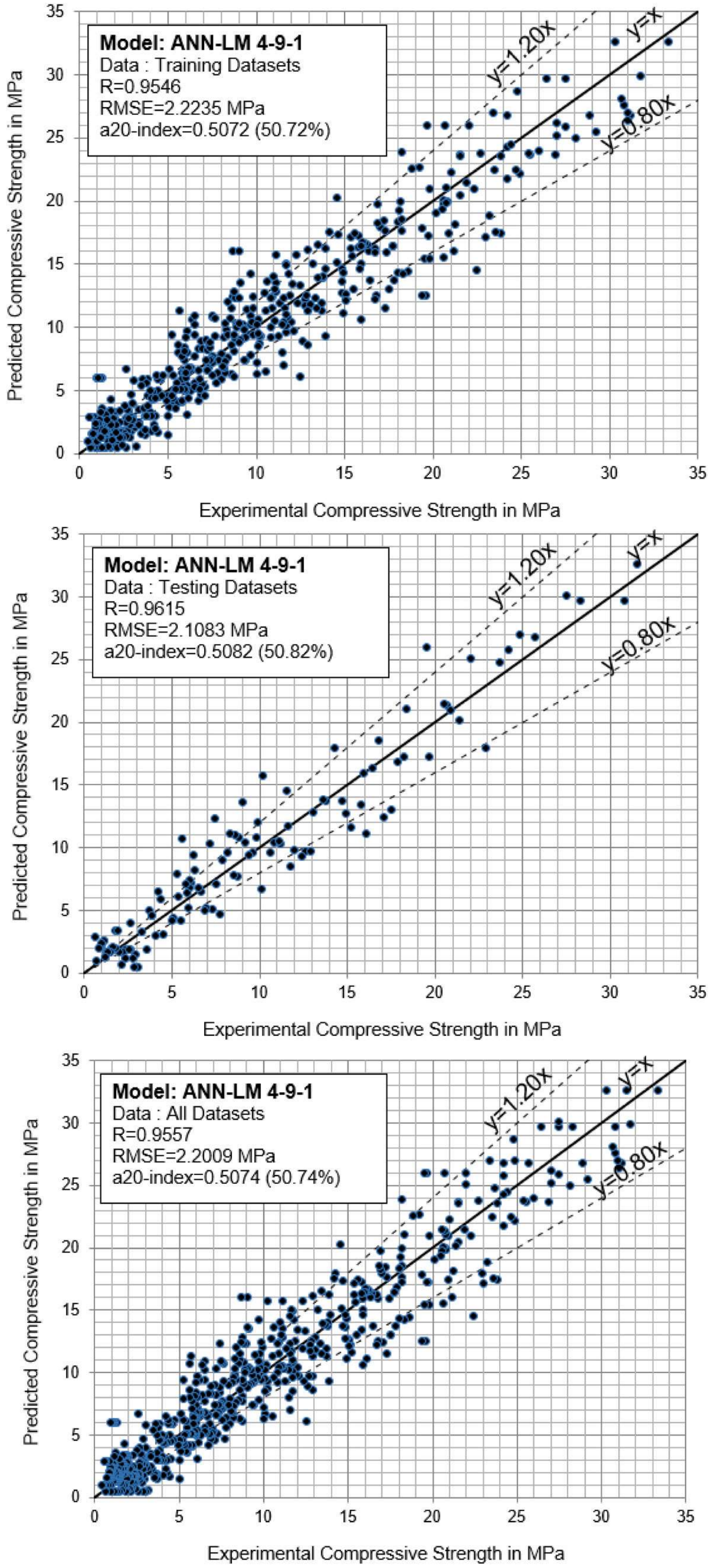


Fig. 4. Experimental ('true') vs Predicted values of the masonry compressive strength for the developed and proposed optimal ANN-LM 4–9–1 model.

Table 9

Final values of weights and biases of the optimal ANN-LM 4–9-1 model.

$[I_W]$ (9 ×4)				$[L_W]^T$ (9 ×1)	$[b_i]$ (9 ×1)	$[b_0]$ (1 ×1)
11.94815	−87.50208	−16.91220	0.08752	−30.40946	−0.78908	63.16957
−0.16434	−0.08828	0.21761	0.09588	−63.74307	−0.10795	
−8.70208	3.29864	7.36559	−15.03860	−1.04069	15.96637	
5.43286	0.23739	1.15692	−0.49509	80.31513	0.55455	
0.79220	1.02951	−3.32502	−26.76595	−1.19873	11.62022	
−1.95824	−1.65733	4.54037	−6.22362	−1.22580	3.17310	
−62.05147	16.63407	−51.35296	41.16826	0.89023	6.06927	
−1.60589	1.32127	79.66294	44.47994	1.28249	−38.00081	
4.35289	0.17715	0.83315	−0.37139	−163.47946	0.94408	

Table 10

Summary of prediction capability of the developed optimal model against the five highest-performing proposals in literature, based on a20-index and testing datasets.

