Analytical approach extending the Granier method to radial sap flow patterns

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Abstract

The Granier thermal dissipation (TD) method is probably the most applied method to compute the transpiration flux of trees, due to its simplicity and effective compromise between theory and data availability. Starting from the heat transfer equations at the basis of Granier's method, the objective of this paper is to derive an analytical solution for the transpiration flux to extend the sap flow equations to the radial domain. We adopted a flexible approach to cope with the differences in radial sapflow density (SFD) profile shapes that are known to occur in relation to wood anatomy (diffuse porous vs. ring- or non-porous xylem). With this purpose, we investigated the robustness of the equations developed on some experimental and reliable radial SFD measurements available in literature to test the influence of considering or not considering the active zone close to the cambium, where most of the species-specific differences are likely to be observed. Moreover, the parameters derived by the extended formulation, are interpreted as descriptive of species-specific radial sap flow patterns. The reliability of the suggested procedure was checked against several experimental SFD profiles from literature: i) monotonically increasing SFD from the centre of the stem towards the cambium, ii) increasing SFD from the centre of the stem and then constant SFD towards the cambium, and iii) increasing SFD from the centre of the stem to a maximum SFD and then decreasing towards the cambium. Results show that according to the suggested procedure, an increasing number of parameters depending on the SFD profile complexity are required to synthetically describe the transpiration flux of different tree species. For the simplest case of monotonically increasing SFD, which could be assumed as standard under conditions of a diffuse porous tree structure, only two parameters with a clear physical meaning are required.

Key-words: Radial sap flow patterns, Granier method, Analytical solutions

1. Introduction

The transpiration component of an agro-ecosystem's evapotranspiration (ET) can be estimated by agro-meteorological models or, indirectly, from micro-meteorological measurements by solving the energy balance, with a good degree of accuracy in many types of agro-ecosystems (Verstraetenet al., 2008; Kool et al., 2014). However, the direct measurement of transpiration is still necessary, especially for ecosystems with a high degree of complexity. For example, while the FAO56 methodology (Allen et al., 1998) has proven effective in the estimation of evapotranspiration for many field crops, it is widely recognized (Kool et al., 2014; Rallo et al., 2014) that the ET estimation of many tree-crops can still be improved due to the complexities of the soil-plant-atmosphere continuum characteristics of such systems (partial soil coverage, tree-size, water capacitance, non-uniform light distribution within the canopy in widely spaced stands, etc.) (Allen et al., 1998; Allen and Pereira, 2009). Furthermore, given their large size, trees' individual features as well as the effects of inter-individual competition might generate significant tree-to-tree variation in transpiration fluxes (Sperry et al., 2008; Lehnebach et al., 2018).

An almost direct measurement of transpiration on a per-individual basis can be obtained by a number of sap flow estimation methods, with a widely varying degree of complexity. A variety of methods are available to calculate the mass flow of water in the transpiration stream by using heat as a tracer. Measurements can be taken both in herbaceous and woody plants and in any conductive organ, including roots. Depending on the method used, measurements are taken in the part of the conductive organ where the sensors are located (Smith and Allen 1996), or in the whole perimeter of the conductive organ (Sakuratani, 1987; Baker and Nieber, 1989).

Some methods integrate the sap flow in the whole sapwood, while others give information to calculate sap flow at different depths below the cambium. Some methods are invasive, with the sensors located within the sapwood; others are non-invasive, with the sensors located outside, although in close contact to the conductive organ. Some methods are suitable for small-diameter stems (Heat balance method), while others can be used in large trees. A number of variations, based on thermal exchange principles, have been developed with a variable degree of empirical approximation (Smith and Allen, 1996). Overall, this wide range of methodological options and different implementations available for sap flow estimation by using thermal methods constitutes a valuable and flexible toolset for the estimation of transpirational fluxes and thus has been widely adopted in ecological research (Wilson et al., 2001).

In the heat-tracer approach, i.e. the injection of heat into the xylematic flow, the most commonly adopted methods have been based on measuring the velocity of a heat pulse (HP) injection (Huber, 1932; Marshall, 1958, Swanson and Whitfield, 1981) or through an analysis of the thermal dissipation (TD) of a constant-rate continuously injected amount of heat (Granier, 1985).

Variations on the HP method are based on the heat-transfer modelling that was established by Huber (1932) and Marshall (1958) and further developed by Swanson and Whitfield (1981) and more, recently, by Green et al. (2003), Burgess et al. (2001) and Nadezhdina (2017). On the other hand, the thermal dissipation (TD) method (Granier, 1985) has often been considered to be empirical and in need of a species-specific calibration (Smith and Allen, 1996; Clearwater et al., 1999). This issue has been recently addressed by novel methodological approaches, both through heat modelling and through intermittent heat injection (Mahjoub et al., 2009; Do et al., 2011), and by following both numerical (Trcala and Čermák, 2014) and analytical approaches (Fuchs et al., 2017). Much like other analytical approaches suggested in different contexts (Baiamonte, 2016; Baiamonte and Singh, 2016), the latter have made it easier to understand the factors affecting the studied process.

Lu et al. (2004) and Wullschleger et al. (2011) analysing the TD methods, showed that since they are intrinsically based on the fundamental heat-transfer laws, they are still an important and challenging aspect of ecological application studies. However, due to the non-linear relationship between sap flux density and the heat transfer coefficient, the Granier method requires a species-specific calibration (Smith and Allen, 1996), and should be performed within a defined portion of the sapwood. Moreover, the uncertainty in determining the zero-flow condition constitutes one of the main limits in using the TD method (Regalado and Ritter, 2007; Rabbel et al., 2016).

Since sap flux density (SFD) estimates obtained through HP or TD are intrinsically localized because only a small portion of sapwood is sampled, upscaling SFD to the whole xylematic tissue is a challenging problem. In many circumstances, local SFD estimates have been extrapolated by simply multiplying by the sapwood area, implicitly assuming an azimuthal homogeneity of the SFD throughout the sapwood. However, this methodology has received an attentive and thorough criticism (Smith and Allen, 1996; Kostner et al., 1998; Čermák and Nadezhdina, 1998). TD methods are usually applied to the external sapwood area near the cambium, and thus they do not consider the radial sap flow patterns shown in many measurements. Moreover, neglecting such patterns could lead to the over/under estimation of the transpiration flux if integrated into the entire sapwood area. Contrary to the TD methods, HP methods can easily explore the radial changes of xylem sap flow by the simple insertion of additional temperature sensors, suitably spaced inside each probe. These sensors are considered independent, on the basis of the assumption that the heat injected by the short pulses is dissipated in the vertical direction through mass flow and that radial, sensor-to-sensor, diffusion of heat is negligible (one-dimensional model, Swanson and Whitfield, 1981).

