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Infrastructure planning, fleet sizing, and scheduling for E-public transport

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Abstract

In this work, we analyze the integrated problem of sizing and locating charging infrastructure as well as sizing a fleet of traditional and electric buses to cover a set of scheduled trips. The available buses include the following: traditional diesel vehicles (TV), standard electric vehicles (EV), and electric vehicles equipped with ultra-fast recharging (UV). The latter exploit the latest available technology, and therefore, their purchasing cost and the cost of their compatible charging infrastructure are very high. However, their recharging time, which is 12 times less than that of EV, makes them useful when dealing with a tight schedule. This study aims to determine how many EV and UV should be purchased and how many chargers of each type should be installed in each recharging station. Also, it aims to outline a feasible schedule for each bus, taking into consideration the eventual stops for recharging, in order to minimize the overall cost. To achieve the study's aims, we create an innovative and elegant integer programming formulation in which we model multiple visits to recharging stations and propose an exact method based on Combinatorial Benders Cuts, in which the master problem is modeled as a temporal bin packing problem with side constraints. In this computational study, we determine the conditions in which the exploitation of ultra-fast technology is useful and analyze the advantages of exploiting a mixed fleet.

Keywords E-public transport · Vehicle scheduling · Mixed fleet · Charging stations location · Fleet sizing

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1 Introduction

In the last decade, the electrification of bus fleets for use as public transport has been the subject of debates worldwide The use of electric vehicles is expected to increase due to its significant environmental benefits. However, this technology also has some drawbacks which have slowed its adoption. In fact, electric buses have a limited driving range and long recharging times, which are not compatible with tight bus schedules. Additionally, electric buses require the installation of expensive charging infrastructure, which overall makes them less flexible compared to the traditional fuel-propelled buses. In the last few years, a new type of electric vehicle, which is equipped with ultra-fast charging technology, has been introduced to the market. This ultra-fast recharge can be carried out by connecting the vehicle to a special recharging infrastructure by means of a pantograph which is installed on the top of the bus. (see Fig. 1).

This new technology reduces the recharging time to about a tenth of that of the traditional fuel-propelled buses, providing 200 km of range in less than ten minutes. However, the purchasing and infrastructure costs of these vehicles are very high. Moreover, the required chargers are more bulky than traditional ones and therefore cannot be installed in stations with relatively little space, such as those situated at city centers. However, ultra-fast recharging vehicles (UV) are already in use in some cities such as Milan, Italy. Despite their limitations, electric vehicles can be used to design an environment-friendly, cost-effective public transport system; to achieve this goal, all strategic decisions, such as charging infrastructure location and sizing, fleet sizing, and vehicle and charging scheduling, must be optimized and coordinated. This creates complex decision problems which need to be addressed with specific and powerful optimization tools (Behnia et al. 2024). This study aimed to expand the current literature in which these decision problems have been addressed separately and provide a combined decision problem which simultaneously addresses all these issues. To this end, we formulate a combined problem mathematically and provide an innovative and elegant way of modeling multiple visits to recharging stations without making dummy clones of them. Also, we provide an efficient solution based on Combinatorial Benders Cuts (CBC), in which the master problem is modeled as a temporal bin packing problem with side constraints. We are the first to combine four different features of CBC namely non-standard relaxation, Minimum Infeasible Sets,

Fig. 1 Ultra-fast recharging vehicles in Milan (source: www.hdmotori.it)

optimality cuts, and infeasibility cause detection. Further, we contribute to the existing literature by conducting an extensive computational study which we use to gain several managerial insights into the usefulness of ultra-fast recharging infrastructure and mixed fleets in e-public transport.

The remaining parts of the paper are organized as follows. In Sect. 2, we provide an extensive literature review on the optimization of e-public transport networks. In Sect. 3, we provide a description of the problem, while in Sect. 4, we present the mathematical formulation of the problem. We devote Sect. 5 to describing the proposed CBC-based solution. The computational results are discussed and analyzed in Sect. 6. In Sect. 7, we summarize our conclusions and discussed possible future developments.

2 Literature review

Designing a cost effective e-public transport network is a complex and challenging task, and several decision problems need to be simultaneously addressed. These problems can be grouped in four categories: (i) dimensioning and location planning for the charging infrastructure, (ii) fleet sizing, (iii) electric vehicle scheduling, and (iv) charging scheduling. These problems are strictly interconnected. Therefore, solving them sequentially, could yield suboptimal solutions.

The problem of locating recharging infrastructures has been widely addressed also in the context of private transportation. However, these problems are profoundly different from ours as they do not have to manage a fixed schedule for the vehicles involved in the network. Therefore our literature review focuses on problems related to public transport networks with electric vehicles. However, for a broad survey on charging infrastructure locations, we refer interested readers to Ullah et al. (2024).

This topic is extensively discussed in the literature; however, previous studies do not combine all four decision problems addressed in our work (Perumal et al. 2022). To the best of our knowledge, there is a gap in the literature with regards to the combination of the four of them. This gap is also highlighted in Behnia et al. (2024). Our goal is to fill this gap and be the first to provide a mathematical formulation and an exact and efficient solution approach to simultaneously handle all of them.

The following literature review is divided into four subsections, each of which discusses works related to a specific decision problem. Papers that address more than one problem are assigned to the section that discusses the problem on which they are mainly focused.

2.1 Charging infrastructure sizing and location planning

The strategic problem of optimizing the distribution of the charging infrastructure for electric buses within the city networks is addressed for the first time in Xylia et al. (2017). The authors considered the schedule of line buses to be the input data, and their goal is to minimize the installation costs of the charging slots needed to cover all the lines. A single charger can be installed at any bus stop, with stop-dependent costs. However, problems regarding simultaneous visits to stops are neglected, and the authors do not allow different buses to cover trips for different lines. Therefore, the resulting problem becomes rather simplified, since the only decision to be made is whether to install a charger at a bus stop or not. A real case in the city of Stockholm is reported. Kunith et al. (2017) conduct their study in a similar setting but consider the ability to install a battery on each vehicle as a decision variable itself. Three types of batteries, each with a different autonomy and cost, are considered as alternatives in their study. A case study in the city of Berlin is also discussed. In this case study, stations' capacity is assumed to be unitary, and no consideration is made about avoiding simultaneous visits to stations. Liu and Song (2017) study the location of wireless recharging pads in the road, where vehicles can recharge their battery by passing over these pads. Battery size is also considered as a decision variable in this study. However, the problem of handling simultaneous visits to the pads is neglected. Their proposed model is tested in Utah State University, which has four bus lines. This approach seems to be viable for small contexts, but it could be too expensive and difficult to realize on larger scale networks. Luo and Fan (2023) tackle the case of jointly optimized design processes. The design of fast-charge systems for urban public transport is addressed in Battaïa et al. (2023). The authors aim at maximizing the route-weighted total passenger capacity of electric buses. A mathematical model and a heuristics solution approach are presented.

Iliopoulou and Kepaptsoglou (2019) investigate the integrated transit route network design and charging infrastructure location problem. They propose a sequential approach in which optimal routes are first designed to maximize the service efficiency for passengers and then optimal charging points are provided to minimize costs. Such costs are used to guide the construction of the routes in order to achieve a balance between the two objectives. He et al. (2019) optimize charging station location and the installation of energy storage systems (ESS) which can accumulate energy outside peak hours (when it is cheaper) and supply it to the stations when needed. The goal is to minimize the overall cost of the system. An et al. (2020) provide a model to determine the optimal number of batteries, chargers, and swapping robots, as well as the type of chargers needed at battery-swapping stations, to satisfy the swapping and charging demand of a fleet of electric buses on a fixed schedule. Uncertainties on this demand are considered, and the problem is modeled as a two-stage stochastic model. A case study related to the city of Melbourne is also presented. The authors are the first to consider different types of chargers (i.e., slow and fast ones) in a public transport context. Fast chargers are four times faster but 30 times more expensive than slow ones. Experiments indicate that slow chargers are more appropriate and that in order to make fast chargers more competitive, a significant price reduction is needed.

Rogge et al. (2018) analyze data from a bus company in Los Angeles. Thee authors find that the cost competitiveness of different charging infrastructures mainly depends on the service frequency, trip length, and operating speed of the transport system. For a system with high operating speed and low service frequency, swapping stations are preferable with respect to charging lanes and stations, while for systems characterized by very low service frequency and short trips, charging stations become competitive. Charging lanes tend to be less competitive when they have high construction costs and low charging power.

He et al. (2023) address the combined charging infrastructure planning, vehicle scheduling, and charging scheduling for a fleet of electric buses. They provide a non-linear formulation but they solve the problem by means of Genetic Algorithms. Differently from our work, they do not address the fleet sizing problem but consider a fixed size fleet composed of homogeneous vehicles.

Interested readers are also pointed to a comprehensive review on charging facility planning and scheduling problems for battery electric bus systems presented by Zhou et al. (2024).

2.2 Fleet sizing

In this section, we review papers that deal with the fleet sizing problem for a mixed fleet of vehicles, including diesel vehicles (DV) and electric vehicles (EV), or a pure electric fleet with different types of EV. We do, however, not consider problems regarding a homogeneous fleet of vehicles in which the number of vehicles to purchase (or use) is a decision variable since such problems are not clearly related to our problem.

Chen et al. (2018) study a combined fleet sizing and charging facilities location problem. Different types of buses with different sizes and autonomy, as well as different recharging options, such as charging stations, inductive charging lanes, and battery swapping, are considered. Pelletier et al. (2019) study an electric bus fleet transition problem to determine bus replacement plans for transport companies to meet their electrification targets in a cost-effective way. Given the different electric bus and charger types available, the problem considers investment decisions such as the number of buses (based on the bus type being purchased) and the number of chargers (based on the charger type to be installed) over a 30-year time period (2020-2050). Li et al. (2016) solve a mixed-fleet sizing problem in which compressed-natural-gas, electric, diesel, and hybrid vehicles are considered. Their proposed model considers operating, emissions-related, and purchasing costs. However, the problem is simplified with respect to the real-world problem, as battery recharging issues for EV are ignored. They assume that vehicle autonomy is large enough to cover for the daily operations of the buses. The authors present a case study related to the city of Hong Kong and provide insights about the advantages of exploiting a mixed fleet.

Islam and Lownes (2019) study the bus fleet-replacement problem over a time period. They minimize the life cycle cost of owning and operating a fleet of buses that require charging infrastructure installation while simultaneously reducing emissions. The authors present a case study in Connecticut, in which they show the advantage of exploiting a mixed fleet.

