Supply chain finance: the role of credit rating and retailer effort on optimal contracts

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Abstract

Supply chain finance aims at finding the best financing arrangements within a given buyer-supplier dyad. The source of capital can be internal (buyer or supplier) or external (financial institution) to the supply chain. So far, many studies have investigated the optimal mix of the sources of capital; our study aims at contributing to the recent literature that explores the interface of operations and finance extending the supplier-based financing models. As the Covid-19 pandemic hits economic activity, the financial constraints have ever greater importance; knock-on effects of the Covid-19 crisis urges on the critical role of a supply chain that should provide financial resources, along to the flow of goods, in the more efficient way.

The proposed model considers a supply chain formed by a supplier and a retailer, both capital-constrained that can ask for a loan to a financial institution or resort on their internal reciprocal sources. Demand is uncertain, retailer, that acts as a price taker, may also affect her product demand applying some effort in increasing sales. Both retailer and supplier can fail. The model optimizes the supplier and retailer's' profit varying their contract parameters; doing this, the research allows to understand the interplay between supply chain operational and financial issues when retailer's sales effort is at work, and its findings can support retailer in the suppliers selection basing on their credit rating. Our findings show how operational (retailer's effort) and financial (trade credit conditions) issues can synergically interact also in supply chain with low working capital or conversely how retailer with high working capital can perform better working with low rating supplier, however boosting the chance to successfully compete in the Covid-19 pandemic era.

Keywords: supply chain finance, trade credit, retailer effort, Covid-19

1. Introduction

In the world of interlocking accounts payables (the money owed by customers in the supply chain which represents a large fraction of working capital) and receivables (the money owed to suppliers further up in the supply chain), firms borrow from their suppliers and lend to their customers, thus creating a trade credit chain that runs parallel to the flow of goods along the supply chain (Boissay et al., 2020). The pandemic has shocked global supply chains, straining business cash flows and working capital. For most firms, a large fraction of working capital is categorised as “accounts receivable” that are matched to some extent by “accounts payable” on the liabilities side of the balance sheet. The interlocking chain of account receivables and payables can be seen as the glue that binds supply chains together in the real economy and sustains their operation, both domestically and internationally (Carstens, 2020). Since financial resources represent a strict constraint in firms’ business, it’s not always possible to borrow loans from third parts. The trade credit represents a way to overcome this problem or an alternative to cope with financial constraints: supplier may decide to finance their partners (or the other way around) in order to allow them to expand their activities with benefits for the whole chain. An emblematic example is constituted by small and medium enterprises that, even though they participate actively to the GDP of their countries, face with serious difficulties in borrowing capital because the small warranties they can offer, especially in economic crisis period; this is why trade credit now more than even can draw the attention of firms and practitioners.

During an economic crisis firms have more difficult to access financing from banks due to their lack of collateral, the weak nature of their business establishments. Liquidity constraints on inventory decisions represents a critical issue that could ignite a negative spiral: less inventory, less sold products, less profit, even less liquidity. Karmaker et al. 2021 find that financial support from the supply chain partners is the most important driver after governmental financial support to tackle the immediate shock on supply chain sustainability due to COVID-19. In this scenario, non-necessary activities are dismissed as maintenance, promotion, customer care: we wonder if a counter-intuitive strategy can allow to invert the direction towards a positive cycle. Therefore, we want to investigate if a risk sharing perspective can help the supply chain to patch its weakness: commitment on both sides and on the operational and financial floor, can cover the lack of liquidity and discover in ancillary activities as sales effort a tool to stop a negative economic spiral.

Building on the existing literature about trade credit, the study introduces a model that considers different financial options available to the Supply Chain (SC)’s members and aims at finding out the optimal values of their decision variables. In particular, huge attention was given to Kouvelis and Zhao (2018), about the impact of credit rating on retailer’s financing decisions, Yan and Zaric (2016), for their organization of coordinating contracts’ families, and Wu, Cheng and Zhou (2018) that analysed the impact of sales effort on ordered quantities. The proposed model tackles with a SC environment with a retailer (she) and a supplier (he), both capital constrained and a bank that may offer a loan to both parts. The retailer may also ask financial support to the supplier that, based on his credit rating, may offer a delayed payment or may ask for an early payment with discount. The model aims at finding the quantity that the retailer buys from the supplier, the level of effort that allows the retailer to increase her sales and the interest rate asked by the supplier to the retailer in case of delayed payment (or the offered discount in case of early payment) that maximize the retailer and supplier's profit. The study aims to contribute to the literature introducing a model where the optimal configuration depends on the credit rating of both parts and the level of the retailer' effort, comparing the retailer's advantages in working with a low credit or a high credit supplier, according to her available working capital and other exogenous variables like the selling price of the product in the market, the producing costs and the interests asked by the bank to its loans, taking into account the possibility of incurring in failure. Our research contributes to the literature in three ways.

First, we introduce in the supply chain financing set up an investment that cannot be financed by trade credit: the effort cost. In previous research, for example Kouvelis and Zhao (2012) that represents a reference point for our study, retailer's investment could be financed alternatively by bank or by supplier. In our study, according to the retailer's working capital, bank loan cannot be avoided by the retailer if she wants to put some effort in the supply chain and therefore for this investment trade credit is not a suitable option. Bankruptcy can therefore occur and impacts the whole supply chain: risk sharing plays a stronger role in a context like this and, as in Yang and Birge (2018), this issue highlights the importance of trade credit, that is, how trade credit enhances supply chain efficiency by allowing the retailer to partially share the risk with the supplier. So far the importance of trade credit as risk sharing mechanism has been related to risk of a "closed" supply chain: our study, introducing the effort, adds investments that need external source of financing and therefore opens to additional consequences of demand uncertainty. Our study shows how effort "can take care of itself": indeed, it leverages the consequences of demand uncertainty because of bank loan but at the same time increases supply chain efficiency.

Our second contribution arises combining the effort consequences above described with the credit rating: in particular, building on Kouvelis and Zhao (2018) and Yan and Zaric (2016), we obtain different contract and therefore different profit varying the supplier credit rating. Low rating supplier can enhance supply chain efficiency: this result is the combined effect of effort and early payment with discount.

Combining effort, supplier credit rating and retailer’s working capital the third contribution of our study arises: retailer’s working capital is the main driver in the supply chain contract and this conclusion allows to provide managerial support in the supplier selection and retailer’s budget allocation among her suppliers. With a numerical example we show that the retailer may find convenient to work with a low rating supplier achieving a higher profit thanks to the higher mark-up due to the supplier discount if her working capital allows an earlier payment.

These main contributions open up to one ancillary contribution: cross-financing among supply chain dyads that opens to a new direction in this field of research that can take into account the relationship between different dyads of supplier-retailer as anticipated by Chod et al. (2016).

The paper is organized as follows: in the following section, we review the related literature while in the third the proposed model will be described. The optimal solutions that maximize retailer and supplier’s expected profit will be analysed in the fourth section and a numerical example in section five allows to put in evidence the model implications in terms of supplier’s selection. Finally, in the last section we conclude discussing the main research’s findings and proposing future developments.

2. Literature review

The trade credit’s benefits have been widely acknowledged and classified by Petersen and Rajan (1997) in three groups: financial, commercial and operational advantages.

Financial advantages cover three aspects: advantage in information acquisition (even if Burkart and Ellingsen, 2004, said that banks are more specialised in assess the creditworthiness of firms), advantage in controlling the buyer, advantage in salvaging value from existing assets.

Commercial advantages come from the possibility offered by trade credit to reduce the price of a product, increasing its demand, using discriminatory pricing without fear of revenge by competitors. (Lavine, 2002).

Operational advantages are related to control better inventory position, especially in case of seasonality issues (Emery, 1987), reducing warehousing costs.

Literature provides also additional motivations (Brick and Fung, 1984; Bellouma, 2014). Recently, Kouvelis and Zhao (2018) investigate the relationship between suppliers’ selection and their credit rating in the supply chain financing landscape. In the absence of any such credit rating information, Kouvelis and Zhao (2012) establish that the best financing solution is to have the supplier exclusively finance all his production requirements through bank loans and then extend trade credits at cheap rates to the retailer. The retailer exclusively uses trade credit to finance all her inventory requirements. Kouvelis and Zhao (2018) introduce parties' credit rating enlarging the cross-financing opportunities: supplier may finance retailer but also the other way around, while bank represents a financial source for both parties. They find a hole in the credit rating range where a retailer doesn't find convenient dealing with a supplier. In fact, suppliers with a higher rating may decide to allow delay payments without asking interests; suppliers with a lower rating may encourage the retailers to anticipate payment offering a discount in the order amount, while suppliers with an average credit rating could incur in higher financing costs in borrowing bank loans, deciding to transfer some of them to the retailer through a higher wholesale price. In some cases, retailer can find more profitable working with a low rated supplier than a higher (but still medium) rated one. In this way, low rated supplier could gain an advantage in his horizontal competition.

Several scholars have investigated the impact of firm's characteristics on trade credit and sometimes their findings are controversial: table 1 provides a summary of their studies (+ means increasing value of the characteristic stimulates the adoption of trade credit, - the opposite relationship, -/+ a U-shaped relationship).

Determinants of a firm	Empirical results
Age	(+) Al Dohaiman (2018) (-/+) Canto-Cuevas et al. (2018) (+) Petersen and Rajan (1997) (-) Rodriguez and Rodriguez (2006)
Buyer's market share	(+) Lee, Zhou and Wang (2018)
Capacity utilization	(+) Fisman (2001)
Current assets level	(+) Al Dohaiman (2018)
Ethnic ties	(+) Hermes et al. (2015)
Financial debt	(-) Petersen and Rajan (1997)
Frequency of purchase	(+) Hermes et al. (2015)
Length of relationship	(+) Hermes et al. (2015)
Liquidity	(-) Hermes et al. (2015)
Profitability	(-) Al Dohaiman (2018)
Retained earnings	(-) Ojenike and Olowoniyi (2014),
Sales growth	(-) Al Dohaiman (2019)

	(+) Petersen and Rajan (1997)
Size	(-) Al Dohaiman (2018) (-) Hermes et al. (2015) (+) Petersen and Rajan (1997) (-) Rodriguez and Rodriguez (2006)
Supplier's bargaining power	(+) Lee, Zhou and Wang (2018)
Supplier's market share	(-) Hermes et al. (2015) (-) Lee, Zhou and Wang (2018)

Table 1 Literature findings about the impact of firm's characteristics on trade credit

It is also interesting considering the suppliers' willingness to provide trade credit financing: competion among suppliers plays an important role in this context and Chod et al. (2016) show that retailers with dispersed suppliers obtain less trade credit than those whose suppliers are more concentrated because of free-rider related issue. Moreover, competition impacts also on the relationship between trade credit and firms' performance: Lee et al. (2018) find that usually trade credit is positively associated with both parties' performance but, when suppliers are more aggressive in their trade credit strategy, then the excess trade credit is negatively associated with buyer performance.

However, trade credit has to be analysed considering other involved variables in the supplier-retailer relationship, such as retailer effort in increasing the sales. In order to make safer agreements, firms use contract as a tool to reduce risk and, to increase retailer's commitment in sales effort and the final demand. Kraiselburd et al. (2004) focuses on the importance of sales effort on the supply chain performance. Sales effort is defined as the time and the resources that retailer deploys to increase demand and concretizes for example in better advertising or product quality improvement. If the supplier doesn't give an appropriate incentive, the retailer may be not encouraged to keep the effort level (Krishnan, Kapuscinski and Butz, 2004). The expected retailer's profit has a parabolic concave downward relationship with sales effort since it increases the demand but simultaneously the effort costs increase (Wu, Cheng and Zhou, 2018).

Mixing the retailer's effort with the supply chain financing exploit the potentiality of operational and financial issues in a supply chain. As observed by Cachon and Lariviere (2005) revenue sharing contracts is undermined by the effort effect that weakens their coordination power. Cachon and Lariviere argue that to fully exploit the effort incentive, the retailer has to bear the cost and the benefit of effort itself; therefore, the author suggests a quantity discount contract instead of a revenue sharing one, in presence of retailer's effort. We wonder if an indirect risk sharing mechanism incorporated in late payment to the supplier may act as a balance between the Cachon and Lariviere' instances and the need to involve the supplier in the demand uncertainty risk. Yang and Birge (2018) attempt to better understand the risk-sharing role of trade credit—that is, how trade credit enhances supply chain efficiency by allowing the retailer to partially share the demand risk with the supplier. However, as demonstrated by Jing et al., 2012, when both trade and bank credits are viable for the capital-constrained buyer and when production cost is above a certain threshold, the manufacturer prefers to have the bank market to bear the retailer's demand uncertainty risk and bank credit financing becomes the unique financing equilibrium.

Actually, the COVID-19 outbreak puts other elements on the supply chain financing puzzle, and we refer to Karmaker et al. 2021 finding about the importance of financial support from the supply chain partners to tackle with the pandemic challenges. We believe that in a period when uncertainty is the new normal, uncertainty sharing becomes a must when we look for an equilibrium in contracts. This is why we investigate the interaction between effort and supply chain financing.

Building on these findings we set up a model that embeds these contributions and allows to determine the optimal retailer-buyer contract's parameters, namely order quantity, retailer's sales effort and supplier trade rating, when the financing source is both internal (trade credit) and external (bank). We also developed a numerical example to compare the optimal retailer's profit when she works with supplier with different credit rating in order to support supplier selection. The obtained optimal order quantity is affected by the retailer's effort and, as a consequence, influences the retailer's profit and then her supplier selection: this result highlights the importance of the interplay between the effort and the financing side of the supply chain and its impact on the supplier selection. These findings are relevant in a pandemic stricken-market characterised by increased uncertainty and financial constraints.