On the contrary, TD methods using standard Granier probes neglect radial sapflow patterns, assuming that the largest contribution to the sapflow transfer is limited to the external xylematic area

(James et al., 2002; Granier, 1985). However, radial variation in the SFD has been observed in many studies (Lu et al., 2000; Nadezhdina et al., 2007; Alvarado-Barrientos et al., 2013, as well as many others), including numerical modelling approaches (Wullschleger et al 2011), with an increasing SFD pattern from the centre of the stem to the cambium, within the sapwood area. The reported patterns of radial changes in SFD are highly variable, even within the same study and the same species (Nadezhdina et al., 2007; Poyatos et al., 2007; Hérnandez-Santana et al., 2008), revealing uncertainties, especially in the sapwood area close to the cambium (Lu et al., 2004); some dependence on tree size has also been reported (Delzon et al., 2004). In general, an increase in the SFD from the centre towards the xylem portion closer to the cambium seems to be the most frequently reported pattern, in agreement with what could be expected for tree species with annual ring growth such as dicotyledons and conifers, with differences depending on wood types (diffuse porous, ring porous, non-porous; Phillips et al., 1996; Bush et al., 2010; Berdanier et al., 2014). The sap velocity peak has been reported to occur at some depth from the cambium in non-porous, (tracheid xylem anatomy) conifer species (Ford et al., 2004a; 2004b), whereas a monotonic increase from the centre towards the cambium is more frequently observed in diffuse-porous species (Fernández et al. 2001; González-Altozano et al., 2008; Berdanier, 2014; Chang et al., 2014; Fan et al., 2018).

A lower SFD in the most exterior parts of the xylem, close to cambium, has been reported in a number of studies (Ford et al., 2004a; Fernández et al., 2006), which attribute it to a number of factors, such as: the lower functionality of these xylem regions; changes in xylem hydraulic conductivity in relation to embolism events (Jiménez et al., 2000; Fernández et al. 2001; Melcher and Zwieniecki, 2013); the vascular connection of specific xylem zones to other parts of the tree, namely the canopy and root system (Phillips et al., 1996; Nadezhdina and Čermák, 2003; Brooks et al., 2003; Fernández et al., 2008; Cao et al., 2018); or even diurnal dynamics (Ford et al., 2004a; Fernández et al., 2006; Nadhezhdina et al., 2007; Saveyn et al., 2008; Motisi et al., 2012; Bai et al., 2015; Hérnandez-Santana et al., 2016). On rare occasions, opposite patterns (i.e.: a decreasing SFD from the centre to the cambium) were reported, but with no clear interpretation (Asian pear, Fernández et al., 2008).

Radial SFD gradients implies that some uncertainties in upscaling remain . However this has been addressed in terms of the magnitude and significance of sampling errors in very few studies (Dye et al., 1991; Hatton et al., 1995; Wullschleger and King, 2000). Tree-to-tree variation in SFD has also been observed to change with tree hydraulic architecture (Sperry et al., 2008; Kumagai et al., 2005) and size, introducing a further uncertainty in upscaling SFD to a whole stand. This supports the idea that an accurate SFD estimation on a per-tree basis must include a sufficient number of observations to account for the individual variability within a stand. Thus, the radial SFD variability in the simple TD method is still an open issue that could be addressed by developing simple analytical approaches, providing initial insight on the studied process prior to the application of time-consuming numerical methods.

The objective of this paper is to derive an analytical solution for the transpiration flux, E_s, under radial sap flow patterns following Granier's method, and to investigate the suitability of the derived solution to cope with both monotonic and non-monotonic SFD profiles, according to the different shapes that they can assume. Moreover, an attempt is made to interpret the parameters derived by the extended formulation, as descriptive of species-specific radial sap flow patterns.

2. Theory

Granier (1985) assumed that for a constant 0.2 W of electric power delivered to heated probes via a coiled, constantly heated element, and under a one-dimensional steady-state thermal regime between the heating element and the medium (wood + sap), the contribution of heat power by the Joule effect is equal to the amount of heat dissipated at the sensor wall:

$$hS(T - T_\infty) = Ri^2 \qquad (1)$$

where h (W m^{-2} K^{-1}) is the heat transfer coefficient, S (m^2) is the surface exchange area, R (Ω) is the electric resistance, i (A) is the electric current, T (°C) is the probe temperature, and T_∞ (°C) is the temperature of the wood material in the absence of any heating. It was also assumed that the coefficient h is linearly related to the sap flux density, ν (m/s), according to an α coefficient, by the following equation (Lu et al. 2004):

$$h = h_0\left(1+\alpha \nu^b\right) \tag{2}$$

where in the original formulation the exponent b equals 1 to account for the assumed linearity, and h_0 is the heat transfer coefficient under zero flux. If the probe temperature at zero flux is denoted as T_{max}, Eq. (2) and Eq. (1) make it possible to determine the coefficient h_0:

$$h_0 = \frac{R\,i^2}{S\left(T_{max} - T_\infty\right)} \tag{3}$$

By using Eq. (1) and Eq. (3), for constant $\nu > 0$, Eq. (2) yields:

$$\nu = \frac{1}{\alpha^{1/b}}\left(\frac{h}{h_0} - 1\right)^{1/b} = \frac{1}{\alpha^{1/b}}\left(\frac{T_{max} - T}{T - T_\infty}\right)^{1/b} \tag{4}$$

The right hand ratio in bracket is a dimensionless number that was denoted by Granier (1985) as the flux index, K. Granier (1985) experimentally showed the relationship between K vs. ν, giving $K = 0.0206\ \nu^{0.8124}$. Inversion of the latter with respect to ν and a comparison with Eq. (4) shows that $1/\alpha^{1/b} = 118.99\ 10^{-6}$ m s^{-1} and $1/b = 1.231$, which are denoted here as c_g and k_g, respectively, where the subscript "g" of both k and c denotes the value of the k and c parameters obtained by Granier (1985) via calibration. It is important to notice that the experimental value $b = 1/k_g \neq 1$ shows that Eq.

(2) is nonlinear.

Eq. (4) can be rewritten by introducing the temperature difference between the heated and the reference probe, ΔT (°C), and the temperature difference between the sensor probes under zero flow conditions, ΔT_{max} (°C) (Lu et al., 2004):

$$\nu = c_g \left(\frac{\Delta T_{max} - \Delta T}{\Delta T} \right)^{k_g} \qquad (5)$$

where c_g is the scale coefficient (0.119) expressed in (Kg m^{-2} s^{-1}), k_g is the shape exponent (1.231) suggested by Granier (1985), and the sap flux density, ν, is also expressed in (Kg m^{-2} s^{-1}). We express the ν unit in (Kg m^{-2} s^{-1}) rather than in (m s^{-1}), since the latter usually refers to the sap flow velocity, whose notation has been discouraged in sap flow studies (Lemeur et al., 2009).