Yıldırım and Yıldız (2021) address the electric buses fleet sizing and scheduling problem with different battery size options, while Lee et al. (2021) deal with the combined fleet and charging infrastructure sizing.

2.3 Vehicle scheduling

Vehicle scheduling problems with route constraints (VSP-RC) have been widely studied in the last four decades. However, to the best of our knowledge, all these studies simply impose a strict constraint on the vehicles' autonomy without allowing for battery recharging operations. Wang and Shen (2007) overcome this issue by adding a fixed extra service time whenever a charging operation took place after a trip. However, they do not consider partial recharges, recharging time (which is dependent on the amount of energy recharged), the time required to reach recharging stations, and the stations' capacity.

Chao and Xiaohong (2013) consider a simplified problem in which fully charged batteries are available for swapping at depots. The problem aims to minimize the number of vehicles needed to service all trips, the number of standby batteries maintained for a vehicle fleet, and the amount of power required by the charge station to meet the energy demand of the electric bus system. A similar problem is addressed by Li (2013), in which the authors assume that the number of batteries available for swapping within a specific time slot is limited. In fact, this is the first study to consider capacity issues at recharging stations.

Modelling and solution approaches for mixed-fleet single-terminal bus scheduling problems are discussed in Rinaldi et al. (2020). An exact solution approach for this problem can also be found in Alvo et al. (2021). The case of shared chargers is addressed in Ji et al. (2023), while Gkiotsalitis et al. (2023) extend the problem to multiple depots and include time windows.

Liu and Ceder (2020) study a bi-objective E-VSP. Their first objective is to minimize the total number of EV exploited, while their second objective is to minimize the number of chargers installed. Chargers have a fixed charging rate and can only be located at depots. Li et al. (2019) address the multi-depot extension of the E-VSP, also taking emission-related costs into account. Specifically, this is done in addition to vehicle purchasing and charger installation costs. Yao et al. (2020) consider vehicle purchasing costs, charging infrastructure installation costs, and costs related to deadhead arcs (i.e., arcs traveled by empty buses in order to reach the starting point of a trip from the ending point of the precedent trip). Tang et al. (2019) propose a robust version of the E-VSP to handle uncertainties regarding trip completion times due to traffic congestion. Olsen and Kliewer (2020) study the E-VSP with non-linear charging functions.

The recent comprehensive review by Bertossi et al. (1987) lists scheduling strategies and future directions in the field of electric buses.

2.4 Charging scheduling

In charging scheduling (CS) problems, the goal is to optimize recharging operations while keeping the trip assigned to each vehicle and its scheduling fixed. In other words, one has to decide between which pair of trips to insert a recharging activity. Generally, electricity prices vary as the day goes by, and this impacts charging scheduling decisions, as discussed by Leou and Hung (2017). Yang et al. (2018) study the CS for a wireless charging system under consumption uncertainty, with the aim to minimize the total operating electricity cost. According to them, the charging decision making process takes place in two stages. In the first stage, energy demand forecasting, computed on historical data, is used to determine the amount of energy to reserve for the next day at a fixed discounted price. In the second stage, the company pays the retail electricity bill if the actual energy demand exceeds that which had been reserved. Jahic et al. (2019) propose a model to minimize the demand peak height. The authors aim to smoothly distribute recharging operations in order to have a more balanced energy demand over time. Bagherinezhad et al. (2020) introduce the spatial component of the CS problem. They do not only decide when charging operations must take place but also at which station. Abdelwahed et al. (2020) study the option of installing fast-charging slots at line terminal stations in order to entirely use idle times for recharging, without requiring to travel deadhead arcs to reach the stations. However, since the number of available slots at each terminal is very limited, not all the vehicles can be charged during their idle time. The goal of the problem is to decide which vehicles must be charged and at which point in time in order to minimize charging costs, considering the fluctuations in electricity prices. Ke et al. (2020) study a system in which vehicles can be either charged or discharged at the stations. The discharged energy can then be sold to electricity companies. This way, the transportation company can modify its CS to exploit electricity price variations across time and reduce its cost or even generate an extra profit eventually. An application of this system in the city of Phengu, Taiwan, is presented. He et al. (2020) introduce a model that not only decides how much, when, and where to recharge but also the recharging speed based on speed prices and schedule constraints. Wu et al. (2023) optimize the charging scheduling in response to time-of-use recharging price considering capacitated charging stations. Klein and Schiffer (2023) study the joint vehicle scheduling and charging problem take into account battery degradation, non-linear charging time, and time-of-use dependent energy tariffs. Electric bus charging scheduling problems considering charging infrastructure integrated with solar photovoltaic and energy storage systems are addressed in Liu et al. (2024).

Dolgui et al. (2024) use classic scheduling terminology to formulate charging operations. The authors provide a notation/classification scheme for regular charging scheduling problems.

In Table 1, we provide a list of the decision problems addressed by each of the above mentioned papers. What clearly emerges from this is that our work is the first to simultaneously handle all the four decision problems arising in this context.

Our work is the first to combine fleet sizing, charging station location and sizing, vehicle scheduling, and charging stops scheduling in the context of public transport with mixed fleets. This work extends the literature by addressing this complex combined decision problem. We present a mathematical formulation and provide an innovative exact method based on a CBC framework. The proposed method also provides a methodological contribution to the literature of CBC, since it is the first to exploit an infeasibility cause detection procedure, which identifies the cause of the infeasibility and provides an ad-hoc designed strong cut for each cause. Furthermore, it combines it with a routine to detect whether the estimation of the objective function provided by the master problem was correct, and, if not, provides a no-good cut to take the underestimation into account. Furthermore, the framework builds on an innovative reformulation for the master problem, which, in contrast to classical approaches, is not obtained by relaxing some constraints, but exploits a relaxation of the original problem which can be modeled as a temporal knapsack problem with side constraints. From a managerial point of view, we provide a detailed discussion

Table 1 Overview of decision problems and the used solution approach for E-public transport optimization

	Infrastructure and location	Fleet sizing	Vehicle scheduling	Charging scheduling	Method
Xylia et al. (2017)	X				Dynamic MIP
Kunith et al. (2017)	X				MIP
Liu and Song (2017)	X				Robust Opt.
He et al. (2019)	X				MIP
Luo and Fan (2023)	X				MIP
An et al. (2020)	X				Stoch. Progr.
Rogge et al. (2018)	X				MIP&GA
Chen et al. (2018)	X	X			MIP
Pelletier et al. (2019)	X	X			MIP
Lee et al. (2021)	X	X			Non-linear Progr.
He et al. (2023)	X		X	X	MIP&GA
Li et al. (2016)		X			MIP
Islam and Lownes (2019)		X			MIP
Rinaldi et al. (2020)		X			MIP
Alvo et al. (2021)		X			MIP&Benders Decomp.
Ji et al. (2023)		X			MIP&BaP
Gkiotsalitis et al. (2023)		X			MIP
Yıldırım and Yıldız (2021)		X	X		Column Gen.
Wang and Shen (2007)			X	X	Ant Colony
Chao and Xiaohong (2013)			X	X	NSGA-II
Li (2013)			X	X	Column Gen.
Liu and Ceder (2020)			X	X	MIP
Li et al. (2019)			X	X	MIP
Yao et al. (2020)			X	X	Heuristics
Tang et al. (2019)			X	X	Robust Opt. &BaP
Olsen and Kliewer (2020)			X	X	Nonlinear Opt.
Klein and Schiffer (2023)			X	X	Nonlinear Progr. &BaP
Leou and Hung (2017)				X	Nonlinear Progr.
Yang et al. (2018)				X	Nonlinear Progr.
Jahic et al. (2019)				X	Heuristics
Bagherinezhad et al. (2020)				X	Simulation
Abdelwahed et al. (2020)				X	MIP
Ke et al. (2020)				X	GA
He et al. (2020)				X	MIP&Nonlinear Progr.
Wu et al. (2023)				X	Simulation
Liu et al. (2024)				X	MIP
This work	X	X	X	X	MIP&CBC

on the benefits and limits of the integration of ultra-fast charging electric vehicles in a public transport system.

3 Problem description

The presented problem relates to decisions that have to be made by providers of public transportation systems with existing networks. The resulting integrated decision problem encompasses four types of decisions: (i) fleet sizing, (ii) recharging station location and sizing, (iii) vehicle scheduling, and (iv) charging stops scheduling. We denote the combined problem as Integrated Fleet Sizing, Charging Station Location and Sizing, and Vehicle- and Charging Scheduling (IFS_CSLS_VCS). Basing on data related to the vehicles considered, we can assume that the battery autonomy allows to cover a whole working day with one single full recharge. To avoid equivalent solutions which indicate different combinations of short recharging stops, and for providing the drivers a smoother and simpler schedule, we impose that at most one recharging stop per day can be carried out by a vehicle. Each trip represents a single run. If we have a line which repeats multiple times the same loop we generate a separate trip for each run. The advantage of this representation is that we can assign different runs of the same service to different vehicles and model recharging stops during the breaks between consecutive trips.

The goal of the problem is to cover a set of scheduled bus trips *I* at the minimum global cost. Each trip *i* was characterized by a starting and an arrival location, s_i and a_i, and a starting and arrival time, t_i^s and t_i^a. The duration of the trip d_i was defined as t_i^s-t_i^a. The travel time between trips *i* and *j*, t_{ij}, is defined as the time taken to reach the starting location of trip *j* from the arrival location of trip *i*. These values are all expressed as integer multiples of the time unit considered, which is 5 min. This way, we are able to round up times, allowing flexibility for small expected delays. As our work assumes already scheduled trips, we do not take traffic management strategies into consideration. These would, however, be of high relevance if customer- or provider choices on using or offering lanes would be included (Magnanti and Wong 1984; Kapitanov et al. 2018). Also, operational decisions, such as crew scheduling are clearly out of scope of this study.

Three set of vehicles are available to cover all trips: TV, EV, and UV, denoted as K^T, K^E, and K^U, respectively. The whole set of available vehicles is defined as $K = K^T \cup K^E \cup K^U$. All the vehicles are assumed to be initially located at a depot 0, to which they must return at the end of the day.