3. The proposed model

The proposed model consists in a supply chain formed by a supplier (*s*), a retailer (*r*), both capital-constrained with an initial working capital named respectively s_s and s_r, and a financial institution (the bank, *b*), that has no constraints. For both kind of firms, the owned working capital may be not enough to face with all the considered supply chain transactions' costs; in this case they need to borrow money. They can borrow money from the bank or from their partner using trade credit in two directions: the retailer may pay later the supplier that will apply an interest to the wholesale price, or the supplier may ask for an early payment with discount. Every firm has a credit rating that indicates their ability in paying back debts and that is given by external and independent credit rating companies: retailer's and supplier's ratings will be respectively marked as R_r and R_s. Being the bank free from constraints, it will give a loan L_{br}/ L_{bs} (respectively to the retailer and to the supplier), asking an interest rate equal to $r_{br}(R_r)/r_{bs}(R_s)$ that is assumed as the risk-free interest rate, plus a risk premium related to the credit rating R_r/R_s. The supplier produces a single type of product: it is assumed a two-period system where the retailer can order only once at time 0 and sell at time 1. The discount factor between these two periods is named δ. All the three parties are risk neutral. Let be c the supplier's unit production cost, w the wholesale price, q the retailer's order quantity and p the unit retail price. Demand is uncertain and is indicated by ξ, which follows a probability distribution function $f(\cdot)$ and a cumulative distribution function $F(\cdot)$. **Retailer**, that acts as a price taker, may also affect her own demand applying some effort in increasing sales: the effort is indicated with e and it has a cost expressed by the function $g(e)$, convex in e with $g(0) = 0$. This effort can be considered as an investment in promotion (like advertising), product management and/or distribution aimed to develop the consumers' awareness of the retailer's brand, in order to increase the probability to reach higher sales. Since the effort may affect retailer's demand, the function $\tilde{q}(e)$ expresses the maximum demand in the market for a given effort, with $\tilde{q}(0) = 0$. The retailer's demand ξ is uncertain and the portion she can satisfy is distributed between 0 and $\tilde{q}(e)$. The model doesn't consider understock and overstock costs and salvage value for unsold pieces neither taxes nor bankruptcy costs (for the effect of bankruptcy costs the reader can refer to Kouvelis and Zhao (2011): essentially, they show how the retailer's financial constraints and the potential of a costly bankruptcy lead to a decrease of the supply chain's efficiency and profits). Even if, Kouvelis and Zhao (2018) state that the salvage value of unsold items and goodwill loss for unmet demand do not change the nature of the problem and therefore, without loss of generality, they assume them equal to zero, in our model understock will determine a loss in revenue (and therefore in retailer's profit) and overstock a lower retailer's profit because of unsold products. As said before the retailer can pay cash when she orders the quantity q or later after the selling. The supplier may concede a *delayed payment* at time 1 with an interest that reflects the shifting of the risk from retailer to supplier.

The retailer may also resort to another form of trade credit: *an early payment with discount at time 0* (maintaining a positive supplier's mark-up), that acts as a financing source for low rating supplier that can't easily obtain a bank loan.

According to Yan and Zaric (2016), the supplier can loan the retailer, through a delayed payment, an amount equal to the wholesale cost plus an additional payment, marked by the authors with *A*. Their work establishes that, since the value of *A* may be affected by the ordered quantity and/or the retailer's effort, in the contract with the best performances in terms of efficiency, flexibility and level of required information, the additional payment *A* depends only on the ordered quantity q; we will import this result in our model.

According to Kouvelis and Zhao (2018), supplier may behave in a different way depending on his credit rating, so in our model we assume that:

- if supplier's credit rating is low, he will ask for an early payment with a discount ($A < 0$, or to generalise with an interest rate $r_s < 0$);
- if supplier's credit rating is high, he will lend a loan (delayed payment) to be paid back in period 1 without interest rate thanks to his financial wellness ($A = 0$);
- if supplier's credit rating stays in the middle, he will lend a loan to be paid back in period 1 with an interest rate $r_s > 0$ ($A > 0$).

We combine Kouvelis and Zhao (2012) findings, showing how under the optimally priced trade credit is cheaper than bank credit, with the trade credit rating findings of Kouvelis and Zhao (2018): the retailer will prefer trade credit, when available and if optimally priced, to bank loan. Table 2 summarizes the external financing options, others than the owned working capital, that may be used by supply chain actors, according to the supplier credit rating (low or medium/high).

	Working with a low rating supplier (A < 0) – Early payment	Working with a medium/high rating supplier ($A \geq 0$) – Late payment
Retailer	$L_{br} = \max(0, wq + A + g(e) - s_r)$ to finance early payment and investment in effort.	Bank loan, equal to $L_{bs} = \max(0, g(e) - s_r)$ to finance investment in effort;

		Trade credit as delayed payment equal to $wq + A$ (A=0 in case of high rating supplier).
Supplier	Early payment from retailer, in order to start the production, equal to $wq + A$. No bank loan is allowed.	$L_{bs} = \max(0, cq - s_s)$ to finance the production.

Table 2 Financial alternatives for the retailer and the supplier

Moreover, the model takes into account the possibility that the retailer may face bankruptcy risk as a consequence of one of the two possible events:

- A default risk related to transactions exogenous to the considered supply chain, modelled as an exogenous Bernoulli random event, with occurrence probability reflected by her credit rating $p_r(R_r)$ as hypothesized by Kouvelis and Zhao (2018), at time 0;
- An endogenous default risk if the revenues cannot cover her loan obligations from the current supply chain transaction at time 1.

In case the retailer fails, bank will have a seniority position in taking retailer's remaining assets, while the supplier will have a lower priority and may face failure. Given the loans from suppliers and/or bank, the retailer has to face two thresholds, $q_1$and q_2, that allow her to pay her debts: q_1 represents the minimum quantity that the retailer has to sell in order to repay her debt to the bank, while q_2 is the minimum quantity that permits to pay back the debt to both the bank and the supplier.

The supplier also may fail as a consequence of two analogous possible events:

- A default risk related to transactions exogenous to the considered supply chain, modelled as an exogenous Bernoulli random event, with occurrence probability reflected by his credit rating $p_s(R_s)$ as hypothesized by Kouvelis and Zhao (2018), at time *0*;
- An endogenous default risk if his revenues (that may be dependent on retailer's revenues) cannot cover his loan obligations from the current supply chain transaction at time *1*. Therefore, he may also face default risk if he cannot pay his debt to the bank because of retailer's insolvency.

It's clear that the failure of one firm, in case of trade credit, has an impact on the whole chain in terms of failure or profit.

Table 3 summarizes the introduced variables

s, r, b	supplier, retailer and bank
s_i	Initial working capital of firm i for $i = r, s$
R_i	Credit rating of firm i for $i = r, s$
q	Retailer's ordered quantity
δ	Discount factor
c	Supplier's unit production cost
w	Wholesale price
A	Additional payment from retailer to supplier in case of trade credit
p	Unit retail price
e	Retailer's effort in promoting demand
$g(e)$	Effort's function cost
$\tilde{q}(e)$	Maximum demand that the retailer can achieve for a given effort
ξ	Uncertain demand
$F(\cdot)$	Demand's cumulative distribution function
$f(\cdot)$	Demand's probability distribution function
$r_{bi}(R_i)$	Bank's interest rate when she offers loan to a firm i, depending on its credit rating for $i = r, s$
L_{bi}	Loan given from a bank to a firm i $i = r, s$
L_{sr}	Loan given from the supplier to the retailer
$r_s(R_r)$	Supplier's interest rate when he offers trade credit to the retailer, it depends on the retailer's credit rating
$p_i(R_i)$	Failure probability of a firm, expressed as a Bernoulli random events depending on firm's credit rating $i = r, s$
$\Pi_i(\cdot)$	Expected profit from firm i for the retailer *i = r_lowrate-s, r_mediumrate-s, r_highrate-s;* for the supplier *i=lowrate-s, mediumrate-s, highrate-s.*
M	Probability that no firm fails because of Bernoulli random events, $[(1 - p_r(R_r))(1 - p_s(R_s))]$

M_r	Probability that only retailer fails because of Bernoulli random events, $(p_r(R_r))(1 - p_s(R_s))]$
M_s	Probability that only supplier fails because of Bernoulli random events, $[(p_s(R_s))(1 - p_r(R_r))]$
q_1	The minimal demand for the retailer to repay the bank loan obligations
q_2	The minimal demand for the retailer to repay the trade credit obligations after the bank loan obligations have been fully paid

Table 3 Model's variables

In the following two subsections we will calculate the retailer's and the supplier's profit according to their working capital and supplier's credit rating that influences the financial relationship with the retailer. In order to provide a framework of the different subcases the reader can find in Appendix F two different schemas for the supplier and retailer's profit functions. The first schema illustrates through two trees (Figure F1 and Figure F2) how the profit function for the retailer and the supplier are determined according to the proposed supply chain model. The second schema summarizes the functions in tables (Tables F1 and Tables F2) citing the corresponding equation in the text.

3.1 Retailer's expected profit

The retailer's expected profit function is affected by the demand and the probability of endogenous or exogenous failure of both firms. In fact, the retailer may sell products only if both companies don't fail between the two periods: if the retailer fails at $t = 0$ for exogenously reasons she can't pay the supplier that, in case of agreed late payment, may face failure; if the random failure occurs to the supplier, he stops the production and the retailer won't receive the order and, in case of early payment, will incur in a loss or may also face failure in case of bank loan.

According to Kouvelis and Zhao (2018) considerations, it's appropriate to separate the case where the retailer works with a low rating supplier and the one in which the retailer works with a medium or a high rating supplier. Furthermore, in our model it is helpful to distinguish the occurrence of random failure in three separate cases: none of the firms fail, only the supplier fails, only the retailer fails.

Working with a Low Rating Supplier (early payment)

In this case, the supplier will ask for an early payment with discount at time 0; this represents his only accessible source of financing. On her side, the retailer needs a loan from the bank only if $s_r < \big(wq + A + g(e)\big)$, where $A = wqr_s$ and $r_s < 0$ because of the discount in the early payment. Otherwise, she will finance the whole investment with her own capital. As said before we will calculate the profits considering three different scenarios:

None of the firms fails because of random events: this happens with probability $M = (1 - p_r(R_r))(1 - p_s(R_s))$.

If $s_r < \big(wq + A + g(e)\big)$ the retailer borrows money from the bank, pays the supplier at time *0* and sells the products at time 1, when she will also pay back the bank debt. The profit will be

$$P^{r1} = \delta[p \min(\xi\ , q) - \ (wq + A + g(e)\ - s_r)(1 +\ r_{br}(R_r))] - s_r\ . \tag{1}$$

In the case $s_r \geq \big(wq + A + g(e)\big)$ the retailer won't ask for any loan and she will pay all liabilities at time 0, the profit becomes:

$$P^{r1\prime} = \delta[p \min(\xi\ , q)] - \ \big(wq + g(e)\big). \tag{2}$$

Only the supplier fails, after the payment of the order, because of random event: that happens with probability $p_s(R_s)(1 - p_r(R_r))$, that is indicated as M_s. In this case, even though she has already paid, the retailer won't receive the order quantity and she won't be able to sell it during period 1, suffering a loss that could bring her to failure if capital constrained. Since she won't have any revenue, the profit will be (negative) equal to

$$P^{r2} = \delta \cdot \big[-\ (wq + A + g(e)\ - s_r)\big(1 + r_{br}(R_r)\big)\big] - s_r, \tag{3}$$

that includes the payment of the bank loan, when $s_r < wq + A + g(e)$, and

$$P^{r2\prime} = -\ (wq + A + g(e)\ ), \tag{4}$$

if $s_r \geq wq + A + g(e)$, because the retailer pays all the investments at time 0 with her own capital.

Only the retailer fails because of random event: in this case, which occurs with probability $p_r(R_r)$, she doesn't sell any product and her expected profit will be 0.

The overall retailer's expected profit when the retailer works with a low rating supplier is:

$$\Pi_{r\_lowrate-s}\ (q, e, r_s) = \begin{cases} M \cdot\ P^{r1} +\ \ M_s \cdot P^{r2} & if\ s_r < wq(1 + r_s) + g(e)\ (bank\ loan) \\ M \cdot\ P^{r1\prime} +\ \ M_s \cdot P^{r2\prime} & if\ s_r \geq wq(1 + r_s) + g(e) (no\ bank\ loan) \end{cases} \tag{5}$$

Working with a Medium and High Rating Supplier (late payment)

In this case, the supplier concedes a loan to the retailer (L_{sr}) that will be paid back in period 1; therefore, $A = L_{sr} r_s$ and $r_s \geq 0$. In this case too, three scenarios may happen:

None of the firms fails because of random events: this happens with probability M. The retailer sells the ordered quantity at time 1 and pays the supplier at the same time, when she will also pay back (if she borrowed money) the debt to bank. Since it could be assumed that $r_s < r_{br}(R_r)$ (and equal to 0 when the supplier has a high credit rating), the retailer prefers to reduce the amount of the bank loan. So, if $s_r \geq g(e)$, the retailer finances the investment in effort only with her capital making q_1 equal to 0, while if $s_r \geq g(e) + wq$ she doesn't even postpone the payment with the medium rating supplier. The approach in the case of high rating supplier is different: in fact, given that the high rating supplier doesn't ask interests on liabilities and that "a euro today is worth more than a euro tomorrow", the retailer finds convenient the delayed payment even though $s_r \geq g(e) + wq$.