By normalizing the temperature difference with respect to its maximum value, ΔT_{max}, and generalizing for any k and c values, Eq. (5) can be further rewritten as:

$$\nu = c \left(\frac{1 - \Delta T / \Delta T_{max}}{\Delta T / \Delta T_{max}} \right)^{k} \qquad (6)$$

For a fixed scale parameter, c, Eq. (6) allows the sap flow density (SFD), ν, to be expressed versus the dimensionless ratio $\Delta T/\Delta T_{max}$, with the shape factor, k, as a parameter. Values of c and k that differ from those suggested by Granier (c_g and k_g) were also determined for a number of species, probe geometry and trees' hydraulic conductivity (Bush et al., 2010; Peters et al., 2018). However, the procedure described in the following can be applied for any pair (c , k).

Contrary to c, which represents the scale factor of Eq. (6) , k influences the shape of the ν relationship. For $c = c_g$, **Figure 1** graphs ν versus $\Delta T/\Delta T_{max}$, for $k = k_g = 1.231$, and for different k

values around k_g (±10 % k_g, ±20 % k_g). As expected, at increasing $\Delta T/\Delta T_{max}$, ν decreases, approaching zero for zero flux conditions ($\Delta T = \Delta T_{max}$). It is interesting to observe that for $\Delta T = 0.5\ \Delta T_{max}$, the same $\nu = c_g$ value occurs, for any k value. The latter, which of course can also be derived by Eq. (6), has some important implications when applying the Granier method. First, the c (i.e. c_g) constant has a clear physical meaning, since it represents the sap flow density that occurs for any k value, when $\Delta T = 0.5\ \Delta T_{max}$. Second, for $\Delta T/\Delta T_{max} > 0.5$, the sap flow density is less sensitive to k, whereas for $\Delta T/\Delta T_{max} < 0.5$ the sap flow density is more affected by k. Therefore, from a practical point of view, an experimental estimation of k should be performed at high SFDs (low $\Delta T/\Delta T_{max}$).

2.1 Determining transpiration flux for monotonic radial sapflow patterns

By assuming a constant ΔT_{max} , i.e. constant thermal properties, throughout the whole sapwood area, Eq. (6) can be used to extend the Granier method to different radial sap flow patterns. In other words, it is assumed that the temperature difference profile along the radius attains its maximum at the internal sapwood area where zero flux conditions actually take place, without the restriction of zero night-time sap flow (Regalado and Ritter, 2007).

Moreover, it is assumed here that the temperature difference, ΔT, between the heated and the reference probe, varies along the radial direction of the trunk or of the branch, according to a simple power-law. The latter was used because of its simplicity, which has also made it widely considered in many other contexts, e.g. to describe the pressure head – flow rate relationship in drip laterals design (Baiamonte, 2018), to describe the allometric scale in the hydraulic scaling of biophysical phenomena (West et al., 1999) or rainfall intensity-duration-frequency curves (Baiamonte and Singh, 2017; Baiamonte, 2019). The simple-power law applied to the radial sap flow pattern is written in dimensional and dimensionless forms here, respectively:

$$\varDelta T = \varDelta T_{min}\, r_*^{m_1} \tag{7a}$$

$$\frac{\varDelta T}{\varDelta T_{max}} = \varDelta T_*\, r_*^{m_1} \tag{7b}$$

where $\varDelta T_{min}$ is the minimum value of the temperature difference along the trunk radius (i.e. $\varDelta T$ at maximum sap flow density), $\varDelta T_*$ denotes the ratio $\varDelta T_{min} / \varDelta T_{max}$, $r_* = r/r_c$ is the distance between the measuring point and the trunk or branch centre, r, normalized with respect to trunk radius (below the cambium), r_c, and m_1 is the power-law shape factor (≤ 0, since ν increases with r_*).

If experimental pairs (r_*, $\varDelta T$) are available, m_1 and $\varDelta T_{min}$ (i.e. $\varDelta T_*$) can be determined through a linear regression of the corresponding logarithmic values. As an example, for a fixed $\varDelta T_{min}$ = 6 °C and for different m_1 values, **Figure 2a** plots the temperature difference versus the normalized radius, showing that for any m_1 value, for r_* approaching to one, $\varDelta T \sim \varDelta T_{min}$. Moreover, the figure shows that the selected m_1 values cover different $\varDelta T$ profile shapes.

The assumption of Eq. (7b) covers a multitude of $\varDelta T$ profiles occurring in heat dissipation probe measurements and, as will be shown, makes it possible to characterize radial sap flow patterns according to the already defined two parameters with a clear physical meaning, ΔT_* and m_1.

Substituting Eq. (7b) in Eq. (6) yields:

$$\nu = c\left(\frac{1-\Delta T_*\, r_*^{m_1}}{\Delta T_*\, r_*^{m_1}}\right)^k = c\left(\frac{\Delta T_{max} - \Delta T_{min}\, r_*^{m_1}}{\Delta T_{min}\, r_*^{m_1}}\right)^k \tag{8}$$

For the same m_1 values used in **Figure 2a**, for $c = c_g$ and $k = k_g$, and, as an example, for a fixed $\varDelta T_*$ = 0.2, Eq. (8) is plotted in **Figure 2b**, indicating that using the power-law to describe the $\varDelta T$ profile makes it possible to reproduce a multitude of radial SFD patterns, including the constant and the linear SFD profiles that are achieved for m_1 = 0 and $m_1 \cong$ -0.66, respectively. Interestingly, m_1 =

0 (dashed line) corresponds to a uniform SFD over the whole sap flow area. Moreover, **Figure 2b** shows that for $m_1 < -0.66$, the SFD profile is characterized by a concave shape, whereas for $m_1 > -0.66$ the SFD profile is characterized by a convex shape. Both shapes could be useful for reproducing experimental radial sap flow patterns (Ford et al., 2004a; Berdanier et al., 2016).

By putting in Eq. (8) ν = 0, and solving by r_*, it is also possible to derive the normalized measuring point that corresponds to the zero flux condition, r_{*0}:

$$r_{*0} = \frac{1}{\Delta T_*^{1/m_1}} \tag{9}$$

whereas, for $r_* = 1$, Eq. (8) provides the maximum SFD, ν_{max}, which, under monotonically increasing SFD vs. r_*, could be attained at the cambium (Bush et al., 2010; Berdanier et al., 2014).