Each electric vehicle (i.e., EV and UV) *k* is associated with a purchasing cost f_k^A and a battery capacity Q_{max}^k which is expressed in units of time. If the total workload of the vehicle is larger than Q_{max}^k, then a recharging stop to a compatible slot must be planned. The electricity cost is not explicitly considered, since large public transportation companies can negotiate advantageous flat tariff contracts with electricity companies. For simplicity we assume that this cost is embedded in the vehicle purchasing cost.

The recharging rate of a vehicle is denoted as δ_k. EV can only be recharged at standard recharging points and at a recharging rate $\delta_k = r^E$, whereas UV can only be recharged at ultra-fast recharging points, at a recharging rate $\delta_k = r^U$, with $r^E << r^U$, obviously. The length of the recharging stop is defined as an integer of time units and must be long enough to provide the minimum amount of charge required by the vehicle to complete its workload. We also assume that a slow-charging plug-in system is available at the depot and that each bus parking slot is equipped with a plug. Therefore, all the vehicles can be simultaneously recharged overnight in order to have a full battery availability for the next day.

Since a fully packed daily schedule can be completed with only one intermediate recharge, we propose, without loss of generality, that each vehicle make at most one recharging stop along its daily trip. This allows us to exclude all the unrealistic solutions in which vehicles stop several times for very short recharges. A set, L, of potential locations for recharging stations is assumed. Each location *l* has a maximum number of EV recharging points, ξ_l^E, and UV recharging points, ξ_l^U, to be installed. Moreover, a global maximum number of recharging points, $\bar{\xi_l}$, is defined. This could be less than ξ_l^E+ξ_l^U. Each EV recharging point is associated with an installation cost f^E, while f^U represents the installation cost of UV recharging points. For each potential station location, the travel time to and from each trip's arrival and starting locations, expressed as t_{il} and t_{li}, respectively, is known. We assume that the transportation company owns all recharging points, for which it pays a flat rate which we consider to be included in the installation cost. We define T_{max} as the latest time at which a vehicle can return to the depot, where $T_{max} = \max_{i \in I} t_i^A + t_{i0}$ and 0 is the depot. The amount of energy consumed in completing each trip, $\bar{q}_i$, and the amount of charge needed to reach the starting point of the next task, the depot, and each charging station, q_{il}, are assumed to all be known. Different from EV and UV, TV are considered to be part of the fleet already owned by the company, so no purchasing costs arise. However, due the high price of fuel, they have a usage cost per time unit c^T that could not be neglected.

Since TV are propelled by fuel, deadheads are also considered in order to determine the operational costs of the vehicle. The fixed cost for a time unit of travel with TV is denoted as γ^T. Note that since we are addressing a tactical decision problem, in order to make comparable purchasing costs which are only paid once and traveling costs which are paid every day, γ^T include the cost for a single time unit, c^T, multiplied by the number of days in the life-cycle of the vehicle. The objective is to minimize the global cost, which is expressed as the sum of vehicle purchasing costs, recharging stations installation cost, and traveling cost. An example problem is depicted in Fig. 2, while the optimal solution for this problem is given in Fig. 3. To perform trips B, C, and D with the same vehicle an UV is needed since the schedule is tight and there are only few minutes available for recharging. Instead, for E, F, and G an EV can be used since there is a long idle time between the end of F and the start of G, which can be used to recharge. Trip A is quite short and near to the depot, therefore it is more convenient to perform it with a TV than to install an additional EV.

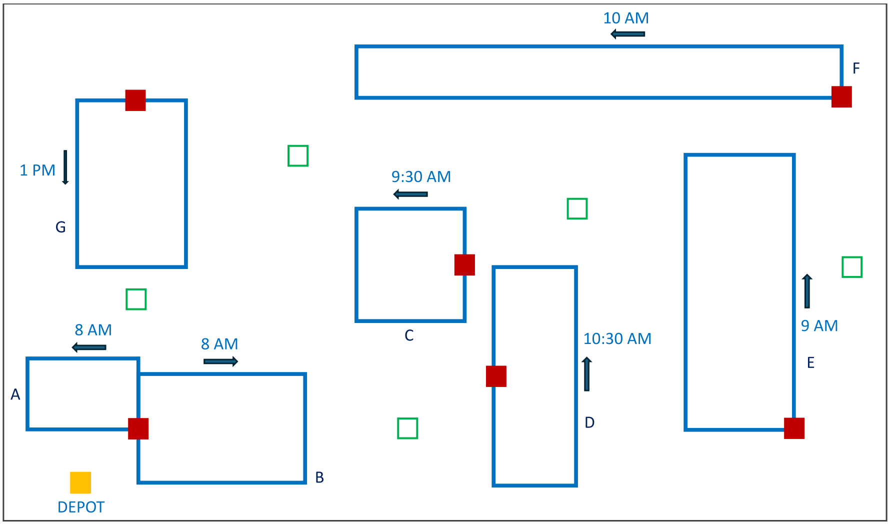

Fig. 2 Example of the decision problem. Red squares are starting and ending points of trips, while black boxes depict the respective trips (starting times and directions are given next to the trip). Green boxes give potential locations of charging stations

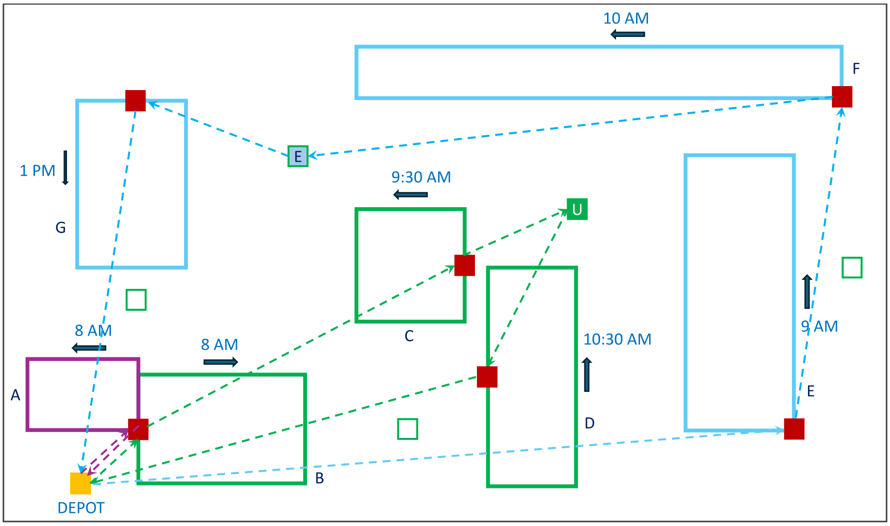

Fig. 3 Example of the decision problem. Red squares are starting and ending points of trips, while black boxes depict the respective trips (starting times and directions are given next to the trip). Green boxes give potential locations of charging stations. Green dashed lines show the trips of UV, black ones those of EV, and purple ones TV trips

4 Mathematical model

The decision problem described in the previous section turns out to be an integrated fleet sizing, facility location, vehicle scheduling, and charging scheduling problem on a tactical level. This problem is $\mathcal{NP}$-hard since it is a combination of several $\mathcal{NP}$ -hard problems, and therefore, it is very challenging to solve. As commonly done in the literature, a vehicle scheduling problem can be modeled as a vehicle routing problem with time windows (VRPTW) on a virtual network, in which nodes represent trips (or, more generally, tasks). Service time windows, where the start and end of the service are included, are used to impose a fixed starting time for each trip. Although this kind of formulation is broadly used in vehicle scheduling and allows for the handling of realistic instances, things become more complicated when dealing with EV, which may need to stop at charging stations to recharge their battery so as to have enough autonomy to complete their list of trips. These problems are by far more difficult to model since special nodes, representing charging stations, must be included in the network, with the feature that they can be visited several times by different vehicles.

In this study, the virtual network includes three type of nodes: the depot, trips' virtual nodes, and charging stations. Trip nodes are virtual since they do not correspond to a single physical location, whereas charging stations are real nodes with fixed coordinates. The depot, 0, is a special virtual node with $t_0^A = t_0^E = 0$ and starting and arrival locations equal to it. Distances to and from virtual nodes are computed as the distance between the arrival point of the trip associated with the first node to the starting point of the trip associated with the second node. The resulting problem become similar to the Green Vehicle Routing Problem introduced by Erdoğan and Miller-Hooks (2012), who model it with a 2-index arc formulation in which the variable y_{ij} indicates whether a vehicle visits node *j* immediately after node *i* by cloning charging stations' nodes and imposing that each cloned node can only be visited once. This formulation has been shown to provide very poor bounds, and it has a very limited efficiency, only allowing for the handling of very small instances.

In this paper, we provide a mixed 2-index and 3-index formulation. In fact, we use 3-index variables for arcs traveled by electric vehicles (EV and UV), and 2-index variables for arcs traveled by TV, which do not need to make multiple visits to nodes. Furthermore, while arc variables for EV and UV are defined over the whole network (including depot, trips, and recharging stations), those for TV are defined over a restricted network (including only depot and trips), helping to further reduce the number of variables involved.

We define x_{ijk} as a binary variable with a value of 1 if arc *ij* is covered by vehicle *k*, and a value of 0 otherwise. It is worth noting that such variables are only defined for EV and UV and not for TV. For arcs traveled by TV, we use 2-index binary variables, y_{ij}, taking a value of 1 if the arc *ij* is covered and a value of 0 otherwise. The arrival time at the starting node of a trip is tracked using the 1-index variable τ_i, whereas, since charging stations may be visited multiple times by different vehicles, we track the arrival time of vehicle *k* at station *l* using a 2-index variable T_{lk}. The number of charging slots for EV and UV are tracked using η_l^E and η_l^U, respectively. Finally, the time spent by vehicle *k* recharging at station *l* is tracked by r_{tk}

Another important novelty, with respect to the literature, is how charging stations' capacity is handled. This capacity issue has been addressed in a study by Bruglieri et al. (2019), where the authors introduce variables that indicated whether two vehicles simultaneously visit a charging station and added constraints to ensure that the number of simultaneous visits is smaller or equal to the number of recharging slots. However, this number of constraints grows with the binomial coefficient $\binom{\xi_l}{|K|}$ and makes the problem intractable for large values of these two parameters Bruglieri et al. (2021b). To overcome this issue, we split the time horizon into time slots and introduced variables, ρ_{lkh}, which indicate whether a vehicle *k* is recharging at station *l* in time slot *h*. This way, we are able to add a single constraint to each station and time-slot to handle the capacity constraint, which strongly reduces the global number of constraints. Also, we introduce variable, s_{lkh}, which indicates whether vehicle *k* has started its recharging operation at station *l* in time slot *h*. These variables allow for the imposition that each vehicle can only recharge once per day.

