



# State guarantees to counteract the financial effects of the COVID-19 pandemic on industrial supply chains

Giuseppe Drago<sup>\*</sup>, Giuseppe Aiello, Alberto Lombardo, Rossana Mangiapane

University of Palermo, Italy

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## ABSTRACT

As the recent COVID-19 pandemic has clearly demonstrated, appropriate state support policies are crucial for supporting industrial supply chains during crises to prevent viable businesses from defaulting. In this context, this study proposes a hybrid credit risk model to appropriately size public interventions for viable (worthy) businesses through systematic risk assessment during a period of turmoil. This study discusses the effects of the credit crunch-based economic downturn and proposes a methodology to assist policymakers in managing limited public resources to effectively support industrial supply chains. The proposed approach initially focuses on the dynamics of credit risk during economic recession periods, identifying the conditions that may justify a public intervention strategy based on public guarantees. Subsequently, a hybrid credit risk model is developed to appropriately size public interventions by quantifying systematic risk. Finally, a numerical application is presented to demonstrate the effectiveness of the proposed approach.

## 1. Introduction

The recent COVID-19 pandemic was an unpredictable event with unprecedented social and economic impact. The containment measures enforced by institutions to limit the diffusion of the virus substantially affected the global economy, lowered consumption and investments, and caused a reduction in exchange and production. These effects can substantially threaten the economies of industrialised countries and generate economic depression in countries characterised by high public debt. According to a recent survey by the Financial Stability Board (FSB), 41% of British SMEs interrupted their operations, and most likely 35% of them would be unable to restart [1]. In Germany, the Deutscher Industrie und Handelskammertag (DIHK) reported that 50% of the SMEs expected negative market effects from the pandemic, while 30% forecasted a decline in revenue of 10% [2]. According to the Italian Confederation of Craft Trade and Small- and Medium-Sized Enterprises, more than 70% of active enterprises in Italy were directly damaged by the crisis [3]. Indeed, during the COVID-19 pandemic period, industrial companies faced unexpected financial turmoil with the risk of being unable to meet their commercial or financial obligations in the short and medium terms. This situation was worsened by the credit crunch, with the revenues of financial institutions decreasing because of an increase in insolvency and to an increase in Not Performing Loans (NPL) [4]. Additionally, financial institutions must consider the lower effectiveness of real or personal guarantees with consequent double-effect defaults, involving a reduction in the estimate of the Recovery Rate (RR) and an increase in the complementary Loss Given Default (LGD). In such a situation, the risk exposure of enterprises to short-term default has substantially

<sup>\*</sup> Corresponding author.

E-mail address: [giuseppe.drago01@community.unipa.it](mailto:giuseppe.drago01@community.unipa.it) (G. Drago).

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increased, charging institutions with the responsibility of counteracting the economic crisis by triggering appropriate measures to support enterprises in withstanding the impact of the pandemic [5]. Such interventions are mostly aimed at facilitating the disbursement of credit to companies (at null interest rates) with their commitment to avoiding dismissals. For this purpose, the actions enforced by national governments aim to offer institutional guarantees on loans, thus generating public debt not conditioned by ordinary disbursement criteria or limited by normal financial constraints. However, such a policy must be accurately considered because it can substantially increase the probability of default (PD) of states with high public debt. Hence, these states must not consider the loosening of budgetary constraints (allowed by the European Government) as a pretext to jeopardise the keeping of public accounts, with the result of indiscriminately financing any operation aimed at supporting private companies.

In April 2020, the European Commission provided a subsidised credit line, the European Stability Mechanism (ESM), which consisted of resources earmarked for direct and indirect pandemic costs. Such credit lines draw financial resources from the European Investment Fund (€ 100 billion from the SURE Fund to supplement national integration funds), European Investment Bank (€ 200 billion for SMEs), and European Central Bank (€ 750 billion Pandemic Emergency Purchase Program (PEPP)). The PEPP allows increased liquidity to be made available to financial institutions, supporting the protection of economic systems from financial turmoil. The European Commission also suspended the Stability and Growth Pact, activating the “escape clause” and allowing the self-financing of individual states through public debt. Thus, specific financial and fiscal policies have been enforced by individual states to delay the payment of national taxes while allowing access to credit granted by public loan guarantees. The strategies are based on the use of capital, which is a scarce and expensive resource, therefore requiring efficient management and transparent and rational allocation.

Thus, a substantial debate originates on the criteria for the selection of the most appropriate support strategies, and a research gap can be identified in the lack of reliable methodologies to support decision makers in this regard. Moreover, the underlying open issues concern the financial instruments to strengthen equity capital, increase companies’ future debt capacity, and the most appropriate exit strategies. In this context, the aim of his study is to first analyse the main variables that characterise credit risk (PD, LGD, and RR) and their impact on financial disbursement credit in the current economic context, and subsequently to focus on the relationship between PD and LGD in an economic recession to justify an adequate intervention strategy based on public guarantees. Overall, the final goal of this research is to propose a hybrid credit risk model combining a structural approach (discriminant analysis) and a market approach for discriminating companies’ worth public intervention, thus reducing the non-performing loans, and optimising the assignment of public resources.

Based on these elements, evidence that political intervention is necessary during unpredictable crisis events, such as the COVID-19 pandemic, is provided, and a methodology is formulated to quantify the financial effects of an unpredictable crisis and discriminate worthy companies from those that are not profitable. The following research questions were addressed.

1. What is the impact of state guarantees on the LGD and RR of companies and the financing capacity of companies in the presence of systematic risks?
2. How can credit be distributed optimally by discriminating between worthy companies and those that are no longer viable?