As a conclusion, the retailer's profit when $s_r < g(e)$ is:

$$P^{r3} = \delta \cdot \left[p \min(\xi , q) - (wq + A) - (g(e) - s_r)\left(1 + r_{br}(R_r)\right)\right] - s_r \tag{6}$$

and $L_{sr} = wq$. When $s_r \geq g(e)$, the retailer's profit is

$$P^{r3\prime} = \delta \cdot [p \min(\xi , q) - (L_{sr} + A)] - \left(L_{sr} - wq - g(e)\right) \tag{7}$$

where $L_{sr} = wq - \left(s_r - g(e)\right)$ if the retailer has a medium credit rate and $L_{sr} = wq$ if the retailer is high rated. If the retailer works with a medium rating supplier and her working capital is enough $(s_r \geq g(e) + wq)$ it isn't convenient to ask for a delayed payment therefore $L_{sr} = 0$ and the profit becomes

$$P^{r3\prime\prime} = \delta[p \min(\xi , q)] - (\, g(e) + wq) \tag{8}$$

Only the supplier fails, after the payment of the order, because of random event: in this case the retailer won't receive the order quantity, and she will have a loss equal to the amount paid in advance. In particular, if $s_r < g(e)$ the loss will be

$$P^{r4} = \delta \cdot (-(g(e) - s_r)(1 + r_{br}(R_r)) - s_r \tag{9}$$

$$P^{r4\prime} = -\left(L_{sr} - wq - g(e)\right) \tag{10}$$

where $L_{sr} = wq - \left(s_r - g(e)\right)$ if the retailer has a medium credit rate and $L_{sr} = wq$ if the retailer is high rated, and if $s_r \geq \; g(e) + wq$ and the retailer works with a medium rating supplier, the loss will be

$$P^{r4\prime\prime} = -wq - g(e). \tag{11}$$

Only the retailer fails because of random event: the profit associated to this eventuality is 0 since she would have payed at t = 1after the failure.

So, the retailer's expected profit can be expressed as

$$\Pi_{r\_mediumrate-s}\ (q,e,r_s)$$
$$= \begin{cases} M \cdot P^{r3} + M_s \cdot P^{r4} & if\ s_r < g(e)\ (bank\ loan\ and\ late\ payment) \\ M \cdot P^{r3'} + M_s \cdot P^{r4'} & if\ g(e) + wq > s_r \geq g(e)\ (partial\ or\ total\ late\ payment) \\ M \cdot P^{r3''} + M_s \cdot P^{r4''} & if\ s_r \geq g(e) + wq\ \ (neither\ bank\ loan\ nor\ late\ payment) \end{cases} \tag{12}$$

when she works with a medium rating supplier, and:

$$\Pi_{r\_highrate-s}\ (q,e,r_s) = \begin{cases} M \cdot P^{r3} + M_s \cdot P^{r4} & if\ s_r < g(e)\ (bank\ loan\ and\ late\ paymer \\ M \cdot P^{r3'} + M_s \cdot P^{r4'} & if\ s_r \geq g(e)\ (late\ payment) \end{cases} \tag{13}$$

when she works with a high rating supplier.

In addition, since the high reliability of the high rating supplier, his default risk for Bernoullian event may be considered so small that it can be considered negligible making the profit approximately equal to:

$$\Pi_{r\_highrate-s}\ (q,e,r_s) \cong \begin{cases} (1 - M_r) \cdot P^{r3} & if\ s_r < g(e) \\ (1 - M_r) \cdot P^{r3'} & if\ s_r \geq g(e) \end{cases} \tag{13bis}$$

According to the literature analysis, in all the scenarios the profit can be considered concave in q and in e and this consideration suggests that there are (and they are unique) an optimal value of q and e that maximize her profit function.

3.2 Supplier's expected profit

For the supplier also we separately deal with the profit of a low rating supplier, because of his preference in early payments, and the one of medium and high rating suppliers that can offer trade credit with late payments. We therefore will calculate the profit for the three failure events.

<u>*Low Rating Supplier (early payment)*</u>

The low rating supplier receives an early payment at time 0, that represents, jointly to his insufficient working capital, the only source of financing for the production.

None of the firms fails because of random events: as we will explain after, this is the only case when the supplier profit is not 0. Therefore, indicating with *lowrate-s* the low rating supplier, his profit may be written as:

$$\prod_{lowrate-s}(q, r_s) = M[wq + A - cq] \tag{14}$$

With A<0 expressing the discount in the ordering price.

Only the supplier fails because of random events: the supplier won't produce, and the received payment will be taken by the bank, therefore his profit will be equal to 0.

Only the retailer fails because of random events: the retailer won't pay the supplier that won't start the production because of his low working capital: indeed, the low credit rating of the supplier suggests that his working capital never assumes a value that allow him to produce without an external financial source that can be represented only by early payment since his low credit rating doesn't allow to borrow money from the bank. So, in this case the low rating supplier's profit will be 0.

Medium and High Rating Supplier (trade credit)

This case is quite different from the previous one. The repayment of trade credit will happen during period 1, leaving the supplier more sensible to the retailer's failure. Being capital constrained, he borrows money from the bank if $s_s < cq$. The value of the loan is equal to $(cq - s_s)$ because it represents the amount of money that the supplier needs in order to start the production; then, $L_{bs} = (cq - s_s)$. If the supplier fails, the bank will take his assets. If $s_s \geq cq$ the production is financed by the supplier's working capital at time 0. In this case, the supplier's profit is affected also by the retailer's working capital, s_r: if she borrows money from the bank, she will pay the retailer only after she pays her bank's liabilities. Therefore, if the product sold volume is lower than q_1, no money will leave for the supplier; if it is greater than q_1 and lower than q_2 the retailer will pay partially the supplier, only when the volume is greater than q_2 the retailer's revenues are enough to refund totally the supplier. The following equations show q_1 and q_2.

$q_1 = \dfrac{L_{br}(1 + r_{br})}{p}$	(15)
$q_2 = \dfrac{L_{br}(1 + r_{br}) + L_{sr}(1 + r_s)}{p}$	(16)

Again, the following scenarios may happen:

None of the firms fails because of random events: the retailer will sell the quantity $min(\xi\ , q)$ and the supplier will be paid for the whole trade credit with probability $F(\xi \geq q_2)$. Otherwise, the supplier will receive retailer's remaining income (revenues from sales minus the debt to the bank), with probability $F(q_1 < \xi < q_2)$, and 0 with probability $F(\xi \leq q_1)$. It's obvious that medium and high rating supplier's performance is strongly affected by retailer's one. Indeed, in this case, debts, to the bank and to the supplier, are paid during period 1. The profit generated by this scenario, that will be indicated as P^{s1} with a probability equal to $M = (1 - p_r(R_r))(1 - p_s(R_s))$, may be split in three parts equal to:

if $s_s < cq$, the supplier has to borrow money from the bank

$$P^{s11} = (F(\xi > q_2))\left[\delta \cdot \left(wq + A - (cq - s_s)\left(1 + r_{bs}(R_s)\right)\right) - s_s\right] \tag{17}$$

$$P^{s12} = F(q_1 < \xi < q_2)\left[\delta \cdot \left(p\min(\xi\ , q) - L_{br}\left(1 + r_{br}(R_r)\right) - (cq - s_s)\left(1 + r_{bs}(R_s)\right)\right) - s_s\right] \tag{18}$$

$$P^{s13} = F(\xi \leq q_1)\left[-\delta \cdot (cq - s_s)\left(1 + r_{bs}(R_s)\right) - s_s\right] \tag{19}$$

while if $s_s \geq cq$, (5), (6) and (7) become:

$$P^{s11'} = (F(\xi > q_2))[\delta \cdot (wq + A) - cq] \tag{20}$$

$$P^{s12'} = F(q_1 < \xi < q_2)\left[\delta\left(p\min(\xi\ , q) - L_{br}\left(1 + r_{br}(R_r)\right)\right) - cq\right] \tag{21}$$

$$P^{s13'} = F(\xi \leq q_1)[-cq] \tag{22}$$

As said before in case of late payment the supplier's profit is affected by the retailer's working capital that affects the value of q_1 and q_2. Indeed, when $s_r \geq g(e), q_1 = 0\ and\ P^{s13}/P^{s13'} = 0$, when $s_r \geq g(e) + wq$, $q_1 = q_2 = 0\ and\ P^{s13}/P^{s13'} = P^{s12}/P^{s12'} = 0$ Notificating with the upper script *mediumrate-s* the medium rating supplier and with *highrate-s* the high rating one, when $s_s < cq$ supplier's expected profit may be written as:

$$P^{mediumrate-s1} = \begin{cases} P^{s11} + P^{s12} + P^{s13} & if\ s_r < g(e) \\ P^{s11} + P^{s12} & if\ g(e) + wq > s_r \geq g(e) \\ P^{s11} & if\ s_r \geq g(e) + wq \end{cases} \tag{23}$$

and:

$$P^{highrate-s1} = \begin{cases} P^{s11} + P^{s12} + P^{s13} & if\ s_r < g(e) \\ P^{s11} + P^{s12} & if\ s_r \geq g(e) \end{cases} \tag{24}$$

while when $s_s \geq cq$ it is:

$$P^{mediumrate-s1'} = \begin{cases} P^{s11'} + P^{s12'} + P^{s13'} & if\ s_r < g(e) \\ P^{s11'} + P^{s12'} & if\ g(e) + wq > s_r \geq g(e) \\ P^{s11'} & if\ s_r \geq g(e) + wq \end{cases} \quad (25)$$

and:

$$P^{highrate-s1'} = \begin{cases} P^{s11'} + P^{s12'} + P^{s13'} & if\ s_r < g(e) \\ P^{s11'} + P^{s12'} & if\ s_r \geq g(e) \end{cases} \quad (26)$$

- *Only the retailer fails for random event, after the production started and before the payment of the order*: in this case, that occurs with probability $p_r(R_r)(1 - p_s(R_s))$, that it's also indicated as M_r, the supplier won't receive any payment at period 1, getting a loss equal to

$$P^{s2} = -\delta \cdot (cq - s_s)\left(1 + r_{bs}(R_s)\right) - s_s \text{ if } s_s < cq \quad (27)$$

and

$$P^{s2'} = -cq \text{ if } s_s \geq cq \quad (28)$$

Only the supplier fails because of random event: in this last and quite unlikely scenario, the supplier will receive a profit equal to 0.

The profit of medium and high rating supplier can finally be written respectively as:

$$\prod\nolimits_{mediumrate-s} (q, e, r_s) = M \cdot P^{mediumrate-s1} + M_r \cdot P^{s2} \quad (29)$$

$$\prod\nolimits_{highrate-s} (q, e, r_s) = M \cdot P^{highrate-s1} + M_r \cdot P^{s2} \quad (30)$$

in the presence of bank's loan ($s_s < cq$)

$$\prod\nolimits_{mediumrate-s} (q, e, r_s) = M \cdot P^{mediumrate-s1'} + M_r \cdot P^{s2'} \quad (31)$$

$$\prod\nolimits_{highrate-s} (q, e, r_s) = M \cdot P^{highrate-s1'} + M_r \cdot P^{s2'} \quad (32)$$

when the supplier has enough working capital ($s_s \geq cq$).

3.3 General assumptions on the effort

In order to solve the model and find the optimal parameters, some assumptions were made regarding the demand distribution. The effort influences the maximum quantity that the retailer could sell at time 1, we assume the following relationship between the effort and the maximum demand achievable:

$$\tilde{q}(e) = ke \quad (33)$$

where k represents the marginal increment of the quantity increasing the effort: the demand, for hypothesis, follows a *continuous uniform distribution* going from 0 to ke.

Being the demand uniformly distributed, the following formulas hold:

$$f(x) = \begin{cases} \frac{1}{ke} & for\ 0 \leq x \leq ke \\ 0 & otherwise \end{cases} \tag{34}$$

and

$$F(x) = \begin{cases} \frac{x}{ke} & for\ 0 \leq x \leq ke \\ 0 & for\ x \leq 0 \\ 1 & for\ x \geq ke \end{cases} \tag{35}$$

We also assume $g(e)$, the effort cost function, equal to

$$g(e) = a \cdot e^2 \tag{36}$$

where a represents the levering effect on the effort to obtain the increased potential market that could depend on the market characteristics and the retailer market position as well.

<u>*Uncertainty in retailer's profit*</u>

As already said, since the demand is represented by a stochastic variable, the expected profit of both actors is affected by uncertainty, represented by the term $min(\xi,\ q)$. In particular, from the point of view of the retailer two different outcomes may occur:

The demand is higher than the ordered quantity: this eventuality has a probability $F(\xi \geq q) = 1 - \frac{q}{ke}$, and the retailer will sell the whole quantity q;

The demand is lower than the ordered quantity: that can happen with probability $F(\xi < q) = \frac{q}{ke}$. In this case, the revenues would be $p \cdot \xi$, and ξ is assumed as the expected value of this event, equal to $\frac{q}{2}$. Whereas in the first case the retailer gains the maximum possible profit, in the second case she may incur in default risk.

<u>*Uncertainty in supplier's profit*</u>

In the case "working with a medium or a high rating supplier" even the supplier's performance is strongly affected by the effective realized demand, generating three different scenarios:

The demand is higher than the threshold q_2: this event has a probability $F(\xi \geq q_2) = 1 - \frac{L_{br}(1+r_{br})+L_{sr}(1+r_s)}{pke}$, and the retailer will sell the quantity needed to pay all of her debts. In this way, the supplier receives the whole repayment for the conceded loan.

The demand is lower than the threshold q_2 *but higher than the threshold* q_1: with probability $F(q_1 < \xi < q_2) = \frac{L_{br}(1+r_{br})+L_{sr}(1+r_s)}{pke} - \frac{L_{br}(1+r_{br})}{pke} = \frac{L_{sr}(1+r_s)}{pke}$. The retailer will face default risk because her impossibility in paying back all the incurred debts. Since the bank covers a seniority position in retailer's remaining assets, the supplier will receive only part of the conceded loan, making the supplier's revenues equal to $w(1+r_s) \cdot (\xi - q_1)$. The quantity $(\xi - q_1)$ is the expected value of this event $\frac{L_{sr}(1+r_s)}{2p}$. In this case, the supplier will fail if $w(1+r_s)\frac{L_{sr}(1+r_s)}{2p} < (cq - s_s)(1+r_b)$, because of the consequently impossibility for the supplier to pays his debt to the bank.

The demand is lower than the threshold q_1: with probability $F(\xi < q_1) = 1 - \frac{L_{br}(1+r_{br})}{pke}$. In this case too, the retailer faces with default but now she is unable to pay the whole debt to the bank, making null the remaining assets for the supplier, that will therefore face with default risk.

The following section provides the optimal values for the parameters q, e and r_s.

4. Optimal contracts

In this section, we will provide and discuss the optimal contracts parameters for two scenarios: the first, when the retailer has a working capital equal to zero and the second case, when the retailer has a working capital higher than zero. This distinction is helpful because the objective functions and the constrained of the problem vary according to the retailer's working capital. The decision variables for the retailers are the order quantity and the effort, while the supplier will decide the optimal interest rate.