The residual battery charging level of vehicle *k* when reaching station *l* is tracked by variable Q_{lk}, and the residual autonomy when reaching the starting point of its trip *i* is tracked by $\bar{Q}_i$. In this case, as it holds for the time tracker variables, the second index is not needed since trip nodes could only be visited once. Finally, variables z_k indicate whether vehicle *k* was purchased or not. Table 2 lists all the sets, parameters, and variables.

$$\min \sum_{k\in\tilde{K}} f_k^A z_k + \sum_{l\in L} f^E \eta_l^E + \sum_{l\in L} f^U \eta_l^U + D \sum_{i\in I} \sum_{j\in I\cup 0} c_i^T y_{ij} + D \sum_{i\in I\cup 0} \sum_{j\in I\cup 0} c_{ij} y_{ij} \tag{1}$$

$$\sum_{k\in\tilde{K}, j\in N} x_{ijk} + \sum_{j\in I\cup 0} y_{ij} = 1 \quad \forall\, i \in I \tag{2}$$

$$\sum_{i\in N} x_{ijk} \le 1 \quad \forall\, j \in L \tag{3}$$

$$\sum_{j\in I\cup 0} y_{ij} = \sum_{j\in I\cup 0} y_{ji} \quad \forall\, i \in I \tag{4}$$

$$\sum_{j\in N} x_{ijk} = \sum_{j\in N} x_{jik} \quad \forall\, i \in N,\ k \in \tilde{K} \tag{5}$$

$$\sum_{i\in N} \sum_{j\in L} x_{ijk} \le 1 \quad \forall\, k \in \tilde{K} \tag{6}$$

$$\sum_{j\in N} x_{0jk} = z_k \quad \forall\, k \in \tilde{K} \tag{7}$$

$$T_{jk} \ge t_i^A + t_{ij} - 2T_{max}(1 - x_{ijk}) \quad \forall\, j \in L,\ i \in I \cup 0,\ k \in \tilde{K} \tag{8}$$

Table 2 List of sets, parameters, and variables

Sets:	
K^T	set of TV
K^E	set of EV
K^U	set of UV
$\tilde{K}$	$K^E \cup K^U$
K	$K^E \cup K^U \cup K^T$
I	set of trips
L	set of potential charging station locations
N	$I \cup L \cup 0$ set of nodes in the network
H	set of time slots
Parameters:	
t_i^S	starting time *i*
t_i^A	arrival time *i*
$\bar{q}_i$	amount of energy needed to perform trip *i*
T_{max}	latest arrival time at depot
Q_{max}^k	battery capacity for vehicle *k*
c_{ij}	cost for traveling between the arrival point of trip *i* and starting point of trip *j* with a TV
r^E	recharging rate for EV
r^U	recharging rate for UV
ξ_l^E	maximum number of EV recharging slots that can be installed at location *l*
ξ_l^U	maximum number of UV recharging slots that can be installed at location *l*
$\bar{\xi}_l$	maximum number of total recharging slots that can be installed at location *l*
f^E	cost of installing a recharging slot for EV
f^U	cost of installing a recharging slot for UV
f_k^A	purchasing cost for vehicle *k*
D	life-cycle of vehicles
c_i^T	cost for executing trip *i* with a TV
δ_k	recharging rate of vehicle *k*
Variables:	
x_{ijk}	value 1 if arc *ij* is covered with vehicle *k* (only for EV and UV), 0 otherwise
y_{ij}	value 1 if arc *ij* is covered by a TV, 0 otherwise
z_k	value 1 if vehicle *k* is used, 0 otherwise
τ_i	arrival time at node *i*
T_{lk}	arrival time at station *l* with vehicle *k*
Q_{lk}	residual autonomy of vehicle *k* when reaching station *l*
$\bar{Q}_i$	residual autonomy of vehicle *k* when reaching the starting point of trip *i*

Table 2 (continued)

Variables:	
η_l^E	# EV charging slots at node l
η_l^U	# UV charging slots at node l
s_{lkh}	value 1 if vehicle k is starting to recharge at time slot h at station l, 0 otherwise
ρ_{lkh}	value 1 if vehicle k is recharging during time slot h at station l, 0 otherwise
r_{lk}	recharging time spent by vehicle k at station l

$$\tau_j \geq t_i^A + t_{ij} - 2T_{max}(1 - \sum_{k \in \tilde{K}} x_{ijk}) \quad \forall\, j \in I,\ i \in I \cup 0 \tag{9}$$

$$\tau_j \geq T_{lk} + r_{lk} + t_{lj} - 2T_{max}(1 - x_{ijk}) \quad \forall\, j \in I,\ l \in L,\ k \in \tilde{K} \tag{10}$$

$$\tau_j \geq t_i^A + t_{ij} - 2T_{max}(1 - y_{ij}) \quad \forall\, j \in I,\ i \in I \cup 0 \tag{11}$$

$$\tau_j \leq t_j^s \quad \forall\, j \in I \tag{12}$$

$$\sum_{k \in K^T} y_{oj} \leq |K^T| \tag{13}$$

$$T_{lk} \geq (h - 0.99)s_{lkh} \quad \forall\, l \in L,\ h \in H,\ k \in \tilde{K} \tag{14}$$

$$T_{lk} \leq h s_{lkh} \quad \forall\, l \in L,\ h \in H,\ k \in \tilde{K} \tag{15}$$

$$\sum_{h \in H} s_{lkh} = \sum_{j \in I \cup 0} x_{jlk} \quad \forall\, l \in L,\ k \in \tilde{K} \tag{16}$$

$$\sum_{h \in H} \rho_{lkh} \geq r_{lk} \quad \forall\, l \in L,\ k \in \tilde{K} \tag{17}$$

$$\left(\sum_{h \in H} \rho_{lkh} - 0.99\right)\delta_k \leq r_{lk} \quad \forall\, l \in L,\ k \in \tilde{K} \tag{18}$$

$$\sum_{h \in H} \sum_{l \in L} s_{lkh} \leq 1 \ \ \forall\, k \in \tilde{K} \tag{19}$$

$$\rho_{lkh} \leq \rho_{lkh-1} + s_{lkh} \quad \forall\, l \in L,\ h \in H \setminus 1,\ k \in \tilde{K} \tag{20}$$

$$\rho_{lk1} = 0 \quad \forall\, i \in L,\ k \in \tilde{K} \tag{21}$$

$$\sum_{k \in K^E} \rho_{lkh} \leq \eta_i^E \quad \forall\, l \in L,\ h \in H \setminus 1,\ k \in \tilde{K} \tag{22}$$

$$\sum_{k \in K^U} \rho_{lkh} \leq \eta_l^U \quad \forall\, l \in L,\ h \in H \setminus 1,\ k \in \tilde{K} \tag{23}$$

$$\eta_l^E \leq \xi_l^E \quad \forall\, l \in L \tag{24}$$

$$\eta_l^U \leq \xi_l^U \quad \forall\, l \in L \tag{25}$$

$$\eta_l^E + \eta_l^U \leq \bar{\xi}_l \quad \forall\, l \in L \tag{26}$$

$$Q_{lk} \leq \bar{Q}_i - \bar{q}_i - q_{il} + 2Q_{max}^k(1 - x_{ilk}) \quad \forall\, l \in L,\ i \in I \cup 0,\ k \in \tilde{K} \tag{27}$$

$$0 \leq Q_{lk} \leq Q_{max}^k \quad \forall\, l \in L,\ k \in \tilde{K} \tag{28}$$

$$\bar{Q}_j \leq \bar{Q}_i - \bar{q}_i - q_{il} + 2Q_{max}^k(1 - \sum_{k \in \tilde{K}} x_{ijk}) \quad \forall\, j \in I,\ i \in I \tag{29}$$

$$\bar{Q}_j \leq Q_{lk} + r_{lk}\delta_k - q_{ij} - 2Q_{max}^k(1 - x_{ijk}) \quad \forall\, j \in I,\ l \in L,\ k \in \tilde{K} \tag{30}$$

$$\bar{Q}_j \geq \bar{q}_j + q_{j0} - 2Q_{max}^k(1 - \sum_{k \in \tilde{K}} x_{0jk}) \quad \forall\, j \in I \tag{31}$$

$$x_{ijk} \in \{0,1\} \quad \forall\, i \in I,\ j \in I,\ k \in K \tag{32}$$

$$y_{ij} \in \{0,1\} \quad \forall\, i \in I,\ j \in I \tag{33}$$

$$z_k \in \{0,1\} \quad \forall\, k \in K \tag{34}$$

$$\rho_{lkh} \in \{0,1\} \quad \forall\, l \in L,\ k \in K,\ h \in H \tag{35}$$

$$s_{lkh} \in \{0,1\} \quad \forall\, l \in L,\ k \in K,\ h \in H \tag{36}$$

$$\eta_h^E \in \mathbb{Z}^+ \quad \forall h \in H \tag{37}$$

$$\eta_h^U \in \mathbb{Z}^+ \quad \forall h \in H \tag{38}$$

The objective function is given in (1) and is composed of five terms: (i) vehicle purchasing costs, (ii) installation costs for standard electric charging slots, (iii) ultra-fast slots, (iv) traveling costs for trip performed by TV, and (v) costs for deadheads traveled between consecutive trips or to and from the depot. Each trip must be performed either by an electric vehicle (EV or UV) or by a TV, as expressed by Constraint (2).