In order to derive the transpiration rate at the tree scale, E_{tree} (Kg s^{-1}), under the simplified assumption of radial symmetry, and in order to account for the SFD variability versus the radial direction, Eq. (8) mast be integrated:

$$E_{tree} = 2\pi \int_{r_1}^{r_2} \nu\, r\, dr = 2\pi\, r_c^2 \int_{r_{*1}}^{r_{*2}} \nu\, r_*\, dr_* \tag{10}$$

where r_{*1} and r_{*2} are the normalized nearest and the farthest measurement points from the trunk centre, respectively.

Integration Eq. (10) provides the following solution for the transpiration rate E_s (Kg s^{-1} m^{-2}), normalized with respect to the cross sectional area below the cambium, πr_c^2 (see **Appendix A**):

$$E_s = \frac{E_{tree}}{\pi r_c^2} = \frac{2c}{m_1 \Delta T_*^{2/m_1}} \left(B\left(r_{*2}^{m_1} \Delta T_*, \frac{2}{m_1} - k, 1+k \right) - B\left(r_{*1}^{m_1} \Delta T_*, \frac{2}{m_1} - k, 1+k \right) \right) \tag{11a}$$

where B (.,.,.) is the incomplete Beta function. Eq. (11a) shows that if c and k are known or are fixed as equal to those suggested by Granier (1985), c_g and k_g, E_s depends on four parameters, ΔT_*, m_1, r_{*1}, r_{*2}. Eq. (11a) in agreement with Eq. (7b), is delimited by the real solutions domain, i.e. the E_s threshold, $E_{s,th}$, obtained by putting the constraint $\Delta T_* = r_{*1}^{-m_1}$ in Eq. (11a), yielding:

$$E_{s,th} = \frac{2 c r_{*1}^2}{m_1} \left(B\left(\frac{r_{*2}^{m_1}}{r_{*1}^{m_1}}, \frac{2}{m_1} - k, 1+k \right) - B\left(\frac{2}{m_1} - k, 1+k \right) \right) \qquad (11b)$$

As an example, for $r_{*1} = 0.5$ and $r_{*2} = 1$, and for Granier's parameters ($c = c_g$ and $k = k_g$), **Figure 3a** shows the transpiration rate, $E_{s,g}$ (mm h^{-1}), versus ΔT_* with different m_1 values as a parameter, calculated according to Eq. (11a). **Figure 3a** shows the high $E_{s,g}$ variability (four orders of magnitude), including all the possible $E_{s,g}$ values according to m_1 and ΔT_* parameters. However, the lowest range of ΔT_* corresponds to very high and unrealistic SFD fluxes, which are quite far from the usual range. As expected, with increasing ΔT_*, $E_{s,g}$ decreases approaching to the zero flux condition, for ΔT_* =1 ($\Delta T_{min} = \Delta T_{max}$). Whereas for fixed ΔT_*, $E_{s,g}$ increases with m_1, showing the influence of the shape parameter m_1 on $E_{s,g}$, which, according to what would be expected, is less and less evident for very high and very low ΔT_* values.

Similar considerations are valid for **Figure 3b**, where the vertical axis is delimited to the most common transpiration rate values (1-1000 mm h^{-1}) that are most likely to be useful for applications compared to that of **Figure 3a**. In both **Figure 3a** and **Figure 3b**, Eq. (11b)'s delimitation of the real solutions domain expressed is also represented. In conclusion, the fusiform shape delimited by the curves displayed in these two figures cover the transpiration rate that could be expected for any tree species, if exploring the [r_{*1}, r_{*2}] domain.

As an example, for an experimental SFD profile, which will be considered in section 3 (ID 5,

Table 1), **Figure 4** illustrates the normalized measuring point corresponding to the zero flux condition r_{*0}, the normalized nearest and farthest measurement points from the trunk centre, r_{*1} and r_{*2}, together with the monotonically increasing SFD profile ABC (Eq. 8), which was derived by ignoring the measurement point at $r_{*3} \in [r_{*2}, 1]$, v_3. In fact, the latter occurrence requires further derivations for non-monotonic radial sapflow patterns (profile ABE), which will be considered in the next section. Eq. (11a) makes it possible to describe the transpiration rate for any monotonic radial sapflow pattern and for any domain delimited by r_{*1} and r_{*2}. If r_{*1} and r_{*2} are located around the zero flux zone and around the cambium, Eq. (11a) provides a good approximation of the transpiration rate. However, if this condition does not occur, Eq. (11a) could underestimate the transpiration rate. In this case, a transpiration rate close to the actual one could be determined by extrapolating the SFD profile on both r_{*0} and r_{*2} =1 sides. Towards this aim, by putting the constraints $r_{*1} = r_{*0}$ (Eq. 9) and $r_{*2} = 1$ in Eq. (11a), E_s could be derived for the entire range $[r_{*0}, 1]$ (see Eq. (A.6 in **Appendix A**):

$$E_s = \frac{2c}{m_1\, \Delta T_*^{2/m_1}} \left(B\left(\Delta T_*, \frac{2}{m_1} - k, 1+k \right) - B\left(\frac{2}{m_1} - k, 1+k \right) \right) \qquad (12)$$

where for this case, the last term B (.,.) in the right hand matches the complete Beta function (Caylor and Dragoni, 2009; Alvarado-Barrientos, 2013). Interestingly, for a fixed c and k, E_s calculated in the range $[r_{*0}, 1]$ only depends on m_1 and ΔT_*.

2.2 Determining transpiration flux for non-monotonic radial sapflow patterns

Eq. (11a) can only be used for monotonic radial sapflow patterns (e.g. diffuse pore species). Non-monotonic radial sapflow patterns can be considered by introducing two more simple cases. In the first case, if the farthest measurement points from the trunk centre r_{*2} is far from the cambium, rather than extrapolating the monotonic SFD increase as for Eq. (11a), a constant SFD was assumed in [r_{*2}, 1]. In the second case, if experimental measurements show a SFD decrease in [r_{*2}, 1], a theoretical linear decreasing SFD profile was taken into account. These last two occurrences are also illustrated in **Figure 4**, where the SFD profile ABD and ABE are plotted, respectively.

