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A generalized correlation for pressure drop prediction in helically coiled tubes across different geometries and operating conditions

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Abstract. In the nuclear sector, helically coiled tube heat exchangers (HCTs) are considered promising for steam generators in Small Modular Reactors (SMRs). However, their complex geometry makes the accurate prediction of pressure losses more challenging, especially in two-phase flows. Existing correlations are generally valid within their specific range of applicability but often lack generality across different geometries, pressures, and flow conditions. This study aims to develop an improved correlation for the pressure drop estimation in HCTs, extending its applicability to various geometric configurations and operating conditions. The proposed correlation is validated against experimental data from the literature and compared with existing models to assess their accuracy and reliability.

1 Introduction

For SMR steam generators, HCTs represent a promising technology, particularly because these systems require highly compact designs while maintaining strong performance. Despite their advantages, the complex geometry of helically coiled tubes poses significant difficulties in accurately predicting pressure drops (PDs), particularly in two-phase flow conditions.

A key factor influencing pressure losses is the secondary flow induced by the tube's curvature. While it enhances mixing and heat transfer, it also increases PDs compared to straight tubes. In the context of two-phase flow PD estimation, several correlations have been developed by fitting experimental data [1-5].

Some authors suggest that correlations proposed for straight tubes, e.g. the Lockhart-Martinelli (LM) model [6], can be adapted to helical geometries by applying correction factors to account for the different frictional behavior. Others propose correlations specifically developed for HCTs, but these often have a narrow validity range or lack dimensional homogeneity.

Despite the large number of correlations available in the literature, their predictive capability remains limited to narrow operating conditions. When applied outside their original validation range, they often yield divergent results, indicating a lack of robustness in capturing parameter variations [3, 7]. As a result, engineers cannot fully rely on these correlations and must account for significant uncertainties when estimating two-phase PDs in HCTs.

The objective of this study is to formulate a more general and reliable correlation for predicting PD in HCTs for two-phase flows. The correlation is validated against experimental data reported in the literature and benchmarked against existing models to assess its performance.

2 Two-phase frictional PDs in HCT

Several studies have examined the frictional PD for two-phase flows in HCTs.

Colombo et al. [1] developed a correlation grounded in the LM method [6]. Since the LM correlations were originally developed for horizontal straight tubes, changes were made to ensure good agreement with experimental data for helical geometries.

As is well known, PD data can be correlated using the liquid-alone two-phase friction multiplier, ϕ_l, as follows:

$$\phi_l^2 = \left(\frac{dP}{dz}\right)_{tp,f} \Big/ \left(\frac{dP}{dz}\right)_l \tag{1}$$

where $(dP/dz)_{tp,f}$ is the two-phase frictional PD, $(dP/dz)_l$ is the PD of the liquid phase flowing alone in the channel at the total mass flow rate.

The parameter, ϕ_l, in case of straight pipes, is represented by the relationship:

$$\phi_l^2 = 1 + C/X + 1/X^2 \tag{2}$$

where *X* represents the LM parameter and C is a constant determined by the flow regime of the vapor and liquid phases.

The parameter *X* in Eq. (2) results as:

$$X=(\rho_v/\rho_l)^{0.5}(\mu_l/\mu_v)^{0.1}\,[(1-x)/x]^{0.9} \tag{3}$$

where x is the quality, ρ_l and ρ_v are the liquid and gas density [kg/m^3] and μ_l and μ_v the gas and liquid dynamic viscosity [Pa s], respectively.

Colombo et al. [1] proposed a correction factor for Eq. (2) to better account for the influence of helical geometry, operating conditions, and boundary effects on PD. This correction is based on the Dean number and the ratio of mixture density to liquid density

The proposed correction is as follows:

$$\phi_{l corr}^2 = 0.0986\, \phi_l^2\, De_l^{0.19} (\rho_m/\rho_l)^{-0.4} \tag{4}$$

where $De_l = Re_l(d/D)^{0.5} = G(1-x)d/\mu_l(d/D)^{0.5}$ is the Dean number of the liquid-phase as flowing alone in the channel, G the two-phase mass flow rate [kg/m^{2}s], d and D are the pipe and the coil dimeter [m], respectively, ϕ_l^2 is evaluated by Eq. (2) and ρ_m is determined as follows:

$$\rho_m = \left[\frac{x}{\rho_v} + \frac{(1 - x)}{\rho_l}\right]^{-1} \tag{5}$$

Eq. (5) is based on the Homogeneous Equilibrium Model (HEM) that considers vapor and liquid phases travelling at the same velocities inside the tubes (slip is 1).