The effort, the ordered quantity and the interest rate the supplier ask to the retailer that maximize the profit of both parts in the SC will be found when the retailer works with a low rating supplier and when she works with a high rating one. In fact, it is reasonable to consider that the partnership with a medium rating supplier is dominated by the partnership with a high rating supplier, because the retailer prefers an interest rate in trade credit equal to zero.

The optimal contract is the one where the parameters q and e maximize the retailer's expected profit, while the parameter r_s maximizes the supplier's expected profit. The sequence of the events is presented in Figure 1:

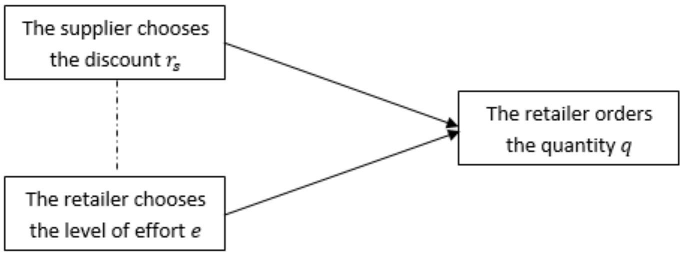

Figure 1 Sequence of decisions in defining the optimal contract

It's therefore possible to see the optimal quantity as the retailer's best response after the decision of the other two parameters. Given this point of view, in order to find all optimal values can be applied a backward induction where in the first stage q is found, as the quantity that maximizes the retailer's expected profit. After that, substituting the optimal quantity in the profit function of both players, the optimal effort is the one that maximizes the retailer's profit, while the optimal discount rate r_s maximizes the supplier's profit (if the supplier is high rated this value is assumed equal to 0).

Lastly, the value of p, w, $r_{br}(R_r)$ (that hereafter will be simply called r_b) and c are considered known and exogenously defined. Actually, in case of low rating supplier, deciding the discount is as the supplier decides the wholesale price *w* with an upper bound (when the discount is equal to 0) equal to the assigned *w* itself.

4.1 Optimal solution when $\boldsymbol{s_r = 0}$

When the retailer has no working capital, she needs an external financial source. Reminding the previous section, the absence of retailer's working capital makes simpler the profit functions where only the first term holds in equations 5, 12 and 13/13bis.

<u>*Working with a low rating supplier*</u>

As in section 3.1, the expected profit of a retailer without working capital, who works with a low rating supplier is expressed in the first part of eq. 5 and shown below:

$$\Pi_{r\_lowrate-s}\,(q, e, r_s) = M \cdot P^{r1} + M_s \cdot P^{r2} \tag{37}$$

While the low rating supplier's expected profit is expressed in eq. 14.

Substituting eq. 36 in eq. 1 and 3 we obtain:

$$P^{r1} = \delta[p \cdot \min(\xi, q) - (wq + wqr_s + ae^2)(1 + r_b)] \tag{1bis}$$

$$P^{r2} = -\delta[(wq + wqr_s + ae^2)(1 + r_b)] \tag{3bis}$$

The revenue $p\min(\xi, q)$ is computed as its expected value, P^{r1} therefore becomes:

$$P^{r1} = \delta \cdot \left[p\frac{q}{2}\left(\frac{q}{ke}\right) + pq\left(1 - \frac{q}{ke}\right) - (wq + wqr_s + ae^2)(1 + r_b)\right] \tag{1ter}$$

Optimal ordered quantity

Substituting 1ter and 3bis in 37, we can find the value of q that maximizes the retailer's profit deriving the obtained profit function. The optimal q for the retailer's profit function of eq. (37) is: [1]

$$q = ken \tag{38}$$

Where n is equal to:

$$n = 1 - \frac{w\,(1 + r_s)(1 + r_b)}{(1 - p_s(R_s))p} \tag{39}$$

The ordered quantity is lower than the maximum demand ke: given the demand uncertainty, the retailer orders a quantity lower than *ke* in order to reduce overstock risks. It doesn't depend on the retailer's failure rate and it will be higher the higher are the effort, the selling price, the supplier's discount and the lower are the wholesale price, the bank interest rate for the retailer's loan and the supplier's failure probability.

Optimal level of effort

According to Yan and Zaric (2016), in the most efficient contract the additional payment *A* depends only on the order quantity; we therefore replace the optimal ordered quantity in the expected profit functions of supplier and retailer to optimize the effort level. Given that the retailer's working capital is set to zero, she won't be able to buy anything without any financing source. As already said, the optimal level of effort influences the optimal ordered quantity and so the first step is to replace the optimal q, that was previously found, in the retailer's profit function obtaining the optimal effort[2]:

$$e = \frac{(1 - p_s(R_s))\left[p \cdot kn - p \cdot \frac{kn^2}{2}\right] - \left(w \cdot kn(1 + r_s)\right)(1 + r_b)}{2a \cdot [(1 + r_b)]} \tag{40}$$

The optimal effort doesn't depend on the retailer's failure rate; it increases with the selling price and the supplier's discount, conversely decreases with the supplier's failure probability (a more reliable partner convinces the retailer in investing more in the collaboration), the

[1] For mathematical details see Appendix D- Case 1

[2] For mathematical details see Appendix D- Case 2

wholesale price, the bank interest rate since the effort becomes costlier and the term a, the cost per unit of effort.

Optimal supplier's discount rate

Finally, to find the optimal discount rate, we have to put the optimal q (eq. 38) in the supplier's expected profit (eq. 14) and maximize it; in this way, we find out the optimal supplier's discount rate[3]:

$$r_s = \frac{c(1+r_b)(1-p_s(R_s))+p}{2(1+r_b)} - 1 \tag{41}$$

The optimal discount rate price doesn't depend on the effort e and that's consistent with the job of Yan and Zaric (2015), and the hypothesis of our model. Moreover, it increases with the supplier's failure probability (a more uncertain supplier has to offer a higher discount in order to convince retailer to start a collaboration) and decreases with the production cost (a higher production cost requires that the supplier asks for a higher wholesale price in order to gain profit) and the selling price (a higher selling price makes the retailer more willing to pay for the product).

Analysing the optimal discount rate, it may happen that for certain values of c, w, p and r_b the optimal value turns higher than 0, despite, according to the hypothesis, the discount rate has to be between 0 and -1. So, in this situation the optimal solution has to be rejected. Reminding the *Extreme Value Theorem*, made by Weierstrass, since the profit function is continuous (because sum of continuous elements) and defined in a closed interval for $-1 \leq r_s \leq 0$, even though there's no value that makes the first derivative equal to zero, a maximum point exists and lies on the interval's extreme points. In this case, the optimal value exists when the discount rate assumes the value of 0, when the optimal value according to the formula is not acceptable.

Working with a high rating supplier

Similar reasoning allows to find the optimal value of the considered variables when the supplier has a high rating (please see Appendix A for detailed explanation).

4.2 Optimal solution when $\boldsymbol{s_r > 0}$

The procedure used to find the optimal parameters is the same of the previous section, whereas the profit function of the retailer changes according to her working capital.

[3] For mathematical details see Appendix D- Case 3

Therefore, we found the optimal parameter for $s_r < wq(1+r_s) + g(e)$ and for $s_r \geq wq(1+r_s) + g(e)$ when the retailer works with a low rating supplier (please see Appendix B for detailed explanation) and for $s_r < g(e)$ and $s_r \geq g(e)$ when the retailer works with a high rating supplier (please refer to Appendix C for detailed explanation).

4.3 Optimal contract parameters (q, e, r_s): analysis and implications

Comparing the optimal values, we noticed that what matters at the end of the day is not the working capital but the presence of a bank loan. Therefore, we can summarize the optimal value according to three level of working capital that determines the presence of the bank loan:

low working capital, that is when the retailer needs a bank loan ($s_r < g(e)$). The optimal parameters are reported in Table 3;

medium working capital when the retailer needs a bank loan only if the supplier is low rating ($g(e) \leq s_r < wq(1+r_s) + g(e)$). The optimal parameters are reported in Table 4;

high working capital when the retailer doesn't need any financial support from the bank ($s_r \geq wq(1+r_s) + g(e)$) The optimal parameters are reported in Table 5.

Parameter	low rating supplier	high rating supplier
Order quantity q	ken	$ke\frac{p-w}{p}$
Effort e	$kn\frac{p(1-p_s(R_s))\left[1-\frac{n}{2}\right]-\left(w(1+r_s)\right)(1+r_b)}{2a[(1+r_b)]}$	$\frac{k(p-w)^2}{4ap(1+r_b)}$
Interest rate r_s	$\min\left(0;\frac{c(1+r_b)+p(1-p_s(R_s))}{2w(1+r_b)}-1\right)$	0

Table 3 Optimal parameters when s_r is low ($s_r < g(e)$)

Parameter	low rating supplier	high rating supplier
Order quantity q	ken	$ke\frac{p-w}{p}$
Effort e	$kn\frac{p\left(1-p_s(R_s)\right)\left(1-\frac{n}{2}\right)-w(1+r_s)(1+r_b)}{2a[(1+r_b)]}$	$\delta k\frac{(p-w)^2}{4ap}$
Interest rate r_s	$min\left(0;\frac{(1-p_s(R_s))p+c(1+r_b)}{2w\,(1+r_b)}-1\right)$	0

Table 4 Optimal parameters when s_r is medium ($g(e) \leq s_r < wq(1+r_s) + g(e)$)

Parameter	low rating supplier	high rating supplier
Order quantity q	ken'	$ke\frac{p-w}{p}$
Effort e	$kn' \frac{\delta p(1-p_s(R_s))\left(1-\frac{n'}{2}\right)-w(1+r_s)}{2a}$	$\delta k\frac{(p-w)^2}{4ap}$
Interest rate r_s	$\min\left(0;\frac{c+\delta p(1-p_s(R_s))}{2w}-1\right)$	0

Table 5 Optimal parameters when s_r is high ($s_r \geq wq(1+r_s)+g(e)$)

Where,

$$n' = \left(1-\frac{1}{(1-p_s(R_s))}\cdot\frac{w(1+r_s)}{\delta \cdot p}\right) \tag{43}$$

The analysis of this optimal parameters allows to draw some conclusions about the supply chain considered in our study according to the variables that influence the optimal parameters and comparing the same parameters varying the retailer's working capital (low, medium and high) and the supplier's credit rating (low and high).

We can summarize these theoretical findings in the following three prepositions, one for each contract parameters:

Proposition 1: The optimal q is lower than the maximum achievable (ke) regardless the retailer's working capital and the supplier's failure rate for random event. The retailer orders a quantity lower than *ke* in order to reduce overstock risks, being the demand uncertain. *Moreover, the optimal q will increase with the retailer's working capital.* However, if the supplier is low rating the marginal increasing rate is higher. Indeed, when the retailer is capital constraint and works with a low rating supplier, bank's loan is necessary. The effect of a bank loan is similar to an increase in the wholesale price and therefore a decrease in the retailer's markup, the higher the working capital the lower the bank loan and its effect on the markup reduction.

Proposition 2: The optimal e increases with the retailer's working capital both in case of low and high supplier's credit rating. A higher working capital lets a higher investment on effort that will increase the potential revenue for the retailer. *Moreover, the optimal e, and as a consequence the optimal q, in case of low rating supplier, increases with the supplier's credit rating* (and therefore it decreases when the supplier's default risk for random event $\left(1-p_s(R_s)\right)$ increases). Indeed, with a low rating supplier early payment is required and therefore the retailer's profit/loss depends on the supplier default risk.

Proposition 3: The optimal discount rate $|r_s|$ increases as the retailer's working capital is lower and a lower working capital asks for a bank loan that in turn will reduce the discount rate. Moreover, the optimal discount rate $|r_s|$ increases when the supplier's default risk for random event $(1 - p_s(R_s))$, increases. The retailer fixes the optimal q so that the more uncertain suppliers will offer a higher discount.

5. Numerical example

In order to better understand the study implications, we developed a numerical example. The example aims at comparing the optimal parameters and retailer's profit when some key inputs variables change under the two possible scenarios of suppliers' credit rating. As emerged from the conclusions C1-C6, the mark up for both supply chain actors and the supplier's credit rating impact on the optimal parameters. Therefore, we started considering a retailer with a low working capital and found the optimal parameters when the inputs' model are the ones shown in Table 6. Figure 2 shows the optimal effort and order quantity varying the wholesale price *w* when the supplier is low rating ($p_s(R_s)$=0,25) and high rating ($p_s(R_s)$=0,1): varying *w* we can study the optimal parameters varying contemporarily the supplier's mark up (*w*-*c*) and the retailer's mark up (*p*-*w*).