Deriving the transpiration rate solution by assuming a constant SFD in [r_{*2}, 1] also makes it possible to study the influence of extrapolating the SFD profile in an E_s estimate, according to the farthest measurement point from the trunk centre, for $r_* \in$ [r_{*2}, 1] (ABD in **Figure 4**). Therefore, for the ABD profile illustrated in **Figure 4**, the following solution for the transpiration rate was derived (see **Appendix B**):

$$E_s = \frac{2c}{m_1 \, \Delta T_*^{2/m_1}} \left(B\left(r_{*2}^{m_1} \, \Delta T_*, \frac{2}{m_1} - k, 1+k \right) - B\left(\frac{2}{m_1} - k, 1+k \right) \right) + c \left(\frac{1 - \Delta T_* \, r_{*2}^{m_1}}{\Delta T_* \, r_{*2}^{m_1}} \right)^k \left(1 - r_{*2}^2 \right) \quad (13)$$

For the second SFD profile (ABE in **Figure 4**), this time including the measurement points at r_{*3}, close to the cambium, for $r_* \in$ [r_{*2}, 1] (BE), it is assumed that the SFD linearly decreases with the dimensionless radius. Accordingly, the following solution for the transpiration rate was derived (see **Appendix C**):

$$E_s = \frac{2c}{n \, \Delta T_*^{2/n}} \left(B\left(r_{*2}^{n} \, \Delta T_*, \frac{2}{n} - k, 1+k \right) - B\left(\frac{2}{n} - k, 1+k \right) \right) + \frac{2 m_2}{3} \left(1 - r_{*2}^3 \right) n \left(1 - r_{*2}^2 \right) \quad (14)$$

where m_2 and n are the gradient and the ν-intercept of the ν - r_* line in the domain [r_{*2}, 1]. Two cases can be distinguished when determining m_2 and n, depending on the availability of only one or more than one measurement point in the domain [r_{*2}, 1].

If only one SFD measurement in the (r_{*2}, 1) domain is available, meaning that only a pair (r_{*3}, ν_3) is known (see **Figure 4**), m_2 and n can be easily calculated as:

$$m_2 = \frac{\nu_2 - \nu_3}{r_{*2} - r_{*3}} \tag{15}$$

$$n = \frac{r_{*2}\,\nu_3 - \nu_2 r_{*3}}{r_{*2} - r_{*3}} \tag{16}$$

where ν_2 and ν_3 are the transpiration fluxes corresponding to r_{*2} and r_{*3}, respectively.

If more than one SFD measurement in the [r_{*2}, 1] domain is available, m_2 can be calculated according to a simple constrained linear regression for the experimental pairs ($r_* - r_{*2}$, $\nu - \nu_2$) and by imposing n, so that in r_{*2}, the corresponding ν_2 value equals that calculated according to Eq. (8):

$$n = \nu_2 - m_2\, r_{*2} = c\left(\frac{1 - \Delta T_*\, r_{*2}^{m_1}}{\Delta T_*\, r_{*2}^{n}}\right)^{k} - m_2\, r_{*2} \tag{17}$$

3. Deriving m_1 and ΔT_* parameters from radial sap flow density measurements

Eq. (8) can be used to extend our analysis to SFD profiles, where actual ΔT data is not available, since methods other than TD such as HPV were used. Differently from what was observed in Section 2.1, where the experimental pairs (r_*, ΔT) were available (Eq. 7b), m_1 and ΔT_* can be as well determined directly from SFD profile data by rearranging Eq. (8):

$$r_*^{m_1} = \frac{1}{\Delta T_*\left(1 + \nu^{1/k}\, c^{-1/k}\right)} \tag{18}$$

Taking the logarithm of both sides of Eq. (18) yields:

$$m_1 \, ln\, r_* = ln \frac{1}{\Delta T_* \left(1 + \nu^{1/k} c^{-1/k}\right)} \quad (19a)$$

Rearranging Eq. (19a) once again gives the linear form:

$$ln\left(1+\nu^{1/k} c^{-1/k}\right) = -n_1 \, ln\, r_* - ln\, \varDelta T_* \; ln\left(1+\nu^{1/k} c^{-1/k}\right) = -m_1 \, ln\, r_* - ln\, \Delta T_* \quad (19b)$$

Thus, from the simple linear regression of the pairs ($ln\ r_*$, $ln\ (1+\nu^{1/k} c^{-1/k})$), m_1 and $\varDelta T_*$ can be determined. It is interesting to observe that if the method used to detect radial SFD patterns differs from that of Granier, using Eq. (19b) provides the same parameters (m_1, $\varDelta T_*$) that would have been obtained, if the Granier method had been used.

3.1 Experimental literature data to test the proposed method

The SFD measurements published by Nadezhdina et al. (2007), obtained in olive trees by the heat field deformation method (HFD) (Nadezhdina et al., 1998; Nadezhdina and Čermák, 2000; Čermák et al., 2004; Nadezhdina et al., 2006), were used as a dataset to derive the corresponding parameters (m_1, $\varDelta T_*$) that would have been obtained under Granier method.

Eleven radial sap flow profiles with different shapes were selected from Nadezhdina et al. (2007) and the corresponding pairs (r_*, ν) were obtained from the figures through digitizing (Engauge digitizer software, Mitchell et al., 2019). An application was carried out in order to check the ability of Eq. (12) to calculate, under the radial symmetry assumption, the transpiration rate normalized with respect the cross sectional area, E_s. Moreover, 2 SFD profiles measured for peach trees (González-Altozano et al., 2008) were also selected for a total of 13 SFD profiles (**Table 1**).

Towards this aim, for the 13 selected SFD profiles, **Table 1** reports: the sap flow location measurements (trunk or branch); the trunk or branch radius r_c (mm); the figure number corresponding to Nadezhdina et al. (2007) and González-Altozano (2008) references; m_1 and ΔT_*, calculated by fitting the linear regression of Eq. (19b); and the normalized radius corresponding to the zero flux, r_{*0} (Eq. 9).

In **Table 1**, for all of the cases in which SFD profiles decrease along the radius in $r_* \in [r_{*2},1]$ (SFD # 1, 5, 8, 9, 10 and 11), the ΔT_* and m_1 parameters, which refer to the monotonic radial profile in $[r_{*0},1]$ (Eq. 12, **Figure 4**, ABC), were calculated by excluding the experimental pairs in $r_* \in [r_{*2},1]$.

Figure 5 graphs the experimental pairs (r_*,ν) and the theoretical SFD expressed by Eq. (8) with the m_1 and ΔT_* parameters reported in **Table 1**. As can be observed in **Figure 5**, Eq. (8) appears to suitably explain the investigated radial SFD patterns. Indeed, the selected profiles cover different radial SFD patterns with different m_1 values, including the almost constant SFD corresponding to the ID10 profile, for which the shape parameters among those associated with the selected profiles (**Table 1**) equals the maximum value $m_1 = -0.091$, which is close to zero (constant SFD profile).