Each recharging station can be visited at most once by each vehicle, as stated by Constraint (3). In the virtual network, whichever vehicle enters a node must also be the vehicle that exits the node. This is true for both TV (4) and both types of EVs (5). Each trip can only be covered by one vehicle as stated by Constraint (6). Constraint (7) allows for the identification of whether an EV or UV has been used. As expressed by Constraint (8), the arrival time of vehicle k to charging station j must be greater than the sum of the time taken to end trip i executed by k immediately before visiting that station and the time taken to reach the station from the ending location of i (i.e., the time needed to connect nodes i and j in the virtual network). If trip j is performed by a vehicle immediately after trip i, the arrival time at the starting location of j cannot be less than the sum of the ending time of trip i and the time taken to reach the starting location of j from the ending location of i (or, more simply, the time needed to reach node j from node i on the virtual network), as imposed by Constraint (9) for EV and by Constraint (11) for TV. If a trip j is performed right after a visit to recharging station l, the arrival time at j must be greater than the sum of the arrival time at l of the vehicle performing j, the time spent by k recharging at l, and the time needed to reach node j from node l in the virtual network (10). The arrival time at trip node j must be less than or equal to the starting time of trip j, as imposed by Constraint (12). Constraint (13) ensures that the number of TV exploited is not higher than the number of available TV. The combination of Constraints (14) and (15) allows for the identification of the time slot h in which a vehicle k starts recharging at station l. Constraint (16) imposes that if a vehicle reaches a station, it must recharge there. This allows us to exclude all dummy solutions in which a vehicle visits a station without stopping for a recharge. The amount of charge gotten by a vehicle is proportional to the number of time slots spent recharging. This is imposed by Constraints (17) and (18). Each vehicle can only recharge once along its route, as imposed by Constraint (19). A vehicle can only be in recharging mode at time slot h if it is already recharging in time slot $h-1$ or if it starts its recharging in time slot h. This allows us to impose recharge continuity. No vehicle can recharge in time slot 1 since all EV and UV are supposed to start their route with a completely full battery. This is shown in Constraint (20). The number of vehicles recharging in the same time slot at the same station cannot exceed the number of recharging slots installed at that station. This holds true both for EV as given in Constraint (22) and UV given in Constraint (23). The number of each slot type installed at a station cannot exceed the maximum number allowed, which is indicated by Constraints (24) and (25). Moreover, the total number of slots installed at a station cannot exceed the total number allowed, which is shown in Constraint (26). The residual charge of vehicle k on arrival at station l must be less than or equal to the residual charge at node i, associated with the last trip performed by k before visiting l, minus the amount of energy consumed by performing trip i and that needed to reach node l from node i. This dependency is formulated in Constraint (27). Vehicles may never run out of battery, and the total amount of charge can never exceed the battery capacity, which is ensured by Constraint (28). If an EV performs trip j immediately after trip i, its residual charge when starting trip j must be less than or equal to its residual charge when starting trip i minus the amount of energy consumed by performing trip i and that needed to reach node j from node i in the virtual network. This is given in Constraint (29). Conversely, if trip j is executed immediately after a visit to recharging station l, the residual charge before starting j must be less than the sum of the residual charge when visiting station l and the amount of charge gotten at l minus the energy consumed to reach node j from node l in the virtual network. This is ensured by Constraint (30). The residual charge when starting trip j must be sufficient for the vehicle to complete the trip and return to the depot, provided j is the last trip to be executed by the vehicle. This is formulated in Constraint (31). Finally, Constraints (32)– (38) specify variable domains. Note that the problem is $\mathcal{NP}$-hard and can therefore not be solved in reasonable time for realistically sized problems as addressed in our work.

5 A combinatorial benders cuts-based approach

The proposed solution approach, CBC, which was introduced by Codato and Fischetti (2006), is suitable for solving combinatorial optimization problems, especially those involving a large number of logical implications among variables. According to this approach, the original problem (OP) can be split into a master problem (MP), which involves only the variables contributing to the objective function, and a subproblem (SP), which contains only the variables responsible for feasibility. This way, the SP becomes a pure feasibility problem, and MP and SP are iteratively executed. The optimal solution of MP is injected into SP to check its feasibility. If SP indicates infeasibility, the so called *feasibility cut* is generated and added to MP. This way, it cuts-off this solution from MP, and MP is solved again. The procedure terminates as soon as a solution is proven to be feasible by SP. The method has been shown to converge to an optimal solution, if it exists, or to prove OP's infeasibility, in a finite number of steps.

CBC has been successfully applied to problems arising in different fields, including logistics (Bai and A. Rubin 2009; Bruglieri et al. 2021a), transportation (Mancini and Gansterer 2021a; Leutwiler and Corman 2022; Eom and Kim 2023), packing (Côté et al. 2014), port operations (Chen et al. 2012; Verstichel et al. 2015), healthcare (Taşkin and Cevik 2013), production (Akpinar et al. 2017; Furugi 2022; Huang et al. 2022; Li et al. 2022), and cloud computing (Mancini et al. 2021, 2022). Also, many extensions of the basic version of CBC are proposed in the literature, in order to speed-up the method's performance. The additional features exploited in these extensions can be grouped into 4 categories:

- **Non-standard relaxation**: In the standard CBC, the MP is obtained by simply deleting some constraints from the OP model. However, in some cases, better results are obtained by using non-standard relaxations to construct MP, such as reformulating the problem as a simpler combinatorial optimization problem. For instance, if OP is a scheduling problem, MP can be described as a *knapsack problem*.
- **Minimum Infeasible Sets (MIS)**: In the standard CBC, the cut added to exclude the infeasible solution forces at least one of the variables currently set to 1 to take the value 0. When the number of variables involved is very large, the cut can become very weak. Stronger cuts can be obtained by identifying a smaller subset

of variables responsible for infeasibility. This set is known as MIS. Further, the standard cut can cut-off just a single solution from the solution space. This slows down the convergence of the standard CBC. Stronger cuts help to significantly speed-up the convergence towards the optimal solution. However, the search for MIS could be computationally expensive. Therefore, it is of crucial to design efficient methods of computing MIS.

- **Optimality cuts:** In some cases, MP does not compute the exact value of the objective function and only gives a rough estimate of it. This may happen when some variables, which potentially contribute to the objective function, are relegated to SP in order to make MP easier to solve. In such cases, SP's role is twofold: it checks the feasibility of the solution provided by MP and computes the actual value of the objective function corresponding to that solution. If this value is lower then the estimation made by MP, an optimality cut is added such that if all the variables equal to 1 in the optimal solution of MP are selected, then a penalty term is added to the objective function to compensate for the estimation error made at the previous iteration. If SP indicates that the solution of MP is infeasible, a standard feasibility cut is added to MP.
- **Infeasibility cause detection (ICD):** In some cases, the infeasibility of the MP solution may be caused by different factors. Detecting the actual cause of infeseability allows us to identify ad-hoc stronger cuts involving larger portions of the solution space. Furthermore, instead of checking the overall feasibility of a solution by solving a mathematical model for SP, which can be time consuming, we can use ad-hoc efficient algorithms to detect the exact cause of the infeasibility, speeding up the process. In Table 3, we provide references to these additional features exploited in the literature. Note that the usage of MIS is quite common, as most of the algorithms contain this feature, whereas only a few of them exploit non-standard relaxation and/or optimality cuts. ICD is a largely unexplored

Table 3 Features exploited by papers proposing CBC algorithms

	Non-std. relax	MIS	Optim. cuts	ICD
Bai and A. Rubin (2009)	X			
Côté et al. (2014)		X		
Chen et al. (2012)		X		
Verstichel et al. (2015)		X		
Taşkin and Cevik (2013)		X		
Akpinar et al. (2017)				
Mancini et al. (2021)	X		X	
Bruglieri et al. (2021a)		X		
Mancini and Gansterer (2021a)	X	X		X
Mancini et al. (2022)	X		X	
Huang et al. (2022)		X		
Furugi (2022)				
Eom and Kim (2023)		X		
Li et al. (2022)		X	X	
Leutwiler and Corman (2022)		X		
This work	X	X	X	X

concept as it is only addressed in a study by Mancini and Gansterer (2021a). Our study is the first to simultaneously make use of all four of the above-mentioned features, and therefore, our approach is also innovative from a methodological point of view.

5.1 Master problem

In our methodology, we used a non-standard relaxation to derive the MP. In fact, the MP is modeled as a multi-dimensional *temporal bin packing* with activity-dependent capacity, where there are two possible activities to be performed: traveling (i.e., executing trips) and recharging. Temporal bin packing is an extension of the classical bin packing problem, in which items consume the bins' capacity only for a restricted period of time (Dell'Amico et al. 2020). The problem we introduce is an extension of temporal bin packing, in which we identify each vehicle as a bin with unitary usage capacity for each time slot h (i.e., it can perform at most one trip in each time slot) and a residual autonomy Q_{kh} which depends on the activities performed in the previous time slots (i.e., traveling or charging). Traveling reduces the residual autonomy, whereas recharging increases it. If a vehicle does not perform any activity, its residual autonomy does not change. Only one trip can be executed by a vehicle in each time slot. We define trips as items which have a unitary demand for all the time slots included between their start and ending times and 0 for the remaining time slots. Recharging activities are defined as special items which have a unitary demand for each time slot in which they are performed and 0 otherwise. These items do not have a fixed duration or starting/arrival time and are not compulsory. Their role is to increase the residual autonomy of the vehicle by a constant rate of r^U (for UV) or r^E (for EV) for each time slot in which they are in use. While the recharging rate per unit of time is constant for each vehicle of the same type, the unitary consumption rate is item-dependent, and it is defined for each trip as g_{ih}, which is obtained by dividing the total amount of energy required by a trip, $\hat{q}_i$, by the trip's duration $t_i^A - T_i^S$. Only one special item can be assigned to each vehicle, i.e., only one daily recharging stop is allowed per vehicle. Special items cannot be assigned to TV (because they are not needed), which are represented as mono-dimensional knapsacks having a unitary usage capacity. In fact, Q_{kh} was only defined for EV and UV. Two new decision variables, λ^E and λ^U, representing the number of slots for EV and UV recharging, respectively, are introduced to deal with recharging slot installation. Note that in the original problem, we have variables indicating the number of slots installed at each station. In this relaxation instead, we are not explicitly treating the location aspect. Hence, we only decide on the number of slots to install and not on where to install them. The number of slots of a specific type installed must be greater than or equal to the maximum number of vehicles of that type performing a recharging activity in the same time slot. An additional set of binary decision variables y_{ik} is introduced to model the assignment of trips to vehicles.