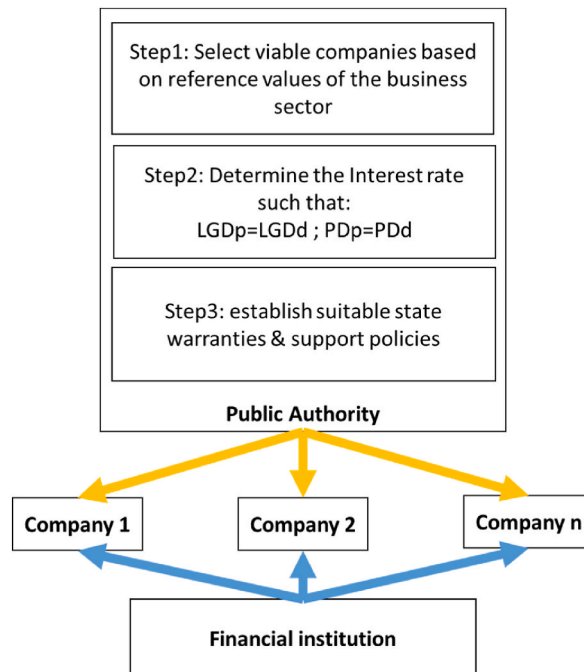


Fig. 1. Research model.

### 3. How can the state guarantee be appropriately sized?

The establishment of appropriate methodologies to address such questions will ultimately help decision makers efficiently manage public funds.

The remainder of the paper reports a brief introduction in Section 1, where the research model is given (fig.1); subsequently, in Section 2, a literature review is provided, and the methodology is discussed in Section 3. Subsection 3.1 discusses the pricing model for managing credit risk which monitors the interest rate and considers PD, LGD, and RR as independent stochastic variables. In subsection 3.2, in line with the literature and considering the current economic context, the hypothesis of stochastic independence is removed coherently with the assumption that the value of the company's assets depends not only on specific factors but also on systematic factors. During financial turmoil, the effect of the latter is more relevant, and the credit risk variables PD, LGD, and RR are statistically correlated. Therefore, the guarantees provided by businesses are less effective, resulting in reluctance to lend funds. In Subsection 3.3, a model is proposed to demonstrate that the state guarantee has a positive impact on GDD/RR. In Subsection 3.4, an approach to credit rationalization is discussed based on a hybrid enterprise model. Finally, Section 4 presents the results and discusses the theoretical implications and limitations of the study, while the last section suggests future developments, managerial insights, and conclusions.

## 2. Literature review

This section provides an overview of the existing models for estimating the key variables (PD, LGD, and LGD) for measuring credit risk which is a topic of high scientific and practical relevance, particularly in industrial contexts where small and medium enterprises (SMEs) constitute the backbone of the economy (e.g. Italy). Owing to their structure and limited financial and technical resources, small and medium enterprises (SMEs) are widely exposed to market perturbations and are thus more vulnerable than big companies. Economic crises, such as those caused by the COVID-19 pandemic, are likely to have devastating effects on SMEs [6]. However, the consequences of the pandemic on the economic performance of SMEs do not equally affect all business sectors, nor are they homogeneously distributed across the territories of EU member states. Therefore, effective and specific political intervention is required to rationalise the distribution of capital in support of SMEs without jeopardising the solvency of the state. This objective can be pursued by analysing the key variables, PD, LGD, and RR, and their reciprocal relationships. These elements are discussed in detail in the following subsections.

### 2.1. Theoretical basis and fundamentals of credit risk

Credit risk is the possibility that an unexpected change in a counterparty's creditworthiness generates a corresponding unexpected change in the current value of its credit exposure [7]. To quantify credit risk, it is necessary to identify the value-at-risk (VaR), that is, the maximum potential loss a financial institution may incur. Assuming that loss is a random variable, its expected value represents the expected loss (EL, Expected Loss), whereas the unexpected loss measures the magnitude of variation or dispersion (UL, Unexpected Loss).

PD, RR, or equivalently, LGD, are the key variables for quantifying credit risk. The four variables mentioned above correspond to the risk parameters upon which the Basel II IRB approach [8] is built.

- PD is the likelihood that a borrower will fail to pay back debt and is the estimated amount of money a financial institution loses in the event of a borrower's default, expressed as a percentage of the total capital at risk. LGD is the complementary value of the RR.

The relevance of these parameters is related to their employment in the risk assessment phase, formulation and design of recovery policies, determination of loan portfolios, and calculation of regulatory capital levels [9].

Technically, the Expected Loss Rate (ELR) is the product of the PD and LGD. EL is obtained by multiplying the ELR with Exposure at default (EAD). The EAD provides an estimate of the outstanding amount if the debtor defaults. UL is the difference between VaR and EL. While EL is estimated ex-ante by the lender, which "covers" it by adding an appropriate spread over  $r_{free}$ , UL translates into a capital requirement (also known as regulatory capital) and is usually expressed as a capital adequacy, which is the ratio of equity and of the risk-weighted assets.

### 2.2. Models for estimating PD

PD is a fundamental element of credit analysis and risk-management frameworks. In scientific literature, PD Estimation models can be subdivided into two categories: scoring models and market approaches. The scoring models were based on key performance indicators referred to as the balance sheet of a company. Early approaches [10] suggested the employment of discriminant analysis to estimate the PD of commercial companies, proposing a methodology based on a priori PD, considering general market conditions. Pacelli and Azzollini [11] proposed an advanced approach for investigating the ability of neural networks to approximate PD. Finlay [12] proposed a method based on the survival of the fittest which was implemented in a Genetic Algorithm (GA), applied to a dataset of UK credit applications of known performance, and compared the results with traditional methods such as linear regression and logistic regression. Finally, Antunes et al. [13] approach the problem of bankruptcy prediction from a probabilistic perspective by applying Gaussian processes (GP).

Market approaches are based on information from the capital market and can be further subdivided into intensity [14] and

structural models [15,16]. The former assumes default conditions as an exogenous event originating from the market, whereas the latter employs modern option pricing theory to evaluate corporate debt. Merton's model is considered the cornerstone of all other structural models and provides a way of relating credit risk to the capital structure of the firm based on the assumption that insolvency occurs at the time of debt expiration, causing the firm to default. This assumption is however questionable; therefore, other models have been proposed based on the assumption that insolvency occurs when firm's value falls below a threshold which in general is a function of time. Another common approach is to assume that default occurs when the value of long-term debt is higher than that of assets [17]. In this regard, Cossin and Pirotte [18] suggest that the value should range between short-term debt and total debt, whereas Longstaff and Schwartz [19] suggest a methodology that considers a deterministic risk-free rate, a constant exogenous RR, and the possibility of bankruptcy occurring at any time with the inability to fulfil the performance obligation [20]. Finally, several studies [21, 22] employ artificial intelligence and machine learning methods to select critical variables for bankruptcy prediction, whereas Boughaci et al. [23] propose a credit scoring model based on clustering techniques and random forests.