As mentioned above, the aim of this application was also to detect the effect of extrapolating the SFD profile until the cambium or of assuming the SFD profile to be constant or decreasing from the furthest measurement from the centre trunk ($r_* = r_{*2}$) to the cambium ($r_* = 1$), according to Eq. (13) and Eq. (14), respectively.

For this purpose, the ν value calculated at r_{*2} by Eq. (8) was assumed to be constant in $[r_{*2}, 1]$ for all of the considered 13 SFD profiles, whereas the m_2 and n parameters were only calculated for the radial SFD measurements (ID # 1, 5, 8, 9, 10 and 11, **Figure 5 a, b, d, e**), where a decreasing SFD profile was observed in $[r_{*2},1]$.

Moreover, for SFD profiles ID # 8 and 9, which had more than two measurements presenting a decreasing SFD in $[r_{*2}, 1]$ as observed above, m_2 was determined by linear regression, whereas the n parameter was calculated by Eq. (17). For the remaining profiles with a decreasing SFD with only one measurement in $[r_{*2}, 1]$, m_2 and n were calculated by Eq. (15) and Eq. (16), respectively. Of course, for SFD profiles ID # 2, 3, 4, 6, 7, 12 and 13, SFD was observed to increase monotonically, Eq. (14) was not applied and m_2 and n were not estimated (**Table 1**). In **Table 1**, the SFD profiles where m_2 and n parameters were obtained by linear regression, are indicated by a star symbol.

4. Results of the applications

Figure 6a plots the 13 theoretical SFD profiles under the assumption of a monotonic radial sapflow pattern in $[r_{*0}, 1]$, indicating that they actually cover a variety of SFD shapes in agreement with the wide range of variability of the m_1 parameter reported in **Table 1** (-0.892, -0.091). In particular, Eq. (8) also describes the sap flow profile ID10 (m_1 = -0.091) well, for which an almost constant SFD profile occurs until it approaches the case in which radial SFD variability is neglected (ν = constant). However, it might be easier to calculate the transpiration rate for this simple case according to $E_s = \nu\,(r_{*2}^2 - r_{*0}^2)$, whereas the extended formulation is required for all of the others, which refer to different radial sap flow patterns.

Once the m_1 and ΔT_* parameters have been estimated, Eq. (7b) makes it possible to derive the corresponding temperature difference profiles that would have been determined if the extended Granier method had been used to determine the radial SFD patterns presented in this study. The corresponding ΔT profiles are plotted in **Figure 6b**, showing that, as expected, for $r_* \sim 1$, the ratio $\Delta T_* = \Delta T_{min}/\Delta T_{max}$ for each profile is achieved, whereas $\Delta T_* = 1$ in r_{*0} (**Table 1**).

For the SFD profiles described in Section 3, in order to investigate the suitability of the derived solution to cope with both monotonic and non-monotonic SFD profiles, according to the different shapes that they may assume, the transpiration rate was calculated by considering the following three radial patterns: i) monotonically increasing SFD in the domain $[r_{*0}, 1]$ (Eq. 12, ABC in **Figure 4**), ii) monotonically increasing SFD in the domain $[r_{*0}, r_{*2}]$ and constant SFD in $[r_{*2}, 1]$ (Eq. 13, ABD in **Figure 4**) and iii) monotonically increasing SFD in the domain $[r_{*0}, r_{*2}]$ and decreasing SFD in $[r_{*2}, 1]$ (Eq. 14, ABE in **Figure 4**).

Figure 7 reports the estimates of $E_{s,g}$, according to Eq. (12), Eq. (13) and Eq. (14) for the three selected SFD profile shapes. The transpiration rate can be observed to be in the range 50 -320 mm/h. However, the hypotheses assumed in Eq. (12), Eq. (13) and Eq. (14) have a certain influence on the computation of $E_{s,g}$. As expected, differences in $E_{s,g}$ are linked to the values of the m_1 parameter and to the location of the furthest measurement point, r_{*2} (see **Figure 4**). For ID #2, 3, 4, 6, 7, 10, 12 and 13, differences in the $E_{s,g}$ estimate appear small because of the high r_{*2} or m_1 values, which have a negligible effect on the extrapolation of the SFD to the cambium. In particular, the latter condition occurs for ID #10 with a high m_1 value (m_1 = -0.091).

The greatest differences in calculating $E_{s,g}$ by extrapolating the SFD profile in [r_{*2}, 1, Eq. 12] or by assuming the SFD in [r_{*2}, 1, Eq. 13] to be constant can be observed for ID #9, which is characterized by low m_1 and r_{*2} values (see **Table 1**). For ID #8, and 9, with a similar r_{*2} value, differences in $E_{s,g}$ are also due to the different m_2 values that shows more abrupt decrease in SFD for ID # 9 than ID #8, as can be observed in **Figure 5d**.

In conclusion, when SFD measurements do not cover the entire radial sap flow pattern ($r_{*2} \ll 1$), and when the m_1 shape factor is low, using Eq. (12) could overestimate the transpiration rate and thus might not be recommended. The use of Eq. (13), which assumes the SFD to be constant from the furthest point measurement to the cambium, could be a good compromise between extrapolating the SFD profile in [r_{*2}, 1, Eq. 12] and calculating the transpiration rate with Eq. (14), especially if the SFD decrease in [r_{*2}, 1] is not supported by an adequate number of observations.

On the contrary, a monotonically increasing SFD profile (Eq. 12) should be preferred if the range of measurements cover the area from the sapwood until the cambium, especially for low m_1 values, with the advantage of describing the transpiration flux with only two parameters (m_1 and ΔT_*).

Moreover, with respect to Eq. (13) and to Eq. (14), Eq. (12) could be more advantageous when applied in a wider context, and when the transpiration rate has to be upscaled. In fact, in order to calculate $E_{s,g}$, Eq. (12) only requires the two parameters, m_1 and ΔT_*, which could be associated with the crop age and with the species characteristic, to arrive at an indirect $E_{s,g}$ estimation. In other words, the availability of a very wide numbers of SFD profiles or of temperature difference profiles, which both make it possible to derive m_1 and ΔT_* parameters, could help delimit the $E_{s,g}$ domain under reference environmental conditions, suggesting an indirect criteria to derive $E_{s,g}$.

In order to locate the selected 13 SFD profiles in the $E_{s,g}$ - ΔT_* plane, for the case of monotonic radial sapflow patterns, Eq. (12) is graphed in **Figure 8** for fixed m_1 values. In the same figure, the calculated $E_{s,g}$ values are plotted for the 13 SFD profiles. Notwithstanding the fact that SFD measurements are limited to only two species, olive (ID #1-11) and peach (ID #12-13), the points cover a wide $E_{s,g}$ - ΔT_* domain as a consequence of the high variability in the selected SFD profiles probably characterized by different observation times, and soil moisture conditions. Moreover, measurements that are often taken under different azimuthal positions and climate conditions (Nadezhdina et al., 2002) may also affect $E_{s,g}$ and ΔT_* variability.