For TV, we introduce variables τ_i, T_i, and σ_i, which represent the duration of the deadhead arc connecting trip from the depot to the starting point of i (in cases where i is the first trip taken by the vehicle), the duration of the deadhead arc connecting trip i and the successive trip, and the duration of the deadhead arc connecting trip from the arrival point of i to the depot (in cases where i is the last trip executed by the vehicle), respectively. Recall that for EV and UV, only the purchasing cost, f^A, is paid since electricity costs are not explicitly addressed but embedded in slot installation costs which include a flat tariff that has been agreed upon with the electricity company. Therefore, we do not need to explicitly consider deadhead arcs traveled by EV and UV in the objective function. Nevertheless, such arcs consume a certain amount of energy, which cannot be neglected if the feasibility of vehicle scheduling is to be ensured. Therefore, we introduce variables α_i, γ_i, and β_i, which play a similar role as τ_i, T_i, and σ_i for TV, except that instead of tracking deadheads duration, they track battery consumption. It is worth noting that the TV schedules provided so far are certainly feasible, while the feasibility for EV and UV schedules obtained using this relaxed problem is only guaranteed for vehicles that do not require recharging along their routes. In fact, deadhead arcs needed to reach and depart from a recharging station are not considered in this relaxed problem, and this can make the schedules obtained by MP infeasible for OP. A graphical representation of MP is shown in Fig. 4. Note that due to the applied non-standard relaxation, only constraints (19), (20), and (28) remain in the MP in their original form. SP, however, checks if the MP, which is formulated as temporal bin packing is feasible and whether the actual objective function value corresponds to that of MP. As traveling and charging can be seen as temporal occupation of vehicles, where the former increases capacity (i.e., energy), while the latter decreases capacity, formulating MP as temporal bin packing problem seems to be a reasonable choice. Details on SP are given in the following.

The resulting MP can be modeled as follows.

$$\min \sum_{k\in\tilde{K}} f_k^A z_k + f^E\lambda^E + f^U\lambda^U + D * c_i^T \sum_{k\in Kt} y_{ik} + D\sum_{i\in I}(\tau_i + T_i + \sigma_i) \tag{39}$$

$$\sum_{k\in K} y_{ik} = 1 \quad \forall\, i \in I \tag{40}$$

$$\sum_{i\in I} g_{ih} y_{ik} + \rho_{kh} \leq 1 \quad \forall\, i \in I \tag{41}$$

$$0 \leq Q_{kh} \leq Q^k_{max} \quad \forall\, k \in K,\ h \in H \tag{42}$$

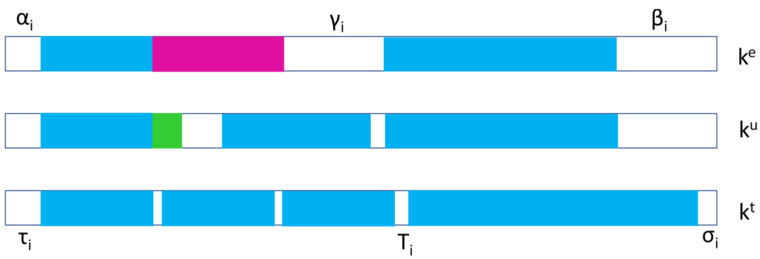

Fig. 4 A graphical representation of the MP. Trips are depicted in cyan and recharging activities in magenta (for EV) and in green (for UV)

$$\sum_{h\in H} s_{kh} \leq 1 \quad \forall\, k \in \tilde{K} \tag{43}$$

$$\rho_{kh} \geq s_{kh} \quad \forall h \in H \setminus 1,\ k \in \tilde{K} \tag{44}$$

$$\rho_{kh} \leq \rho_{kh-1} + s_{kh} \quad \forall\, h \in H \setminus 1,\ k \in \tilde{K} \tag{45}$$

$$Q_{kh} \leq Q_{kh-1} + \delta_k \rho_{kh-1} - \sum_{i\in I} g_{ih-1} y_{ik} \quad \forall\, h \in H \setminus 1,\ k \in \tilde{K} \tag{46}$$

$$z_k \geq \frac{1}{|I|} \sum_{i\in I} y_{ik} \quad \forall\, k \in \tilde{K} \tag{47}$$

$$\sum_{k\in K^E} \rho_{kh} \leq \lambda^E \tag{48}$$

$$\sum_{k\in K^U} \rho_{kh} \leq \lambda^U \tag{49}$$

$$\tau_i \geq c_{0i}(y_{ik} - \sum_{u\in 1..i-1} y_{uk}) \quad \forall\, k \in K^T,\ i \in I \tag{50}$$

$$\sigma_i \geq c_{i0}(y_{ik} - \sum_{u\in i+1..|I|} y_{uk}) \quad \forall\, k \in K^T,\ i \in I \tag{51}$$

$$T_i \geq c_{ij}(y_{ik} + y_{jk} - \sum_{u\in i+1..j-1} y_{uk} - 1) \quad \forall\, k \in K^T,\ i \in I,\ j \in I | j > i \tag{52}$$

$$\alpha_i \geq q_{0i}(y_{ik} - \sum_{u\in 1..i-1} y_{uk}) \quad \forall\, k \in \tilde{K},\ i \in I \tag{53}$$

$$\beta_i \geq q_{i0}(y_{ik} - \sum_{u\in i+1..|I|} y_{uk}) \quad \forall\, k \in \tilde{K},\ i \in I \tag{54}$$

$$\gamma_i \geq q_{ij}(y_{ik} + y_{jk} - \sum_{u\in i+1..j-1} y_{uk} - 1) \quad \forall\, k \in \tilde{K},\ i \in I,\ j \in I | j > i \tag{55}$$

$$\sum_{h\in H} s_{kh} \geq \frac{1}{Q_{max}} \left[\sum_{i\in I} qt_i y_{ik} + \sum_{i\in I} (\alpha_{ik} + \beta_{ik} + \gamma_{ik}) \right] \quad \forall\, k \in \tilde{K} \tag{56}$$

The objective function given in (39) aims to minimize the total costs for the company, which is a sum of the vehicle purchasing, recharging slots installation, and TV traveling costs. Each trip must be performed by a vehicle as stated in Constraints (40). A vehicle can perform at most one activity in each time slot. This means that it can either recharge or execute a trip (see Constraint (41)). A vehicle can never neither run out of battery nor achieve a residual autonomy larger than its maximum autonomy. This is ensured by Constraint (42). Only one recharge stop per vehicle is allowed, which is indicated by the combination of Constraints (43), (44), and (45). Constraint (46) tracks the residual battery of the vehicle at the beginning of each time slot, which cannot be greater than the difference between the residual battery at the beginning of the previous time slot and the quantity consumed plus the amount of energy recharged in that time slot. This is only valid from the second time slot on. The residual battery at the first time slot is only bounded by Q^k_{max}, as implied by Constraint (42). Constraint (47) states that if at least one trip has been assigned to a vehicle, it is marked as used, and therefore, the corresponding purchasing cost must be paid. The number of slots installed for each type of vehicle must be greater than or equal to the maximum number of vehicles of that type simultaneously recharging in the same time slot, as stated by Constraints (48) and (49), respectively. Constraint (50) allows for the computation of the duration of the deadhead arcs between the depot and the starting point of the first trip executed by a vehicle. Such constraints are only binding for trip *i* and vehicle *k* if *k* goes on trip *i* (i.e., if y_{ik}=1) but not on the trips starting before *i*. Similarly, we track the duration of the deadhead between the arrival location of the last trip executed by a vehicle and the depot using Constraint (51). In this case, the constraint is only binding for trip *i* and vehicle *k* if *k* performs trip *i* (i.e., if y_{ik}=1) but none of the trips starting after *i*. The duration of the deadhead among two consecutive trips is computed by Constraint (52). This constraint is only binding for a pair of trips *i* and *j* and vehicle *k* if *k* performs both *i* and *j* but none of the trips whose starting times fall between the starting time of *i* and that of *j*. The last three sets of constraints require the trips to be ordered by a non-decreasing starting time in order to work properly. Similarly, it is possible to track the amount of battery consumed by deadhead arcs for EV and UV (see Constraints (53), (54), and (55)). Finally, Constraint (56) forces a vehicle to perform a stop if the amount of battery required to perform all the trips assigned to the vehicle plus all the deadheads is larger than the vehicle's battery capacity. Constraints are not required for these since battery capacity violations are already forbidden by Constraint (42); however, they can act as valid inequalities to speed up the solution process.

5.2 The proposed combinatorial benders cut framework

As mentioned above, the role of SP in our CBC framework is twofold. First, it checks the feasibility of the solution provided by MP, and second, it checks if the actual objective function value corresponds to the one provided by MP. By this, our framework follows the idea to split OP into MP, which involves only the variables contributing to the objective function, and SP, which contains only the variables responsible for feasibility. We then iteratively solve MP and SP, where the optimal solution of MP is injected into SP to check for feasibility. In case an infeasibility is detected, a *feasibility cut* is generated and added to MP. The latter is then solved again, while the infeasible region is not available anymore. The procedure terminates as soon as a solution is proven to be feasible by SP. A technical description of all steps is provided in the following.

In each iteration *N*, after solving MP to optimality, we pass the obtained partial solution to the mathematical model of OP by fixing the assignment of trips to vehicles. Since trips have mandatory starting times, imposing their assignment to vehicles, this implicitly imposes the sequence in which the trips are executed by the vehicles. However, the model is allowed to decide where and when to perform recharging stops. Using w_{ik} as a binary variable that indicates whether trip *i* is assigned to vehicle *k* or not, we can impose the assignment of trips by adding to OP the following sets of constraints.

$$\sum_{j\in N} x_{ijk} = w_{ik} \tag{57}$$

$$w_{ik} = \hat{y}_{ik}^{N} \tag{58}$$

Note that $\hat{y}_{ik}^{N}$ is a binary variable indicating whether trip *i* has been assigned to vehicle *k* in the N^{th} iteration of MP or not.

Thus, this constrained version of OP is solved to optimality. If the problem is feasible and the value of the obtained solution, ξ, is equal to the value computed by MP, z^*, then z^* is optimal for OP as well. Otherwise, we add a penalty $\pi_n = f(z^*) - f(\xi)$, which corresponds to the solution z^* of the objective function. This discrepancy between the two values can occur if the number of slots to be installed has been underestimated by MP. In fact, MP only computes a lower bound of this value. For example, if simultaneous recharging is not required, MP states that 1 slot is sufficient to serve all the vehicles, but for OP, there may be no location for this unique slot, which is suitable for all the vehicles, because the vehicles' schedules do not allow for a long detour to reach that location. In this case, we would need to locate more slots across different locations, and this would increase installation costs. Thus, the objective function of MP in each iteration after the first, becomes

$$\begin{aligned} \min \sum_{k\in\tilde{K}} f_k^A z_k + f^E\lambda^E + f^U\lambda^U + \gamma^T \sum_{k\in Kt} y_{ik} \\ + \sum_{i\in I}(\tau_i + T_i + \sigma_i) + \sum_{n\in 1..N-1} \pi^n U^n, \end{aligned} \tag{59}$$

where U^n is a binary variable indicating whether the trip-to-vehicle assignment configuration obtained by MP in iteration *N* has been selected. To activate U^n, we need to add the following optimality cut (O-CUT) to MP:

$$U^n \geq \sum_{k\in\tilde{K}} \sum_{i\in I|\hat{y}_{ik}^{N}=1} \frac{w_{ik}}{\epsilon^N - 0.99}, \tag{60}$$

where ϵ^N is the number of trips assigned to EV or UV in the N^{TH} solution of MP.