### 2.3. Models for estimating RR and LGD

The estimation of RR and LGD is complicated because it involves several exogenous (macroeconomic factors, such as inflation rate, real GDP growth rate, unemployment rate, etc.) and endogenous factors (firm characteristics, such as age, number of creditors, total assets, etc.) [24–26]. Approaches for estimating RR and LGD can be subdivided into two main classes: market and workout. The former is based on market information, whereas the latter involves models based on historical data. The market approach has substantial limitations; therefore, a workout approach is generally considered preferable [27]. These classes involve methods such as regression, decision trees, neural networks, and hybrid models. Linear regression approaches are most frequently employed in the scientific literature [28,29], although their application has been questioned by many authors [30,31] who argue that the distribution of LGD/RR is not normal. Some authors thus proposed the employment of a beta distribution, while others [32] proposed non-parametric models, such as decision trees, neural networks [33], and mixture regression [34]. Over the last two decades, several studies [35,36] have estimated the average LGD and RR for different sectors of debtor activity.

### 2.4. The systematic elements and the dependency of PD, LGD and RR

The aforementioned models for estimating credit risk are based on the general assumption that PD, LGD, and RR are statistically independent, thereby neglecting their possible correlations. Consequently, credit risk is influenced only by specific factors mainly attributable to the nature and methods of conducting business. However, this assumption is questionable, as Allen and Saunders [37] suggest that systematic elements influence credit risk. Frye and Jacobs [38] proposed a methodology involving a single systematic factor, referred to as the economic situation, thus considering the possibility of a statistical correlation between corporate default and economic uncture. In addition, Altman et al. [39] demonstrated that the presence of a correlation between PD and LGD can significantly affect both expected loss (EL) and unexpected loss (UL). Rösch and Kaserer [40] proves the existence of the “flight to quality” phenomenon, involving the investors in requesting a higher rate during crisis or catastrophic events. This situation generated an increase in the spread between rating classes. Finally, Buncic and Melecky [41] employ data from the 2008 financial crisis to conduct stress tests.

## 3. Methodology

This section introduces the proposed methodology to appropriately size state intervention during a crisis period. According to the general framework given in fig. 2, the consequences of an economic crisis on the financial performance of companies belonging to a supply chain are discussed, and the proposed model for appropriately sizing state intervention is formulated.

Merton's model is employed to demonstrate how the presence of systematic risk affects PD and RR simultaneously and to investigate the influence of a guarantee on the pricing model of financial institutions and credit risk. In addition, the loss of effectiveness of real and personal guarantees promoted by companies in the context of loan applications is proven, and how state guarantees influence financial institutions' reluctance to lend when significant economic downturns are likely is shown.

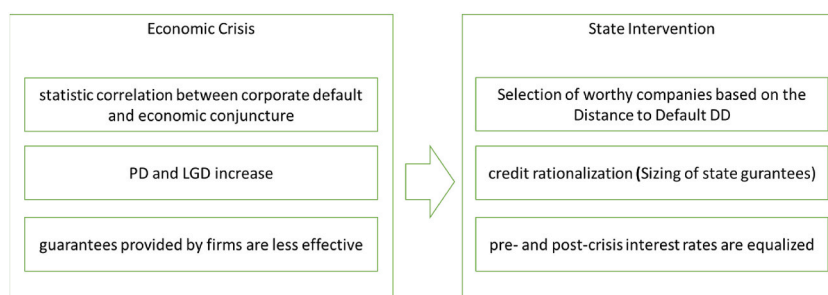


Fig. 2. Methodological framework.

Based on these results, Altman's Z score was employed to determine a company's creditworthiness, considering both retrospective measures such as financial performance and prospective measures such as the industry's average distance to default ( $DD_M$ ) before December 31, 2019 ex -COVID-19. The proposed methodology aims to overcome one of the limitations of Altman's structural model and exclude companies that were already close to default before the crisis from state guarantees. For firms classified as creditworthy, it is possible to transform the Altman  $Z_{score}$  into the probability of default according to the Altman model [42].

Finally, considering the pricing model in the presence of systematic risk, an appropriate level of government guarantee is determined to make the cost of borrowing money equivalent in the ex-ante and ex-post scenarios.

The following nomenclature is introduced.

- PD: probability of default.
- RR: recovery rate.
- LGD: loss given default.
- EAD: Exposure at default; the predicted amount of loss which a financial institution may be exposed to when a debtor defaults on a loan.
- EL: Expected loss given by the products of EAD, PD, and LGD.
- $S_{EL}$ : spread required by the financial institution for EL.
- VaR: measure of the risk of loss for investments.
- UL: unexpected loss
- $S_{UL}$ : spread required by the financial institution for UL.
- $r_{free}$ : risk-free rate.
- IRT: internal interest transfer rates.
- i: interest rates required by financial institution.
- A: amount of principal and interest.
- $r_e$ : cost of equity.
- um: monetary unit.
- $V_t$ : firm value at time t

In general, the pricing model of financial institutions considers the variables IRT,  $r_{free}$  and EAD to be deterministic, whereas the others are stochastic.