These results indicate the need to establish standard environmental conditions, including: soil moisture content, sampling time, stand age, establishing a fixed experimental setup that could lead to locate in the $E_{s,g}$ - ΔT_* plane the transpiration rate domains corresponding to different species, towards extending the potential evapotranspiration concept, which is also defined under reference conditions, to the sole transpiration component of the water mass balance.

5. Conclusions

An analytical approach extending the Granier method to radial sap flow density (SFD) patterns, showing high flexibility with parameters with a clear physical meaning, is suggested. The proposed procedure made it possible to develop a general framework to analyse different radial SFD patterns, with an increasing number of parameters, depending on the SFD profile complexity. These parameters were able to synthetically describe the transpiration flux, E_s, of different tree species.

For the simplest case, with a monotonically increasing SFD, which could be assumed for diffuse porous tree structure, only two distinctly physical parameters are required, the ratio between the minimum and the maximum temperature difference, ΔT_*, and a shape factor, m_1, which could be derived by SFD profiles measured by methods other than Granier. Therefore, the proposed procedure could be suitable as a meta-analysis tool, for homogenizing SFD profiles of different species by means of ΔT_* and m_1 parameters, irrespective of the measurement method used.

The totality of transpiration flux conditions are constrained within a fusiform plane, E_s - ΔT_*, delimited by the analytical solutions derived, thus covering the transpiration flux that could be expected for any tree species. The high variability of the transpiration flux, located in the E_s - ΔT_* plane, and that of the corresponding m_1 and ΔT_* parameters, which was observed for two different species (olive and peach trees), suggested that fixing experimental setup, associated with standard environmental conditions, could lead to locate in the E_s - ΔT_* plane, according to m_1 and ΔT_*, more restricted transpiration rate domains, corresponding to different species, towards extending the potential evapotranspiration concept to the sole transpiration component of the water mass balance.

Appendix A - *Deriving the transpiration rate E_s, for monotonically increasing SFD vs. r_*, from the zero flux circle (r*0), to the outer region below the cambium [r*1 = r*0, r*2 = 1] (see ABC Fig.4).*

The radial variation of the sap flux density, ν, and the transpiration rate at the tree scale, E_{tree} (Kg s^{-1}), are expressed according to Eq. (8) and Eq. (10), respectively:

$$\nu = c\left(\frac{1-\Delta T_*\, r_*^{m_1}}{\Delta T_*\, r_*^{m_1}}\right)^k \tag{A.1}$$

$$E_{tree} = 2\pi\int_{r_1}^{r_2} v\, r\, dr = 2\pi\, r_c^2 \int_{r_{*1}}^{r_{*2}} v\, r_*\, dr_* \tag{A.2}$$

Substituting Eq. (A.1) in Eq. (A.2) yields:

$$E_{tree} = 2\pi c\, r_c^2 \int_{r_{*1}}^{r_{*2}} \left(\frac{1-\Delta T_*\, r_*^{m_1}}{\Delta T_*\, r_*^{m_1}} \right)^k r_*\, dr_* \qquad \text{(A.3)}$$

Rewriting the integrand of Eq. (A.3) as:

$$\int_{r_{*1}}^{r_{*2}} \left(\frac{1}{\Delta T_*\, r_*^{m_1}} - 1 \right)^k r_*\, dr_* = \int_{r_{*2}}^{r_{*1}} r_* \left(1 - \frac{1}{\Delta T_*\, r_*^{m_1}} \right)^k dr \qquad \text{(A.4)}$$

shows that it corresponds to the incomplete Beta function, $B_z(a,b) = B(z;a,b) = \int_0^z t^{a-1}(1-t)^{b-1} dt$, with $t = r_*$, $a = 2$, $b = k + 1$, which provides an analytical solution for the transpiration rate E_s (Kg s^{-1} m^{-2}), denoted as the ratio between the transpiration at the tree scale and the cross sectional area below the cambium, πr_c^2:

$$E_s = \frac{E_{tree}}{\pi r_c^2} = \frac{2c}{m_1\, \Delta T_*^{2/m_1}} \left(B\left(r_{*2}^{m_1}\, \Delta T_*, \frac{2}{m_1} - k, 1+k \right) - B\left(r_{*1}^{m_1}\, \Delta T_*, \frac{2}{m_1} - k, 1+k \right) \right) \qquad \text{(A.5)}$$

By putting the constraint $r_{*1} = r_{*0} = \Delta T_*^{-1/m_1}$ (Eq. 9) in Eq. (A5), meaning that the SFD is extrapolated to the zero flux circle (r_{*0}), E_s could be derived for the range [r_{*0}, 1], which must be considered if the extrapolation is also extended to the outer region below the cambium ($r_{*2} = 1$):

$$E_s = \frac{2c}{m_1\, \Delta T_*^{2/m_1}} \left(B\left(\Delta T_*, \frac{2}{m_1} - k, 1+k \right) - B\left(\frac{2}{m_1} - k, 1+k \right) \right) \qquad \text{(A.6)}$$

where in this case, the last term on the right, B (.,.), matches the complete Beta function (Caylor and Dragoni 2009, Alvarado-Barrientos 2013).

Appendix B - *Deriving the transpiration rate E_s, for monotonically increasing SFD vs. r_*, in the domain $[r_{*0}, r_{*2}]$ and constant SFD in $[r_{*2}, 1]$ (see ABD Fig.4).*

In order to study the influence of assuming the SFD profile to be constant, after the farthest measurement from the centre, $r_* \in [r_{*2}, 1]$ (ABD in **Figure 4**), thus excluding SFD profile extrapolation, which Eq. (A.6) takes into account, the additional transpiration rate in the domain $[r_{*2}, 1]$ should be considered:

$$E_s^{(r_{*2},1)} = 2c \int_{r_{*2}}^{1} \left(\frac{1 - \Delta T_* \, r_{*2}^{m_1}}{\Delta T_* \, r_{*2}^{m_1}} \right)^k r_* \, dr_* = 2c \left(\frac{1 - \Delta T_* \, r_{*2}^{m_1}}{\Delta T_* \, r_{*2}^{m_1}} \right)^k \left. \frac{r_*^2}{2} \right|_{r_{*2}}^{1} \qquad \text{(B.1)}$$

that can be solved as:

$$E_s^{(r_{*2},1)} = c \left(\frac{1 - \Delta T_* \, r_{*2}^{m_1}}{\Delta T_* \, r_{*2}^{m_1}} \right)^k \left(1 - r_{*2}^2 \right) \qquad \text{(B.2)}$$