This way, the constraint is only active if all the trip assignments to EV and UV correspond to those provided by the N^{TH} solution of MP. It is worth noting that this constraint is active regardless of the assignment of trips to TV. This is true since assignments to TV have no influence on the slots installation costs. This way, we are able to correct the underestimated installation costs not only of a single solution but also for a potentially large number of solutions, (all those sharing the same assignment of trips to each single EV or UV but different assignment of trips among the other vehicles), speeding up the solution process. For other infeasiblities, a cause detector is applied as described in the following.

If the solution of the OP injected with the MP solution turns out to be infeasible, we activate a procedure to detect the cause of this infeasibility. We solve a simplified version of OP for each EV (EV or UV) and only pass to the model those trips that were assigned to the vehicle, letting the model check if a feasible charging schedule exists. If not, it means that the schedule is so tight that there is not enough time to reach a station, recharge, and reach the starting point of the next trip, and that the assignment is not feasible. In this case, we identified an MIS of variables responsible for infeasibility, and we can add an ad hoc feasibility cut (V-CUT):

$$\sum_{i \in I | \hat{y}^N_{ik}=1} w_{ik} \leq \eta^N_k, \tag{61}$$

where η^N_k is the number of trips assigned to vehicle *k* on the N^{th} iteration of MP. Note that this feasibility check procedure is executed only for vehicles for which a recharging stop is needed because if no recharging stops are required, the schedule provided by MP for that vehicle is certainly feasible.

If all the single vehicle schedules are feasible, this means that the cause of infeasibility is that the number of slots required at a specific station exceeds the maximum number of slots that can be installed at that station. In this case, we have to exclude the combination of all the assignments to EV and UV from MP. This can be done by adding the following global cut (G-CUT).

$$\sum_{k \in \tilde{K}} \sum_{i \in I | \hat{y}^N_{ik}=1} w_{ik} \leq \epsilon^N \tag{62}$$

A resuming scheme of the overall CBC framework is depicted in Fig. 5.

6 Computational experiments

In this section, we discuss the results of the computational experiments performed on different sets of instances with 10, 20, and 30 trips. Each set is composed of 20 instances, showing similar characteristics. For simplicity, we refer to these sets as small, medium, and large instances. The number of recharging stations is fixed to 4 for the first (small instances). 5 (medium instances), and 6 (large instances). They are spread over a given area. Due to the limited space available, central stations have stricter limitations on the number of slots per type that can be installed. Each time

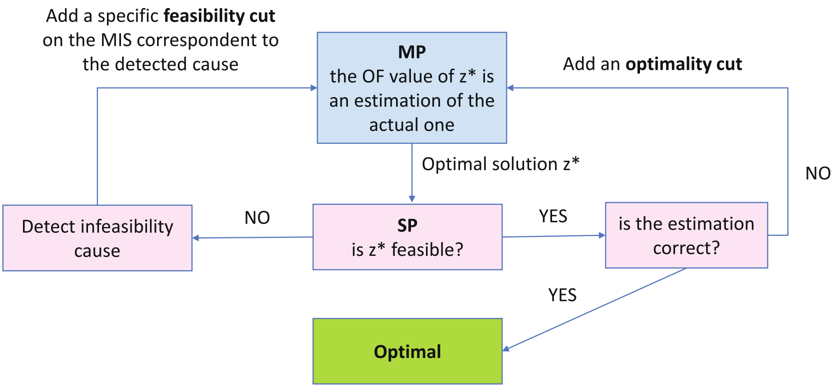

Fig. 5 A graphical scheme of the CBC framework

unit corresponds to 5 min. Trips are given a duration of 45 to 90 min (corresponding to 9 and 18 time units, respectively), which is realistic for urban public lines. Starting and arrival times are defined as multiples of the time unit. For example, we assume that a trip's starting time can be 9:05 or 9:10 but not a value in between. Vehicles' autonomy is assumed to be equal to 200 time units for both EV and UV. A complete recharge for EV takes 25 time units (2 h and 5 min), while UV can be fully recharged in 2 time units (10 min). Thus, ultra-fast chargers are more than 12 times faster than standard ones. The purchasing price for EV is assumed to be €100,000, while that for UV is assumed to be €400,000. These data corresponds to the innovative vehicles introduced in 2021 in the public transport fleet of the city of Milan (source: www.hdmotori.it).

Slot installation costs are equal to €2000 for EV and €15.000 for UV. TV are assumed to already be owned by the company, so their purchasing cost are not considered. Operating cost for a TV is assumed to be €0.27 Euro per km, which corresponds to €0.675 per time unit. For instances with 10, 20, and 30 trips, we consider 2, 3, and 5 available vehicles for each type, respectively. Generally, all instances are built upon publicly available information and are available in Mancini and Gansterer (2024).

We perform experiments to compare the behaviour of the mathematical model (from now on referred to as MODEL) and the CBC approach. We run both algorithms within 3600 s. In Tables 4, 5, and 6, we report the results obtained by MODEL from instances with 10, 20, and 30 tasks, respectively. In each table, We provide the best objective function value obtained (UB) within the given time limit. Furthermore, we indicate the best lower bound (LB), the percentage optimality gap (GAP), the time elapsed (TIME), the number of recharging slots installed for EV (#E-SLOTS) and for UV (#U-SLOTS), and the number of vehicles used for each type (#TV, #EV, #UV). Optimality gaps are calculated as $\frac{OF-LB}{LB}$ with OF being the objective function value and LB the value of the lower bound.

Note that, when #E-SLOTS (#U-SLOTS) are equal to 0 but #EV (#UV) are positive, this means that no installation of recharging slots of type #EV (#UV) is required

Table 4 Results (MODEL) on small instances

UB	LB	GAP	TIME	#E-SLOTS	#U-SLOTS	#TV	#EV	#UV
1,144,447	1,144,447	0.0%	80.4	0	0	0	2	2
299,000	299,000	0.0%	10.1	0	0	0	2	0
321,000	321,000	0.0%	13.2	0	0	0	2	0
379,966	379,966	0.0%	8.7	0	0	0	2	0
352,376	352,376	0.0%	8.3	0	0	0	2	1
717,367	717,367	0.0%	16.2	0	0	0	2	1
368,367	368,367	0.0%	6.3	0	0	0	2	0
368,376	368,376	0.0%	6.4	0	0	0	2	0
745,187	745,187	0.0%	73.4	0	0	0	2	1
738,367	738,367	0.0%	15.1	0	0	0	2	1
543,445	543,445	0.00%	23.8	0	0	0	2	0.6

Table 5 Results (MODEL) on medium instances

UB	LB	GAP	TIME	#E-SLOTS	#U-SLOTS	#TV	#EV	#UV
683,178	683,178	0.0%	194.1	0	0	3	3	0
599,939	599,939	0.0%	35.9	1	0	3	3	0
523,966	509,755	2.8%	3,600.0	1	0	3	3	0
547,376	547,376	0.0%	1,204.8	1	0	3	3	0
513,183	513,183	0.0%	2,665.0	1	0	3	3	0
471,583	471,583	0.0%	265.5	1	0	3	3	0
438,000	371,000	18.1%	3,600.0	1	0	3	3	0
394,934	361,000	9.4%	3,600.0	1	0	3	3	0
464,183	464,183	0.0%	379,312.0	0	0	3	3	0
395,069	360,183	9.7%	3,600.0	2	0	3	3	0
503,141	488.138	4.0%	1,914.5	0.9	0	3	3	0

Table 6 Results (MODEL) on large instances

UB	LB	GAP	TIME	#E-SLOTS	#U-SLOTS	#TV	#EV	#UV
923,253	662,041	39.46%	3,600	0	0	4	4	0
722,630	294,921	145.00%	3,600	0	0	4	4	0
1,254,447	581,183	115.8%	3,600	1	1	5	5	1
123.944,7	776,782.6	59.6%	3,600	1	1	5	5	1
1,233,927	1,206,744	2.3%	3,600	0	1	5	5	1
772,551	494,000	56.4%	3,600	3	0	4	5	0
751,551	744,551	0.9%	3,600	0	0	4	5	0
852,229	852,229	0.0%	3,600	0	0	5	5	0
703,874	360,000	95.5%	3,600	1	0	4	4	0
713,183	408,762	74.5%	3,600	2	1	4	4	0
916,709	638,121	59.0%	3,600	0.8	0.4	4.4	4.6	0.3

because the related vehicles do not need en-route recharge during the day but are recharged overnight by means of traditional slow-recharging plug-in systems available at the depot. We assume that each bus parking slot at the depot is equipped with a plug, therefore all the vehicles can be simultaneously charged overnight to have a full battery availability for the next day.

MODEL is able to solve small instances to optimality within a short time (less than 24 s), but CBC is even faster, requiring less than 4 s. EV are the most convenient option, since the 2 available EV were always used. UVs are also used but only for instances for which the 2 EV are not sufficient. TV are not competitive and are never used. No recharging slots are needed since the battery of the vehicle is sufficient to cover all the trips without recharging.

As the size of the instances increases, the performance of MODEL decreases considerably. In fact, on medium-sized instances, MODEL is able to solve to optimality only 6 out of 10 instances, and the computational time required increases to more than 1900 s. In this case, both EV and TV are fully exploited whereas UV are never selected. This depends on the utilization rate of vehicles. In fact, UV cause very high fixed cost even when they are used for a limited amount of time, while TV costs are proportional to their usage. Therefore, a fully used TV costs more than a UV, but if their workload is limited, TV are more cost efficient. The selected TV have a limited workload; however, this workload cannot be carried out by a single vehicle due to the overlapping trip scheduling. The largest portion of the trips is performed by EV. In this case, TV are more cost efficient than UV. The overall workload increased with respect to the small instances and the vehicles' need to recharge. However, since the schedules are not so tight, visits to recharging stops can be planned to avoid overlappings, and therefore, a single slot is sufficient in most cases. MODEL is not able to handle large instances. None of the instances can be solved to optimality, and the optimality gaps are huge. Regarding vehicle usage, the trend is similar to that of the medium instances. EV and TV are preferred, and UV are only used when the other vehicles are not sufficient to cover all the trips. The charging scheduling problem become more challenging, and in some cases, 2 or 3 slots are required.