The validation activity reported in [1] is based on experiment covering the operational conditions of different mass flow per unit area 200÷800 kg/m²s, pressures 2÷6 MPa, and curvature ratio (CR), d/D, of 0.01253.

Ferraris and Marcel [3] investigated the effect of geometric parameters, including CR and pitch, on flow behaviour and proposed a correlation to predict the two-phase friction factor, f_{tp}, in the calculation of the PD:

$$\left(\frac{dP}{dz}\right)_{tp,f} = \frac{f_{tp}}{2}\frac{G^2}{\rho_m d} \tag{6}$$

with ρ_m calculated by Eq. (5) and f_{tp} by the following equations:

$$f_{tp}=\Psi\ [xf_{vo} + (1 - x)f_{lo}] \tag{7}$$

$$\Psi = 1+ 0{,}207\ x^{1.3}(1 - x)^{2/3}\ Re_{lo}^{0.27} \tag{8}$$

where f_{lo} and f_{vo} are the liquid and gas friction factors, respectively, calculated using the Ito correlation for turbulent flow [8], considering each phase flowing alone through the tube.
For instance, for the liquid phase, f_{lo}, is:

$$f_{lo}=0.304Re_{lo}^{-0.25} + 0.029\ (d/D)^{0.5} \tag{9}$$

where Re_{lo} is the liquid-only Reynolds number:

$$Re_{lo}= Gd/\mu_l \tag{10}$$

The correlation to evaluate f_{tp} is validated for mass flow per unit area ranging from 150 to 1100 kg/m²s, pressures between 1 and 8 MPa, CR in the range from 0.0125 to 0.081.

Moradkhani et al. [4] proposed a model for frictional PD in two-phase flow within smooth helically coiled tube heat exchangers, aiming to enhance the accuracy of existing correlations through the integration of machine learning techniques and a wide range of experimental results.

Based on the application of this method, the following dimensionless parameters were chosen as the optimized input factors for modelling f_{tp}:

$$f_{tp} = f(Re_{lo}, Re_{vo}, P_{red}\ , It\ ,d/D, X) \tag{11}$$

where It=Tan(-γ/2) is the inclination factor with γ the inclination angle of coil axis to horizontal (0 horizontal, −π/2 vertical downflow and +π/2 vertical upflow), $P_{red}=P/P_{crit}$ represents the ratio of the system pressure to the critical pressure, and X is evaluated by Eq. (3). Re_{lo} and Re_{vo} are the liquid-only and gas-only Reynolds numbers, e.g. Eq. (10) for liquid-only phase.

The proposed correlation to be used in Eq (6) is as follows:

$$f_{tp}=0.077+ 0.0016\frac{Re_{lo}}{Re_{go}} -1{,}29\times10^{-6}(\frac{d}{D})^2+0.074\ P_{red}\ (It\ -1.67)+0.44\ A_1\ |It| - 0.043\ It \tag{12}$$

$$A_1=\min(0.053, X) \tag{13}$$

The parameter ranges used for modeling two-phase flow are as follows: Re_{lo} between 3592 and 143266; Re_{vo} from 55143 to 811688; P_{red} from 0.03 to 0.32; It, between -1 and +1. The CR falls between 0.005 and 0.092, while the LM parameter X ranges from 0.006 to 2.76.

While numerous other models exist in the literature, this review focused on the most recent and advanced contributions, which have been widely studied and used for validation activities.

2.1 A new correlation

By analyzing the literature and the dimensionless parameters studied for f_{tp} modeling, the dependencies of these parameters were studied by applying the Cluster Analysis (CA) [9], a powerful statistical technique used to identify relationships among experimental data.

CA is a statistical technique that groups data into clusters based on their similarity. Elements within the same cluster share similar characteristics, whereas elements in different clusters are distinctly separated. This methodology allows the identification of relevant trends and interrelations within the dataset, thereby improving the accuracy of the proposed correlation by capturing underlying patterns that are not immediately evident.