Variable	Value
p	180 euro
a	0,05
δ	0,97
k	2
c	30 and 50 euro
r_b	0,05

Table 6 Variables values for the numerical example

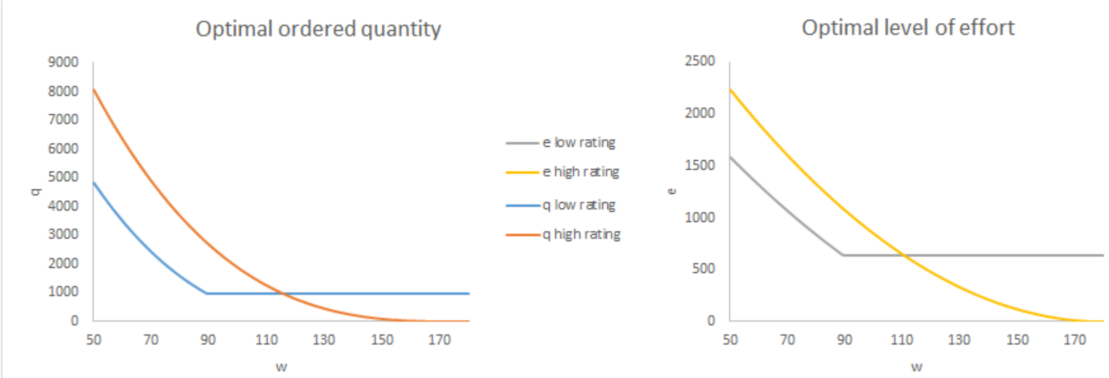

Figure 2 Optimal q and e for the numerical example in table 6 (c = 50 euro)

The retailer's expected profit is shown in Figure 3

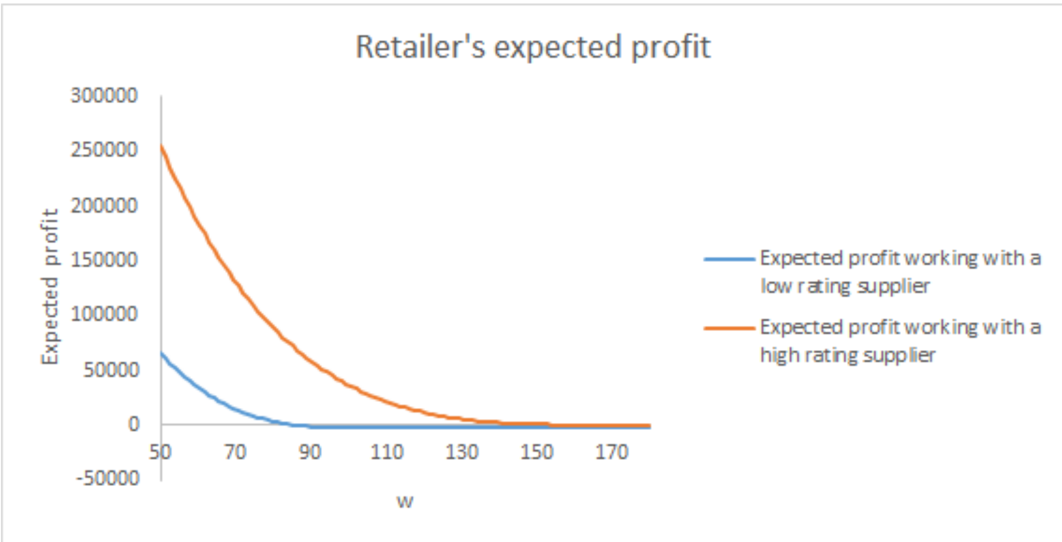

Figure 3 Retailer's expected profit with high and low rating supplier (c = 50 euro)

From figure 2 we can observe that for higher values of w, the retailer may order a higher quantity and invest more on effort when the supplier is low rating; however as shown in figure 3, the uncertainty about actual supplier's performance makes the expected profit lower than the profit achievable working with a high rating supplier.

This consideration may change decreasing the production cost: in fact, with a value of c equal to 30 euro, even if we obtained the same trend for the optimal value of q and e, the profit shows an interesting behaviour: figure 4 shows the retailer's expected profit when she works with a low rating and a high rating supplier and we can notice that there is a threshold value of w beyond which it is more convenient for the retailer to work with a low rating supplier. This finding is aligned with Jing et al. 2012 that find a threshold cost condition which separates the two credit types they consider (bank credit financing and trade credit financing) in equilibrium.

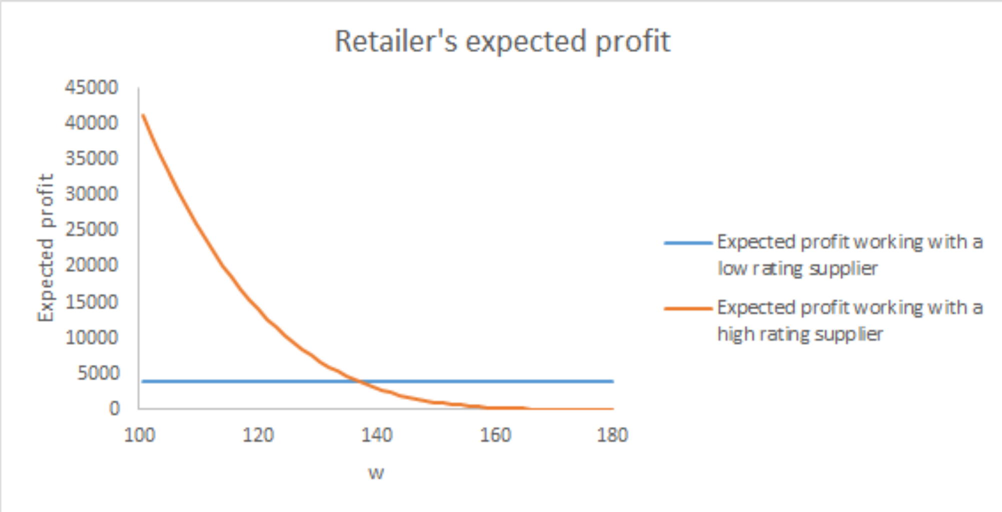

Figure 4 Retailer's expected profit with high and low rating supplier (c = 30 euro)

We therefore can find a threshold for *c* that delimits the convenience (with high value for *w*) to work with a low rating supplier rather than with a high rating one (Appendix E shows the threshold for *c* for the three level of the retailer's working capital): this threshold increases when the retailer's working capital increases, a higher threshold represents a wider range of value of *c* where the low rating supplier partnership offers higher profit. This finding means that low rating supplier, that can appear less attractive for the retailer, may gain appealing if they work on their internal efficiency and reduce the production cost *c*. Similar trends (in *q*, *e* and *profit*) have been obtained considering medium and high retailer's working capital.

The second part of the numerical example analyses how the retailer's profit changes according to her supplier failure rate (with c = 30 euro): Figure 5 shows the obtained results for a low level of retailer's working capital, in Figure 6 the working capital is medium and in Figure 7 it is high.

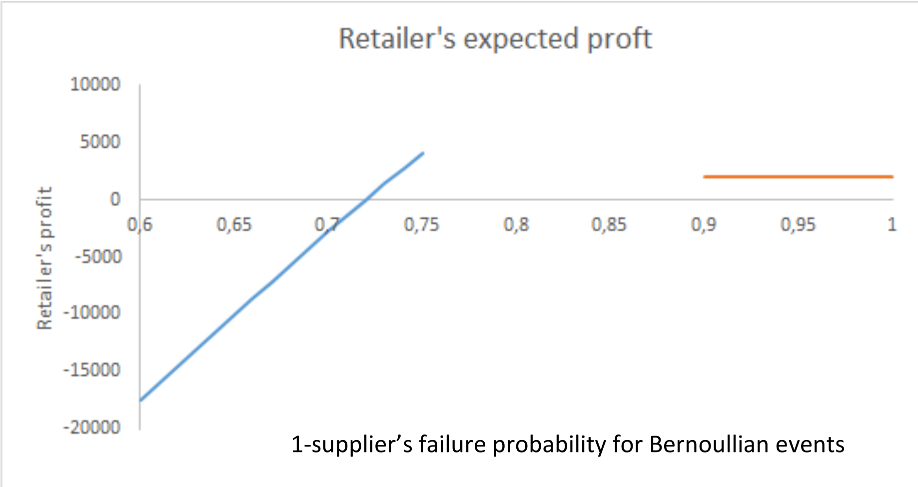

Figure 5 Retailer's expected profit when her working capital is low

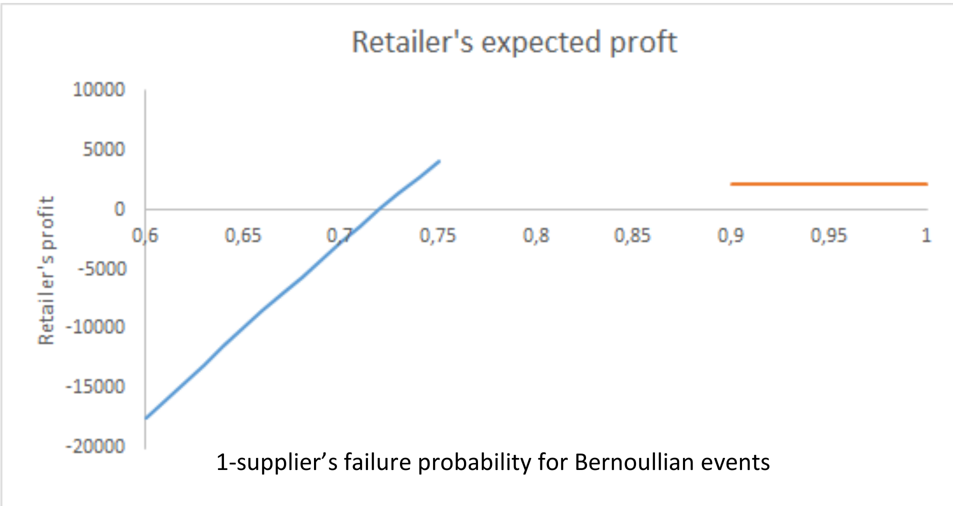

Figure 6 Retailer's expected profit when her working capital is medium

Basing on the results shown in Figures 2-7 we can draw some conclusions:

C1 (figures 2-4): If the overall supply chain mark-up, p-c, is low the retailer gets a higher profit working with a high rating supplier;

C2 (figures 2-4): When the retailer's mark-up, p-w, is low, she may have a higher profit working with a low rating supplier. In fact, the discount on the wholesale price made by the low rating supplier, allows the retailer to gain an extra mark-up. The discount depends on the supplier's mark up (*w-c*); beyond a mark-up threshold (in correspondence of the *c* threshold) the discount is high enough to make the retailer's profit working with a low rating supplier higher than as she works with a high rating one.

C3 (figures 2-4): Also the supplier failure rate for Bernoullian events influences the optimal parameters and therefore the retailer's profit (as already observed in section 4.3).

C4 (figures 5 and 6): *When the mark-up is low, a higher working capital doesn't change much the expected profit of a collaboration with a high rating supplier, while low rating retailers with a moderate failure rate for random events allow the retailer to achieve higher profit than working with high rating supplier*. This can be explained if we remember that in our model low rating suppliers offer a discount with early payment while high rating supplier offer late payment with zero interest: therefore, a retailer with high working capital doesn't benefit from a late payment while it does from a discount.

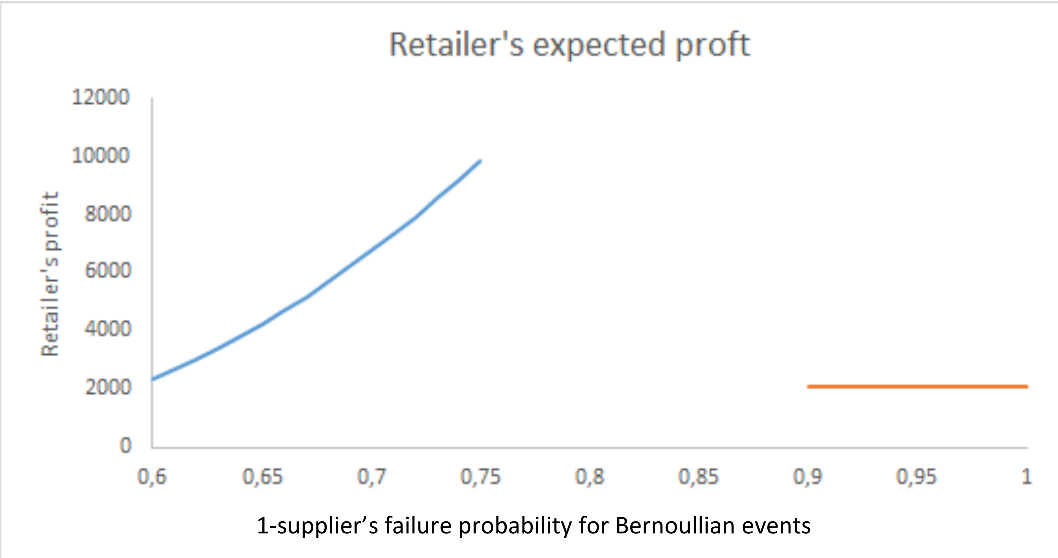

Figure 7 Retailer's expected profit when her working capital is high

C5 (figure 7): The high working capital retailer's profit is higher when she works with a low rating supplier than working with a high rating supplier. Indeed, when the retailer's mark-up is low, a higher working capital owned by the retailer incentives more the collaboration with a low rating supplier. This is the combined result of two different concurrent effects at work: on the one hand the working capital is enough to finance the retailer's supply chain transactions then she doesn't need any bank loan; on the other hand, the early payment required by the low rating supplier represents a discount in the wholesale price. This is therefore the cheaper configuration for the retailer that induces her to put more effort, buy a higher volume and look for a higher profit. The financial good health is then confirmed as a powerful leverage in the analysed supply chain model (as in Kouvelis and Zhao, 2012). However, as shown in figure 5 also a financial constrained retailer may find her convenience (even if less remarkable) in working with a low rating supplier.

The retailer could therefore decide to allocate her budget according to the supplier's credit rating investing more in the working capital of supply chains with low rating suppliers and exploit the late payment free of charge offered by high rating suppliers.

6. Conclusions

In a supply chain with a supplier and a retailer, parts may combine their working capital with a bank loan or, from the retailer's point of view, with a supplier's loan (trade credit) and, from a supplier's point of view, with a retailer loan (early payment). The proposed model combines models features already available in the related literature: in particular it introduces the retailer's effort in a supply chain financing scenario. The study focuses on supply chain with high and low rating suppliers, since the collaboration with a medium rating supplier was considered dominated by the collaboration with a high rating supplier (that offers to the retailer a free interest late payment). The optimal contract parameters (the order quantity, the retailer's effort and the supplier's discount rate) vary according to the retailer's working capital, supplier's credit rating and supply chain mark-up. In particular, even if the effort influences the optimal order quantity, the retailer will never order the maximum achievable market demand: however, the effort affects positively the supply chain performance and the supplier's selection when the retailer uses her profit as a criterion. If in the literature trade credit has been shown as a way to share the retailer's risk related to uncertain demand, this research introduces a new element that can redistribute the supply chain risk. Indeed, the retailer' effort in increasing the market demand leverage the order quantity and therefore the profit of the supplier, that, in turn increases his willingness to extend credit to his buyer. Therefore, if previously studies show trade credit increases supply chain efficiency and the retailer's effort encourages supplier to finance his buyer, we can conclude that this new element is worthy of further consideration especially in market with lower and lower liquidity and increasing uncertainty.