### 3.1. Relation between pricing model and collateral

This section discusses the relationship between i and LGD, referring to a financial supply chain composed of a financial institution, company, authority, and stakeholders. The financial institution finances the company and receives a loan paid by the company, considering the capital requirements imposed by the authority and the expectations of the stakeholders to estimate i. The model is based on the assumption that the financial institution is a price-taker operating in perfect competition, and the interest rate is calculated based on EL, UL, and IRT, considering the spreads  $S_{IRT}$ ,  $S_{EL}$  and  $S_{UL}$  according to equation (1). Assuming that the financial institution accepts the price, the interest rate is endogenous and the internal interest transfer rates represent the cost of the loan, which, in general, is related to the financial institution's rating. In this study, this rate is considered equal to the risk-free rate, and the financial institution is risk-neutral; thus, a risk-free investment A is equal to a risky investment with an expected value equal to A. From these assumptions, the following conditions can be derived:

$$i = (IRT + S_{EL} + S_{UL}) = (r_{free} + S_{EL} + S_{UL}) \quad (1)$$

$$(1 + r_{free} + S_{EL} + S_{UL})[(1 - PD) + (1 - LGD)PD] = (1 + r_{free}) + VaR(r_e - r_{free}) \quad (2)$$

eq. (2) suggests that the expected value of the financing compound is equal to the amount of investment plus the shareholder premium ( $r_e - r_{free}$ ). These premiums reward shareholders by minimising the risk of investment loss (VaR). Financial managers' main goal is to maximise the value of stock shares (to create value for stakeholders). Hence, shareholder returns should outperform certain benchmarks, such as the cost of capital. The idea is that shareholders' money should be used to earn a higher return than they can earn by investing in other assets involving the same risk. If financial institutions require an interest rate to cover financial costs and the profit rate, this creates value for shareholders. The interest rate is obtained using the following equation:

$$i = \frac{r_{free} + LGD * PD + VaR(r_e - r_{free})}{1 - (PD * LGD)} = \frac{r_{free} + (1 - RR) * PD + VaR(r_e - r_{free})}{1 - (PD * (1 - RR))} \quad (3)$$

The following assumptions hold: PD depends on the borrower's rating, LGD depends on the collateral provided by businesses, VaR depends on the characteristics of the loan, and  $r_e$  and  $r_{free}$  are exogenous variables that depend on the capital market and the financial institution's risk.

Considering all factors as constants, according to eq.3, when the LGD rises, the rate requested by financial institutions also increases; hence, financial institutions create value for stakeholders. However, financial institutions cannot increase their interest rates above the maximum, as represented by the usury rate. The RR has a symmetrical trend; therefore, if the RR increases, the financial

institution demands a lower interest rate,  $i$ . Accordingly, the guarantee can reduce the financial institution’s interest rate. In other words, financial institutions lose investments when a double default occurs for both the lender and borrower. The double default effect is rare when PD and LGD are independent statistical variables; however, it is more likely to occur when there is a downturn trend or an economic crisis, in which case the systematic factor (e.g. ripple effects) impacts both PD and LGD/RR, thus reducing the effectiveness of the collateral.

3.2. The relationship between the PD and LGD of firms in the presence of systematic factors

In this subsection, Merton’s model was employed to examine the relationship between PD and LGD. Merton’s model assumes that there are no bankruptcy charges, the liquidation value equals the firm value, and debt and equity are frictionless tradeable assets. In addition, large and medium firms are funded by shares (“equity”) and bonds (“debt”). Merton’s model further assumes that debt consists of a single bond with face value  $D$ , market value  $B$ , and maturity  $T$ . Finally, Merton’s model refers to a simple debt structure, and assumes that the total market value  $V_t$  (the asset value at time  $t$ ) follows a geometric Brownian motion:

$$dV_t = \mu V_t dt + \sigma_v V_t dZ_t \tag{4}$$

where  $\mu$  is the mean rate of return on assets,  $\mu$  is asset volatility, and  $dZ_t$  is the Wiener process. The stochastic differential equation (4) can be solved explicitly, yielding a unique solution:

$$\ln V_t = \ln V_0 + \left( \mu - \frac{\sigma_v^2}{2} \right) T \tag{5}$$

This is a normal distribution with the following mean (eq. (5a)) and variance (eq. 5b):

$$\mu^* = \ln V_0 + \left( \mu - \frac{\sigma_v^2}{2} \right) T \tag{5a}$$

$$\sigma^{*2} = \sigma_v^2 T \tag{5b}$$

According to eq. (5), the return on assets follows a stochastic process, with the uncertainty of  $V_t$  increasing over time. In addition, Merton’s model assumes that  $\mu^* = r_{free}$  thus obtaining a risk-neutral probability. At maturity  $T$ , if  $V_t > D$ , the latter is paid in full, and the remainder is distributed among the shareholders. If  $V_t < D$ , then default is deemed to occur, and the bondholders exercise a debt covenant allowing them to liquidate the firm and receive the liquidation value (equal to the total firm value because there are no bankruptcy costs) in lieu of the debt. The RR is equal to the ratio of  $V_t$  to liability  $D$ . In this case, shareholders do not receive anything, but according to the principle of limited liability, they are not required to inject additional funds to repay the debt.

Based on such considerations, the shareholders have a cash flow at  $T$  equal to  $(V_T - D)$ , therefore the equity can be regarded as an institutional put option on the  $V_t$ . However, the bondholder receives the minimum value between  $V_t$  and  $D$ . In general, investors, such as bondholders, may hedge their credit risk by purchasing credit derivatives, thus transferring the risk from the lender to the seller in exchange for payment. If lenders purchase the institutional option, and the investment is risk-free. Mathematically terms we obtain, eq. 6:

$$B_0 + P_0 = D * e^{-Tr_{free}} \tag{6}$$

where  $P_0$  is the value of the put option and  $D e^{-Tr_{free}}$  is a risk-free bond that repays  $D$  with absolute certainty.