Ranking	Source	Model	Method	Performance Indices				
				a20-index	R^2	RMSE (MPa)	MAPE	VAF
1	This article	ANN-LM 4–9–1	ANNs	0.5082	0.9615	2.1083	0.3391	92.0625
2	Lumantarna et al., [81]	$f_{wc} = 0.75 f_{bc}^{0.75} f_{mc}^{0.31}$	RG	0.3033	0.7454	6.0370	0.4176	46.8621
3	Engesser [49]	$f_{wc} = \frac{1}{3} f_{bc} + \frac{2}{3} f_{mc}$	RG	0.3033	0.7338	6.5148	0.4295	47.3341
4	Tassios [129]	$f_{wc} = \frac{2}{3} f_{bc} + 0.1 f_{mc}$	RG	0.2951	0.7774	7.9218	0.4099	15.4996
5	Moayedian, Hejazi [94]	$f_{wc} = 0.6 f_{bc}^{0.7} f_{mc}^{0.4}$	RG	0.2705	0.7876	4.7425	0.4388	61.0820
6	Mann [84]	$f_{wc} = 0.83 f_{bc}^{0.66} f_{mc}^{0.18}$	RG	0.2377	0.8239	4.7257	0.5028	64.0042

RG: Regression Analysis

the masonry height to thickness ratio (h_w/t_w), and the mortar thickness to masonry unit thickness ratio (t_m/t_b) is given in matrix form by:

$$f_{wc} = 33.05 \times logsig([L_W] \times [\text{radbas}([I_W] \times [IP] + [b_i])] + [b_0]) + 0.45 \tag{6}$$

where [IP] is a 4 × 1 vector with the normalized values of the four input parameters given by Eq. (7):

$$[IP] = \left[\begin{array}{c} \dfrac{f_{bc} - 2.30}{72.6} \\ \dfrac{f_{mc} - 0.30}{31.7} \\ \dfrac{(h_w/t_w) - 1.15}{7.45} \\ \dfrac{(t_m/t_b) - 0.01}{0.32} \end{array} \right] \tag{7}$$

In this equation, logsig is the Log-sigmoid transfer function, and radbas refers to the Radial Basis transfer function. $[I_W]$ is a 9 × 4 containing the weights of the hidden layer, $[L_W]$ is a 1 × 9 vector containing the weights of the output layer, $[b_i]$ is a 9 × 1 vector containing the bias values of the hidden layer, and $[b_0]$ is a 1 × 1 vector containing the bias values of the output layer. All these matrices are provided in Table 9.

The above proposed equation, which is based on the developed and proposed ANN 4–9–1 optimal model, can be integrated into any software and any programming environment. This helps to disprove the widely accepted idea that ANN models are "black boxes", while also making the proposed model a particularly useful tool for researchers, and even more so for practicing engineers.

4.4. Comparison of the developed models and available proposals in the literature

The optimal machine learning model of this study presented in previous sections (ANN-LM 4–9–1) is compared here with available proposals that predict the compressive strength of masonry walls, found in literature. This comparison is performed according to the selected performance indices. A full comparison is provided in the supplementary document entitled *Prediction Performance Comparison.* This comparison is curtailed in Table 10, which summarises the performance of the proposed ANN model and the five highest performing semi-empirical models. The optimal, ANN-LM 4–9–1 model proposed in this study leads to the highest a-20 index which at least 65 % higher, when compared to the remaining studies, indicating an improved performance of the proposed ANN architecture.