In this case, the total amount of the transpiration rate can be derived by putting $r_{*1} = r_{*0}$ (Eq. 9) into Eq. (A.6), i.e. considering the domain $[r_{*0}, r_{*2}]$ and by adding Eq. (B.2), giving:

$$E_s = \frac{2c}{m_1 \, \Delta T_*^{2/m_1}} \left(B\left(r_{*2}^{m_1} \, \Delta T_*, \frac{2}{m_1} - k, 1 + k \right) - B\left(\frac{2}{m_1} - k, 1 + k \right) \right) + c \left(\frac{1 - \Delta T_* \, r_{*2}^{m_1}}{\Delta T_* \, r_{*2}^{m_1}} \right)^k \left(1 - r_{*2}^2 \right) \qquad \text{(B.3)}$$

Appendix C - *Deriving the transpiration rate E_s, for monotonically increasing SFD vs. r_*, in the domain [r_{*0}, r_{*2}] and decreasing SFD in [r*2, 1] (see ABE Fig.4).*

For the third considered case (ABE in **Figure 4**), this time including the measurement points close to the cambium (BE), it is assumed that the SFD linearly decreases with the dimensionless radius. Thus, the additional contribution of the SFD to the transpiration rate in the domain [r_{*2}, 1] can be calculated as:

$$E_s^{(r_{*2},1)} = 2\int_{r_{*2}}^{1} \nu\, r_*\, dr_* = 2\int_{r_{*2}}^{1} \left(m_2\, r_* + n\right) r_*\, dr_* = \frac{2\,m_2}{3}\left(1 - r_{*2}^{3}\right) n \left(1 - r_{*2}^{2}\right) \qquad \text{(C.1)}$$

where m_2 and n are the gradient and the ν-intercept of the ν - r_* line in the domain [r_{*2}, 1]. Analogously to the second case, the total amount of the transpiration rate can be derived by putting $r_{*1} = r_{*0}$ (Eq. 9) into Eq. (A.6), i.e. by considering the domain [r_{*0}, r_{*2}] and by adding Eq. (C.1), giving:

$$E_s = \frac{2c}{n\,\Delta T_*^{2/n}}\left(B\left(r_{*2}^{n}\,\Delta T_*, \frac{2}{n} - k, 1+k\right) - B\left(\frac{2}{n} - k, 1+k\right)\right) + \frac{2\,m_2}{3}\left(1 - r_{*2}^{3}\right) n \left(1 - r_{*2}^{2}\right) \quad \text{(C.2)}$$

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Nomenclature

b	=	shape coefficient ($b = 1/k_g$)	
c	=	scale coefficient ($c = 1/\alpha^{1/b}$)	[Kg m^{-2} s^{-1}]
c_g	=	scale coefficient (Eq. 5) according to Granier (0.119)	[Kg m^{-2} s^{-1}]

E_s = transpiration rate [Kg m^{-2} s^{-1}]

E_{tree} = transpiration rate at the tree scale [Kg s^{-1}, mm h^{-1}]

$E_{s,th}$ = E_s threshold value, delimiting the real solutions domain [Kg s^{-1}, mm h^{-1}]

$E_{s,g}$ = transpiration rate when the Granier's parameters (c_g, k_g) are fixed [Kg s^{-1}, mm h^{-1}]

h = heat transfer coefficient [W m^{-2} K^{-1}]

h_0 = heat transfer coefficient under zero flux [W m^{-2} K^{-1}]

i = electric intensity [A]

k = shape coefficient

K = flux index

k_g = shape coefficient (Eq. 5) according to Granier (1.231)

m_1 = power-law shape factor, for $r_* \in [r_{*0}, r_{*2}]$ (Eq. 7a and Eq. 7b, A-B in Fig. 4)

m_2 = linear shape factor, for $r_* \in [r_{*0}, r_{*2}]$ (Eq. 14, B-E in Fig. 4)

n = linear scale factor, i.e. ν-intercept of the ν-r_* line, for $r_* \in [r_{*2}, 1]$ (Eq. 14, B-E in Fig. 4) [Kg m^{-2} s^{-1}]

r = distance between the measuring point and the trunk or branch centre [m]

r_c = trunk radius below the cambium [m]

r_{*0} = normalized distance between the measuring point and the zero flux point [m]

r_{*1} = normalized distance between the nearest measuring point in [r_{*0},r_{*2}] from the trunk or branch centre (Fig. 4)

r_{*2} = normalized distance between the farthest measuring point in [r_{*0},r_{*2}] from the trunk or branch centre (Fig. 4)

r_{*3} = normalized distance between the farthest measuring point in [r_{*2},1] from the trunk or branch centre (Fig. 4)

r_* = ratio between the distance from the measuring point to the trunk or branch centre, r, and trunk radius below the cambium, r_c.

R = electric resistance [Ω]

S = area of the exchange surface [m^2]

T = probe temperature [°C]

T_{max} = probe temperature at zero flux [°C]

T_{∞} = temperature of the wood material in the absence of heating [°C]

α = scale coefficient, $\alpha = 1/c_g$, (Eq. 2 and Eq. 4) [s/m]

ΔT = temperature difference between the heated and the reference probe [°C]

ΔT_{max} = temperature difference between the sensor probes under zero flow conditions [°C]

T_{min} = minimum value of the temperature difference along the trunk radius [°C]

ΔT_* = ratio between ΔT_{min} and ΔT_{max}, corresponding to the power-law scale factor, for $r_* \in [r_{*0}, r_{*2}]$ (Eq. 7a and Eq. 7b, A-B in Fig. 4)

ν = sap flow density (SFD) [$Kg\ m^{-2}\ s^{-1}$]

ν_{max} = maximum value of the sap flow density (SFD) [$Kg\ m^{-2}\ s^{-1}$]

ν_1 = sap flow density (SFD) at the nearest measuring point in $[r_{*0}, r_{*2}]$ from the trunk or branch centre (Fig. 4) [$Kg\ m^{-2}\ s^{-1}$]

ν_2 = sap flow density (SFD) at the farthest measuring point in $[r_{*0}, r_{*2}]$ from the trunk or branch centre (Fig. 4) [$Kg\ m^{-2}\ s^{-1}$]

ν_3 = sap flow density (SFD) at the farthest measuring point in $[r_{*2}, 1]$ from the trunk or branch centre (Fig. 4) [$Kg\ m^{-2}\ s^{-1}$]