In this work, the CA procedure was carried out through an RCRAN script, which analyzed multiple experimental datasets grouped using k-means clustering algorithm. [10]. The optimal number of clusters was determined using the Silhouette index, which measures how well each observation fits within its assigned cluster relative to the others [11]. By assessing both cohesion and separation, this index allows for the identification of the most consistent clustering configuration.

The least squares method was subsequently applied to adjust the correlation parameters, aiming to minimize the difference between predicted and experimental values.

To perform this, the BFGS (Broyden-Fletcher-Goldfarb-Shanno) optimization method was employed to find the best-fitting parameters that minimize the Root Mean Square Error (RMSE). BFGS helps find the best solution by approximating the second derivatives of the objective function, which speeds up the process of finding the minimum error [12].

The study resulted in the following relationship to evaluate f_{tp} in Eq. (6):

$$f_{tp} = 0.00306 + [x\, f_{go} + (1-x)\, f_{lo}] + (0.0271 De_l^{0.19} - 0.178\, P_{red})[x^{1.948}(1-x)^{0.856}] \quad (14)$$

In Eq. (14), De_l number is calculated as follows:

$$De_l = Re_{lo}(d/D_c)^{0.5} \quad (15)$$

with D_c =D(1 + tg(β)] and β the helix angle.

3 Comparison results

3.1 Pressure range 2-6 MPa and CR of 0.01253

The experimental setup consists of an HCT characterized by CR of 0.01253 (*d*=0.01253 m, D=1m), a coil pitch of 0.8 m, and a total tube length of 32 m [1, 13].

The thermo-hydraulic behaviour was examined for two-phase flow conditions under both adiabatic and diabatic operating modes.

Tests were conducted with mass flow per unit area ranging from 200 to 800 kg/m²s, pressures of 2, 3, 4, and 6 MPa, and vapor quality varying between 0.1 and 0.9.

Tab.1 shows the results for RMSE and Mean Absolute Percentage Error (MAPE) based on experimental data obtained under diabatic conditions.

Table 1. RMSE and MAPE computed for the correlations using experimental data [1, 13].

G	P	Present		Colombo [1]		Ferraris [3]		Moradkhani [4]	
[kg/m²s]	MPa	RMSE%	MAPE%	RMSE%	MAPE%	RMSE%	MAPE%	RMSE%	MAPE%
200	2	9.8	7.8	16.6	14.6	9.1	7.5	23.4	20.2
	4	7.2	5.9	19.8	17.25	5.0	4.0	6.2	4.6
	6	8.7	6.8	23.0	19.4	11.4	9.3	10.8	8.7
400	2	9.2	8.1	12.6	10.7	10.4	8.5	22.8	15.1
	4	10.4	8.9	12.5	10.0	6.0	4.4	13.8	9.1
	6	11.8	10.	14.3	10.6	9.6	7.4	11.2	8.2
600	2	4.8	3.6	19.4	13.6	13.7	11.8	34.9	24.4
	4	13.0	10.2	22.8	14.4	12.2	9.1	49.1	30.1
	6	10.8	8.4	23.3	14.6	7.6	5.8	20.4	10.7
800	2	7.8	5.5	25.7	16.7	14.7	12.3	42.6	28.7
	4	7.0	5.8	17.8	15.4	10.0	8.6	24.6	13.1
	6	9.3	7.0	8.7	7.3	6.3	4.8	7.7	6.4

It should be highlighted that the present correlation stands out for its good reliability across many configurations. The RMSE and MAPE remain below 13% and 10%, respectively.

The Colombo et al. correlation [1] exhibits more variable performance, showing good agreement for data with 400 kg/m²s and all pressures but its accuracy declines for the other mass flow rates. The RMSE and MAPE are below 26% and 17%, respectively.

The correlation proposed by Ferraris and Marcel [3] produces results very similar to those of the present correlation, with RMSE and MAPE below 15% and 12%, respectively.

The application of the Moradkhani et al. correlation [4] exhibits the weakest performance across all configurations, with higher errors compared to the other models. The RMSE and MAPE are below 49% and 30%, respectively.

Very similar results are observed when performing the comparison with experiments conducted under adiabatic operating conditions, which are not reported here for brevity.