The value of  $P_0$  can be obtained using the Black and Sholes pricing model according to eq.7:

$$P_0 = D * e^{-Tr_{free}} N(-d_2) - N(-d_1) V_0 \tag{7}$$

where  $N(XX)$  is the cumulative normal distribution, and  $d_2$  and  $d_1$  are defined according to eqs. (7a)–(7b):

$$d_1 = \frac{\ln\left(\frac{V_0}{D}\right) + (r_{free} + \frac{1}{2}\sigma_v^2)T}{\sigma_v \sqrt{T}} = \frac{\frac{1}{2}\sigma_v^2 T - \ln(L)}{\sigma_v \sqrt{T}} \tag{7a}$$

$$d_2 = \frac{\ln\left(\frac{V_0}{D}\right) + (r_{free} - \frac{1}{2}\sigma_v^2)T}{\sigma_v \sqrt{T}} = \frac{\frac{1}{2}\sigma_v^2 T + \ln(L)}{\sigma_v \sqrt{T}} = d_1 - \sigma_v \sqrt{T} \tag{7b}$$

$L$  is the financial leverage and is equal to  $\frac{De^{-Tr_{free}}}{V_t}$ ,

$I$  according to eq. 8, the physical probability of default at time  $T$ , measured at time  $t$ , is

$$PD = P(V_t < D) \tag{8}$$

This probability is equal to the probability of exercising an institutional put option (eq. (8a)). Thus, the probability of exercising in a risk-neutral world is:

$$N(-d_2) = 1 - N(d_1) \tag{8a}$$

The risk neutral probability can thus be calculated as:

$$PD = P(V_t < D) = N(-d_2) = (1 - N(-d_1)) = N\left[-\frac{\ln\left(\frac{V_0}{D}\right) + (r_{free} - \frac{1}{2}\sigma_V^2)T}{\sigma_V\sqrt{T}}\right] \tag{9}$$

This model is based on the assumption that the investor does not request a risk premium, the investor is thus risk neutral and  $\mu$  is equal to  $r_{free}$ . However, investors expect to yield more than the risk-free rate of return because they request a risk premium when investing in risky activities. The actual probability of default,  $PD^*$  is obtained when it uses rate of return on the assets  $\mu$  instead of  $r_{free}$ . In this case, investor require a rate  $\mu > r_{free}$ . In according to eqs. (9a)–(9b) as a result:

$$d_1^* = \frac{\ln\left(\frac{V_0}{D}\right) + (\mu + \frac{1}{2}\sigma_V^2)T}{\sigma_V\sqrt{T}}, d_1^* > d_1, N(-d_1^*) < N(-d_1) \tag{9a}$$

$$d_2^* = \frac{\ln\left(\frac{V_0}{D}\right) + (\mu - \frac{1}{2}\sigma_V^2)T}{\sigma_V\sqrt{T}}, d_2^* < d_2, N(-d_2^*) > N(-d_2) \text{ and } PD^* < PD \tag{9b}$$

In according to eq. (10), the expected RR is equal to:

$$E\left(\frac{V_t}{D} \middle| V_t < D\right) = \frac{1}{D} E(V_t | V_t < D) \tag{10}$$

In other words,  $E\left(\frac{V_t}{D} \middle| V_t < D\right)$  is equal to  $1/D$  times the mean of the truncated log-normal.

$$E\left(\frac{V_t}{D} \middle| V_t < D\right) = e^{\mu^*} + \frac{\sigma^2}{2} \left( \frac{N\left(\frac{\ln D - \mu^*}{\sigma} - \sigma^*\right)}{N\left(\frac{\ln D - \mu^*}{\sigma}\right)} \right) \tag{11}$$

By substituting  $\mu^*$  and  $\sigma^*$  into eq. (11) we obtain eq. (12):

$$E\left(\frac{V_t}{D} \middle| V_t < D\right) = e^{\ln V_0 + \mu T} + \frac{N\left(-\frac{\ln\left(\frac{V_0}{D}\right) + (\mu + \frac{1}{2}\sigma_V^2)T}{\sigma_V\sqrt{T}}\right)}{N\left(-\frac{\ln\left(\frac{V_0}{D}\right) + (\mu - \frac{1}{2}\sigma_V^2)T}{\sigma_V\sqrt{T}}\right)} = V_0 e^{\mu T} \frac{N(-d_1^*)}{N(-d_2^*)} = E(V_t) \frac{N(-d_1^*)}{N(-d_2^*)} \tag{12}$$

Hence, the RR is equal to:

$$RR = E\left(\frac{V_t}{D} \middle| V_t < D\right) = V_0 e^{\mu T} \frac{\Phi(-d_1^*)}{\Phi(-d_2^*)} = E\left(\frac{V_t}{D}\right) \frac{\Phi(-d_1^*)}{\Phi(-d_2^*)} \tag{13}$$

Considering the value of assets and volatility as independent variables and RR and PD as dependent variables, eqs. (9) and (13) show that a change in the  $V_t$  affects PD, RR, and LGD. When  $V_t$  increased, PD and LGD decreased, whereas RR increased. However, if the asset’s value falls, a default is likely to occur; hence, if PD and LGD increase, the guarantees provided by firms are less effective. In conclusion, there is a correlation between PD and LGD/RR in the presence of systematic risks. Indeed, during an economic downturn, the liquidation value of a company is likely to decrease. If PD increases (due to the economic crisis) and part of the corporate assets of insolvent companies consist of receivables from other bankrupt companies, RR decreases.

Such considerations can be referred to as the economic crisis experienced during the COVID-19 pandemic and the related business interruptions, reductions in both supply and demand, and supply chain disruptions originating from social distancing policies and quarantines enforced on a global scale. Thus, COVID-19 has slowed the global economy, influencing the systemic risk of supply chains and increasing the probability of corporate default. The confluence of sharp economic losses and historic levels of corporate debt risk has generated substantial stress for financial institutions, with unprecedented levels of liquidity risk, loan defaults, and loss of intermediation revenues.

### 3.3. Impact of the state guarantee on the RR and LGD

Based on the model presented in Section 3.2, the impact of a state guarantee on RR and LGD is investigated in this section, discussing its effectiveness in relieving financial institutions from their stress situation and avoiding the default of business companies and the disruption of globalised supply chains. Due to crises such as the COVID-19 pandemic, the demand for bank loans has soared to record levels since March 2020, driven by a decline in the capacity of firms to finance their current expenses through operating cash flows due to the fall in revenues during the lockdown period. This situation resulted in an acute need for liquidity to finance working capital and the necessary investments. Moreover, in the context of high uncertainty, firms seek loans to build precautionary liquidity buffers or adapt their businesses to the new environment. To support banks in accommodating the surge in loan demand under favourable conditions, most European governments have implemented special policies to provide public guarantees for bank loans, thus transferring a percentage of the credit risk and potential credit losses from banks to the government, thereby mitigating the costs

for banks.