The study provides also a numerical example that allows to disentangle the retailer convenience in working with a low or high rating supplier. The obtained results show that, when the retailer is capital constrained and she has to ask for a financial support, the low rating supplier allows the retailer to get higher expected profits when the wholesale price, considered also exogenous, is close to the selling price and far from the supplier production cost: in fact, a lower retailer's mark-up makes more convenient the low rating supplier discount that can increase when his mark-up increases. When the retailer has enough working capital to finance the investment, the collaboration with a low rating supplier is even more advantageous for the retailer if compared with the one with high credit rating partners. Also in this case, the retailer's effort plays an important role amplifying the mark up influence on the profit according to its effect on the order quantity: when the optimal effort with a low rating supplier is higher than the one with a high rating supplier a low rating supplier is preferred for a wider range of mark up.

Our setting is multifaceted including variables considered at once in previous studies, therefore our results are partially aligned with previous studies' ones departing from them as illustrated in the three propositions that summarize the theoretical implications of our study and as demonstrated with the numerical example that provides managerial implications through the conclusions (*C1-C5*) drew up in the previous section.

Obviously, in the reality, the decision is also strongly affected by the retailer's risk tolerance, especially when uncertainty is exacerbated by a global economic crisis: given that this work assumes that all the members of the supply chain are risk neutral, actually, a risk-adverse retailer may have a higher utility in working with a high rating supplier even though the expected profit is lower, but more certain. Therefore, future development could investigate how the risk aversion affects the partnership decisions. It could be also interesting to hypothesize a demand with a different distribution than the uniform one as well as considering the wholesale and/or the selling price not exogenous.

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Appendix A

In this appendix the optimal parameters are discussed when the retailer works with a high rating supplier and s_r=0.

The following Figure A1 explains the sequence of actions during a collaboration between supplier and a medium or a high rating supplier.

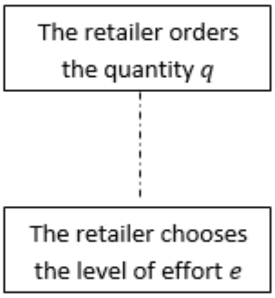

Figure A1 Sequence of actions in the case "working with a high rating supplier"

The main difference in a collaboration with a medium or a high supplier is in the interest rate r_s, that is higher than 0 in the first case and equal to 0 in the second case.

Therefore, the sequence of actions in this case consists only in the retailer that chooses the optimal level of effort and the optimal ordered quantity since she already knows that the loan from supplier doesn't require an interest.

A.1 Optimal ordered quantity

The first optimal parameter to be found is the optimal ordered quantity maximizing the retailer's expected profit function that may be written as in eq. 12 (details in section 3.1 of the main manuscript).

The optimal ordered quantity is:[4]

$$q = ke \cdot \frac{p-w}{p} \tag{A1}$$

- *The higher the effort, the higher the optimal ordered quantity, even if it is lower than the maximum demand ke*: the same considerations made in the section 4.1 hold.
- *The higher the retailer's percentage mark up $\frac{p-w}{p}$, the higher the optimal ordered quantity*.

[4] For mathematical details see Appendix D– Case 4

A.2 Optimal level of effort

Substituting the optimal q in the profit function of eq. 12 and maximizing it with the same procedure as before, the optimal level of effort is found:[5]

$$e = \frac{k(p-w)^2}{4ap(1+r_b)} \tag{A2}$$

- *The higher the retailer's mark-up the higher the optimal level of effort*: a higher mark-up incentive the retailer to sustain investments in effort, in order to increase her expected profit.
- *The higher the bank's interest rate the lower the optimal level of effort*: a higher interest increases the burden of the bank loan and this penalizes the effort.
- *Finally, it is straightforward to notice that the optimal effort will increase with k and decrease with a*.

[5] For mathematical details see Appendix D– Case 5

Appendix B

In this appendix the optimal parameters are discussed when the retailer works with a low rating supplier and s_r>0.

B.1 Optimal parameters when $\mathbf{0 < s_r < wq(1 + r_s) + g(e)}$

Optimal solutions give the same results as the case $s_r = 0$ (section 4.1).[6]

B.2 Optimal ordered quantity when $\mathbf{s_r \geq wq(1 + r_s) + g(e)}$

In this case the retailer profit is detailed in section 3.1 eq. 5.

The optimal q is equal to:[7]

$$q = ke \cdot \left(1 - \frac{w(1 + r_s)}{(1 - p_s(R_s))\delta \cdot p}\right) \tag{B1}$$

A higher working capital adds a consideration on the optimal quantity:

- *The ordered quantity is higher if compared to the one when* $s_r < wq(1 + r_s) + g(e)$: in fact, the term $(1 + r_b)$ is missing, so not asking the bank for a loan, having no interest to pay allows the retailer to order a higher quantity. Moreover, the quantity increases also for the effect of the *discount factor* δ: in fact, a higher discount factor requires a higher volume of sold quantity in order to repay bank debt incurred at time 0. Indeed, when the retailer asks for a bank debt both revenue and payment are at *t* equal to 1 therefore the discount effect is neutral, instead, when the retailer pays the supplier in advance and the revenue inflow materializes at time *t* equal to 1 the discount factor matters.

B.3 Optimal level of effort when $\mathbf{s_r \geq wq(1 + r_s) + g(e)}$

Again, substituting the optimal order quantity in the retailer's profit reported below, putting equal to 0 the partial derivative, the optimal effort is:[8]

$$e = k\frac{\delta \cdot (1 - p_s(R_s))\left[p \cdot n' - p \cdot \frac{n'^2}{2}\right] - \left(w \cdot n'(1 + r_s)\right)}{2a} \tag{B2}$$

- *The optimal effort is higher than as* $s_r < wq(1 + r_s) + g(e)$: this result is consistent with the observation regarding the ordered quantity. The lower are the amount of bank interests, and so the liabilities, the higher the free cash flow and therefore the investment in effort

[6] For mathematical details see Appendix D- Cases 1-3
[7] For mathematical details see Appendix D- Case 6
[8] For mathematical details see Appendix D- Case 7

B.4 Optimal discount rate when $s_r \geq wq(1 + r_s) + g(e)$

Substituting the optimal quantity in the supplier's profit function (section 3.2, eq. 14) and maximizing it, the optimal discount rate is:[9]

$$r_s = \frac{c(1 + r_b) + \delta \cdot p\big(1 - p_s(R_s)\big)}{2w(1 + r_b)} - 1 \qquad \text{(B3)}$$

As expected, also the discount rate is affected by the discount factor. In particular:

- *The higher the discount factor, the lower the discount rate*: in absence of bank debt/ithout any need to borrow money from bank, the higher the discount factor the lower the difference between the value of the today cost and tomorrow revenue and as a consequence the higher the mark up present value therefore the possibility for the supplier to increase the net wholesale price. The same reasoning for the ratio between *p* and *w* and the ratio between *c* and *w*.
- *The higher the supplier credit rating and therefore the lower his default risk* $\big(1 - p_s(R_s)\big)$, *the higher the discount rate:* a more uncertain supplier has to offer a higher discount in order to push the retailer in working with him.
- *The consideration about Weierstrass' theorem is valid also in this case.*

[9] For mathematical details see Appendix D– Case 8

Appendix C

In this appendix the optimal parameters are discussed when the retailer works with a high rating supplier and $s_r > 0$

The retailer's expected profit is discussed in section 3.1, eq. 13 and eq. 13bis.

As already said, being the supplier in a good financial health the interest r_s is set to 0, so the research is limited to q and e.

C.1 Optimal parameters when $s_r < g(e)$

The research of optimal parameters gives the same results of the case $s_r = 0$ treated in Appendix A.

C.2 Optimal ordered quantity when $s_r \geq g(e)$

In this case the retailer's profit is is expressed in the second part of equation 13 and in a simplified version , in eq 13bis.

Due to the negligibility of the loss function and the nature of the trade credit, the optimal q when $s_r \geq g(e)$ follows the same formulation of the case $s_r < g(e)$:[10]

$$q = ke \cdot \left(1 - \frac{w}{p}\right) \tag{C1}$$

- *The optimal ordered quantity is expressed in the same way also with a higher working capital.*

C.3 Optimal level of effort when $s_r \geq g(e)$

The optimal value of the effort, after the substitution of the optimal quantity (C1) in the derivative is: [11]

$$e = \delta \cdot k \frac{(p - w)^2}{4ap} \tag{C2}$$

- *The optimal effort is higher than the optimal effort when $s_r < g(e)$*
- *The higher the discount factor the higher the optimal level of effort*: even though the discount factor reduces the effect of the selling price, the higher the factor the higher the level of effort.
- *The optimal level of effort is higher than as $s_r < g(e)$*: thanks to lower liabilities, the retailer may invest more.

[10] For mathematical details see Appendix D – Case 9

[11] For mathematical details see Appendix D – Case 10

Appendix D

In this appendix all the proofs of solutions in Appendices A, B and C are reported.

Case 0: resolution of $\frac{M}{M+M_s}$ and $\frac{M_s}{M+M_s}$

These ratios and their inverse are often present during the resolution of the derivatives.

As already defined, $M = (1 - p_r(R_r))(1 - p_s(R_s))$ and $M_s = (p_s(R_s))(1 - p_r(R_r))$, so:

$$\frac{M}{M + M_s} = \frac{(1 - p_r(R_r))(1 - p_s(R_s))}{(1 - p_r(R_r))(1 - p_s(R_s)) + (p_s(R_s))(1 - p_r(R_r))} =$$

$$= \frac{(1 - p_r(R_r))(1 - p_s(R_s))}{(1 - p_s(R_s) + p_s(R_s))(1 - p_r(R_r))} = \frac{(1 - p_s(R_s))}{1} = (1 - p_s(R_s))$$

And

$$\frac{M_s}{M + M_s} = \frac{p_s(R_r)(1 - p_r(R_r))}{(1 - p_r(R_r))(1 - p_s(R_s)) + (p_s(R_s))(1 - p_r(R_r))} =$$

$$= \frac{p_s(R_r)}{(1 + p_s(s))}$$

Case 1: working with a low rating supplier, optimal ordered quantity when $0 \leq s_r < wq(1 + r_s) + g(e)$.

The retailer's profit function is (eq. 5):

$$\prod\nolimits_{r\_lowrate-s}(q, e, r_s) = M \cdot P^{r1} + M_s \cdot P^{r2}$$

where:

$$P^{r1} = \delta \cdot \left[p \cdot \frac{q}{2} \cdot \left(\frac{q}{ke}\right) + p \cdot q \cdot \left(1 - \frac{q}{ke}\right) - (wq(1 + r_s) + (ae^2) - s_r)(1 + r_b)\right] - s_r;$$

$$P^{r2} = \delta \cdot [-wq(1 + r_s) + (ae^2) - s_r](1 + r_b) - s_r$$

$$\frac{d\prod_{r\_lowrate-s}}{dq} = M \cdot \frac{dP^{r3}}{dq} + M_s \cdot \frac{dP^{r3}}{dq}$$

Putting the derivative equal to 0:

$$\frac{d\prod_{r\_lowrate-s}}{dq} = M \cdot \delta \cdot \left[p - \frac{pq}{ke} - w(1+r_s)(1+r_b)\right] + M_s \cdot \delta \cdot [-w(1+r_s)(1+r_b)] = 0$$

$$M \cdot p - (M + M_s) \cdot [w(1+r_s)(1+r_b)] = M \cdot \frac{pq}{ke}$$

As anticipated, the optimal quantity is:

$$q = ke\left(\frac{M \cdot p - (M + M_s) \cdot [w(1+r_s)(1+r_b)]}{M \cdot p}\right)$$

$$q = ke\left(1 - \frac{(M + M_s)}{M} \cdot \frac{[w(1+r_s)(1+r_b)]}{p}\right)$$

$$q = ke\left(1 - \frac{1}{(1 - p_s(R_s))} \cdot \frac{w(1+r_s)(1+r_b)}{p}\right) = ken \tag{D1}$$

Where $n = \left(1 - \frac{1}{(1-p_s(R_s))} \cdot \frac{w(1+r_s)(1+r_b)}{p}\right)$

Case 2: working with a low rating supplier, optimal effort when $0 \leq \boldsymbol{s_r} < \boldsymbol{wq(1 + r_s) + g(e)}$.

The profit function is the same of case 1 that, after the substitution of the optimal ordered quantity, becomes:

$P^{r1} = \delta \cdot \left[p \cdot \frac{ken}{2} \cdot \left(\frac{ken}{ke}\right) + p \cdot ken \cdot \left(1 - \frac{ken}{ke}\right) - (w \cdot ken \cdot (1+r_s) + (ae^2)(1+r_b) - s_r\right] - s_r;$

$$P^{r2} = \delta \cdot [-w \cdot ken(1+r_s) + (ae^2) - s_r](1+r_b) - s_r;$$

Then,

$$\frac{d\prod_{r\_lowrate-s}(q, e, r_s)}{de} = M \cdot \frac{dP^{r1}}{de} + M_s \cdot \frac{dP^{r2}}{de} =$$

$$= M \cdot \delta \cdot \left[p \cdot kn - p \cdot \frac{kn^2}{2} - (w \cdot kn \cdot (1+r_s) + (2ae)(1+r_b)\right] - M_s \cdot \delta \cdot ((2ae)(1+r_b) - w \cdot k \cdot n \cdot (1+r_s))$$

Putting the derivative equal to 0:

$$M \cdot \left[p \cdot kn - p \cdot \frac{kn^2}{2} - w \cdot kn \cdot (1+r_s)\right] - M_s \cdot w \cdot kn \cdot (1+r_s)(1+r_b)$$
$$= (M + M_s) \cdot (2ae)(1+r_b)$$

The optimal effort is therefore:

$$e = kn \frac{p\big(1 - p_s(R_s)\big)\left[1 - \frac{n}{2}\right] - \big(w(1+r_s)\big)(1+r_b)}{2a[(1+r_b)]} \quad \text{(D2)}$$

Case 3: working with a low rating supplier, optimal discount rate when 0≤ $\boldsymbol{s_r} < \boldsymbol{wq(1+r_s)+g(e)}$.

