systemic risk in the finance and insurance sectors. *Journal of Financial Economics, 104*(3), 535–559.

Borio, C., & Drehmann, M. (2011). Toward an operational framework for financial stability: "fuzzy" measurement and its consequences: Ch. 04. In R. Alfaro (Ed.), *Financial stability, monetary policy, and central banking* (1st ed., Vol. 15, pp. 063–123). Chile: Central Bank of Chile.

Borio, C., Furfine, C., & Lowe, P. (2001). "Procyclicality of the financial system and financial stability: Issues and policy options," in marrying the macro- and micro- prudential dimensions of financial stability. *BIS Papers, 1,* 1–57.

Braouezec, Y., & Wagalath, L. (2019). Strategic fire-sales and price-mediated contagion in the banking system. *European Journal of Operational Research, 274,* 1180–1197.

Brownlees, C., & Engle, R. (2016). SRISK: A conditional capital shortfall index for systemic risk measurement. *Review of Financial Studies* (**forthcoming**).

Brunnermeier, M. K., & Cheridito, P. (2013). Measuring and allocating systemic risk. *Risk, 7,* 46.

Brunnermeier, M. K., & Sannikov, Y. (2015). International credit flows and pecuniary externalities. *American Economic Journal: Macroeconomics, 7*(1), 297–338.

Bruno, V., & Shin, H. (2014). Cross-border banking and global liquidity. Monetary and Economic Department, BIS Working Papers (458).

Bruno, V., & Shin, H. S. (2015a). Cross-border banking and global liquidity. *Review of Economic Studies, 82*(2), 535–564.

Bruno, V., & Shin, H. S. (2015b). Capital flows and the risk-taking channel of monetary policy. *Journal of Monetary Economics, 71,* 119–132.

Calabrese, R., Elkink, J. A., & Giudici, P. S. (2017). Measuring bank contagion in Europe using binary spatial regression models. *Journal of the Operational Research Society, 68*(12), 1503–1511.

Calabrese, R., & Osmetti, S. A. (2019). A new approach to measure systemic risk: A bivariate copula model for dependent censored data. *European Journal of Operational Research, 279*(3), 1053–1064. https://doi.org/10.1016/j.ejor.2019.06.027.

Castrén, O., & Rancan, M. (2014). Macro-networks: An application to euro area financial accounts. *Journal of Banking & Finance, 46,* 43–58.

Cerutti, E., Claessens, S., & McGuire, P. (2012). *Systemic risks in global banking: What available data can tell us and what more data are needed?* Technical report, National Bureau of Economic Research.

Chakrabarti, D., Wang, Y., Wang, C., Leskovec, J., & Faloutsos, C. (2008). Epidemic thresholds in real networks. *ACM Transactions on Information and System Security, 10*(4), 1:1-1:26.

Cont, R., Moussa, A., & Santos, E. B. (2012). Network structure and systemic risk in banking systems. In *Handbook of systemic risk*. Cambridge University Press.

Degryse, H., Elahi, M., & Penas, M. F. (2010). Cross-border exposures and financial contagion. *International Review of Finance, 10*(2), 209–240.

Diebold, F. X., & Yılmaz, K. (2014). On the network topology of variance decompositions: Measuring the connectedness of financial firms. *Journal of Econometrics, 182*(1), 119–134.

Eisenberg, L., & Noe, T. H. (2001). Systemic risk in financial systems. *Management Science, 47*(2), 236–249.

Elad, F., & Bongbee, N. (2017). Event study on the reaction of stock returns to acquisition news. *International Finance and Banking, 4*(1), 33–43.

Engle, R., Jondeau, E., & Rockinger, M. (2015). Systemic risk in Europe. *Review of Finance, 19*(1), 145–190.

Engle, R., & Sheppard, K. (2001). Theoretical and empirical properties of dynamic conditional correlation multivariate GARCH. Working Paper UCSD 15.

Fagiolo, G. (2007). Clustering in complex directed networks. *Physical Review E: Statistical, Nonlinear, and Soft Matter Physics, 76*(2), 1–8.

Feinstein, Z. (2020). Capital regulation under price impacts and dynamic financial contagion. *European Journal of Operational Research, 281*(2), 449–463. https://doi.org/10.1016/j.ejor.2019.08.044.

Furfine, C. H. (2003). Interbank exposures: Quantifying the risk of contagion. *Journal of Money, Credit and Banking, 35*(1), 111–128.

Gauthier, C., Lehar, A., & Souissi, M. (2012). Macroprudential capital requirements and systemic risk. *Journal of Financial Intermediation, 21*(4), 594–618.

Glasserman, P., & Young, H. P. (2015). How likely is contagion in financial networks? *Journal of Banking & Finance, 50,* 383–399.

Haldane, A. (2009). *Rethinking the financial network*. Speech delivered at the Financial Student Association, Amsterdam.

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica, 57,* 357–384.

Hattori, M., & Suda, Y. (2007). Developments in a cross-border bank exposure "network". In BIS (Ed.), *Research on global financial stability: The use of BIS international financial statistics*. CGFS Papers. Bank for International Settlements, vol. 29 (pp. 16–31).

Hautsch, N., Schaumburg, J., & Schienle, M. (2015). Financial network systemic risk contributions. *Review of Finance, 19*(2), 685–738.