3.2 Pressure range 0.1-0.4 MPa and CR of 0.012 and 0.01875

The HCT geometries considered in this study comprise two coils with CRs of 0.01875 and 0.012 and pitches of 0.485 m and 0.79 m, respectively. Both configurations share the same inner tube diameter of d=0.012 m [14]. Each coil has an overall length of approximately 10 m. The two-phase flow conditions were established using water flow rates between 95 and 230 g/s, while the air flow rate varied from 0.04 to 0.3 g/s for each water flow rate, at operating pressures between 0.1 and 0.4 MPa. PD measurements were obtained through nine pressure taps distributed along the HCT and monitored by eight differential pressure transducers connected between consecutive taps.

The results are presented in Fig. 1, where PD experimental data are compared with predictions obtained using the present correlation (Fig. 1a), as well as the correlations proposed by Colombo et al. (Fig. 1b), Ferraris and Marcel (Fig. 1c), and Moradkhani et al. (Fig. 1d).

From the results analysis, the highest prediction accuracy is achieved with both the present correlation and the one introduced by Ferraris and Marcel. However, the present correlation produces errors generally below, or at most approximately, 30% across all considered cases.

3.3 Pressure at 2.14 MPa and CR of 0.0246

The test section investigated by Kozeki et al. [15] has CR of 0.0246 (d = 0.0155 m, D = 0.628 m). The helical pitch was not reported. The operational pressure is 2.14 MPa with mass flow per unit area of 161, 325, and 486 kg/m²s.

The experimental PDs as a function of quality are reported in Fig. 2, together with the present predictions and those of Ferraris and Marcel (Fig. 2a), as well as the predictions of Colombo et al. and Moradkhani et al. (Fig. 2b). Error bars indicate an uncertainty of ±15%.

Although the experimental data exhibit some dispersion, all correlations generally provide comparable predictions of the pressure gradient. The present correlation, however, shows improved agreement with the measurements, particularly at a mass flow per unit area of 486 kg/m²s.

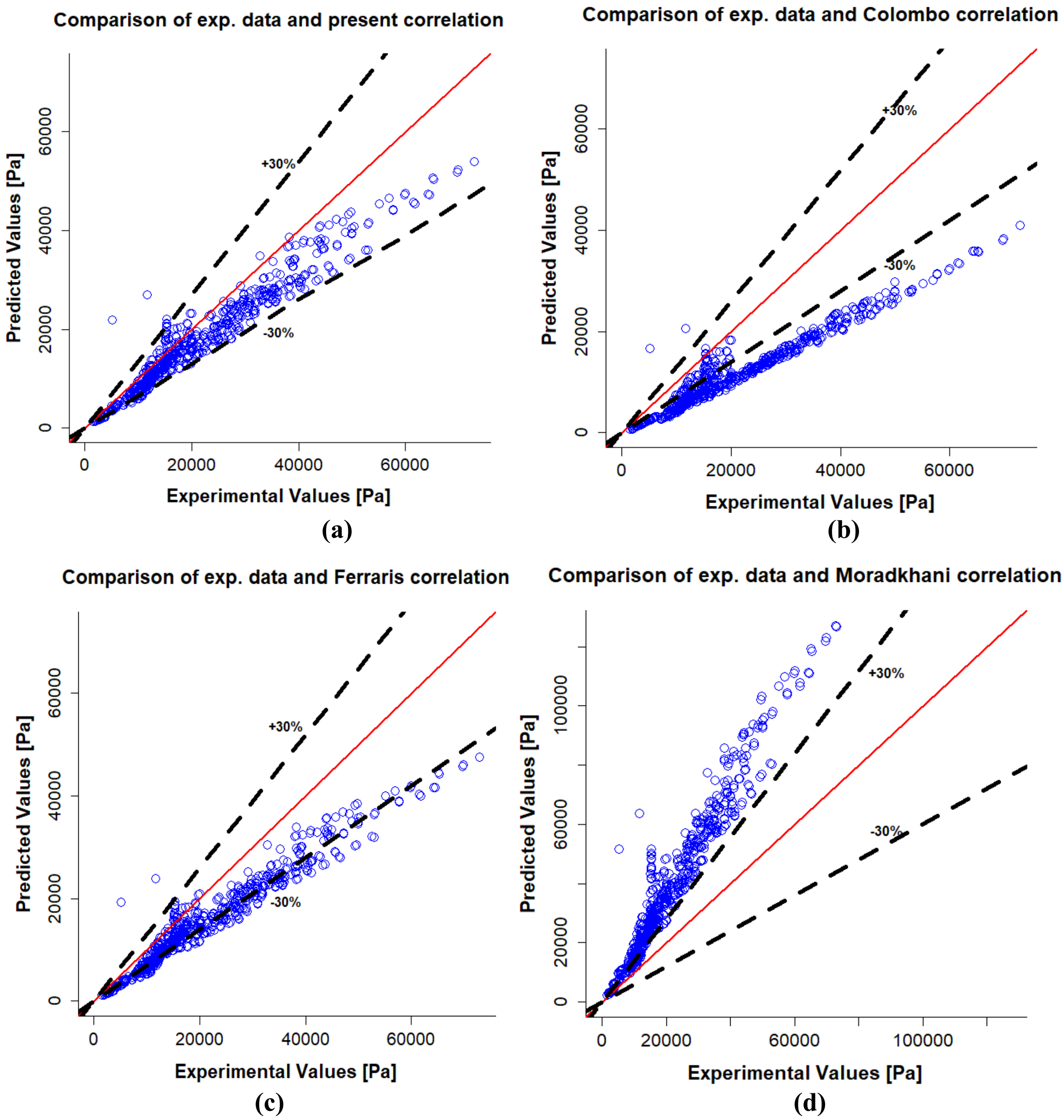
3.4 Pressure at 4.8 and 8 MPa and CR of 0.033 and 0.081