The results show that CBC is strongly outperforming MODEL. CBC takes only a few seconds to solve small instances to optimality. This is because the initial MP always turns out to be feasible and therefore no cuts are added, and only one iteration is performed. This was expected since the optimal solution does not require any recharging stops and therefore the MP solution is certainly feasible.

Medium instances can all be solved to optimality by CBC within small computational times (less than 15 s). Most of the instances are solved at the first iteration, with only one instance requiring an additional 10 iterations and 27 cuts. All the cuts are V-CUTS, which means that the cause of the infeasibility is always related to vehicle scheduling being too tight to allow for a detour to a station for a battery recharge.

CBC is also able to solve all the large instances to optimality within reasonable computational times (126 s). The number of iterations required is similar to that required for medium instances. Most of the instances are solved at the first iteration, while some require more iterations and several cuts. Similarly, in this case, all the cuts are V-CUTS. Based on our findings, we argue that the most common cause of infeasibility is related to single vehicle scheduling. Therefore, the process of detecting the cause of the infeasibility is very useful since it can detect and isolate the groups of trips that cannot be assigned to the same EV or UV. Hence, we can simultaneously cut off a large number of solutions from the solutions space, which strongly speeds up the process. For example, in a 30-trips instance, if one vehicle is serving trips 1, 2, and 3 but this assignment is detected to be infeasible with standard cuts from Codato and Fischetti (2006), we would only cut the current solution and then, at the following iteration, likely have another infeasible solution in which 1,2, and 3 are served together while the assignment of the other trips change. However, with V-CUTS, we simultaneously cut all the solutions which include the assignment of 1, 2, and 3 to the same vehicle, whichever is the assignment of the other trips. The number of such solutions can be extremely large. Moreover, the temporal bin packing relaxation provides results that are so tight that it allows us to obtain a solution that is optimal for OP in most cases, which is a clear advantage. It is worth noting that in instances that are not solved to optimality by MODEL, the solution obtained by MODEL can be significantly worse than that obtained by CBC, which is able to improve the scheduling, reducing not only TV costs (by decreasing their workload) but also installation costs by reducing the number of slots to be installed. Related studies, where only few cuts are needed to achieve optimal solutions are, for instance, Mancini et al. (2021) and Mancini and Gansterer (2021b). We also tried to solve larger instances with 40 trips but in this case also the master problem becomes very difficult to solve, therefore we did not manage to solve any of them without a reasonable computational time. This results suggest the exploitation of heuristic methods or decomposition approaches to handle very large instances. In the latter case, clustering techniques could be used to partition the set of trips in homogeneous clusters which can be solved independently by means of our CBC approach.

We also perform managerial analyses in which we assume that subsidies were received from public authorities in order to reduce UV purchasing costs and therefore provide incentives for the implementation of this technology. First, we reduce UV costs by 25% and then by 50%. In both cases, the optimal solution does not change (only its value changes) and the usage of UV is not increased. This is because UV only becomes preferable to EV in cases where the schedule is so tight that an ultra-fast recharge is feasible while a standard one is not. This happens very rarely since the amount of time required to fully charge an empty EV are quite large but a full day’s workload can be performed with a small partial recharge which only requires a few time units more than an ultra-fast recharge. Since UV’s advantage of a higher recharging speed is quite limited, the only criterion of preference is its purchasing cost. Therefore UV would only become preferable if their cost becomes lower than that of EV, which is not realistic. Results on small, medium, and large instances are reported in Tables 7, 8, and 9, respectively. In each table we provide the value of the objective function (OF), which is optimal for all the instances since the time limit is never reached. Furthermore, we indicate the elapsed time (TIME), the number of iterations (ITER) performed by the algorithm, the number of cuts related to a single vehicle scheduling (#V-CUTS), the number of global feasibility cuts (#G-CUTS) and optimality cuts (#O-CUTS), and the number of charging slots installed per each type of vehicle (#E-SLOTS and #U-SLOTS).

Table 7 Results (CBC) on small instances

UB	TIME	ITER	#V-CUTS	#G-CUTS	#O-CUTS	#E-SLOTS	#U-SLOTS
1,144,447.0	1.1	1	0	0	0	0	0
299,000.0	4.7	1	0	0	0	0	0
321,000.0	7.2	1	0	0	0	0	0
379,966.4	3.6	1	0	0	0	0	0
352,376.1	1.5	1	0	0	0	0	0
717,367.5	0.7	1	0	0	0	0	0
368,367.5	3.9	1	0	0	0	0	0
368,376.1	2.9	1	0	0	0	0	0
745,187.1	6.5	1	0	0	0	0	0
738,367.5	1.9	1	0	0	0	0	0
543,445.5	3.4	1	0	0	0	0	0

Table 8 Results (CBC) on medium instances

UB	TIME	ITER	#V-CUTS	#G-CUTS	#O-CUTS	#E-SLOTS	#U-SLOTS
683,178.6	34.4	1	0	0	0	0	0
599,939.1	13.7	1	0	0	0	1	0
509,755.4	10.4	1	0	0	0	1	0
547,376.1	10.2	1	0	0	0	1	0
513,183.8	19.3	4	9	0	0	1	0
471,583.8	12.6	1	0	0	0	1	0
371,000.0	18.9	10	27	0	0	1	0
361,000.0	5.9	1	0	0	0	1	0
464,183.8	13.2	1	0	0	0	0	0
380,183.8	8.9	1	0	0	0	1	0
490,138.4	14.7	2.2	3.6	0	0	0.8	0

Table 9 Results (CBC) on large instances

OF	TIME	ITER	#V-CUTS	#G-CUTS	#O-CUTS	#E-SLOTS	#U-SLOTS
914,965.6	198.1	1	0	0	0	0	0
460,183.8	43.0	10	10	0	0	1	0
1,254,447	132.1	1	0	0	0	1	1
1,239,447	59.4	1	0	0	0	0	0
1,233,927	70.5	1	0	0	0	0	0
751,551.3	65.7	1	0	0	0	0	0
751,551.3	63.6	1	0	0	0	0	0
852,229.7	101.3	1	0	0	0	0	0
471,000.0	140.7	9	24	0	0	0	0
486,000.0	393.8	5	12	0	0	1	0
841,530.3	126.8	3.1	4.6	0	0	0.3	0.1

Regarding the comparison of TV and UV, a reduction in UV costs can make them more attractive than TV, even with a medium workload. However, TV are still preferable to UV since they are used for a very small workload. However, there are particular instances in which UV would be more advantageous than EV, such as during very long trips (4 h) starting and arriving at the same terminal every 4 h and 20 min. In this case, there is only time for an ultra-fast UV recharge and not an EV one. Therefore, the number of EV needed would be greater than that of UV, and UV would be more convenient.

Globally, we can state that the market is not yet ready for this innovative ultra-fast recharging technology which is very expensive and would only be useful in specific situations. Further, EV are much more preferable to TV from both an environmental and economic perspective. The market should go in this direction, progressively substituting TV fleets with EV. However, a limited number of TV could still be useful to cover very small workload residuals from the EV schedules, which do not justify the purchase of an additional EV.

7 Conclusions and future research

We addressed the integrated problem of sizing and locating charging infrastructure, as well as sizing a fleet of traditional and electric buses to cover a set of scheduled trips. Our study is the first to tackle all these decision problems in an integrated way; these problems have, in practice, been solved sequentially, leading to potentially suboptimal solutions.

In this study, we considered a mixed fleet composed of TV, standard EV, and EVs equipped with an ultra-fast charging technology (UV). We proposed a mixed integer programming formulation and an exact method based on combinatorial benders cuts (CBC). We extended the classical CBC framework by proposing a non-standard relaxation based on the temporal bin packing problem. Furthermore, we provided a method for identifying the cause of infeasibility and deriving ad-hoc cuts which turned out to be stronger than the classical ones. In addition, we found the role of the subproblem to twofold: checking the feasibility of the MP solution (as in standard CBC) and verifying whether the estimation of the objective function value provided by MP corresponds to the actual value.

Our computational experiment results showed that CBC strongly outperformed the mathematical model and that the process of detecting the cause of the infeasibility is very effective. We also provided managerial insights on the usage of ultra-fast technology. Our conclusion is that the market is not yet ready for this technology, given the very high purchasing costs of the vehicles and their compatible charging infrastructure. Ultra-fast recharging technology actually only impacts on very tight schedules which are not common in practice. Standard EV seem to be the better choice since they have low purchasing costs and even lower operational costs compared to TV. Therefore, we suggest a transition of the fleet in public transport from TV to EV. Future studies in this field could address the uncertainty in vehicle consumption and trip completion time due to traffic conditions or adverse weather conditions. This could be practically highly relevant especially in cases of very tight schedules in which a small delay in an early morning trip could propagate and affect the whole day's schedule. Also, non-linear charging rates could be considered to make the problem even more realistic. Another study could address the use of fleets composed of vehicles with different passenger capacities and trips with requirements on the minimum and maximum capacity allowed. Also, investigating the combination of charging and discharging activities at stations could be worthwhile.

For large-scale employment of the proposed charging system, technological impediments might still exist. We encourage future research also in this direction. From a more methodological perspective, heuristic algorithms could be developed to address large and challenging instances particularly if systems of large metropolitan cities are to be optimized. Also, the newly proposed CBC framework could be generalized and adapted to efficiently solve other complex combinatorial problems.

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Data availability Data are publicly available as Mendeley dataset. Mancini, Simona; Gansterer, Margaretha (2024), "Infrastructure Planning, Fleet Sizing, and Scheduling for E-public Transports", Mendeley Data, V1, doi: 10.17632/vgb55fg4k8.1 This reference is also cited in the manuscript.

Declarations

Conflict of interest None of the authors have any conflict of interest with this paper.

Consent to participate All persons who meet authorship criteria are listed as authors, and all authors certify that they have participated sufficiently in the work to take public responsibility for the content, including participation in the concept, design, analysis, writing, or revision of the manuscript. Furthermore, each author certifies that this material or similar material has not been and will not be submitted to or published in any other publication.

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