The optimal solution maximizes the supplier's expected profit, that in this case is assumed as in eq. 14:

$$\Pi_{lowrate-s}\ (q, r_s) = M[(w(1+r_s) - c) \cdot q]$$

After the substitution of the optimal ordered q, the profit function becomes:

$$\Pi_{lowrate-s}\ (r_s) = M\left[(w(1+r_s) - c) \cdot ke \cdot \left(1 - \frac{w\,(1+r_s)(1+r_b)}{(1 - p_s(R_s))p}\right)\right]$$

$$\frac{d\Pi_{lowrate-s}\ (r_s)}{dr_s} = M \cdot ke \cdot \left[w - 2\frac{w^2\,(1+r_s)(1+r_b)}{(1-p_s(R_s))p} + c\frac{w\,(1+r_b)}{(1-p_s(R_s))p}\right] = 0$$

$$2w\,(1+r_s)(1+r_b) = (1 - p_s(R_s))p + c(1+r_b)$$

The optimal discount rate is:

$$r_s = \frac{c(1+r_b) + p\big(1 - p_s(R_s)\big)}{2w(1+r_b)} - 1 \quad \text{(D3)}$$

Case 4: working with a high rating supplier, optimal ordered quantity when $s_r < g(e)$.

The retailer's profit that works with a high rating supplier is as in eq. 13:

$$\Pi_{r\_high-s}\ (q, e, r_s) = M \cdot P^{r3} + M_s \cdot P^{r4}$$

Where the term $M_s \cdot P^{r2}$ is negligible, M in this case may be considered equal to 1. Therefore, the profit gets its maximum value when:

$$\frac{d\prod_{r\_high-s}(q,e,r_s)}{dq} = \frac{dP^{r3}}{dq} = \delta \cdot \left[\frac{pq}{ke} + p - 2 \cdot \frac{pq}{ke} - w\right] = 0$$

$$p - \frac{pq}{ke} - w = 0$$

the optimal ordered quantity is:

$$q = ke \cdot \frac{p-w}{p} \qquad \text{(D4)}$$

Case 5: working with a high rating supplier, optimal effort when $s_r < g(e)$.

The profit function to maximize is the same of case 4, where we substituted the optimal q:

$$\frac{d\prod_{r\_high-s}(e)}{de} = \left[-\frac{p\cdot\left(ke\cdot\frac{p-w}{p}\right)^2}{2\cdot ke^2} + \frac{p\cdot q\left(ke\cdot\frac{p-w}{p}\right)^2}{ke^2} - (2 \cdot a \cdot e)(1+r_b)\right] = 0;$$

$$\frac{d\prod_r(q,e,H)}{de} = \left[-\frac{k(\cdot p-w)^2}{2\cdot p} + \frac{k(\cdot p-w)^2}{p} - (2 \cdot a \cdot e)(1+r_b)\right] = 0;$$

$$(2 \cdot a \cdot e)(1+r_b) = \frac{k(\cdot p-w)^2}{2p};$$

The optimal level of effort is:

$$e = \frac{k(p-w)^2}{4ap(1+r_b)} \qquad \text{(D5)}$$

Case 6: working with a low rating supplier, optimal ordered quantity when $s_r \geq wq(1+r_s) + g(e)$.

The retailer’s profit function is also in this case expressed in the second part of eq. 5:

$$\prod_{r\_lowrate-s}(q,e,r_s) = M \cdot P^{r1'} + M_s \cdot P^{r2'}$$

Where, as in eq. 3 and 4:

$$P^{r1'} = \delta \cdot \left[p \cdot \frac{q}{2} \cdot \left(\frac{q}{ke}\right) + p \cdot q \cdot \left(1 - \frac{q}{ke}\right)\right] - (wq(1+r_s) + (ae^2));$$

$$P^{r2'} = [-wq(1+r_s) + (ae^2)]$$

$$\frac{d\prod_{r\_lowrate-s}(q,e,r_s)}{dq} = M \cdot \frac{dP^{r1'}}{dq} + M_s \cdot \frac{dP^{r2'}}{dq}$$

$$\frac{d\prod_{r\_lowrate-s}(q,e,r_s)}{dq} = M \cdot \delta \cdot \left[p - \frac{pq}{ke}\right] - M \cdot w(1+r_s) + M_s \cdot [-w(1+r_s)] = 0$$

$$M \cdot \delta \cdot p - (M + M_s) \cdot [w(1+r_s)] = M \cdot \delta \cdot \frac{pq}{ke}$$

The optimal q is:

$$q = ke\left(\frac{M \cdot p - (M + M_s) \cdot [w(1+r_s)]}{M \cdot p}\right)$$

$$q = ke\left(1 - \frac{(M + M_s)}{M} \cdot \frac{[w(1+r_s)]}{\delta \cdot p}\right)$$

$$q = ke\left(1 - \frac{1}{(1 - p_s(R_s))} \cdot \frac{w(1+r_s)}{\delta \cdot p}\right)$$

$$q = ken' \tag{D6}$$

Where:

$$n' = \left(1 - \frac{1}{(1 - p_s(R_s))} \cdot \frac{w(1+r_s)}{\delta \cdot p}\right)$$

Case 7: working with a low rating supplier, optimal effort when $s_r \geq wq(1+r_s) + g(e)$.

Again, after the substitution of the optimal ordered quantity of case 6, the previous eq. 3 and 4 become:

$$P^{r1'} = \delta \cdot \left[p \cdot \frac{ken'}{2} \cdot \left(\frac{ken'}{ke}\right) + p \cdot ken' \cdot \left(1 - \frac{ken'}{ke}\right)\right] - (w \cdot ken' \cdot (1+r_s) + (ae^2);$$

$$P^{r2'} = [-w \cdot ken'(1+r_s) + (ae^2)];$$

$$\frac{d\prod_{r\_lowrate-s}(e,r_s)}{de} = M \cdot \frac{dP^{r1}}{de} + M_s \cdot \frac{dP^{r2}}{de} =$$

$$= M \cdot \delta \cdot \left[p \cdot kn' - p \cdot \frac{kn'^2}{2}\right] - (M + M_s) \cdot \left(w \cdot kn' \cdot (1+r_s)\right) - (M + M_s) \cdot (2ae) = 0$$

$$= M \cdot \delta \cdot \left[p \cdot kn' - p \cdot \frac{kn'^2}{2}\right] - (M + M_s) \cdot (w \cdot kn' \cdot (1 + r_s)) = (M + M_s) \cdot (2ae)$$

The optimal effort is:

$$e = kn' \frac{\delta p\big(1 - p_s(R_s)\big)\left(1 - \frac{n'}{2}\right) - w(1 + r_s)}{2a} \quad \text{(D7)}$$

Case 8: working with a low rating supplier, optimal discount rate when $s_r \geq wq(1 + r_s) + g(e)$.

As in case 6 the supplier's profit function is the one in eq. 14:

$$\prod\nolimits_{lowrate-s} (q, r_s) = M[(w(1 + r_s) - c) \cdot q]$$

After the substitution of the optimal ordered *q* of case 6, the profit function becomes:

$$\prod\nolimits_{lowrate-s} (r_s) = M\left[(w(1 + r_s) - c) \cdot ke \cdot \left(1 - \frac{w\,(1 + r_s)(1 + r_b)}{(1 - p_s(R_s))p}\right)\right]$$

$$\frac{d\prod_{lowrate-s} (r_s)}{dr_s} = M \cdot ke \cdot \left[w - 2\frac{w^2\,(1 + r_s)(1 + r_b)}{(1 - p_s(R_s))p} + c\frac{w\,(1 + r_b)}{(1 - p_s(R_s))p}\right] = 0$$

$$2w\,(1 + r_s)(1 + r_b) = (1 - p_s(R_s))p + c(1 + r_b)$$

The optimal discount rate is:

$$r_s = \frac{c(1 + r_b) + p\big(1 - p_s(R_s)\big)}{2w(1 + r_b)} - 1 \quad \text{(D8)}$$

The optimal solution maximizes the supplier's expected profit, that in this case is expressed in eq.
:

$$\prod\nolimits_{sL} (r_s) = M[(w(1 + r_s) - c) \cdot q]$$

After the substitution of the optimal ordered q, the profit function becomes:

$$\prod_{lowrate-s}(r_s) = M\left[(w(1+r_s)-c)\cdot ke\cdot\left(1-\frac{w\,(1+r_s)}{\left(1-p_s(R_s)\right)\cdot\delta\cdot p}\right)\right]$$

That gets its maximum value when:

$$\frac{d\prod_{lowrate-s}(r_s)}{dr_s} = M\cdot ke\cdot\left[w-2\frac{w^2\,(1+r_s)}{\left(1-p_s(R_s)\right)\cdot\delta\cdot p}+c\frac{w}{\left(1-p_s(R_s)\right)\cdot\delta\cdot p}\right]=0$$

$$\left(1-p_s(R_s)\right)\cdot\delta\cdot p-2w\,(1+r_s)+c=0$$

Therefore, the optimal discount rate is:

$$r_s = \frac{c+\delta p(1-p_s(R_s))}{2w}-1 \qquad \text{(D9)}$$

Case 9: working with a high rating supplier, optimal ordered quantity when $s_r \geq g(e)$.

The retailer's profit working with a high rating supplier is expressed in the second part of equation 13:

$$\prod_{r\_high-s}(q,e,r_s) = M\cdot P^{r3'}+M_s\cdot P^{r4'}$$

Where the term $M_s\cdot P^{r2}$ is negligible, M in this case may be considered equal to 1 and $P^{r3\prime} = \delta\cdot\left[p\cdot\frac{q}{2}\cdot\left(\frac{q}{ke}\right)+p\cdot q\cdot\left(1-\frac{q}{ke}\right)-wq\right]-ae^2$

The optimal value can be found solving the following:

$$\frac{d\prod_{r\_high-s}(q,e,r_s)}{dq}=\frac{dP^{r3\prime}}{dq}=\delta\cdot\left[\frac{pq}{ke}+p-2\cdot\frac{pq}{ke}-w\right]=0$$

$$p-\frac{pq}{ke}-w=0$$

the optimal ordered quantity is:

$$q = ke\cdot\frac{p-w}{p} \qquad \text{(D10)}$$

that gives the same results of the capital constrained retailer.

Case 10: working with a high rating supplier, optimal effort when $s_r \geq g(e)$.

The retailer's profit is the same of case 9,

The optimal $q = ke \cdot \frac{p-w}{p}$ is substituted in the derivative, obtaining:

$$\frac{d\prod_{r\_high-s}(e)}{de} = \delta \cdot \left[-\frac{p \cdot \left(ke \cdot \frac{p-w}{p}\right)^2}{2 \cdot ke^2} + \frac{p \cdot q\left(ke \cdot \frac{p-w}{p}\right)^2}{ke^2} \right] - (2 \cdot a \cdot e) = 0;$$

$$\delta \cdot \left[-\frac{k(\cdot p-w)^2}{2 \cdot p} + \frac{k(\cdot p-w)^2}{p} \right] = (2 \cdot a \cdot e);$$

$$(2 \cdot a \cdot e) = \delta \cdot \frac{k(\cdot p-w)^2}{2p};$$

The optimal level of effort is:

$$e = \delta k \frac{(p-w)^2}{4ap} \qquad \text{(D11)}$$

Appendix E

In this section, the threshold for the parameter c is reported. It is calculated comparing the retailer's profit working with a low rating and a high rating supplier. For value lower than the threshold when she works with a low rating supplier her profit is higher than as she works with a high rating one. Three main cases arise as for the optimal parameters ($p_s(R_s)$ refers always to the low rating supplier):

Caso 1: The retailer needs a bank loan - $s_r < g(e)$

$$c < p\left\{\frac{M\left[q_L\left(1-\frac{n}{2}\right)-q_H\left(1-\frac{p-w}{p}\right)\right]-wq_H(1+r_b)-a({e_L}^2-{e_H}^2){e_L}^2+a{e_H}^2}{q_L} \cdot 2-(1-p_s(R_s))\right\}$$

Where (please refer to table 3 of the main manuscript)

q_L	ke_Ln
n	$1-\frac{w\,(1+r_s)(1+r_b)}{p(1-p_s(R_s))}$
e_L	$\frac{pkn\left(1-p_s(R_s)\right)\left[1-\frac{n}{2}\right]-wkn(1+r_s)(1+r_b)}{2a\cdot[(1+r_b)]}$
q_H	$ke_H\cdot\frac{p-w}{p}$
e_H	$\frac{(p-w)^2}{4akp(1+r_b)}$

- **Caso 2: The retailer needs a bank loan if working with a low rating supplier, while she doesn't need a bank loan if the supplier is high rating -** $g(e) \le s_r < wq(1 + r_s) + g(e))$

$$c < p\left[\frac{M\delta p\left[q_L\left(1-\frac{n}{2}\right) - q_H\left(1-\frac{p-w}{p}\right)\right] - wq_H - a\delta {e_L}^2 + a{e_H}^2}{q_L(1+r_b)} \cdot 2(1+r_b) - p(1-P_s(R_s))\right]$$

Where (please refer to table 4 of the main manuscript)

q_L	$ke_L \cdot \left(1 - \frac{w\,(1+r_s)(1+r_b)}{(1-p_s(R_s))\,p}\right)$
n	$1 - \frac{w\,(1+r_s)(1+r_b)}{(1-p_s(R_s))\,p}$
e_L	$\frac{pkn\left(1-p_s(R_s)\right)\left[1-\frac{n}{2}\right] - wkn(1+r_s)(1+r_b)}{2a\cdot[(1+r_b)]}$
q_H	$ke_H \cdot \frac{p-w}{p}$
e_H	$\delta \cdot k\,\frac{(p-w)^2}{4ap}$

- **Caso 3: The retailer doesn't borrow money from the bank -** $s_r \ge wq(1 + r_s) + g(e)$

$$c < p\left[\frac{M\delta p\left[q_L\left(1-\frac{n}{2}\right)-q_H\left(1-\frac{p-w}{p}\right)\right]-wq_H-a\delta {e_L}^2+a{e_H}^2}{q_L}\cdot 2-\delta\cdot p(1-P_s(R_s))\right]$$

Where (please refer to table 5 of the main manuscript):

q_L	$ke\cdot\left(1-\frac{w(1+r_s)}{p\delta(1-p_s(R_s))}\right)$
n	$1-\frac{w(1+r_s)}{p\delta(1-p_s(R_s))}$
e_L	$k\frac{\delta pn(1-p_s(R_s))\left[1-\frac{n}{2}\right]-w\cdot n\cdot(1+r_s)}{2a}$
q_H	$ke_H\cdot\frac{p-w}{p}$
e_H	$\delta\cdot k\frac{(p-w)^2}{4ap}$

Appendix F

This appendix provides two different schemas for the supplier and retailer's profit functions. The first schema illustrates through two trees (Figure F1 and Figure F2) how the profit function for the retailer and the supplier are determined according to the proposed supply chain model. The second schema summarizes the functions in tables (Tables F1 and Tables F2) citing the corresponding equation in the text.