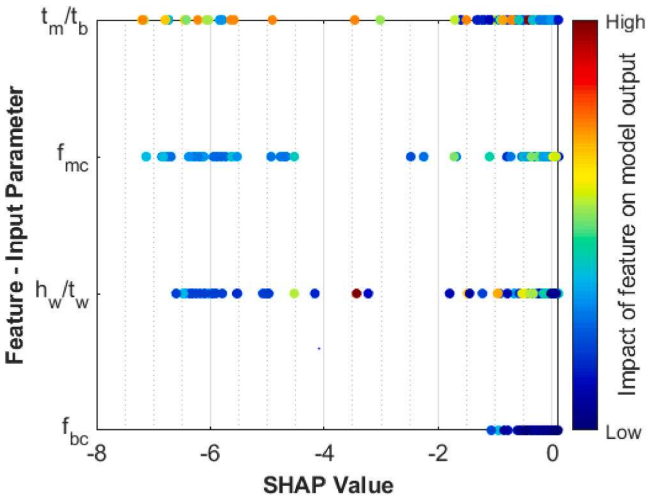
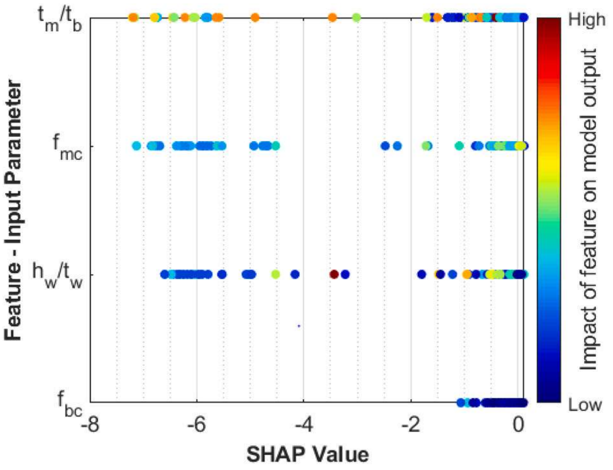
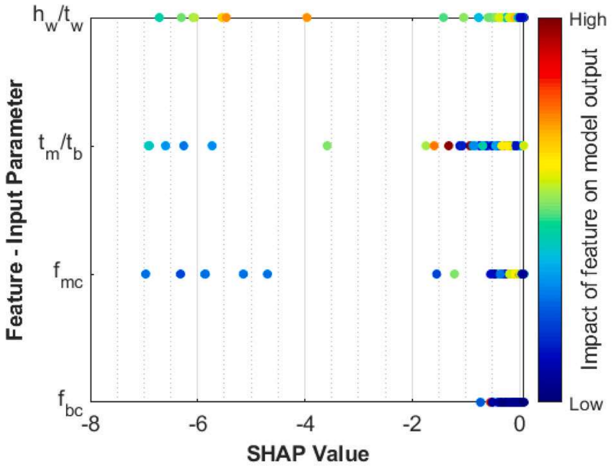


Fig. 5. Assessment of the parameters influencing the masonry compressive strength based on optimal ANN-LM 4–9–1 model and using SHapley Additive exPlanations (SHAP) method: SHAP value for each one dataset of input parameters influencing the masonry compressive strength.

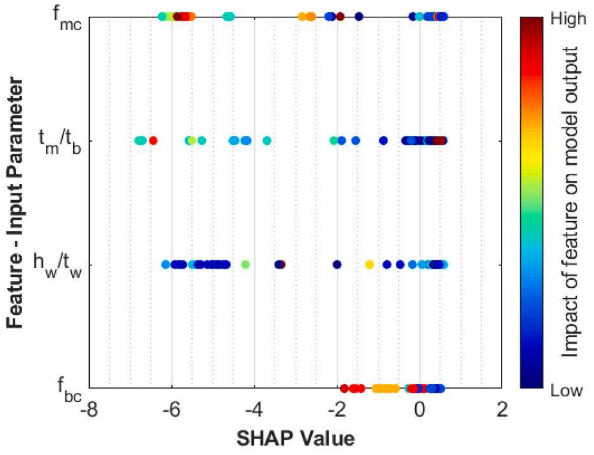
It is worth noticing that the other indices are also significantly improved in comparison with the literature findings, further emphasizing the capacity of the proposed machine learning model to predict the compressive strength of masonry walls. For instance, the R^2 index of the ANN-LM 4–9–1 is equal to 0.9615, while the highest R^2 index



a) All samples; masonry compressive strength up to 33 MPa



b) Samples with masonry compressive strength up to 15 MPa



c) Samples with masonry compressive strength greater than 15 MPa

Fig. 6. Assessment of the parameters influencing the masonry compressive strength based on optimal ANN-LM 4–9–1 model and using SHapley Additive exPlanations (SHAP) method.

obtained from literature is 0.8262 [66]. RMSE and MAPE values obtained from this study are also significantly lower when compared to existing literature while the VAF value of this study is the highest among the ones presented in Table 10.

4.5. *Assessment of the parameters affecting masonry compressive strength*

Aiming to reveal the nature of masonry materials, this subsection seeks to evaluate the parameters that influence the compressive strength of masonry and to rank them based on their impact. For this purpose, the optimal developed ANN-LM 4–9–1 model and the SHapley Additive exPlanations (SHAP) method, recently proposed [31,82], are used to determine the significance of each input parameter on the output parameter. These parameters are then ranked from the most to the least influential.