The tests described in [16] were conducted using a test facility that consists of helically coiled tubes made from stainless steel, with heated lengths of about 8 m. The key geometrical parameters of the test sections are coil diameters, D, 180 mm, 280 mm, and 380 mm; inner diameters, d, 12.5 mm and 14.5 mm, thus the CR range are from 0.033 to 0.081, The helical angle was 6.0°.

The system pressure ranged from 2 to 8 MPa, with mass flow per unit area rates between 300 and 1100 kg/m²s.

For the sake of brevity, some results are reported in Fig. 3, which shows the comparison with experimental PD data obtained at a mass flow rate of 600 kg/m²s and at pressures of 4.8 MPa (Fig. 3a) and 7.6 MPa (Fig. 3b) as a function of quality. In these figures, the experimental data are presented with 20% error bars.

All the correlations, except for the one by Colombo, provide a good prediction of the experimental data, with an error of about 20% or less.



(a) (b) (c) (d)

Figure 1. Predicted versus experimental pressure drops for the examined correlations [14].

3.5 Pressure at 7 MPa and CR of 0.107

The experimental configuration reported in [5] consists of a helically coiled tube with an overall length of about 5.3 m, an inner diameter d = 12 mm, and a coil diameter D = 112 mm, corresponding to CR of 0.107.

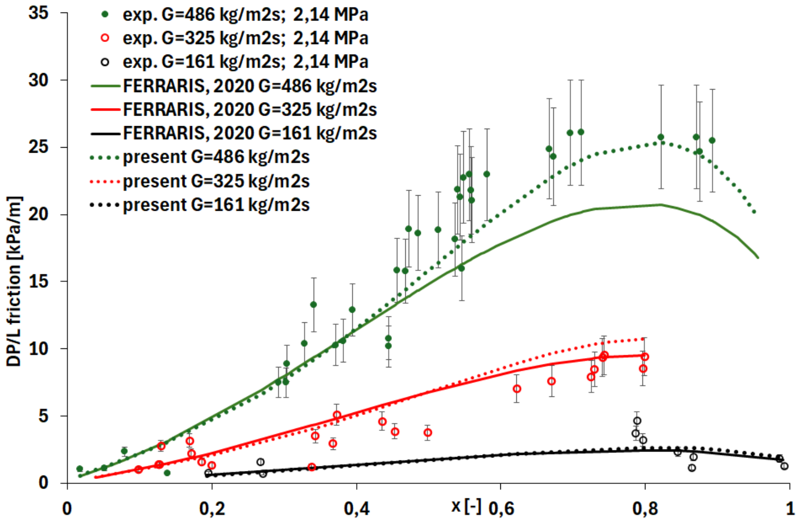
The coil pitch is 22.5 mm. Tests were conducted under system pressures ranging from 3.5 MPa to 7 MPa and mass fluxes between 320 and 1100 kg/m²s.