In the following, a model is proposed to demonstrate that state guarantees have a positive impact on LGD/RR; therefore, counteracting the effects of an economic downturn if the effects on sovereign risk are neglected is a viable financial strategy. Through this approach, the uncertainty of the stochastic variables LGD/RR can be reduced because the state guarantee is assumed to be certain. Equations (9) and (13) suggest that when  $V_t$  decreases, PD and LGD increase, and RR decreases. In the case of default, the state guarantee allows the financial institution to recover a percentage  $\delta$  of the loan, which can thus be considered deterministic, thereby reducing the overall risk. This allows financial institutions to substitute the weighting coefficient of the debtor (company) with that of the guarantor (state) for the percentage of exposure hedged.

In mathematical terms, eq. (13) becomes:

$$RR = E\left(\frac{V_t}{D} \mid V_t < D\right) = \left(\frac{V_0}{D}\right) \frac{e^{\rho_1 T} \Phi(-d_1^*)}{\Phi(-d_2^*)} + \delta = E\left(\frac{V_t}{D}\right) \frac{\Phi(-d_1^*)}{\Phi(-d_2^*)} + \delta \tag{14}$$

As previously demonstrated (eq. (14)), the state guarantee allows to reduce the cost of LGD to  $(1-\delta)$  and to increase the RR to  $\delta$ . Hence, a state guarantee has a significant impact on reducing the active interest rate required by businesses, thereby facilitating their financing in the presence of systematic risk.

### 3.4. Hybrid enterprise model: an approach to credit rationalization

Appropriate loan guarantee schemes are crucial for supporting the financing needs of firms during a crisis, such as in the early COVID-19 period, contributing jointly to other policy measures to prevent viable businesses from facing defaults. An institutional support strategy is necessary to mitigate the risk of severe liquidity reduction for business companies and triggering bankruptcy, which would, in turn, deplete bank capital. This would ultimately result in a sudden reduction in credit flows and in a tightening of credit conditions, thereby instigating more bankruptcies and hampering the financing of surviving firms' adjustment towards a "new normal" way of conducting business. At the same time, if the policy support provided in the crisis led to a permanent large-scale expansion of the government's role in steering economic outcomes, it may hamper allocative efficiency and reduce the productive capacity of the Euro area over a longer horizon by artificially keeping afloat firms that are not viable or sufficiently profitable. Moreover, the specific design and calibration of guaranteed schemes may entail side effects, for instance in the form of incentives for excessive indebtedness and imprudent risk allocation. Public integrity is of paramount importance for ensuring adequate resiliency in response to a dramatic crisis such as the COVID-19 pandemic. In a context where public debt weighs heavily on governments, financial support programs must reflect a compromise between a rapid response to a crisis and the maintenance of a sufficient level of prudence. Therefore, it is necessary to rationalise public funds and create conditions for an efficient and transparent allocation system that can discriminate companies that are no longer viable from deserving ones, thus maximising their value. Indeed, a generalised and indiscriminate granting of state guarantees would result in a waste of public funds.

A hybrid enterprise model was proposed to discriminate between companies that deserve financial support based on their Distance to Default. The distance to default (DD<sub>i</sub>) of a generic company is the difference between  $V_i$  and the company's Default Point level (DP), expressed as a multiple of the standard deviation ( $\sigma$ ). DP<sub>i</sub> is the sum of the short-term debt ( $s$ ) and 50% of the long-term debt. Analytically, we obtain eqs. (15) and (16):

$$DP_i = s + \frac{1}{2} \tag{15}$$

$$DD_i = \frac{V_i - DP_i}{V\sigma} \tag{16}$$

The DD<sub>M</sub> is calculated using the average DD of listed companies in the sector, according to eq. (17).

$$DD_M = \sum_{i \in M} DD_i / M \tag{17}$$

Initially, the worthy/non-worthy classification did not consider the effects of the pandemic. The model determined creditworthiness in the absence of a crisis and excluded companies already close to default before the crisis. For companies recognised as worthy, it is possible to calculate the relative probability of default. Referring to the COVID-19 pandemic, for example, this translates into considering the financial performance of a company as of December 31, 2019. The authors further assumed that independent variables were distributed according to a multiple normal distribution by transforming Altman's z-score to PD. In particular, Company A's PD belongs to a cluster of undeserving companies if

$$PD = p(A|x_j) = \left( \frac{1}{1 + \frac{1-\pi}{\pi} e^{\mathcal{L}_{score} - \alpha}} \right) \tag{18}$$

where  $\alpha$  is the cut-off point and  $\alpha$  is an a priori PD derived purely by deductive reasoning.

This probability allows to create a model capable of recognising the "sentiment" of the capital market based on the economic



situation. The  $\pi$  parameter in eq. (18) was estimated using the probability empirically associated with the  $DD_M$  indicator of the sector before and after the crisis. Considering the variables reported in Subsection 3.1, to avoid burdening the discussion, the subscript p is attributed to all pre-crisis variables, while the subscript d indicates the post-crisis variables. In addition, we identify with  $\pi_p$  the pre-crisis probability associated with the  $DD_M$  indicator of the sector and with  $\pi_d$  the post-crisis probability.  $\pi_p$  can be easily calculated as an empirical PD associated with the sector  $DD_M$  indicator in 2019. Hence, a reasonable representation could be to attribute an empirical probability of default, which is referred to as the 2008 crisis.