The resulting average SHAP values for the input parameters are: 0.5574 for t_m/t_b, 0.4952 for f_{mc}, 0.4764 for h_w/t_w and 0.1079 for f_{bc}. Additionally, the SHAP values for each individual sample (dataset) are presented in Fig. 5. Based on these results, it is evident that the most important parameter affecting the compressive strength of masonry is the mortar thickness to masonry unit thickness ratio (t_m/t_b) followed in order by the mortar compressive strength (f_{mc}), the masonry height to thickness ratio (h_w/t_w) and the parameter with the smallest influence, which is the masonry unit compressive strength (f_{bc}).

To further explore the complex behaviour and nature of masonry materials, an analysis was conducted to assess the influence of these four parameters, with a distinction made between whether the masonry is "strong" or "weak," using the compressive strength of 15 MPa as threshold. The results are presented in Fig. 6, both for the entire dataset of 611 experimental samples and for the subsets of "weak" masonry with compressive strength up to 15 MPa (475 samples) and "strong" masonry with compressive strength greater than 15 MPa (136 samples). Based on this analysis, the following observations can be made:

- For the entire dataset of 611 samples, with compressive strength ranging from 0.45 to 33.30 MPa, the critical parameter is the mortar thickness to masonry unit thickness ratio (t_m/t_b),

- For "weak" masonry, the main parameter affecting compressive strength is the masonry height to thickness ratio (h_w/t_w),
- For "strong" masonry, the primary parameter influencing compressive strength is the mortar compressive strength (f_{mc}),
- In all cases, the parameter with the least influence is the masonry unit compressive strength (f_{bc}).

4.6. *Mapping and revealing the nature of masonry compressive strength*

In this section, using the proposed optimal ANN-LM 4–9–1 model, an attempt will be made to map and reveal the complex nature of masonry through the production of a set of graphs. These graphs will also demonstrate that the well-known and frequently encountered problem of overfitting did not occur during the model's training. Specifically, Fig. 7 depicts the relationship between masonry compressive strength and the compressive strength of the masonry unit for masonry height to thickness ratios of 5.00, 6.00, 7.00, and 8.00 using the optimal ANN-LM 4–9–1 model, with a mortar thickness to masonry unit thickness ratio of 0.10. Additionally, Fig. 8 illustrates the curves of masonry compressive strength versus the compressive strength of mortar for a mortar thickness to masonry unit thickness ratio of 0.10, with mortar unit strength ranging from 10.00 to 70.00 MPa in increments of 10.00 MPa, using the optimal ANN-LM 4–9–1 model.

Based on these mappings, the following points can be made:

- The smoothness of the contours confirms that the proposed computational methodology avoids the overfitting of data, a common issue in the development of prognostic soft computing models. In cases of