Fig. 4 presents the comparison with experimental PD data obtained at mass fluxes of 320 kg/m²s (Fig. 4a) and 1100 kg/m²s (Fig. 4b).

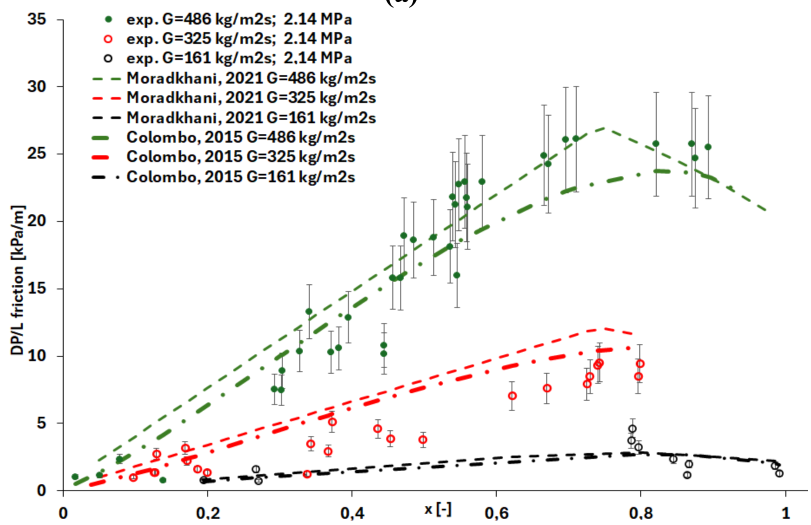
The corresponding evaluation parameters, expressed in terms of RMSE and MAPE, are summarized in Table 2.

An overestimation of the experimental data is noted when applying the Colombo et al. correlation. Unlike the previous cases, the best results are obtained both by using the present correlation and by employing the correlations proposed by Ferraris and Marcel and Moradkhani et al.

Moreover, the Moradkhani et al. correlation provides excellent results for high mass flow rates of 820 and 1100 kg/m²s.



(a)

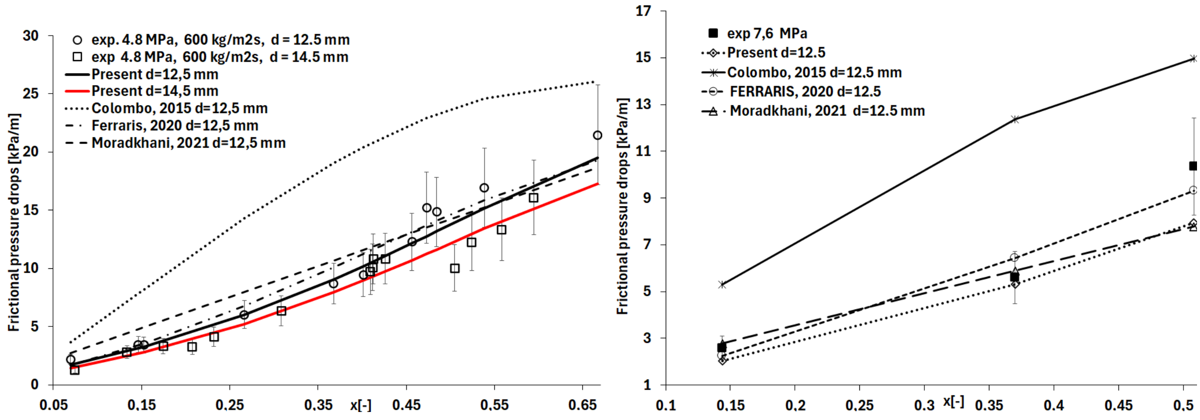


(b)

Figure 2. Comparison of helically coiled tube correlation predictions with the experiments [15].