Assuming that the effects of the crisis are reflected in Factor  $\pi$ , it is possible to assume that  $\pi_d$  is greater than  $\pi_p$ . Therefore, the same company shows a higher PD. In analytical terms, we obtain Equation (19):

$$\pi_p < \pi_d \rightarrow PD_p < PD_d \tag{19}$$

Considering that eq. Three and considering the  $r_{free}$ ,  $r_e$ , and LGD constants, with a higher PD ( $\pi_d$ ), intermediaries should grant loans at a higher rate. In fact, financing a riskier company leads to an increase in the VaR required of intermediaries and the risk premium required by shareholders, or to requests for greater guarantees. However, as demonstrated in Section 3.2, in the presence of economic downturns, the real and personal guarantees provided by companies lose their reliability. Therefore, the only possible solution to mitigate these effects is a state guarantee.

Given eq. 3 it is possible to calculate the interest rates before ( $i_p$ ) and after ( $i_d$ ) the crisis as:

$$i_p = f(PD; LGD; VaR; r_{free}; r_e)_p = \frac{r_{free} + LG D_p * PD(\pi_p) + VaR(PD(\pi_p); LG D_p) \cdot (r_{e_p} - r_{free})}{1 - (PD(\pi_p) * LG D_p)} \tag{20}$$

Where  $VaR$  is  $f(PD; LGD)_p$  and  $r_e = f(PD; LGD; VaR; r_{free})_p$ ,

$$i_d = f(PD; LGD; VaR; r_{free}; r_e)_d = \frac{r_{free} + LG D_d * PD(\pi_d) + VaR(PD(\pi_d); LG D_d) \cdot (r_{e_d} - r_{free})}{1 - (PD(\pi_d) * LG D_d)} \tag{21}$$

Where is  $VaR$  is  $f(PD; LGD)_d$  and  $r_e = f(PD; LGD; VaR; r_{free})_d$ ,

Considering the demonstrations in section 2 and section 3 we obtain inequation 22:

$$PD_p < PD_d; LGD_p < LGD_d; RR_p > RR_d \tag{22}$$

Hence, it is possible to determinate inequation 23:

$$i_p = f(PD; LGD/RR; VaR; r_{free}; r_e)_p < i_d = f(PD; LGD/RR; VaR; r_{free}; r_e)_d \tag{23}$$

For the financial institution to finance the loan to the same  $i_p$  against an increase in  $PD_d$ ,  $LGD_d$ ,  $VaR_d$  and  $r_{e_d}$  more collateral  $RR_r$  is required.

$$LGD_r < LGD_d; RR_r > RR_d \tag{24}$$

In according to eq. (24), the state must increase the RR from  $RR_d$  to  $RR_r$  and reduce the cost of LGD from  $LG D_d$  to  $LG D_r$ .

To maximise the state guarantee, the pre- and post-crisis interest rates are equalized (eq. (25)).

$$i_p = i_d \tag{25}$$

$$i_p = \frac{r_{free} + (1 - RR_r) * PD(\pi_d) + VaR(PD(\pi_d); (1 - RR_r)) \cdot (r_{e_d} - r_{free})}{1 - (PD(\pi_d) * (1 - RR_r))} \tag{25a}$$

Using eq. (25a),  $RR_r$  the estimated by the intermediary against the recognition at the same rate can be derived. The level of RR that the State (RRs) should guarantee in a policy of "post crisis rate containment" can finally be determined:

$$RR_s = RR_r - RR_d \tag{26}$$

#### 4. Results

To validate the proposed model, we consider the case of a company characterised by the following post-crisis and pre-crisis creditworthiness variables, according to the data in Table 1.

**Table 1**  
Pre&post crisis credit parameters.

Pre-crisis	Post- crisis
$PD_p = 0.02$	$PD_d = 0.052$
$LGD_p = 0.45$	$LGD_d = 0.6$
$RR_p = 0.55$	$RR_d = 0.4$
$VaR_p = 0.092$	$VaR_d = 0.112$

Using eq. (20) and eq. (21), a financial institution with  $r_{ep} = 0.16$  and  $r_{ed} = 0.18$ , willing to finance a company in the case of  $r_{free} = 0.03$ , would charge interest rates of  $i_b = 0.0542$  and  $i_d = 0.0805$ , respectively. If the state intervenes with a  $\delta$  of 100%, the pre- and post-crisis interest rates are equated thus, using Eqs. (25a) and (26), the  $RR_r$ ,  $RR_s$  values obtained were 0.915 and 0.515, respectively, demonstrating how the state guarantee is a valid financial strategy to counteract the effects of an economic downturn, provided that the effects on sovereign risk are neglected. Equation (14) in Section 3.3 shows that the state guarantee reduces the stochastic uncertainty of the RR and LGD variables; therefore, the state guarantee has a significant impact on the reduction of the lending rate required by companies, thus facilitating the financing of enterprises in the presence of systematic risk.

## 5. Discussion

These results highlight the importance of appropriate banking policies and financial support from governments for the survival of SMEs during periods of financial turmoil. In line with existing research [43,44] and according to the results, in the presence of systematic risks, companies have less access to traditional financing channels due to the loss of the effectiveness of guarantees; therefore, public guarantees can be used as a financial strategy to mitigate these effects. In contrast to similar researches [45–47], however, the proposed approach considers the correlation between PD and LGD, and thus involves the following elements.

- (i) the presence and the degree of effectiveness of collateral.
- (ii) The characteristics of the debtor include sector, country in which it operates, and speed of liquidation of assets in the event of a default.
- (iii) state of the economic cycle (i.e. the effects of correlation).

When PD increases due to the economic crisis and part of the corporate assets of insolvent companies are made up of receivables from other bankrupt companies, RR decreases and LGD increases; therefore, the effectiveness of the collateral provided is reduced due to ripple effects. Neglecting the correlation between PD and LGD/RR results in underestimating the level of public intervention in relation to companies' real financial needs.