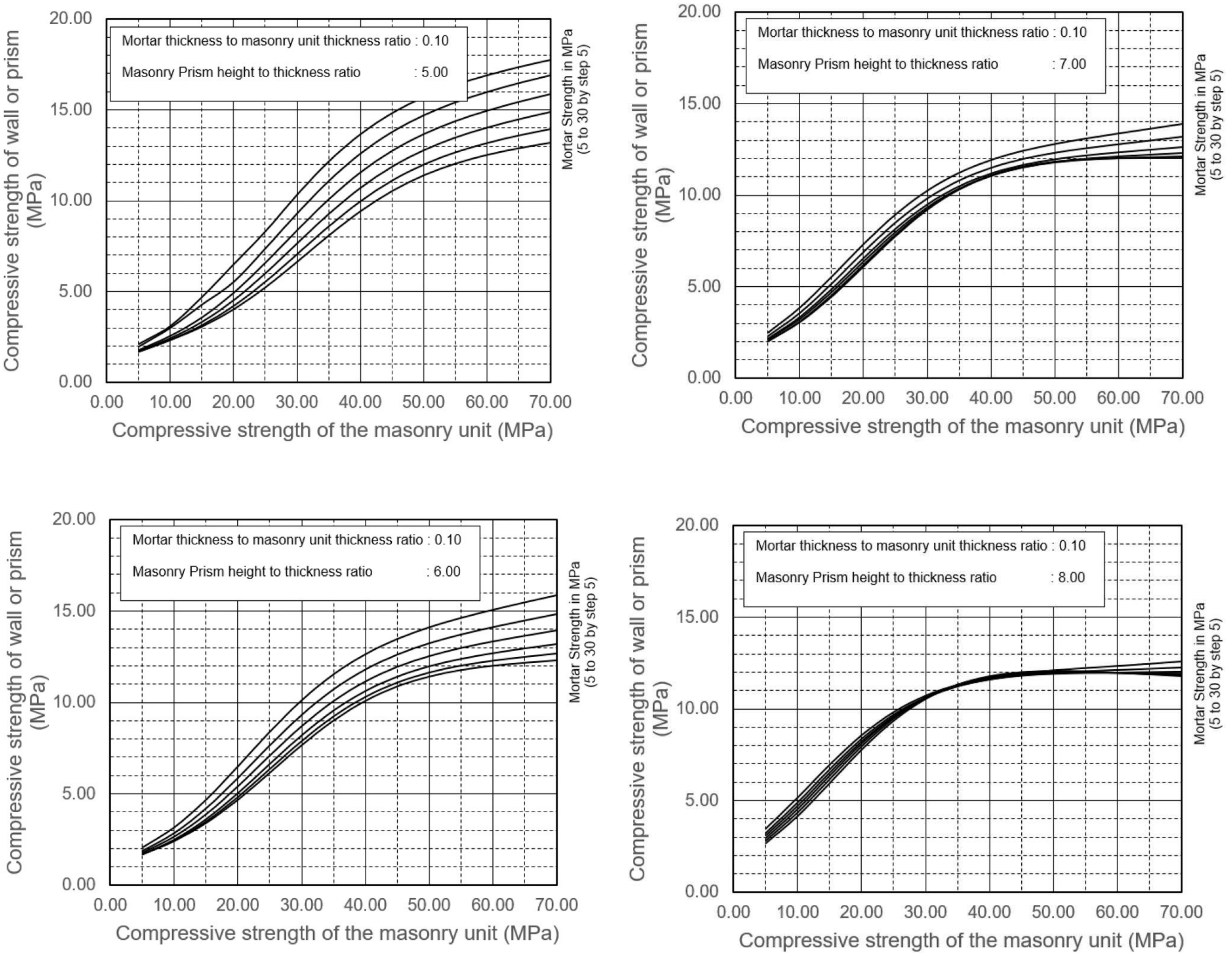


Fig. 7. Masonry compressive strength vs Compressive strength of the masonry unit for mortar thickness to masonry unit thickness ratio 0.10 and masonry height to thickness ratio 5.00, 6.00, 7.00 and 8.00 using the optimal ANN-LM 4–9–1 model.

overfitting, the model may appear to fit very closely to the experimental data used for its training; however, for slightly perturbed data ranges, the predictions become significantly worse,

- The relationship between masonry compressive strength and the compressive strength of the masonry unit is nonlinear, and this nonlinearity becomes particularly pronounced for masonry unit compressive strength values exceeding 20.00 MPa (Fig. 7),
- In contrast, the relationship between masonry compressive strength and mortar strength is linear (Fig. 8),
- According to Fig. 7, for mortar thickness to masonry unit thickness ratio (t_m/t_b) equal to 0.1, an increase of the mortar strength results in increase of the masonry compressive strength. This is more intense for higher compressive strength values of the masonry unit.
- According to Fig. 7, for $t_m/t_b = 0.10$ the compressive strength of masonry becomes less sensitive to f_{mc} for higher values of h_w/t_w (higher slenderness), highlighting the effect of geometric slenderness on the compressive strength of masonry.

5. Limitations and future research

The database used for the development and training of a predictive model defines the model limitations and determines the range of parameter values for which updates are necessary. Specifically, every predictive computational model is valid for input parameter values that fall between the minimum and maximum values defined by the database. Conversely, for parameter values outside these limits, the reliability of the predictions is compromised, as the model has not been trained on such values.

In light of the above, the optimal ANN-LM 4–9–1 model proposed in this study is valid for parameter values within the minimum and maximum ranges specified in Table 6. Additionally, as previously mentioned, the database defines the parameter value ranges that require further research and corresponding updates. These ranges are specifically indicated by the histograms of parameters used for predicting masonry compressive strength, which were presented in Fig. 2. Based on these histograms, the reliability of the proposed model is uncertain for parameter value ranges where the data is insufficient, such as for values of the mortar thickness to masonry unit thickness ratio (t_m/t_b) greater than 0.20, where the number of data points is quite small.

Although the database used in this study is the largest ever compiled and utilized for the development of predictive mathematical models in this context, the authors have set immediate research goals to update it. The aim is to formulate a more reliable computational model for predicting masonry compressive strength, thereby contributing to the design of more reliable and safe masonry structures.