	Retailer's profit working with a low rating supplier	
	Bank loan ($s_r < \left(wq + A + g(e)\right)$	No bank loan ($s_r \geq \left(wq + A + g(e)\right)$
No one fails	P^{r1} $(eq. 1)$	$P^{r1\prime}$ $(eq. 2)$
The supplier fails	P^{r2} $(eq. 3)$	$P^{r2\prime}(eq. 4)$
The retailer fails	*0*	
$\prod_{r\_lowrate-s}(q, e, r_s)$ $(eq. 5)$	$M \cdot P^{r1} + M_s \cdot P^{r2}$	$M \cdot P^{r1'} + M_s \cdot P^{r2'}$

Table F1a

	Retailer's profit working with a medium rating supplier		
	Bank loan ($s_r < g(e)$)	No bank loan, L_{rs}>0 ($g(e) \leq s_r < \left(wq + A + g(e)\right)$	No bank loan, L_{rs}=0 ($s_r \geq g(e) + wq$)
No one fails	P^{r3} $(eq. 6)$	$P^{r3\prime}$ $(eq. 7)$	$P^{r3\prime\prime}$ $(eq. 8)$
The supplier fails	P^{r4} $(eq. 9)$	$P^{r4\prime}$ $(eq. 10)$	$P^{r4\prime\prime}$ $(eq. 11)$
The retailer fails	*0*		
$\prod_{r\_medium-s}(q, e, r_s)$ $(eq. 12)$	$M \cdot P^{r3} + M_s \cdot P^{r4}$	$M \cdot P^{r3'} + M_s \cdot P^{r4'}$	$M \cdot P^{r3''} + M_s \cdot P^{r4''}$

Table F1b

	Retailer's profit working with a high rating supplier	
	Bank loan ($s_r < g(e)$)	No bank loan ($s_r \geq g(e)$)
No one fails	P^{r3} $(eq. 6)$	$P^{r3\prime}$ $(eq. 7)$
The supplier fails	P^{r4} $(eq. 9)$	$P^{r4\prime}$ $(eq. 10)$
The retailer fails	*0*	
$\prod_{r\_high-s}(q, e, r_s)$ $(eq. 13 \cong 13bis)$	$M \cdot P^{r3} + M_s \cdot P^{r4} \cong (1 - M_r) \cdot P^{r3}$	$M \cdot P^{r3'} + M_s \cdot P^{r4'} \cong (1 - M_r) \cdot P^{r3'}$

Table F1c

Tables F1: Retailer's profit according to credit rating supplier (F1a, b, c), failure events and retailer's working capital (in brackets the corresponding equation discussed in section 3.1)

	Low rating supplier's profit
	Bank loan $(s_r < \big(wq + A + g(e)\big)$
No one fails	$\prod_{lowrate-s}\ (q, r_s) = M[wq + A - cq](eq. 14)$
The supplier fails	0
The retailer fails	0
$\prod_{lowrate-s}$ (eq. 14)	$\prod_{lowrate-s}$

Table F2a

	Medium rating supplier's profit and $s_s < cq$ $(s_s \geq cq)$		
	Bank loan $(s_r < g(e))$	No bank loan, $L_{rs}>0$ $(g(e) \leq s_r < \big(wq + A + g(e)\big)$	No bank loan, $L_{rs}=0$ $(s_r \geq g(e) + wq)$
No one fails	$P^{s11} + P^{s12} + P^{s13}\ (eq. 17, 18, 19)$ $\big(P^{s11'} + P^{s12'} + P^{s13'}(eq. 20, 21, 22)\big)$	$P^{s11} + P^{s12}\ (eq. 17, 18)$ $(P^{s11'} + P^{s12'}(eq. 20, 21))$	$P^{s11}\ (eq. 17)$ $(P^{s11'}(eq. 20))$
The supplier fails	0		
The retailer fails	$P^{s2}\ (eq. 27)\ (P^{s2'}\ (eq. 28))$		
$\prod_{mediumrate-s}\ (eq. 29)(eq. 31)$	$M(P^{s11} + P^{s12} + P^{s13}) + M_r P^{s2}$ $\big(M\big(P^{s11'} + P^{s12'} + P^{s13'}\big) + M_r P^{s2}\big)$	$M(P^{s11} + P^{s12}) + M_r P^{s2}$ $(M\big(P^{s11'} + P^{s12'}\big) + M_r P^{s2})$	$MP^{s11} + M_r P^{s2}$ $(MP^{s11'} + M_r P^{s2})$

Table F2b

	High rating supplier's profit and $s_s < cq$ $(s_s \geq cq)$	
	Bank loan $(s_r < g(e))$	No bank loan $(s_r \geq g(e))$
No one fails	$P^{s11} + P^{s12} + P^{s13}\ (eq. 17, 18, 19)$	$P^{s11} + P^{s12}\ (eq. 17, 18)$ $(P^{s11'} + P^{s12'}(eq. 20, 21))$

	$(P^{s11'}+P^{s12'}$ $+P^{s13'}(eq.20,21,22))$	
The supplier fails	0	
The retailer fails	P^{s2} $(eq.27)$ $(P^{s2'}$ $(eq.28))$	
$\prod_{highrate-s}(eq.30)(eq.32)$	$M(P^{s11}+P^{s12}+P^{s13})$ $+M_r P^{s2}$ $\left(M\left(P^{s11'}+P^{s12'}+P^{s13'}\right)\right.$ $\left.+M_r P^{s2}\right)$	$M\cdot P^{r3'}+M_s\cdot P^{r4'}$ $\cong(1-M_r)\cdot P^{r3'}$

Table F2c

Tables F2: Supplier's profit according to his credit rating, failure events and retailer and supplier's working capital (in brackets the corresponding equation discussed in section 3.2)

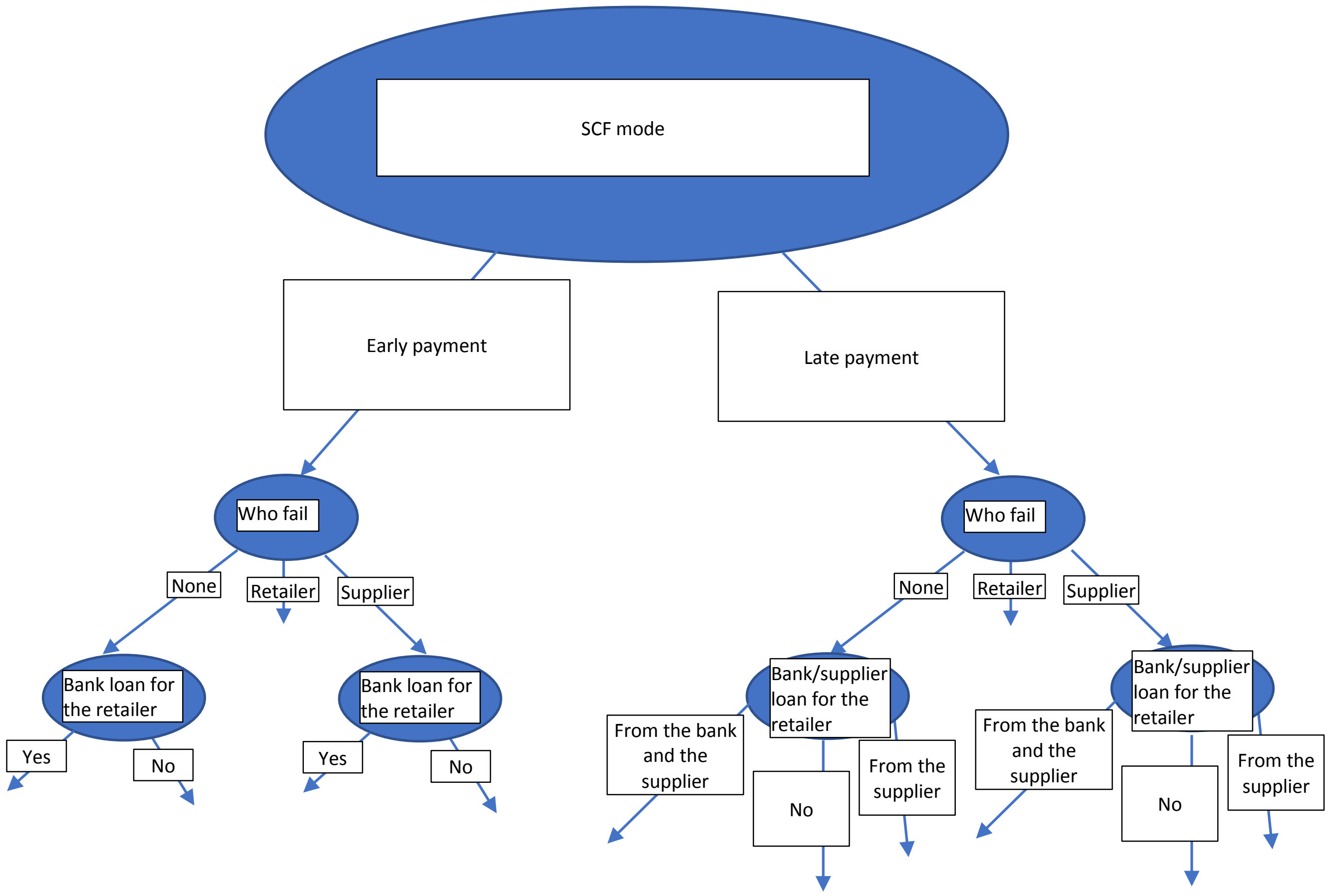

Figure F1: Retailer's profit tree: for each final branch of the tree a proper profit function is formulated in section 3.1

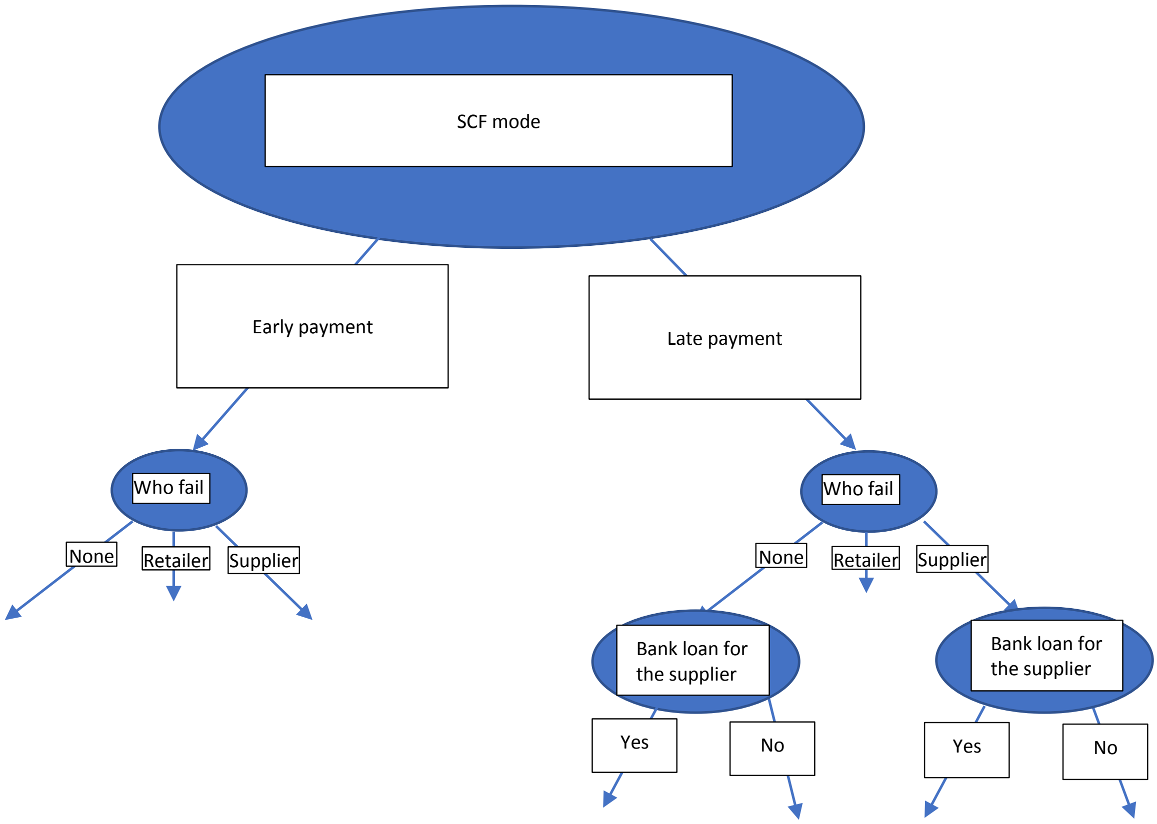

Figure F2: Supplier's profit tree: for each final branch of the tree a proper profit function is formulated in section 3.2