Heath, A., Kelly, G., Manning, M., Markose, S., & Shaghaghi, A. R. (2016). CCPs and network stability in OTC derivatives markets. *Journal of Financial Stability, 27,* 217–233.

Heesterbeek, J. A. P., & Dietz, K. (1996). The concept of R_o in epidemic theory. *Statistica Neerlandica, 50*(1), 89–110.

IMF-BIS-FSB. (2009). Financial stability board international monetary fund-bank for international settlements (2009) guidance to assess the systemic importance of financial institutions, markets and instruments: Initial considerations.

Ipsen, I., & Wills, R. M. (2005). Analysis and computation of Google's pagerank. In *7th IMACS international symposium on iterative methods in scientific computing*. Toronto: Fields Institute.

Karolyi, G. A., Sedunov, J., & Taboada, A. G. (2017). Cross-border bank flows and systemic risk.

Leslé, V. L., & Avramova, S. (2012). Revisiting risk-weighted assets. IMF Working Papers WP/12/90.

Mackinlay, A. C. (1997). Event studies in economics and finance. *Journal of Economic Literature, 53,* 13–39.

Mahdavi Ardekani, A., Distinguin, I., & Tarazi, A. (2020). Do banks change their liquidity ratios based on network characteristics? *European Journal of Operational Research*. https://doi.org/10.1016/j.ejor.2020.02.011.

Markose, S., Giansante, S., & Shaghaghi, A. (2012). "Too interconnected to fail" financial network of US CDS market: Topological fragility and systemic risk. *Journal of Economic Behavior and Organization, 83*(3), 627–646.

Markose, S., Giansante, S., & Shaghaghi, A. R. (2017). A systemic risk assessment of OTC derivatives reforms and skin-in-the-game for CCPs. *Banque de France Financial Stability Review, 21*(April), 111–126.

Markose, S. M. (2012). Systemic risk from global financial derivatives: A network analysis of contagion and its mitigation with super-spreader tax. IMF Working Paper (12/282).

Markose, S. M. (2013). Systemic risk analytics: A data-driven multi-agent financial network (MAFN) approach. *Journal of Banking Regulation, 14*(3–4), 285–305.

May, R. M. (1972). Will a large complex system be stable? *Nature, 238*(5364), 413–414.

May, R. M. (1974). *Stability and complexity in model ecosystems*. Princeton: Princeton University Press.

Minoiu, C., Kang, C., Subrahmanian, V., & Berea, A. (2013). Does financial connectedness predict crises? IMF Working Papers WP/13/267.

Minoiu, C., & Reyes, J. A. (2013). A network analysis of global banking: 1978–2009. *Journal of Financial Stability, 9*(2), 168–184.

Minsky, H. (1986). *Stabilizing an unstable economy*. Yale: Yale University Press.

Mistrulli, P. E. (2011). Assessing financial contagion in the interbank market: Maximum entropy versus observed interbank lending patterns. *Journal of Banking & Finance, 35*(5), 1114–1127.

Newman, M. (2010). *Networks: An introduction*. Oxford: Oxford University Press.

Page, L., & Brin, S. (1998). The anatomy of a large-scale hypertextual Web search engine. *Computer Networks, 30,* 107–117.

Saramäki, J., Kivelä, M., Onnela, J. P., Kaski, K., & Kertész, J. (2007). Generalizations of the clustering coefficient to weighted complex networks. *Physical Review E: Statistical, Nonlinear, and Soft Matter Physics, 75*(2), 2–5.

Savona, R. (2014). Hedge fund systemic risk signals. *European Journal of Operational Research, 236,* 282–291.

Schularick, M., & Taylor, A. M. (2012). Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870–2008. *American Economic Review, 102*(2), 1029–61.

Simaan, M., Gupta, A., & Kar, K. (2020). Filtering for risk assessment of interbank network. *European Journal of Operational Research, 280*(1), 279–294.

Soramäki, K., Bech, M., Arnold, J., Glass, R., & Beyeler, W. (2007). The topology of interbank payment flows. *Physica A, 379*(1), 317–333.

Torri, G., Giacometti, R., & Paterlini, S. (2018). Robust and sparse banking network estimation. *European Journal of Operational Research, 270*(1), 51–65. https://doi.org/10.1016/j.ejor.2018.03.041.

van den Driessche, P. (2017). Reproduction numbers of infectious disease models. *Infectious Disease Modelling, 2,* 288–303.

von Mises, R., & Pollaczek-Geiringer, H. (1929). Praktische verfahren der gleichungsauflösung. *ZAMM - Zeitschrift für Angewandte Mathematik und Mechanik, 9,* 152–164.

Von Peter, G. (2007). International banking centres: A network perspective. *BIS Quarterly Review*.

Wang, Y., Chakrabarti, D., Wang, C., & Faloutsos, C. (2003). Epidemic spreading in real network: An eigenvalue viewpoint. In *22nd International symposium on reliable distributed systems (SRDS'03)* (pp. 25–34). Los Alamitos, CA: IEEE Computer Society.

Ye, W., Liu, X., & Miao, B. (2012). Measuring the subprime crisis contagion: Evidence of change point analysis of copula functions. *European Journal of Operational Research, 22,* 96–103.

Ye, W., Luo, K., & Liu, X. (2017). Time-varying quantile association regression model with applications to financial contagion and VaR. *European Journal of Operational Research, 256,* 1015–1028.