The first insight provided by this study is the establishment of a methodology to discriminate between worthy and non-worthy companies in response to the need for public funds related to the risk of unjustified debt accumulation and large deficits due to deep recessions [48]. In this regard, the proposed model allows for the calculation of the  $Z_{score}$ , considering back-looking structural variables (performance indices) considered by other authors [10] and a forward-looking ( $DD_M$ ), as in KMV. Exclusively for worthy companies, the transition from  $Z_{score}$  to PD is made by assuming that the indices are characterised by a multivariate normal distribution [42]. By considering back- and forward-looking variables simultaneously, the proposed approach overcomes a substantial limitation of structural models related to their unresponsiveness to changes in economic and financial conditions.

A further contribution of this study is the proposal of a methodology to determine the level of state guarantees that can mitigate the increase in rates due to economic crises. The proposed approach considers the maximum percentage ( $\delta$ ) of state intervention to achieve identical pre-crisis and post-crisis rates and limits the use of financial resources. The percentage  $\delta$  can be determined considering the general macroeconomic balance, and it will therefore depend upon the available funds, the number of companies deserving the public guarantee and their financial parameters (rating, level of deficit), and the reputation of the state (referred to the sovereign risk). This methodology does not explicitly consider sovereign risk; however, public managers should consider macroeconomic equilibrium when choosing the intervention percentage.

Finally, this research provides relevant insights for financial supply chain managers who must be aware of the relationship between specific, systemic, and financial risks and their effects during crises such as the COVID-19 pandemic. Effective risk management requires public managers to consider the macroeconomic balance and the trade-off between extraordinary interventions to support firms, public deficit, rationing of public funds, and sovereign risk [49]. Practically point of view, public managers must consider the amount of available funds, the number of enterprises that deserve the state guarantee, and possible additional budget constraints. The methodology presented in this regard suggests supporting only enterprises whose PD has increased owing to the pandemic. This condition justifies the deficit increment and adherence to best practices for optimal capital allocation. Managers should also be aware of the impact of credit risk on the balance sheet in the presence of systematic risk; effective risk management along the SC requires the formulation of contingency plans and attention to the company's internal processes [50]. Similarly, for public managers, effective risk management involves assessing the overall risk of rising interest rates, considering that the financial imbalances of the credit crunch could create a chain of failures that could ultimately affect the financial institution itself [51].

## 6. Limitations

The proposed model presents some inherent limitations derived from assuming the state as an entity that meets its obligations with certainty; however, this assumption is generally accepted in financial models. In addition, IRT,  $r_{free}$  and EAD are considered deterministic variables; therefore, the risk exposure of financial institutions is neglected. Generally, EAD is considered an estimate of the extent to which a bank may be exposed to a counterparty in the event and at the time of the counterparty's default. EAD is equal to the current outstanding amount in the case of fixed exposures, such as term loans. According to Ref. [8], EAD measures the credit line that is likely to be drawn further in the event of a default. This model has the following theoretical limitations.

- (i) The assumption that the matrices of variances and covariances of the independent variables (financial indices) are equal for both groups in the sample used for the discriminant analysis, and the assumption that the distribution of the firms' performance indices is multivariate normal can be unrealistic.
- (ii) qualitative factors such as reputation, management quality, industry outlook are neglected.
- (iii) Estimation of the-parameter could be complicated in the case of recent crises such as COVID-19, due to inherent difficulties in retrieving the data. Referring to parameters related to past crises (e.g. the 2008 crisis) can be a solution in this regard.

## 7. Conclusions

In a complex historical moment, such as the COVID-19 pandemic, public intervention is necessary to support the financing needs of firms, together with other support policy measures, to prevent viable businesses from becoming illiquid. Institutions failing assisting valuable enterprises, could generate frequent bankruptcies resulting in a sudden reduction in credit flows and in the tightening of credit conditions, thereby causing additional bankruptcies.

In such emergency periods, extraordinary public intervention is justified by Keynesian theory as long as it does not result in a waste of public funds or an increase in sovereign risk. In such a situation, the effective management of public resources is required to adequately size credit. However, during financial turmoil, because of the correlation between PD, LGD, and RR in the presence of systematic risks and considering the related loss of effectiveness of companies, a negative impact on the credit crunch is likely to occur. By contrast, the public loan guarantee scheme positively impacts the pricing model of financial institutes because they can recover the loan in the event of a business default, thus lowering their risk.

However, the recent pandemic has demonstrated the difficulty public institutions face in establishing financial support policies that can effectively protect valuable businesses from the risk of default. In some cases, such difficulties have led public decision-makers to indiscriminately support all active businesses without promoting precise targeted financial support measures, eventually wasting valuable capital.

According to the proposed methodology, to establish an effective public intervention strategy during a crisis, companies deserving public financial support must be preliminarily identified through a transparent and solid methodology. This study proposes an approach based on both Altman's scoring model ( $Z_{score}$ ) and Merton's model (market approach) to discriminate between worthy and non-worthy businesses. In addition to its solid theoretical foundation, the proposed approach is practical because the decision parameters referring to the financial performance of the company can be easily calculated using the data reported in the balance sheet (e.g. working capital, total assets, retained earnings, earnings before interest and tax, market value of equity, total liabilities and sales, and total assets). The DD can be calculated using the KMV approach.

Once the companies that deserve public financial support are selected, the  $Z_{score}$  can be translated into a probability of default by attributing an a priori probability to the DD of the companies in the same business sector, calculated by referring to past crises (e.g. the 2008 crisis). Using the intermediary's pricing model, it is theoretically possible to establish an appropriate level of state guarantee which makes the cost of borrowed money equivalent in ex-ante and ex-post scenarios.

This study contributes to the literature on risk management, optimal capital allocation, and supply chain financing by proposing a methodology that can be used in future research. Further developments in the methodology are currently addressing the refinement of the model to overcome the current limitations and its practical application considering real data.

## Author contribution statement

Giuseppe Drago: Conceived and designed the experiments; Wrote the paper.  
 Giuseppe Aiello: Conceived and designed the experiments.  
 Alberto Lombardo: Contributed reagents, materials, analysis tools or data.  
 Rossana Mangiapane: Contributed reagents, materials, analysis tools or data.

## Data availability statement

No data was used for the research described in the article.

## Declaration of interest's statement

The authors declare no competing interests.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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