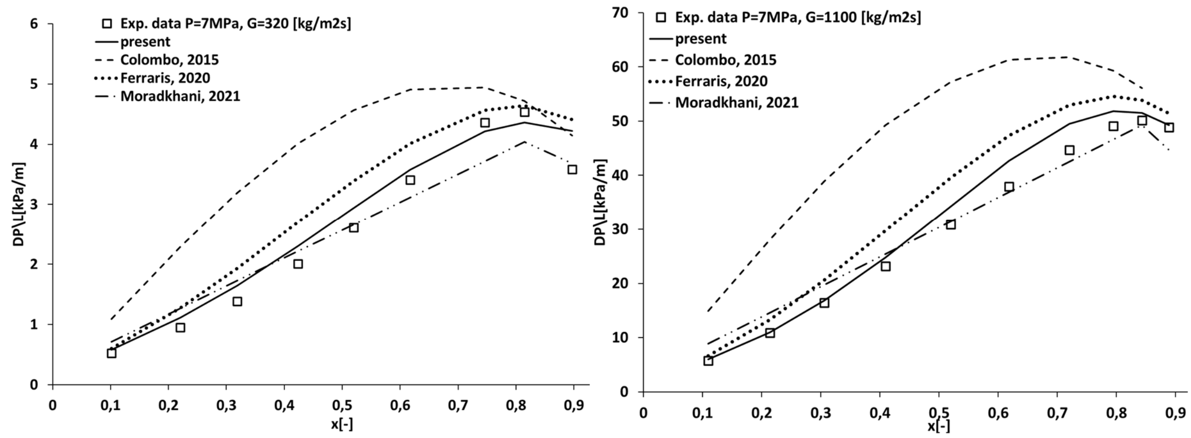
Table 2. RMSE and MAPE computed for the correlations using experimental data [17].

G [kg/m²s]	Present RMSE%	Present MAPE%	Colombo,2015[1] RMSE%	Colombo,2015[1] MAPE%	Ferraris, 2020[3] RMSE%	Ferraris, 2020[3] MAPE%	Moradkhani,2021[4] RMSE%	Moradkhani,2021[4] MAPE%
320	11.8	10.7	85.1	69.3	25.1	21.7	19.7	15.8
550	11.0	8.9	97.6	77.21	23.2	19.8	27.3	20.5
820	7.0	6.2	96.4	78.6	18.6	16.4	18.6	12.9
1100	10.28	9.2	96.5	78.2	19.9	18.2	21.6	14.2



(a) (b)

Figure 3. Comparison between correlation results and experimental data [16].



(a) (b)

Figure 4. Comparison between correlation results and experimental data [5].

4 Conclusions

The present study introduces a new correlation for predicting two-phase frictional PDs in helically coiled tubes, enhancing accuracy and applicability across a broad range of operating conditions. A comparison with experimental data and some literature correlations has highlighted the following key findings:

- the proposed correlation improves predictive capability, particularly at lower and intermediate pressures (2-4 MPa) and across various mass flow rates, with a MAPE below 10%;
- the present correlation, compared to the one proposed by Colombo et al. [1], exhibits greater reliability, avoiding the overestimations observed in some test cases with higher CRs;
- the performance of the present correlation is comparable to that of Ferraris and Marcel [3], which also provides very accurate results. However, for cases with lower pressures (0.4 MPa) and higher curvature ratios, the new correlation offers improved predictions;
- the correlation by Moradkhani et al. [4] tends to produce the least accurate results, particularly at lower pressures, though it performs well at high pressures and mass flow rates.

Given these results, the proposed correlation represents an improvement in the prediction of PDs in helically coiled tubes. Its accuracy, wide applicability, and dimensional consistency make it a valuable tool for the design and optimization of steam generators in advanced nuclear reactors and other applications.

Table 3 summarizes the ranges of operating conditions and CRs covered by the considered models. The present work extends the applicability range in terms of pressure, mass flux, and CR. In addition, it ensures consistently low MAPE values across a broader domain of operating conditions, confirming its improved generalization capability and predictive robustness.

Table 3. Applicability ranges (pressure, mass flux, and CR) and corresponding MAPE values compared for the selected literature correlations and the present model.

Model	Pressure [MPa]	Mass Flux rate [kg/m²s]	Curvature Ratio [d/D]	MAPE%
Colombo et al. (2015)	2–6	200–800	0.0125	5-80%
Ferraris & Marcel (2020)	0.5–8	150–1100	0.0125–0.081	3-35%
Moradkhani et al. (2021)	0.6–7.5	90-2000	0.005–0.092	10-40%
Present work	0.1 –8	70–2500	0.01–0.1	2-30%

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